TWO-CAVITY MASER WITH OPPOSED MOLECULAR BEAMS

A. F. Krupnov, V. A. Skvortsov, and L. A. Sinegubko

Izvestiya VUZ. Radiofizika, Vol. 11, No. 2, pp. 244–250, 1968

UDC 621.376.33

A Ramsey-type opposed-beam ammonia maser is described in which the effective line Q is increased by a factor of 2.5 over that of a conventional single-cavity maser. A study is made of the effect of saturation on the line Q and on the magnitude of the zone over which synchronous operation is possible. It is shown that the line Q is a maximum when the maser is constructed and operated in a symmetrical manner.

The possibility of increased frequency stability of molecular masers resulting from the narrowing of the spectral line in Ramsey systems was investigated in [4], where it was confirmed that the line was indeed narrowed in comparison with that produced by a single-cavity device. However, the work reported in these papers related to the case of short cavities ($l_0 \sim 3$ cm) with a short spacing between them ($l \sim 15$ cm). As a result, in comparison with conventional masers with a single cavity of length around 10 cm, there was no absolute improvement in the line Q. Clearly, the use of the more complex two-cavity systems will be justified only if they increase the absolute Q of the line. In the experiment described below, maser frequency pulling when the cavities were detuned simultaneously was reduced as a result of spectral line narrowing by a factor of around 2.5 in comparison with the case of a conventional single-cavity maser of cavity length 10 cm.

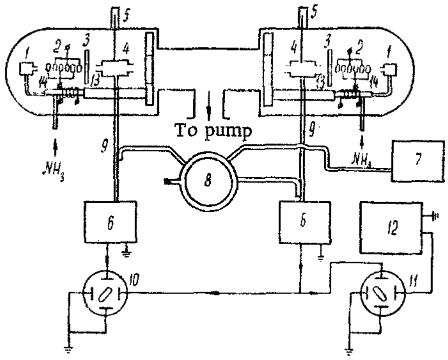

Fig. 1. Block diagram of the two-cavity opposed-beam ammonia maser.

1. Figure 1 shows a block diagram of our two-cavity opposed-beam maser, which utilized the J = 3, K = 3 line of ammonia $N^{14}H_3$. When designing the device use was made of preliminary results on the shaping of long beams of ammonia molecules [5], and allowance was made for the falling off in intensity of the beam with increasing distance [6]. In accordance with [5] the molecular beams were produced by sources 1, in which the outlet channel diameter and length equaled 0.15 mm, spaced 20 mm from a ring separator system of length 40 mm and inner diameter 3.5 mm. The rings were separated by a distance equal to their radius. The separator voltage was maintained at around 25–30 kV. To reduce molecular collisions in the cavity cold diaphragms 3 with apertures of around 4 mm were placed in front of the cavities. Cavities 4 were 10 cm long and had a Q of around $6-8 \cdot 10^3$. The distance between the cavities was 35 cm. Each cavity could be tuned by a tuning mechanism 5 and was coupled to a separator receiver 6. Beats were obtained by supplying signals from an auxiliary maser 7 to the two receivers. Decoupling was by means of a hybrid ring 8 and directional couplers 9. After detection the beat signals from the receivers were fed to the vertical and horizontal plates, respectively, of an oscilloscope 10, by means of which the oscillations in the maser cavities could be checked for synchronization. When the oscillations in the two cavities are in synchronism, the beat frequencies from the two receivers are the same and a stationary ellipse is observed on the oscilloscope screen. The beat signal from one of the receivers is also fed to one pair of plates of an oscilloscope 11, to the second plates of which is fed a signal from an audio generator 12. The beat frequency can thus be measured by the Lissajous-figures technique.

Ammonia was supplied in much the same manner as described in [7]. To purify the ammonia it was frozen in tubes 14, which were warmed up by electric heaters during operation to evaporate pure ammonia for supply to the sources. The ammonia reservoir was sufficient for 8–10 hr of operation. The device was evacuated by a single TsVL-100 pump at the center of the device and by liquid nitrogen dewars 13. With no beam the vacuum was $6-7 \cdot 10^{-7}$ mm Hg; under operating conditions with the two beams present it was $2-3 \cdot 10^{-6}$ mm Hg. The transit space between the cavities was screened against electric and magnetic fields for the reasons given in [8].

