A diagnostic test has 90% sensitivity and 80% specificity for a disease with prevalence 1 in 10,000 individuals. Calculate the probability that an individual with a positive test has the disease.

diagnostic test has 90% sensitivity and 80% specificity for a disease with a prevalence of 1 in 10,000 individuals

calculate $p(A=1|B=1)$, $A = disease$, $B = test$

sensitivity = true positive proportion $p(B=1|A=1) = 0.9$

specificity = true negative proportion $p(B=1|A=0) = 1 - 0.8 = 0.2$

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

$p(A)$ = marginal probability of A — prior probability of A

$p(A|B)$ = conditional probability of A given B — posterior probability of A

$p(B|A)$ = conditional probability of B given A — likelihood function

$p(B)$ = marginal probability of B — normalizing constant

$$\begin{aligned} p(A=1|B=1) &= \frac{p(B=1|A=1)p(A=1)}{p(B=1)} \\ &= \frac{p(B=1|A=1)p(A=1)}{p(B=1|A=1)p(A=1) + p(B=1|A=0)p(A=0)} \\ &= \frac{0.9 \times 0.0001}{0.9 \times 0.0001 + 0.2 \times 0.9999} = 0.0004498 \end{aligned}$$

The probability of actually having the disease when testing positive is 0.0004498.

Let θ denote the proportion of Texans who support the death penalty. From past experience, it is known that E(θ) = 0.6 and V (θ) = 1/25.

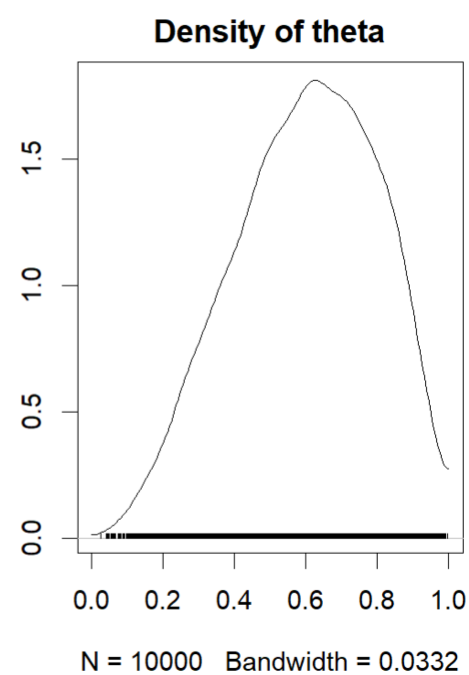
$$E(\theta) = 0.6$$
$$V(\theta) = 0.04$$

Choose a Beta prior distribution for θ, with appropriate hyper-parameters α1 and α2.

$$mean = \frac{a}{a+b} = 0.6$$
$$var = \frac{ab}{(a+b)^2(a+b+1)} = \frac{1}{25}$$

prior distribution: $P(\theta) = dbeta(a, b), a = 3, b = 2$

Simulate a sample of 10,000 values from this distribution using rjags and generate the empirical density.



Estimate the probability that θ > 0.8.

From 10,000 Monte Carlo simulations, there is a 0.1753 probability that $\theta > 0.8$.

Use rjags to estimate the probability that in a set of 50 Texans, more than 66% of them (n = 33) support the death penalty.

From 10,000 Monte Carlo simulations, there is a 0.4044 probability that in a set of 50 Texans, more than 33 will support the death penalty.

A random sample of 20 subjects with sickle cell anemia of African ancestry was genotyped at a locus in chromosome 2 that is supposed to be associated with less severe disease. The data summary is

Genotype AA	Genotype AB	Genotype BB
13	**6**	**1**

We wish to estimate the prevalence of carriers of one or more alleles B (hence genotype AB or BB) in the population of sickle cell anemia patients of African ancestry. A priori, we assume that the prevalence of carriers of one or more alleles B is approximately 20%, based on data from 10 sickle cell anemia subjects of African ancestry.

Describe a Bayesian Beta-Binomial model that could be used to describe the data and describe a prior distribution that represents the prior knowledge.

A beta-binomial model can be used to model this data because the probability for outcome of interest, carrier of the B allele, is unknown. Only from sampling the population of those with African ancestry and tallying the yes/no binomial outcome can θ be measured.

$$mean = \frac{a}{a+b} = \frac{2}{10}$$

prior distribution: $P(\theta) = dbeta(a, b), a = 2, b = 8$

Compute the Bayesian estimate of the prevalence of carriers in sickle cell anemia patients of African ancestry.

data: $Y = 7, n = 20$

posterior distribution: $P(\theta) = dbeta(a + y, b + n - y) = \ P(\theta) = dbeta(9,21)$

From 10,000 Monte Carlo simulations, the prevalence of carriers in sickle cell anemia patients of African ancestry is 0.2989.

Estimate the probability that in a sample of 100 sickle cell anemia patients of African ancestry more than 50 will be carriers of one or more alleles B.

From 10,000 Monte Carlo simulations, there is a 0.0267 chance of observing more than 50 carriers in a sample of 100 sickle cell anemia patients of African ancestry

Estimate the probability that in a sample of 100 sickle cell anemia patients of African ancestry the number of carriers of one or more alleles B is more than 30 but less than 70.

From 10,000 Monte Carlo simulations, there is a 0.4632 chance of observing more than 30 but less than 70 carriers in a sample of 100 sickle cell anemia patients of African ancestry.