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Bremsstrahlung from Collisions of Low-Energy Electrons with Positive Ions in a Magnetic Field

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Abstract—Bremsstrahlung from electron–ion collisions in a magnetic field is studied for low energies at which the Larmor radius of an electron is smaller than the characteristic impact parameter of close collisions in zero magnetic field. It is shown that the magnetic field does not qualitatively change the bremsstrahlung power at low frequencies smaller than the reciprocal time of electron transit in the vicinity of an ion in close collision in zero magnetic field. At high frequencies, the radiation intensity decreases in accordance with a power law, attains its minimal value, and then increases in accordance with a power law up to frequencies on the order of the electron cyclotron frequency. At such frequencies, the spectral power attains typical power values in zero magnetic field. At frequencies lower than the cyclotron frequency considered here, bremsstrahlung is polarized predominantly linearly in the plane formed by the magnetic field and the direction of radiation.

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1. INTRODUCTION

Magnetic fields qualitatively change electron–ion collisions [1–9] when the Larmor radius of an electron,

$$r_B = v_0/\omega_B \tag{1}$$

becomes smaller than not only the Debye radius of electrostatic screening of a scattering center, but also the characteristic impact parameter r_s of close collisions in zero magnetic field,

$$r_s = Ze^2/mv_0^2. \tag{2}$$

Here, v_0 is the initial electron velocity, $\omega_B = eB/mc$ is the cyclotron frequency, B is the magnetic field, $e > 0$ is the elementary charge, $Z > 0$ is the charge number of an ion, and m is the electron mass. At distance (2), the energy of the Coulomb interaction becomes on the order of the initial kinetic energy of the electron.

The relation

$$r_B \ll r_s \tag{3}$$

between spatial scales considered here for thermal velocity $v_T = \sqrt{k_B T/m}$ (k_B is the Boltzmann constant) has the form

$$\frac{r_B}{r_s} = \frac{1.3}{Z}\left(\frac{T}{10^4\ \text{K}}\right)^{3/2}\left(\frac{B}{10^7\ \text{G}}\right)^{-1} \ll 1$$

and is observed in the photospheres of magnetic white dwarfs [10] with a temperature of $T \sim 10^4$ K and a magnetic field of $B > 10^7$ G, as well as in experiments on the production of antihydrogen [11, 12] in a magnetic field of $B > 3 \times 10^4$ G at temperature $T < 15$ K; this was attained in experiment [13] with electron–electron collisions ($T = 30$ K, $B = 6 \times 10^4$ G), and it probably holds in magnetars [14] for proton–proton collisions ($B \sim 10^{15}$ G).

If condition (3) is satisfied, deceleration of an electron in the direction of the magnetic field is controlled by effective close collisions, in which the electron passes by an ion over a time on the order of or shorter than the cyclotron rotation period ω_B^{-1} [1, 2]. Impact parameters p_h of effective collisions have an upper bound [3]:

$$p_{\text{cr}} = \left(\frac{12\sqrt{\pi}}{\Gamma^2(1/4)}\right)^{2/3}\ln^{2/3}\left[\left(\frac{r_s}{r_B}\right)^{1/3}\right]L_u, \tag{4}$$

where $\Gamma(x)$ is the gamma function and distance

$$L_u = (Zmc^2/B^2)^{1/3} = r_s^{1/3}r_B^{2/3} \tag{5}$$

is independent of the initial electron velocity and falls in the interval

$$r_B \ll L_u \ll r_s \tag{6}$$

under condition (3). By virtue of relation (6), the Coulomb field in effective collisions accelerates an electron approaching an ion to velocities much higher than its initial velocity. In the vicinity of the ion, the drift of the guiding center of cyclotron rotation is violated and the kinetic energy accumulated during the electron's approach to the ion is effectively redistributed between the motion along and across the magnetic field. When the electron moves away from the ion, the Coulomb field decelerates the electron along the magnetic field. If the kinetic energy of the longitudinal motion after the first passage is insufficient to overcome the attraction of the ion, the electron repeatedly returns to the ion until it overcomes the ion's attraction.

In close collisions, the final pitch angle (between the instantaneous velocity of the particle and the magnetic field) depends irregularly (chaotically) on the impact parameter, magnetic field, and particle energy [5, 6]. Conversely, in distant collisions ($p_h \gg L_u$), the pitch angle of the electron is conserved to a high degree of accuracy (with an error on the order of $\exp(-0.62 p_h^{3/2}/L_u^{3/2})$) [3].

Thus, in view of quasi-bound chaotic motion of the electron, effective close collisions qualitatively differ from collisions in the case of a weaker magnetic field or a higher particle energy. The qualitative change in collisions under conditions (3) must affect the frequency spectrum and polarization of bremsstrahlung. In this study, we analyze bremsstrahlung under condition (3) for frequencies

$$\omega \ll \omega_B. \tag{7}$$

It will be shown in Section 2 that electron bremsstrahlung in a magnetic field is a superposition of radiation from three incoherent sources. At frequencies (7) under investigation, radiation from a source in the form of a dipole oriented along the magnetic field dominates. The bremsstrahlung spectrum from distant collisions will be calculated analytically in Section 3; in Section 4, the spectrum from close collisions will be analyzed and frequency intervals in which radiations from distant and close collisions dominate will be indicated. The bremsstrahlung and cyclotron radiation power levels for various sources will be considered in Section 5. The main results are formulated in the final section.

2. DESCRIPTION OF BREMSSTRAHLUNG IN A MAGNETIC FIELD

The frequency spectrum of the energy fluence with polarization vector **e** directed into a unit solid angle about direction **n** from a single electron–ion collision is defined as (see Eq. (3.17) in [15])

$$\begin{aligned} W_{\mathbf{e}} &= \frac{e^2}{(2\pi)^2 c^3} |\mathbf{e}^* \cdot [\mathbf{n} \times [\mathbf{n} \times \mathbf{F}]]|^2 \\ &= \frac{e^2}{(2\pi)^2 c^3} |\mathbf{e}^* \cdot \mathbf{F}|^2 \end{aligned} \tag{8}$$

("*" indicates complex conjugation) and is proportional to spectrum **F** of electron acceleration $\mathbf{a}(t)$,

$$\mathbf{F} = \lim_{\gamma \to 0} \int_{-\infty}^{\infty} \mathbf{a}(t) \exp(i\omega t - |\gamma t|) dt. \tag{9}$$

Passing to the second equality in (8), we take into account the orthogonality of vectors **n** and **e**. The electron acceleration in a magnetic field remains finite at a long distance from the ion due to the action of the Lorentz force. For the integral in Eq. (9) to converge to a certain limit, we introduced in the integrand a factor $\exp(-|\gamma t|)$ slowly decreasing for $t \longrightarrow \pm\infty$. We attach the origin of the Cartesian system of coordinates (x, y, z) to the ion and direct unit vector $\mathbf{z}^\circ$ along the magnetic field and unit vectors $\mathbf{x}^\circ$ and $\mathbf{y}^\circ$ at right angles to it.

