Planar data classification with one hidden layer

Welcome to your week 3 programming assignment. It's time to build your first neural network, which will have a hidden layer. You will see a big difference between this model and the one you implemented using logistic regression.

You will learn how to:

- Implement a 2-class classification neural network with a single hidden layer
- Use units with a non-linear activation function, such as tanh
- Compute the cross entropy loss
- Implement forward and backward propagation

1 - Packages

Let's first import all the packages that you will need during this assignment.

- <u>numpy (www.numpy.org)</u> is the fundamental package for scientific computing with Python.
- <u>sklearn (http://scikit-learn.org/stable/)</u> provides simple and efficient tools for data mining and data analysis.
- <u>matplotlib (http://matplotlib.org)</u> is a library for plotting graphs in Python.
- testCases provides some test examples to assess the correctness of your functions
- planar_utils provide various useful functions used in this assignment

```
In [ ]: # Package imports
    import numpy as np
    import matplotlib.pyplot as plt
    from testCases_v2 import *
    import sklearn
    import sklearn.datasets
    import sklearn.linear_model
    from planar_utils import plot_decision_boundary, sigmoid, load_planar_o
    %matplotlib inline
    np.random.seed(1) # set a seed so that the results are consistent
```

2 - Dataset

First, let's get the dataset you will work on. The following code will load a "flower" 2-class dataset into variables X and Y.

```
In [ ]: X, Y = load_planar_dataset()
```

Visualize the dataset using matplotlib. The data looks like a "flower" with some red (label y=0) and some blue (y=1) points. Your goal is to build a model to fit this data.

```
In [ ]: # Visualize the data:
   plt.scatter(X[0, :], X[1, :], c=Y, s=40, cmap=plt.cm.Spectral);
```

You have:

```
- a numpy-array (matrix) X that contains your features (x1, x2
)
- a numpy-array (vector) Y that contains your labels (red:0, b
lue:1).
```

Lets first get a better sense of what our data is like.

Exercise: How many training examples do you have? In addition, what is the shape of the variables x and y?

Hint: How do you get the shape of a numpy array? (help) (https://docs.scipy.org/doc/numpy/reference/generated/numpy.ndarray.shape.html)

```
In [ ]: ### START CODE HERE ### (≈ 3 lines of code)
    shape_X = None
    shape_Y = None
    m = None  # training set size
    ### END CODE HERE ###

print ('The shape of X is: ' + str(shape_X))
    print ('The shape of Y is: ' + str(shape_Y))
    print ('I have m = %d training examples!' % (m))
```

Expected Output:

```
shape
of X (2, 400)
shape
of Y (1, 400)
m 400
```

3 - Simple Logistic Regression

Before building a full neural network, lets first see how logistic regression performs on this problem. You can use sklearn's built-in functions to do that. Run the code below to train a logistic regression classifier on the dataset.

```
In [ ]: # Train the logistic regression classifier
    clf = sklearn.linear_model.LogisticRegressionCV();
    clf.fit(X.T, Y.T);
```

You can now plot the decision boundary of these models. Run the code below.

Expected Output:

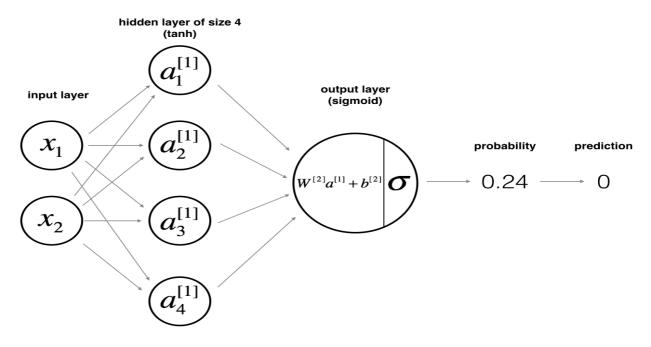
Accuracy 47%

Interpretation: The dataset is not linearly separable, so logistic regression doesn't perform well. Hopefully a neural network will do better. Let's try this now!

4 - Neural Network model

Logistic regression did not work well on the "flower dataset". You are going to train a Neural Network with a single hidden layer.

