The structure of the field at the mirrors (TEM_{q02} mode) is evident from Figs. 2a, 2b. At mirror 1, where the beams converge, we see an interference pattern in the form of broad vertical bands, and at mirror 2, the "traces" of the two individual beams. Introduction of losses into one of the beams inside the resonator disrupted emission; this indicates the presence of a single oscillatory process forming an "angular" emission structure.

When one of the spherical mirrors is replaced by a plane one, it is possible to excite a double-beam mode or "angle," with its apex at the spherical mirror (see Fig. 1d). In this case, from the plane mirror side we get two beams (with a certain structure), whose axes are strictly parallel and about 10 mm apart. At a distance of several meters from the mirror the beams give a corresponding distinct interference pattern.

In our experiments we excited a single emission structure of up to eight beams by means of a plate and up to four beams by means of a piece of wire. At the same time, we observed the generation of independent "partial" beam systems similar to those described in [1]: a direct beam and an "angle" (see Fig. 1a, where the direct beam is indicated by the broken line), two "angles" in different planes, etc. All the beams of each "partial" mode lay in the same plane. Upon excitation of a single system of beams, the plane in which they lay was in most cases perpendicular to the edge of the taper.

Figures 1b, 1c show the paths of the beams in a laser for four- and three-beam modes, respectively. Figure 2c shows intensity distribution for simultaneous generation of an "angular" structure (TEM_{q01}) and a direct beam (TEM_{q01} with degeneracy – "doughnut") at the surface of mirror 1 (see Fig. 1a); Fig. 2d shows the intensity distribution at mirror 2 for simultaneous generation of an "angular" structure (TEM_{q00}) and a direct beam (TEM_{q23}).

In our experiments the multibeam regimes were stable with respect to change in the dimensions of the resonator. Thus, its length d (and the ratio d/r, where r is the radius of curvature of the mirrors) can be varied within wide limits when the field is disturbed with a tapered plate. Multibeam conditions were observed for $0.3 \leqslant d/r \leqslant 2$; in this case the position of the interfering plate affected only the angle between the beams.

Upon excitation of a single n-beam field distribution in the resonator the frequencies of the nearest longitudinal modes should differ by $\Delta\nu = c/2nd$. We verified this by observing from the output of a photodiode the oscillations of the difference frequency – the result of the beats of the longitudinal modes of the laser emission.

We also simultaneously observed the beats, very close in frequency, of adjacent modes in the direct beam and the beats of corresponding modes in "angle" emission. It was found that the fluctuations in the frequency and amplitude of these beats are not correlated. Moreover, within the limits of accuracy of our experiments, overlapping of one of the beams (inside the resonator) had no effect on the frequency of the beat signal, the transverse intensity distribution, or the output power of the other mode. Note that in [1] in the case of a confocal resonator different beam systems were observed to interact.

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EXCITATION PARAMETER OF A BEAM MASER

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In designing a maser based on some quantum transition it is important to make a preliminary estimate of the maximum attainable excitation parameter η_{max} (the condition $\eta \geq 1$ defines the self-excitation regime). The characteristics of the resonator and the working levels of the molecule entering into the expression for η are usually known with a good degree of accuracy, and the problem of finding η_{max} largely reduces to finding the maximum number of active molecules. In [1] it was noted that the maximum beam density is limited by collisions and that the microwave collision diameter is greater than the gas-kinetic. Using an ammonia generator, the authors of [2, 3] experimentally obtained dependences of the amplitude of the emission on the pressure in the beam source, illustrating disruption of the beam due to collisions. However, they did not take the collisions quantitatively into account. Here we shall attempt to make a quantitative estimate of the influence of collisions in the beam; the results obtained are compared with the data of an experi-

ment on a formaldehyde maser. The type of dependence of η on the molecular flux and the length of the sorting system, with collisions taken into consideration, is also found.

Consider a parallel molecular flux N_0 (sec^{-1}) impinging on a sorting system of length L and radius R, half of the molecules being at the upper levels and half at the lower levels. The parallel beam is sorted as follows: active molecules entering the sorting system fly inside the system, but passive molecules are thrown out by the field. We are interested in the difference in the fluxes of active and passive molecules at the working levels N_a^* at a distance l from the sorting system (i.e., in the resonator) as a function of the molecular flux at the input of the system and its length.