2. We were interested in the monochromatic mode of operation in cavities mutually synchronized in frequency.* To determine the optimum relationship between the maser parameters for which the gain in the effective line Q is greatest, an analysis was carried out on the equations of a two-cavity opposed-beam molecular maser (see Appendix 1). The analysis showed that the optimum conditions corresponded to a

*A large number of modes of oscillation exist in a two-cavity molecular oscillator. In our case, for example, with both beams switched on and with a slight difference in the cavity tunings, a biharmonic mode of oscillation could be produced. The difference frequency could be tuned over a range of several kilocycles.

symmetrical maser arrangement, when all its elements (beam sources, separators, etc.) are identical. In the small-signal approximation [9] the gain in the effective line Q for a symmetrical maser is

$$G = \frac{1 + 7\chi + 6\chi T/\tau}{1 + 3\chi},$$

where $\chi(l)$ is the attenuation of the beam in traversing the distance l between the cavities, $T = l/v$ is the time required by a molecule to traverse the intercavity distance, and τ is the time required by a molecule to traverse the length of a cavity. Starting from this expression and the experimentally obtained beam attenuation [6], it is possible to calculate the gain to be expected in the effective line Q. For our arrangement $\chi = 0.13$, $T/\tau = 3.5$, and $G = 3.3$, which is less than the maximum value $G = 3.5$ [6] since for constructional reasons the distance between the cavities had to be somewhat greater than the optimum. The gain obtained from this formula implies that if the two cavities are simultaneously detuned in the same direction by a certain amount, the frequency pulling of the line should be smaller by a factor G than in a single-cavity resonator for the same detuning.

The following technique was used to measure the gain in the line Q. Firstly, from the beats produced using the auxiliary maser, the slope of the curve of the maser frequency shift against cavity detuning determined for each cavity with the device operated as a single-beam single-cavity system (one of the beams switched off). The frequency pulling of each of the cavities turned out to depend linearly on the displacement of the cavity tuning rod. The original slope for the one cavity was 450 cps for a 0.01-mm displacement of the rod and for the second cavity it was 272 cps. The difference is due to constructional differences in the tuning mechanisms. The synchronous mode of operation was obtained by tuning each cavity to the center of the spectral line by maximizing the amplitude of the oscillations with the oppositely incident beam switched off. Such tuning was sufficient to ensure mutual synchronization of the oscillations in the cavities when the two beams were switched on. A stationary ellipse was observed on the screen of oscilloscope 10.

After the two beams were switched on and the cavities synchronized, the cavities were detuned in turn by a prescribed amount and the resulting shift in the maser frequency measured. For each cavity the slope of the resulting frequency-detuning curve was compared with the original G_1 and G_2 and the reduction obtained was calculated. It can be shown that if the cavity Q's are approximately the same, the gain in the effective line Q is given by this measuring technique as

$$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2}.$$

For arbitrary intensities of the two beams, the frequency-detuning slope is reduced as a rule by different amounts for the two cavities. Conditions were often observed when the frequency shift was almost completely determined by one of the cavities, detuning of the other cavity having only a very slight effect on the maser frequency.

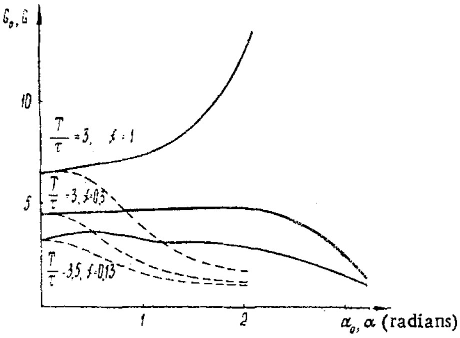

Fig. 2. Gain in effective line Q of two-cavity maser as a function of amplitude of oscillation $\mathcal{E}$. The solid lines correspond to a single-energy beam and the broken lines to a beam with a velocity distribution $((\bar{v})^2/v^3)\exp(-\bar{v}/v)$. Here $\alpha_0 = |\mu_{12}|\mathcal{E}\tau_0/\hbar$, $\alpha = |\mu_{12}|\mathcal{E}\bar{\tau}/\hbar$, τ_0 is the time of flight through the cavity for the single-energy beam; $\bar{\tau}$ refers to the case of molecular velocity distribution.

The frequency-detuning slope for the cavity determining the frequency was often reduced under such conditions to around one-half of its original value. To symmetrize the maser, the relationship between the beam intensities was arranged so that the gains G_1 and G_2 measured for each cavity were the same. This balances out any accidental asymmetry in the device, originating, for instance, during adjustment or as a result of slight differences in its elements.

