# MATHEMATICS

Formulas:-

## SET-1:-

1) $\frac{d}{dx}(k) = 0$

2) $\frac{d}{dx} x^n = nx^{n-1}$

3) $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

4) $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{-1}{2x\sqrt{x}}$

5) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

6) $\frac{d}{dx} e^x = e^x$

7) $\frac{d}{dx} a^x = a^x \log a$

8) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

9) $\frac{d}{dx} x^{-n} = \frac{-n}{x^{n+1}}$

10) $\frac{d}{dx}(\ln x) = \frac{1}{x}$

11) $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

12) $\frac{d}{dx}\sqrt[n]{x} = \frac{1}{n\sqrt[n]{x^{n-1}}}$

13) $\frac{d}{dx}(\log_x a) = \frac{-\log a}{x(\log x)^2}$

## SET-2

① $\frac{d}{dx}\sin x = \cos x$

② $\frac{d}{dx}\cos x = -\sin x$

③ $\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}$

④ $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

⑤ $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$

⑥ $\frac{d}{dx}\sec x = \sec x \cdot \tan x$

## SET-3:-

① $\frac{d}{dx}\sinh x = \cosh x$

② $\frac{d}{dx}\cosh x = \sinh x$

③ $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$

④ $\frac{d}{dx}\coth x = -\operatorname{cosech}^2 x$

⑤ $\frac{d}{dx}\operatorname{cosech} x = -\operatorname{cosech} x \cdot \coth x$

⑥ $\frac{d}{dx}\operatorname{sech} x = \operatorname{sech} x \cdot \tanh x$

## SET-4

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$

## SET-5

$$\frac{d}{dx}\sinh^{-1}x = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}\cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tanh^{-1}x = \frac{1}{1-x^2}$$

$$\frac{d}{dx}\coth^{-1}x = \frac{1}{x^2-1}$$

$$\frac{d}{dx}\operatorname{sech}^{-1}x = \frac{-1}{|x|\sqrt{1-x^2}}$$

$$\frac{d}{dx}\operatorname{cosech}^{-1}x = \frac{-1}{|x|\sqrt{1+x^2}}$$

## SET-6

$$\frac{d}{dx}x^x = x^x(1+\ln x)$$

$$\frac{d}{dx}\sqrt{x^2+a^2} = \frac{x}{\sqrt{x^2-a^2}}$$

$$\frac{d}{dx}\sqrt{a^2-x^2} = \frac{-x}{\sqrt{a^2-x^2}}$$

$$\frac{d}{dx}\ln(x+\sqrt{x^2+a^2}) = \frac{1}{\sqrt{x^2+a^2}}$$

$$\frac{d}{dx}\ln(x-\sqrt{x^2+a^2}) = \frac{-1}{\sqrt{x^2+a^2}}$$

$$\frac{d}{dx}\ln(x+\sqrt{x^2-a^2}) = \frac{1}{\sqrt{x^2-a^2}}$$

$$\frac{d}{dx}\sqrt{f} = \frac{f'}{2\sqrt{f}}$$

$$\frac{d}{dx}\left(\frac{1}{f}\right) = \frac{-f'}{f^2}$$

$$\frac{d}{dx}(\ln\sec x) = \tan x$$

$$\frac{d}{dx}(\ln\sin x) = \cot x$$

$$\frac{d}{dx}(\ln(\sec x+\tan x)) = \sec x$$

$$\frac{d}{dx}(\ln(\operatorname{cosec}x+\cot x)) = -\operatorname{cosec}x$$

SET - 7

$$\frac{d}{dx}\left(\ln\left(\cot\frac{x}{2}\right)\right) = -\operatorname{cosec} x$$

$$\frac{d}{dx}\left(\ln\left(\tan\frac{x}{2}\right)\right) = -\operatorname{cosec} x \cdot \cot x$$

$$\frac{d}{dx}\left(\frac{\sin mx}{m}\right) = \cos mx$$

$$\frac{d}{dx}\left(\frac{-\cos mx}{m}\right) = \sin mx$$

$$\frac{d}{dx}\left(\frac{a^x}{\ln a}\right) = a^x$$

$$\frac{d}{dx}\left(\frac{e^{mx}}{m}\right) = e^{mx}$$

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n}\right) = x^n$$

SET - 8

$$\frac{d}{dx}(u \pm v) = \cancel{uv' + vu'}\ \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}(k f(x)) = k \frac{d}{dx} f(x)$$

$$\frac{d}{dx}(uv) = u'v + v'u \quad v'u + u'v$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \cancel{\frac{u'v - v'u}{v^2}} \quad \cancel{\frac{vu - u'v}{v^2}} \quad \frac{vu' - uv'}{v^2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\frac{d^2x}{dy^2} = -\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$$

## SHORT TRICKS :-

**chainrule :-**

$$\frac{d}{dx}\left(e^{x}f(x)\right) = e^{x}\left(f(x) + f'(x)\right)$$

$$\frac{d}{dx}\left(\frac{af(x)+b}{cf(x)+d}\right) = \frac{ad-bc}{(cf(x)+d)^2} \times f'(x)$$

$$\frac{d}{dx}\left(\frac{1+f(x)}{1-f(x)}\right) = \frac{2f'(x)}{(1-f(x))^2}$$ also for $\frac{d}{dx}\left(\frac{1-f(x)}{1+f(x)}\right) = \frac{-2f'(x)}{(1+f(x))^2}$

$$\frac{d}{dx}\left(\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\right) = \frac{-4}{(e^{x}-e^{-x})^2} = -\operatorname{cosec}^2 hx$$

$$\frac{d}{dx}\left(\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\right) = \frac{4}{(e^{x}+e^{-x})^2} = \sec^2 hx$$

$$\left[\sinh x = \frac{e^{x}-e^{-x}}{2}\ ;\ \cosh x = \frac{e^{x}+e^{-x}}{2}\ ;\ \tanh x = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\right]$$

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$$\frac{d}{dx}(\sin^{n}x) = n(\sin x)^{n-1}\cdot\cos x$$

$$\frac{d}{dx}(\cos^{n}x) = -\sin x\cdot n(\cos x)^{n-1}$$

$$\frac{d}{dx}(\sin nx) = n\cos nx$$

$$\frac{d}{dx}(\cos nx) = -n\sin nx$$

$$\frac{d}{dx}\sin x^{\circ} = \frac{\pi}{180}\cos x^{\circ}$$

$$\frac{d}{dx}\cos x^{\circ} = \frac{-\pi}{180}\sin x^{\circ}$$

$\frac{d}{dx}\sin^2 x^\circ = \frac{\pi}{180} 2\sin x^\circ \cos x^\circ$

$\frac{d}{dx}\sin(x^\circ+30^\circ) = \frac{\pi}{180}\sec(x^\circ+30^\circ)\tan(x^\circ+30^\circ)$

$\frac{d}{dx}\{x\} = 1$ → Fractional part

$\frac{d}{dx}[x] = 0$ → greatest integer

$\frac{d}{dx}|x| = \frac{|x|}{x}$

$$\boxed{\begin{aligned}\sinh^{-1}x &= \log(x+\sqrt{x^2+1})\\ \cosh^{-1}x &= \log(x+\sqrt{x^2-1})\\ \tanh^{-1}x &= \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)\end{aligned}}$$

series

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$\frac{d}{dx}(1+x+x^2+x^3\dots x^n) = n\left(\frac{n+1}{2}\right)$

## Substitution:

In the given function put if

$f(x) = \frac{2x}{1+x^2}$ put $x = \tan\theta$

$f(x) = \frac{1-x^2}{1+x^2}$ $x = \tan\theta$

$2x^2-1 ; 3x-4x^3$ ~~$x = \cos\theta$~~

~~$1-2x^2 ; 4x^3-3x$~~ $x = s$

$f(x) = 1 - 2x^2 ; 3x-4x^3$ put $x = \sin\theta$

$2x^2-1 ; 4x^3-3x$ $x = \cos\theta$

$$\boxed{\begin{aligned} 3\sin^{-1}x &= \sin^{-1}(3x-4x^3) \\ 3\cos^{-1}x &= \cos^{-1}(4x^3-3x) \\ 3\tan^{-1}x &= \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) \end{aligned}}$$

$$\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x} = \frac{\cos x+\sin x}{\cos x-\sin x} = \sec 2x + \tan 2x$$

$$\tan\left(\frac{\pi}{4}-x\right) = \frac{1-\tan x}{1+\tan x} = \frac{\cos x-\sin x}{\cos x+\sin x} = \sec 2x - \tan 2x$$

$$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

$f(x) = \sqrt{1-x^2}$ put $x = \sin\theta$ (or) $\cos\theta$

$= \sqrt{1+x^2}$ put $x = \tan\theta$ (or) $\cot\theta$
for $\sin^{-1}$ for $\cos^{-1}$