The decrease in the flux of both active and passive molecules, at one of the working levels, during passage through a layer dz will be $dN = -Nd\nu$, where $d\nu$ is the number of collisions that a single molecule experiences in traversing the layer dz. The number of collisions per unit time $\nu = \left(\frac{\sqrt{2}}{b}\right)\bar{v}\sigma n(z)$, where $\bar{v}$ is the mean velocity in the gas, σ is the collision cross section, n(z) is the density of the molecules, and b = 1 for the gas and b = 3 for a unidirectional beam [4]. We shall take b = 2. Hence the number of collisions in the transit time $dt = dz/v$, $d\nu = \nu dt = \frac{\sigma n(z)}{\sqrt{2}}\,dz$. In turn, n(z) = $= N_0(z)/Sv$, where $N_0(z)$ is the total molecular flux at the section of the sorting system in question, and S is the cross-sectional area of the system. Finally,

$$dN = -N\frac{\sigma N_0(z)}{\sqrt{2}\,Sv}\,dz.$$

Integrating with respect to z from 0 to L + 1, at the output we get

$$N = N(0)\exp\left[-\frac{\sigma}{\sqrt{2}\,Sv}\int_0^{L+l} N_0(z)\,dz\right]. \qquad (1)$$

Note that in radiospectroscopy, where the quantum energy $\hbar\omega \ll kT$, the cross section of collisions in which the quantum state of the molecule changes but the trajectory does not is greater than the gas-kinetic by about one order [5]. Therefore in the estimates σ should be taken from radiospectroscopic data and the gas-kinetic collisions in the beam neglected. The quantity $\int_0^{L+l} N_0(z)\,dz$ is determined by the choice of passive molecule scattering mechanism. For instantaneous scattering the integral assumes the value $(1/2)N_0(L + l)$, and in the absence of scattering the value $N_0(L + l)$. For estimates the value of the integral may be assumed to be $(3/4)N_0(L + l)$.

At the input the number of active molecules at the working level is $N_a(0) = \delta N_0$, where δ is the population of the level. Consider the case where a deflecting force proportional to the distance from the axis of the sorting system $F = \varkappa r$ acts on the passive molecules of the working level. A number of real cases are closely approximated by this law. Under the action of the linear force a passive molecule traveling through the sorting system moves according to the law $r = r_0 \operatorname{ch}\Omega z$, where $\Omega = (1/v)\sqrt{\varkappa/m}$, v and m are the velocity and mass of the molecule. The output of the sorting system can be reached only by those passive molecules whose trajectories lie within a certain limiting trajectory defined by the condition $r_0\operatorname{ch}(\Omega L) = R$. Their flux at the input $N_n(0) = \delta N_0/\operatorname{ch}^2(\Omega L)$. For the difference in the fluxes of active and passive molecules, taking (1) into consideration, we get

$$N_a^* = \delta N_0 th^2\Omega L\,\exp\left[-\frac{3\sigma N_0(L+l)}{4\sqrt{2}\,Sv}\right].$$

The optimum values of L and N_0 (for which N_a^* is maximum) are found from the equations $\partial N_a^*/\partial N_0 = 0$ and $\partial N_a^*/\partial L = 0$ or

$$N_0 = \frac{1}{A(L+l)};$$

$$\Omega = AN_0\operatorname{sh}(2\Omega L),$$

where

$$A = 3\sigma/4\sqrt{2}\,Sv.$$

The principal characteristics of the formaldehyde transitions investigated are presented in [6, 7]. For the transition 1_{01}-0_{00}, given a voltage on the sorting system of ~8 kV (in this case the deflecting force is almost linear) and T = 300°K, the value of Ω is 0.4 cm^{-1}. Note that when the voltage on the sorting system is increased above 8 kV the excitation parameter, as is evident from the experimental curve in Fig. 1, does not change in view of the nature of the Stark effect

of the given transition [6]. For the transition 5_{14}-5_{15} the linearity of the force is preserved up to the breakdown voltage (~ 18 kV). In this case $\Omega = 0.19$ cm^{-1}.