The gain in the effective line Q measured for small amplitudes of oscillation turned out to equal 5 for each resonator, while the over-all gain $G = 2.5$. The difference between the experimental and the calculated ($G = 3.3$) gains can probably be ascribed to the composite structure of the $J = 3$, $K = 3$, $\Delta F = 0$ line of $N^{15}H_3$ ammonia, which was not taken into account when calculating the gain. The structure of this line was investigated in [8], where it was found to give rise in the separated beam to two components spaced by 1586 ± 80 cps. The Ramsey width of the components in our case is 1000 cps. In this manner, the line shape in our arrangement is determined by the superposition of two unresolved Ramsey interference cuves, which must bring about a reduction in the effective line Q. If isotropic ammonia $N^{15}H_3$ were used in our arrangement, it would probably be possible to approach the calculated gain of 3.3.

3. In real molecular oscillators conditions may exist under which saturation effects become important. The theoretical results for such conditions depend strongly on the nature of the function taken to represent the velocity distribution of the molecules in the separated beam, the true form of which is unknown.

This is illustrated in Fig. 2, which shows the calculated gain as a function of the amplitude of oscillation for a single-energy beam (solid lines) and for a beam distribution of the form

$$[(\bar{v})^2/v^3]\exp(-\bar{v}/v).$$

The calculations for a single-energy beam are cited in Appendix 2; the calculations for a beam distribution of the form $[(\bar{v})^2/v^3]\exp(-\bar{v}/v)$ are given in [9].

With a view to determining the optimum operating conditions of the maser, its operation was investigated experimentally for various amplitudes of oscillation. The slope of the frequency-detuning curve was investigated as a function of beam intensity, the symmetry condition always being maintained. The gain scarcely altered for a beam intensity variation of more than an order of magnitude, corresponding to a range from the threshold of oscillation to the maximum amplitude of oscillation. The absence of any appreciable change in the gain with increasing saturation indicates that the molecular velocity distribution in a real two-cavity maser is closer to a single-energy distribution than to the form $[(\bar{v})^2/v^3]\exp(-\bar{v}/v)$ normally employed in calculations (see Fig. 2).

Increasing the beam intensity had no effect on the gain but had an appreciable effect on the range over which synchronous operation was possible. At small amplitudes of oscillation each cavity could be detuned without interruption of synchronous operation over a range corresponding to the detuning of a single-cavity maser by 500–800 cps. If, however, the amplitude of oscillation was increased to its maximum value, the zone over which synchronous operation was possible was reduced by more than an order of magnitude and tuning to synchronous operation became very difficult.

Our investigations showed that the main factor limiting the gain in the effective line Q in such systems is the reduction in beam intensity with increasing beam length. Thus, despite the fact that oscillation was possible with the cavity spaced 70 cm from the separator [5], the fall-off in the beam attenuation with increasing distance limits the optimum cavity separation to 25–30 cm [6]. In our opinion the method for further improving such systems lies in removing the effects of beam attenuation.

We note that the main factor responsible for the attenuation of the beam of active molecules is, in our opinion, the angular divergence of the beam in the transit space between the cavities. A "lens" description of the separator [12] leads to a dependence $\chi(l)$ which is in satisfactory agreement with experiment [6]. Calculation leads to a hyperbolic dependence of χ on the cavity spacing, although the experimental curve falls off somewhat more rapidly. Imperfect focusing of the molecules in the remote cavity is explained by the different interaction of molecules with different quantum numbers M_J with the field of the separator and by the already existing velocity scatter of the molecules in the beam. Also, it must not be forgotten that the separator is not a perfect "collecting lens," which leads to further defocusing of the beam.

In principle, modes of operation are possible such that $\chi \geq 1$. This sort of increase in χ would enable the gain in the effective Q to approximate the limiting value $\sim 2T/\tau$. Efforts to realize values of $\chi \geq 1$ could be made, for example, by employing very short separators; active molecules passing through the separator would receive an impulse towards the axis of the system and passive molecules an impulse towards the periphery, i. e., the velocity vectors of the molecules would change. However, thanks to the presumed small length of the separator system at its outlet, it would scarcely effect any spatial separation of active and passive molecules and the excess active molecules in the first cavity will be small. The passive molecules have time to leave the beam in the space between the cavities, so that it is mainly active molecules that will enter the second cavity. But since χ is defined, in effect, as the ratio of the absolute excess of active over passive molecules in the second cavity to that in the first, it follows that χ may in this case be greater than unity. The practical realization of such conditions will, of course, call for special investigations.