To average radiation spectrum (8) over trajectories taking into account the axial symmetry of the problem, we use the basis vectors

$$\begin{gathered} \hat{\mathbf{e}}_z = \mathbf{z}^\circ, \quad \hat{\mathbf{e}}_r = (\mathbf{x}^\circ + i\mathbf{y}^\circ)/\sqrt{2}, \\ \hat{\mathbf{e}}_l = (\mathbf{x}^\circ - i\mathbf{y}^\circ)/\sqrt{2}. \end{gathered} \tag{10}$$

These vectors correspond to Fourier harmonics

$$F_z = \hat{\mathbf{e}}_z^* \cdot \mathbf{F} = \lim_{\gamma \to 0} \int_{-\infty}^{\infty} \ddot{z}(t) \exp(i\omega t - |\gamma t|) dt, \tag{11}$$

$$\begin{aligned} F_{r,l} &= \hat{\mathbf{e}}_{r,l}^* \cdot \mathbf{F} \\ &= \lim_{\gamma \to 0} \int_{-\infty}^{\infty} \frac{\ddot{x}(t) \mp i\ddot{y}(t)}{\sqrt{2}} \exp(i\omega t - |\gamma t|) dt. \end{aligned} \tag{12}$$

The upper and lower signs in notation "∓" correspond to F_r and F_l, respectively. We express radiation spectrum (8) in terms of Fourier harmonics (11) and (12),

$$W_{\mathbf{e}} = \frac{e^2}{(2\pi)^2 c^3} \sum_{\alpha, \beta} (\mathbf{e}^* \cdot \hat{\mathbf{e}}_\alpha) F_\alpha F_\beta^* (\hat{\mathbf{e}}_\beta^* \cdot \mathbf{e}), \tag{13}$$

where subscripts α and β run through values z, r, and l.

Let us suppose that the electron can move in a trajectory $\mathbf{r}(t) = [z(t), \rho(t), \phi(t)]$, where $\rho = \sqrt{x^2 + y^2}$, and azimuth angle ϕ is measured from the x axis. Since the electric and magnetic fields in the problem are stationary and axisymmetric, the particle can also move in trajectory $\mathbf{r}'$, which can be obtained by rotation of $\mathbf{r}$ through angle ϕ_0 about the z axis and by a time delay in the motion of the particle of $\Delta t = \phi_0/\omega_B$:

$$\mathbf{r}'(t) = [z(t - \Delta t), \rho(t - \Delta t), \phi(t - \Delta t) + \phi_0].$$

The electron velocities on trajectories **r** and **r'** coincide before and after the collision when the electron performs a Larmor rotation: $\dot{\mathbf{r}}'(t) = \dot{\mathbf{r}}(t)$ for $t \longrightarrow \pm\infty$. Fourier harmonics (11) and (12) on trajectories **r** and **r'** differ only in complex arguments:

$$\begin{aligned} F_z' &= F_z \exp(i\omega \Delta t), \\ F_{r,l}' &= F_{r,l} \exp(i\omega \Delta t \mp i\phi_0). \end{aligned} \tag{14}$$

After averaging over trajectories **r'** with different angles of rotation $0 < \phi_0 < 2\pi$, all products $F_\alpha F_\beta^*$ with indices

$\alpha \neq \beta$ in expression (13) vanish because of factors $\exp(\mp i\phi_0)$ in harmonics F'_r and F'_l (14):

$$\langle W_{\mathbf{e}}\rangle_{\phi_0} = \frac{e^2}{(2\pi)^2 c^3}\sum_\alpha |\mathbf{e}^*\cdot\hat{\mathbf{e}}_\alpha|^2 |F_\alpha^2|$$
$$= \frac{e^2}{(2\pi)^2 c^3}\Big[|F_z^2||e_z^2| + \frac{1}{2}(|F_r^2| + |F_l^2|)(|e_x^2| + |e_y^2|) \quad (15)$$
$$+ \frac{i}{2}(|F_r^2| - |F_l^2|)(e_x e_y^* - e_x^* e_y)\Big],$$

where subscript α runs through values z, r, and l; e_x, e_y, and e_z are the Cartesian coordinates of vector $\mathbf{e}$. It should be noted that moduli $|F_\alpha^2|$ in expression (15) are independent of the directions of unit vectors $\mathbf{x}°$ and $\mathbf{y}°$ and are identical for all trajectories $\mathbf{r}'$ and $\mathbf{r}$.

Integration of relation (15) with respect to impact parameters gives the spectral power of bremsstrahlung from an electron with polarization vector $\mathbf{e}$ into the unit solid angle about direction $\mathbf{n}$

$$I_{\mathbf{e}} = I_{z0}|e_z^2| + \frac{1}{2}(I_{r0} + I_{l0})(|e_x^2| + |e_y^2|)$$
$$+ \frac{i}{2}(I_{r0} - I_{l0})(e_x e_y^* - e_x^* e_y), \quad (16)$$

where

$$I_{\alpha 0} = \frac{n_i e^2 v_{0\parallel}}{(2\pi)^2 c^3}\int_0^{2\pi} d\varphi_h \int_0^\infty dp_h p_h |F_\alpha^2|, \quad (17)$$

index α assumes values of z, r, and l; n_i is the number density of ions; $v_{0\parallel}$ is the initial electron velocity in the direction of the magnetic field; and impact parameter p_h is the initial distance from the center of cyclotron rotation of the electron to the z axis. Angle φ_h is defined by the position of the electron on the Larmor circle at the instant of intersection of plane $z = 0$ in the case when the particle continues its motion in the initial helical trajectory unperturbed by the ion [3]. This angle is equal to the angle between the segment connecting the ion and the center of rotation (impact parameter) and the segment connecting the center of rotation with the electron (Larmor radius).

In accordance with expressions (15) and (16), radiation from the electron is the superposition of radiation from three incoherent sources with dipole moments directed along $\hat{\mathbf{e}}_z$, $\hat{\mathbf{e}}_r$, and $\hat{\mathbf{e}}_l$ (10). The powers of radiation from the sources integrated over all directions are $8\pi I_{z0}/3$, $8\pi I_{r0}/3$, and $8\pi I_{l0}/3$. Thus, the problem of bremsstrahlung from an electron in a magnetic field is reduced to calculation of powers (17).

At the same time, the magnetic field at frequencies (7) considered here suppresses the transverse high-frequency conductivity of electrons. The suppression of conductivity is not associated with the change in collisions and is due to the fact that the wave polarized across the magnetic field causes only a slow drift of the electron in crossed electric field of the wave and the external uniform magnetic field. Consequently, at frequencies (7), the wave polarized in the plane containing the magnetic field and the direction of radiation $\mathbf{n}$ is mainly absorbed and emitted [16, 17]. For this reason, we can expect that the values of power I_{z0} at frequencies (7) are much higher than I_{r0} and I_{l0}. Thus, we will confine our analysis to calculation of I_{z0} alone. The contributions to I_{z0} from distant and close collisions will be considered separately.

3. DISTANT COLLISIONS

In distant collisions with impact parameters $p_h > p_{\text{cr}}$ (4), an electron passes through the region near an ion over a characteristic time longer than ω_B^{-1} and, hence, moves under the conditions of the magnetic drift approximation. The center of cyclotron rotation is virtually attached to a magnetic field line and moves with an instantaneous acceleration

$$\ddot{z} \approx -\frac{Ze^2 z}{m(p_h^2 + z^2)^{3/2}}. \quad (18)$$

In the plane perpendicular to the magnetic field, the electron only slowly drifts over the azimuth in crossed electric and magnetic fields. The change in azimuth angle ϕ is small and becomes on the order of $\pi/2$ only at the boundary between distant and close collisions.

