Chapters Included
- Limits & continuity
- Differention
- Applications of Differention
-

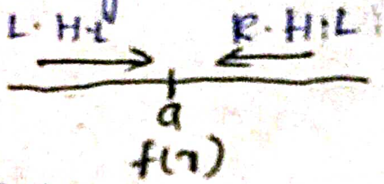
LIMITS

objectives
- Left and Right Hand Limits
- Existence of Limit
- Indeterminate Forms
- Algebra of Limits
- sandwich Theorm [Not in portion but learnt)
- Different methods of solving limits
  - Direct substitution
  - Factorisation
  - Rationalisation
  - Standard limits
  - Algebraic limits at infinity
- Trigonometric, Logarthmic, exponential Limits
- L Hospital's Rule

References :-
- Mohit Tyagi lectures ✓

Left Hand Limit — LHL of a functio... $x \to a$ (if exist) towards which function approaches or tales as x is taken towards a from LH side

Right Hand Limit — R·H·L of a function $f(x)_{x\to a}$ is the value towards x is taken toward a from right



Limit of a function — It exists at $x=a$ if $L.H.L_{x=a} = R.H.L_{x=a} = l$

then $\lim_{x\to a} f(x) = l$

(l is called limiting value of $f(x)$ at $x=a$)

Note:-

1) If L·H·L & R·H·L exists but not equal then limit do not exists.

Form of representation

$$L.H.L_{x=a} = \underset{x\to a^-}{Lt} f(x) \quad \begin{matrix} x = a+h \\ h = x-a = 0^- \end{matrix}$$

$$= \underset{h\to 0^-}{Lt} f(a+h)$$

$$= \underset{h\to 0^+}{Lt} f(a-h)$$

$$R.H.L\Big]_{x=a} = \underset{x\to a^+}{Lt} f(x)$$

$$= \underset{h\to 0^+}{Lt} f(a+h)$$

$$= \underset{h\to 0^-}{Lt} f(a-h)$$

COMBINATION $\underset{x\to a}{Lt} f(x) = \underset{h\to 0}{Lt} (a+h)$

Examples

1) Find LHL & RHL of function $f(x)$, where $f(x) = \begin{cases} \frac{|x-4|}{x-4} & x \neq 4 \\ 0 & x = 4 \end{cases}$

L.H.L

$x = a - h$

$$\lim_{x \to 4} \frac{|x-4|}{x-4} = \lim_{h \to 0^+} \frac{|4-h-4|}{4-h-4}$$

$$= \lim_{h \to 0^+} \frac{|-h|}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h}$$

$$= \lim_{h \to 0} -1 = -1$$

R.H.L

$x = a + h$

$$\lim_{x \to 4} \frac{|x-4|}{x-4} = \lim_{h \to 0} \frac{|4+h-4|}{4+h-4}$$

$$= \lim_{h \to 0} 1 = 1$$

2) $f(x) = \begin{cases} 1+x^2 & 0 \leq x \leq 1 \\ 2-x & < 1 \end{cases}$

L.H.L

$x = a - h$
$(x = 1 - h)$

$$\lim_{x \to 1} 1+x^2 = \lim_{h \to 0} 1+(1-h)^2$$

$$= \lim_{h \to 0} 1+1^2-2h+h^2$$

$$= \lim_{h \to 0} h^2-2h+2$$

$$= 2$$

R.H.L

$$\lim_{x \to 1} 2-x = \lim_{h \to 0} 2-(1+h)$$

$$= \lim 1-h$$

Note

Indeterminate forms	$\lim_{x \to A} f(x)$ & $\lim_{x \to B} g(x)$ are indetermite where A, B

1) $\frac{\infty_1}{\infty_2} = \frac{1/\infty_1}{1/\infty_2} = \frac{0}{0}$

2) $0 \times \infty = 0 \times \frac{0}{1/\infty} = \frac{0}{0}$

3) 0^0 $\left(y = 0^0 \atop \log y = \log 0^0 \Rightarrow 0 \log 0 \Rightarrow 0 \times \infty \right)$

4) ∞^0 $(\log y = 0 \log \infty = 0 \times \infty)$

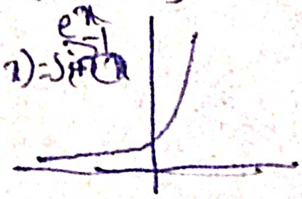
5) 1^∞ $(\infty \log 1 = \infty \times 0)$

6) $\infty_1 - \infty_2$

Reasons of Nonexistence of limits

1) If $L \cdot H \cdot L \neq R \cdot H \cdot L$
2) If $L \cdot H \cdot L$ (or) $R \cdot H \cdot L$ not exists
3) If function fluctuates

for (2) Ex :- 1) $f(x) = e^x$



$LHL = 0$
$R \cdot H \cdot L = \infty$

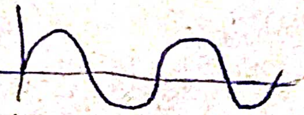
2) $\frac{1}{x^2 - 9} = f(x)$ $\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 3^+} f(x) = \infty$

Note

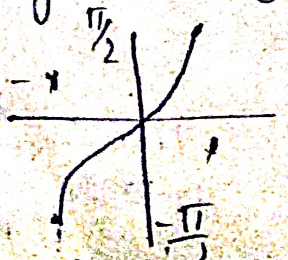
$\boxed{\frac{1}{x \to -0} = -\infty}$

$\sin x =$



Speciality of $\sin^{-1} x$

* If $L \cdot H \cdot L$ (or) $R \cdot H \cdot L$ not valued than any one of it ~~has~~ ~~its~~ equal to limit



$\lim_{x \to 1^+} =$ not valued.