The authors wish to thank B. V. Gromov for his assistance with the experiments.

APPENDIX

1. The equations for a two-cavity opposed beam molecular oscillator will be obtained in the small-signal approximation for steady-state conditions. The derivation utilizes the general solution of the equations describing the state of a two-level quantum-mechanical system subjected to a harmonic perturbation:

$$\Delta\omega_1\mathcal{E}_1 = (\omega/2)R_1\Big[-2\chi_1\mathcal{E}_2\tau_2\sin\varphi + 6\chi_1\mathcal{E}_2\tau_2\varepsilon T\cos\varphi + 3\chi_1\mathcal{E}_2\tau_2(\tau_1+\tau_2)\varepsilon\cos\varphi + \chi_1\mathcal{E}_1\tau_1^2\varepsilon + \mathcal{E}_1\tau_1^2\varepsilon\Big]; \tag{A1a}$$

$$\frac{\mathcal{E}_1}{Q_1} = R_1(\chi_1\mathcal{E}_1\tau_1 + 2\chi_1\mathcal{E}_2\tau_2\cos\varphi + \chi_1\mathcal{E}_2\tau_2\varepsilon T\sin\varphi + \mathcal{E}_1\tau_1); \tag{A1b}$$

$$\Delta\omega_2\mathcal{E}_2 = (\omega/2)R_2\Big[2\chi_2\mathcal{E}_1\tau_1\sin\varphi + 6\chi_2\mathcal{E}_1\tau_1\varepsilon T\cos\varphi + 3\chi_2\mathcal{E}_1\tau_1(\tau_1+\tau_2)\varepsilon\cos\varphi + \chi_2\mathcal{E}_2\tau_2^2\varepsilon + \mathcal{E}_2\tau_2^2\varepsilon\Big]; \tag{A1c}$$

$$\frac{\mathcal{E}_2}{Q_2} = R_2(\chi_2\mathcal{E}_2\tau_2 + 2\chi_2\mathcal{E}_1\tau_1\cos\varphi + \chi_2\mathcal{E}_1\tau_1\varepsilon T\sin\varphi + \mathcal{E}_2\tau_2). \tag{A1d}$$

These equations were obtained in [11] in another form and by another method.

Here $\Delta\omega_i = \omega_{12} - \omega_i$, $\varepsilon = \omega_{12} - \omega$ is the cavity detuning and the shift in the maser frequency from the transition frequency ω_{12}; Q_i, $\mathcal{E}_i$, τ_i are, respectivly, the cavity Q-factor, field amplitude, and time of flight; T is the time of flight between cavities; φ is the phase difference between the oscillations in the cavities; $R_i = 4\pi\rho_i|\mu_{12}|^2\hbar^{-1}$, where μ_{12} is the dipole moment matrix element of the transition, and ρ_i is the density of molecules passing directly from the separator into the cavity; and $\chi_i = \chi\rho_j/\rho_i$, where χ is the beam attenuation in traversing the distance between the cavities. In the case of practical interest $\tau/T \ll 1$ for equal cavity detunings $\Delta\omega_1 = \Delta\omega_2 = \Delta\omega$ (in this case $|\varphi| \ll 1$), we obtain from (A1a) and (A1c) the following expression for the frequency of oscillation:

$$\varepsilon = \frac{\Delta\omega Q_2}{12\chi Q_R}\left[2\chi(1+n_q) + \chi\left(n_q n_{\mathcal{E}} n_\tau + \frac{1}{n_{\mathcal{E}} n_\tau}\right) + \right.$$

$$+ n_q n_\rho n_{\mathcal{E}} n_\tau + \frac{1}{n_\rho n_{\mathcal{E}} n_\tau} \Bigg],$$

where

$$n_q = \frac{Q_1}{Q_2}, \quad n_\rho = \frac{\rho_1}{\rho_2}, \quad n_{\mathcal{E}} = \frac{\mathcal{E}_1}{\mathcal{E}_2}, \quad n_\tau = \frac{\tau_1}{\tau_2}, \quad Q_R = \frac{\omega T}{2}.$$

