Journal of Communications Technology and Electronics, Vol. 50, No. 12, 2005, pp. 1400–1408. Translated from Radiotekhnika i Elektronika, Vol. 50, No. 12, 2005, pp. 1509–1517.

Determination of Loss in a Fabri–Perot Resonator from Its Response to Exciting Radiation with a Fast-Scanned Frequency

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Received October 22, 2004

Abstract—The response of a high-Q Fabri–Perot resonator to exciting radiation with a frequency that is fast digitally scanned and a phase that exhibits no jumps during switchings is simulated. The inverse problem of determination of the parameters of the resonator from the shape of its response is solved. Solution of the problem is important for precision measurements of the decay parameter of millimeter- and submillimeter-wave radiation in dielectrics. Such measurements have become possible with the advent of modern fast frequency synthesizers. The developed algorithm is applied to optimize the experimental conditions.

INTRODUCTION

At present, spectrometers based on open Fabri–Perot resonators are the highest precision tools for investigation of attenuation of millimeter- and submillimeter-wave electromagnetic radiation in liquid, solid, and gas dielectrics, including atmospheric air. These investigations are important for both fundamental theoretical studies (of intermolecular interaction and propagation of electromagnetic radiation in various media) and applications (remote sensing of the atmosphere, radio astronomy, meteorology, radar, shortwave telecommunications, gas analysis, etc.). In particular, the most accurate values of the parameters of the main microwave atmospheric absorption lines that are used for monitoring the Earth's atmosphere from satellites and ground-based observatories have been obtained by means of a spectrometer with a Fabri–Perot resonator [1, 2]. Such a spectrometer has also been used to measure the dielectric parameters of plates made from artificial diamonds that exhibit the lowest absorption among all known dielectrics and serve as a material of the output windows of superhigh-power gyrotrons [3]. Known alternative methods do not ensure measurements of such low decay parameters.

A resonator spectrometer operating in a band of 45–200 GHz [1, 4, 5] has recently been designed at the Institute of Applied Physics, Russian Academy of Sciences (IAP RAS). The schematic of this spectrometer, an apparatus which is used for investigating the attenuation of microwave radiation in dielectrics, is displayed in Fig. 1.

The decay parameter is measured with the spectrometer on the basis of a change in the loss in a Fabri–Perot resonator caused by a sample to be examined that is placed into the resonator. The resonator loss is determined from the width of the resonance response of the resonator. This width is measured by mathematically fitting the theoretical shape of the resonance curve to the response observed during the experiment. The experimental response is recorded while the frequency of exciting radiation is changed stepwise.

A frequency synthesizer that is capable of fast digital frequency scan (with a switching time of 200 ns and a minimum interval between switchings of 58 μs) and exhibits no phase jumps during switchings is employed as a radiation source. The aforementioned features are supplied by a Radio Frequency (RF) Direct Digital Synthesizer (DDS) used as a reference-signal source [4, 5]. The use of a fast DDS makes it possible to record the resonator's response during a time interval that is substantially shorter than the characteristic time of noises distorting the shape of the response. At the same time, precision frequency control is retained; therefore, the accuracy of the measured width of the response is improved significantly. As a consequence, the decay parameter is measured by more than an order of magnitude more accurately than the parameter determined by

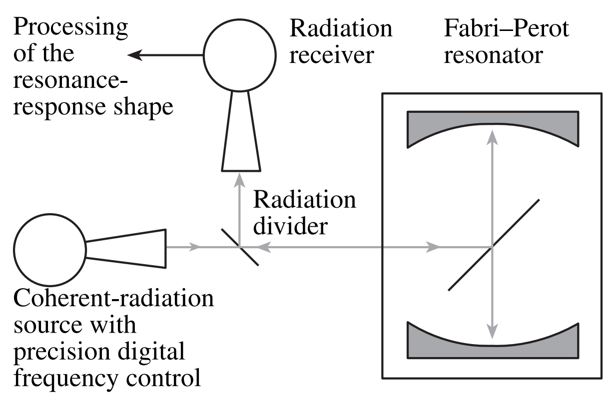

Fig. 1. Simplified schematic of a spectrometer based on a Fabri–Perot resonator.

existing analogous devices and the sensitivity of the spectrometer is enhanced [4, 5]. The developed measurement technique ensures detecting of a relative change of 10^{-4} (or 20 Hz in absolute units) in the resonance response curve width of 200 kHz. Such a high accuracy imposes very severe restrictions on the accuracy of the model shape of the resonance response. The existing specific spectrometer guarantees a frequency-scan speed of 60 μs/step. The response's amplitude is measured just before frequency switching. The resonator's time constant is ≈1.6 μs; that is, stationary oscillations are formed in the time interval $t \gg \tau$. The frequency step is usually 0.002 of the width of the resonance curve. The resonator is excited by radiation whose phase is retained during frequency switchings. All of the aforementioned factors provide for the resonance response such that its shape is described well by the known Lorentz curve. A single resonance response is recorded at 500 points over 30 ms. Highly accurate precision measurements necessitate about 1000 records. Recording a fine attenuation profile in a dielectric sample over a frequency interval of 50 GHz involves measurements at nearly 100 frequencies. Thus, recording resonance responses takes nearly an hour; therefore, the spectrometer has limited applications. It is technically possible to reduce the recording time substantially by increasing the frequency-scan speed. This possibility is provided by fast RF DDSs of the next generation (see, e.g., the website http://www.analog.com/dds). At present, the researchers of the IAP RAS are developing a frequency synthesizer whose performance-speed parameters exceed those of the preceding model by an order of magnitude. Ensuring such a significant increase of the frequency-scan speed and simultaneously retaining the measurement precision necessitate development of an efficient theoretical model. This problem is solved in this paper.