$\lim \sin^{-1}(x) = \lim \sin^{-1} x = \pi$

Difference between $\lim_{x\to a} f(x)$, $f(x)$ at $x \to a$

<u>cases</u>

1) If $\lim_{x\to a} f(x)$ exist and $f(a)$ does not exist

Ex: $f(x) = \frac{x^2 - a^2}{x - a}$ at $x = a$ $= \frac{0}{0}$ (not exist)

$\lim_{x\to a} f(x) = \cancel{(x+a)} = (a+a) = 2a$

2) If $f(x)$ at $x = a$ exists but $\lim_{x\to a} f(x)$ not exists

Ex: $f(x) = [x]$ — integral part

$\lim_{x\to a} [x]$ does not exist [L.H.L $\neq$ R.H.L]

But $f(a) = a$

3) Both $f(a)$, $\lim_{x\to a} f(x)$ exist

$f(x) \begin{cases} \sin x & x < 0 \\ x & x \geq 0 \end{cases}$

4) If both exist and not equal too.

conclusion: $\lim_{x\to a} f(x)$ need not to be equal to $f(a)$

<u>Note</u>

1) $\frac{1}{0^-} = -\infty$ $\quad \frac{1}{0^+} = +\infty$

2) $\lim_{x\to\infty} a^x = 0 \quad (0 < a < 1)$

$= 1 \quad (a = 1)$

$= \infty \quad (a > 1)$

Basics of Evaluation

1) $1+2+3+4+\dots+n = \frac{n(n+1)}{2}$

2) $1^2+2^2+3^2+4^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

3) $1^3+2^3+3^3+4^3+\dots n^3 = \left(\frac{n(n+1)}{2}\right)^2$

4) $\sqrt{x}-\sqrt{y} = \frac{x-y}{\sqrt{x}+\sqrt{y}}$

5) $\sqrt[3]{x}-\sqrt[3]{y} = \frac{x-y}{\sqrt[3]{x^2}+\sqrt[3]{x\cdot y}+\sqrt[3]{y^2}}$

6) $\sin\alpha + \sin(\alpha+2\beta) + \sin(\alpha+3\beta) + \dots \sin(\alpha+(n-1)\beta)$

$$= \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \cdot \sin\left(\frac{2\alpha+(n-1)\beta}{2}\right)$$

7) $\cos\alpha + \cos(\alpha+2\beta) + \cos(\alpha+3\beta) \dots \cos(\alpha+(n-1)\beta)$

$$= \cancel{\frac{\cos\left(\frac{n\beta}{2}\right)}{\cos\left(\frac{\beta}{2}\right)}} \; \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \cdot \cos\left(\frac{2\alpha+(n-1)\beta}{2}\right)$$

Methods of evaluation of limits

1) Direct Substitution Method
2) Rationalization method
3) standard formulaes
4) Trigonometric, Exponential, Logarthimic limits
5) L Hospital rule

Basic Problems

1) $\lim_{x\to 1} \frac{x^2+5x-6}{x^2-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+6)}{\cancel{(x-1)}(x+1)} = \frac{7}{2}$

2) $\lim_{x\to 1} \frac{x^3+5x^2+7x-13}{x^5+7x^3-8} = \frac{\cancel{(x-1)}(x^2+6x+13)}{\cancel{(x-1)}(x^4+8x^3+8x^2+8x+8)}$

TRICK

L.H.S | R.H.S

$1 \; | \; 1$

$-13 \; | -8$ $\quad -1+13 \; | +8$

$4+5$ $\quad -1+6$

$= \frac{1+6+13}{1+8+8+8+8} = \frac{20}{26}$

3) $\lim_{x\to 2} \frac{5x^2+x-21}{\sqrt{x}-\sqrt{2}} = \frac{1}{0}$ = not exist

4) $\lim_{x\to 2} \frac{3x^2+4x-20}{\sqrt{x}-\sqrt{2}} = \frac{0}{0}$ (Indeterminant)

$x-2)\,3x^2+4x-20\,(3x+10$

$\underline{3x^2-6x}$

$10x-20$

$\underline{10x-20}$

(0)

$\lim_{x\to 2} \left(\frac{(x-2)(3x+10)}{\frac{x-2}{\sqrt{x}+\sqrt{2}}} \right)$

5)

$= \lim_{x\to 2} (3x+10)(\sqrt{x}+\sqrt{2}) = 16(2\sqrt{2}) = 32\sqrt{2}$

5) $\lim_{x\to\frac{\pi}{2}} \tan^2 x(\sin x-1) = \lim_{x\to\frac{\pi}{2}} \frac{\sin^2 x}{\cos^2 x}(\sin x-1) = \lim_{x\to\frac{\pi}{2}} \frac{\sin^2 x(\sin x-1)}{1-\sin^2 x}$