Substituting acceleration (18) into expression (11) for Fourier harmonic F_z and replacing integration with respect to time by integration over $dz = \dot{z}\,dt$, where $\dot{z}$ can be determined from the approximate energy conservation law (disregarding small displacement $\Delta\rho$ of the electron across the magnetic field, $\Delta\rho \ll p_h$),

$$|\dot{z}| \approx \left[v_{0\parallel}^2 + \frac{2Ze^2}{m(p_h^2 + z^2)^{1/2}}\right]^{1/2}, \quad (19)$$

we obtain the contribution to spectral power I_{z0} (17) from distant collisions:

$$I_{z0}^{(d)} = \frac{2n_i Z^2 e^6}{\pi m^2 c^3 v_{0\parallel}}\int_{\eta_{\text{cr}}}^\infty d\eta\eta$$
$$\times\left(\int_0^\infty d\zeta\zeta\frac{\sin\left[\int_0^\zeta d\zeta'(1 + 2\Omega/\sqrt{\eta^2 + \zeta'^2})^{-1/2}\right]}{(1 + 2\Omega/\sqrt{\eta^2 + \zeta^2})^{1/2}(\eta^2 + \zeta^2)^{3/2}}\right)^2. \quad (20)$$

Here, the dimensionless integration variables are $\eta = p_h\omega/v_{0\parallel}$ and $\zeta = z\omega/v_{0\parallel}$. The lower integration limit in relation (20),

$$\eta_{\text{cr}} = p_{\text{cr}}\omega/v_{0\parallel},$$

can be set at zero for conditions (3) and frequencies (7) considered here. Indeed, quantity η_{cr} corresponds to the boundary between distant and close collisions at which an electron passes through the region near the ion over a time on the order of ω_B^{-1}. However, the value of the integral with respect to η in expression (20) is mainly controlled by numerous more distant collisions,[1] where the electron passes through the region near the ion over a time on the order of $\omega^{-1} \gg \omega_B^{-1}$. As a result, the values of integrals in formula (20) depend only on normalized frequency

$$\Omega = Ze^2\omega/mv_{0\|}^3 \equiv \omega/\omega_s(v_{0\|}), \tag{21}$$

i.e., the ratio of ω to the reciprocal time of electron flight near the ion at a distance r_s (2),

$$\omega_s(v_{0\|}) = mv_{0\|}^3/Ze^2. \tag{22}$$

Under conditions (3) considered here, frequency ω_s falls into the range (7) under investigation. Hence, we can find the limiting values of intensity (20) at low frequencies,

$$\omega \ll \omega_s(v_{0\|}) \ll \omega_B \tag{23}$$

and at high frequencies,

$$\omega_s(v_{0\|}) \ll \omega \ll \omega_B. \tag{24}$$

At low frequencies (23), spectrum $I_{z0}^{(d)}$ is flat (independent of ω):

$$\begin{aligned} I_{z0}^{(d)} &= \frac{2n_iZ^2e^6}{\pi m^2c^3v_{0\|}} \\ &\times \int_0^\infty d\eta\,\eta\left(\int_0^\infty \frac{\zeta\sin\zeta d\zeta}{(\eta^2+\zeta^2)^{3/2}}\right)^2 = \frac{n_iZ^2e^6}{\pi m^2c^3v_{0\|}}. \end{aligned} \tag{25}$$

To derive this relation, we set parameter $\Omega \ll 1$ in expression (20) equal to zero. Approximation $\Omega = 0$ corresponds to the fact that radiation intensity at frequencies (23) is controlled by very distant collisions with impact parameters $p_h \sim v_{0\|}/\omega \gg r_s$, for which the ion perturbs the electron velocity only slightly, and coordinate z is a linear function of time: $z \approx v_{0\|}t$. In expression (25), the integral with respect to ζ is equal to the Macdonald function $K_0(\eta)$ in accordance with formula (2.5.9.11) from [18] and corresponds to the second term in formula (3.58) from [15], in which radiation from distant collisions in zero magnetic field is considered. Resultant integral $\int_0^\infty d\eta\,\eta K_0^2(\eta)$ is equal to 1/2 in accordance with formula (2.16.33.2) from [19].

It should be noted that in zero magnetic field, radiation at frequencies (23) is mainly determined by the change in the direction of the particle velocity due to the Coulomb force component perpendicular to the initial electron velocity. The corresponding radiation is described by the first term in formula (3.58) from [15]. In a strong magnetic field, the action of the transverse component of the Coulomb force is compensated by the Lorentz force associated with azimuthal drift in crossed electric and magnetic fields. As a result, radiation is due only to successive acceleration and deceleration of the electron in the Coulomb field along the z axis. As a result, the Coulomb logarithm does not appear in expression (25).

At high frequencies (24), parameter $\Omega \gg 1$; consequently, we can replace radicals $(1 + 2\Omega/\sqrt{\eta^2+\zeta^2})^{1/2}$ in formula (20) by the approximate expression $(2\Omega/\sqrt{\eta^2+\zeta^2})^{1/2}$. Such a substitution corresponds to the fact that radiation at frequencies (24) is controlled by collisions with impact parameters $p_h \ll r_s$. In these collisions, the Coulomb field strongly accelerates the electron so that its velocity near the ion is almost independent of its initial value. Expression (20) for the spectral power assumes the form

$$\begin{aligned} I_{z0}^{(d)} &= \frac{2n_iZ^2e^6}{\pi m^2c^3v_{0\|}}(2\Omega)^{-2/3} \\ &\times \int_0^\infty d\bar\eta\,\bar\eta\left(\int_0^\infty \frac{\bar\zeta\sin\left[\int_0^{\bar\zeta}(\bar\eta^2+\bar\zeta'^2)^{1/4}d\bar\zeta'\right]d\bar\zeta}{(\bar\eta^2+\bar\zeta^2)^{5/4}}\right)^2 \\ &= 0.41\frac{n_iZ^2e^6v_{0\|}}{\pi m^2c^3(\omega Ze^2/m)^{2/3}}, \end{aligned} \tag{26}$$

where the dimensionless integration variables are given by

$$\begin{aligned} \bar\eta &= (2\Omega)^{-1/3}\eta = p_h\omega^{2/3}/(2Ze^2/m)^{1/3}, \\ \bar\zeta &= (2\Omega)^{-1/3}\zeta = z\omega^{2/3}/(2Ze^2/m)^{1/3}. \end{aligned} \tag{27}$$

It should be noted that in zero magnetic field, the radiation spectrum would attain a constant value at high frequencies (24) (see formula (70.22) from [20]). This spectrum is controlled by hyperbolic trajectories in which the electron approaches the ion to much shorter distances as compared to the impact parameter. Under given conditions (3), the magnetic field prevents the displacement of the electron across the z axis and the electron does not approach the ion to a

[1] The change in integration limit η_{cr} can also be interpreted as the "inclusion" of close collision in calculations as if the electron passes by the ion only once without changing the direction of its velocity and without multiple returns to the scattering center. However, the intensity of radiation emitted during close collisions becomes on the order of and higher than the intensity of radiation from distant collisions at corresponding frequencies due to multiple returns of the electron to the ion (see Section 4.4). Thus, the change in integration limit η_{cr} does not affect the accuracy of calculation of radiation from distant as well as close collisions.

distance shorter than the impact parameter. As a result, spectral power (26) decreases with increasing frequency.