1. MODELING OF THE SHAPE OF A RESONANCE RESPONSE

If the frequency of resonator-exciting radiation is scanned sufficiently slowly, the response of a high-Q resonator is described quite accurately by the Lorentz curve. The faster the response recording, the weaker the effect of noises (mechanical vibrations, voltage jumps, and temperature drift). In addition, the duration of the experiment is reduced. However, distortions caused by the finite resonator's response time become more pronounced and the use of the Lorentz curve as a model shape results in systematic errors. In this section, the problem of modeling the shape of the resonator's response is solved in the case when the frequency of exciting radiation is scanned stepwise in the absence of phase jumps during frequency switchings. Note that, in the situation considered, the continuousness of the phase, which is a natural boundary condition in oscillation problems, follows from the principle of the DDS's operation. This property is not common to all synthesizers. All other synthesizers exhibit a phase jump during frequency switching. When a resonator is excited by such radiation, the time of recording the undistorted resonance response is many times greater than that in the case when the radiation frequency is scanned continuously using an analog technique. The use of a DDS for with fast frequency scanning at a fine step resembles analog scanning and, additionally, provides for the exact value of frequency at each instant. All aforementioned factors allow precision measurements of the resonator's parameters and recording of the resonator's response during a time interval comparable to the resonator's time constant.

Let us consider a resonator excited by coherent radiation as a harmonic oscillator with decay that is driven by a harmonic external force. This oscillator is described by the known second-order differential equation

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = \exp(i\omega t), \tag{1}$$

where x is the electric-field intensity at a certain point of the resonator normalized by the amplitude of exciting radiation, γ is the decay parameter of the resonator field, ω is the circular frequency of the exciting-radiation source, ω_0 is the circular eigenfrequency of the resonator field in free space, and t is the current time. A general solution to Eq. (1) is considered in [6]. This solution implies that a high-Q resonator ($\gamma \ll \omega_0$) which is excited at frequencies close to the resonance frequency ($\omega \approx \omega_0$) has a response corresponding to the Lorentz function that has the full width $\Delta\nu = \Delta\omega/2\pi = \gamma/\pi$ at half the maximum amplitude. When the exciting-radiation source is turned off, the energy accumulated in the resonator and the field amplitude decay as $\exp(-2\gamma t)$ and $\exp(-\gamma t)$, respectively. Therefore, the time constant of the resonator can be represented as $\tau = 1/\gamma = \pi/\Delta\nu$. Relative energy loss P observed during a single traversing of radiation through the resonator (for brevity, referred to as the resonator loss) is determined by the formula $P = 0.5(W_0 - W_T)/W_0$, where W_0 is the energy accumulated in the resonator $W_T = W_0\exp(-2\gamma T)$, $T = 2L/c$ is the time interval of the complete radiation cycle in the resonator, L is the length of the resonator, and c is the velocity of light. Since $2\gamma T \ll 1$ for a high-Q resonator, we have $W_T \approx W_0(1 - 2\gamma T)$ and the resonator loss can be expressed through the resonance width as $P = 2\pi L\Delta\nu/c$.

A general solution to Eq. (1) yields the resonator field at all instants, thus providing for a model of the resonator's response. However, solution of the formulated problem necessitates only the field amplitudes at the instants immediately before frequency switchings. A redundant general solution is obtained via redundant computations; hence, the duration of the experiment increases. Therefore, it is expedient to apply the following approximations. Assume that all oscillations occur at one frequency equal to the center frequency of the resonance response ($\omega = \omega_0$) and have a variable phase.

The error caused by this assumption is small because the experiment uses radiation at frequencies of about 100 GHz and the frequency varies within less than 2 MHz during recording of one resonance response. Then, a wave in the resonator and a source wave can be added as two coherent waves (the complex-amplitude method).

Consider the transient process (evolution of the field amplitude) developing in the resonator at a constant radiation frequency during the interval between frequency switchings. Let us introduce the following notation: φ is the phase shift between, one, the field radiated by a source and, two, the real part of the amplitude of the resonator field and $\lambda_0 = \omega_0/2\pi$ and $\lambda = \omega/2\pi$ are the resonance wavelength and the wavelength of the radiation source, respectively. In that case, the phase changes during one oscillation period by the quantity

$$d\varphi_T = (\lambda - \lambda_0)k = ck\left(\frac{1}{\nu} - \frac{1}{\nu_0}\right) = \frac{2\pi}{\nu_0}(\nu_0 - \nu), \quad (2)$$

where $k = 2\pi\nu/c$ is the wave number, $\nu = c/\lambda$ is the radiation frequency, and $\nu_0 = c/\lambda_0$ is the center frequency of the resonance response.