Here is our model:



Mathematically:

For one example $x^{(i)}$:

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$
(1)

$$a^{[1](i)} = \tanh(z^{[1](i)}) \tag{2}$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$
(3)

$$y^{(i)} = a^{[2](i)} = \sigma(z^{[2](i)}) \tag{4}$$

$$y^{(i)} = a^{[2](i)} = \sigma(z^{[2](i)})$$

$$y_{prediction}^{(i)} = \begin{cases} 1 & \text{if } a^{[2](i)} > 0.5\\ 0 & \text{otherwise} \end{cases}$$
(5)

Given the predictions on all the examples, you can also compute the cost J as follows:

$$J = -\frac{1}{m} \sum_{i=0}^{m} \left(y^{(i)} \log(a^{[2](i)}) + (1 - y^{(i)}) \log(1 - a^{[2](i)}) \right)$$
 (6)

Reminder: The general methodology to build a Neural Network is to:

- 1. Define the neural network structure (# of input units, # of hidden units, etc).
- 2. Initialize the model's parameters
- - Implement forward propagation
 - Compute loss
 - Implement backward propagation to get the gradients
 - Update parameters (gradient descent)

You often build helper functions to compute steps 1-3 and then merge them into one function we call nn model(). Once you've built nn model() and learnt the right parameters, you can make predictions on new data.

4.1 - Defining the neural network structure

Exercise: Define three variables:

```
n_x: the size of the input layer
n_h: the size of the hidden layer (set this to 4)
n y: the size of the output layer
```

Hint: Use shapes of X and Y to find n_x and n_y. Also, hard code the hidden layer size to be 4.

```
In [ ]: # GRADED FUNCTION: layer sizes
        def layer_sizes(X, Y):
             .....
            Arguments:
            X -- input dataset of shape (input size, number of examples)
            Y -- labels of shape (output size, number of examples)
            Returns:
            n x -- the size of the input layer
            n h -- the size of the hidden layer
            n y -- the size of the output layer
            ### START CODE HERE ### (≈ 3 lines of code)
            n_x = None # size of input layer
            n h = None
            n y = None # size of output layer
            ### END CODE HERE ###
            return (n x, n h, n y)
```

Expected Output (these are not the sizes you will use for your network, they are just used to assess the function you've just coded).

```
n_x 5n_h 4n_y 2
```

4.2 - Initialize the model's parameters

Exercise: Implement the function initialize parameters().

Instructions:

- Make sure your parameters' sizes are right. Refer to the neural network figure above if needed.
- You will initialize the weights matrices with random values.
 - Use: np.random.randn(a,b) * 0.01 to randomly initialize a matrix of shape (a,b).
- You will initialize the bias vectors as zeros.
 - Use: np.zeros((a,b)) to initialize a matrix of shape (a,b) with zeros.

```
In [ ]: GRADED FUNCTION: initialize parameters
        ef initialize_parameters(n_x, n_h, n_y):
           Argument:
           n x -- size of the input layer
           n h -- size of the hidden layer
           n y -- size of the output layer
           Returns:
           params -- python dictionary containing your parameters:
                           W1 -- weight matrix of shape (n h, n x)
                           b1 -- bias vector of shape (n h, 1)
                           W2 -- weight matrix of shape (n y, n h)
                           b2 -- bias vector of shape (n y, 1)
           np.random.seed(2) # we set up a seed so that your output matches our
           ### START CODE HERE ### (\approx 4 lines of code)
           W1 = None
           b1 = None
           W2 = None
           b2 = None
           ### END CODE HERE ###
           assert (W1.shape == (n h, n x))
           assert (b1.shape == (n h, 1))
           assert (W2.shape == (n_y, n_h))
           assert (b2.shape == (n_y, 1))
           parameters = {"W1": W1,
                         "b1": b1,
                         "W2": W2,
                         "b2": b2}
           return parameters
```

| W1 | [[-0.00416758 -0.00056267] [-0.02136196 0.01640271] [-0.01793436 -0.00841747] [0.00502881 -0.01245288]] |
|----|--|
| b1 | [[0.] [0.] [0.] [0.] |
| W2 | [[-0.01057952 -0.00909008 0.00551454 0.02292208]] |
| b2 | [[0.]] |

[[0 00440750 0 00050007] [0 00400400

4.3 - The Loop

Question: Implement forward propagation().

