

A variationally calculated room temperature line-list for H<sub>2</sub>O<sub>2</sub>

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## ABSTRACT

A room temperature line list for hydrogen peroxide is computed using a high level *ab initio* potential energy surface by Małyszczek and Koput (2013) with a small adjustment of the equilibrium geometry and height of the torsional barrier and a new *ab initio* dipole moment surface (CCSD(T)-f12b/aug-cc-pv (T+d)Z). In order to improve further the *ab initio* accuracy, the vibrational band centers were shifted to match experimental values when available. The line list covers the wavenumber region up to 8000 cm<sup>-1</sup> with the rotational excitations  $J \leq 40$ . Room temperatures synthetic spectra of H<sub>2</sub>O<sub>2</sub> are generated and compared to the spectra from the HITRAN and PNNL-IR databases showing good agreement.

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## 1. Introduction

Hydrogen peroxide is a trace species in Earth [1–3] atmospheric chemistry and plays a role in stratospheric ozone production. There have been multiple detections of H<sub>2</sub>O<sub>2</sub> in the Martian atmosphere [4–7], where it is possibly formed by triboelectricity in dust devils and dust storms [6] and it may well act as an agent in the oxidization of the Martian surface. A recent detection in the interstellar medium [8] gives insight to the formation of water in space. Finally H<sub>2</sub>O<sub>2</sub> has been detected in the atmosphere of Europa [9] in the 3.5 μm (≈2587 cm<sup>-1</sup>) region.

Hydrogen peroxide is an asymmetric prolate rotor molecule. Its most interesting characteristic is that it is the simplest molecule that exhibits internal (torsional) rotation. This gives it a double-minimum in its torsional potential as well as two alignments of the O–H bonds: *cis* and *trans*. The torsional mode therefore contains four ‘sub-levels’ for each excitation and requires an additional internal-rotation quantum number ( $\tau = 1, 2, 3, 4$ ) [10] to properly characterize it. It also only exhibits c-type transitions which consequently means it has no pure rotational transitions. H<sub>2</sub>O<sub>2</sub> has six vibrational modes:  $\nu_1$  and  $\nu_5$  represent the symmetric and asymmetric O–H stretching respectively,  $\nu_3$  and  $\nu_6$  represent the O–H

bending respectively,  $\nu_2$  represents the O–O stretch and the  $\nu_4$  mode represents the torsional excitation and is commonly represented in literature as  $n$ .

The HITRAN 2012 database [11] only contains transitions for hydrogen peroxide below 1800 cm<sup>-1</sup>. This region covers the torsional, O–H bending modes and O–O stretch but misses the O–H stretches at the 3750 cm<sup>-1</sup> region. Only a few studies deal with spectra of H<sub>2</sub>O<sub>2</sub> in the mid-infrared and near-infrared [12–14] regions. Available experimental studies mostly concern torsional rovibrational transitions involving the ground and excited states [15], and the  $\nu_3$  [16] and  $\nu_6$  [14] vibrational modes. Whilst there are ample data available in the literature for the torsional and bending bands, the fundamental stretching modes are more difficult to obtain accurate term values for. The O–H stretching modes,  $\nu_1$  and  $\nu_5$ , in particular have been especially problematic. The separation between the two bands is about 8–10 cm<sup>-1</sup> and torsional splitting from the double minimum of the potential gives rise to doubling [17] in the form of ‘quasi’-degenerate states [18] that are difficult to resolve accurately. Olsen et al. [15] give an estimate of 3610–3618 cm<sup>-1</sup> for  $\nu_5$  and 3601–3617 cm<sup>-1</sup> for  $\nu_1$  whilst a Raman study gives a lower value of 3607 cm<sup>-1</sup> [17] for the  $\nu_1$  band-center but determining the accuracy to better than 0.1 cm<sup>-1</sup> is difficult. A theoretical line-list may provide a way of characterizing the confusing spectra of H<sub>2</sub>O<sub>2</sub> above 1800 cm<sup>-1</sup> but so far none have been produced.

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In this work we present a room temperature line list generated using the variational approach TROVE [19,20] based on a high quality *ab initio* potential energy [21,22] and new dipole moment surfaces.

## 2. Method

The accuracy of the line positions is determined by the quality of the PES. The PES is based of the high-accuracy *ab initio* calculations of Małyszczek and Koput [21]. The PES was computed using the CCSD(T)-F12 method with parts of the PES utilizing aug-cc-pV7Z basis-sets. These calculations can be considered state-of-the-art for this problem. We include the small adjustment to the *ab initio* equilibrium geometry and height of the torsional barrier proposed by Polyansky et al. [22]. The PES boasts a root-mean-squares (rms) difference of  $0.02 \text{ cm}^{-1}$  for rotational levels up to  $J = 35$  within low-lying vibrational states [22].