Let Y_1 and Y_2 be the real and imaginary parts of the resonator-field amplitude. Then, the energy of the field can be expressed as

$$W = Y_1^2 + Y_2^2. \quad (3)$$

With the use of the form of a solution to Eq. (1), the dynamic equations can be represented as

$$\begin{cases} \dot{\varphi} T = d\varphi_T & (4) \\ \dot{Y}_1 = -\gamma Y_1 + \cos\varphi & (5) \\ \dot{Y}_2 = -\gamma Y_2 + \sin\varphi, & (6) \end{cases}$$

where $T = 1/\nu_0$ is the period of field oscillations. A solution to Eq. (4) has the form

$$\varphi = 2\pi(\nu_0 - \nu)t + \varphi_0, \quad (7)$$

where, at the initial instant ($t = 0$), φ_0 is the phase that is determined from the solution to the dynamic equations describing the preceding transient at the preceding radiation-source frequency. Phase φ is retained during frequency switching owing to the condition for retaining the phases of the radiation source and of the resonator field.

Solutions to Eqs. (5) and (6) can be represented as

$$\begin{aligned} Y_{1,2} = {} & C_{1,2}\exp(-\gamma t) + A_{1,2}\sin(\alpha t + \varphi_0) \\ & + B_{1,2}\cos(\alpha t + \varphi_0), \end{aligned} \quad (8)$$

where $\alpha = 2\pi(\nu_0 - \nu)$. Coefficients C_1 and C_2 are determined from the boundary conditions in the case when amplitudes $Y_{1,2}$ and phase φ are retained during phase switching. Substituting solutions (8) into Eqs. (5) and (6) and equating the coefficients at identical functions of time in the obtained equations, we find coefficients $A_{1,2}$ and $B_{1,2}$. At each step, two nondegenerate linear systems of two equations for two unknowns are solved to obtain

$$\begin{aligned} A_1^n &= \frac{\alpha_n}{\alpha_n^2 + \gamma^2}, \quad B_1^n = \frac{\gamma}{\alpha_n^2 + \gamma^2}, \\ A_2^n &= \frac{\gamma}{\alpha_n^2 + \gamma^2}, \quad B_2^n = \frac{-\alpha_n}{\alpha_n^2 + \gamma^2}. \end{aligned} \quad (9)$$

Index n is the step number for the scanned radiation-source frequency, $\alpha_n = 2\pi(\nu_0 - \nu_n)$.

Thus, if Y_1, Y_2, and φ_0 are known at a certain instant when the driving-radiation frequency is switched, it is possible to find these quantities at the next switching instant:

$$\begin{aligned} Y_{1,2}^{n+1} = {} & C_{1,2}^{n+1}\exp(-\gamma T_S) \\ & + A_{1,2}^{n+1}\sin(\alpha_{n+1}T_S + \varphi_0^{n+1}) \\ & + B_{1,2}^{n+1}\cos(\alpha_{n+1}T_S + \varphi_0^{n+1}), \end{aligned} \quad (10)$$

where $C_{1,2}^{n+1} = Y_{1,2}^n - A_{1,2}^n\sin(\varphi_0^{n+1}) - B_{1,2}^n\cos(\varphi_0^{n+1})$, $\varphi_0^{n+1} = \alpha_n T_S + \varphi_0^n$, and T_S is the time interval between frequency switchings. Using recurrence relationships (10) and expressions (9) and assuming that the resonator field amplitude is negligible at frequencies far from the resonance value (at the first step, all coefficients A, B, and C are equal to zero), we can obtain Y_1 and Y_2 at all frequencies of exciting radiation. The resonator's radiation energy W, which determines the resonance response, is obtained by formula (3) at each frequency step. Thus, we arrive at a numerical model of the resonator's response.

When the frequency of exciting radiation is fast scanned, the shape of the resonance response may be distorted owing to the finite response time of the receiving circuits. The analysis of this aspect is of practical importance for the formulated problem. Hot-electron bolometers that are cooled by liquid helium (see, e.g., http://www.terahertz.co.uk/QMCI/qmc.html) are the most promising detectors for use in millimeter- and submillimeter-wave resonator spectrometers. These bolometers operate efficiently over wider frequency bands, are more sensitive, and have more uniform amplitude–frequency characteristics as compared to other types of receivers. However, the speed of the response of these bolometers is limited by frequencies lower than 1 MHz. The best modern low-noise amplifiers used as video amplifiers of detected radiation signals also have a bandwidth that is no larger than 1 MHz.

In addition, in the experiment, the receiver's passband is sometimes narrowed purposely for filtering out of high-frequency noises. However, when the passband is narrowed substantially, the useful signal amplitude is also decreased. It is shown below that the bandwidth of receiving circuits can be optimized at a given scan speed by simulating distortions of the signal shape. The optimum bandwidth corresponds to the maximum signal-to-noise ratio (SNR), i.e. the ratio of the desired signal to noise.