4. CLOSE COLLISIONS

4.1. Division of Trajectory

In close collisions with impact parameters $p_h < p_{cr}$ (4), an electron multiply returns to an ion and its motion is chaotic [2, 4–7]. Plane $z = 0$ "cuts" the trajectory of the particle into the initial and final segments of its arrival at and departure from the ion and into segments of bound motion from plane $z = 0$ to turning points (at which $\dot{z} = 0$) and back to plane $z = 0$. We will refer to the segments of bound motion as "loops."[2]

We divide Fourier harmonic F_z (11) into the sum of integrals over the above segments of the trajectory. Integrands $\ddot{z}$ at the initial and final segments can be represented in the form

$$\ddot{z} = \frac{d(\dot{z} - v_{0\|})}{dt}, \quad \ddot{z} = \frac{d(\dot{z} - v_{f\|})}{dt},$$

respectively, where constants $v_{0\|}$ and $v_{f\|}$ are the projections of the initial and final electron velocities onto the magnetic field direction. The integrals over the initial and final segments will be evaluated by parts, while the integrals over the loops will be taken twice,

$$\begin{aligned} F_z = &- v_{0\|}\exp(i\omega t_0) - i\omega\int_{-\infty}^{t_0}(\dot{z} - v_{0\|})\exp(i\omega t)dt \\ &- \sum_{q=1}^{N}\omega^2\int_{t_{q-1}}^{t_q} z\exp(i\omega t)dt \\ &- i\omega\int_{t_N}^{\infty}(\dot{z} - v_{f\|})\exp(i\omega t)dt + v_{f\|}\exp(i\omega t_N), \end{aligned} \tag{28}$$

where t_q are the instants of intersection of plane $z = 0$ ($q = 0, 1, 2, \ldots, N$) and N is the total number of loops. It should be noted that integrands $\dot{z} - v_{0\|}$ and $\dot{z} - v_{f\|}$ in Eq. (28) tend to zero for $|t| \longrightarrow \infty$ so that the integrals over the initial and final segments converge.

Further, let us suppose than an electron impinges on an ion from half-space $z < 0$ and $v_{0\|} > 0$. Then coordinate z is positive on odd (first, second, etc.) loops and negative on even loops. Velocity $\dot{z}$ is positive on the final segment for an even N and negative for an odd N. For this reason, we can explicitly single out the signs of integrals in Eq. (28):

$$\begin{aligned} F_z = &\, g_i\exp(i\omega t_0) + \sum_{q=1}^{N}(-1)^q h_q\exp(i\omega t_q) \\ &+ (-1)^{N+1}g_f\exp(i\omega t_N), \end{aligned} \tag{29}$$

where

$$g_i = -|v_{0\|}| - i\omega\int_0^{\infty}|\dot{z}(t_0 - \tau) - v_{0\|}|e^{-i\omega\tau}d\tau, \tag{30}$$

$$g_f = -|v_{f\|}| + i\omega\int_0^{\infty}|\dot{z}(t_N + \tau) - v_{f\|}|e^{i\omega\tau}d\tau, \tag{31}$$

$$h_q = \omega^2\int_0^{t_q - t_{q-1}}|z(t_q - \tau)|e^{-i\omega\tau}d\tau, \quad q = 1, 2, \ldots, N. \tag{32}$$

The contribution to power I_{z0} (17) from close collisions assumes the form

$$\begin{aligned} I_{z0}^{(c)} = \frac{n_i e^2 v_{0\|}}{(2\pi)^2 c^3}\Bigg[&\mathcal{I}\{|g_i^2|\} + \mathcal{I}\{|g_f^2|\} + \sum_{q=1}^{\infty}\mathcal{I}_{N \ge q}\{|h_q^2|\} \\ &+ 2\mathrm{Re}\sum_{r=0}^{\infty}(-1)^{r+1}\mathcal{I}_{N=r}\{g_i g_f^*\exp[i\omega(t_0 - t_N)]\} \\ &+ 2\mathrm{Re}\sum_{r=0}^{\infty}(-1)^{r+1}\mathcal{I}_{N>r}\{g_i h_{1+r}^*\exp[i\omega(t_0 - t_{1+r})]\} \\ &+ 2\mathrm{Re}\sum_{r=0}^{\infty}(-1)^{r+1}\mathcal{I}_{N>r}\{g_f h_{N-r}^*\exp[i\omega(t_N - t_{N-r})]\} \\ &+ 2\mathrm{Re}\sum_{r=0}^{\infty}(-1)^{r+1} \\ &\times\sum_{q=1}^{\infty}\mathcal{I}_{N>q+r}\{h_q h_{q+1+r}^*\exp[i\omega(t_q - t_{q+1+r})]\}\Bigg]. \end{aligned} \tag{33}$$

In this relation, we interchange integration with respect to impact parameters and summation over segments of the trajectory and regroup the terms so that summation index $r \ge 0$ denotes the number of loops connecting two segments of the trajectory corresponding to the product in the braces. As a result, $\mathcal{I}$ denotes the integral of the quantity in braces over the range of impact parameters p_h and φ_h, in which the expression being integrated has sense. This domain is specified by the condition for N indicated in the subscript on $\mathcal{I}$ and reflects the presence of corresponding segments connected by r loops. Notation $\mathcal{I}$ without a subscript indicates an integral with respect to all impact parameters of close collisions, $p_h \le p_{cr}$, and $0 \le \varphi_h < 2\pi$. In the expansion of $|F_z^2|$ (29), interference terms of type

[2] A typical example of a trajectory with multiple loops is given in Figs. 2 and 3 from [7].

$h_q h^*_{q+1+r} \exp[i\omega(t_q - t_{q+1+r})]$ appear paired with a complex conjugate quantity, which explains coefficient "2" of the sums and the fact that we take the real parts of these sums.

4.2. Summation of Spectra from Different Parts of Trajectories

The alternating form of the terms in the interference sums over r in expression (33) allows us to estimate these sums as their first terms multiplied by coefficients on the order of unity. Indeed, if the terms sharply decrease upon an increase in r by unity, it is sufficient to take only the first term. If, however, the corresponding quantities vary insignificantly upon a change in r, we can use the identity

$$\left(\sum_{r'=0}^{\infty}(-1)^{r'}c_{r'}\right) - \frac{c_0}{2} = \sum_{r'=0}^{\infty}(-1)^{r'}\frac{c_{r'} - c_{r'+1}}{2}$$

for an arbitrary sum $\sum_{r'=0}^{\infty}(-1)^{r'}c_{r'}$ being estimated. The terms in the sum on the right-hand side are much smaller in absolute value than the terms of the sum on the left-hand side; consequently, $\sum_{r'=0}^{\infty}(-1)^{r'}c_{r'} \approx c_0/2$. Thus, expression (33) for the sought power assumes the form