$= \sqrt{x^2-1}$ put $x = \sec\theta$ (or) $\operatorname{cosec}\theta$

$= \sqrt{a^2-x^2}$ put $x = a\sin\theta / a\cos\theta$

$= \sqrt{a^2+x^2}$ put $x = a\tan\theta / a\cot\theta$

$= \sqrt{x^2-a^2}$ put $x = a\sec\theta / a\operatorname{cosec}\theta$

$\sqrt{1+x}$ $\sqrt{1-x}$

put $x = \cos 2\theta$

$\sqrt{\frac{1+x}{1-x}}$ $\sqrt{\frac{1-x}{1+x}}$

$\frac{1}{2}\cos^{-1}x = \sin^{-1}\sqrt{\frac{1-x}{2}}$

$\cos^{-1}\sqrt{\frac{1+x}{2}}$

$\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

$\cot^{-1}\sqrt{\frac{1+x}{1-x}}$

$\sec^{-1}\sqrt{\frac{2}{1+x}}$

$\operatorname{cosec}^{-1}\sqrt{\frac{2}{1-x}}$

$\sqrt{(x-a)(b-x)}$

~~$\sqrt{a+x} + \sqrt{a-x}$~~

$x = a\sin^2\theta + b\cos^2\theta$

$\sqrt{(x-a)(x-b)}$

$\sqrt{(a-x)(b-x)}$

$x = a\sec^2\theta - b\tan^2\theta$

$= |b-a|\sin\theta \cdot \cos\theta$

$|a-b|\sec\theta \cdot \tan\theta$

If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$ then $\frac{dy}{dx} = \frac{-x}{y}$

$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$ then $\frac{dy}{dx} = \frac{-x}{y}$

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

→ If $ax^2 + 2hxy + by^2 = c$ then $\frac{d^2y}{dx^2} = \frac{c(h^2-ab)}{(hx+by)^3}$

→ If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ then $\frac{d^2y}{dx^2} = \frac{\Delta}{(hx+by+f)^3}$

## Important Results

1) If $x^2+y^2=a^2$ then $\frac{dy}{dx}=\frac{-x}{y}$

2) If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$

3) If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then $\frac{dy}{dx}=\frac{-b^2x}{a^2y}$

4) If $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ then $\frac{dy}{dx}=\frac{b^2x}{a^2y}$

5) $\frac{d}{dx}\left(\frac{ax+b}{cx+d}\right)=\frac{ad-bc}{(cx+d)^2}$

6) $\frac{d}{dx}\left(\frac{af(x)+b}{c(fx)+d}\right)=\frac{(ad-bc)\times f'(x)}{(cf(x)+d)^2}$

*7) $\frac{d}{dx}\left(f(x)^{g(x)}\right)=(f(x))^{g(x)}\left[\frac{g(x)\cdot f'(x)}{f(x)}+g'(x)\log f(x)\right]$

* 8) $\frac{d}{dx}\left[f(x)^{f(x)}\right]=f(x)^{f(x)}\left[1+\log f(x)\right]\cdot f'(x)$

9) If $y=\log\sqrt{\frac{1+\sin mx}{1-\sin mx}}$ then $\frac{dy}{dx}=m\sec mx$

10) If $y=\log\sqrt{\frac{1-\sin mx}{1+\sin mx}}$ then $\frac{dy}{dx}=-m\sec mx$

11) If $y=\log\sqrt{\frac{1+\cos mx}{1-\cos mx}}$ then $\frac{dy}{dx}=-m\operatorname{cosec} mx$

12) If $y=\log\sqrt{\frac{1-\cos mx}{1+\cos mx}}$ then $\frac{dy}{dx}=m\operatorname{cosec} mx$

13) $\frac{d}{dx}(\sin^n x\cdot\sin nx)=n\sin^{n-1}x\cdot\sin(n+1)x$

14) $\frac{d}{dx}(\cos^n x\cdot\cos nx)=n\cos^{n-1}x\cos(n+1)x$

15) $\frac{d}{dx}(\sin^n x\cdot\cos nx)=-n\sin^{n-1}x\cos(n+1)x$

16) $\frac{d}{dx}(\cos^n x\sin nx)=-\cos^{n-1}x\cdot\sin(n+1)x$

If $y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) \ldots \infty}}}$ $\dfrac{dy}{dx} = \dfrac{f'(x)}{2y-1}$

18) $\tan^{-1}\left(\dfrac{x+a}{1-ax}\right) = \tan^{-1}x + \tan^{-1}a$

19) $\tan^{-1}\left(\dfrac{x-a}{1+ax}\right) = \tan^{-1}x - \tan^{-1}a$

20) $\cot^{-1}\left(\dfrac{ax-1}{a+x}\right) = \cot^{-1}x + \cot^{-1}a$

21) $\cot^{-1}\left(\dfrac{ax+1}{a-x}\right) = \cot^{-1}x - \cot^{-1}a$

If $y = (f(x))^y$ then $\dfrac{dy}{dx} = \dfrac{y^2 f'(x)}{f(x)\,[1 - y\log f(x)]}$

24) If $y = (f(x))^x$ then $\dfrac{dy}{dx} = y\left[\log f(x) + \dfrac{f'(x)}{f(x)}\right]$

25) If $x^m y^n = a^{m+n}$ then $\dfrac{dy}{dx} = \dfrac{-my}{nx}$

26) If $a^x + a^y = a^{x+y}$ then $\dfrac{dy}{dx} = -a^{y-x}$

27) If $x^m \cdot y^n = (x+y)^{m+n}$ then $\dfrac{dy}{dx} = \dfrac{y}{x}$

28) If $x^2 + y^2 + 2gx + 2fy + c = 0$ then $\dfrac{dy}{dx} = -\left(\dfrac{g+x}{f+y}\right)$

29) If $ax^2 + 2hxy + by^2 = 0$ then $\dfrac{dy}{dx} = -\left(\dfrac{ax+hy}{hx+by}\right)$

If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
then $\dfrac{dy}{dx} = \dfrac{-(ax + hy + g)}{hx + by + f}$

A Hot Girl Friend Has Boy Friend

31) If $x = at^2$ ; $y = 2at$ then $\dfrac{dy}{dx} = \dfrac{1}{t}$

If $x = a\cos^3\theta$ $y = a\sin^3\theta$ then $\dfrac{dy}{dx} = -\tan\theta$

## Shortcut formulae

1) If $y = \tan^{-1}\left[\dfrac{f(x) \pm g(x)}{1 \pm f(x)\cdot g(x)}\right]$ then $\dfrac{dy}{dx} = \dfrac{f'(x)}{1+(f(x))^2} \pm \dfrac{g'(x)}{1+(g(x))^2}$

2) If $y = \tan^{-1}\left[\dfrac{a\cos f(x) \pm b\sin f(x)}{b\cos f(x) \pm a\sin f(x)}\right]$ then $\dfrac{dy}{dx} = \pm f'(x)$

3) If $y = \cot^{-1}\left[\dfrac{a\cos f(x) \pm b\sin f(x)}{b\cos f(x) \pm a\sin f(x)}\right]$ then $\dfrac{dy}{dx} = \mp f'(a)$

4) If $y = \tan^{-1}\left(\dfrac{\sqrt{1+f(x)} \pm \sqrt{1-f(x)}}{\sqrt{1-f(x)} \mp \sqrt{1-f(x)}}\right)$ then $\dfrac{dy}{dx} = \dfrac{\mp f'(x)}{2\sqrt{1-(f(x))^2}}$

# INVERSE TRIGONOMETRY

I

| | Domain | Range |
|---|---|---|
| $\sin^{-1}$ | - | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ |
| $\cos^{-1}$ | | $[0, \pi]$ |
| $\tan^{-1}$ | | $(-\frac{\pi}{2}, \frac{\pi}{2})$ |
| $\cot^{-1}$ | | $(0, \pi)$ |
| $\operatorname{cosec}^{-1}$ | | $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$ |
| $\sec^{-1}$ | | $[0, \pi] - \{\frac{\pi}{2}\}$ |

| | Domain | Range |
|---|---|---|
| $\sin$ | - | $[-1, 1]$ |
| $\cos$ | . | $[-1, 1]$ |
| $\tan$ | - | $R$ |
| $\cot$ | - | $R$ |
| $\operatorname{cosec}$ | - | $R - \{-1, 1\}$ |
| $\sec$ | . | $R - \{-1, 1\}$ |