$= \lim_{x\to\frac{\pi}{2}} \frac{\sin^2 x(\sin x-1)}{(1-\sin x)(1+\sin x)}$

$= \lim_{x\to\frac{\pi}{2}} \frac{-\sin^2 x(1-\sin x)}{(1-\sin x)(1+\sin x)}$

$= \frac{-1}{2}$

6) $\lim_{x\to\infty} \dfrac{x^4+5x^2+7x+9}{x^3+8x^4+5x+3}$

$= \lim_{x\to\infty} \dfrac{1+\frac{5}{x^2}+\frac{7}{x^3}+\frac{9}{x^4}}{\frac{1}{x}+8+\frac{5}{x^3}+\frac{3}{x^4}}$

[If $x\to\infty$
$\frac{1}{x}\to 0$]

$= \frac{1}{8}$

7) $\lim_{x\to\infty} (x^3-5x^2)$ $\quad \infty-\infty$

$= \lim_{x\to\infty} x^2(x-5)$: does not exist

8) $\lim_{x\to\infty} (\sqrt{x+3}-\sqrt{x-6})$ $\quad (\infty-\infty)$

$= \lim_{x\to\infty} \dfrac{x+3-x+6}{\sqrt{x+3}+\sqrt{x-6}}$

$= \lim_{x\to\infty} \dfrac{9}{\sqrt{x+3}+\sqrt{x-6}}$ $\quad \left(\frac{9}{\infty}\right)$

$= 0$

9) $\lim_{x\to\infty} \sqrt{x^2-5x-7} - \sqrt{x^2+7x-5}$ $\quad (\infty-\infty)$

$= \lim_{x\to\infty} \dfrac{x^2-5x-7-x^2-7x+5}{\sqrt{x^2-5x-7}+\sqrt{x^2+7x-5}}$

$= \lim_{x\to\infty} \dfrac{-12x-2}{\sqrt{x^2-5x-7}+\sqrt{x^2+7x-5}}$

$= \lim_{x\to\infty} \dfrac{\frac{-12x}{x}-\frac{2}{x}}{\sqrt{\frac{x^2}{x^2}-\frac{5x}{x^2}-\frac{7}{x^2}}+\sqrt{\frac{x^2}{x^2}+\frac{7x}{x^2}-\frac{5}{x^2}}}$

$\frac{1}{x}\to 0$

$= \frac{-12}{2} = -6$

Algebra of Limits

1) $\lim_{x \to a} (k f(x)) = k \lim_{x \to a} f(x)$

2) $\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

[RHS should not be indeterminate form]

3) $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

[Any one term of R.H.S should not zero (or) indeterminate form without knowing other ($\infty \cdot 0$ - Indeterminate)]

4) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$

[for go with effective degree]

5) $\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$

Shortcut keys

1) when $x \to 0$ then w.r.t effective degree

$\dfrac{H}{L} = 0$, $\dfrac{L}{H} = \infty$ Doesnot exist, $\dfrac{E}{E}$ = Nonzero Finite

Ex: ⓐ $\lim_{x \to 0} \dfrac{3x^5 + 7x^2 + 5x^4}{5x + 8x^3 + x^9} = \dfrac{\frac{3x^5}{x^5} + \frac{7x^2}{x^5} + \frac{5x^4}{x^5}}{\frac{5x}{x^5} + \frac{8x^3}{x^5} + \frac{x^9}{x^5}}$

$\dfrac{2}{1} = \dfrac{H}{L} = 0$, $\dfrac{5}{\infty} = 0$

2) When $x \to \infty$ then w.r.t effective degree

$\dfrac{H}{L}$ = Doesnot exist, $\dfrac{L}{H} = 0$, $\dfrac{E}{E}$ = N.Z.F

ⓑ $\lim_{x \to \infty} \dfrac{5x^3 + x^8 + 9}{8x^3 + 3x^2 + 7x^5} = \dfrac{8}{5} = \dfrac{H}{L}$ = DNE

Effective Degree

If $x \to 0$ · least power of polynomial
$x \to \infty$ · highest power of polynomial

If $x = 0.000 \ldots 1$
$x^2 = 0.0 \ldots \ldots 1$
x^2 is very small compared to x, so it can be neglected when $x \to 0$

If $x = 100 \ldots 0$
$x^2 = 10 \ldots \ldots 0$
x^2 is very large than x, so its can be neglected when $x \to \infty$

Ex-1) $\lim_{x \to 0} \dfrac{3x^2 + 5x^3 + 7x^8}{x^9 + x^{12} + 7x}$

$= \dfrac{3x^2}{7x} = \dfrac{3x}{7} = 0 \quad (x = 0)$

2) $\lim_{x \to \infty} \dfrac{8x^3 + 7x^2 + x}{9x + 12x^2 + 13x^3}$

$= \dfrac{8x^3}{13x^3} = \dfrac{8}{13}$

Note :- If Effective degree term is getting cancelled in sum form then do not use this technique