Instructions:

- Look above at the mathematical representation of your classifier.
- You can use the function sigmoid(). It is built-in (imported) in the notebook.
- You can use the function np.tanh(). It is part of the numpy library.
- The steps you have to implement are:
 - 1. Retrieve each parameter from the dictionary "parameters" (which is the output of initialize parameters()) by using parameters[".."].
 - 2. Implement Forward Propagation. Compute $Z^{[1]}, A^{[1]}, Z^{[2]}$ and $A^{[2]}$ (the vector of all your predictions on all the examples in the training set).
- Values needed in the backpropagation are stored in "cache". The cache will be given as an input to the backpropagation function.

```
In [ ]: # GRADED FUNCTION: forward propagation
        def forward propagation(X, parameters):
            Argument:
            X -- input data of size (n x, m)
            parameters -- python dictionary containing your parameters (output
            Returns:
            A2 -- The sigmoid output of the second activation
            cache -- a dictionary containing "Z1", "A1", "Z2" and "A2"
            # Retrieve each parameter from the dictionary "parameters"
            ### START CODE HERE ### (≈ 4 lines of code)
            W1 = None
            b1 = None
            W2 = None
            b2 = None
            ### END CODE HERE ###
            # Implement Forward Propagation to calculate A2 (probabilities)
            ### START CODE HERE ### (≈ 4 lines of code)
            Z1 = None
            A1 = None
            Z2 = None
            A2 = None
            ### END CODE HERE ###
            assert(A2.shape == (1, X.shape[1]))
            cache = {"Z1": Z1,}
                      "A1": A1,
                      "Z2": Z2,
                      "A2": A2}
            return A2, cache
```

0.262818640198 0.091999045227 -1.30766601287 0.212877681719 Now that you have computed $A^{[2]}$ (in the Python variable "A2"), which contains $a^{[2](i)}$ for every example, you can compute the cost function as follows:

$$J = -\frac{1}{m} \sum_{i=0}^{m} \left(y^{(i)} \log(a^{[2](i)}) + (1 - y^{(i)}) \log(1 - a^{[2](i)}) \right)$$
 (13)

Exercise: Implement compute_cost() to compute the value of the cost J.

Instructions:

 There are many ways to implement the cross-entropy loss. To help you, we give you how we would have implemented

```
-\sum_{i=0}^{m}y^{(i)}\log(a^{[2](i)}): \log probs = np.multiply(np.log(A2),Y) cost = -np.sum(logprobs)  # no need to use a for loop!
```

(you can use either np.multiply() and then np.sum() or directly np.dot()).

```
In [ ]: # GRADED FUNCTION: compute cost
        def compute_cost(A2, Y, parameters):
            Computes the cross-entropy cost given in equation (13)
            Arguments:
            A2 -- The sigmoid output of the second activation, of shape (1, nu
            Y -- "true" labels vector of shape (1, number of examples)
            parameters -- python dictionary containing your parameters W1, b1,
            Returns:
            cost -- cross-entropy cost given equation (13)
            m = Y.shape[1] # number of example
            # Compute the cross-entropy cost
            ### START CODE HERE ### (≈ 2 lines of code)
            logprobs = None
            cost = None
            ### END CODE HERE ###
            cost = np.squeeze(cost) # makes sure cost is the dimension we
                                        # E.g., turns [[17]] into 17
            assert(isinstance(cost, float))
            return cost
```

cost 0.693058761...

Using the cache computed during forward propagation, you can now implement backward propagation.

Question: Implement the function backward_propagation().

Instructions: Backpropagation is usually the hardest (most mathematical) part in deep learning. To help you, here again is the slide from the lecture on backpropagation. You'll want to use the six equations on the right of this slide, since you are building a vectorized implementation.

Summary of gradient descent

$$\begin{split} dz^{[2]} &= a^{[2]} - y \\ dW^{[2]} &= dz^{[2]}a^{[1]^T} \\ db^{[2]} &= dz^{[2]} \\ dz^{[2]} &= dz^{[2]} \\ dz^{[2]} &= \frac{1}{m}dZ^{[2]}A^{[1]^T} \\ dz^{[2]} &= \frac{1}{m}np. sum(dZ^{[2]}, axis = 1, keepdims = True) \\ dz^{[1]} &= W^{[2]T}dz^{[2]} * g^{[1]'}(z^{[1]}) \\ dW^{[1]} &= dz^{[1]}x^T \\ db^{[1]} &= dz^{[1]} \\ db^{[1]} &= \frac{1}{m}np. sum(dZ^{[1]}, axis = 1, keepdims = True) \\ &= \frac{1}{m}np.$$