II

$\sin^{-1}(-x) = -\sin^{-1}x$ $\qquad$ $\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$ $\qquad$ $\cot^{-1}(-x) = \pi - \cot^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$ $\qquad$ $\sec^{-1}(-x) = \pi - \sec^{-1}x$

III

$\sin^{-1}x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

$\cos^{-1}x = \sec^{-1}\left(\frac{1}{x}\right)$

$\tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right)$

IV

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ ; $x \in [-1, -1]$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ ; $x \in R$

$\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$ ; $|x| \geq 1$

V

$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) = \cos^{-1}\left(\sqrt{1-y^2}\cdot\sqrt{1-x^2} - xy\right)$

$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right) = \cos^{-1}\left(\sqrt{1-y^2}\cdot\sqrt{1-x^2} + xy\right)$

$\cos^{-1}x + \cos^{-1}y = \sin^{-1}\left(y\sqrt{1-x^2} + x\sqrt{1-y^2}\right) = \cos^{-1}\left(xy - \sqrt{1-y^2}\cdot\sqrt{1-x^2}\right)$

$\cos^{-1}x - \cos^{-1}y = \sin^{-1}\left(y\sqrt{1-x^2} - x\sqrt{1-x^2}\right) = \cos^{-1}\left(xy + \sqrt{1-y^2}\cdot\sqrt{1-x^2}\right)$

All of these are valid if $x \geq 0$ $y \geq 0$ & $x^2+y^2 \leq 1$

But if $x, y \geq 0$ $x^2+y^2 \not\leq 1$

then $\pi - ($ $)$

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VI $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ ; $x>0, y>0$ $xy \not< 1$

$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ $x>0$ $y>0$ $xy>1$

$\tan^{-1}x + \tan^{-1}y = -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ $x<0$ $y>0$ $xy>1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ $x>0$ $y>0$ $xy<1$

$\tan^{-1}x - \tan^{-1}y = -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ $x<0$ $y>0$ $xy>1$

VII 2θ forms

$2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ ; if $x \in [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$

$= \pi - \sin^{-1}(2x\sqrt{1-x^2})$ ; if $x$ is $\frac{1}{\sqrt{2}} \leq x \leq 1$

$= -\pi - \sin^{-1}(2x\sqrt{1-x^2})$ ; if $-1 \leq x \leq \frac{-1}{\sqrt{2}}$

$= \cos^{-1}(1-2x^2)$

$2\cos^{-1}x = \cos^{-1}(2x^2-1)$ ; if $0 \leq x \leq 1$

$= 2\pi - \cos^{-1}(2x^2-1)$ ; if $-1 \leq x < 0$

&

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ if $-1<x<1$

$= \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ if $x>1$

$= -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ if $x<1$

$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right.$

1 3θ form

$3\sin^{-1}x = \sin^{-1}(3x-4x^3)$

$3\cos^{-1}x = \cos^{-1}(4x^3-3x)$

$3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Note:- If $x_1, x_2, x_3 \dots x_n \in R$ then

$\tan^{-1}x_1 + \tan^{-1}x_2 + \tan^{-1}x_3 \dots \tan^{-1}x_n = \tan^{-1}\left[\frac{S_1 - S_3 + S_5 - S_7 \dots}{1 - S_2 + S_4 + S_6 \dots}\right]$

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SHORTCUTS

1 radian = 56.29°

$1° = \frac{\pi}{180} = 0.017$

$$\sin^{-1}(\sin\theta) = \begin{cases} -\pi-\theta & \text{if } \theta \in \left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right] \\ \theta & \text{if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \pi-\theta & \text{if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \\ -2\pi+\theta & \text{if } \theta \in \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right] \end{cases}$$

$$\cos^{-1}(\cos\theta) = \begin{cases} -\theta & \text{if } \theta \in [-\pi, 0] \\ \theta & \text{if } \theta \in [0, \pi] \\ 2\pi-\theta & \text{if } \theta \in [\pi, 2\pi] \\ -2\pi+\theta & \text{if } \theta \in [2\pi, 3\pi] \end{cases}$$

$$\tan^{-1}(\tan\theta) = \begin{cases} \pi-\theta & \text{if } \theta \in \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \\ \theta & \text{if } \theta \in \left(-\frac{\pi}{2}, \pi/2\right) \\ \theta-\pi & \text{if } \theta \in \left(\pi/2, \frac{3\pi}{2}\right) \\ \theta-2\pi & \text{if } \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) \end{cases}$$

→ $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

→ ~~if~~ $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \frac{\pi}{4}$

---

1) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right]$

$= \tan^{-1}\left[\frac{S_1 - S_3}{1 - S_2}\right]$

2) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ then $xy + yz + zx = 1$

3) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$. then $x+y+z = xyz$

4) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2}$ then $x^2+y^2+z^2+2xyz = 1$

★ 5) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ then

$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

6) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$ then $xy+yz+zx = 3$

7) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ then $x^2+y^2+z^2+2xyz = 1$

8) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ then $xy+yz+zx = 3$

★ 9) If $\sin^{-1}x + \sin^{-1}y = \theta$ then $\cos^{-1}x + \cos^{-1}y = \pi - \theta$

★ 10) If $\cos^{-1}x + \cos^{-1}y = \theta$ then $\sin^{-1}x + \sin^{-1}y = \pi - \theta$

## Trigonometry :-

* $\sin^2 A + \cos^2 A = 1 \Rightarrow \dfrac{\sin A}{1+\cos A} = \dfrac{1-\cos A}{\sin A}$

* $\sec^2 A - \tan^2 A = 1 \qquad \sec A + \tan A = \dfrac{1}{\sec A - \tan A}$

* $\operatorname{cosec}^2 A - \cot^2 A = 1 \qquad \operatorname{cosec} A + \cot A = \dfrac{1}{\operatorname{cosec} A - \cot A}$

- $\tan^2 x \cdot \sin^2 x = \tan^2 x - \sin^2 x \;|\; \cot^2 x \cdot \cos^2 x = \cot^2 x - \sin^2 x$

* $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ } reverse the sign

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

* $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

$\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot A - \cot B}$

$\tan A + \tan B = \dfrac{\sin(A+B)}{\cos A \cos B}$

$\tan A - \tan B = \dfrac{\sin(A-B)}{\cos A \cos B}$

* $\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

$\tan(A+B) \cdot \tan(A-B) = \dfrac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \cdot \tan^2 B}$

$\cot(A+B)\cot(A-B) = \dfrac{\cot^2 A \cot^2 B - 1}{\cot^2 B - \cot^2 A}$

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

$\sin x + \cos x = \sqrt{2}\sin(45 + x) = \sqrt{2}\cos(45 - x)$

$\cos x - \sin x = \sqrt{2}\cos(x + 45) = \sqrt{2}\sin(45 - x)$

* $\sin 2\theta = 2\sin\theta\cos\theta = \cancel{2\cos^2\theta - 1 = 1 - 2\sin^2\theta} = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$$\cos 2\theta = \cos^2 A - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$$
$$= \frac{\cot^2 A - 1}{\cot^2 A + 1}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$= \frac{2\cot\theta}{\cot^2\theta - 1}$$

$$\cot 2\theta = \frac{\cot^2 A - 1}{2\cot A}$$

* $\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

* $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

* $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) = 2SC$

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) = 2CS$

$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) = 2CC$

$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) = -2SS$

$\sin(A+B+C) = \sin A\cos B\cos C + \cos A\sin B\cos C + \sin C\cos A\cos B - \sin A\sin B\sin C$

$\cos(A+B+C) = \cos A\cos B\cos C - \sin A\sin B\cos C - \sin A\cos B\sin C - \cos A\sin B\sin C$

Short tricks:-

1) $\sqrt{1+\sin 2A} = \pm(\cos A + \sin A)$

$\sqrt{1-\sin 2A} = \pm(\cos A - \sin A)$

2) $\sqrt{1+\cos KA} = 2\cos^2\left(\frac{KA}{2}\right)$

$\sqrt{1-\cos KA} = 2\sin^2\left(\frac{KA}{2}\right)$

$1+\cos 2A = 2\cos^2 A$

$1-\cos 2A$ ~~$1+\sin 2A$~~ $= 2\sin^2 A$

$\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$

pr * 3) $\cot A + \tan A = 2\operatorname{cosec} 2A$

$\cot A - \tan A = 2\cot 2A$

SA 4) $\tan\left(\frac{\pi}{4}+\theta\right) = \frac{1+\tan\theta}{1-\tan\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \cot(45^\circ-\theta)$