Ex :- $\lim_{x \to \infty} \sqrt{x^2 + x + 4} - \sqrt{x^2 + 3x + 1}$

$[\sqrt{x^2} - \sqrt{x^2}]$ cancelled

so

$\lim_{x \to \infty} \dfrac{x^2 + x + 4 - x^2 - 3x - 1}{\sqrt{x^2 + x + 4} + \sqrt{x^2 + 3x + 1}}$

$\lim_{x \to \infty} \dfrac{-2x + 3}{\sqrt{x^2 + x + 4} + \sqrt{x^2 + 3x + 1}} = \lim_{x \to \infty} \dfrac{-2 + \frac{3}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2} + \frac{4}{x^2}} + \sqrt{\frac{x^2}{x^2} + \ldots}}$

$= \dfrac{-2}{2} = -1$

Examples for Shortkeys

1) $\underset{x \to 0}{Lt} \dfrac{3x^3+7x^2+5x^4}{3x^3+8x^3+x^9} = \dfrac{3}{3} = 1$

Substitution (changing of limits)

1) If $x \to a$ then let $t = x-a$
$t \to 0$
$x = t+a$

2) If $x \to -\infty$ then $t = -x \; (x = -t)$
$t \to \infty$

3) If $x \to \infty$ then $x = \dfrac{1}{t} \Rightarrow t = \dfrac{1}{x}$
$t \to 0$

Binomial Theorm

$(1+x)^n = 1 + \dfrac{nx}{1!} + \dfrac{n(n-1)x^2}{2!} + \dfrac{n(n-1)(n-2)}{3!}x^3 \dots$

If $x \to 0$

assume

$(1+x)^n = 1 + \dfrac{nx}{1!}$

(or)

$1 + \dfrac{n(n-1)x^2}{2!} + nx$

- Expand until it gets cancelled and only one term remains

Example

$\underset{x \to 0}{Lt} \dfrac{(1+3x)^{1/4} - 1}{x^2} \quad \left[\dfrac{0}{0}\right]$

$= \underset{x \to 0}{Lt} \dfrac{\cancel{1} + \frac{3}{4}x - \cancel{1}}{x^2}$

$= \infty$ (Does not exist)

STANDARD FORMULA

1) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

PROOF :- let $t = x - a$

$x = a + t$

and $t \to 0$

$= \lim_{t \to 0} \frac{(a+t)^n - a^n}{a+t-a} = \frac{a^n\left(1+\frac{t}{a}\right)^n - a^n}{t}$

$= \lim_{t \to 0} \frac{a^n\left(\cancel{1} + \frac{nt}{a} \cancel{- 1}\right)}{t}$

$= a^n \lim_{t \to 0} \frac{\frac{n\cancel{t}}{a}}{\cancel{t}} = \boxed{a^{n-1} \cdot n}$

Example

$\lim_{x \to 2} \frac{x^5 - 32}{x - 2} = \lim_{x \to 2} \frac{x^5 - 2^5}{x - 2}$

$= 5(2)^4 = 80$

2) If $a = 1$

$\lim_{x \to 1} \frac{x^n - 1}{x - 1} = n$

Ex:- 1) $\lim_{x \to 1} \frac{x^5 - 1}{x - 1} = 5$

$\boxed{\text{Imp}}$ 2) $\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{(x^2 - 1)} = \lim_{x \to 1} \frac{x^{\frac{1}{3}} - 1}{(x-1)(x+1)}$

$= \lim_{x \to 1} \frac{1}{x+1} \cdot \lim_{x \to 1} \frac{x^{\frac{1}{3}} - 1}{x - 1}$

$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

3) $\lim_{x \to 1} \frac{x+x^2+x^3+\dots+x^{20}-20}{x-1}$ $\left[\frac{0}{0}\right]$

$= \lim_{x \to 1} \frac{x+x^2+x^3+\dots -1-1-1\dots-1 \text{ (20 times)}}{x-1}$

$= \lim_{x \to 1} \frac{(x-1)+(x^2-1)+(x^3-1)\dots(x^{20}-1)}{x-1}$

$= \lim_{x \to 1} \frac{x-1}{x-1} + \lim_{x \to 1} \frac{x^2-1}{x-1} + \dots \lim_{x \to 1} \frac{x^{20}-1}{x-1}$

$= 1+2+3+\dots 20 = \frac{\cancel{20}^{10}(21)}{\cancel{2}} = 210$

4) $\lim_{x \to \infty} \frac{1+2+3+4\dots+n}{n^2+4n+7}$

as $x \to \infty$ considering highest term $= \lim_{x \to \infty} \frac{\frac{\cancel{n}(n+1)}{2}}{n^{\cancel{2}}}$ (approximation)

$= \lim_{x \to \infty} \frac{n+1}{2n} = \lim_{x \to \infty} \frac{1}{2}\left(1+\frac{1}{n}\right)$

$\frac{1}{n} = 0$ as $n \to \infty$

$= \frac{1}{2}$

5) $\lim_{n \to \infty} \frac{1^2+2^2+3^2+4^2+\dots 16n^2}{(1+2+3+4+\dots 7n)(2n+5)}$

$n \to 4n$ $= \frac{\frac{(4n)(4n+1)(8n+1)}{6}}{\frac{(7n)(7n+1)}{2}(2n)}$

$n \to 7n$ neglecting (5) n^0 term as $n \to \infty$ $= \frac{2}{6}\left(\frac{(4)\left(4+\frac{1}{n}\right)\left(8+\frac{1}{n}\right)}{7\left(7+\frac{1}{n}\right)(2)}\right)$