The rovibrational energies were calculated using the TROVE [19] computer suite. TROVE is a variational nuclear-motion solver and can be employed in all steps of line-list production for molecules of arbitrary structure. It has been successful to produce hot line lists for  $\text{CH}_4$  [23],  $\text{PH}_3$  [24],  $\text{H}_2\text{CO}$  [25] and room temperature ones for  $\text{PH}_3$  [26] and  $\text{SO}_3$  [27]. TROVE was also used to simulate a room temperature spectrum for another non-rigid, chain molecule HSOH based on a high level *ab initio* potential energy and dipole moment surfaces [28,29].

Within TROVE, the nuclear-motion Hamiltonian is represented as an expansion around a reference configuration where the bond lengths for the O–O bond ( $R$ ), the O–H<sub>1</sub> bond ( $r_1$ ), the O–H<sub>2</sub> bond ( $r_2$ ) and the bond angles for O–H<sub>1</sub> ( $\theta_1$ ) and O–H<sub>2</sub> ( $\theta_2$ ) are frozen at their equilibrium and the torsional angle  $\tau$  varies on a grid of 10,000 values ranging from  $0^\circ$  to  $720^\circ$ . The internal co-ordinates are described in Fig. 1.

TROVE utilizes an approximate kinetic energy operator (KEO). Comparisons with exact KEO codes such as WAVR4 [30] show TROVE converging just as well [22] whilst being less computationally demanding. Convergence is obtained for expansion orders between 6 and 8 [19].

Here, the kinetic energy is expanded around the reference geometry in terms of five linearized co-ordinates of the form:

$$\zeta_i = x_i^l - x_i^e, \quad (1)$$

where  $x_i^l$  and  $x_i^e$  represents linearized version and equilibrium geometry of the bond lengths and angles respectively. Here,  $i = 1, i = 2, i = 3, i = 4$  and  $i = 5$  represent  $R, r_1, r_2, \theta_1$  and  $\theta_2$  respectively and  $i = 6$  is the sixth co-ordinate,  $\zeta_6 = \tau$ . The potential is also expanded in terms of the Morse-type coordinates  $\zeta_1 = 1 - \exp(-a_1\zeta_1)$  ( $a_i$  are parameters,  $i = 1, 2, 3$ ) for the stretches and the displacements or  $\zeta_4$  and  $\zeta_5$  for the two bends. For this work

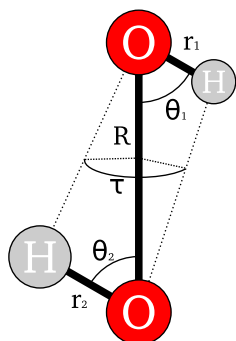


Fig. 1. The internal co-ordinates of HOOH.

the kinetic energy expansion order is 6 and the potential expansion order is 8.

A symmetry-adapted basis-set is constructed by a multi-step contraction scheme that is truncated via polyad number  $P_{\max} = 42$  [22]. The primitive vibrational basis-set is constructed by solving the 1D Schrodinger equation for each basis-function  $\phi_{v_i}(\zeta_i)$  ( $i = 1, 2, \dots, 6$ ) associated with the vibrational quantum number  $v_i$  via the Numerov–Cooley method [31,32] for each mode allowed by the polyad  $P$ :

$$P = 4v_1 + 8(v_2 + v_3 + v_4 + v_5) + v_6 \leq P_{\max}. \quad (2)$$

The six dimensional co-ordinate space is then divided into four reduced subspaces:  $(\zeta_1)$ ,  $(\zeta_2, \zeta_3)$ ,  $(\zeta_4, \zeta_5)$  and  $(\zeta_6)$  based on symmetry. The reduced Hamiltonian is solved using the primitives  $\phi_{v_i}$  as basis-functions to obtain the contracted vibrational basis-functions  $\Phi_{n_1}(\zeta_1)$ ,  $\Phi_{n_2}(\zeta_2, \zeta_3)$ ,  $\Phi_{n_3}(\zeta_4, \zeta_5)$  and  $\Phi_{n_4}(\zeta_6)$ . These basis-functions are then symmetrised according to the  $D_{2h}(M)$  Molecular Group symmetry [33] and the final vibrational basis-set is formed from the product of the four contracted basis-functions which are truncated via Eq. (2) and symmetrized again. The general form of the Hamiltonian operator is:

$$H = H_{vib} + \frac{1}{2} \sum_{\alpha\beta} J_{\alpha} G_{\alpha\beta} J_{\beta} + \frac{1}{2} \sum_{\alpha\lambda} (p_{\lambda} G_{\lambda\alpha} + G_{\alpha\lambda} p_{\lambda}) J_{\alpha}, \quad (3)$$

where  $J_{\alpha}$  and  $p_{\lambda}$  are the rotational and vibrational momentum operators respectively and  $H_{vib}$  is the pure vibrational ( $J = 0$ ) Hamiltonian given as:

$$H_{vib} = \frac{1}{2} \sum_{\lambda\mu} p_{\lambda} G_{\lambda\mu} p_{\mu} + V + U, \quad (4)$$