$= \sec 2\theta + \tan 2\theta$

$\tan\left(\frac{\pi}{4}-\theta\right) = \frac{1-\tan\theta}{1+\tan\theta} = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \cot(45^\circ+\theta)$

$= \sec 2\theta - \tan 2\theta$

5) $\tan\left(\frac{\pi}{4}+\frac{A}{2}\right) = \frac{1+\tan\frac{A}{2}}{1-\tan\left(\frac{A}{2}\right)} = \frac{\cot\left(\frac{A}{2}\right)+1}{\cot\left(\frac{A}{2}\right)-1} = \frac{\cos\left(\frac{A}{2}\right)+\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)-\sin\left(\frac{A}{2}\right)}$

$\frac{\sin x}{1+\cos x}$ $= \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{\operatorname{cosec} A+1}{\operatorname{cosec} A-1}} = \frac{1+\sin A}{\cos A} = \frac{\cos A}{1-\sin A}$

$= \frac{\operatorname{cosec} A+1}{\cot A} = \frac{\cot A}{\operatorname{cosec} A-1} = \sec A + \tan A = \cot\left(\frac{\pi}{4}-\frac{A}{2}\right)$

Note:- If $x\sin A - y\cos A = z$ then $x\cos A - y\sin A = \sqrt{x^2+y^2-z^2}$

$\tan A + \cot A = 2\operatorname{cosec} 2A$ $\quad \cot A - \tan A = 2\cot 2A$

$\cot A = \cot 2A + \operatorname{cosec} 2A$

6) $\tan A + 2\tan 2A + \ldots 2^{n-1}\tan^{n-1}A + 2^n\cot 2^n A = \cot A$

7) $\sin\theta \cdot \sin(\alpha-\theta)\sin(\alpha+\theta) = \frac{1}{4}\sin 3\theta$

$\cos\theta \cdot \cos(\alpha-\theta)\cos(\alpha+\theta) = \frac{1}{4}\cos 3\theta$ $\frac{1}{4}$

$\tan\theta \cdot \tan(\alpha-\theta)\tan(\alpha+\theta) = \tan 3\theta$

$\cot\theta \cdot \cot(\alpha-\theta)\cot(\alpha+\theta) = \cot 3\theta$

$\sec\theta \cdot \sec(\alpha-\theta)\cot(\alpha+\theta) = 4\sec 3\theta$

$\operatorname{cosec}\theta \cdot \operatorname{cosec}(\alpha-\theta)\operatorname{cosec}(\alpha+\theta) = 4\operatorname{cosec} 3\theta$

where $\alpha = 60°, 120°, 240°, 300°, 480°$

8) $\sin^2\theta + \sin^2(\alpha-\theta) + \sin^2(\alpha+\theta) = \frac{3}{2}$

$\cos^2\theta + \cos^2(\alpha-\theta) + \cos^2(\alpha+\theta)$

where $\alpha = 60°, 120°, 240, 300$.

~~9) $\sin^3\theta \pm \sin^3(\alpha+\theta) \pm \sin^3(\alpha+\theta) \pm$ 3~~

~~where~~

9) $\sin^3\theta + \sin^3(60-\theta) - \sin^3(60+\theta)$

$\sin^3\theta - \sin^3(120-\theta) + \sin^3(120+\theta)$

$\sin^3\theta - \sin^3(240-\theta) + \sin^3(240+\theta)$ $= \frac{3}{4}\sin 3\theta$

$\sin^3\theta + \sin^3(300-\theta) - \sin^3(300+\theta)$

$\sin^3\theta - \sin^3(60+\theta) + \sin^3(120+\theta)$

$\sin^3\theta + \sin^3(120+\theta) + \sin^3(240+\theta)$ $= -\frac{3}{4}\sin 3\theta$

$\sin^3\theta + \sin^3(240+\theta) - \sin^3(300+\theta)$

$\sin^3\theta - \sin^3(60+\theta) - \sin^3(300+\theta)$

same for $\cos\theta$ too... but reversed sign

10) $\tan\theta + \tan(\theta-\alpha) + \tan(\theta+\alpha) = 3\tan\theta$

$\cot\theta + \cot(\theta-\alpha) + \cot(\theta+\alpha) = 3\cot\alpha$

where $\alpha = 60°, 120°, 240°, 300°$

11) $\tan\theta + \tan(\alpha+\theta) + \tan(\alpha-\theta) = 3\tan 3\theta$

$\cot\theta + \cot(\alpha+\theta) + \cot(\alpha-\theta) = 3\cot 3\theta$

$\alpha = 60°, 120°, 240°, 300°$

$\cos\theta + \cos(\theta-\alpha) + \cos(\theta+\alpha) = 0$

12) $\cos\alpha + \cos\beta + \cos\gamma = 4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right) - \cos(\alpha+\beta+\gamma)$

4CCC − C

$\sin\alpha + \sin\beta + \sin\gamma = 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\gamma+\alpha}{2}\right) + \sin(\alpha+\beta+\gamma)$

4SSS + S

n- no. of repetitions (terms)

13) $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \dots \sin\theta_n = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \times \sin\left(\frac{f+l}{2}\right)$

$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \dots \cos\theta_n = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \times \cos\left(\frac{f+l}{2}\right)$

14) $\frac{\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \dots \sin\theta_n}{\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \dots \cos\theta_n} = \tan\left(\frac{f+l}{2}\right)$

odd → $\cos\frac{\pi}{2n+1} + \cos\frac{3\pi}{2n+1} + \dots \cos\left(\frac{(2n-1)\pi}{2n+1}\right) = \frac{1}{2}$

even → $\cos\frac{2\pi}{2n+1} + \cos\frac{4\pi}{2n+1} + \dots \cos\left(\frac{2n\pi}{2n+1}\right) = \frac{-1}{2}$

16) $\cos\alpha \cdot \cos\beta \cdot \cos\gamma = \frac{1}{4}\left[\cos(\alpha+\beta+\gamma) + \cos(\beta+\gamma-\alpha) + \cos(\gamma+\alpha-\beta) + \cos(\alpha+\beta-\gamma)\right]$

$\sin\alpha \cdot \sin\beta \cdot \sin\gamma = \frac{1}{4}\left[\sin(\alpha+\beta-\gamma) + \sin(\beta+\gamma-\alpha) + \sin(\alpha+\beta+\gamma-\alpha) - \sin(\alpha+\beta+\gamma)\right]$

17) $\cos\left(\frac{\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right)\cos\left(\frac{3\pi}{n}\right)\cdots\cos\left(\frac{(n-1)\pi}{n}\right) = \frac{(-1)^{\frac{n-1}{2}}}{2^{n-1}}$ (n is odd)

$= 0$ (n is even)

$\cos\left(\frac{\pi}{2n+1}\right)\cos\left(\frac{2\pi}{2n+1}\right)\cos\left(\frac{3\pi}{2n+1}\right)\cdots\cos\left(\frac{n\pi}{2n+1}\right) = \frac{1}{2^n}$

$\cos\left(\frac{\pi}{2n}\right)\cdot\cos\left(\frac{2\pi}{2n}\right)\cos\left(\frac{3\pi}{2n}\right)\cdots\cos\left(\frac{(n-1)\pi}{2n}\right) = \frac{\sqrt{n}}{2^{n-1}}$

18) $\sin\left(\frac{\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)\cdot\sin\left(\frac{3\pi}{n}\right)\cdots\sin\left(\frac{(n-1)\pi}{n}\right) = \frac{n}{2^{n-1}}$

$\sin\left(\frac{\pi}{2n+1}\right)\sin\left(\frac{2\pi}{2n+1}\right)\cdot\sin\left(\frac{3\pi}{2n+1}\right)\cdots\sin\left(\frac{n\pi}{2n+1}\right) = \frac{\sqrt{2n+1}}{2^n}$

$\sin\left(\frac{\pi}{2n}\right)\sin\left(\frac{2\pi}{2n}\right)\cdot\sin\left(\frac{3\pi}{2n}\right)\cdots\sin\left(\frac{(n-1)\pi}{2n}\right) = \frac{\sqrt{n}}{4^{n-1}}$

19) $\cos\theta\cdot\cos2\theta\cdot\cos4\theta\cdot\cos8\theta\cdots\cos2^{n-1}\theta = \frac{\sin(2^n\theta)}{2^n\sin\theta}$