$= \frac{\cancel{2} \times 4^{2} \times 4 \times 8}{\dots} = \frac{64}{147}$

6) $\lim_{n\to\infty} \frac{1+2+3+\dots+n}{n^2}$

$= \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^2}$

$= \lim_{n\to\infty} \frac{1}{2}\left(1+\frac{1}{n}\right) = \frac{1}{2}$

(or)

Important

$\cancel{= \lim_{n\to\infty} \frac{1}{n^2} + \lim_{n\to\infty} \frac{2}{n^2} + \dots \lim_{n\to\infty} \frac{1}{n}}$

$\cancel{= 0 + 0 + 0 + 0 + 0 \quad = 0}$

It should not be done like this since $0+0+0\dots = 0 + \infty =$ (indeterminate)

[If no. of terms $\to \infty$ then we can not separate terms (if sum of terms is infinite).

7) $\lim_{n\to\infty} \left(\frac{1}{1\cdot 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \dots \frac{1}{n(n-1)}\right)$

$\frac{1}{n(n-1)} = \frac{1}{n-1} - \frac{1}{n}$

$= \lim_{n\to\infty} \left(1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \frac{1}{4} + \dots \cancel{\frac{1}{n-1}} - \frac{1}{n}\right)$

$= \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)$

$= 1$

Terms to remember

$\prod$ - Product

$\sum$ - sum.

π - Pie

8) $\lim_{n\to\infty} \prod_{r=2}^{n} \left(1-\frac{1}{r^2}\right)$

$= \lim_{n\to\infty} \left(\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right) \right)$

$= \lim_{n\to\infty} \left(\frac{2^2-1}{2^2} \cdot \frac{3^2-1}{3^2} \cdot \frac{4^2-1}{4^2} \cdot \frac{n^2-1}{n^2} \right)$

$2^2-1 = (2-1)(2+1) = 1\cdot 3$

$= \lim_{n\to\infty} \left(\frac{1\cdot 3}{2^2} \cdot \frac{2\cdot 4}{3^2} \cdot \frac{3\cdot 5}{4^2} \cdots \frac{(n-1)(n+1)}{n^2} \right)$

$= \lim_{n\to\infty} \frac{1}{2}\left(\frac{n+1}{n}\right) = \lim_{n\to\infty} \frac{1}{2}\left(1+\frac{1}{n}\right) = \frac{1}{2}$

9) $\lim_{x\to 2} \left(\frac{1}{x^2-4} - \frac{1}{x^2+3x-10} \right)$ $[\infty-\infty]$

$= \lim_{x\to 2} \left(\frac{1}{(x-2)(x+2)} - \frac{1}{(x-2)(x+5)} \right)$

$= \lim_{x\to 2} \frac{1}{x-2}\left(\frac{1}{x+2} - \frac{1}{x+5} \right)$

$= \lim_{x\to 2} \frac{1}{x-2}\left(\frac{1}{4} - \frac{1}{7} \right) \to \frac{3}{24}$

$\left[\frac{1}{0}\cdot\left(\frac{3}{24}\right) \to \infty\right]$

$= \lim_{x\to 2} \frac{3}{24(x)-2(24)}$

$= \frac{3}{0}$ (does not exist)

[x may be y too]

Trigonometric Standard Formulas

1) $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Ex:- 1) $\lim_{x\to 3} \frac{\sin x^2-9}{x^2-9} = 1$

2) $\lim_{x\to 3} \frac{\sin x^2-9}{x-3}$

$= \lim_{x\to 3} \frac{x+3}{x+3} \cdot \frac{\sin x^2-9}{x-3}$

$= \lim_{x\to 3} x+3 \cdot \lim_{x\to 3} \frac{\sin x^2-9}{x^2-9}$

$= 6 \cdot 1 = 6$

Important

$x \to 0$

$\pi x \not\to 0$

so it is written $(2\pi - \pi x)$

3) $\lim_{x\to 2} \frac{\sin \pi x}{x^2-4} \quad \left[\frac{0}{0}\right]$

$= \lim_{x\to 2} -\frac{\sin(2\pi - \pi x)}{x^2-4}$

$= \lim_{x\to 2} \frac{\sin(\pi x - 2\pi)}{(x^2-4)(\pi x - 2\pi)} \cdot (\pi x - 2\pi)$

$= \lim_{x\to 2} \frac{\sin(\pi x - 2\pi)}{\pi x - 2\pi} \cdot \lim_{x\to 2} \frac{\pi x - 2\pi}{x^2-4}$

$= \lim_{x\to 2} \frac{\sin(\pi x - 2\pi)}{\pi x - 2\pi} \cdot \lim_{x\to 2} \frac{\pi \cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$= 1 \cdot \frac{\pi}{4}$

$= \frac{\pi}{4}$

(2) $\lim\limits_{x\to 0} \dfrac{\tan x}{x} = 1$

Ex: $\lim\limits_{x\to 4} \dfrac{\tan (x^2-16)}{x^2-16} = 1$

(3) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} = \dfrac{1}{2}$

Ex:- 1) $\lim\limits_{x\to 0} \dfrac{1-\cos 3x}{x^2} = \lim\limits_{x\to 0} \dfrac{1-\cos 3x}{9x^2} \times \dfrac{9}{9}$