It is not hard to show that the slope of the frequency-detuning curve is a minimum when $n_{\mathcal{E}} n_\tau = 1/\sqrt{n_q}$ and $n_\rho = 1$. In this case

$$\varepsilon = \frac{\Delta\omega Q_2}{6\chi Q_R} \left[\chi (1 + n_q) + \sqrt{n_q}(1 + \chi) \right], \qquad (A2)$$

and the conditions for self-excitation (A1b) and (A1d) take the form

$$\frac{1}{Q_1} \leqslant R\tau_1 (1 + \chi + 2\chi \sqrt{n_q}),$$
$$\frac{1}{Q_2} \leqslant R\tau_2 \left(1 + \chi + 2\chi \frac{1}{\sqrt{n_q}}\right). \qquad (A3)$$

If the cavity Q-factors are different ($n_q \neq 1$) and the self-excitation conditions (A3) are satisfied, it is not hard to show that for the same beam intensity (R = const) the self-excitation conditions (A3) will also be satisfied for a symmetrical maser in which the cavity with higher Q is replaced by one of lower Q identical with that already in the device. It can be seen from (A2) that the slope of the frequency-detuning curve of the symmetrical maser is less than that of the original nonsymmetrical maser.

In this manner, the minimum slope of a maser with two cavities is obtained when $n_q = 1$, $n_\rho = 1$, and $n_{\mathcal{E}} n_\tau = 1$. These conditions are most easily satisfied in a completely symmetrical arrangement: $n_q = n_\rho = n_{\mathcal{E}} = n_\tau = 1$, i. e., when all the elements of the maser (the separator system, cavities, and so on) and the intensities of the molecular beams are identical.

2. The effect of saturation was investigated only for a symmetrical maser; the problem was solved in the same manner as for the small-signal case. For a single-energy beam the expression for the gain in the effective line Q has the form

$$G_0 = \frac{1 + \cos\alpha_0}{1 - (\sin\alpha_0)/\alpha_0} \Bigg[1 - (\sin\alpha_0)/\alpha_0 +$$
$$+ \chi \left[1 + (\sin\alpha_0)/\alpha_0 - (\sin 2\alpha_0)/\alpha_0 + (T\alpha_0 \sin\alpha_0)/\tau\right] \Bigg] \times$$
$$\times \left[1 - \cos\alpha_0 + \chi(\cos\alpha_0 - \cos 2\alpha_0) \right]^{-1}.$$

Here $\alpha_0 = |\mu_{12}|\mathcal{E}\tau_0/\hbar$, $\mathcal{E}$ is the field amplitude in the cavity, and τ_0 is the time of flight of molecules through the cavity. If the above expression is averaged with respect to velocities distributed according to $(\bar{v})^2 v^{-3} \exp(-\bar{v}v^{-1})$, the resulting expression agrees with that obtained for the gain in [9].

REFERENCES

1. F. Holuj, H. Daams, and S. N. Kalra, J. Appl. Phys., 33, 2370, 1962.
2. A. F. Mukhamedgalieva and G. M. Strakhovskii, Izv. VUZ. Radiofizika [Soviet Radiophysics], 8, no. 4, 824, 1965.
3. A. F. Mukhamedgalieva, A. N. Oraevskii, and G. M. Strakhovskii, Pis'ma, ZhETF, 1, 22, 1965.
4. A. F. Mukhamedgalieva, A. N. Oraevskii, and G. M. Strakhovskii, Izv. VUZ. Radiofizika [Soviet Radiophysics], 9, no. 2, 302, 1966.
5. A. F. Krupnov and V. A. Skvortsov, Izv. VUZ. Radiofizika [Soviet Radiophysics], 9, no. 4, 824, 1966.
6. A. F. Krupnov, V. A. Skvortsov, and L. A. Sinegubko, Izv. VUZ. Radiofizika [Radiophysics and Quantum Electronics], 10, no. 1, 141, 1967.
7. G. M. Strakhovskii, V. M. Tatarenkov, and V. Yu. Sudzilovskii, Pribory i tekhnika eksperimenta, no. 3, 245, 1965.
8. A. F. Krupnov, V. A. Skvortsov, and L. A. Sinegubko, Izv. VUZ. Radiofizika [Radiophysics and Quantum Electronics], 10, no. 1, 142, 1967.
9. E. M. Belenov and A. N. Oraevskii, ZhTF, 36, 557, 1966.
10. N. Ramsey, Molecular Beams [Russian translation], IL, Moscow, 1964.
11. A. N. Oraevskii, Molecular Oscillators [in Russian], Nauka, Moscow, 1964.
12. G. Becker and B. Fischer, Zs. für angew, Phys., 19, 537, 1965; 20, 71, 1965.

9 January 1967

Scientific-Research Radiophysical Institute, Gor'kii University