$$\begin{aligned} I^{(c)}_{z0} &= \frac{n_i e^2 v_{0\parallel}}{(2\pi)^2 c^3} \\ &\times \Bigg[\mathcal{I}\{|g_i^2|\} + \mathcal{I}\{|g_f^2|\} + \sum_{q=1}^{\infty}\mathcal{I}_{N\ge q}\{|h_q^2|\} \\ &- 2c_{i-f}\mathrm{Re}\,\mathcal{I}_{N=0}\{g_i g_f^*\} \\ &- 2c_{i-l}\mathrm{Re}\,\mathcal{I}_{N>0}\{g_i h_1^* \exp[i\omega(t_0 - t_1)]\} \\ &- 2c_{f-l}\mathrm{Re}\,\mathcal{I}_{N>0}\{g_f h_N^*\} - 2c_{l-l} \\ &\times \mathrm{Re}\sum_{q=1}^{\infty}\mathcal{I}_{N>q}\{h_q h^*_{q+1}\exp[i\omega(t_q - t_{q+1})]\}\Bigg], \end{aligned} \qquad (34)$$

where c_{i-f}, c_{i-l}, c_{f-l}, and c_{l-l} are numerical coefficients on the order of unity. Let us show that the interference terms with coefficients c_{i-f}, c_{i-l}, c_{f-l}, and c_{l-l} at frequencies (7) considered here are smaller than all the remaining terms in relation (34).

For example, the kinetic energy in the vicinity of the ion assumes the characteristic value

$$E_u = Ze^2/L_u = m\omega_B^2 L_u^2, \qquad (35)$$

where L_u is the characteristic impact parameter (5) of close collisions. It is natural to expect that an electron usually makes a loop over a time $T \sim L_u/\sqrt{E_u/m} = \omega_B^{-1}$ (see relation (35)) and that turning points are separated from the ion by a distance on the order of L_u. Then Fourier harmonic $|h_q|$ (32) has a value on the order of $\omega^2 L_u T = \omega^2 L_u \omega_B^{-1}$, and most trajectories with $N \ge q$ make a contribution to term $\mathcal{I}_{N\ge q}\{|h_q^2|\}$ in relation (34) on the order of

$$J_{\text{short}} = (\omega^2 L_u \omega_B^{-1})^2 S,$$

where $S < \pi p_{\text{cr}}^2$ is the "area" of impact parameters of trajectories with $N \ge q$.

At the same time, a few trajectories have a qth loop extended along the magnetic field, in which an electron moves to turning point $z_t \gg L_u$ and back over a time $T \sim \omega^{-1} \gg \omega_B^{-1}$. In these trajectories, Fourier harmonic $|h_q|$ (32) attains its maximal values on the order of $\omega^2 z_t T = \omega z_t$. Distance z_t fixes the kinetic energy of transverse motion at the turning point, $E_\perp = E_0 - Ze^2/z_t$, where $E_0 = m v_0^2/2$. In turn, the value of $E_\perp$ varies only slightly at long distances $|z| \gg L_u$ from the ion, at which the Coulomb field decelerates the electron only along the magnetic field. In our estimates, we assume that the electron velocity after its next passage near the ion can be directed in a wide cone with the opening angle on the order of π and, hence, $E_\perp$ assumes values from 0 to E_u distributed (on the average) uniformly over impact parameters. With such a uniform distribution, the "area" S_{long} of impact parameters of trajectories with a long qth loop, normalized to area S of all trajectories with $N \ge q$, is equal to the ratio of the corresponding energy intervals

$$\frac{E_\perp - E_0}{E_u} = \frac{Ze^2}{z_t}\frac{1}{E_u} = \frac{L_u}{z_t}$$

(see relation (35)). We obtain the following contribution from the trajectories with a long qth loop to term $\mathcal{I}_{N\ge q}\{|h_q^2|\}$ in relation (34):

$$J_{\text{long}} = (\omega z_t)^2 S_{\text{long}} = \left(\frac{\omega_B}{\omega}\right)^2 \frac{z_t}{L_u} J_{\text{short}} \gg J_{\text{short}}$$

(here, $(\omega_B/\omega)^2 \gg 1$ and $z_t/L_u \gg 1$).

Thus, radiation emitted from the bound segment of a trajectory at frequencies (7) considered here is controlled by sparse long loops.

In this case, we can disregard interference sum $-2c_{l-l}\mathrm{Re}\sum_{q=1}^{\infty}\mathcal{I}_{N>q}\{h_q h^*_{q+1}\exp[i\omega(t_q - t_{q+1})]\}$ in relation (34) as compared to $\sum_{q=1}^{\infty}\mathcal{I}_{N\ge q}\{|h_q^2|\}$. The interference sum could attain values on the order of $\sum_{q=1}^{\infty}\mathcal{I}_{N\ge q}\{|h_q^2|\}$ only if long loops appeared predominantly in coupled pairs (as neighboring segments). However, we can naturally expect that for most trajectories, a long loop is coupled with more probable short loops.

At the same time, we can disregard "interference" terms $-2c_{i-l}\mathrm{Re}\,\mathcal{I}_{N<0}\{g_i h_1^* \exp[i\omega(t_0 - t_1)]\}$ and $-2c_{f-l}\mathrm{Re}\mathcal{I}_{N>0}\{g_f h_N^*\}$ in formula (34) as compared to $\mathcal{I}\{|g_i^2|\} + \mathcal{I}\{|g_f^2|\}$. Indeed, the motion on the initial and final segments can be conditionally represented as the motion over halves of loops infinitely extended along z. Accordingly, harmonics $|g_i|$ and $|g_f|$ are not smaller in order of magnitude than harmonic $|h_q|$ on a single long loop. In this case, the above-mentioned interference terms could attain values on the order of $\mathcal{I}\{|g_i^2|\} + \mathcal{I}\{|g_f^2|\}$ only if the initial and final segments were usually coupled with long loops. We can naturally expect, however, that the initial and final segments are coupled with more probable short loops.

Finally, in expression (34), we can disregard interference term $-2c_{i-f}\mathrm{Re}\,\mathcal{I}_{N=0}\{g_i g_f^*\}$ as compared to $\mathcal{I}\{|g_i^2|\} + \mathcal{I}\{|g_f^2|\}$ since in the definition of the former term, we have to integrate over a small domain of impact parameters of sparse close collisions with $N = 0$, which have no loops.

Thus, we have demonstrated that "interference" of the amplitudes of the radiation spectra from different parts of the trajectory is leveled out as a result of integration over impact parameters. This effect is qualitatively associated with the fact that the main contribution to radiation at frequencies (7) considered here comes from the initial and final segments of the trajectory and from sparse long loops. These key segments are connected by numerous short loops. Consequently, if we take a pair of key segments, we can naturally expect (on the average over impact parameters) that the numbers of trajectories on which the two selected key segments lie on the same side and on different sides of plane $z = 0$ are approximately equal. Trajectories with opposite configurations of segments in a pair give interference terms with opposite signs, which are mutually compensated in integration with respect to impact parameters.

Consequently, the spectral power of radiation (33) and (34) from close collisions is controlled by the sum of "incoherent" terms:

$$I_{z0}^{(c)} = \frac{n_i e^2 v_{0\|}}{(2\pi)^2 c^3} \times \left[\mathcal{I}\{|g_i^2|\} + \mathcal{I}\{|g_f^2|\} + \sum_{q=1}^{\infty} \mathcal{I}_{N\geq q}\{|h_q^2|\}\right]. \tag{36}$$

Let us estimate these terms and retain only those that can be on the order of or larger than the radiation power (25) and (26) from distant collisions.