The finite response time of receiving circuits can be described by the following equation:

$$\dot{W}_R = \frac{1}{\tau_R}(W - W_R), \tag{11}$$

where W is the energy in the resonator; W_R is the received signal; and τ_R is the time constant of receiving circuits, which is related to receiver's bandwidth $\Delta\nu_R$ by the formula $\tau_R = 1/(\pi\Delta\nu_R)$. The substitution of Eqs. (8) into (3) yields the time dependence of the resonator energy

$$\begin{aligned} W = {} & (C_1^2 + C_2^2)\exp(-2\gamma t) + \frac{1}{\alpha^2 + \gamma^2} \\ & + [A_3 \sin(\alpha t + \varphi_0) + B_3 \cos(\alpha t + \varphi_0)]\exp(-\gamma t), \end{aligned} \tag{12}$$

where

$$A_3 = \frac{2}{\alpha^2 + \gamma^2}(\alpha C_2 + \gamma C_1) \text{ and } B_3 = \frac{2}{\alpha^2 + \gamma^2}(\gamma C_2 - \alpha C_1).$$

A general solution to Eq. (11) at $2\gamma \neq \gamma_R$, where $\gamma_R = 1/\tau_R$, can be represented as

$$\begin{aligned} W_R = {} & \frac{1}{\alpha^2 + \gamma^2} \\ & + [A_4 \sin(\alpha t + \varphi_0) + B_4 \cos(\alpha t + \varphi_0)] \\ & \times \exp(-\gamma t) + C_4 \exp(-2\gamma t) + C_5 \exp(-\gamma_R t), \end{aligned} \tag{13}$$

where

$$A_4 = \gamma_R \frac{\alpha B_3 + (\gamma_R - \gamma)A_3}{\alpha^2 + (\gamma_R - \gamma)^2}, \quad B_4 = \gamma_R \frac{(\gamma_R - \gamma)B_3 - \alpha A_3}{\alpha^2 + (\gamma_R - \gamma)^2},$$

and

$$C_4 = \frac{\gamma_R(C_1^2 + C_2^2)}{\gamma_R - 2\gamma}.$$

The substitution of the time interval between switchings for t in (13) yields the intensity of the received signal at an instant of measurement. Constant C_5 is found from the boundary conditions with allowance for the fact that W_R is retained during frequency switching. Thus, we arrive at the recurrence relation that yields the received signal at each step of the exciting-radiation frequency:

$$\begin{aligned} W_R^{n+1} = {} & \Big[W_R^n - \Big(\frac{1}{\alpha_n^2 + \gamma^2} + A_4^n \sin(\varphi_0^{n+1}) \Big) \exp(-\gamma T_S) \\ & + B_4^n \cos(\varphi_0^{n+1}) \exp(-\gamma T_S) + C_4^n \exp(-2\gamma T_S) \Big] \\ & \times \exp(-\gamma_R T_S) + \frac{1}{\alpha_{n+1}^2 + \gamma^2} \\ & + A_4^{n+1} \sin(\alpha_{n+1} T_S + \varphi_0^{n+1}) \\ & \times \exp(-\gamma T_S) + B_4^{n+1} \cos(\alpha_{n+1} T_S + \varphi_0^{n+1}) \\ & \times \exp(-\gamma T_S) + C_4^{n+1} \exp(-2\gamma T_S). \end{aligned} \tag{14}$$

Figure 2 shows the resonance response calculated at various scan speeds V for exciting radiation and various bandwidths of receiving circuits. Figures 2a–2c correspond to scan speeds of 0.027, 0.5, and 2 relative units meaning the number of halfwidths of the undistorted resonance response $\Delta\nu/2$ that are scanned during the time interval equal to the time constant of the resonator, τ. The receiver's bandwidth in kilohertz is indicated in the figure near the peak of the corresponding response. The responses obtained without considering the receiving-circuit effect are marked with stars. The ordinate is the response amplitude normalized by the amplitude of the corresponding undistorted response. The abscissa is the relative detuning between the current and resonance frequencies: $\delta\nu = (\nu - \nu_0)/\Delta\nu$. The calculations assumed the following parameters of a real Fabri–Perot resonator incorporated into the spectrometer [4, 5]: a resonance frequency of 100 GHz, a Q factor of 5×10^6, and an decay parameter of 6.2831×10^5 s^{-1}. Each of the records contains 200 frequency steps equal to 10 kHz. As has been expected, under slow frequency scanning ($V \ll 1$), the response is well described by the Lorentz curve. The deviation of the response marked by the star in Fig. 2a is only 10^{-14} rel. units. With an increase in the scan speed to $V \approx 1$, the amplitude of the response decreases, its width increases, and the peak shifts in the direction of frequency scanning. At $V \approx 0.5$, the drop-down wing of the response exhibits oscillations whose amplitude and number increase with the speed. This change in the response's shape is in agreement with the results of study [7]. In [7], the resonator's response is calculated and compared to experimental data for fast continuous scan of the exciting-radiation frequency. As