$= 9 \times \dfrac{1}{2} = \dfrac{9}{2}$

2) $\lim\limits_{x\to 3} \dfrac{1-\cos(x^2-9)}{(x-3)^2}$

$\cancel{\lim\limits_{x\to 3} \dfrac{1-\cos(x^2-9)}{x^2+9-6x}}$

Important step

$= \lim\limits_{x\to 3} \dfrac{1-\cos(x^2-9)}{(x-3)^2} \cdot \dfrac{(x+3)^2}{(x+3)^2}$

$= \lim\limits_{x\to 3} \dfrac{1-\cos(x^2-9)}{(x^2-9)^2} \cdot (x+3)^2$

$= \dfrac{36}{2} = 18$

3) $\lim\limits_{x\to 4} \dfrac{\tan(\pi x)}{\sin(2\pi x)}$

$= \lim\limits_{x\to 4} \dfrac{-\tan(4\pi - \pi x)}{-\sin(8\pi - 2\pi x)}$

$= \lim\limits_{x\to 4} \cancel{\dfrac{\tan(\pi x - 4\pi)}{\sin(\pi x - 8\pi)}} \quad \dfrac{\tan(4\pi - \pi x)}{\sin(8\pi - 2\pi x)}$

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

$\lim\limits_{x\to 0} \dfrac{\tan x}{x} = 1$

$= \lim\limits_{x\to 4} \dfrac{\tan(4\pi - \pi x)}{4\pi - \pi x} \cdot \dfrac{4\pi - \pi x}{8\pi - 2\pi x} \cdot \dfrac{8\pi - 2\pi x}{\sin(8\pi - 2\pi x)}$

(1) (1/2) (1)

$= 1/2$

3) $\lim_{x\to 3} \frac{1+\cos \pi x}{(\sqrt{7+x^2}-4)(x^2-9)}$

4) If $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

if $\sin\theta = x$

$\theta = \sin^{-1}x$

$\lim_{x\to 0} \frac{x}{\sin^{-1}x} = 1$ (or) $\lim_{x\to 0} \frac{\sin^{-1}x}{x} = 1$

$\lim_{x\to 0} \frac{x}{\tan^{-1}x} = 1$ (or) $\lim_{x\to 0} \frac{\tan^{-1}x}{x} = 1$

COMBINATION:- Let $A, B = x, \sin x, \sin^{-1}x, \tan x, \tan^{-1}x$

then $\lim_{x\to 0} \frac{A}{B} = 1$ (also $\frac{\sin x}{\sin^{-1}x}$ (or) $\frac{\tan x}{\tan^{-1}x}$)

Ex:- 1) $\lim_{x\to 0} \frac{\sin^{-1}(3x)}{\tan^{-1}(2x)}$

$= \lim_{x\to 0} \frac{\sin^{-1}(3x)}{3x} \cdot \frac{3x}{2x} \cdot \frac{2x}{\tan^{-1}2x}$

$= \frac{3}{2}$

2) $\lim_{x\to 0} \frac{\sin 3x - \sin x}{\tan^{-1}5x}$

$= \lim_{x\to 0} \frac{2\sin x \cos 2x}{\tan^{-1}(5x)}$

$\cos 2(0) = 1$ $= 2 \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{x}{5x} \cdot \frac{5x}{\tan^{-1}5x}$

$= 2 \times \frac{1}{5}$

3) $\lim_{x\to 0} \frac{\cos x - \cos 5x}{x^2}$

$= \lim_{x\to 0} \frac{2\sin 3x \sin 2x}{x^2}$

$= 2 \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \frac{3 \cdot 2}{\cancel{5x}} \cdot \frac{\sin 2x}{2x}$

$= 6$

4) $\lim_{x\to\infty} x\cdot\sin\left(\frac{1}{x}\right)$

let $y = \frac{1}{x}$

$= \lim_{y\to 0} \frac{1}{y}\cdot\sin y$

$= 1$

5) $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$

$= 0\cdot[-1,1] = 0$

6) $\lim_{x\to\frac{\pi}{3}} \frac{\cancel{3}\sin x - \sqrt{3}}{\cos\frac{3x}{2}}$

$= \lim_{x\to\frac{\pi}{3}} \frac{2\left(\sin x - \frac{\sqrt{3}}{2}\right)}{\cos\frac{3x}{2}}$

$= \lim_{x\to\frac{\pi}{3}} \frac{2(\sin x - \sin 60^\circ)}{\sin\left(\frac{\pi}{2} - \frac{3x}{2}\right)}$

$= \lim_{x\to\frac{\pi}{3}} \frac{2\left(2\sin\left(\frac{x-\pi/3}{2}\right)\cdot\cos\left(\frac{x+\pi/3}{2}\right)\right)}{\frac{\sin(\pi/2 - \frac{3x}{2})}{\pi/2 - \frac{3x}{2}}\cdot \pi/2 - \frac{3x}{2}}$