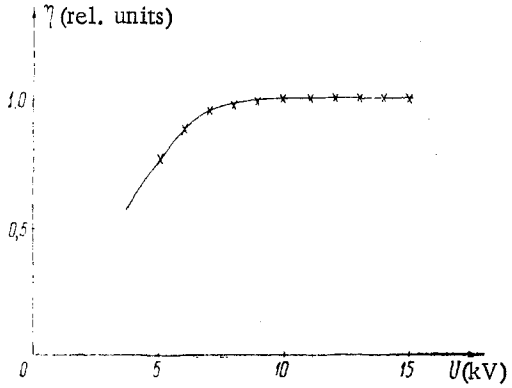

Fig. 1. Excitation parameter as a function of the sorting voltage.

Experiments were conducted on a beam maser with the 1_{01}-0_{00} and 5_{14}-5_{15} transitions of formaldehyde ($\lambda = 4$ mm). The resonators (E_{010} mode) had a length $l = 3$ cm and quality factors of 3500 and 2500, respectively. The inside radius of the quadrupole sorting system was 0.1 cm. A source 0.6 mm in diameter was placed sufficiently far away from the sorting system (~ 1.7 cm). To increase the excitation parameter, the source was cooled to about 216°K.

The population δ is proportional to $T^{-3/2}$, $\tau^2 \sim T^{-1}$ and, in general $\eta \sim T^{-5/2}$. The method previously in [9] was used to measure η. The calculated dependence $\eta(T)$ and the experimental points are shown in Fig. 2.

The dependences $\eta(L)$ and $\eta(N_0)$ were recorded for a generator without diaphragms, for the transition 1_{01}-0_{00}. A set of quadrupole sorting systems of identical design but different length L was employed. The voltage on the sorting systems was kept at ~13 kV. In each case the optimum beam was selected. In measuring values of the excitation parameter greater than unity, we heated the source until emission ceased and found the value of η for 216°K from the graph of $\eta(T)$. Figure 3 shows the calculated curve and the experimental points for values of the excitation parameter as a function of the length L.

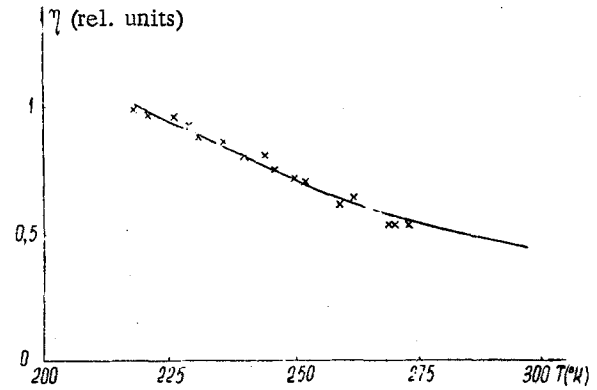

Fig. 2. Excitation parameter as a function of the temperature of the source.

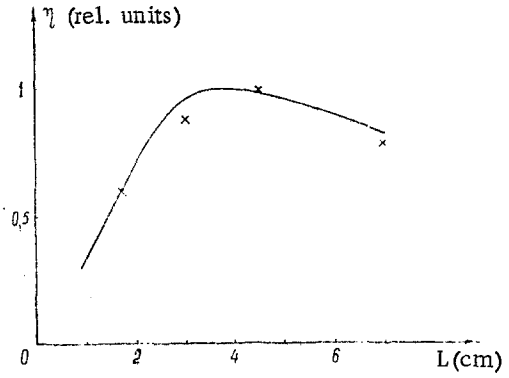

Fig. 3. Excitation parameter as a function of the length of the sorting system.

Note that N_0 is very difficult to measure, since only a fraction of the total molecular flux from the source enters the sorting system. For our type of source, with a single aperture, the directional pattern remains constant over the working range of flow rates [8]. In view of the lack of a suitable flowmeter, the dependence recorded experimentally was not $\eta(N_0)$ but $\eta(\Delta P)$ – on the increase in pressure ΔP. Earlier, while working with ammonia, it was established that the increase in pressure depends linearly on the molecular flux. The sorting system was of optimum length. The maxima of the calculated and experimental curves coincided, and the character of the dependence $\eta(N_0)$ was the same. The corresponding calculated curve and experimental points are presented in Fig. 4.