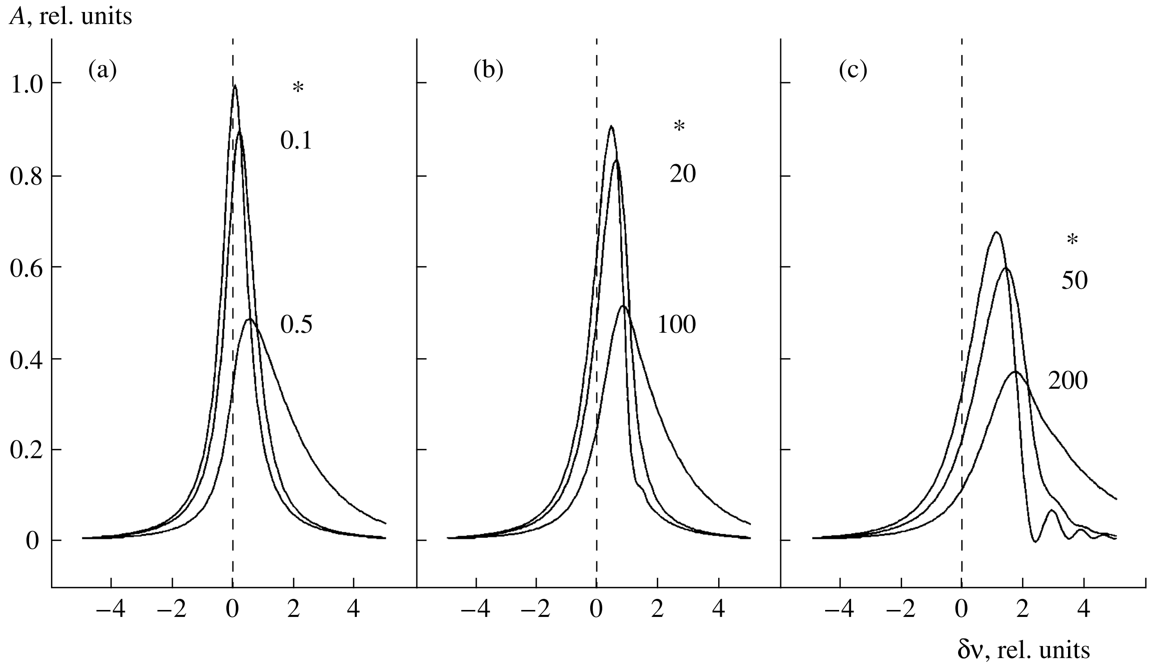

Fig. 2. Resonance response calculated at various frequency-scan speeds of exciting radiation and various bandwidths of receiving circuits. A is the response amplitude normalized by the amplitude of the corresponding undistorted response and $\delta\nu$ is the relative mismatch between current and resonance frequencies. The scan speeds are (a) 0.027, (b) 0.5, and (c) 2 relative units. In each figure, the receiver's bandwidth in kilohertz is indicated near the peak of the corresponding response. The responses marked by stars are obtained without considering the receiving-circuit effect.

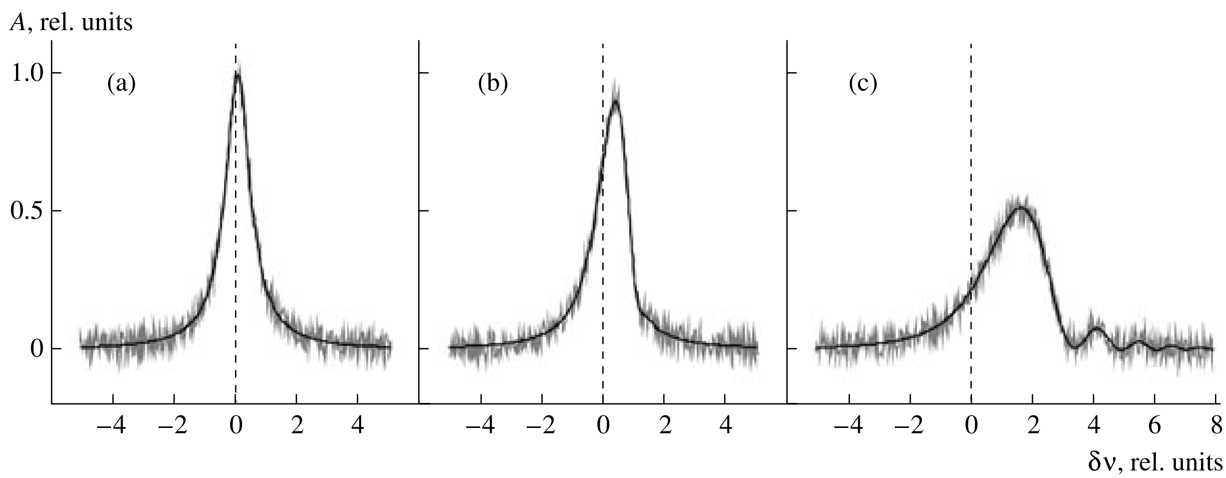

Fig. 3. Simulation of recording the resonance response at various scan speeds of exciting radiation (curves with noise components) and model responses fitting these data (smooth curves). The scan speeds are (a) 0.013, (b) 0.5, and (c) 4 rel. units. The experimental parameters are presented and the results are discussed in the text.

