010 - Sampling Distributions

EPIB 607

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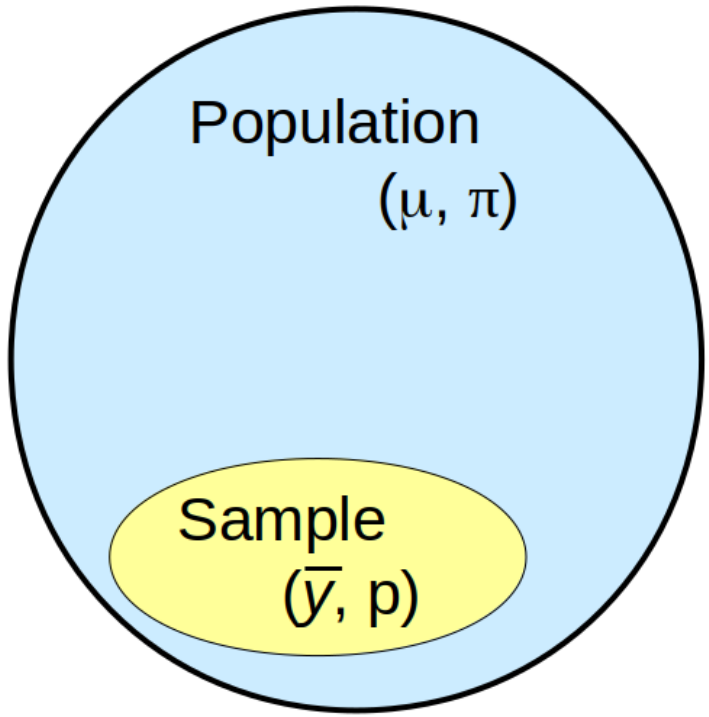
Parameters and Statistics

- **Paramter**: An unknown numerical constant pertaining to a population/universe, or in a statistical model.
  - μ: population mean π: population proportion
- **Statistic**: A numerical quantity calculated from a sample. The empirical counterpart of the parameter, used to *estimate* it.
  - $\bar{y}$: sample mean p: sample proportion



Examples

Proportions:

- Proportion of Earth's surface covered by water
- Proportion who saw a medical doctor last year
- Proportion of Québécois who don't have a family doctor

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Means:

- Mean depth in n randomly selected ocean locations
- Mean household size in n randomly selected households.
- Median number of persons under-5 in a sample of n households

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- When conducting a study, it is always better to seek statistical advice sooner rather than later. Get a statistician involved at the *planning* stage of the study... by the analysis stage, it may be too late!

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- Taking 5 people from the same household to estimate
  - proportion of Québécois who don't have a family doctor
  - who saw a medical doctor last year
  - average rent

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- Sampling the depth of the ocean only around Montreal to estimate
  - proportion of Earth's surface covered by water

Collecting data takes effort

In general

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Sampling Distributions

- Given a sample of n observations from a population, we will be calculating estimates of the population mean, proportion, standard deviation, and various other population characteristics (parameters)
- Prior to obtaining data, there is uncertainty as to which of all possible samples will occur
- Because of this, estimates such as $\bar{y}$ (the sample mean) will vary from one sample to another

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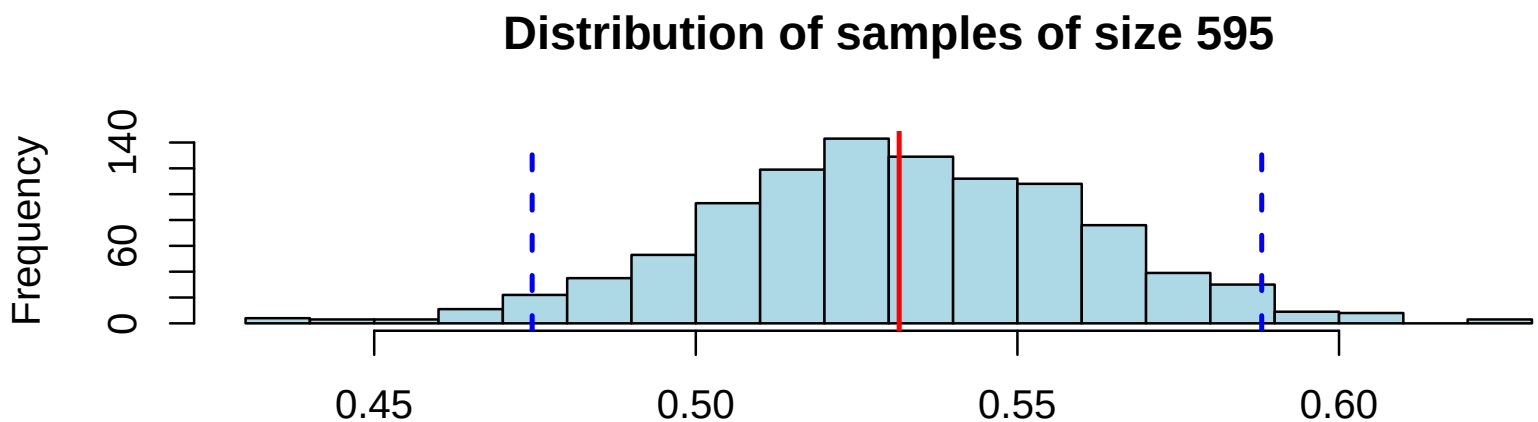
- DVB definition: If we could see all the statistics (means, proportions, ect.) from all possible samples (Chapter 18, page 432)

