$1 - \sin ax \qquad \cos ax \quad = \quad a\sec ax.$

(2) $y = \log\sqrt{\dfrac{1-\sin ax}{1+\sin ax}}$

Sol

$$\frac{dy}{dx} = \frac{1}{\cancel{2}} \times \frac{\cancel{(1+\sin ax)}}{1-\sin ax} \times \frac{-\cancel{2}\cos ax}{(1+\sin ax)^{\cancel{2}}} \times a$$

$$= \frac{-a\cancel{\cos ax}}{1-\sin^2 ax} = \frac{-a}{\cos ax} = -a\sec ax.$$

$\cos^{2} ax$

① $y = \log\sqrt{\dfrac{1+\sin x}{1-\sin x}} = \sec x.$

Sol $\dfrac{dy}{dx} = \dfrac{1}{2}\left[\log(1+\sin x) - \log(1-\sin x)\right]$

$= \dfrac{1}{2}\left[\dfrac{\cos x}{1+\sin x} - \dfrac{\cos x}{(1-\sin x)}\right]$

$= \dfrac{1}{2}\left[\dfrac{\cos x - \sin x\cdot\cos x - \cos x - \sin x\cdot\cos x}{1-\sin^2 x}\right]$

$= \dfrac{1}{\cancel{2}}\left(\dfrac{-\cancel{2}\sin x\cdot\cos x}{1-\sin^2 x}\right) = \dfrac{-\sin x\cdot\cos x}{\cos^2 x}$ ~~$= -\tan x$~~

② $y = \log\sqrt{\dfrac{1+\cos x}{1-\cos x}}$

Sol $\dfrac{dy}{dx} = \dfrac{1}{2}\log\left(\dfrac{1+\cos x}{1-\cos x}\right) = \dfrac{1}{\cancel{2}} \times \dfrac{1-\cos x}{1+\cos x} \times \dfrac{-\cancel{2}\sin x}{(1-\cos x)^2}$

$= \dfrac{-\sin x}{1-\cos^2 x} = \dfrac{-1}{\sin x}$

$= -\operatorname{cosec} x.$

③ $y = \log\sqrt{\dfrac{1-\sin x}{1+\sin x}}$

$\dfrac{dy}{dx} = \dfrac{1}{2}\log\left(\dfrac{1-\sin x}{1+\sin x}\right) = \dfrac{1}{\cancel{2}} \times \dfrac{1+\sin x}{1-\sin x} \times \dfrac{-\cancel{2}\cos x}{(1+\sin x)^2}$

$= \dfrac{-\cos x}{1-\sin^2 x} = \dfrac{-1}{\cos x}$

$= -\sec x.$

④ $y = \log\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$\dfrac{-\cos x+1}{\cos x+1}$

Sol $\dfrac{dy}{dx} = \dfrac{1}{\cancel{2}} \times \dfrac{1+\cos x}{1-\cos x} \times \dfrac{\cancel{2}\sin x}{(1+\cos x)^2}$

$= \dfrac{\sin x}{1-\cos^2 x} = \dfrac{1}{\sin x} = \operatorname{cosec} x.$

$\dfrac{\sin ax+1}{-\sin ax+1}$

$1-(-1)$

① $y = \log\sqrt{\dfrac{1+\sin ax}{1-\sin ax}}$

Sol $\dfrac{dy}{dx} = \dfrac{1}{\cancel{2}} \times \dfrac{1-\sin ax}{(1+\sin ax)} \times \dfrac{\cancel{2}\cos ax}{(1-\sin ax)^2} \times a$

$= \dfrac{a\cos ax}{1-\sin^2 ax} = \dfrac{a}{\cos ax} = a\sec ax.$

② $y = \log\sqrt{\dfrac{1-\sin ax}{1+\sin ax}}$

Sol $\dfrac{dy}{dx} = \dfrac{1}{\cancel{2}} \times \dfrac{(1+\sin ax)}{1-\sin ax} \times \dfrac{-\cancel{2}\cos ax}{(1+\sin ax)^2} \times a$

$= \dfrac{-a\cos ax}{1-\sin^2 ax} = \dfrac{-a}{\cos ax} = -a\sec ax.$

$\cos^2 ax$

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① $y = \log\sqrt{\dfrac{1+\sin x}{1-\sin x}} = \sec x$

Sol

$$\frac{dy}{dx} = \frac{1}{2}\left[\log(1+\sin x) - \log(1-\sin x)\right]$$

$$= \frac{1}{2}\left[\frac{\cos x}{1+\sin x} - \frac{\cos x}{(1-\sin x)}\right]$$

$$= \frac{1}{2}\left[\frac{\cos x - \sin x\cdot\cos x - \cos x - \sin x\cdot\cos x}{1-\sin^2 x}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{-\cancel{2}\sin x\cdot\cos x}{1-\sin^2 x}\right] = \frac{-\sin x\cdot\cancel{\cos x}}{\cos^2 x} \cancel{= -\tan x}$$

② $y = \log\sqrt{\dfrac{1+\cos x}{1-\cos x}}$

Sol

$$\frac{dy}{dx} = \frac{1}{2}\log\left(\frac{1+\cos x}{1-\cos x}\right) = \frac{1}{\cancel{2}} \times \frac{\cancel{1-\cos x}}{1+\cos x} \times \frac{-\cancel{2}\sin x}{\cancel{(1-\cos x)^2}}$$

$$= \frac{-\sin x}{1-\cos^2 x} = \frac{-1}{\sin x}$$

$$= -\operatorname{cosec} x.$$

③ $y = \log\sqrt{\dfrac{1-\sin x}{1+\sin x}}$

$$\frac{dy}{dx} = \frac{1}{2}\log\left(\frac{1-\sin x}{1+\sin x}\right) = \frac{1}{\cancel{2}} \times \frac{\cancel{1+\sin x}}{1-\sin x} \times \frac{-\cancel{2}\cos x}{(1+\sin x)^{\cancel{2}}}$$

$$= \frac{-\cos x}{1-\sin^2 x} = \frac{-1}{\cos x}$$

$$= -\sec x.$$

④ $y = \log\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$$\frac{-\cos x+1}{\cos x+1}$$

Sol

$$\frac{dy}{dx} = \frac{1}{\cancel{2}} \times \frac{\cancel{1+\cos x}}{1-\cos x} \times \frac{\cancel{2}\sin x}{(1+\cos x)^{\cancel{2}}}$$

$$= \frac{\sin x}{1-\cos^2 x} = \frac{1}{\sin x} = \operatorname{cosec} x.$$

$$\frac{\sin ax+1}{-\sin ax+1} \qquad 1-(-1)$$

① $y = \log\sqrt{\dfrac{1+\sin ax}{1-\sin ax}}$

Sol

$$\frac{dy}{dx} = \frac{1}{\cancel{2}} \times \frac{\cancel{1-\sin ax}}{(1+\sin ax)} \times \frac{\cancel{2}\cos ax}{(1-\sin ax)^{\cancel{2}}} \times a$$

$$= \frac{a\cos ax}{1-\sin^2 ax} = \frac{a}{\cos ax} = a\sec ax.$$

② $y = \log\sqrt{\dfrac{1-\sin ax}{1+\sin ax}}$

Sol

$$\frac{dy}{dx} = \frac{1}{\cancel{2}} \times \frac{\cancel{(1+\sin ax)}}{1-\sin ax} \times \frac{-\cancel{2}\cos ax}{(1+\sin ax)^{\cancel{2}}} \times a$$

$$= \frac{-a\cos ax}{1-\sin^2 ax} = \frac{-a}{\cos ax} = -a\sec ax.$$

$\cos^2 ax$

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③ $y = \log\sqrt{\dfrac{1+\cos ax}{1-\cos ax}}$

$\dfrac{\cos ax + 1}{-\cos ax + 1}$ $1-(-1) \to$

$\dfrac{dy}{dx} = \dfrac{1}{\cancel{2}} \times \dfrac{\cancel{(1-\cos ax)}}{1+\cos ax} \times \dfrac{-\cancel{2}\sin ax \cdot a}{(1-\cos ax)^{\cancel{2}}}$

$= \dfrac{-a \sin ax}{\dfrac{1-\cos^2 ax}{\sin^2 ax}} = -a \operatorname{cosec} ax$

④ $y = \log\sqrt{\dfrac{1-\cos ax}{1+\cos ax}}$

$\dfrac{dy}{dx} = \dfrac{\cancel{1}}{\cancel{2}} \times \dfrac{\cancel{2}\sin ax \times a}{(1+\cos ax)^{\cancel{2}}} \times \dfrac{\cancel{(1+\cos ax)}}{1-\cos ax}$

$= \dfrac{2a \sin ax}{1-\cos^2 ax} = \dfrac{2a \sin ax}{\sin^2 ax} = 2a \operatorname{cosec} ax$

① $y = \log\left(\sqrt{\operatorname{cosec}x + 1} + \sqrt{\operatorname{cosec}x - 1}\right)$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{\operatorname{cosec}x+1} + \sqrt{\operatorname{cosec}x-1}} \cdot (\quad)$

$\dfrac{dy}{dx} = \dfrac{-\operatorname{cosec}x \cdot \cot x}{2\sqrt{\operatorname{cosec}x+1}} + \dfrac{-\operatorname{cosec}x \cdot \cot x}{2\sqrt{\operatorname{cosec}x-1}}$