4.3. Radiation at the Initial and Final Segments of Trajectories

In Fourier harmonic g_i (30), the contribution to the integral mainly comes from region $|z| \lesssim r_s$ (2), in which velocity $\dot{z}$ can be approximated by the expression

$$|\dot{z}| \approx \sqrt{\frac{2Ze^2}{m|z|}} \gg v_{0\|}.$$

Let us pass in expression (30) from integration with respect to time to integration over $dz = \dot{z}\,d\tau$; using formula (2.5.5.3) from [18], we obtain the following estimate:

$$\begin{aligned} g_i &\approx -|v_{0\|}| - i\omega\int_0^{\infty} dz \exp\left[-i\int_0^z \frac{\omega dz'}{\sqrt{2Ze^2/mz'}}\right] \\ &= -|v_{0\|}| - \frac{(\omega Ze^2/m)^{1/3}(\sqrt{3}+i)}{6^{1/3}/\Gamma(2/3)}. \end{aligned} \tag{37}$$

An analogous estimate can be obtained for Fourier harmonic (31):

$$g_f \approx -|v_{0\|}| - \frac{(\omega Ze^2/m)^{1/3}(\sqrt{3}-i)}{6^{1/3}/\Gamma(2/3)}. \tag{38}$$

At low frequencies (23), the first terms dominate in harmonics (37) and (38): $|g_i^2| \approx v_{0\|}^2$ and $|g_f^2| \approx v_{f\|}^2$. In the definition of $\mathcal{I}\{|g_i^2|\}$, integration with respect to impact parameters can be reduced to multiplication of $|g_i^2|$ by πp_{cr}^2, where quantity p_{cr} is defined in (4). Determining $\mathcal{I}\{|g_f^2|\}$, we replace analogous integration by integration with respect to $v_{f\|}^2$ using the differential cross section [3]: $2\pi p_h dp_h = \pi p_{\mathrm{cr}}^2 d(v_{f\|}^2)/v_0^2$. We obtain the sought estimate for radiation power in (36) from the initial and final segments of trajectories at low frequencies (23):[3]

$$\begin{aligned} I_{z0}^{(i+f)} &= \frac{n_i e^2 v_{0\|}}{(2\pi)^2 c^3}[\mathcal{I}\{|g_i^2|\} + \mathcal{I}\{|g_f^2|\}] \\ &= \frac{n_i e^2 p_{\mathrm{cr}}^2 v_{0\|}}{4\pi c^3}\left(v_{0\|}^2 + \frac{v_0^2}{2}\right) \sim \frac{n_i e^2 L_u^2 v_0^3}{\pi c^3}. \end{aligned} \tag{39}$$

This quantity is smaller than radiation power (25) from distant collisions by approximately a factor of $r_s^2/L_u^2 = (r_s/r_B)^{4/3} \gg 1$ (see relation (5)).

At high frequencies (24), the second terms dominate in expressions (37) and (38), and the radiation

[3] We would have obtained precisely expression (39) for sought power (17) if we initially confined our analysis to the instantaneous collision approximation.

power at the initial and final segments of trajectories can be estimated as

$$I_{z0}^{(i+f)} = \frac{n_i e^2 p_{\rm cr}^2 (\omega Z e^2/m)^{2/3} v_{0\parallel}}{(9/2)^{1/3} \pi c^3/\Gamma^2(2/3)}$$
$$= \frac{2.1 n_i Z^2 e^6 v_{0\parallel}}{\pi m^2 c^3 (\omega Z e^2/m)^{2/3}} \left(\frac{\omega}{\omega_B}\right)^{4/3} \ln^{4/3}\left[\left(\frac{r_s}{r_B}\right)^{1/3}\right]. \quad (40)$$

This quantity attains values of radiation power (26) from distant collisions only at frequencies on the order of ω_B outside range (7) under investigation.

Thus, radiation at the initial and final segments is insignificant for calculating the sought radiation power (17). It remains for us to consider radiation from loops.

4.4. Radiation at a Bound Segment of Trajectories

The main contribution to Fourier harmonic h_q (32) on a long loop accumulates at a long distance from the ion, at which $|z| \gg L_u$. Here, z is an even function of time τ' measured from instant $t_{tq} \approx (t_q + t_{q-1})/2$ of passage through the turning point. Integral (32) can be reduced to the double integral over a half of the loop, which we take once by parts:

$$h_q \approx 2\omega \exp[-i\omega(t_q - t_{q-1})/2]$$
$$\times \int_0^{(t_q - t_{q-1})/2} d\tau' |\dot{z}(t_{tq} - \tau')| \sin\omega\tau'. \quad (41)$$

In domain $|z| \gg L_u$, the motion of the electron is equivalent to the Kepler motion along an ellipse strongly extended along the z axis. We carry out the change of variables [Section 70 in [20])

$$|z| = z_{tq}(1 + \cos w)/2, \quad (42)$$

where $z_{tq} \approx Ze^2/(\mathcal{E}_{\perp q} - E_0)$ is the distance between the qth turning point and the ion, $\mathcal{E}_{\perp q} > E_0$ is the kinetic energy of the transverse motion at the turning point, and $E_0 = m v_0^2/2$. Velocity

$$|\dot{z}| = \sqrt{\frac{2Ze^2}{m z_{tq}}} \tan\frac{w}{2} \quad (43)$$

can be obtained by substituting relation (42) into the energy conservation law for the "one-dimensional" Coulomb potential: $m|\dot{z}|^2/2 - Ze^2/|z| = -Ze^2/z_{tq}$. Determining time τ' in Eq. (41) and substituting relations (42)–(44) into (41),

$$\tau' = \int_{|z|}^{z_{tq}} \frac{d|z'|}{|\dot{z}'|} = \frac{(z_{tq}/2)^{3/2}}{\sqrt{Ze^2/m}}(w + \sin w), \quad (44)$$

we find that harmonic

$$|h_q| \equiv \mathfrak{h}(\mathcal{E}_{\perp q}) = 2\left(\frac{\omega Z e^2}{m}\right)^{1/3}$$
$$\times \bar{\zeta}_t \int_0^{\pi} \sin[\bar{\zeta}_t^{3/2}(w + \sin w)] \sin w dw \quad (45)$$

is controlled by energy $\mathcal{E}_{\perp q}(p_h, \varphi_h)$ which, in turn, is fixed uniquely by the normalized remoteness of the turning point:

$$\bar{\zeta}_t = \frac{z_{tq}\omega^{2/3}}{2(Ze^2/m)^{1/3}} = \frac{(\omega Z e^2/m)^{2/3}}{2(\mathcal{E}_{\perp q} - E_0)/m}. \quad (46)$$

In this case, to calculate $\sum_{q=1}^{\infty} \mathcal{I}_{N \ge q}\{|h_q^2|\}$ in relation (36), we pass from integration with respect to impact parameters to integration with respect to $\mathcal{E}_{\perp q}$. For this purpose, we change the order of summation over loop numbers q and integration with respect to p_h and φ_h:

$$\sum_{q=1}^{\infty} \mathcal{I}_{N \ge q}\{|h_q^2|\} = \int_0^{2\pi} d\varphi_h \int_0^{\infty} dp_h p_h$$
$$\times \sum_{q=1}^{N(p_h,\varphi_h)} |\mathfrak{h}(\mathcal{E}_{\perp q}(p_h, \varphi_h))|^2. \quad (47)$$