Sampling distribution of correlations[1]

Lets create a pseudo population from the 595 observations by sampling **with replacement**, and calculate the correlation. Lets repeat this process 1000 times:

```
library(oibiostat); data("famuss"); B <- 1000; N <- 595
R <- replicate(B, {
  dplyr::sample_n(famuss, size = N, replace = TRUE) %>%
  dplyr::summarize(r = cor(height, weight)) %>%
  dplyr::pull(r)
})
```



[1] from 004-exploring-data-2

Why are sampling distributions important?

- Modeling how sample statistics vary from sample to sample is one of the most powerful ideas we'll see in this course.
- A sampling distribution *model* for how a sample statistics varies from sample to sample allows us to quantify that variation and to talk about how likely it is that we'd observe a sample statistic in any particular interval.
- Thus, they are used in confidence intervals for parameters. Specific sampling distributions (based on a null value for the parameter) are also used in statistical tests of hypotheses.

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- We will get a sense of what a sampling distribution is in Exercise 1

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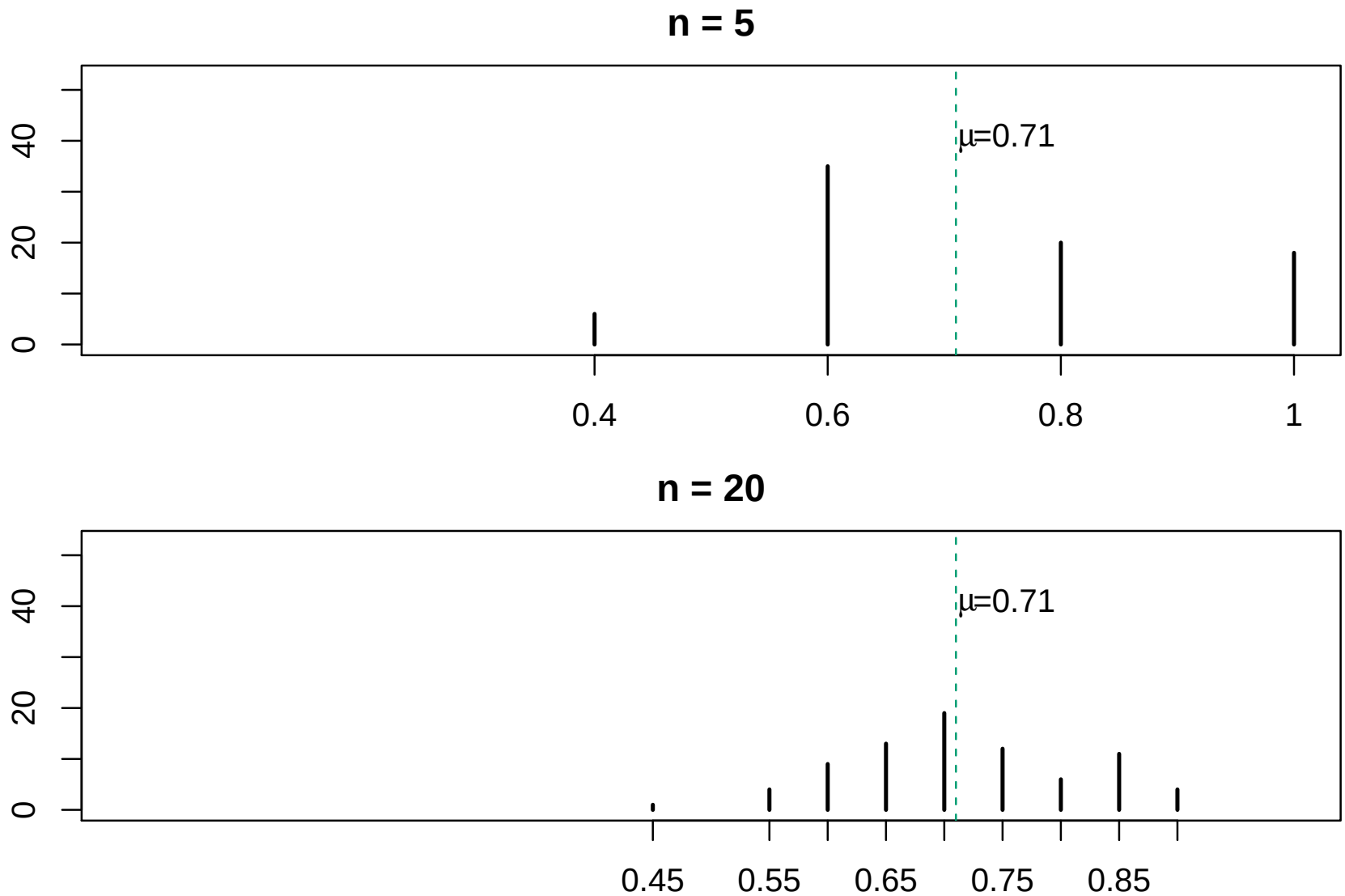
- **CAVEAT**: This is a luxury using a toy example. In actual studies, we only get one shot!