~~② $y = \log$~~

$= \dfrac{-\operatorname{cosec}x \cdot \cot x}{2}\left[\dfrac{\sqrt{\operatorname{cosec}x-1} + \sqrt{\operatorname{cosec}x+1}}{\sqrt{\operatorname{cosec}^2 x - 1}}\right]$

$= \dfrac{-1}{2}\operatorname{cosec}x \cdot \cot x \left[\dfrac{\sqrt{\operatorname{cosec}x-1} + \sqrt{\operatorname{cosec}x+1}}{\sqrt{\cot^2 x}}\right]$

$= \dfrac{-1}{2}\operatorname{cosec}x \cdot \cancel{\cot x}\ \dfrac{\left(\sqrt{\operatorname{cosec}x-1} + \sqrt{\operatorname{cosec}x+1}\right)}{\cancel{\cot x}}$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{\operatorname{cosec}x+1} + \sqrt{\operatorname{cosec}x-1}} \cdot \left(\sqrt{\operatorname{cosec}x+1} + \sqrt{\operatorname{cosec}x-1}\right) \cdot \dfrac{-1}{2}\operatorname{cosec}x$

$\dfrac{dy}{dx} = \dfrac{-\operatorname{cosec}x}{2}$

② $y = \log\left(\sqrt{\operatorname{cosec}x + 1} - \sqrt{\operatorname{cosec}x - 1}\right)$

Sol $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{\operatorname{cosec}x+1} - \sqrt{\operatorname{cosec}x-1}}\left(\sqrt{\operatorname{cosec}x-1} - \sqrt{\operatorname{cosec}x+1}\right) \times \dfrac{-\operatorname{cosec}}{2b}$

$= \dfrac{\sqrt{\operatorname{cosec}x+1} - \sqrt{\operatorname{cosec}x-1}}{\sqrt{\operatorname{cosec}x+1} - \sqrt{\operatorname{cosec}x-1}}\left(-\dfrac{1}{2}\operatorname{cosec}x\right) \Rightarrow -\dfrac{1}{2}\operatorname{cosec}x$

$\dfrac{d}{dx}\left(\sqrt{\operatorname{cosec}x+1} - \sqrt{\operatorname{cosec}x-1}\right) = \dfrac{1(-\operatorname{cosec}x\cot x)}{2\sqrt{\operatorname{cosec}x+1}} - \dfrac{1(-\operatorname{cosec}x \cdot \cot x)}{2\sqrt{\operatorname{cosec}x-1}}$

$= -\dfrac{1}{2}\operatorname{cosec}x \cdot \cot x\left[\dfrac{1}{\sqrt{\operatorname{cosec}x+1}} - \dfrac{1}{\sqrt{\operatorname{cosec}x-1}}\right]$

$= -\dfrac{1}{2}\operatorname{cosec}x \cdot \cancel{\cot x}\left[\dfrac{\sqrt{\operatorname{cosec}x-1} - \sqrt{\operatorname{cosec}x+1}}{\sqrt{\operatorname{cosec}^2 x - 1}}\right]$

1) $y = \sin^{-1}\left(\frac{x}{a}\right)$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \times \frac{1}{a} \Rightarrow \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}} \times \frac{1}{a}$$

$$= \frac{1}{\sqrt{a^2-x^2}}$$

2) $y = \cos^{-1}\left(\frac{x}{a}\right)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{x^2}{a^2}}} \times \frac{1}{a} = \frac{-1}{\frac{\sqrt{a^2-x^2}}{\cancel{a}}} \times \frac{1}{\cancel{a}} = \frac{-1}{\sqrt{a^2-x^2}}$$

3) $y = \tan^{-1}\left(\frac{x}{a}\right)$

$$\frac{dy}{dx} = \frac{1}{1+\frac{x^2}{a^2}} \times \frac{1}{a} = \frac{1}{\frac{a^2+x^2}{a^2}} \times \frac{1}{\cancel{a}}$$

$$= \frac{a}{a^2+x^2}$$

$$\boxed{\begin{aligned} \frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) &= \frac{1}{\sqrt{a^2-x^2}} \\ \frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) &= \frac{-1}{\sqrt{a^2-x^2}} \end{aligned}}$$

① $y = \sin\sqrt{x}$

Sol $\frac{dy}{dx} = \cos\sqrt{x} \cdot \times \frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{2\sqrt{x}}$

② $y = \cos\sqrt{x}$

$$\frac{dy}{dx} = -\sin\sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$$

③ $y = \sqrt{\sin\sqrt{x}}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin\sqrt{x}}} \times \cos\sqrt{x} \times \frac{1\ \cancel{\sin\sqrt{x}}}{2\sqrt{x}} = \frac{-\cancel{\sin\sqrt{x}}\cdot\cos\sqrt{x}}{4\sqrt{x}\cdot\sqrt{\sin\sqrt{x}}}$$

④ $y = \sqrt{\cos\sqrt{x}}$.

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\cos\sqrt{x}}} \times (-\sin\sqrt{x}) \times \frac{1}{2\sqrt{x}} = \frac{-\sin\sqrt{x}}{4\sqrt{x\cos\sqrt{x}}}$$

⑤ $y = \sin^{-1}(x^2)$

Sol ⊃ $\frac{1}{\sqrt{1-x^4}} \times 2x = \frac{2x}{\sqrt{1-x^4}}$

⑥ $y = \cos^{-1}(x^2)$

Sol ⊃ $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^4}} \times 2x = \frac{-2x}{\sqrt{1-x^4}}$

⑦ $y = \tan^{-1}(x^2)$

Sol ⊃ $\frac{2x}{1+x^4}$

8) $y = \sec^{-1} x^2$

Sol $\frac{dy}{dx} = \frac{1}{|x^{\cancel{2}}|\sqrt{x^4-1}} \times \cancel{2x} = \frac{2}{x\sqrt{x^4-1}}$

9) $y = \operatorname{cosec}^{-1} x^2$

Sol $\frac{-2x}{x^2\sqrt{x^4-1}} = \frac{-2}{x\sqrt{x^4-1}}$

10) $y = \sin^{-1} e^x$

Sol $\frac{1}{\sqrt{1-(e^x)^2}} = \frac{1}{\sqrt{1-e^{2x}}} \times e^x$

11) $y = \cos^{-1} e^x$

Sol $\frac{-1}{\sqrt{1-(e^x)^2}} \times e^x$

12) $y = \tan^{-1} e^x$

Sol $\frac{1}{1+(e^x)^2} \times e^x.$

14) $y = \sec^{-1} e^x$

Sol $\frac{1}{\cancel{e^x}\sqrt{(e^x)^2-1}} \times \cancel{e^x} = \frac{1}{\sqrt{e^{2x}-1}}$

15) $y = \operatorname{cosec}^{-1} e^x$

$\frac{-1}{\cancel{e^x}\sqrt{(e^x)^2-1}} \cancel{\times e^x} = \frac{-1}{\sqrt{e^{2x}-1}}$

16) $y = \sec^{-1} 5^x$

$\frac{dy}{dx} = \frac{1}{\cancel{(5^x)}\sqrt{(5^x)^2-1}} \times \cancel{5^x} \log 5 = \frac{\log 5}{\sqrt{5^{2x}-1}}$

17) $y = \operatorname{cosec}^{-1} 5^x$

$\frac{dy}{dx} = \frac{-1}{\cancel{|5^x|}\sqrt{5^{2x}-1}} \times \cancel{5^x} \log 5 = \frac{-\log 5}{\sqrt{5^{2x}-1}}$

18) $y = \sec^{-1}\sqrt{x}$

$\frac{1}{\sqrt{x}\sqrt{x-1}} \times \frac{1}{2\sqrt{x}} = \frac{1}{x\sqrt{x-1}}$

19) $y = \operatorname{cosec}^{-1}\sqrt{x}$

$\frac{-1}{x\sqrt{x-1}}$

20) $y = \log(\log x)$

$\frac{dy}{dx} = \frac{1}{\log x} \times \frac{1}{x} = \frac{1}{x \log x}$

21) $y = 10^{3x-1}$

$\frac{dy}{dx} = 10^{3x-1} \times \log 10 \times 3 = 3 \times 10^{3x-1}$

22) $y = 5^{-x}$

$\frac{dy}{dx} = 5^{-x} \log 5 \times (-1) = -5^{-x} \log 5$

$$y = \sqrt{\frac{1+\sin x}{1-\sin x}}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{\frac{1+\sin x}{1-\sin x}}} \times \frac{\cancel{2}\cos x}{(1-\sin x)^2}$$

$$= \frac{\cos x}{\sqrt{1+\sin x}\,.\,(1-\sin x)^{-1/2}\,(1-\sin x)^2}$$

$$= \frac{\cos x}{\sqrt{1+\sin x}\,.\sqrt{1-\sin x}\,(1-\sin x)}$$

$$= \frac{\cancel{\cos x}}{\underset{\cancel{\cos x}}{\sqrt{1-\sin^2 x}}\,(1-\sin x)} = \frac{1}{1-\sin x.}$$

$$-\frac{1}{2}+2 = \frac{3}{2}$$

$$y = \sqrt{\frac{1-\sin x}{1+\sin x.}}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{\frac{1-\sin x}{1+\sin x}}} \times \frac{-\cancel{2}\cos x}{(1+\sin x)^2}$$

