The data below is from a case-control study to assess the association between myocardial infarction (MI) and coffee drinking. Smoking is known to be associated with both coffee drinking and MI and it is a possible confounder between that relationship.

	Nonsmoker		*Total*
	5+ Cups	**<5 Cups**	
MI	30	207	237
No MI	33	327	360
Total	63	534	*597*
OR=1.4361			

	Smoker		*Total*
	5+ Cups	**<5 Cups**	
MI	117	216	333
No MI	25	114	139
Total	142	330	*472*
OR=2.47			

	Total Sample		*Total*
	5+ Cups	**<5 cups**	
MI	147	423	570
No MI	58	441	499
Total	205	864	*1069*
OR=2.6423			

Use the data to fit the following models:

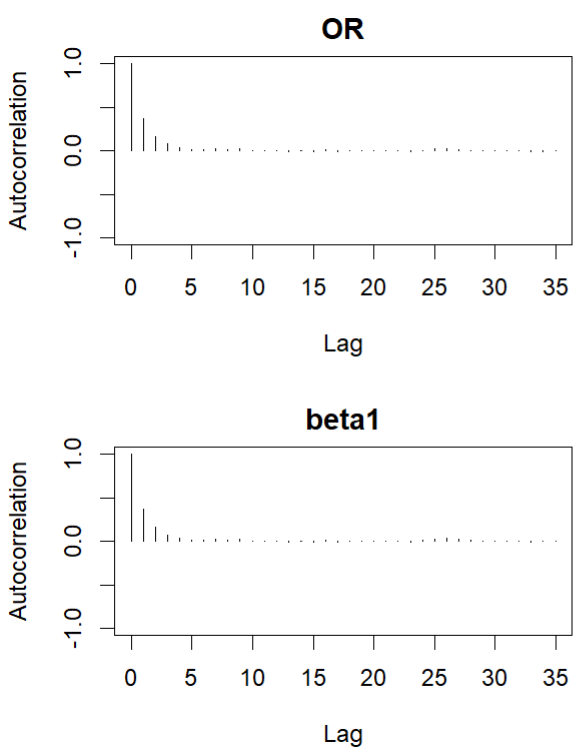
1. The model describing the crude association between MI and coffee drinking
2. The model describing the crude association between MI and coffee drinking adjusted by smoking
3. The model describing the crude association between MI and coffee drinking that interacts with smoking

Report measures of associations and describe monitors of convergence for each model.

Crude Model

From 10,000 Monte Carlo simulations, people who drink more than 5 cups of coffee a day have 2.6509 times the odds of having a myocardial infarction compared to those who drink less than 5 cups a day (95% Credible Interval: 1.8997, 3.7321).

$$Y = -0.0428 + 0.9769coffee$$



The parameter and therefore the OR has about 40% autocorrelation with a lag of 1, which decreases exponentially with greater lags.

Adjusted Model

From 10,000 Monte Carlo simulations, in each smoking status group, people who drink more than 5 cups of coffee a day have 1.9361 times the odds of having a myocardial infarction compared to those who drink less than 5 cups a day (95% Credible Interval: 1.3849, 2.7561).

$$Y = -0.4906 + 0.6607coffee + 1.1898smoking$$

$$OR(5 + cups\ smoker\ vs < 5\ cups\ smoker) = \frac{e^{\beta_0+\beta_1+\beta_2}}{e^{\beta_0+\beta_2}} = e^{\beta_1} = e^{0.6607} = 1.9361$$

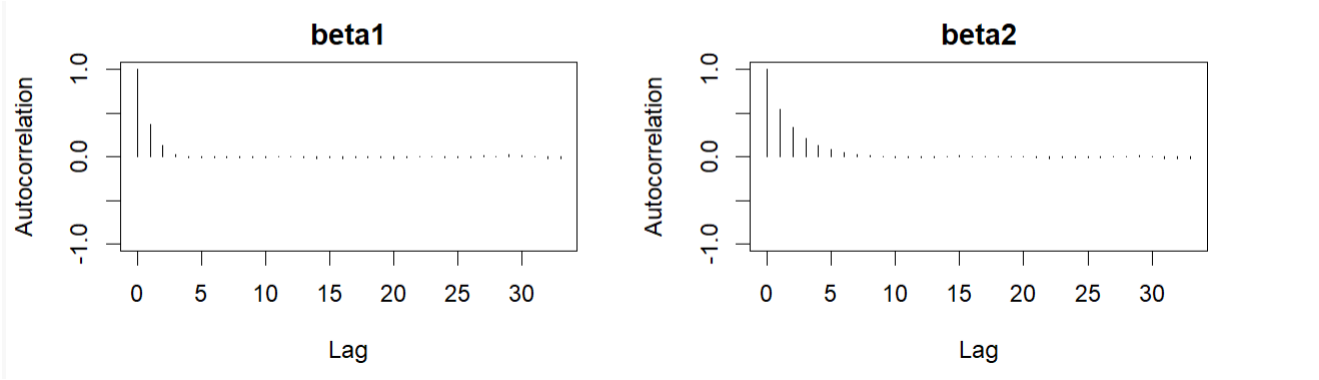
$$OR(5 + cups\ nonsmoker\ vs < 5\ cups\ nonsmoker) = \frac{e^{\beta_0+\beta_1}}{e^{\beta_0}} = e^{\beta_1} = e^{0.6607} = 1.9361$$