Samples

Sampling Distributions

Exercise 1 Results

Sampling distribution: proportion covered by water



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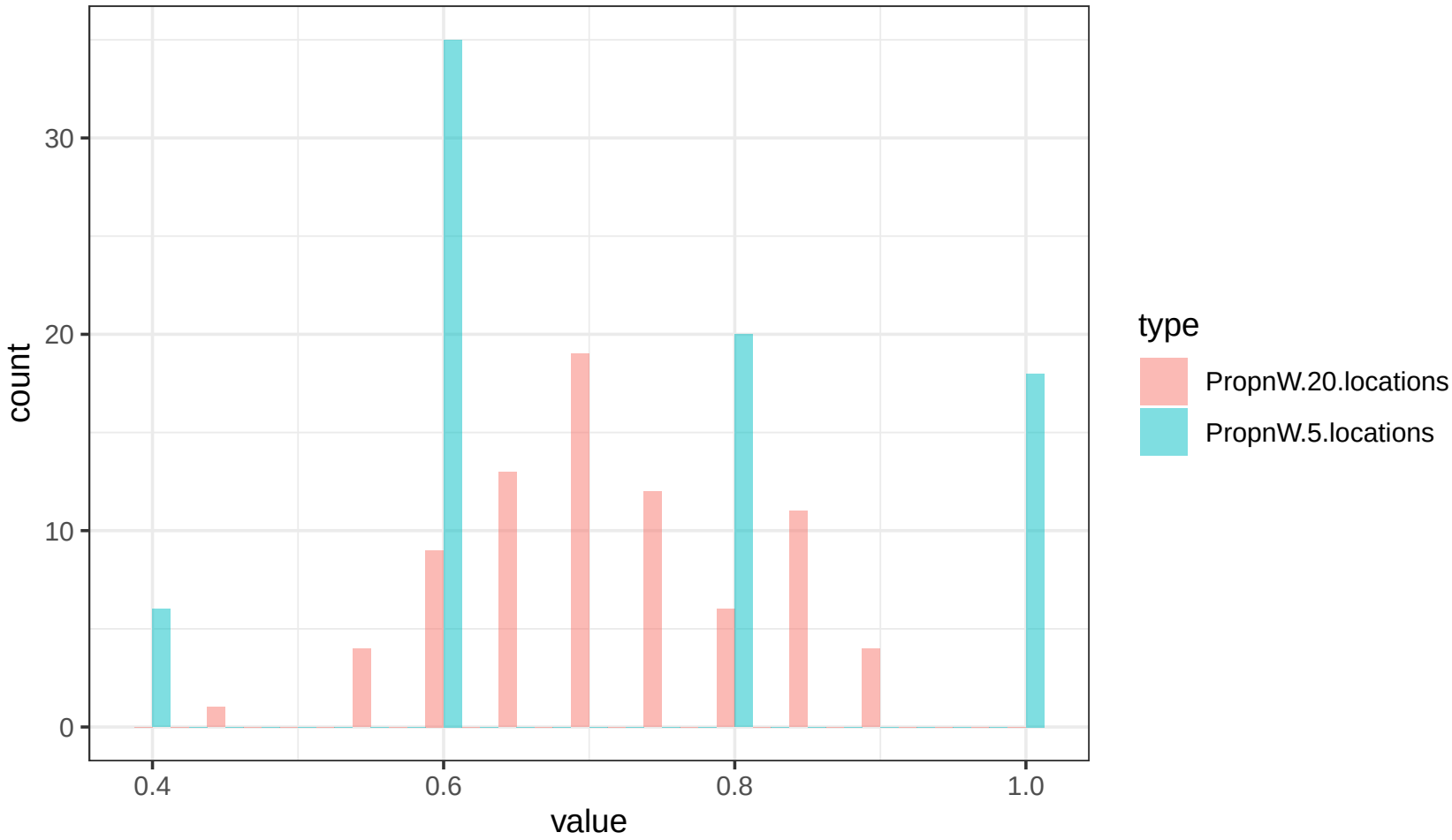
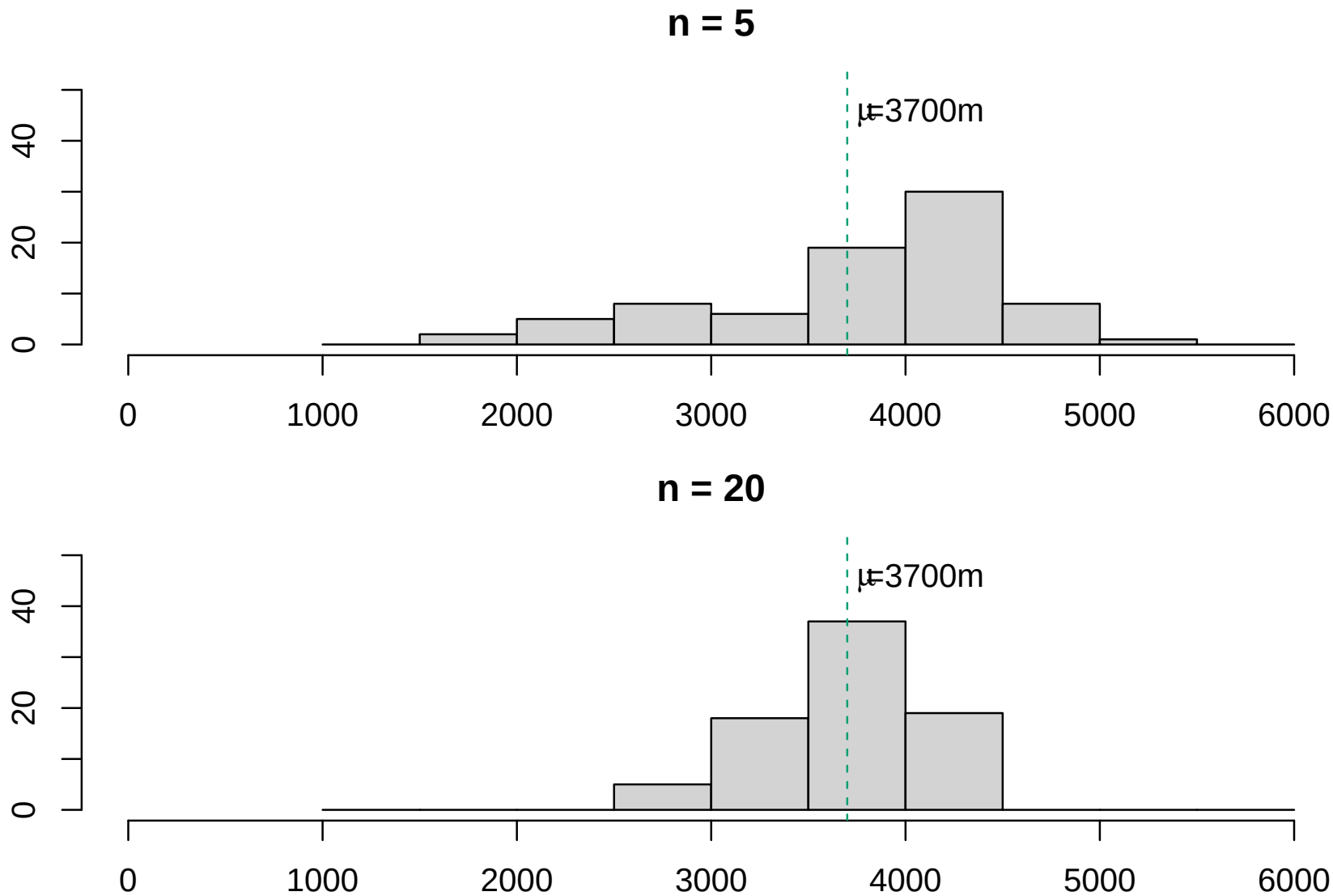


Sampling distribution: mean depth of the ocean



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