$$= \frac{-\cos x}{\sqrt{1-\sin x}\,.\,(1+\sin x)^{-1/2}\,(1+\sin x)^2}$$

$$= \frac{-\cos x}{\sqrt{1-\sin x}\,\sqrt{1+\sin x}\,(1+\sin x)}$$

$$= \frac{-\cancel{\cos x}}{\underset{\cancel{\cos x}}{\sqrt{1-\sin^2 x}}\,(1+\sin x)}$$

$$= \frac{-1}{1+\sin x.}$$

$$y = \sqrt{\frac{1+\cos x}{1-\cos x.}}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{\frac{1+\cos x}{1-\cos x}}} \times \frac{-\cancel{2}\sin x}{(1-\cos x)^2}$$

$$= \frac{-\sin x}{\sqrt{1+\cos x}\,(1-\cos x)^{-1/2}\,(1-\cos x)^2}$$

$$= \frac{-\sin x}{\sqrt{1+\cos x}\,.\,\sqrt{1-\cos x}\,(1-\cos x)} = \frac{-\cancel{\sin x}}{\underset{\cancel{\sin x}}{\sqrt{1-\cos^2 x}}\,(1-\cos x)}$$

$$= \frac{-1}{1-\cos x}$$

(4) $$y = \sqrt{\frac{1-\cos x}{1+\cos x.}}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{\frac{1-\cos x}{1+\cos x}}} \times \frac{\cancel{2}\sin x}{(1+\cos x)^2} = \frac{\sin x}{(1-\cos x)^{1/2}\,(1+\cos x)^{-1/2}\,(1+\cos x)^2}$$

$$= \frac{\sin x}{(1-\cos x)^{1/2}\,(1+\cos x)^{3/2}} = \frac{\sin x}{\sqrt{1-\cos x}\,.\sqrt{1+\cos x}\,(1+\cos x)}$$

$$= \frac{\sin x}{\sqrt{1-\cos^2 x}\,.\,(1+\cos x)}$$

$$= \frac{\cancel{\sin x}}{\cancel{\sin x}\,.\,(1+\cos x)}$$

$$= \frac{1}{(1+\cos x.)}$$

1. $y = \sin^n x = n(\sin x)^{n-1} \cdot \cos x$
2. $y = \cos^n x = -\sin x \cdot n(\cos x)^{n-1}$
3. $y = \sin nx = n \cancel{\cos} \, nx$
4. $y = \cos nx = -\cancel{n \cos nx} \cdot n \sin nx$

1. $y = \sin^n x \cdot \sin nx$

$$\frac{dy}{dx} = \sin^n x \cdot (n \cos nx) + \sin nx \; n(\sin x)^{n-1} \cdot \cos x$$

$$= n(\sin x)^{n-1} \left[\sin x \cdot \cos nx + \sin nx \cdot \cos x \right]$$

$$= n(\sin x)^{n-1} \left[\sin (x + nx) \right] = n(\sin x)^{n-1} \sin (n+1) x$$

2. $y = \sin^n x \cdot \cos nx$

$$\frac{dy}{dx} = \sin^n x \, (-n \sin nx) + n(\sin x)^{n-1} \cdot \cos x \cdot \cancel{\sin} \cos nx$$

$$= n(\sin x)^{n-1} \left[\cos x \cdot \cos nx - \sin x \cdot \sin nx \right]$$

$$= n(\sin x)^{n-1} \left[\cos (x + nx) \right] = n(\sin x)^{n-1} \cos (n+1) x$$

3. $y = \cos^n x \cdot \cos nx$

$$\frac{dy}{dx} = \cos^n x \, (-n \sin nx) + \cos nx \; n(\cos x)^{n-1} \cdot -\sin x$$

$$= -n \cos^{n-1} x \left[\cos x \cdot \sin nx + \cos nx \cdot \sin x \right]$$

$$= -n \cos^{n-1} x \left[\cancel{\cos} \sin (x + nx) \right]$$

4. $y = \cos^n x \cdot \sin nx$

$$\frac{dy}{dx} = \cos^n x \cdot n \cos nx + \sin nx \; n \cos^{n-1} x \cdot -\sin x$$

$$= n \cos^{n-1} x \left[\cos x \cdot \cos nx - \sin x \cdot \sin nx \right]$$

$$= n \cos^{n-1} x \left[\cos (x + nx) \right]$$

$$= n \cos^{n-1} x \left[\cos (n+1) x \right]$$

1. $y = \cancel{e^x \cdot \tan^2 x} \; e^{\tan^2 x}$

$$\frac{dy}{dx} = e^{\tan^2 x} \cdot 2\tan x \cdot \sec^2 x$$

2. $y = e^{\cot^2 x}$

$$\frac{dy}{dx} = e^{\cot^2 x} \cdot 2\cot x \cdot -\mathrm{cosec}^2 x$$

3. $y = \sin^2 x$
$= 2\sin x \cdot \cos x$

4. $y = \sin^3 x$
$= 3\sin^2 x \cdot \cos x$

5. $y = \cos^2 x = 2\cos x \cdot (-\sin x)$

6. $y = \cos^3 x = 3\cos^2 x \, (-\sin x)$

7) $y = \sin x^2$

$\frac{dy}{dx} = \cos x^2 \cdot 2x = 2x \cdot \cos x^2$

8) $y = \cos x^2$

$\frac{dy}{dx} = -\sin x^2 \cdot 2x = -2x \cdot \sin x^2$

9) $y = \sin x^3$

$\frac{dy}{dx} = 3\cos x^3 \cdot 3x^2 = 9x^2 \cdot \cos x^3$

10) $y = \cos x^3$

$\frac{dy}{dx} = -\sin x^3 \cdot 3x^2 = -3x^2 \cdot \sin x^3$

11.) $y = \cancel{\sin}\ \sin x^\circ$

$x^\circ = \frac{\pi}{180} x(x)$

$y = \sin\left(\frac{\pi}{180} x\right)$

$= \cos\left(\frac{\pi}{180} x\right) \times \frac{\pi}{180}$

$= \frac{\pi}{180} \cos x^\circ$

$$\boxed{\begin{aligned} \cos x^\circ &= -\frac{\pi}{180} \sin x^\circ \\ \sin^2 x^\circ &= \frac{\pi}{180} 2\sin x^\circ \cdot \cos x^\circ \\ \sec(x+30^\circ) &= \frac{\pi}{180} \sec(x+30^\circ)\tan(x^\circ+30^\circ) \end{aligned}}$$

① $y = \log \frac{\sin(x-a)}{\sin(x-b)}$

$\frac{dy}{dx} \Rightarrow \log \frac{\sin(x-a)}{\sin(x-b)} = \log\sin(x-b) - \log\sin(x-a)$

$= \frac{\cos(x-a)}{\sin(x-a)} - \frac{\cos(x-b)}{\sin(x-b)}$

$= \cot(x-a) - \cot(x-b)$

② $y = \log \cosh 2x$

$\frac{dy}{dx} = \frac{1}{\cosh 2x} \cdot \sinh 2x \cdot 2 = 2\tanh 2x$

③ $y = a^{\sinh^2 x}$

$\frac{dy}{dx} = a^{\sinh^2 x} \log a \cdot 2\sinh x \cdot \cosh x$

④ $y = \sinh^{-1}\left(\frac{3x}{4}\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{\left(\frac{3x}{4}\right)^2 + 1}} \cdot \frac{3}{4} = \frac{1}{\frac{\sqrt{9x^2+4}}{\cancel{4}}} \times \frac{3}{\cancel{4}\,2}$

$= \frac{3}{2\sqrt{9x^2+4}}$

(5) $y = \cosh^{-1}\left(\frac{5x}{7}\right)$

$$\frac{dy}{dx} = \frac{-1}{\frac{\sqrt{25x^2-49}}{7}} \times \frac{5}{7} = \frac{-5}{\sqrt{25x^2-49}}$$

(6) $y = \sec^{-1} a^x$

$$\frac{dy}{dx} = \frac{1}{a^x\sqrt{a^{2x}-1}} \cdot a^x \log a = \frac{\log a}{\sqrt{a^{2x}-1}}$$

* (7) $y = \tan^{-1}\left(\frac{b\tan x}{a}\right)$

$$\frac{dy}{dx} = \frac{1}{\frac{a^2 + b^2\tan^2 x}{a^2}} \cdot \times \frac{b}{a}\sec^2 x$$

$$= \frac{a\sec^2 x}{a^2 + b^2\tan^2 x}$$

(8) $y = \sin^{-1}\left[\frac{a+b\sin x}{b+a\sin x}\right] = \frac{\sqrt{b^2-a^2}}{b+a\sin x}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{a+b\sin x}{b+a\sin x}\right)^2}} \times \frac{(b^2-a^2)\cos x}{(b+a\sin x)^2}$$

$$= \frac{1}{\sqrt{\frac{b^2+a^2\sin^2 x + 2ab\sin x - a^2 - b^2\sin^2 x - 2ab\sin x}{(b+a\sin x)^2}}} \times ($$

$$\frac{dy}{dx} = \frac{1}{\frac{\sqrt{(b^2-a^2) - \sin^2 x(b^2-a^2)}}{(b+a\sin x)}} \times \frac{(b^2-a^2)\cos x}{(b+a\sin x)^2}$$

$$= \frac{1}{\sqrt{(b^2-a^2)(1-\sin^2 x)}} \times \frac{(b^2-a^2)\cos x}{(b+a\sin x)}$$