$$OR(< 5\ cups\ smoker\ vs < 5\ cups\ nonsmoker) = \frac{e^{\beta_0+\beta_2}}{e^{\beta_0}} = e^{\beta_2} = e^{1.1898} = 3.2864$$

$$OR(5 + cups\ smoker\ vs\ 5 +\ cups\ nonsmoker) = \frac{e^{\beta_0+\beta_1+\beta_2}}{e^{\beta_0+\beta_1}} = e^{\beta_2} = e^{1.1898} = 3.2864$$

$$OR(5 + cups\ smoker\ vs < 5\ cups\ nonsmoker) = \frac{e^{\beta_0+\beta_1+\beta_2}}{e^{\beta_0}} = e^{\beta_1+\beta_2} = e^{0.6607+1.1898} = 1.11898$$

$$OR(5 + cups\ nonsmoker\ vs < 5\ cups\ smoker) = \frac{e^{\beta_0+\beta_1}}{e^{\beta_0+\beta_2}} = e^{\beta_1-\beta_2} = e^{0.6607-1.1898} = 0.5891$$



The parameter for smoking has greater than 50% autocorrelation with a lag of 1, which decreases with subsequent lag periods. The parameter for coffee consumption has around 40% autocorrelation with a lag of 1, around 15% autocorrelation with a lag of 2, and negligible levels of autocorrelation afterwards.

From 10,000 Monte Carlo simulations, the probability of the OR being greater than 1 is 0.9999. With a Bayes Factor of 9999, one can decisively reject the null hypothesis and conclude that there is a positive association between coffee consumption and myocardial infarction, adjusting for smoking status.

$$Bayes\ Factor = \frac{0.9999}{1 - 0.9999} = 9999$$

Interaction Model

From 10,000 Monte Carlo simulations, nonsmokers who drink more than 5 cups of coffee a day have 1.4391 times the odds of having a myocardial infarction compared to nonsmokers who drink less than 5 cups a day (95% Credible Interval: 0.8462, 2.4798). Smokers who drink more than 5 cups of coffee a day have 2.4794 times the odds of having a myocardial infarction compared to smokers who drunk less than 5 cups a day (95% Credible Interval: 1.5508, 4.0989). Comparing smokers to nonsmokers multiplies the odds ratio by a factor of 1.7229 due to the positive interaction between smoking drinking more than 5 cups of coffee a day on myocardial infarction.

$$Y = -0.4583 + 0.3640coffee + 1.1022smoking + 0.5440coffee \times smoking$$

$$OR(5 + cups\ smoker\ vs < 5\ cups\ smoker) = \frac{e^{\beta_0+\beta_1+\beta_2+\beta_3}}{e^{\beta_0+\beta_2}} = e^{\beta_1+\beta_3} = e^{0.3640+0.5440} = 2.4794$$

$$OR(5 + cups\ nonsmoker\ vs < 5\ cups\ nonsmoker) = \frac{e^{\beta_0+\beta_1}}{e^{\beta_0}} = e^{\beta_1} = e^{0.3640} = 1.4391$$