$\cos\left(\frac{\pi}{2^n-1}\right)\cos\left(\frac{2\pi}{2^n-1}\right)\cos\left(\frac{4\pi}{2^n-1}\right)\cdots\cos\left(\frac{2^{n-1}\pi}{2^n-1}\right) = \frac{-1}{2^n}$

$\cos\left(\frac{\pi}{2^n+1}\right)\cos\left(\frac{2\pi}{2^n+1}\right)\cos\left(\frac{4\pi}{2^n+1}\right)\cdots\cos\left(\frac{2^{n-1}\pi}{2^n-1}\right) = \frac{1}{2^n}$

20) $\sqrt{2+\sqrt{2+\sqrt{2+\cdots\sqrt{2+2\cos(k\theta)}}}} = 2\cos\left(\frac{k\theta}{2^n}\right)$

$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\ldots}}}} = 2\cos\frac{\pi}{2^{n+1}}$

21) compendo & dividendo tricks:-

(i) $\frac{\sin\alpha}{\sin\beta} = \frac{m}{n}$ $\quad \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = \frac{m+n}{m-n}$

(ii) $\frac{\cos\alpha}{\cos\beta} = \frac{m}{n}$ $\quad -\cot\left(\frac{\alpha+\beta}{2}\right)\cot\left(\frac{\alpha-\beta}{2}\right) = \frac{m+n}{m-n}$

(iii) $\frac{\tan\alpha}{\tan\beta} = \frac{m}{n}$ $\quad \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{m+n}{m-n}$

(iv) $\cot\alpha\cdot\cot\beta = \frac{m}{n}$ $\quad \frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = \frac{m+n}{m-n}$

22) $\tan A + \tan B + k\tan A\cdot\tan B = k$

$\tan A - \tan B - k\tan A\cdot\tan B = k$

$\tan A - \tan B + k\tan A\tan B = -k$

$\tan A + \tan B - k\tan A\tan B = -k$

$\tan A + \tan B + \tan(A+B)\tan A\tan B = \tan(A+B)$

23) (i) If $A+B = 45°/225°/\cdots(4n+1)\frac{\pi}{4}$

$(1+\tan A)(1+\tan B) = 2$

$(1-\cot A)(1-\cot B) = 2$

$\frac{(1+\cot A)(1+\cot B)}{\cot A\cdot\cot B} = 2$

(ii) If $A+B = 135^\circ / 315^\circ / \cdots (4n+1)\frac{\pi}{4}$ $n \in I$

$$(1-\tan A)(1-\tan B) = 2$$
$$(1-\cot A)(1-\cot B) = 2$$
$$\frac{(\cot A - 1)(\cot B - 1)}{\cot A \cdot \cot B} = 2$$

If $A+B+C = 0 / 180^\circ / 360^\circ / \cdots 2\pi$

(iii) $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

$$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

(iv) If $A+B+C = 0$

$$\cos A + \cos B = 0$$
$$\sin A - \sin B = 0$$
$$\tan A + \tan B = 0$$
$$\cot A + \cot B = 0$$

(v) If $A+B = 90^\circ$

$$\sin^2 A + \sin^2 B = 1$$
$$\cos^2 A + \cos^2 B = 1.$$
$$\tan A \cdot \tan B = 1$$
$$\cot A \cdot \cot B = 1$$
$$\sin A = \cos B$$
$$\cos A = \sin B$$

(vi) If $A+B = 90^\circ$ $\tan A \cancel{+} - \tan B = 2\tan(A-B)$

(vii) If $a\sin\theta + b\cos\theta = c$

$$a\sin\theta - b\cos\theta = \sqrt{a^2+b^2-c^2}$$

# Ratios of special angles :-

1) $\sin 7\frac{1}{2}^\circ = \dfrac{\sqrt{4-\sqrt{6}-\sqrt{2}}}{2\sqrt{2}}$ $\quad \cos 7\frac{1}{2}^\circ = \dfrac{\sqrt{4+\sqrt{6}+\sqrt{2}}}{2\sqrt{2}}$

2) $\sin 9^\circ = \dfrac{\sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}}}{4}$ $\quad \cos 9^\circ = \dfrac{\sqrt{3+\sqrt{5}} + \sqrt{5-\sqrt{5}}}{4}$

✻ 3) $\sin 15^\circ$ (or) $\cos 75^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ $\quad \sin 75^\circ$ (or) $\cos 15 = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$\tan 15 = 2-\sqrt{3} = \cot 75^\circ$

$\cot 15 = 2+\sqrt{3} = \tan 75^\circ$

4) $\sec 15^\circ = \sqrt{6}-\sqrt{2} = \operatorname{cosec} 75^\circ$

$\operatorname{cosec} 15^\circ = \sqrt{6}+\sqrt{2} = se\,75^\circ$

5) $\sin 22\frac{1}{2}^\circ = \sqrt{\dfrac{\sqrt{2}-1}{2\sqrt{2}}} = \frac{1}{2}\sqrt{2-\sqrt{2}} = \cos 67\frac{1}{2}^\circ$

$\cos 22\frac{1}{2}^\circ = \sqrt{\dfrac{\sqrt{2}+1}{2\sqrt{2}}} = \frac{1}{2}\sqrt{2+\sqrt{2}} = \sin 67\frac{1}{2}^\circ$

$\tan 22\frac{1}{2}^\circ = \sqrt{2}-1 = \cos 67\frac{1}{2}^\circ$

$\cot 22\frac{1}{2} = \sqrt{2}+1 = \tan 67\frac{1}{2}^\circ$

✻ 6) $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4} = \cos 72$

$\cos 36 = \dfrac{\sqrt{5}+1}{4} = \sin 54$

$\cos 18^\circ = \dfrac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72$

$\sin 36^\circ = \dfrac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^\circ$

$\tan 18 = \sqrt{4\sqrt{5}-8}$

7) $\tan 7\frac{1}{2}° = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$

$\tan 37\frac{1}{2} = (\sqrt{3}-\sqrt{2})(\sqrt{2}+1)$

$\tan 52\frac{1}{2} = (\sqrt{3}+\sqrt{2})(\sqrt{2}-1)$

$\tan 82\frac{1}{2} = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$

## Transformations:-

1) $\Sigma \sin(2a) = 4\Pi \sin a$
2) $\Sigma \cos(2a) = -1-4\Pi \cos a$
3) $\Sigma \sin A = 4\Pi \cos\left(\frac{A}{2}\right)$
4) $\Sigma \cos A = 1+4\Pi \sin\frac{A}{2}$
5) $\Sigma \tan A = \Pi \tan A$
6) $\Sigma \tan\left(\frac{A}{2}\right) \cdot \tan\left(\frac{B}{2}\right) = 1$
7) $\Sigma \cot A \cdot \cot B = 1$
8) $\Sigma \cot\left(\frac{A}{2}\right) = \Pi \cot\left(\frac{A}{2}\right)$
9) $\Sigma \sin A \cos B \cos C = \Pi \sin A$
10) $\Sigma \sin A \sin B \cos C = 1+\Pi \cos A$

$\Sigma$ = sum  $\Pi$ = Product

| Degree | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° | 210° |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ |

| 225° | 240° | 270° | 300° | 315° | 330° | 360° |
|---|---|---|---|---|---|---|
| $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ | $2\pi$ |

# INTEGRATION

$$\int f(x)\,g(x)\,dx = f(x)\int g(x) - \int f'(x)\int g(x)\,dx$$

## SET-1

$\int k = kx + c$

$\int x^n = \frac{x^{n+1}}{n+1} + c$

$\int \frac{1}{x} = \log|x| + c$

$\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + c$

$\int e^x = e^x + c$

$\int a^x = \frac{a^x}{\log a} + c$

$\int \log x = x\log x - 1$

$\int \sqrt{x} = \frac{2}{3}x\sqrt{x}$

## SET-2

1) $\int \sin x = -\cos x + c$

2) $\int \cos x = \sin x + c$

3) $\int \sec^2 x = \tan x + c$

4) $\int \operatorname{cosec}^2 x = -\cot x + c$

5) $\int \sec x \cdot \tan x = \sec x + c$

6) $\int \operatorname{cosec} x \cdot \cot x = -\operatorname{cosec} x + c$

7) $\int \tan x = \log|\sec x|$ (or) $-\log|\cos x| + c$

8) $\int \cot x = \log|\sin x|$ (or) $-\log|\operatorname{cosec} x| + c$

9) $\int \sec x\,dx = \log|\sec x + \tan x| + c$
$= \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