We substitute this expression into the identity (47)

$$\sum_{q=1}^{N(p_h,\varphi_h)} |\mathfrak{h}(\mathcal{E}_{\perp q}(p_h, \varphi_h))|^2 = \int_0^{\infty} dE_\perp |\mathfrak{h}(E_\perp)|^2$$
$$\times \sum_{q=1}^{N(p_h,\varphi_h)} \delta[E_\perp - \mathcal{E}_{\perp q}(p_h, \varphi_h)],$$

where δ is the delta function, and change the order of integration with respect to $E_\perp$ and p_h, φ_h:

$$\sum_{q=1}^{\infty} \mathcal{I}_{N \ge q}\{|h_q^2|\} = \int_0^{\infty} dE_\perp |\mathfrak{h}(E_\perp)|^2 \frac{d\sigma}{dE_\perp}. \quad (48)$$

We conditionally refer to quantity

$$\frac{d\sigma}{dE_\perp} = \int_0^{2\pi} d\varphi_h \int_0^{\infty} dp_h p_h$$
$$\times \sum_{q=1}^{N(p_h,\varphi_h)} \delta[E_\perp - \mathcal{E}_{\perp q}(p_h, \varphi_h)] \quad (49)$$

as the scattering cross section. Indeed, let us suppose that function $\delta(K)$ differs from zero only on a small interval of its argument $0 < K < \Delta E_\perp$ and is equal to $1/\Delta E_\perp$ on this interval. Then quantity (49) acquires the meaning of the "area" of the impact parameters, from which the electron reaches any turning point with a kinetic energy of transverse motion in the interval from $E_\perp$ to $E_\perp + \Delta E_\perp$, divided by $\Delta E_\perp$.

Cross section (49) differs from zero only for $E_\perp > E_0$ since energy $\mathcal{E}_{\perp q}(p_h, \varphi_h) > E_0 = m v_0^2/2$ at the turning point. Let us extend definition (49) to domain $E_\perp \le E_0$. For this purpose, we complete the definition of $\mathcal{E}_{\perp q}(p_h, \varphi_h)$ as the kinetic energy of transverse motion at the point of maximum distance between the electron and the ion after the qth intersection of plane $z = 0$ (i.e., at the turning point or for $|z| = \infty$). Accordingly, we include the last passage of the electron by the ion into sum (49),

$$\frac{d\sigma}{dE_\perp} = \int_0^{2\pi} d\varphi_h \int_0^\infty dp_h p_h \times \sum_{q=1}^{N(p_h,\varphi_h)+1} \delta[E_\perp - \mathcal{E}_{\perp q}(p_h, \varphi_h)]. \tag{50}$$

In domain $E_\perp > E_0$, quantity (50) preserves the value and the meaning of quantity (49) and coincides with the conventional cross section of scattering to the final state with energy $E_\perp$ of transverse motion per unit energy interval in domain $E_\perp \le E_0$.

Energy $\mathcal{E}_{\perp q}(p_h, \varphi_h)$ assumes values in a wider range $0 \le E_\perp \lesssim E_u$ (35) as compared to $E_0 = m v_0^2/2$. In turn, dependence of cross section (50) on $E_\perp$ is defined by the properties of function $\mathcal{E}_{\perp q}(p_h, \varphi_h)$ and, hence, changes significantly only for $E_\perp \sim E_u$. If we know the conventional cross section for states with energy $E_\perp \le E_0$, its extrapolation to domain $E_\perp > E_0$ must correctly describe cross section (50) in the energy interval $E_0 < E_\perp \ll E_u$, which corresponds exactly to long loops. For the differential cross section proposed in [3], we have

$$d\sigma/dE_\perp = \sigma_0/E_0, \tag{51}$$

where $\sigma_0 = \pi p_{\rm cr}^2$ is the total cross section of close collisions and $p_{\rm cr}$ is defined by relation (4).[4] In the quantum-mechanical interpretation, cross section (51) corresponds to equiprobable arrival of the electron after a collision at an arbitrary Landau level with energy $E_\perp \le E_0$. Extrapolation of relation (51) to domain $E_\perp > E_0$ corresponds to the assumption of equiprobable arrival of the electron at higher Landau levels ("trapped" by the ion) also after an arbitrary passage near the ion.

Substituting Fourier harmonic (45) and cross section (51) into expression (48), we obtain the required power of radiation emitted by an electron moving over a bound segment of trajectories,

$$\begin{aligned} I_{z0}^{(l)} &= \frac{n_i e^2 \sigma_0 v_{0\parallel}}{(2\pi)^2 c^3 E_0} \int_{E_0}^\infty dE_\perp |\mathfrak{h}(E_\perp)|^2 \\ &= \frac{n_i e^2 p_{\rm cr}^2 v_{0\parallel}}{\pi c^3 v_0^2} \left(\frac{\omega Z e^2}{m}\right)^{4/3} \\ &\times \int_0^\infty d\bar\zeta_t \left(\int_0^\pi \sin[\bar\zeta_t^{3/2}(w + \sin w)] \sin w dw \right)^2 \\ &= \frac{4.78 n_i Z^2 e^6}{\pi m^2 c^3 v_0} \frac{v_{0\parallel}}{v_0} \left(\frac{\omega}{\omega_B}\right)^{4/3} \ln^{4/3}\left[\left(\frac{r_s}{r_B}\right)^{1/3}\right], \end{aligned} \tag{52}$$

where integration with respect to $E_\perp$ is transformed into integration with respect to $\bar\zeta_t$ in accordance with (46). Spectral power (52) exceeds power (26) of radiation from distant collisions at frequencies higher than

$$\begin{aligned} \omega_i &= 0.29\omega_s(v_0) \frac{(r_s/r_B)^{2/3}}{\ln^{2/3}[(r_s/r_B)^{1/3}]} \\ &\equiv \frac{0.29\omega_B}{(r_s/r_B)^{1/3} \ln^{2/3}[(r_s/r_B)^{1/3}]}, \end{aligned} \tag{53}$$

where quantity $\omega_s(v_0) = m v_0^3/Ze^2$ is defined in (22). Frequency (53) falls in the interval of frequencies (7) considered here and corresponds to high frequencies (24).