$$= \frac{\sqrt{b^2-a^2}}{b+a\sin x}$$

(2) $y = \sin^{-1}\left[\frac{b+a\sin x}{a+b\sin x}\right] = \frac{\sqrt{a^2-b^2}}{a+b\sin x}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{b+a\sin x}{a+b\sin x}\right)^2}} \times \frac{(a^2-b^2)\cos x}{(a+b\sin x)^2}$$

$$= \frac{1}{\frac{\cos x\sqrt{(a^2-b^2)}}{(a+b\sin x)}} \times \frac{(a^2-b^2)\cos x}{(a+b\sin x)^2}$$

$$= \frac{\sqrt{a^2-b^2}}{(a+b\sin x)}$$

(3) $y = \cos^{-1}\left[\frac{a+b\cos x}{b+a\cos x}\right] \quad \frac{\sqrt{b^2-a^2}}{b+a\cos x}$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\quad)^2}} \times \frac{(b^2-a^2)-\sin x}{(b+a\sin x)^2}$$

$$= \frac{1}{\sqrt{\frac{b^2+a^2\cos^2 x + 2ab\cos x - a^2 - b^2\cos^2 x - 2ab\cos x}{(b+a\cos x)^2}}} \times \frac{(b^2-a^2)\sin x}{(b+a\sin x)^2}$$

$$= \frac{1}{\sqrt{(b^2-a^2) - \cos^2 x\,(b^2-a^2)}} \times \frac{(b^2-a^2)\sin x}{(b+a\cos x)}$$

$$= \frac{1}{\sqrt{(b^2-a^2)(1-\cos^2 x)}} \times \frac{(b^2-a^2)\sin x}{b+a\cos x}$$

$$= \frac{1}{\cancel{\sin x}\sqrt{b^2-a^2}} \times \frac{(b^2-a^2)\cancel{\sin x}}{b+a\cos x} = \frac{\sqrt{b^2-a^2}}{b+a\cos x}$$

④ VI. $y = \cos^{-1}\left[\frac{b+a\cos x}{a+b\cos x}\right]$

$$= \frac{-1}{\sqrt{1-\left(\frac{b+a\cos x}{a+b\cos x}\right)^2}} \times \left(\frac{a\cos x+b}{b\cos x+a}\right)$$

$$= \frac{-1}{\sqrt{\quad}} \times \frac{(a^2-b^2)(-\sin x)}{(b\cos x+a)^2}$$

$$\frac{-1}{\sqrt{\frac{(a+b\cos x)^2 - (b+a\cos x)^2}{(a+b\cos x)^2}}} = \frac{-1}{\sqrt{\frac{a^2+b^2\cos^2 x+\cancel{2ab\cos x} - b^2 - a^2\cos^2 x - 2ab\cancel{\cos x}}{(a+b\cos x)^2}}}$$

$$= \frac{-1}{\sqrt{\frac{(a^2-b^2) - \cos^2 x\,(a^2-b^2)}{(a+b\cos x)^2}}} = \frac{-1}{\sqrt{\frac{(a^2-b^2)(1-\sin^2 x)}{(a+b\cos x)^2}}}$$

$$= \frac{\cancel{-}1}{\frac{\sin x\cdot\sqrt{a^2-b^2}}{(a+b\cos x)}} \times \frac{(a^2-b^2)(\cancel{-}\sin x)}{(a+b\cos x)^{\cancel{2}}}$$

$$= \frac{a^2-b^2}{\sqrt{a^2-b^2}\,(a+b\cos x)} = \frac{\sqrt{a^2-b^2}}{(a+b\cos x)}$$

① $y = \cancel{\frac{1}{b\tan^{-1}}}\ \frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) - \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$y = \cancel{\frac{1}{b}}\,\frac{\cancel{b}}{b^2+x^2} - \cancel{\frac{1}{a}}\cdot\frac{\cancel{a}}{a^2+x^2}$

$$= \frac{1}{b^2+x^2} - \frac{1}{a^2+x^2}$$

$$= \frac{a^2+\cancel{x^2}-b^2-\cancel{x^2}}{(b^2+x^2)(a^2+x^2)} = \frac{a^2-b^2}{(b^2+x^2)(a^2+x^2)}$$

$1-\cos x = 2\sin^2\frac{x}{2}$

② $y = \frac{1+\sin x-\cos x}{1+\sin x+\cos x} = \tan\frac{x}{2}$ $\cancel{\frac{u'}{v'} = \frac{\cos x+\sin x}{\cos x-\sin x}}$ $1+\cos x = 2\cos^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}$

$$\frac{dy}{dx} = \cancel{\frac{(1+\sin x+\cos x)(\cos x+\sin x) - (1+\sin x-\cos x)(\cos x-\sin x)}{(1+\sin x+\cos x)^2}}$$

$$= \frac{1}{2}\sec^2\frac{x}{2}.$$

③ $y = \log\left(\frac{1+x+x^2}{1-x+x^2}\right) = \log(1+x+x^2) - \log(1-x+x^2)$

$\cancel{\frac{1-x+x^2}{1+x+x^2}}$ $(x^2+x+1)(x^2-x+1) = x^4+x^2+1$

Answer: $\frac{-2x^2+2}{x^4+x^2+1}$

(4) $y = \underset{u}{e^{ax}} \cdot \underset{v}{\sin(bx+c)}$

$$\frac{dy}{dx} = ae^{ax}.\sin(bx+c) + e^{ax}.\cos(bx+c).b$$

$$= e^{ax}\left[a\sin(bx+c) + b\cos(bx+c)\right]$$

(5) $y = e^{ax}.\cos(bx+c)$

$$\frac{dy}{dx} = ae^{ax}.\cos(bx+c) - e^{ax}.\sin(bx+c)\,b$$

$$= e^{ax}\left[a\cos(bx+c) - b\sin(bx+c)\right]$$

(6) $y = \frac{1}{\sqrt{ax+b}}$

$$\frac{dy}{dx} = \frac{-1}{2(ax+b)\sqrt{ax+b}} \times a = \frac{-a}{2(ax+b)\sqrt{ax+b}}$$

(7) $y = \sqrt{\sin(m\sin^{-1}x)}$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{\sin(m\sin^{-1}x)}} \times \cos(m\sin^{-1}x) \times \frac{m}{\sqrt{1-x^2}}$$

(8) $y = \sqrt{\cos(m\cos^{-1}x)}$

$$\frac{dy}{dx} = \frac{\cancel{-}1}{2\sqrt{\cos(m\cos^{-1}x)}} \times \cancel{-}\sin(m\cos^{-1}x) \times \frac{-m}{\sqrt{1-x^2}}$$

$$= \frac{m\sin(m\cos^{-1}x)}{2\sqrt{\cos(m\cos^{-1}x)}\sqrt{1-x^2}}$$

(9) $y = \tan^{-1}(\cos\sqrt{x})$

$$\frac{dy}{dx} = \frac{1}{1+(\cos\sqrt{x})^2} \times -\sin\sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}(1+\cos^2\sqrt{x})}$$

(10) $y = \left(x^2 + \frac{1}{x^2}\right)^2$

$u' = 3x^2 \quad v' = 2x$

$$\frac{dy}{dx} = \left(\frac{x^4+1}{x^2}\right)^2 = 2\left(\frac{x^4+1}{x^2}\right)\left(\frac{x^2\,3x^2 - (x^4+1)\,2x}{x^4}\right)$$

$$= 2\left(\frac{x^4+1}{x^2}\right)\left(\frac{3x^4 - 2x^5 - 2x}{\cancel{x^4}\,x^4}\right)$$

(11) $y = \frac{x+2-2}{x+2}$

$$\frac{dy}{dx} = \frac{1\cancel{x+2}}{\cancel{x+2}} - \frac{2}{x+2} = +\frac{2}{(x+2)^2}$$

(12) $y = \frac{x^5+6-6}{x^5+6}$

$$\frac{dy}{dx} = \cancel{\frac{x^5+6\,1}{x^5+6}} - \frac{6}{x^5+6} = 0 - \left(\frac{\cancel{-}6}{(x^5+6)^2} \times 5x^4\right)$$

$$= \frac{30x^4}{(x^5+6)^2}$$

(13) $y = \frac{1+\tan x}{1-\tan x}$

$\frac{\tan x+1}{\tan x - 1}$

$-ab-bc$

$y = \tan\left(\frac{\pi}{4}+x\right)$

$= \sec^2\left(\frac{\pi}{4}+x\right)$ 2

(b) $\frac{-2}{(1-\tan x)^2}(\sec^2 x)$

$$\frac{-2\sec^2 x}{1+\tan^2 x - 2\tan x}$$

(14) $y = \dfrac{1-\tan x}{1+\tan x} = \tan\left(\dfrac{\pi}{4}-x\right)$

$\dfrac{dy}{dx} = \sec^2\left(\dfrac{\pi}{4}-x\right)(-1)$

$= -\sec^2\left(\dfrac{\pi}{4}-x\right)$

(15) $y = 2^{2^x}$

$\dfrac{dy}{dx} = 2^{2^x}\log 2 \cdot 2^x \log 2 = 2^x \cdot 2^{2^x}(\log 2)^2$

(16) $y = \log\left(\dfrac{1+x}{1-x}\right)^{1/4} - \dfrac{1}{2}\tan^{-1}x$

Sol $\dfrac{dy}{dx} = \dfrac{1}{\cancel{4}_2} \times \cancel{\dfrac{1-x}{1+x}} \times \dfrac{\cancel{2}}{(1-x)^{\cancel{2}}} - \dfrac{1}{2}\dfrac{1}{(1+x^2)}$