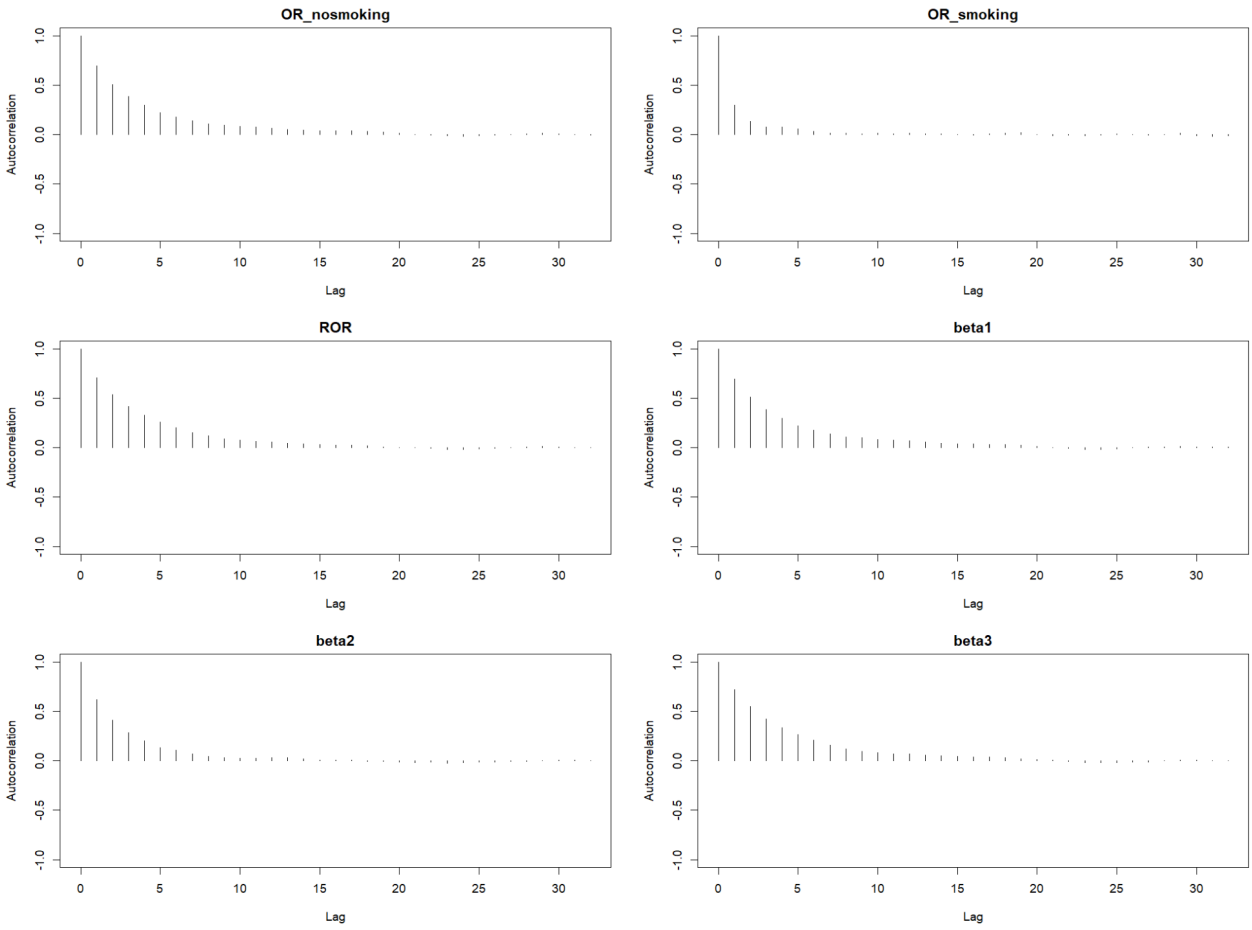
$$ROR = \frac{OR(5 + cups\ smoker\ vs < 5\ cups\ smoker)}{OR(5 + cups\ nonsmoker\ vs < 5\ cups\ nonsmoker)} = \frac{\frac{e^{\beta_0+\beta_1+\beta_2+\beta_3}}{e^{\beta_0+\beta_2}}}{\frac{e^{\beta_0+\beta_1}}{e^{\beta_0}}} = \frac{e^{\beta_1+\beta_3}}{e^{\beta_1}} = e^{\beta_3} = 1.7229$$

$$OR(< 5\ cups\ smoker\ vs < 5\ cups\ nonsmoker) = \frac{e^{\beta_0+\beta_2+\beta_3}}{e^{\beta_0}} = e^{\beta_2+\beta_3} = e^{1.1022+0.5440} = 5.1872$$

$$OR(5 + cups\ smoker\ vs\ 5 +\ cups\ nonsmoker) = \frac{e^{\beta_0+\beta_1+\beta_2+\beta_3}}{e^{\beta_0+\beta_1}} = e^{\beta_2+\beta_3} = e^{1.1022+0.5440} = 5.1872$$

$$OR(5 + cups\ smoker\ vs < 5\ cups\ nonsmoker) = \frac{e^{\beta_0+\beta_1+\beta_2+\beta_3}}{e^{\beta_0}} = e^{\beta_1+\beta_2+\beta_3} = e^{0.3640+1.1022+0.5440} = 7.4648$$

$$OR(5 + cups\ nonsmoker\ vs < 5\ cups\ smoker) = \frac{e^{\beta_0+\beta_1}}{e^{\beta_0+\beta_2}} = e^{\beta_1-\beta_2} = e^{0.3640-1.1022} = 0.4780$$



All three parameters are extremely correlated with themselves, almost 75% with a lag of 1, which decrease slowly until they become somewhat negligible past a lag of 20.

From 10,000 Monte Carlo simulations, the probability of the interaction term being greater than 0 is 0.9282. With a Bayes Factor of infinity, there is strong evidence supporting the alternative hypothesis of a positive interaction between coffee consumption and smoking on myocardial infarction.

$$Bayes\ Factor = \frac{0.9282}{1 - 0.9282} = 12.9276$$

Which model do you think provides the best result? What are the conclusions from your analysis?

The interaction model provides the best results because it allows the association between coffee consumption and myocardial infarction to change based on smoking status. The Bayesian model with an interaction term comes closest to the odds ratios of the data gathered.

Nonsmokers who drink more than 5 cups of coffee a day have 1.4391 times the odds of having a myocardial infarction compared to nonsmokers who drink less than 5 cups a day (95% Credible Interval: 0.8462, 2.4798). Smokers who drink more than 5 cups of coffee a day have 2.4794 times the odds of having a myocardial infarction compared to smokers who drunk less than 5 cups a day (95% Credible Interval: 1.5508, 4.0989).