where  $G_{\alpha\beta}$  are kinetic energy factors,  $U$  is a pseudo-potential [19] and  $V$  is the molecular potential energy. The contracted Hamiltonian is solved up to an energy eigenvalue threshold of  $24,000 \text{ cm}^{-1}$ . A final contraction step can be performed by solving the  $J = 0$  problem given by Eq. (4) and replacing the bulky primitive vibrational basis-sets with the more compact  $J = 0$  wavefunctions. This has the added benefit of making the computation of the Hamiltonian matrix elements for  $J > 0$  more efficient as the  $H_{vib}$  contribution becomes:

$$\langle \Psi_{J=0,i}^{\Gamma} | H_{vib} | \Psi_{J=0,i'}^{\Gamma} \rangle = E_i \delta_{ii'}, \quad (5)$$

Table 1

Experimental [15,16,36,37] band centers used in the empirical shift.

$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$	$\tau$	Symmetry	Ab initio ( $\text{cm}^{-1}$ )	Shifted/Obs ( $\text{cm}^{-1}$ )
0	0	0	0	0	0	4	$A_u$	11.312	11.437
0	0	0	0	0	0	3	$B_{2u}$	11.312	11.437
0	0	0	1	0	0	1	$A_g$	255.529	254.55
0	0	0	1	0	0	2	$B_{2g}$	255.532	254.55
0	0	0	1	0	0	4	$A_u$	371.589	370.893
0	0	0	1	0	0	3	$B_{2u}$	371.590	370.893
0	0	0	2	0	0	1	$A_g$	570.809	569.743
0	0	0	2	0	0	2	$B_{2g}$	570.818	569.743
0	0	0	2	0	0	4	$A_u$	777.432	776.1221
0	0	0	2	0	0	3	$B_{2u}$	777.458	776.1148
0	0	1	0	0	0	1	$A_g$	865.539	865.939
0	0	1	0	0	0	2	$B_{2g}$	865.539	865.939
0	0	1	0	0	0	4	$A_u$	877.470	877.934
0	0	1	0	0	0	3	$B_{2u}$	877.470	877.934
0	0	0	3	0	0	1	$A_g$	1002.666	1000.882
0	0	0	3	0	0	2	$B_{2g}$	1002.869	1000.930
0	0	0	0	0	1	1	$B_{1u}$	1265.003	1264.583
0	0	0	0	0	1	3	$B_{1g}$	1285.879	1285.121
0	0	0	1	0	1	1	$B_{1u}$	1506.164	1504.872
0	0	0	1	0	0	3	$B_{1g}$	1649.977	1648.367
0	0	0	2	0	1	1	$B_{1u}$	1855.823	1853.634
0	0	0	2	0	1	3	$B_{1g}$	2075.366	2072.404

where  $\Psi_{J=0,i}^{\Gamma}$  is the symmetrized wavefunction of a particular state  $i$  obtained from Eq. (4). The vibrational contribution is both diagonal and simply the eigenvalue of  $\Psi_{J=0,i}^{\Gamma}$ . Additionally, experimental band centers can be substituted instead of the eigenvalues, this ‘shifts’ the band center onto the experimental value and allows further rotational excitations to be computed around the experimental band centers. Here the  $J = 0$  wavefunctions with eigenvalues up to  $8000 \text{ cm}^{-1}$  are utilized further reducing the size of the Hamiltonian. The original primitive basis-set was of size 2,789,400, this was reduced to 23,078 in the contracted and finally to 2875 using the  $J = 0$  ‘form’. A more detailed explanation of this methodology is given by Yurchenko et al. [34].

Using these symmetrized wavefunctions also has the benefit that the Hamiltonian matrix is factorized into independent blocks according to the  $\mathcal{D}_{2h}(\text{M})$  symmetry.  $\mathcal{D}_{2h}(\text{M})$  is isomorphic to the  $\text{C}_{2h}^+(\text{M})$  symmetry group which best describes the torsional splitting caused by the *cis* and *trans* tunneling [35]. The irreducible representations of this group are  $A_g, A_u, B_{1g}, B_{1u}, B_{2g}, B_{2u}, B_{3g}$  and  $B_{3u}$ . However, the states corresponding to  $B_{2g}, B_{2u}, B_{3g}$  and  $B_{3u}$  have zero statistical weight and therefore their matrix blocks are not constructed and diagonalized for  $J > 0$ . It is usual to describe the  $\text{H}_2\text{O}_2$  torsional modes using the notation  $(n, \tau)$ , where  $n$  is the excitation of the torsional mode. The excitations of the torsional  $(\nu_4/n)$  mode are represented by  $A_g, A_u, B_{2g}$  or  $B_{2u}$  symmetry which correspond to the quanta  $\tau = 1, \tau = 4, \tau = 2$  and  $\tau = 3$ , respectively. This needs to be taken into account when manipulating the band-centers as it requires modifying the  $B_{2g}$  or  $B_{2u}$  eigenvalues in order to properly represent the splitting at higher  $J$ . Table 1 lists

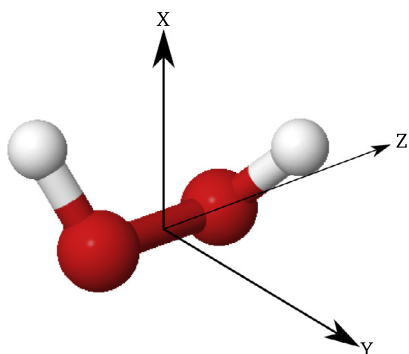


Fig. 2. The principal axis for HOOH used in the DMS expansion.