$= \dfrac{1}{2(1-x^2)} - \dfrac{1}{2(1+x^2)}$

$= \dfrac{1}{2}\left[\dfrac{\cancel{1}+x^2 \cancel{-1}+x^2}{1-x^4}\right] = \dfrac{\cancel{2}x^2}{\cancel{2}(1-x^4)} = \dfrac{x^2}{1-x^4}$

(17) $y = \tanh^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)$

$\dfrac{dy}{dx} = \dfrac{1}{1-\left(\dfrac{x^2-1}{x^2+1}\right)^2} \times \dfrac{2\cdot 2x}{(x^2+1)^2}$

$= \dfrac{1}{\dfrac{(x^2+1)^2-(x^2-1)^2}{\cancel{(x^2+1)^2}}} \times \dfrac{4x}{\cancel{(x^2+1)^2}} = \dfrac{4x}{(x^2+1)^2-(x^2-1)^2}$

* $y = (x+a)(x^2+a^2)(x^4+a^4)(x^8+a^8)$

$y = \dfrac{(x-a)(x+a)(x^2+a^2)(x^4+a^4)(x^8+a^8)}{(x-a)}$

$y = \dfrac{x^{16}-a^{16}}{x-a}$ $\quad u' = 16x^{15}$, $v' = 1$

$\dfrac{dy}{dx} = \dfrac{(x-a)\,16x^{15} - (x^{16}-a^{16})(1)}{(x-a)^2}$

$= \dfrac{16x^{15} - 16ax^{15} - x^{16} + a^{16}}{(x-a)^2}$

If $f(x) = \sqrt{ax} + \dfrac{a^2}{\sqrt{ax}}$ then $f'(a) =$ ______

$$f'(x) = \frac{a}{2\sqrt{ax}} + \frac{(-a^2)}{2\sqrt{ax}\cdot ax}\cdot a$$

$$f'(a) = \frac{\cancel{a}}{2\sqrt{a\cdot \cancel{a}}} - \frac{\cancel{a^2}}{2\sqrt{a\cdot a}\cdot \cancel{a\cdot a}}\times \cancel{a}$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

If $f(x) = \dfrac{1}{1+x^{\beta-\alpha}+x^{\gamma-\alpha}} + \dfrac{1}{1+x^{\gamma-\beta}+x^{\alpha-\beta}} + \dfrac{1}{1+x^{\alpha-\gamma}+x^{\beta-\gamma}}$

Then $f'(x) =$ <u>0</u>

$$f(x) = \frac{1}{1+\frac{x^\beta}{x^\alpha}+\frac{x^\gamma}{x^\alpha}} + \frac{1}{1+\frac{x^\gamma}{x^\beta}+\frac{x^\alpha}{x^\beta}} + \frac{1}{1+\frac{x^\alpha}{x^\gamma}+\frac{x^\beta}{x^\gamma}}$$

$$= \frac{x^\alpha}{x^\alpha+x^\beta+x^\gamma} + \frac{x^\beta}{x^\beta+x^\gamma+x^\alpha} + \frac{x^\gamma}{x^\gamma+x^\alpha+x^\beta}$$

$$= \frac{x^\alpha+x^\beta+x^\gamma}{x^\alpha+x^\beta+x^\gamma} = 1$$

$$f'(x) = \frac{d}{dx}(1)$$

$$= 0$$

If $f(x) = e^x$ Then $\dfrac{h'(x)}{h(x)} =$ ______

$g(x) = \sin^{-1}x$

$h(x) = f(g(x))$

$h(x) = e^{g(x)}$

$h(x) = e^{\sin^{-1}x}$

$$\frac{h'(x)}{h(x)} = \frac{\frac{\cancel{e^{\sin^{-1}x}}}{\sqrt{1-x^2}}}{\frac{\cancel{e^{\sin^{-1}x}}}{1}} = \frac{1}{\sqrt{1-x^2}}$$

* If $f(x) = \left(\dfrac{x^a}{x^b}\right)^{a+b}\cdot\left(\dfrac{x^b}{x^c}\right)^{b+c}\cdot\left(\dfrac{x^c}{x^a}\right)^{c+a}$. Then $f'(x) =$ <u>0</u>

$$(x^a\cdot x^{-b})^{a+b}\cdot(x^b\cdot x^{-c})^{b+c}\cdot(x^c\cdot x^{-a})^{c+a}$$

$$(x)^{a^2-b^2}\,(x)^{b^2-c^2}\,(x)^{c^2-a^2}$$

$$f(x) = x^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}} = 1$$

$$f'(x) = 0$$

* If $y = 2^{ax}$ and $y' = \log 256$ at $x=1$ then $a =$ <u>2.</u>

$y' = 2^{ax}\log 2\cdot a$

$a\cdot 2^{ax}\log 2 = \log 2^8$

$a\cdot 2^{ax}\cancel{\log 2} = 8\cancel{\log 2}$

$a\cdot 2^a = 8.$

$\boxed{a=2}$

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If $h(x) = e^{e^x}$ Then $\frac{h'(x)}{h(x)} = \underline{\log h(x)}$

1) $\log h(x)$
2) $-\log h(x)$
3) $2\log h(x)$
4) $3\log h(x)$

$h(x) = e^{e^x}$

$\frac{h'(x)}{h(x)} = \frac{\cancel{e^{e^x}} \cdot e^x}{\cancel{e^{e^x}}}$

$= e^x$

$= \log h(x)$

$\log h(x) = \log e^{e^x}$

$= e^x \log e$

$= 1 \times e^x$

$= e^x$

DIFFERENTIATON OF DETERMINANTS

1) If $y = \begin{vmatrix} f(x) & g(x) \\ \phi(x) & \psi(x) \end{vmatrix}$ then $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) \\ \phi(x) & \psi(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) \\ \phi'(x) & \psi'(x) \end{vmatrix}$

Eg: $y = \begin{vmatrix} e^x & a^x \\ \sin x & \cos x \end{vmatrix} = \begin{vmatrix} e^x & a^x \log a \\ \sin x & \cos x \end{vmatrix} + \begin{vmatrix} e^x & a^x \\ \cos x & -\sin x \end{vmatrix}$

2) If $y = \begin{vmatrix} f_1(x) & g_1(x) & h_1(x) \\ f_2(x) & g_2(x) & h_2(x) \\ f_3(x) & g_3(x) & h_3(x) \end{vmatrix}$

$= \begin{vmatrix} f_1'(x) & g_1'(x) & h_1'(x) \\ f_2(x) & g_2(x) & h_2(x) \\ f_3(x) & g_3(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} - & - & - \\ f_2'(x) & g_2'(x) & h_2'(x) \\ - & - & - \end{vmatrix} + \begin{vmatrix} - & - & - \\ - & - & - \\ f_3'(x) + & g_3'(x) + & h_3'(x) \end{vmatrix}$

* If $y = \begin{vmatrix} f(x) & g(x) \\ a & b \end{vmatrix}$ where a & b are constants then

$\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) \\ a & b \end{vmatrix}$

* If $y = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ a & b & c \\ p & q & r \end{vmatrix}$ where a, b, c, p, q and r constants.

$\frac{dy}{dx} = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ a & b & c \\ p & q & r \end{vmatrix}$

* If $f(x) = \begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta\sec x & \tan x & x \\ 1 & \tan x - \tan\theta & 0 \end{vmatrix}$ then $f'(0) = \underline{-1}$

$f'(x) = \begin{vmatrix} \cancel{0} & \cancel{0} & \cancel{0} \\ & & \\ & & \end{vmatrix}$ $0 + \begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta \cdot \sec x \cdot \tan x & \sec^2 x & 1 \\ 1 & \tan x - \tan\theta & 0 \end{vmatrix}$

$+ \begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta\sec x & \tan x & x \\ 0 & \sec^2 x & 0 \end{vmatrix}$

$f'(0) = 0 + \begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ 0 & 1 & 1 \\ 1 & 0-\tan\theta & 0 \end{vmatrix} + \begin{vmatrix} \sec\theta & \tan^2\theta & 1 \\ \theta & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$

$\cancel{f'(0) = \frac{d}{}\,0}$ [sec θ]

1) $\frac{d}{dx}|x|$ for $x<0$, $\frac{d}{dx}|x| = \frac{d}{dx}(-x) = -1$

for $x>0$, $\frac{d}{dx}|x| = \frac{d}{dx}(x) = 1$

$$\boxed{\frac{d}{dx}|x| = \frac{|x|}{x}}$$

2) $y = \log|x|$

$= \frac{1}{|x|} \times \frac{|x|}{x} = \frac{1}{x} \quad (x \neq 0)$

Note: Odd function $f(-x) = -f(x)$

even function $f(-x) = f(x)$

The derivate of odd function is even and

" " " even function is "

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\sinh^{-1}x = \log\left(x + \sqrt{x^2+1}\right)$

$\cosh^{-1}x = \log\left(x + \sqrt{x^2-1}\right)$

Series.

1. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

2. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

3. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \dots$

* If $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$ find $\frac{dy}{dx}$

$y = \sin x$ [∵ From above]

$\frac{dy}{dx} = \cos x.$

$$y = \log \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}$$

$\cancel{780}$

$(b-a)(a-b) = ab - a^2 - b^2 + ab$
$= a^2 - b^2 + ab$
$= ab - a^2 - a^2 - ab$
$= -2a^2$
$= -((a-b)^2)$

$$y = \log\left(\sqrt{1+x} + \sqrt{1-x}\right) - \log\left(\sqrt{1+x} - \sqrt{1-x}\right)$$

$$= \frac{1}{\left(\sqrt{1+x} + \sqrt{1-x}\right)}\left(\frac{1}{2\sqrt{1+x}} + \frac{(-1)}{2\sqrt{1-x}}\right) - \frac{1}{\left(\sqrt{1+x} - \sqrt{1-x}\right)}\left(\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{1-x}}\right)$$

$$= \frac{1}{2}\left[\left(\frac{\frac{1}{\sqrt{1+x}} - \frac{1}{\sqrt{1-x}}}{\sqrt{1+x} + \sqrt{1-x}}\right) - \left(\frac{\frac{1}{\sqrt{1+x}} + \frac{1}{\sqrt{1-x}}}{\sqrt{1+x} - \sqrt{1-x}}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x^2}}}{\sqrt{1+x} + \sqrt{1-x}}\right) - \left(\frac{\frac{\sqrt{1-x} + \sqrt{1+x}}{\sqrt{1-x^2}}}{\sqrt{1+x} - \sqrt{1-x}}\right)\right]$$

$$= \frac{1}{2\sqrt{1-x^2}}\left[\frac{\left(\sqrt{1-x} - \sqrt{1+x}\right)\left(\sqrt{1+x} - \sqrt{1-x}\right) - \left(\sqrt{1+x} + \sqrt{1-x}\right)\left(\sqrt{1-x} + \sqrt{1+x}\right)}{\left(\sqrt{1+x}\right)^2 - \left(\sqrt{1-x}\right)^2}\right]$$

$$= \frac{1}{2\sqrt{1-x^2}}\left[\left(\sqrt{1-x}\right)\left(\sqrt{1+x}\right) - \left(\sqrt{1-x}\right)^2\left(\cancel{\sqrt{1-x}}\right) - \left(\sqrt{1+x}\right)^2 + \left(\sqrt{1+x}\right)\left(\sqrt{1-x}\right)\right.$$

$$= \frac{-1}{x\sqrt{1-x^2}}$$

$$y = \log \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} = \frac{-1}{x\sqrt{1-x^2}}$$

$$= \frac{+1}{x\sqrt{1-x^2}}$$

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DIFFERENTIATION BY SUBSTITUTION METHOD

1) $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $\quad x = \tan\theta$, $\theta = \tan^{-1}x$.

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\tan^{-1}x$

$\frac{dy}{dx} = \frac{2}{1+x^2}$

2) $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta) = 2\theta$

$y = 2\tan^{-1}x$

$\frac{dy}{dx} = \frac{2}{1+x^2}$

3) $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \tan^{-1}(\tan 2\theta)$

$= 2\theta = 2\tan^{-1}x$

$= \frac{2}{1+x^2}$

$$\left\{\begin{matrix} \sin^{-1}\frac{2x}{1+x^2} \\ \cos^{-1}\frac{1-x^2}{1+x^2} \\ \tan^{-1}\frac{2x}{1-x^2}\end{matrix}\right. = 2\tan^{-1}x \quad \frac{dy}{dx} = \frac{2}{1+x^2}\Bigg\}$$

4) $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

$y = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\sin 2\theta) = \cos^{-1}(\cos(90^\circ - 2\theta))$

$= 90^\circ - 2\theta$

$y = 90^\circ - 2\tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{-2}{1+x^2}$

5.) $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$y = \sin^{-1}(\cos 2\theta)$

$= \sin^{-1}[\sin(90-2\theta)] = 90^\circ - 2\theta = 90^\circ - 2\tan^{-1}x$

$= \frac{-2}{1+x^2}$

① $y = \sin^{-1}(2x^2-1)$ $\quad x = \cos\theta$, $\theta = \cos^{-1}x$.

$y = \sin^{-1}(2\cos^2\theta - 1)$

$= \sin^{-1}(\cos 2\theta) = \sin^{-1}[\sin(90^\circ - 2\theta)] = 90^\circ - 2\theta$

$= 90^\circ - 2\cos^{-1}x = \cancel{\frac{+2}{1+x^2}}$ $\quad -2 \times \frac{-1}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$

② $y = \cos^{-1}(2x^2-1)$

$y = \cos^{-1}(2\cos^2\theta - 1) = \cos^{-1}(\cos 2\theta) = 2\theta = 2\cos^{-1}x$

$\frac{dy}{dx} = \cancel{\frac{2}{1+x^2}}$ $\quad \frac{-2}{\sqrt{1-x^2}}$

③ $y = \text{cosec}^{-1}\left(\frac{1}{2x^2-1}\right)$ $\quad x = \cos\theta$

$= \text{cosec}^{-1}\left(\frac{1}{\cos 2\theta}\right) = \text{cosec}^{-1}(\sec 2\theta) = \text{cosec}^{-1}(\text{cosec}(90^\circ - 2\theta))$

$= 90^\circ - 2\theta = 90^\circ - 2\cos^{-1}x$

$= \frac{+2}{\sqrt{1-x^2}}$

④ $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$= \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta) = 2\theta$

$= 2\cos^{-1}x$

$= \frac{-2}{\sqrt{1-x^2}}$

① $y = \sin^{-1}(1-2x^2)$ put $x = \sin\theta$, $\theta = \sin^{-1}x$

$y = \sin^{-1}(1-2\sin^2\theta)$

$= \sin^{-1}(\cos 2\theta) = 90° - 2\sin^{-1}x$

$\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$

② $y = \cos^{-1}(1-2x^2)$

$= \cos^{-1}(\cos 2\theta) = 2\theta = 2\sin^{-1}x$

$= \frac{2}{\sqrt{1-x^2}}$

③ $y = \cancel{\frac{1}{\csc^{-1}}}\ \csc^{-1}\left(\frac{1}{1-2x^2}\right)$

$\csc^{-1}\left(\frac{1}{\cos 2\theta}\right) = \csc^{-1}(\csc 2\theta) = 90° - 2\theta = -2\sin^{-1}x$

$= \frac{-2}{\sqrt{1-x^2}}$

④ $y = \sec^{-1}\left(\frac{1}{1-2x^2}\right)$

$y = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta) = 2\theta = 2\sin^{-1}x$

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

① $y = \sin^{-1}(3x - 4x^3)$ put $x = \sin\theta$ $\theta = \sin^{-1}x$

$= \sin^{-1}(\sin 3\theta) = 3\theta = \frac{3}{\sqrt{1-x^2}}$

② $y = \cos^{-1}(3x - 4x^3)$

$= \cos^{-1}(\sin 3\theta) = 90° - 3\theta$

$= \frac{-3}{\sqrt{1-x^2}}$

③ $y = \sin^{-1}(4x^3 - 3x)$ $x = \cos\theta$.

$y = \sin^{-1}(\cos 3\theta) = 90° - 3\theta = -3\cos^{-1}x$

$= \cancel{-}3 \times \frac{\cancel{-}1}{\sqrt{1-x^2}} = \frac{3}{\sqrt{1-x^2}}$

④ $y = \cos^{-1}(4x^3 - 3x)$

$y = \cos^{-1}(\cos 3\theta) = 3\theta = \frac{-3}{\sqrt{1-x^2}}$

⑤ $y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$ $x = \tan\theta$, $\theta = \tan^{-1}x$.

$y = \tan^{-1}(\tan 3\theta) = 3\theta = 3\tan^{-1}x$

$= \frac{3}{1+x^2}$

⑥ $y = \sin^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x)$

$= 3\sin^{-1}x + 3\cos^{-1}x$

$= 3\left[\sin^{-1}x + \cos^{-1}x\right]$

$= 3\left(\frac{\pi}{2}\right)$

$\frac{dy}{dx} = 0$

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

$3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

$3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$

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$\sqrt{1+x}$, $\sqrt{1-x}$

$\sqrt{\frac{1+x}{1-x}}$, $\sqrt{\frac{1-x}{1+x}}$

Put $x = \cos\theta$

$1+\cos 2\theta = 2\cos^2\theta$

$1-\cos 2\theta = 2\sin^2\theta$

$1+\cos\theta = 2\cos^2\frac{\theta}{2}$

$1-\cos\theta = 2\sin^2\frac{\theta}{2}$

① $y = \sin^{-1}\sqrt{\frac{1-x}{2}}$

$y = \sin^{-1}\sqrt{\frac{1-\cos\theta}{2}}$

$= \sin^{-1}\sqrt{\frac{\cancel{2}\sin^2\frac{\theta}{2}}{\cancel{2}}}$

$= \frac{\theta}{2} = \frac{\cos^{-1}x}{2}$

$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

② $y = \cos^{-1}\sqrt{\frac{1+x}{2}}$

$y = \cos^{-1}\sqrt{\frac{1+\cos\theta}{2}} = \cos^{-1}\sqrt{\frac{\cancel{2}\cos^2\frac{\theta}{2}}{\cancel{2}}}$

$= \frac{\theta}{2} = \frac{\cos^{-1}x}{2}$

$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

③ $y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ put $x = \cos\theta$

$= \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\cos^2\theta/2}}$

$= \tan^{-1}(\tan\theta/2)$

$= \theta/2 = \frac{1}{2}\cos^{-1}x$

$= \frac{1}{2}\times\frac{-1}{\sqrt{1-x^2}}$

④ $y = \cot^{-1}\sqrt{\frac{1+x}{1-x}}$

$= \cancel{\cot^{-1}(\cot}\ \frac{1}{2}\cos^{-1}x$

$\frac{dy}{dx} = \frac{1}{2}\times\frac{-1}{\sqrt{1-x^2}}$

⑤ $y = \operatorname{cosec}^{-1}\sqrt{\frac{2}{1-x}}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}\cos^{-1}x\right) = \frac{1}{2}\times\frac{-1}{\sqrt{1-x^2}}$