- Tips:
 - To compute dZ1 you'll need to compute $g^{[1]'}(Z^{[1]})$. Since $g^{[1]}(.)$ is the tanh activation function, if $a = g^{[1]}(z)$ then $g^{[1]'}(z) = 1 a^2$. So you can compute $g^{[1]'}(Z^{[1]})$ using (1 np.power(A1, 2)).

```
In [ ]: # GRADED FUNCTION: backward propagation
        def backward propagation(parameters, cache, X, Y):
            Implement the backward propagation using the instructions above.
            Arguments:
            parameters -- python dictionary containing our parameters
            cache -- a dictionary containing "Z1", "A1", "Z2" and "A2".
            X -- input data of shape (2, number of examples)
            Y -- "true" labels vector of shape (1, number of examples)
            Returns:
            grads -- python dictionary containing your gradients with respect
            m = X.shape[1]
            # First, retrieve W1 and W2 from the dictionary "parameters".
            ### START CODE HERE ### (≈ 2 lines of code)
            W1 = None
            W2 = None
            ### END CODE HERE ###
            # Retrieve also A1 and A2 from dictionary "cache".
            ### START CODE HERE ### (≈ 2 lines of code)
            A1 = None
            A2 = None
            ### END CODE HERE ###
            # Backward propagation: calculate dW1, db1, dW2, db2.
            ### START CODE HERE ### (* 6 lines of code, corresponding to 6 equal
            dZ2 = None
            dW2 = None
            db2 = None
            dZ1 = None
            dW1 = None
            db1 = None
            ### END CODE HERE ###
            grads = {"dW1": dW1,
                      "db1": db1,
                      "dW2": dW2,
                      "db2": db2}
            return grads
```

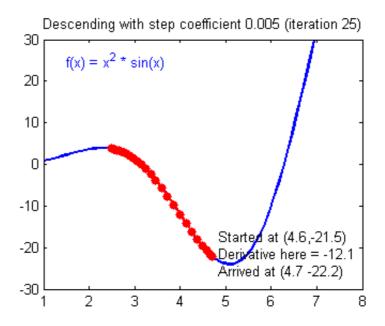
```
In [ ]: parameters, cache, X_assess, Y_assess = backward_propagation_test_case

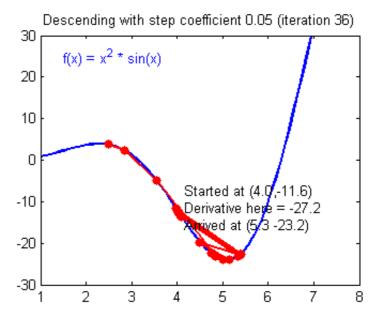
    grads = backward_propagation(parameters, cache, X_assess, Y_assess)
    print ("dW1 = "+ str(grads["dW1"]))
    print ("db1 = "+ str(grads["db1"]))
    print ("dW2 = "+ str(grads["dW2"]))
    print ("db2 = "+ str(grads["db2"]))
```

Question: Implement the update rule. Use gradient descent. You have to use (dW1, db1, dW2, db2) in order to update (W1, b1, W2, b2).

General gradient descent rule: $\theta = \theta - \alpha \frac{\partial J}{\partial \theta}$ where α is the learning rate and θ represents a parameter.

Illustration: The gradient descent algorithm with a good learning rate (converging) and a bad learning rate (diverging). Images courtesy of Adam Harley.





```
In [ ]: # GRADED FUNCTION: update parameters
        def update_parameters(parameters, grads, learning_rate = 1.2):
            Updates parameters using the gradient descent update rule given about
            Arguments:
            parameters -- python dictionary containing your parameters
            grads -- python dictionary containing your gradients
            Returns:
            parameters -- python dictionary containing your updated parameters
            # Retrieve each parameter from the dictionary "parameters"
            ### START CODE HERE ### (≈ 4 lines of code)
            W1 = None
            b1 = None
            W2 = None
            b2 = None
            ### END CODE HERE ###
            # Retrieve each gradient from the dictionary "grads"
            ### START CODE HERE ### (≈ 4 lines of code)
            dW1 = None
            db1 = None
            dW2 = None
            db2 = None
            ## END CODE HERE ###
            # Update rule for each parameter
            ### START CODE HERE ### (≈ 4 lines of code)
            W1 = None
            b1 = None
            W2 = None
            b2 = None
            ### END CODE HERE ###
            parameters = {"W1": W1,
                           "b1": b1,
                           "W2": W2,
                           "b2": b2}
            return parameters
```

```
In [ ]: parameters, grads = update_parameters_test_case()
    parameters = update_parameters(parameters, grads)

print("W1 = " + str(parameters["W1"]))
    print("b1 = " + str(parameters["b1"]))
    print("W2 = " + str(parameters["W2"]))
    print("b2 = " + str(parameters["b2"]))
```