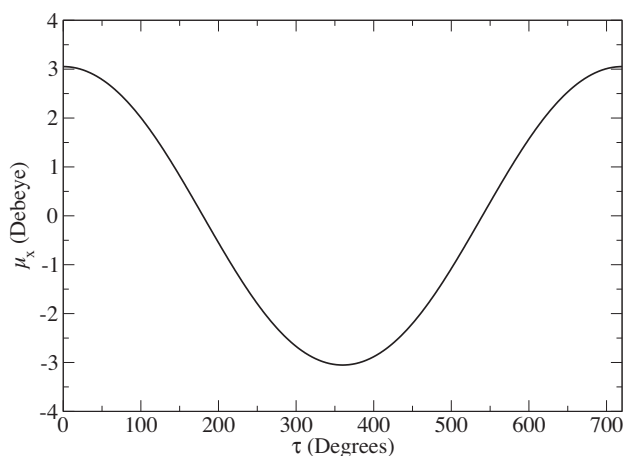


Fig. 3. The  $\mu_x$  dipole moment component for  $\text{H}_2\text{O}_2$ , computed at the torsional geometries shown.

all the band-centers that were utilized in the empirical shifts together with the *ab initio* values before the shift.

The  $\tau$  quantum number can be preserved in the quantum number assignment in TROVE by utilizing the following form:

$$\nu_4 = 4n + i, \quad (6)$$

where  $n$  is the excitation and  $i$  is the symmetry where  $i = 0, 1, 2, 3$  is  $A_g, B_{2g}, B_{2u}$  and  $A_u$ , respectively. Integer quantum numbers  $n$  and  $\tau$  can be simply retrieved:

$$\tau = (\nu_4 \bmod 4) + 1, \quad n = \left\lfloor \frac{\nu_4}{4} \right\rfloor. \quad (7)$$

The final line-list format will extract both numbers and utilize  $n$  as the value for  $\nu_4$  and include a separate  $\tau$  quantum number which should not be confused with the commonly used rotational parity symbol  $\tau_{\text{rot}} = \pm$  or with the torsional angle  $\tau$  (see below). For this line-list, the Hamiltonian matrices up to the limit of  $J = 40$  are constructed and diagonalized using the  $J = 0$  contracted basis set for all eigenvalues and eigenvectors but only eigenvectors up to  $8000 \text{ cm}^{-1}$  are stored and used for producing the transitions.

### 2.1. Dipole moment surface and intensities

An electric dipole moment surface (DMS) is required to calculate the absolute intensities for each transition. An *ab initio* DMS was computed at the CCSD(T)-f12b/aug-cc-pV(T+d)Z [38] level of theory in the frozen-core approximation using CCSD(T) [39] on a grid of 50,000 geometries in conjunction with the finite electric field method and field of 0.005 a.u. We expect this DMS to be of adequate quality, as this level of theory is known to yield accurate intensities (see e.g., review [40]).

The  $\mathcal{D}_{2h}(\text{M})$  symmetry-adapted projections of the dipole moment Cartesian components  $\mu_x, \mu_y$ , and  $\mu_z$  are given in the analytical representations with each component expanded in Taylor series (312 parameters in total) in terms of internal coordinates around the equilibrium configuration using a molecular-fixed axis system as follows. The  $z$  axis is aligned along the O–O bond, and

Table 2

Absolute values of experimental [13] and *ab initio* transition moments, in debye, for  $(n', \tau = 1) \leftrightarrow (n'', \tau = 3)$ .

$n'$	$n''$	Calc	Obs	(Obs-Calc)/Obs (%)
0	0	1.5683	1.5723	0.25
0	1	0.3332	0.3413	−2.40
1	0	0.6031	0.6136	−1.72
1	1	1.1664	1.1751	0.74
2	1	1.1664	1.1628	0.31
2	2	1.2638	1.2825	1.46
3	2	1.3276	1.3535	−1.91