⑥ $y = \sec^{-1}\sqrt{\frac{2}{1+x}}$

$y = \frac{1}{2}\cos^{-1}x$

$\boxed{\frac{dy}{dx} = \frac{-1}{2}\times\frac{1}{\sqrt{1-x^2}}}$

$\frac{1}{2}\cos^{-1}x = \left\{ \begin{array}{l} \sin^{-1}\sqrt{\frac{1-x}{2}} \\ \cos^{-1}\sqrt{\frac{1+x}{2}} \\ \tan^{-1}\sqrt{\frac{1-x}{1+x}} \\ \cot^{-1}\sqrt{\frac{1+x}{1-x}} \\ \operatorname{cosec}^{-1}\sqrt{\frac{2}{1-x}} \\ \sec^{-1}\sqrt{\frac{2}{1+x}} \end{array} \right\}$

① $y = \sin^{-1}(\cos x)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right)$

$y = \frac{\pi}{2} - x$

$\frac{dy}{dx} = 0 - 1 = -1$

② $y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$

$= \sin^{-1}x + \cos^{-1}x$

$y = \frac{\pi}{2}$

$\frac{dy}{dx} = 0$

③ $y = \cos^{-1}x + \cos^{-1}\sqrt{1-x^2}$

$= \cos^{-1}x + \sin^{-1}x$

$y = \frac{\pi}{2}$

$\frac{dy}{dx} = 0$

④ $y = \sin^{-1}x + \cos^{-1}\sqrt{1-x^2}$

$= \sin^{-1}x + \sin^{-1}x$

$y = 2\sin^{-1}x$

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

⑤ $y = \cos^{-1}x + \sin^{-1}\sqrt{1-x^2}$

$y = \cos^{-1}x + \cos^{-1}x = 2\cos^{-1}x$

$\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$

$\sqrt{1-x^2} \Rightarrow$ put $x = \sin\theta$ or $\cos\theta$

$\sqrt{1+x^2} \Rightarrow$ put $x = \tan\theta$ or $\cot\theta$

$\sqrt{x^2-1} \Rightarrow$ put $x = \sec\theta$ or $\operatorname{cosec}\theta$

$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$

$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$

$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

1) $y = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$ put $x = \sin\theta$

$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}(\tan\theta)$

$= \theta = \sin^{-1}x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

2) $y = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$ put $x = \cos\theta$

$y = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \theta = \cos^{-1}x$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$

3) $y = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$ put $x = \cos\theta$

$y = \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$

$y = \theta$

$y = \cos^{-1}x$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$

4) $y = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$ put $x = \sin\theta$

$= \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \theta$

$y = \sin^{-1}x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

1) $$y = \tan^{-1} \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}} \qquad x = \cos\theta$$

$$= \tan^{-1} \frac{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}{\sqrt{1+\cancel{x}\cos\theta} - \sqrt{1-\cos\theta}} = \tan^{-1} \frac{\sqrt{2\cos^2\theta/2} + \sqrt{2\sin^2\theta/2}}{\sqrt{2\cos^2\theta/2} + \sqrt{2\,\cancel{\cos}\sin^2\theta/2}}$$

$$= \tan^{-1}\left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$$

$$y = \frac{\pi}{4} + \frac{\cos^{-1}x}{2}$$

$$\frac{dy}{dx} = \frac{-1}{2} \times \frac{1}{\sqrt{1-x^2}}$$

2) $$y = \tan^{-1} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \qquad x = \cos\theta$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right)$$

$$= \cancel{\tan} \frac{\pi}{4} - \frac{\theta}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

$$= \frac{\cancel{-}1}{2} \times \frac{\cancel{-}1}{\sqrt{1-x^2}}$$

$$= \frac{1}{2\sqrt{1-x^2}}$$

① $$y = \tan^{-1} \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \qquad x^2 = \cos\theta, \quad \theta = \cos^{-1}(x^2)$$

$$= \tan^{-1}\left(\frac{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta/2} + \sqrt{2\sin^2\theta/2}}{\sqrt{2\cos^2\theta/2} - \sqrt{2\sin^2\theta/2}}\right) = \tan^{-1}\left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \theta/2\right)\right) = \frac{\pi}{4} + \frac{\theta}{2}$$

$$y = \frac{\pi}{4} + \frac{\cos^{-1}x^2}{2}$$

$$\frac{dy}{dx} = \frac{-1}{\cancel{2}\sqrt{1-(x^2)^2}} \times \cancel{2}x = \frac{-1}{\sqrt{1-x^4}}$$

② $$y = \tan^{-1} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \qquad x^2 = \cos\theta, \quad \theta = \cos^{-1}(x^2)$$

$$y = \tan^{-1}\left(\frac{\cos\frac{\theta}{2} - \sin\theta/2}{\cos\frac{\theta}{2} + \sin\theta/2}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right)$$

$$= \frac{\pi}{4} - \frac{\theta}{2}$$

$$y = \frac{\pi}{4} - \frac{\cos^{-1}x^2}{2}$$

$$\frac{dy}{dx} = \frac{-1}{\cancel{2}} \times \frac{-1}{\sqrt{1-(x^2)^2}} \times \cancel{2}x$$

$$= \frac{1}{\sqrt{1-x^4}}$$

If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then $\frac{dy}{dx} = -x/y$.

$$\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$$

$$\cos^{-1}\sqrt{1-x^2} + \sin^{-1}y = \frac{\pi}{2}$$

$$y = \sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{1-x^2}} \times -\cancel{2}x = \frac{-x}{y}$$

If $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$, then $\frac{dy}{dx} = -x/y$

$$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$$

$$\sin^{-1}\sqrt{1-x^2} + \cos^{-1}y = \frac{\pi}{2}$$

$$y = \sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{1-x^2}} \times -\cancel{2}x$$

$$= \frac{-x}{\sqrt{1-x^2}} = \frac{-x}{y}$$

$2\tan^{-1}x$	$\Rightarrow$	3
$3\sin^{-1}x$	$\Rightarrow$	1
$3\cos^{-1}x$	$\Rightarrow$	1
$3\tan^{-1}x$	$\Rightarrow$	1
$\frac{1}{2}\cos^{-1}x$	$\Rightarrow$	6
$\sin^{-1}x$	$\Rightarrow$	2
$\cos^{-1}x$	$\Rightarrow$	2

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$\sqrt{a^2-x^2}$: $\Rightarrow$ put $x = a\sin\theta$ (or) $a\cos\theta$.

$\sqrt{a^2+x^2}$

$\sqrt{x^2-a^2}$

$\cos^{-1}\left(\frac{x}{a}\right)$

$\frac{1}{\sqrt{a^2-x^2}}$

If $y = \tan^{-1}\left[\frac{x+\sqrt{a^2-x^2}}{x-\sqrt{a^2-x^2}}\right]$

put $x = a\cos\theta$

$\theta = \cos^{-1}\left(\frac{x}{a}\right)$

$$= \tan^{-1}\left[\frac{a\cos\theta + \sqrt{a^2-a^2\cos^2\theta}}{a\cos\theta - \sqrt{a^2-a^2\cos^2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{a}(\cos\theta+\sin\theta)}{\cancel{a}(\cos\theta-\sin\theta)}\right]$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$$

$$= \frac{\pi}{4}+\theta = \frac{\pi}{4}+\cos^{-1}\left(\frac{x}{a}\right) = 0 - \frac{1}{}$$

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* $y = \tan^{-1}\left[\dfrac{x - \sqrt{a^2-x^2}}{x + \sqrt{a^2-x^2}}\right]$ put $x = a\cos\theta$. $\theta = \cos^{-1}\dfrac{x}{a}$

$= \tan^{-1}\left[\dfrac{a\cos\theta - \sqrt{a^2 - a^2\cos^2\theta}}{a\cos\theta + \sqrt{a^2 - a^2\cos^2\theta}}\right] = \tan^{-1}\left[\dfrac{a\cos\theta - a\sin\theta}{a\cos\theta + a\sin\theta}\right]$

$= \tan^{-1}\left[\tan\left(\dfrac{\pi}{4} - \theta\right)\right]$

$y = \dfrac{\pi}{4} - \theta = \dfrac{\pi}{4} - \cos^{-1}\dfrac{x}{a}$

$\dfrac{dy}{dx} = 0 - \left(\dfrac{-1}{\sqrt{a^2-x^2}}\right) = \dfrac{1}{\sqrt{a^2-x^2}}$

* $y = \sin(3\sin^{-1}x)$

$= \sin(\sin^{-1}(3x - 4x^3))$

$y = 3x - 4x^3$

$\dfrac{dy}{dx} = 3 - 12x^2$

* $y = \cos(3\cos^{-1}x)$

$y = \cancel{\cos}(\cancel{\cos^{-1}}(4x^3 - 3x))$

$y = 4x^3 - 3x$

$\dfrac{dy}{dx} = 12x^2 - 3$

$\dfrac{d}{dx}\cos^{-1}\dfrac{x}{a} = \dfrac{-1}{\sqrt{1-\frac{x^2}{a^2}}} \times \dfrac{1}{a}$

$= \dfrac{-1}{\frac{\sqrt{a^2-x^2}}{\cancel{a}}} \times \dfrac{1}{\cancel{a}}$

$= \dfrac{-1}{\sqrt{a^2-x^2}}$

$\dfrac{d}{dx}\tan^{-1}\dfrac{x}{a} =$

$\dfrac{1}{1+\frac{x^2}{a^2}} \times \dfrac{1}{a}$

$= \dfrac{1}{\frac{a^2+x^2}{a^{\cancel{2}}}} \times \dfrac{1}{\cancel{a}}$

$= \dfrac{a}{a^2+x^2}$

* $y = \sin^{-1} 2x\sqrt{1-x^2}$ put $x = \sin\theta$.