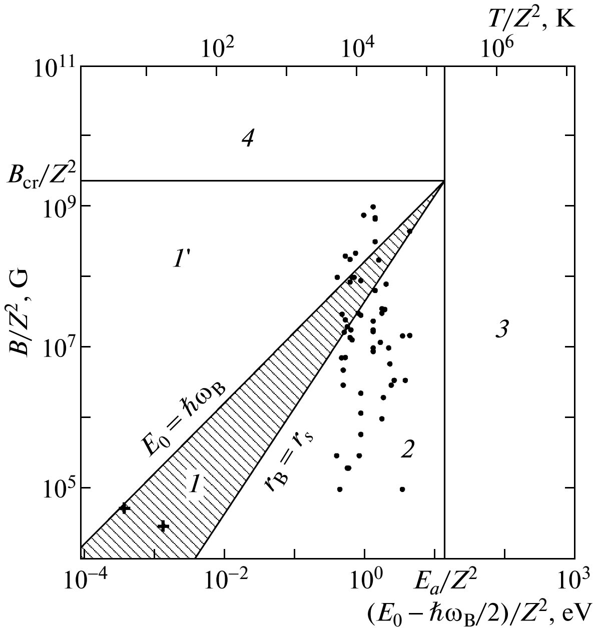
5. DISCUSSION OF BREMSSTRAHLUNG AND CYCLOTRON RADIATION

Under conditions (3) considered here, the Lorentz force exceeds the Coulomb interaction force over the entire path length of distant collisions ($p_h \gg L_u$) and far away from the ion at characteristic distances $r > L_u$ (5) for close collisions. Accordingly, the power of spontaneous cyclotron radiation from electrons exceeds the bremsstrahlung power integrated over frequencies in range (7) under investigation.

Consequently, if the optical thickness of a plasma source is smaller than unity at all frequencies, reabsorption is insignificant and the frequency spectrum of observed radiation is proportional to the spectrum of spontaneous radiation from particles. The radiation power from such a source in the cyclotron line is much higher than the bremsstrahlung power at frequencies (7) considered here. Accordingly, if the cyclotron line falls in the transmission frequency band of a detector, the signal being detected is controlled by cyclotron radiation and the contribution from bremsstrahlung is insignificant.

Conversely, if the optical thickness of the source in the cyclotron line is much greater than unity, the spectral intensity of radiation in this line is limited by reabsorption and is approximately equal to the intensity of blackbody radiation at a temperature equal to the elec-

[4] Cross section (51) recalculated to the cosine of the pitch angle of the final velocity ($\cos\theta_f = v_{f\parallel}/v_0$) assumes the form $d\sigma/d(\cos\theta_f) = \pi p_{\rm cr}^2 |\cos\theta_f| \propto |\cos\theta_f|$, which is in qualitative agreement with the results of numerical calculations (see Fig. 4 in [5]).



Limiting versions of electron–ion collisions in a magnetic field. Circles denote magnetic fields B and thermal energies $E_0 - \hbar\omega_B/2 = k_BT$ in the photospheres of isolated magnetic white dwarfs [10]; crosses denote analogous parameters in experiments [11, 12] on antihydrogen; $E_a = Z^2e^4m/2\hbar^2 = 13.6Z^2$ eV is the ground-state energy of a hydrogenlike ion in zero magnetic field, $B_{cr} = Z^2e^3m^2c/\hbar^3 = 2.35 \times 10^9 Z^2$ G is the magnetic field in which energy $\hbar\omega_B/2$ of the Landau ground level is E_a. Relation (3) considered here is observed in regions *1* and *1'* [8, 9]. The classical description of the motion of an electron used here is applicable only in domain *1*.

tron temperature. The optical thickness in the continuum in this case may remain much smaller than unity so that the spectrum of observed radiation is proportional to the calculated spectral power of bremsstrahlung (25), (26), and (52). Let $\Delta\nu_c$ be the cyclotron linewidth and $\Delta\nu_{rec} \gg \Delta\nu_c$ be a wider transmission frequency band of the detector. If the optical thickness of the source in the continuum exceeds a small quantity $\Delta\nu_c/\Delta\nu_{rec} \ll 1$, the signal at the detector output is controlled by wide-band bremsstrahlung even if the cyclotron line falls in the transmission frequency band of the detector.

Thus, the change in optical thickness of the source due to a change in its linear size or plasma density makes it possible in principle to obtain any relation between the observed levels of cyclotron radiation and bremsstrahlung.

For "laboratory" magnetic fields $B \sim 1$ T $= 10^4$ G, relation (3) considered here is observed at a temperature of $T \ll 80$ K. At such a temperature, background thermal radiation from elements of the experimental setup becomes significant. This strongly limits the possibility of measuring bremsstrahlung from the plasma. For example, if the temperatures of free electrons of the plasma and surrounding elements of the setup are identical, the spectrum of observed radiation may become independent of the presence or absence of the plasma and will correspond to the blackbody radiation spectrum.

In the case of isolated magnetic white dwarfs, the emitting volume of the plasma in the form of a photosphere is optically thick at all frequencies. The temperature of the plasma increases towards the center of the star. The brightness temperature of radiation emitted by the photosphere is approximately equal to the temperature of the medium at a depth at which the optical thickness for radiation at a given frequency is on the order of unity. Cyclotron radiation emitted from the photosphere forms at a smaller depth in the plasma with a lower temperature as compared to radiation in the continuum. Consequently, cyclotron radiation could be observed only in absorption against the background of a brighter continuum. However, the dipole magnetic field of the star is nonuniform—its value is halved as we pass from the pole of the star to its equator. As a result, the observed radiation from the entire star at a certain frequency (outside the hydrogen atom lines) is formed by processes in the continuum almost on the entire surface of the star except a narrow range of latitudes, in which the local cyclotron frequency differs from the frequency of observation by less than the Doppler linewidth. Thus, radiation observed in the case of isolated magnetic white dwarfs (outside the hydrogen atom lines) is mainly controlled by processes in the continuum (photorecombination and bremsstrahlung), while the contribution from cyclotron radiation is small.

6. CONCLUSIONS

We have analyzed bremsstrahlung from an electron colliding with ions in a magnetic field in the case when the Larmor radius (1) of the electron is smaller than characteristic impact parameter (2) of close collisions in zero magnetic field. At frequencies (7) considered here (below electron cyclotron frequency ω_B), the radiation pattern approximately corresponds to that of a dipole oriented along the magnetic field (radiation is polarized predominantly linearly in the plane of the magnetic field and the direction of radiation). Spectral power I_{z0} of radiation in the direction perpendicular to the magnetic field is described by the sum of the radiation powers (20) and (52) from distant and close collisions. At low frequencies (23), the spectrum is flat (25) and coincides in order of magnitude with the radiation spectrum of an electron beam in zero magnetic field. At frequencies $\omega_s(v_{0\|}) \ll \omega \ll \omega_i$, the spectral power decreases in proportion to $\omega^{-2/3}$ (26) and attains its minimal value at $\omega \sim \omega_i$ (53). At frequencies $\omega_i \ll \omega \ll \omega_B$, the radiation spectrum is controlled by quasi-bound motion of the electron in close collisions. The radiation power increases as $\omega^{4/3}$ (52) and returns to the characteristic radiation power in zero magnetic field at frequencies on the order of ω_B (outside the line).

In our calculations, we assumed that the motion of an electron is classic, which is valid for particles with energy $E_0 = m v_0^2/2 > \hbar\omega_B$. The figure shows the classical range *1* of energies and magnetic fields in which relation (3) between the spatial scales considered here holds and our results are applicable. At the same time, electron energy E_0 in this region is higher than the energy of emitted light quanta with frequencies (7); consequently, emission of radiation is not accompanied by photorecombination. In classical region *2* of weaker magnetic fields, bremsstrahlung from thermal particles can be obtained using the Kirchhoff law from the absorption coefficients [21], while in quantum region *3* of high-energy electrons, it can be calculated using the results obtained in [17]. For quantum regions *1'* and *4* corresponding to stronger magnetic fields, further investigation of bremsstrahlung is required.

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