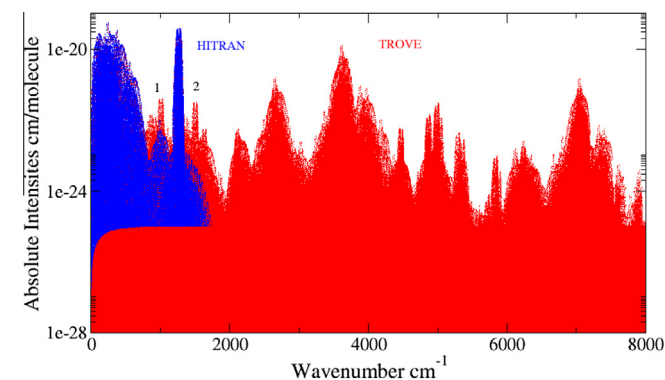
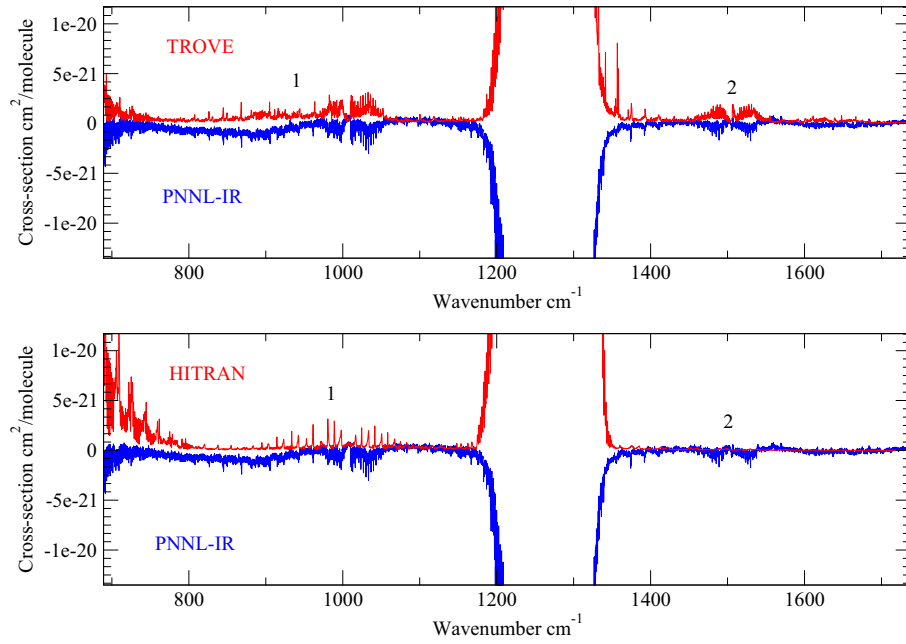


Fig. 4. Overview of our synthetic spectrum at  $T = 296 \text{ K}$  against HITRAN [11].



**Fig. 5.** Cross-section comparison of the room temperature line-list against experimental PNNL-IR [45] data at the  $\nu_1$  and  $\nu_5$  band at 323.15 K with HWHM =  $0.3120 \text{ cm}^{-1}$ .

the  $x$  axis lies in the plane bisecting the two O–O–H planes (i.e. planes containing the O–O and O–H bonds). The  $y$  axis is oriented such that the  $xyz$  axis system is right-handed. This  $xyz$  axes is not exactly but close to the principal axis system shown in Fig. 2. With the chosen axes, the  $x$ ,  $y$ , and  $z$  components of the dipole moment span the  $B_{1u}$ ,  $B_{3u}$ , and  $B_{2u}$  representations, respectively. The three electronically averaged dipole components are represented by the following analytical functions:

$$\bar{\mu}_x = \cos(\tau/2) \sum_{i_1, i_2, \dots, i_6} \mu_{i_1, i_2, \dots, i_6}^{(x)} \zeta_1^{i_1} \zeta_2^{i_2} \zeta_3^{i_3} \zeta_4^{i_4} \zeta_5^{i_5} \zeta_6^{i_6}, \quad (8)$$

$$\bar{\mu}_y = \sin(\tau/2) \sum_{i_1, i_2, \dots, i_6} \mu_{i_1, i_2, \dots, i_6}^{(y)} \zeta_1^{i_1} \zeta_2^{i_2} \zeta_3^{i_3} \zeta_4^{i_4} \zeta_5^{i_5} \zeta_6^{i_6}, \quad (9)$$

$$\bar{\mu}_z = \sum_{i_1, i_2, \dots, i_6} \mu_{i_1, i_2, \dots, i_6}^{(z)} \zeta_1^{i_1} \zeta_2^{i_2} \zeta_3^{i_3} \zeta_4^{i_4} \zeta_5^{i_5} \zeta_6^{i_6}, \quad (10)$$

where

$$\zeta_1 = \Delta R e^{-(\Delta R)^2}, \quad (11)$$

$$\zeta_2 = \Delta r_1 e^{-(\Delta r_1)^2}, \quad (12)$$

$$\zeta_3 = \Delta r_2 e^{-(\Delta r_2)^2}, \quad (13)$$

$$\zeta_4 = \Delta \theta_1, \quad (14)$$

$$\zeta_5 = \Delta \theta_2, \quad (15)$$

$$\zeta_6 = \cos \tau. \quad (16)$$

The expansion parameters of the  $x$ ,  $y$  and  $z$  components of the dipole obey the following permutation rules:

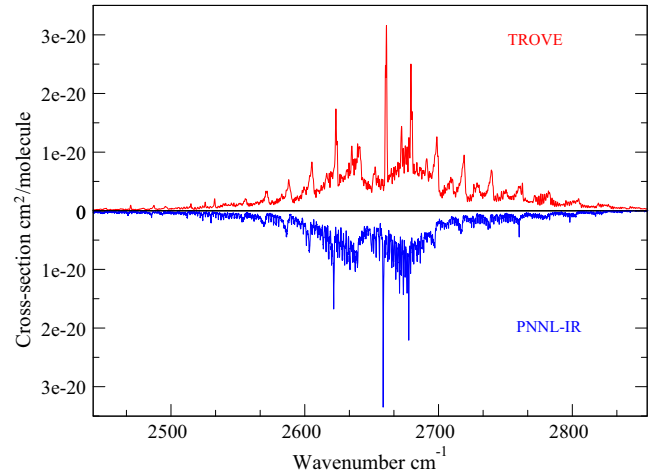
$$\mu_{i_1, i_2, i_3, i_4, i_5, i_6}^{(x)} = \mu_{i_1, i_3, i_2, i_5, i_4, i_6}^{(x)}, \quad (17)$$

$$\mu_{i_1, i_2, i_3, i_4, i_5, i_6}^{(y)} = -\mu_{i_1, i_3, i_2, i_5, i_4, i_6}^{(y)}, \quad (18)$$

$$\mu_{i_1, i_2, i_3, i_4, i_5, i_6}^{(z)} = -\mu_{i_1, i_3, i_2, i_5, i_4, i_6}^{(z)}, \quad (19)$$

corresponding to the permutation of the two hydrogen atoms and therefore  $\mu_{0,0,0,0,0,0}^{(\alpha)} = 0$  ( $\alpha = y, z$ ) for any  $i_6$ . The dependence of  $\mu_x$  component against the torsional angle is shown in Fig. 3.

The 130, 90, and 92 symmetrically independent expansion parameters  $\mu_{i_1, i_2, i_3, i_4, i_5, i_6}^{(\alpha)}$  ( $\alpha = x, y, z$ ) were obtained in a least-squares fit to the  $3 \times 20,842$  *ab initio* dipole moment values

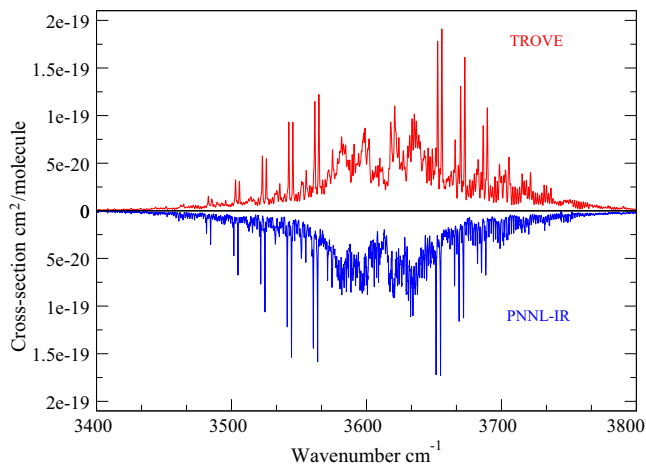


**Fig. 6.** Cross-section comparison of the room temperature line-list against experimental PNNL-IR [45] data at 323.15 K with HWHM =  $0.3120 \text{ cm}^{-1}$ .

(corresponding to energies below  $hc \ 12,000 \text{ cm}^{-1}$ ) with the rms error of 0.0013, 0.0002, and 0.0010 D, respectively.<sup>1</sup> The resulting DMS parameters are given in the [Supplementary material](#).

In TROVE the *ab initio* DMS is re-expanded around the non-rigid reference configuration in terms of the  $\zeta_i$  ( $i = 1 \dots 5$ ) variables as described above, using the numerical finite differences method [19]. At some torsional values close to  $\tau = 360^\circ$  and  $\tau = 720^\circ$  the finite differences produce discontinuities, which lead to meaningless values of the dipole expansion parameters  $\mu_{i,j,k}(\tau)$  for a number of grid points. These points are treated as outliers and replaced by interpolated values. Additionally as the expansion is not an exact representation of the  $\mu_z$  dipole component it may display artificial asymmetry. This is resolved by computing  $\mu_z$  for torsional geometries up to  $\tau = 360^\circ$  and then mirroring the values up to  $\tau = 720^\circ$ .

<sup>1</sup> 1 D =  $3.33564 \times 10^{-30} \text{ C m}$ .



**Fig. 7.** Cross-section comparison of the room temperature line-list against experimental PNNL-IR [45] data at the  $\nu_1$  and  $\nu_5$  band at 323.15 K with HWHM = 0.3120  $\text{cm}^{-1}$ .

