Group 2 - Lab 12 Submission

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Questions

Question 1. Find the equivalent polar and exponential forms of the below complex numbers:

- $5 – 2j$
- $6 + 4j$
- $5 – 5j$
- $2 + 3j$

Polar Form

$z = r(\cos \theta + j \sin \theta)$ where;

$r = \sqrt{x^2+y^2}$ and

$\theta = \tan^{-1} (\dfrac {y}{x})$

- $5.38(\cos 21.80 + j \sin 21.80)$

- $5.38(cos -21.80 + j sin -21.80)$

- $7.21(cos 33.69 + j sin 33.69)$

- $7.07(cos -45.00 + j sin -45.00)$

- $3.06(cos 56.31 + j sin 56.31)$

Exponential Form

$z = r\ e^{j\theta} $ where θ is in radians

- $5.38 e^{0.38j}$

- $5.38 e^{-0.38j}$

- $7.21 e^{0.59j}$

- $7.07 e^{-0.78j}$

- $3.06 e^{0.98j}$

Question 2. Find the equivalent rectangular form of the below complex numbers:

- $2e \dfrac{\pi}{3} j $
- $-4e \dfrac{\pi}{6} j $
- $5(\cos \dfrac{\pi}{3} + j\sin \dfrac{\pi}{3}) $
- $2(\cos \dfrac{\pi}{4} + j\sin \dfrac{\pi}{4}) $

Ans: In rectangular form; $z = x + jy$ and

$\tan \theta = \dfrac{y}{x}$ and $r^2 = x^2 + y^2$

$y \approx \sqrt {r^2 - x^2}$ and $x = \sqrt{\dfrac{r^2}{(\tan \theta)^2 + 1}}$

Respective Rectangular Form are

- $1 + j\sqrt 3$
- $\sqrt12 + 2j$

since $e^{j\theta} = \cos \theta + j \sin \theta$ and $z = r (\cos \theta + j \sin \theta) = re^{j\theta}$

- $\dfrac{5}{2} + j\dfrac {5}{2}\sqrt 3$
- $\sqrt2 + j\sqrt 2$

Question 3. For the two complex numbers given below, find the equivalent polar and exponential forms. Then, calculate Z_1Z_2 and $\dfrac{Z_1}{Z_2}$ for each of the 3 forms and show that they are equal.

- $Z_1 = 2 + 3j$ and $Z_2 = −1 + 4j$

Rectangular Form

$Z_1Z_2 = -14 + 5j$ and $\dfrac{Z_1}{Z_2} = \dfrac{-14 + 5j}{17}$

Polar Form

$z = r(\cos \theta + j \sin \theta)$ where;

$r = \sqrt{x^2+y^2}$ and

$\theta = \tan^{-1} (\dfrac {y}{x})$

$\approx Z_1 = \sqrt 13(\cos 56.31 + j \sin 56.31)$ and $Z_2 = \sqrt 17(\cos -75.96 + j \sin -75.96)$

$Z_1Z_2 = \sqrt 13 \times \sqrt17 (\cos -19.65 + j \sin-19.65)$ and $\dfrac{Z_1}{Z_2} = \dfrac{\sqrt13}{\sqrt17}(\cos 132.27 + j \sin 132.27)$

Exponential Form

$z = r\ e^{j\theta} $ where θ is in radians

$\approx Z_1 = \sqrt 13e^{j0.98}$ and $Z_2 = \sqrt17 e^{j-1.32}$

$Z_1Z_2 = \sqrt 13 \times \sqrt17 e^{j-0.34}$ and $\dfrac{Z_1}{Z_2} = \dfrac{\sqrt13}{\sqrt17}e^{j2.3}$

Prove

Given: $Z_1Z_2 = -14 + 5j$ and $\dfrac{Z_1}{Z_2} = \dfrac{-14 + 5j}{17}$

in polar form; $z = r (\cos \theta + j \sin \theta)$ $r = \sqrt {14^2 + 5^2} = \sqrt221 == \sqrt 13 \times \sqrt 17 (Proved)$

$\dfrac{Z_1}{Z_2} = \sqrt {{\dfrac{-14}{17}}^2 + {\dfrac{5}{17}}^2} = \sqrt \dfrac{13}{17} (Proved) $

$\theta = \tan^{-1}(\dfrac{y}{x}) = \tan^{-1}(\dfrac{5}{-14}) = -19.65 (Proved) $

$\dfrac{Z_1}{Z_2} = \tan^{-1}(\dfrac{\dfrac{5}{17}}{\dfrac{-14}{17}}) = -19.65 (Proved) $

in exponential form; $z = r\ e^{j\theta} $ where θ is in radians

$Z_1Z_2 = {\sqrt13}{\sqrt17} e^{-0.34} (Proved)$

$\dfrac{Z_1}{Z_2} = {\sqrt13}{\sqrt17}e^{-2.3} (Proved)$