$\frac{2\pi}{\frac{3}{2}}\ \frac{2\pi}{3}\ \frac{60}{180}$

let $x = \pi/3$ in cos

$= \lim_{x\to\pi/3} \frac{2\cdot 2\cdot\frac{1}{2}}{1} \cdot \frac{\sin\left(\frac{x-\pi/3}{2}\right)}{\pi/2 - \frac{3x}{2}}$

$$= 2 \lim_{x \to \frac{\pi}{3}} \frac{\sin\left(\frac{x - \pi/3}{2}\right)}{\frac{x - \frac{\pi}{3}}{2}} \cdot \frac{\frac{x - \pi/3}{2}}{\frac{\pi}{2} - \frac{3x}{12}}$$

$$= 2 \lim_{x \to \pi/3} \frac{\frac{x - \pi/3}{2}}{3/2\left(\pi/3 - x\right)}$$

$$= \frac{2 \cdot (-1) \cdot 1/\cancel{2}}{3/\cancel{2}} = \frac{-2}{3}$$

7) $$\lim_{x \to 0} \frac{1 - \cos x \cdot \cos 2x}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - \cos x \cos 2x + \cos x - \cos x}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{x^2} + \lim_{x \to 0} \frac{\cos x - \cos x \cdot \cos 2x}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{x^2} + \lim_{x \to 0} \frac{\cos x\,(1 - \cos 2x)}{x^2}$$

$$= 1/2 + \lim_{x \to 0} 1 \left(\frac{1 - \cos 2x}{4x^2} \cdot 4\right)$$

$$= 1/2 + 1/\cancel{2} \times 4^{2}$$

$$= \frac{5}{2}$$

Method 3

Exponential and logarithmic Standards

1) $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

Ex:- 1) $\lim_{x\to \pi} \frac{e^{\sin x} - 1}{\tan x}$

$= \lim_{x\to\pi} \frac{e^{\sin x} - 1}{\sin x} \cdot \cos x$

$= 1 \times -1 = -1$

2) $\lim_{x\to 3} \frac{e^x - e^3}{x^2 - 9}$

$= \lim_{x\to 3} \frac{e^3(e^{x-3} - 1)}{(x-3)(x+3)}$

$= \frac{e^3}{6}(1)$

2) $\lim_{x\to 0} \frac{a^x - 1}{x} = \ln(a) \; (\log_e a)$

Ex:- $\lim_{x\to 0} \frac{2^x - 1}{x} = \log_e 2 = \ln 2$

3) $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

Ex:- $= \lim_{x\to 0} \frac{\ln(x^2 - 3)}{x^2 - 4}$

$= \lim_{x\to 0} \frac{\ln(1 + x^2 - 4)}{x^2 - 4} = 1$

4) $\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a}$

Ex:- $\lim_{x\to 0} \frac{\ln \cos x}{\sin 3x \cdot \sin^{-1} 2x}$

$= \lim_{x\to 0} \frac{\frac{\ln(1 + \cos x - 1)}{\cos x - 1} \cdot \cos x - 1}{\frac{\sin 3x}{3x} \cdot 3x \cdot \frac{\sin^{-1} 2x}{2x} \cdot 2x}$

$$= \lim_{x\to 0} \frac{1(\cos x - 1)}{6x^2}$$

$$= \lim_{x\to 0} \frac{-1(1-\cos x)}{6\ x^2}$$

$$= \frac{-1}{6} \times \frac{1}{2} = \frac{-1}{12}$$

Ex:-2) $\lim_{x\to e} \frac{\ln(x)-1}{x^2-e^2}$

$$= \lim_{x\to e} \frac{\ln(1+x-1)}{x-1} \cdot x-1 \to 1$$

$$= \lim_{x\to e} \frac{\ln x - \ln e}{(x-e)(x+e)}$$

$$= \lim_{x\to e} \frac{\frac{\ln\left(\frac{x}{e}+1-1\right)}{\frac{x}{e}-1}}{(x-e)(2e)} \quad \frac{x}{e}-1$$

$$= \lim_{x\to e} \frac{\frac{x-e}{e}}{(x-e)(2e)}$$

$$= \frac{1}{2e^2}$$

Ex:3) $\lim_{x\to 0} \frac{e^{\tan x} - e^{\sin x}}{(e^{2x}-1)(\tan^{-1}2x \cdot \sin 3x)}$

$$= \lim_{x\to 0} \frac{e^{\tan x} - 1}{\left(\frac{e^{2x}-1}{2x}\right)\left(\frac{\tan^{-1}2x}{2x}\right)\left(\frac{\sin 3x}{3x}\right) \cdot 12x^3}$$

$$= \lim_{x\to 0} \frac{\left(\frac{e^{\tan x}-1}{\tan x}\right) \cdot \tan x}{12x^3}$$

$$= \lim_{x\to 0} \frac{\tan x}{12x^3}$$

$$= \frac{1}{12} \lim_{x\to 0} \frac{\tan x}{x} \cdot \frac{1}{x^2}$$

$$= \frac{1}{12} \lim_{x\to 0} \frac{1-\cos^2 x}{x^2}$$

$$= \frac{1}{12} \lim_{x\to 0} \frac{1-\cos x}{x^2} \cdot (1+c)$$

$$= \frac{1}{12} \times \frac{1}{2} = \frac{1}{24}$$

1^{∞} form

Ex:- $\lim_{x\to 0} \cancel{\log}(1+x)^{\frac{1}{x}}$

In total mathematics if we find $(\text{variable}_1)^{\text{variable}_2}$ then we will write it as $e^{\ln (v_1)^{v_2}}$