W1 0.03978052] [-0.01936718] [-0.02410458 W1 0.03978052] [-0.01653973 -0.02096177] [0.01046864 -0.05990141]] b1 [[-1.02420756e-06] [1.27373948e-05] [8.32996807e-07] [-3.20136836e-06]] W2 [[-0.01041081 -0.04463285 0.01758031 0.04747113]] b2 [[0.00010457]]

4.4 - Integrate parts 4.1, 4.2 and 4.3 in nn_model()

Question: Build your neural network model in nn_model().

Instructions: The neural network model has to use the previous functions in the right order.

```
In [ ]: # GRADED FUNCTION: nn model
        def nn model(X, Y, n h, num iterations = 10000, print cost=False):
            Arguments:
            X -- dataset of shape (2, number of examples)
            Y -- labels of shape (1, number of examples)
            n h -- size of the hidden layer
            num_iterations -- Number of iterations in gradient descent loop
            print cost -- if True, print the cost every 1000 iterations
            Returns:
            parameters -- parameters learnt by the model. They can then be used
            np.random.seed(3)
            n_x = layer_sizes(X, Y)[0]
            n y = layer sizes(X, Y)[2]
            # Initialize parameters, then retrieve W1, b1, W2, b2. Inputs: "n x
            ### START CODE HERE ### (≈ 5 lines of code)
            parameters = None
            W1 = None
            b1 = None
            W2 = None
            b2 = None
            ### END CODE HERE ###
            # Loop (gradient descent)
            for i in range(0, num iterations):
                ### START CODE HERE ### (≈ 4 lines of code)
                # Forward propagation. Inputs: "X, parameters". Outputs: "A2, c
                A2, cache = None
                # Cost function. Inputs: "A2, Y, parameters". Outputs: "cost".
                cost = None
                # Backpropagation. Inputs: "parameters, cache, X, Y". Outputs:
                grads = None
                # Gradient descent parameter update. Inputs: "parameters, grads
                parameters = None
                ### END CODE HERE ###
                # Print the cost every 1000 iterations
                if print_cost and i % 1000 == 0:
                    print ("Cost after iteration %i: %f" %(i, cost))
            return parameters
```

```
In [ ]: X_assess, Y_assess = nn_model_test_case()
    parameters = nn_model(X_assess, Y_assess, 4, num_iterations=10000, prin
    print("W1 = " + str(parameters["W1"]))
    print("b1 = " + str(parameters["b1"]))
    print("W2 = " + str(parameters["W2"]))
    print("b2 = " + str(parameters["b2"]))
```

| 0.692739 | cost after iteration 0 |
|--|------------------------|
| : | : |
| [[-0.65848169 1.21866811] [-0.76204273 1.39377573] [0.5792005 -1.10397703] [0.76773391 -1.41477129]] | W1 |
| [[0.287592] [0.3511264] [-0.2431246] [-0.35772805]] | b1 |
| [[-2.45566237 -3.27042274 2.00784958 3.36773273]] | W2 |
| [[0.20459656]] | b2 |

4.5 Predictions

Question: Use your model to predict by building predict(). Use forward propagation to predict results.

Reminder: predictions =
$$y_{prediction} = 1\{\text{activation} > 0.5\} = \begin{cases} 1 & \text{if } activation > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

As an example, if you would like to set the entries of a matrix X to 0 and 1 based on a threshold you would do: $X_new = (X > threshold)$

```
In [ ]: # GRADED FUNCTION: predict

def predict(parameters, X):
    """
    Using the learned parameters, predicts a class for each example in
    Arguments:
    parameters -- python dictionary containing your parameters
    X -- input data of size (n_x, m)

Returns
    predictions -- vector of predictions of our model (red: 0 / blue:
    """

# Computes probabilities using forward propagation, and classifies
    ### START CODE HERE ### (~ 2 lines of code)
    A2, cache = None
    predictions = None
    ### END CODE HERE ###

return predictions
```

```
In [ ]: parameters, X_assess = predict_test_case()
    predictions = predict(parameters, X_assess)
    print("predictions mean = " + str(np.mean(predictions)))
```

predictions mean 0.666666666667

It is time to run the model and see how it performs on a planar dataset. Run the following code to test your model with a single hidden layer of n_h hidden units.

```
In [ ]: # Build a model with a n_h-dimensional hidden layer
    parameters = nn_model(X, Y, n_h = 4, num_iterations = 10000, print_cos

# Plot the decision boundary
    plot_decision_boundary(lambda x: predict(parameters, x.T), X, Y)
    plt.title("Decision Boundary for hidden layer size " + str(4))
```