$= \sin^{-1} 2\sin\theta \cdot \sqrt{1-\sin^2\theta}$

$= \sin^{-1}(2\sin\theta\cdot\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$= 2\sin^{-1}x$

$\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-x^2}}$

* $y = \sin(\sin^{-1}x + \cos^{-1}x)$

$= \sin\left(\dfrac{\pi}{2}\right)$

$y = 1$

$\dfrac{dy}{dx} = 0$

* $y = \sin^{-1}\left(\dfrac{3x}{2} - \dfrac{x^3}{2}\right)$ put $\dfrac{x}{2} = \sin\theta$ $\theta = \sin^{-1}\left(\dfrac{x}{2}\right)$

$y = \sin^{-1}\left(3\left(\dfrac{x}{2}\right) - 4\left(\dfrac{x}{2}\right)^3\right) = \sin^{-1}(\sin 3\theta)$

$y = 3\sin^{-1}\left(\dfrac{x}{2}\right)$

$\dfrac{dy}{dx} = \dfrac{3}{\sqrt{4-x^2}}$

* $y = \cos^{-1}\left(\dfrac{4x^3}{27} - x\right)$

$y = \cos^{-1}\left(4\left(\dfrac{x}{3}\right)^3 - 3\left(\dfrac{x}{3}\right)\right)$ put $\dfrac{x}{3} = \cos\theta$ $\theta = \cos^{-1}\dfrac{x}{3}$

$= 3\theta$

$= 3\cos^{-1}\dfrac{x}{3}$

$= \dfrac{-3}{\sqrt{9-x^2}}$

$$y = \tan^{-1}\left(\frac{6x - 8x^3}{1 - 12x^2}\right)$$

$$= \tan^{-1}\left(\frac{3(2x) - (2x)^3}{1 - 3(2x)^2}\right)$$

$$= 3\tan^{-1} 2x$$

$$= 3 \times \frac{1}{1+4x^2} \times 2$$

$$= \frac{6}{1+4x^2}$$

$y =$

$$y = \tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3a^2x}{a^3} - \left(\frac{x}{a}\right)^3}{1 - 3\frac{ax^2}{a^3}}\right)$$

$$= \tan^{-1}\left[\frac{3\left(\frac{x}{a}\right) - \left(\frac{x}{a}\right)^3}{1 - 3\left(\frac{x}{a}\right)^2}\right]$$

$$= 3\tan^{-1}\left(\frac{x}{a}\right)$$

$$= \frac{3a}{a+x^2}$$

$$y = \cos^{-1}\left[\frac{9-x^2}{9+x^2}\right]$$

put $\frac{x}{3} = \tan\theta$

$\theta = \tan^{-1}\frac{x}{3}$

$$y = 9\cos^{-1}\left[\frac{1-\left(\frac{x}{3}\right)^2}{1+\left(\frac{x}{3}\right)^2}\right]$$

$$= 9\cos^{-1}\left[\cos 2\theta\right]$$

$$= 9 \times 2\tan^{-1}\frac{x}{3} = \frac{54}{9+x^2}$$

$$y = \tanh^{-1}\left[\frac{x}{\sqrt{1+x^2}}\right]$$

put $x = \sinh\theta$

$\theta = \sinh^{-1}x$

$\cosh^2 x - \sinh^2 x = 1$

$$= \tanh^{-1}\left[\frac{\sinh\theta}{\sqrt{1+\sinh^2\theta}}\right] = \tanh^{-1}\left[\tanh\theta\right]$$

$$y = \theta = \sinh^{-1}x.$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}$$

$$y = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$$

put $\log x = \tan\theta$

$\theta = \tan^{-1}(\log x)$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \cos^{-1}(\cos 2\theta)$$

$$= 2\theta = 2\tan^{-1}(\log x)$$

$$\frac{dy}{dx} = 2 \times \frac{1}{1+(\log x)^2} \times \frac{1}{x}$$

$$= \frac{2}{x\left(1+(\log x)^2\right)}$$

$$y = \sin^2\left(\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right) \qquad \frac{1}{2}\cos^{-1}x$$

$$= \sin^2\left(\frac{1}{2}\cos^{-1}x\right)$$

$$= \frac{1 - \cos\cancel{2}\left(\frac{1}{\cancel{2}}\cos^{-1}x\right)}{2}$$

$$y = \frac{1-x}{2}$$

$$\frac{dy}{dx} = \frac{-1}{2}$$

$$y = \cos^2\left(\sin^{-1}\sqrt{\frac{1-x}{2}}\right) \qquad \frac{1}{2}\cos^{-1}x$$

$$= \cos^2\left(\frac{1}{2}\cos^{-1}x\right)$$

$$= \frac{1 + \cos\cancel{2}\left(\frac{1}{\cancel{2}}\cos^{-1}x\right)}{2}$$

$$y = \frac{1+x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

$$y = \tan^{-1}\left[\frac{1-\sqrt{1-x^2}}{x}\right] \qquad \text{put } x = \sin\theta$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right] = \tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}{\cancel{2}\,\cancel{\sin\frac{\theta}{2}}\cdot\cos\frac{\theta}{2}}\right] = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} = \frac{\sin^{-1}x}{2} = \frac{1}{2\sqrt{1-x^2}}$$

$$y = \cancel{\tan}\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right] \qquad \text{put } x = \tan\theta$$

$$= \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right] \qquad \frac{1}{\cos} = \tan^{-1}\left[\frac{\frac{1-\cos\theta}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}{\cancel{2}\,\cancel{\sin\frac{\theta}{2}}\cdot\cos\frac{\theta}{2}}\right]$$

$$y = \frac{\theta}{2} = \cancel{\sin}\,\frac{\tan^{-1}x}{2} = \frac{1}{2(1+x^2)}$$

$$y = \tan^{-1}\left[\frac{x}{1+\sqrt{1+x^2}}\right] \quad x = \tan\theta$$

$$= \tan^{-1}\left[\frac{\tan\theta}{1+\sec\theta}\right] \quad \frac{1}{\cos\theta} = \tan^{-1}\left[\frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{\cos\theta+1}{\cancel{\cos\theta}}}\right]$$

$$= \tan^{-1}\left[\frac{2\sin\frac{\theta}{2}\cdot\cancel{\cos\frac{\theta}{2}}}{2\cos^{\cancel{2}}\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{\cot}\ \theta}{\tan\frac{\theta}{2}\ 2}\right] = \frac{\theta}{2}$$

$$= \cancel{90^\circ - \frac{\theta}{2}} = \cancel{90^\circ}\ \frac{\tan^{-1}x}{2} = \frac{1}{2(1+x^2)}$$

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$$y = \tan^{-1}\left[\frac{x}{1-\sqrt{1+x^2}}\right]$$ put $x = \tan\theta$

$$= \tan^{-1}\left[\frac{\tan\theta}{1-\sec\theta}\right] = \tan^{-1}\left[\frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{-1\left[1-\cos\theta\right]}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{2\sin\frac{\theta}{2}}\cdot\cos\frac{\theta}{2}}{-1\left[\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}\right]}\right] = \tan^{-1}\left[-\cot\frac{\theta}{2}\right]$$

$$= \left(90^\circ - \frac{\theta}{2}\right) = -90^\circ + \frac{\theta}{2}$$

$$\frac{dy}{dx} = \frac{\tan^{-1}x}{2} = \frac{1}{2(1+x^2)}$$

$$y = \tan^{-1}\left[\frac{\frac{x}{\cancel{a}}}{\sqrt{\frac{a^2}{\cancel{a}}-\frac{x^2}{\cancel{a}}}}\right]$$ put

$$y = \tan^{-1}\left[\frac{\sqrt{1-x^2}-1}{x}\right]$$ put $x = \sin\theta$

$$= \tan^{-1}\left[\frac{\sqrt{1-\sin^2\theta}-1}{\sin\theta}\right] = \tan^{-1}\left[\frac{\cos\theta-1}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{-(1-\cos\theta)}{\sin\theta}\right]$$

$$= \tan^{-1}\left[\frac{-\cancel{2\sin^2\frac{\theta}{2}}}{\cancel{2\sin\frac{\theta}{2}}\cdot\cos\frac{\theta}{2}}\right] = \tan^{-1}\left[-\tan\frac{\theta}{2}\right]$$

$$= \frac{-\theta}{2} = \frac{-\sin^{-1}x}{2}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$$

$$y = \tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right]$$ put $x = a\sin\theta$, $\theta = \sin^{-1}\frac{x}{a}$

$$= \tan^{-1}\left[\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\cancel{a}\sin\theta}{\cancel{a}\cos\theta}\right]$$

$$= \tan^{-1}(\tan\theta)$$

$$= \theta$$

$$y = \sin^{-1}\frac{x}{a}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}$$