has been mentioned above, this is equivalent to the case under consideration when the number of steps is sufficiently large and the frequency step is sufficiently small. The responses calculated according to our model and to the method proposed in [7] are in good agreement. When the width of an undistorted response accommodates about 1000 steps, the results coincide. If the number of steps is approximately 300, a slight discrepancy is observed: the response is delayed during continuous scanning, and this delay increases as the number of steps decreases. The results obtained in this study and in [7] can be compared at $V = 2$ and 0.5. To this end, let us compare the responses marked with stars in Figs. 2b and 2c to the results displayed in Figs. 4b and 4c in [7]. Additionally note the good qualitative agreement of the shape of the response of a high-Q resonator that is observed experimentally under fast linear analog radiation-frequency scanning in the shortwave

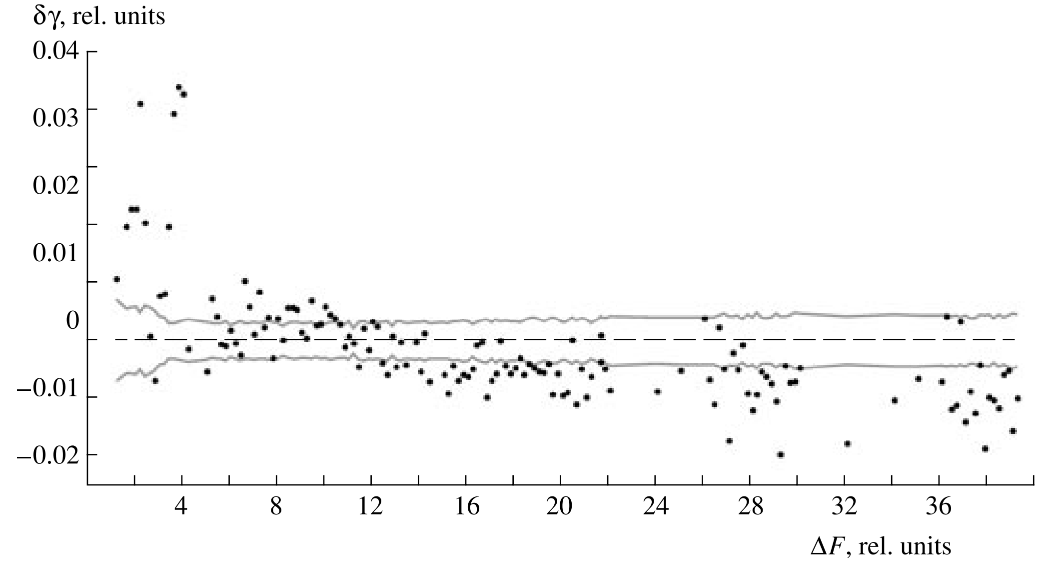

Fig. 4. Discrepancy of the measured decay parameter from the true value (dots) and the confidence interval of measurements (solid lines) vs. the frequency-scan range employed for recording resonance responses. Width ΔF of the scan range is expressed in half-widths of a resonance response.

part of the millimeter-wave band [8] and the shape of the response calculated according to the proposed model.

The effect of the bandwidth of receiving circuits on the shape of the received signal is illustrated in Fig. 2. As has been expected, this effect results in the delay of the response and its smearing in the direction of frequency scanning. The receiver's bandwidth necessary for an undistorted response to be observed increases linearly with *V*.

2. DETERMINATION OF THE RESONATOR'S PARAMETERS FROM THE SHAPE OF THE RESPONSE

The model developed in the previous section yields recurrence relations that provide for simulation of the shape of the resonance response using known parameters. However, solution of the inverse problem, that is, determination of the parameters of a resonator from the shape of its response is of practical importance. Unfortunately, the developed model does not permit employment of numerous available software packages that provide for mathematical determination of the numerical parameters of analytical functions that best fit experimental data. Therefore, it is necessary to develop an algorithm and the corresponding code that provide for fitting the parameters of a model resonator's response. In order to solve this problem, we introduce the estimator

$$E = \sum_{n=1}^{k} (W_n^{\mathrm{mod}} - W_n^{\mathrm{exp}})^2, \tag{15}$$

where k is the number of frequency steps (the number of points in a response), W_n^{mod} is the amplitude of a model response at the nth step of the exciting-radiation frequency, and W_n^{exp} is the amplitude of an experimental response. Function (15) is to be minimized over all its adjustable arguments, which are the decay parameter, amplitude, and frequency shift of the response. Being known from experimental conditions, the frequency step, the time between switchings, the number of frequency steps, and the time constant of receiving circuits are fixed arguments. At the minimum of the estimator, its first-order derivatives with respect to all indeterminate arguments vanish. The zeros of the derivatives of the estimator can be found using the multidimensional Newton method [9]. In the case of 1D function $f(x)$, the method involves seeking successive approximations to a solution to the equation $f(x) = 0$ according to the recurrence formula

$$x_{n+1} = x_n - \frac{f(x_n)}{\left.\frac{df}{dx}\right|_{x = x_n}}. \tag{16}$$