Expected Output:

Cost after 0.218607

```
In [ ]: # Print accuracy
    predictions = predict(parameters, X)
    print ('Accuracy: %d' % float((np.dot(Y,predictions.T) + np.dot(1-Y,1-p))
```

Accuracy90%

Accuracy is really high compared to Logistic Regression. The model has learnt the leaf patterns of the flower! Neural networks are able to learn even highly non-linear decision boundaries, unlike logistic regression.

Now, let's try out several hidden layer sizes.

4.6 - Tuning hidden layer size (optional/ungraded exercise)

Run the following code. It may take 1-2 minutes. You will observe different behaviors of the model for various hidden layer sizes.

```
In [ ]: # This may take about 2 minutes to run

plt.figure(figsize=(16, 32))
hidden_layer_sizes = [1, 2, 3, 4, 5, 20, 50]
for i, n_h in enumerate(hidden_layer_sizes):
    plt.subplot(5, 2, i+1)
    plt.title('Hidden Layer of size %d' % n_h)
    parameters = nn_model(X, Y, n_h, num_iterations = 5000)
    plot_decision_boundary(lambda x: predict(parameters, x.T), X, Y)
    predictions = predict(parameters, X)
    accuracy = float((np.dot(Y,predictions.T) + np.dot(1-Y,1-prediction))
    print ("Accuracy for {} hidden units: {} %".format(n_h, accuracy))
```

Interpretation:

- The larger models (with more hidden units) are able to fit the training set better, until eventually the largest models overfit the data.
- The best hidden layer size seems to be around n_h = 5. Indeed, a value around here seems to fits the data well without also incurring noticable overfitting.
- You will also learn later about regularization, which lets you use very large models (such as n_h = 50) without much overfitting.

Optional questions:

Note: Remember to submit the assignment but clicking the blue "Submit Assignment" button at the upper-right.

Some optional/ungraded questions that you can explore if you wish:

- What happens when you change the tanh activation for a sigmoid activation or a ReLU activation?
- Play with the learning_rate. What happens?
- What if we change the dataset? (See part 5 below!)

You've learnt to:

- Build a complete neural network with a hidden layer
- Make a good use of a non-linear unit
- Implemented forward propagation and backpropagation, and trained a neural network
- See the impact of varying the hidden layer size, including overfitting.

Nice work!

5) Performance on other datasets

If you want, you can rerun the whole notebook (minus the dataset part) for each of the following datasets.

```
In [ ]: # Datasets
        noisy circles, noisy moons, blobs, gaussian quantiles, no structure =
        datasets = {"noisy_circles": noisy_circles,
                     "noisy_moons": noisy_moons,
                     "blobs": blobs,
                     "gaussian quantiles": gaussian_quantiles}
        ### START CODE HERE ### (choose your dataset)
        dataset = "noisy moons"
        ### END CODE HERE ###
        X, Y = datasets[dataset]
        X, Y = X.T, Y.reshape(1, Y.shape[0])
        # make blobs binary
        if dataset == "blobs":
            Y = Y%2
        # Visualize the data
        plt.scatter(X[0, :], X[1, :], c=Y, s=40, cmap=plt.cm.Spectral);
```

Congrats on finishing this Programming Assignment!

Reference:

- http://scs.ryerson.ca/~aharley/neural-networks/ (http://scs.ryerson.ca/~aharley/neural-networks/ (http://scs.ryerson.ca/~aharley/neural-networks/ (http://scs.ryerson.ca/ (<a href=
- http://cs231n.github.io/neural-networks-case-study/ (http://cs231n.github.io/neural-networks-case-study/ (http://cs231n.github.io/neural-networks-case-study/ (http://cs231n.github.io/neural-networks-case-study/ (http://cs231n.github.io/neural-networks-case-study/ (http://cs231n.github.io/neural-networks-case-study/)