10) $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c$
$= \log\left|\tan\frac{x}{2}\right| + c$

## SET-3

$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1}x + c = -\cos^{-1}x + c$

$\int \frac{1}{1+x^2} = \tan^{-1}x + c = -\cot^{-1}x + c$

$\int \frac{1}{|x|\sqrt{x^2-1}} = \sec^{-1}x + c = -\operatorname{cosec}^{-1}x + c$

$\int \frac{1}{\sqrt{1+x^2}} = \sinh^{-1}x + c$

$\int \frac{1}{\sqrt{x^2-1}} = \cosh^{-1}x + c$

## SET-4

$\int \frac{1}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

$\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

$\int \frac{1}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + c$
(or)
$= \log\left|x + \sqrt{x^2+a^2}\right| + c$

$\int \frac{1}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + c$
(or)
$\log\left|x + \sqrt{x^2-a^2}\right| + c$

## SET-5

① $\int \sinh x = \cosh x + c$

② $\int \cosh x = \sinh x + c$

③ $\int \tanh x = \log|\cosh x| + c$

④ $\int \coth x = \log|\sinh x| + c$

⑤ $\int \operatorname{sech} x = 2\tan^{-1}(e^{x}) + c$

⑥ $\int \operatorname{cosech} x = \log|\tanh(\frac{x}{2})| + c$

⑦ $\int \operatorname{sech}^2 x\,dx = \tanh x + c$

⑧ $\int \operatorname{cosech}^2 x\,dx = -\coth x + c$

⑨ $\int \operatorname{sech} x \tanh x\,dx = \operatorname{sech} x + c$

⑩ $\int \operatorname{cosech} x \cdot \coth x\,dx = \operatorname{cosech} x + c$

## SET-6

① $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

② $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

$\Rightarrow \int\sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

$\Rightarrow \int\sqrt{a^2+x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + c$

$\rightarrow \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + c$

## SET-7

① $\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2} + c$

② $\int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1-x^2} + c$

③ $\int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + c$

④ $\int \cot^{-1}x\,dx = x\cot^{-1}x + \frac{1}{2}\log(1+x^2) + c$

⑤ $\int \sec^{-1}x\,dx = x\sec^{-1}x - \cosh^{-1}x + c$

⑥ $\int \operatorname{cosec}^{-1}x\,dx = x\operatorname{cosec}^{-1}x + \cosh^{-1}x + c$

⑦ $\int \sinh^{-1}x\,dx = x\sinh^{-1}x - \sqrt{x^2+1} + c$

⑧ $\int \cosh^{-1}x\,dx = x\cosh^{-1}x - \sqrt{x^2-1} + c$

① $\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\log\left|\frac{x+a}{x+b}\right| + c$

② $\int \frac{1}{(x-a)(x^2+b)}dx = \frac{1}{a^2+b}\left[\frac{1}{x-a} - \frac{x+a}{x^2+b}\right]$

③ $\int \frac{1}{(ax+b)(cx+d)} = \frac{1}{ad-bc}\log\left|\frac{ax+b}{cx+d}\right|$

④ $\int \frac{1}{(x^2+a^2)(x^2+b^2)} = \frac{1}{b^2-a^2}\left[\frac{1}{a}\tan^{-1}\frac{x}{a} - \frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)\right]$

⑤ $\int \frac{x}{(x^2+a^2)(x^2+b^2)} = \frac{1}{2(b^2-a^2)}\log\left|\frac{x^2+b^2}{x^2+a^2}\right| + c$

⑥ $\int \frac{1}{x(x^n+1)} = \frac{1}{n}\log\left|\frac{x^n}{1+x^n}\right|$

⑦ $\int \frac{1}{x(1-x^n)} = \frac{1}{n}\log\left|\frac{x^n}{1-x^n}\right|$

$\cos x| + c$

$\operatorname{osec} x| + c$

$+ c$

$+ c$

$t x| + c$

$c$

$+ c$

$x^2+a^2| + c$

$\frac{x}{a}) + c$

$| + c$

$$\int |x|\,dx = \frac{x|x|}{2} + c$$

$$\int \frac{|x|}{x}\,dx = |x| + c$$

$$\int te^{t}\,dt = (t-1)e^{t} + c$$

$$\int te^{-t}\,dt = -(t+1)e^{-t} + c$$

$$\int e^{x}[f(x) + f'(x)] = e^{x} f(x) + c$$

$$\int e^{x} f(x)\,dx = e^{x}[f(x) - f'(x) + f''(x) \dots]$$

$$\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx] + c$$

$$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx] + c$$

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}} = 2\log\left(\sqrt{x-a} + \sqrt{x-b}\right) + c$$

$$\int \frac{dx}{\sqrt{(x-a)(x+b)}} = 2\log\left(\sqrt{x-a} + \sqrt{x+b}\right) + c$$

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx = \left(\frac{bd+ac}{c^2+d^2}\right)x + \left(\frac{ad-bc}{c^2+d^2}\right)\log(c\cos x + d\sin x) + K$$

## Standard substitutions

| | |
|---|---|
| $\sqrt{a^2-x^2}$ | $x = a\sin\theta$ (or) $a\cos\theta$ |
| $\sqrt{a^2+x^2}$ | $x = a\tan\theta$ (or) $a\sinh\theta$ |
| $\sqrt{x^2-a^2}$ | $x = a\sec\theta$ (or) $a\cosh\theta$ |
| $\sqrt{a-x}$ | $x = a\sin^2\theta$ (or) $a\cos^2\theta$ |
| $\sqrt{a+x}$ | $x = a\tan^2\theta$ (or) |
| $\sqrt{x-a}$ | $x = a\sec^2\theta$ |

| | |
|---|---|
| $\sqrt{\dfrac{a-x}{a+x}}$ | $x = a\cos\theta$ |
| $\dfrac{a-x}{\sqrt{a^2-x^2}}$ | $x = a\sin\theta$ |

$$\frac{1}{\sqrt{a}}\sinh^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)$$

(or)

## NON IMP POINTS

$\int ax^2+bx+c$

$b^2 > 4ac$ $= \dfrac{1}{\sqrt{b^2-4ac}}\log\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}$

$b^2 < 4ac$ $= \dfrac{2}{\sqrt{4ac-b^2}}\tan^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right)$

$a < 0$ $= \dfrac{1}{\sqrt{b^2-4ac}}\log\dfrac{\sqrt{b^2-4ac}-(2ax+b)}{\sqrt{b^2-4ac}+(2ax+b)}$

(or)

$\dfrac{1}{\sqrt{(-a)}}\sin^{-1}\left(\dfrac{-(2ax+b)}{\sqrt{b^2-4ac}}\right)+k$

# MATRICES

- Rectangular arrangement of no. in rows & coloumns $1 \le i \le m$ ; $1 \le j \le n$ m, n - rows & colouns

- Row [ ]
- coloumn [ ]
- Null $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- square $[\ \ ]_{i \times j}$
- Rectangular $[\ \ ]_{i \times j}$

- Diagnol Matrix - unique
- scalar Matrix - same
- Identity Matrix - 1
- Null matrix - 0
- Triangular
  - UTM $i > j$ $a_{ij} = 0$
  - LTM $i < j$ $a_{ij} = 0$

In diagnol matrix min no. of zeros is $n^2 - n$

Min no. of zeros $= \frac{n^2 - n}{2}$

Min no. of distinct entries $= \frac{n^2 + n}{2} + 1$

Trace: sum of principal diagnal elements

$tr(AB) = tr(BA)$

$tr(A \pm B) = tr(A) \pm tr(B)$

$tr(A^T) = A$

$tr(AB) \neq tr(A) \cdot tr(B)$

$tr(KA) = K\, tr(A)$

$\rightarrow Tr(ABC) = Tr(BCA) = Tr(CAB)$

$\rightarrow tr(A + A^T) = 2tr(A)$ | $tr(A - A^T) = 0$

If $A = \begin{bmatrix} a & a \\ a & a \end{bmatrix}$ then $A^n = (\text{trace of } A)^{n-1} \cdot A$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^2 - (tr\ of\ A) \cdot A + |A| \cdot I = 0$

For only 2x2 matrix

Matrix multiplication

$AB \neq BA$

$(AB)C = A(BC) = ABC$ · Associative

$A(B+C) = AB + BC$ - Distributive

$AB = AC$ (B need not to be equal to C)