Vector $\vec{X}$ of the indeterminate arguments of estimator (15) is introduced instead of argument x, which characterizes the 1D case, and the vector of the derivatives of the estimator corresponds to function $f(x)$:

$$\vec{Y}(\vec{X}) = \nabla E(\vec{X}). \tag{17}$$

Instead of the derivatives $\frac{df(x)}{dx}$, we have the matrix

$$\mathbf{F} = \begin{pmatrix} \frac{\partial Y_1}{\partial X_1} & \frac{\partial Y_2}{\partial X_1} & \frac{\partial Y_3}{\partial X_1} \\ \frac{\partial Y_1}{\partial X_2} & \frac{\partial Y_2}{\partial X_2} & \frac{\partial Y_3}{\partial X_2} \\ \frac{\partial Y_1}{\partial X_3} & \frac{\partial Y_2}{\partial X_3} & \frac{\partial Y_3}{\partial X_3} \end{pmatrix}, \tag{18}$$

and recurrence relation (16) takes the form

$$\vec{X}_{n+1} = \vec{X}_n - \mathbf{F}^{-1}(\vec{X}_n)\vec{Y}_n. \tag{19}$$

After the appropriate number of iterations, the estimator's arguments determined by relationship (19) correspond to the parameters of the model resonator's response that best fit the experimental response. The values of the arguments in a small vicinity of expected results serve as initial data and can be obtained from preliminary experiments. The expressions presented above are easy to program.

The efficiency of the algorithm has been tested by the following procedure. First, the resonator's response is simulated using given parameters. Next, white noise is added to the response. The noise intensity is such that the SNR corresponds to a real experimental value. After that, the parameters of this response, which is treated as an experimental response, are recovered with the developed algorithm. In addition to the shape of the resonance response, the model involves both a linear multiplicative term that takes into account a spurious noise signal that is always observed experimentally at the receiver and an additive constant that makes allowance for d.c. voltage components in the receiver and amplifier channels of the spectrometer. Figure 3 shows the resonator's responses obtained with the developed algorithm at three different speeds of radiation-frequency scan. Determined decay parameters ($\gamma = (6.40 \pm 0.15) \times 10^5$, $(6.31 \pm 0.04) \times 10^5$, and $(6.29 \pm 0.15) \times 10^5$ s^{-1} at $V = 0.0013$, 0.5 and 4.0 rel. units, respectively) of the resonator coincide (within the error corresponding to the statistical uncertainty of a parameter at a given SNR) with the decay parameter used in the response simulation ($\gamma_0 = 6.2832 \times 10^5$ s$^{-1)}$.

3. OPTIMIZATION OF THE EXPERIMENTAL PARAMETERS

The decay parameter of the resonator can be measured at various values of such experimental parameters as the range of frequency scan within which the resonator's response is recorded; the time interval between frequency switchings (the time interval corresponding to one step); the bandwidth of the receiver; the step of the synthesizer used in frequency scan; and the total time of the experiment, which is equal to the product of the number of frequency steps, the time interval between frequency switchings, and the number of response records.

Optimization generally includes compromising between two conflicting requirements: a high accuracy of measurements and a short time of the experiment. In particular, the accuracy of determination of the decay parameter should be estimated as a function of the experimental parameters; that is, it is necessary to find the accuracy of the experiment ensured in a given time or the time needed to guarantee a given accuracy. It is necessary to solve the optimization problem in order to find out whether the decay parameter is measured with a higher accuracy when frequency is scanned slowly and the response is recorded once or when frequency is scanned fast many times and the obtained results are averaged. In both cases, the time of the experiment is assumed to be the same.

The developed algorithms can be applied to solve the optimization problems formulated above by means of the following computer simulation. First, N realizations of the recorded resonator's response that differ in noise noises are modeled using given parameters and relationships (9) and (10). Then, these records, regarded as experimental, are processed according to the algorithm described in Section 2. For each realization, decay parameter γ and its mean $\bar{\gamma}$ and variance σ_γ are determined. Next, one of the parameters to be optimized (frequency-scan speed, frequency step, the number of steps, etc.) is changed successively and the procedure is repeated. The same operations are performed for the remaining parameters to be optimized. After that, the discrepancy of the obtained decay parameter from its actual value γ_0 and confidence interval $\sigma_\gamma/\sqrt{N}$ are analyzed as functions of the variable parameter. As an example, Fig. 4 demonstrates optimization of the range of frequency scan used to record the resonance response. In the figure, width ΔF of the scan range is expressed in halfwidths of the undistorted response $\Delta\nu/2$. The calculation is performed at a scan speed of $V = 0.02$ rel. units and an SNR of 20. It is assumed that all responses are recorded at equal numbers of frequency steps (167). Each point is the result of averaging over 100 realizations.