Vibrational transition moment are computed as:

$$\mu_{if} = \sqrt{\sum_{A=X,Y,Z} |\langle \Psi_{J=0,i}^T | \bar{\mu}_A | \Psi_{J=0,f}^T \rangle|^2}. \quad (20)$$

Our vibrationally averaged moment for the ground vibrational state is 1.5683 D and compares well with measured of 1.5728 D [41]. This is very different from the equilibrium value of the *ab initio* dipole moment  $\bar{\mu}_x^e = 1.738$  D (at  $R = 1.4554$  Å,  $r_1 = r_2 = 0.96257$  Å,  $\theta_1 = \theta_2 = 101.083^\circ$ ), showing the strong non-rigid character of the  $\text{H}_2\text{O}_2$  dipole moment.

Information on transition moments for  $\text{H}_2\text{O}_2$  is limited; Table 2 compares available experimentally derived values at different torsional excitations from experiment [13]. Our calculated values reproduce the experimental with a maximum deviation of 2.4%. Some papers report effective transition dipole moments as a torsional expansion in terms of  $\tau$ , e.g.  $\phi_z \cos \tau$  ( $\phi_z$  is a parameter) [13] which are difficult to compare to our fully averaged transition dipoles.

Using the DMS and eigenvectors obtained from diagonalization, the line strengths for each transition can be computed providing they satisfy the selection rules:

$$J' - J'' = 0, \pm 1, J' + J'' \neq 0, \quad (21)$$

$$A_g \leftrightarrow A_u, B_{1g} \leftrightarrow B_{1u}, \quad (22)$$

and the  $B_{2g}$ ,  $B_{2u}$ ,  $B_{3g}$  and  $B_{3u}$  are forbidden due to their zero nuclear statistical weights. The Einstein-A coefficient for a particular transition from the initial state  $i$  to the final state  $f$  is given by:

$$A_{if} = \frac{64 \times 10^{-36} \pi^4 \bar{\nu}_{if}^3}{3h} (2J_i + 1) \sum_{A=X,Y,Z} |\langle \Psi^f | \bar{\mu}_A | \Psi^i \rangle|^2, \quad (23)$$

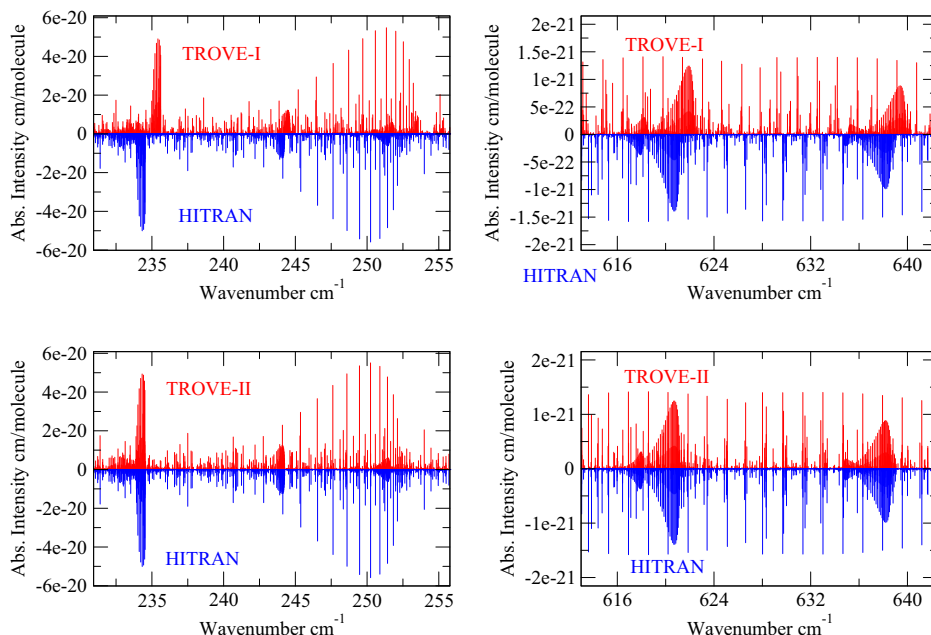
where  $J_i$  is the rotational quantum number for the initial state,  $h$  is Planck's constant,  $\bar{\nu}_{if}$  is the transition frequency ( $hc \bar{\nu}_{if} = E_f - E_i$ ),  $\Psi^f$  and  $\Psi^i$  represent the eigenfunctions of the final and initial states respectively,  $\bar{\mu}_A$  is the electronically averaged component of the dipole moment along the space-fixed axis  $A = X, Y, Z$  (see also Yurchenko et al. [42]). From this the absolute absorption intensity is determined by:

$$I(f \leftarrow i) = \frac{A_{if}}{8\pi c} g_{ns} (2J_f + 1) \frac{\exp\left(-\frac{E_i}{kT}\right)}{Q \bar{\nu}_{if}^2} \left[ 1 - \exp\left(-\frac{hc \bar{\nu}_{if}}{kT}\right) \right], \quad (24)$$