$(A+B)^2 = A^2 + B^2 + AB + BA$

$(A-B)^2 = A^2 + B^2 - AB - BA$

$A^m \cdot A^n = A^{m+n}$

$(A^m)^n = (A^n)^m$

Transpose

~~Trace of~~

$(A \pm B)^T = A^T \pm B^T$

$(KA)^T = KA^T$ ; $(A^T)^T = A$

$(AB)^T = B^T \cdot A^T$

$A = B \Rightarrow A^T = B^T$ ; $|A^T| = |A|$

~~$tr(A + A^T) = 2tr(A)$ $tr(A - A^T) = 0$~~

If $A^T = A$ - symmetric → Diagnol different
$a_{ij} = a_{ji}$

$A^T = -A$ - skew symmetric → Diagnal 0
$a_{ij} = -a_{ji}$

Properties

If A is symmetric

$A + A^T$, $A \cdot A^T$, $A^T \cdot A$

$-A$, $K \cdot A$, $A^T$, $A^{-1}$, $B^TAB$, $A^n$, $A^{2n}$ } symmetric

$A^n$ — odd - skewsym, even sym

$A - A^T$ - Skew symmetric

$A^{2n+1}$ (If A is skewsym)

If A, B are two symmetric

$A \pm B$, $AB + BA$ - symmetric

$AB - BA$ - Skew symmetric

Every square matrix canbe expressed as $\frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

If A is skew symmetric then $B^TAB$ - Skew sym (B - square matrix)
$C^TAC$ - 0 matrix (C - coloum matrix)

- If $AB = BA$ then they commute.
- If a matrix is given and asked to find power of n then substitute $n = 0$ in options and check with question [only for 2x2 square matrix]
- For idempodent matrix $A^2 = A$ ~~and $A^n$~~ — n is odd - A, n is even - I

If A, B are idem then $AB = BA = 0$

$A + B$, $I - A$ are idemp

$AB = A$ ; $BA = B$ ; $A^n + B^n = A + B$

Involuntary matrix - $A^2 = I$ — for n odd - A, even I

Nilpotent matrix - $A^P = 0$ (P is index of nilpotent)

Peridic matrix :- $A^{k+1} = A$

conjugate :- Representing corresponding conjugate elements of a matrix - $(\bar{A})$

$(\overline{A^T}) = (\bar{A})^T$ → Tranposed conjugate

if $(\overline{A^T}) = A$ Hermittan

$(\overline{A^T}) = -A$ skew hermittan

- If $AA^T = I = A^TA$ then $A^{-1} = A^T$ [orthogonal matrix]
- sum of squares of elements in a row/coloum is 1
- products of corresponding elements 0

If A is a square matrix

$A+A^T$, $AA^T$, $A^TA$ - symmetric

$A-A^T$ - skew symmetric

A, B are skew symmetric

$A^n \pm B^n$ — n is odd - skew symmetric
— n is even - symmetric

---

If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ the adj $A = \begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix}$

a - bc
b - ac
c - ab

$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ then Adj $A = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$

$A = \begin{bmatrix} 0 & 0 & a \\ 0 & a & 0 \\ a & 0 & 0 \end{bmatrix}$ then adj $A = -A$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

• If $AB = BA$ then they commute together

If $|A| = 0$ then A is a singular matrix

Properties of Adjoint & Inverse

① $A^{-1} = \frac{adj(A)}{|A|} \Rightarrow A \cdot A^{-1} = I_n$

② $A(adj A) = |A| \cdot I_n = adj(A) \cdot A$

③ $(AB)^{-1} = B^{-1}A^{-1}$

④ $(A^{-1})^T = (A^T)^{-1}$

⑤ $|adj A| = |A|^{n-1}$

⑥ $(adj\ AB) = adj(B) \cdot adj(A)$

⑦ $adj(A^T) = (adj(A))^T$

⑧ $adj(adj(A)) = |A|^{n-2} \cdot A$ ; $|adj(adj(A))| = |A|^{(n-1)^2}$

⑨ $|A^{-1}| = |A|^{-1}$

⑩ $(A^K)^{-1} = (A^{-1})^K$

⑪ $(KA^{-1}) = \frac{A^{-1}}{K}$

⑫ $adj(K(A_n)) = K^{n-1} adj(A_n)$

$AX = B$

(Homo) $B = 0$

- $|A| \neq 0$
  Trivial
  $x = y = z = 0$
- $|A| = 0$
  $\infty$ sol

(NonHomo) $B \neq 0$

- $|A| \neq 0$
  $x = A^{-1}B$
  Unique
- $|A| = 0$
  - $\text{adj}(A)\cdot B \neq 0$
    Inconsistent
  - $\text{adj}(A)\cdot B = 0$
    no sol

From crammers rule

$\Delta \neq 0$ – Unique sol ~~fatleos~~

$\Delta = 0$ $\Delta_1, \Delta_2, \Delta_3$ atleast one of is nonzero inconsistent

$\Delta \neq 0$ $\Delta_1 = \Delta_2 = \Delta_3 = 0$ – Trivial sol

$\Delta = 0$ $\Delta_1 = \Delta_2 = \Delta_3 = 0$ – $\infty$ solutions

$$x = \frac{\Delta_1}{\Delta} \quad y = \frac{\Delta_2}{\Delta} \quad z = \frac{\Delta_3}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

# TRANSFORMATIONS

## Laplace Transformation :-

→ If $f(t)$ is defined $\forall t \in R$ then

$$L\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, K(s,t)\, dt = \bar{f}(s)$$

$K(s,t) \longrightarrow$ Kernel of integral Transform

(i) $K(s,t) = e^{ist}$ (or) $e^{-ist}$ — Fourier Transform

(ii) If $K(s,t) = e^{-st}$ — Laplace

→ If $K(s,t) = e^{-st}$ then $L\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-st} dt = \bar{f}(s)$

— Bilateral L.T

(It fails for exp series)

→ If $f(t)$ is defined $\forall (t \geq 0)$ then

$$L\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)\, dt = \bar{f}(s)$$

— Unilateral L.T

## Necessary condition for existence of L.T :

- Laplace of $f(t)$ exists if $\int_0^{\infty} f(t)\, e^{-st} dt$ is convergent integral

$$\int_0^{\infty} \sin\omega t = [-\cos\omega t]_0^{\infty} = \pm 1 + \cos 0$$ [ossillating (between two finite values)

↓

It is not convergent integral

## Sufficient condition for existency :

* These are sufficient not necessary

- $f(t)$ is continuos
- $f(t)$ is a fn of exponential order
- $\lim_{t \to \infty} e^{-st} f(t) = 0$ (or) finite

NOTE (L.T exists for $\frac{1}{\sqrt{t}}$ even if is not piecewise) because $t \to 0$, $f(t) \to \infty$

$: \left(\sqrt{\frac{\pi}{s}}\right)$ and exponential

# L·T of Elementary Functions

1) $L(1) = 1/s \quad (s>0)$

2) $L(e^{at}) = \frac{1}{s-a} \quad (s>a) \qquad L\{a^t\} = \frac{1}{s-\log a}$

3) $L(e^{-at}) = \frac{1}{s+a} \quad (s>-a)$

4) $L\{\sin at\} = \frac{a}{s^2+a^2} \quad (s>0)$

5) $L\{\cos at\} = \frac{s}{s^2+a^2} \quad (s>0)$

6) $L\{\sinh at\} = \frac{a}{s^2-a^2} \quad (s>|a|)$

7) $L\{\cosh at\} = \frac{s}{s^2+a^2} \quad (s>|a|)$

8) $L\{t^n\} = \frac{n!}{s^{n+1}} = \frac{\Gamma n+1}{s^{n+1}}$

$(n \in z^+) \qquad (n \notin z^+)$

$(s>0)$

* $L\{t\sin at\} = \frac{2as}{(s^2+a^2)^2}$

* $L\{t\cos at\} = \frac{s^2-a^2}{(s^2+a^2)^2}$

$L\{t^2\sin at\} = (-2a)\frac{a^2-3s^2}{(s^2+a^2)^3}$

$L\{t^2\cos at\} = \frac{2s^3-6a^2s}{(s^2+a^2)^3}$

Q)

Gamma function :- use only when $n>-1$

$\Gamma n+1 = n\Gamma n = n(n-1)(n-2)\ldots$ for +ve values of n

$\Gamma n+1 = n! \quad (n \in z^+) \rightarrow \Gamma 4 = 3!\ ;\ \Gamma 5 = 4!\ ;\ \Gamma 6 = 5!\ \ldots$

