Neural Modeling and Computational Neuroscience

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Basic Hebb Rule

$$\tau_w \frac{d\boldsymbol{w}}{dt} = v\,\boldsymbol{u}$$

differential equation

t refers to the learning dynamics
t refers to the different input patters

$$\tau_w \frac{d\boldsymbol{w}}{dn} = v\,\boldsymbol{u}$$

Discrete-time - Basic Hebb Rule (Naïve)

$$\tau_w \frac{d\boldsymbol{w}}{dn} = v\,\boldsymbol{u}$$

differential equation

iterated map

$$\tau_w \Delta\boldsymbol{w}(n) = v(n)\,\boldsymbol{u}(n)$$

$$v(n) = \boldsymbol{w}(n)^T \boldsymbol{u}(n)$$

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \Delta\boldsymbol{w}(\mathrm{n})$$

$$\Delta\boldsymbol{w}(n) = \boxed{\frac{1}{\tau_w}}\, v(n)\,\boldsymbol{u}(n)$$

$$\Delta\boldsymbol{w}(n) = \eta\, v(n)\,\boldsymbol{u}(n)$$

learning rate

Discrete-time - Basic Hebb Rule (Euler)

differential equation

$$\tau_w \frac{d\boldsymbol{w}}{dn} = v\,\boldsymbol{u} \quad \Rightarrow \quad \frac{d\boldsymbol{w}}{dn} = \frac{1}{\tau_w} v\,\boldsymbol{u} = \mathrm{f}(\boldsymbol{w})$$

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + hf(\boldsymbol{w})$$

$$\boldsymbol{w}(n+1) = w(n) + \boxed{h\frac{1}{\tau_w}} v(n)\boldsymbol{u}(n)$$

$$\Delta\boldsymbol{w}(n) = \eta\, v(n)\,\boldsymbol{u}(n)$$

learning rate