Discrepancy $\delta\gamma$ of the obtained decay parameter from the true value is shown by dots, and the boundaries of the confidence interval are shown by solid lines. Both quantities are normalized by the true value of the decay parameter. Both the spread of points observed for a small change in ΔF and the confidence interval indicate the statistical accuracy of the determined decay parameter. The minimum discrepancy of the obtained decay parameter from the true value is observed at $\Delta F \approx 11$–13 halfwidths. Most points are located within the confidence interval. As ΔF decreases to 8–10, the confidence interval decreases but the discrepancy points shift to the region of positive values. This means

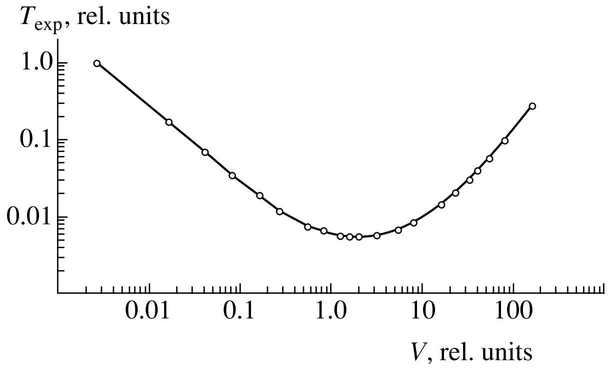

Fig. 5. Experimental time T_{exp} vs. frequency-scan speed V in the case of the preservation of the accuracy of the determined decay parameter.

that the statistical error of measurements decreases, as expected, owing to the relative increase of the number of points in the most informative part of the response, but the systematic error of measurements appears. As ΔF increases, the discrepancy points shift to the region of negative values and leave the confidence interval. This also indicates the systematic error. When $\Delta F \approx 40$, the systematic error is greater than the statistical error by a factor of 3 and amounts to 1.2%.

The optimum scan speed can be estimated using a simpler means. Assume that the experiment is not restricted to the use of narrowband receiving circuits and the SNR is rather high for the recorded resonator's response. Reducing the time of the experiment by increasing the frequency scan speed, we decrease the amplitude and enlarge the width of the resonator's response owing to the finite response time of the latter (see Fig. 2). The SNR decreases proportionally with the amplitude, and, as a consequence, the accuracy of determination of the decay parameter is deteriorated. If a loss in the accuracy is undesirable, the SNR initial level can be maintained by increasing the number of averaging operations and the time of the experiment. Since the width of the response increases, we have to increase the scan range. In order to retain the accuracy of measurements, it is necessary to increase the number of frequency steps; hence, the time of the experiment increases. Figure 5 shows the time of the experiment as a function of the relative frequency-scan speed V on the logarithmic scale. In the figure, duration of the experiment, T_{exp}, is normalized by the duration of an analogous experiment at the maximum frequency-scan speed in the existing variant of the spectrometer [4, 5]. The dependence has a pronounced minimum at a scan speed of 2 relative units. This velocity is optimum under the above assumptions for the experiment described in this paper. It is seen from the Fig. 5 that, at the optimum speed, the gain in the experimental time is higher than two orders of magnitude. Owing to an increase in the number of iterations, this result can be employed, in particular, for real-time monitoring of the decay parameter or for enhancing the sensitivity and accuracy of measurements.

The other experimental parameters can be optimized in a similar fashion. However, although the optimization method is obviously an attractive and efficient tool for choosing the experimental parameters, it is necessary to keep in mind that the results obtained can be regarded only as a good approximation of real results since even the best model cannot take into account the entire variety of experimental factors affecting the shape of a resonance response.

CONCLUSIONS

In this study, a theoretical basis for the next generation of resonator spectrometers with fast digital frequency scan has been developed.

A model is proposed for the response of an open Fabri–Perot resonator to exciting radiation that has both a frequency which is fast stepwise scanned and a phase that exhibits no jumps during frequency switchings. This model allows numerical determination of the shape of the resonance response at given experimental parameters. Agreement with the experimental and theoretical results of study [7], which have been obtained for fast continuously scanned radiation (the limit case for our model), indicates the adequacy of the developed model.

The proposed theoretical model of the response shape and the multidimensional Newton method are used to develop an algorithm for recovering the decay parameter and determining the resonator loss from experimental records of the resonator's response in the presence of fast stepwise frequency scan.

Simulations have demonstrated that the obtained results can be applied to optimize the experimental parameters. In particular, the maximum frequency-scan speed that is the most suitable for the experiment has been estimated. This speed exceeds the speed used in the existing spectrometer [4, 5] by more than two orders of magnitude and allows reduction of the experimental time. This factor may be useful both for real-time monitoring of radiation absorption and for substantial enhancement of the accuracy and sensitivity of measurements.

ACKNOWLEDGMENTS

The authors are grateful to I.M. Ovsyannikov for attention to the study and helpful recommendations, A.F. Krupnov for reading the manuscript and useful remarks, V.V. Dorovskikh for the help in processing of simulation results, and S.E. Tret'yakova for the help in preparing the layout of the manuscript.

This study was supported by the Russian Foundation for Basic Research, project nos. 03-02-16125 and 05-02-17522-a.

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