where  $k$  is the Boltzmann constant,  $T$  the absolute temperature and  $g_{ns}$  is the nuclear spin statistical weight factor.  $Q$ , the partition function, is given by:

$$Q = \sum_i g_i \exp\left(-\frac{E_i}{kT}\right), \quad (25)$$

where  $g_i$  is the degeneracy of a particular state  $i$  with energy  $E_i$ . For  $\text{H}_2\text{O}_2$ ,  $g_i$  is  $g_{ns}(2J_i + 1)$  with  $g_{ns} = 1$  for  $A_g$  and  $A_u$ ,  $g_{ns} = 3$  for  $B_{1g}$  and  $B_{1u}$  and  $g_{ns} = 0$  for the  $B_{2g}$ ,  $B_{2u}$ ,  $B_{3g}$  and  $B_{3u}$  symmetries. The transitions were computed using the energy limits  $hc$  4000 and  $hc$  8000  $\text{cm}^{-1}$  for the lower and upper states, respectively to achieve our target  $\bar{\nu}_{if}$  limit of 8000  $\text{cm}^{-1}$ . The recently developed GAIN (GPU Accelerated INTensities) code was utilized to compute 1,487,073,009 transitions within 6 h by exploiting M2090 nVidia



**Fig. 8.** Comparing two versions of the synthetic spectrum against HITRAN at  $T = 295$  K. TROVE-I is the *ab initio*, TROVE-II is using the empirical band-center shifts.



graphics processing units (GPU) for their high degree of parallelism and computational efficiency. A paper discussing the methodology in detail will be published elsewhere [43].

### 3. Results

Using our line-list we obtained the partition function of 9840.91 at 296 K, which compares well to the HITRAN value 9819.80 [44].

Fig. 4 is our synthetic spectrum at 296 K for all 1.4 billion lines and comparing against the 126,983 lines from HITRAN highlights the significant degree of completeness our line-list provides. However two regions (1) and (2) show disagreement in line intensity, which can be attributed to the lack of the experimental data used for producing the HITRAN intensities by Perrin et al. [13,14].

The PNNL-IR [45] database provides additional cross-sections above 1800 cm<sup>-1</sup>. Fig. 5 compares ours and HITRAN's simulated cross-sections to PNNL using a Gaussian convolution with HWHM at 0.312 cm<sup>-1</sup> at  $T = 323.15$  K and demonstrates that we agree much better in intensity and structure indicating problems with HITRAN intensities for these two regions.

Fig. 6 highlights a band in the 3.5  $\mu$ m region which is a combination of the  $(\nu_3 + 6\nu_4, \tau = 1) \leftarrow (0, \tau = 1)$ ,  $(2\nu_3 + 4\nu_4, \tau = 1) \leftarrow (0, \tau = 1)$ ,  $(3\nu_3, \tau = 1) \leftarrow (0, \tau = 2)$ ,  $(\nu_3 + \nu_4 + \nu_5, \tau = 2) \leftarrow (0, \tau = 2)$ ,  $(\nu_3 + 4\nu_4, \tau = 4) \leftarrow (0, \tau = 3)$  and other weaker hot bands. Good agreement is seen in both structure and overall intensity but our assumed Gaussian line profile utilized in our convolution may not be adequate enough to properly replicate the PNNL-IR cross-section leading to some minor differences in the overall cross-section.

Fig. 7 further illustrates the quality of both our line-positions and absolute intensities by comparing the  $\nu_1$  and  $\nu_5$  (ours vs PNNL-IR's) bands in the 2.7  $\mu$ m region. As this is the region of most contention in the literature, it is to be hoped that this line-list may provide a means with which to assign the confusing spectra in this region.

Finally, the importance of the band shifts previously discussed is illustrated in Fig. 8. Here the TROVE-I spectra is purely using the *ab initio* band centers while TROVE-II utilizes the experimental band centers from Table 1. The *ab initio* deviation of 1.12 cm<sup>-1</sup> reduces significantly to 0.005 cm<sup>-1</sup> using this empirical shifting method.

### 4. Conclusions

Presented here is a high-quality room temperature line-list for H<sub>2</sub>O<sub>2</sub> that covers the region up to 8000 cm<sup>-1</sup>. The complete line-list is available at Exomol website: <http://www.exomol.com>. The ultimate goal is a hot line-list for H<sub>2</sub>O<sub>2</sub> that reaches up to 10,000 cm<sup>-1</sup>. Whilst there is extremely good agreement in both line-positions and intensities, the current strategy is to refine this PES to give a new spectroscopically-determined PES. The band-center shifting does resolve many discrepancies between the theoretical and experimental energy, however, due to the lack of line-positions, it may be not as accurate in predicting the nature of the  $\nu_1$  and  $\nu_5$  bands. By applying the refinement process, it may bring the PES closer to the 'true' form giving better predictions in this troublesome region. Nevertheless, this line-list can produce synthetic spectra that compare well with room temperature.

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### Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.jms.2015.10.004>.

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