$(\ast\, 0! = 1)$

$\Gamma 1 = 0 \quad \& \quad \Gamma 1/2 = \sqrt{\pi} \quad \Gamma{-\tfrac{1}{2}} = -2\sqrt{\pi} \qquad \Gamma{-P} = \text{infinite} \rightarrow P\text{ Integer}$

$\Gamma n = \frac{\Gamma n+1}{n}$ for −ve values of n

$(\Gamma 0, \Gamma{-1}, \Gamma{-2}, \Gamma{-3} \ldots$ are not defined)

$$\boxed{\therefore L(\sqrt{t}) = \frac{\sqrt{\pi}}{2s^{3/2}}\ ;\ L\{\tfrac{1}{\sqrt{t}}\} = \sqrt{\frac{\pi}{s}}}$$

$L\{t^{3/2}\} = \frac{3\sqrt{\pi}}{4s^{5/2}} \qquad L\{t^{5/2}\} = \frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}{s^{7/2}} = \frac{15\sqrt{\pi}}{8s^{7/2}}$

## L.T of Elementary Functions

1) $L(1) = 1/s \quad (s>0)$

2) $L(e^{at}) = \frac{1}{s-a} \quad (s>a)$ $\qquad L\{a^t\} = \frac{1}{s-\log a}$

3) $L(e^{-at}) = \frac{1}{s+a} \quad (s>-a)$

4) $L\{\sin at\} = \frac{a}{s^2+a^2} \quad (s>0)$

5) $L\{\cos at\} = \frac{s}{s^2+a^2} \quad (s>0)$

6) $L\{\sinh at\} = \frac{a}{s^2-a^2} \quad (s>|a|)$

7) $L\{\cosh at\} = \frac{s}{s^2+a^2} \quad (s>|a|)$

8) $L\{t^n\} = \frac{n!}{s^{n+1}} = \frac{\Gamma n+1}{s^{n+1}}$

$(n\in z^+) \quad (n\notin z^+)$

$(s>0)$

* $L\{t\sin at\} = \frac{2as}{(s^2+a^2)^2}$
* $L\{t\cos at\} = \frac{s^2-a^2}{(s^2+a^2)^2}$

$L\{t^2\sin at\} = (-2a)\frac{a^2-3s^2}{(s^2+a^2)^3}$

$L\{t^2\cos at\} = \frac{2s^3-6a^2s}{(s^2+a^2)^3}$

9)

**Gamma function:-** use only when $n>-1$

$\Gamma n+1 = n\Gamma n = n(n-1)(n-2)\ldots$ for +ve values of n

$\Gamma n+1 = n! \quad (n\in z^+) \rightarrow \Gamma 4 = 3!\,;\ \Gamma 5 = 4!\,;\ \Gamma 6 = 5! \ldots$

$0! = 1$

$\Gamma 1 = 0 \quad \& \quad \Gamma 1/2 = \sqrt{\pi} \quad \Gamma -\frac{1}{2} = -2\sqrt{\pi} \qquad \Gamma -p = \text{infinite} \xrightarrow{} \text{p-integer}$

$\Gamma n = \frac{\Gamma n+1}{n}$ for -ve values of n

($\Gamma 0, \Gamma -1, \Gamma -2, \Gamma -3 \ldots$ are not defined)

$$\boxed{\therefore L(\sqrt{t}) = \frac{\sqrt{\pi}}{2s^{3/2}}\ ; \quad L\left\{\frac{1}{\sqrt{t}}\right\} = \sqrt{\frac{\pi}{s}}}$$

$L\{t^{3/2}\} = \frac{3\sqrt{\pi}}{4s^{5/2}} \qquad L\{t^{5/2}\} = \frac{\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}}{s^{7/2}} = \frac{15\sqrt{\pi}}{8s^{7/2}}$

**First shifting Theorm** :- If $L\{f(t)\} = \bar{f}(s)$ then

$$L\{e^{at} f(s)\} = \bar{f}(s-a)$$

$$L\{e^{-at} f(s)\} = \bar{f}(s+a)$$

**Second shifting Theorm** :- If $L\{f(t)\} = \bar{f}(s)$ and

$$G(t) = \begin{cases} f(t-a) ; & t \geq a \\ 0 ; & t < a \end{cases}$$

$$L\{G(t)\} = e^{-as} \bar{f}(s)$$

**Change of scale** :-

$$L\{f(at)\} = \frac{1}{a} \bar{f}\left(\frac{s}{a}\right)$$

$$L\{f(t/a)\} = a\bar{f}(as)$$

**L.T of Derivatives**

If $L[f(t)] = \bar{f}(s)$ then

(i) $L[f'(t)] = s\bar{f}(s) - f(0)$

(ii) $L[f''(t)] = s^2 \bar{f}(s) - s f(0) - f'(0)$

(iii) $L[f'''(t)] = s^3 \bar{f}(s) - s^2 f(0) - s f'(0) - f''(0)$

**L.T of Integrals of a function** :-

If $L\{f(t)\} = \bar{f}(s)$ then

(i) $L\left[\int_0^t f(u)\,du\right] = \frac{1}{s} \bar{f}(s)$

(ii) $L\left[\int_0^t \int_0^t f(v)\,du\,du\right] = \frac{1}{s^2} \bar{f}(s)$

**Laplace of periodic function**

If $f(t) = f(t+T) \; \forall t$

$L\{f(t)\} = \cancel{\frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-st}}}$ $\frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}$

Unit step Fn / Heaviside's Fn.

$u(t-a)$ (or) $u_a(t)$ (or) $H(t-a) = \begin{cases} 1; & t \geq a \\ 0 & t < a \end{cases}$

$L\{u(t-a)\} = \frac{e^{-as}}{s}$

$L\{f(t-a)u(t-a)\} = e^{-as} L\{f(t)\}$

$L\{f(t)\,u(t-a)\} = e^{-as} L\{f(t+a)\}$

$L\{\delta(t)\} = 1$

$L\{\delta(t-a)\} = e^{-as}$

$L\{N(t)\} = 0$
(Nullfunction)

$L\{\sinh at\, f(t)\} = \frac{1}{2}\left(\bar{f}(s-a) - \bar{f}(s+a)\right)$

$L\{\cosh at\, f(t) = \frac{1}{2}\left(\bar{f}(s-a) + \bar{f}(s+a\right)$

Ex:- (i) $L\{\sinh 2t \sin 3t\} = \frac{1}{2}\left[\frac{\cancel{s-2}\,3}{(s-2)^2+9} - \frac{3}{(s+2)^2+9}\right]$

$\cancel{= 3(2s^2+12s}$

$L\{[t]\} = \frac{1}{s(e^s-1)}$ [] – greatest integer

$L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-\frac{1}{4s}} = \frac{1}{2s}\sqrt{\frac{\pi}{s}}\, e^{\frac{-1}{4s}}$

$L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\frac{\pi}{s}}\, e^{\frac{-1}{4s}}$

Initial value Theorem:- $\lim_{t\to 0} F(t) = \lim_{s\to\infty} sf(s)$

Final value theorm :- $\lim_{t\to\infty} F(t) = \lim_{s\to 0} sf(s)$

Evaluation of Integrals :- $f(s) = L\,F(t)$ then

$$\int_0^\infty e^{-st} F(t)\,dt = f(s)$$

as $s \to 0$ $\int_0^\infty F(t)\,dt = f(0)$

Useful formula

$$\int e^{at}\sin\beta t\,dt = \frac{e^{at}(a\sin\beta t - \beta\cos\beta t)}{a^2+\beta^2}$$

$$\int e^{at}\cos\beta t\,dt = \frac{e^{at}(a\cos\beta t + \beta\sin\beta t)}{a^2+\beta^2}$$

- $L\{e^{iat}\} = \dfrac{s+ia}{s^2+a^2}$
- $L\left\{\dfrac{\sin at}{t}\right\} = \tan^{-1}\left(\dfrac{a}{s}\right) = \cot^{-1}\left(\dfrac{s}{a}\right)$

Multip & Division

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$$

$$L\left\{\frac{F(t)}{t}\right\} = \int_0^\infty f(u)\,du$$

Ex:- $\displaystyle\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$

$\left(\tan^{-1}u\right)_0^\infty$

$L\{F(t)\}$ where $F(t) = \begin{cases} \sin t & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases}$

$$= \frac{1}{1-e^{-2\pi s}}\left(\frac{1+e^{-\pi s}}{s^2+1}\right)$$

$$= \frac{1+e^{-\pi s}}{(1-e^{-2\pi s})\,s^2+1}$$