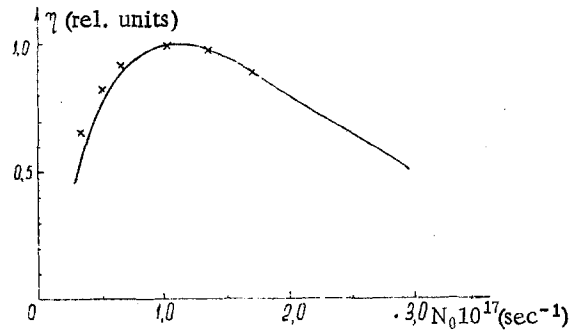

Fig. 4. Excitation parameter as a function of beam intensity.

From an examination of Figs. 3 and 4 it is clear that the above method satisfactorily describes the character of the dependences $\eta(L)$ and $\eta(N_0)$. It is natural to expect, however, that the absolute value of η_{max} thus estimated will be too high, since technical and design limitations on the excitation parameter were not taken into account (the expression $\eta = 4\pi Q N_a^* \mu^2 \tau^2 / \hbar V$, with allowance for the resonator fill factor, was used in the calculations). In fact, the estimate of η_{max} for a sorting system 4.5 cm long gives $\eta_{max} = 3.5$ for the transition 1_{01}-0_{00} and $\eta_{max} = 1.1$ for the transition 5_{14}-5_{15}. However, the values obtained in the experiment with cold diaphragms were about half as great, i.e., $\eta_{max} = 2$ for 1_{01}-0_{00} and $\eta_{max} = 0.5$ for 5_{14}-5_{15}.

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INVESTIGATION OF AUTOMATIC CONTROL SYSTEMS BY MEANS OF AMPLITUDE-PHASE CRITERIA OF NECESSARY STABILITY WITH RESPECT TO MODULUS AND PHASE

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We have a number of stability criteria for automatic control systems (ACS) [1]. The most universal of these is the generalized stability criterion (D-expansions) proposed by Yu. I. Neimark. With the help of D-expansion curves

$$1 + W(j\omega) = 0, \tag{1}$$

the limits of stability of an ACS are determined as a function of the investigated parameters.

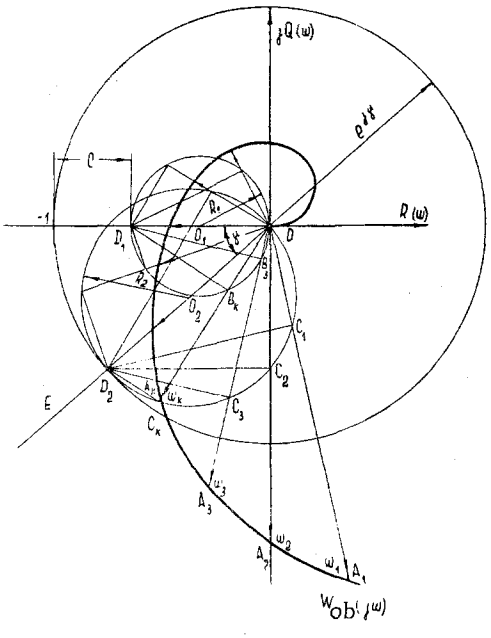

Fig. 1.

However, in the synthesis and analysis of ACS the problem is usually much broader. Not only is it necessary to determine the value of the parameters ensuring the stability of the system, it is also required that the ACS have the necessary reserve of stability. The reserve of stability of a system in the plane of a complex variable is determined by the position of the amplitude-phase characteristic of the open ACS in relation to two points. The point $D_1(c-1, 0)$ determines the reserve of stability with respect to the modulus c, and the point $D_2(-\cos\gamma, -j\sin\gamma)$ determines the reserve of stability with respect to the phase γ (Fig. 1.).

With this taken into consideration, the expressions

$$1 - c + W(j\omega) = 0; \tag{2}$$

$$e^{j\gamma} + W(j\omega) = 0 \tag{3}$$

are the amplitude-phase criteria of the boundary of necessary stability with respect to modulus and phase, respectively. They are more general than the criteria in the method of D-expansions, as they make it possible to determine a family of curves with different reserves of stability and the set of parameters ensuring the prescribed stability of the ACS. In particular, when c and γ are zero, we obtain expression (1) for the D-expansion curves defining the limits of stability as a function of the investigated parameters.

Using (2) and (3) to construct curves in the plane of the investigated parameters of the ACS, in the general case we obtain regions of insufficient stability with respect to modulus and phase, regions of insufficient stability with respect to modulus, regions of insufficient stability with respect to phase, and regions of sufficient stability with respect to modulus and phase.