$$= \lim_{x\to 0} e^{\ln \log(1+x)^{\frac{1}{x}}}$$

$$= \lim_{x\to 0} e^{\frac{\ln(1+x)}{x}}$$

$$= e^1 = e$$

Remember it as standard formulae

$$\boxed{\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e}$$

as $x \to a$ $f(x) \to 0$ then

$$\lim_{x\to 0} (1+f(x))^{\frac{1}{f(x)}} = e$$

as $x \to a$ $f(x) \to 1$, $g(x) \to \infty$

$$\lim_{x\to 0} (1+f(x))^{g(x)} = e^{\lim_{x\to a} g(x)(f(x)-1)}$$

$$= e^{\lim_{x\to a} \text{power}(\text{Base}-1)}$$

Examples

1) $\underset{x\to 0}{lt} (\cos x)^{\cot^2 x}$

$= \underset{x\to 0}{lt}\ e^{\ln \cot^2 x (\cos x - 1)}$

$= \underset{x\to 0}{lt}\ e^{\ln \frac{\cos^2 x}{\sin^2 x}(\cos x - 1)}$ $= \underset{x\to 0}{lt}\ e^{\frac{1}{1-\cos^2 x}\cdot \cos x - 1}$

$= \underset{x\to 0}{lt}\ e^{\frac{-\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}(1+\cos x)}}$ $= \underset{x}{lt}\ e^{\frac{-1}{2}}$

$= \frac{1}{\sqrt{e}}$

2) $\underset{x\to\infty}{lt} \left(\frac{x+2}{x+3}\right)^{x+4}$

$= \underset{x\to\infty}{lt}\ e^{x+4\left(\frac{x+2}{x+3} - 1\right)}$

$= \underset{x\to\infty}{lt}\ e^{x+4\left(\frac{-x}{x+3}\right)}$

$= \underset{x\to\infty}{lt}\ e^{(x+4)(0)}$ $= \underset{x\to\infty}{lt}\ e^{-\frac{x}{x}}$

$0 \times \infty$ Indefinite

$= e^0 = 1$ $= e^{-1} = \frac{1}{e}$

3) $\underset{x\to\infty}{lt} \left(\frac{2x+1}{x+3}\right)^{x+7}$

First step is to check whiel if it is in the form 1^∞

1^∞ then we can proceed

<1 $(0.99)^\infty - 0$

>1 $(1.01)^\infty - D.N.E$

Neglecting lower terms $x\to\infty$

$\underset{x\to\infty}{\lim} \frac{2x}{x} = (2)^\infty - D.N.E$

4) $\lim_{x\to\infty}\left(\frac{2x+3}{x-5}\right)^{4-x} = 2^{-\infty}$

$= \frac{1}{2^{\infty}} = 0$

Shortcuts in limits

1) $\lim_{x\to 3}\frac{\sin(x^2-9)}{x-3}$

$= \lim_{x\to 3}\frac{\sin x^2-9}{x-3}\cdot\frac{x^2-9}{x^2-9}$

$= \lim_{x\to 3}\frac{x^2-9}{x-3}$

2) $\lim_{x\to\pi}\frac{\sin(\sin x)}{x^2-\pi^2}$

$= \lim_{x\to\pi}\frac{\sin x}{x^2-\pi^2}$

$= \frac{\sin(\pi-x)}{(x-\pi)(x+\pi)}$

$= -\frac{1}{2\pi}$

$\Longrightarrow$ $\lim_{y\to 0}\frac{\sin y}{x}$ (Note :- $y\Rightarrow 0$)

$= \frac{y}{x}$

3) $\lim_{x\to\frac{\pi}{6}}\frac{\sin(x)-\frac{1}{2}}{\sin(6x)}$

$= \lim_{x\to\frac{\pi}{6}}\frac{\sin x-\sin\frac{\pi}{6}}{\sin 6x}$

$= \lim_{x\to 0}\frac{2\sin\left(\frac{\frac{x}{6}-\frac{\pi}{6}}{2}\right)\cos\left(\frac{\frac{\pi}{6}+x}{2}\right)}{\sin(\pi-6x)}\quad x-\frac{\pi}{6}$

$\frac{\cancel{2}\left(\frac{\frac{\pi}{6}-x}{\cancel{2}}\right)\frac{\sqrt{3}}{2}}{\pi-6x}$

$x-\frac{\pi}{6}$

$= \frac{\cancel{\left(\frac{\pi}{6}-x\right)}\frac{\sqrt{3}}{2}}{6\cancel{\left(\frac{\pi}{6}-6\right)}}$

$= -\frac{\sqrt{3}}{12}$

$\Longrightarrow$ $\lim_{x\to a}\frac{\sin \overset{\to 0}{f(x)\cdot g(x)}}{h(x)} = \frac{f(x)g(x)}{h(x)}$

Summary :- If argument is tending towards zero and it is in product form then $\sin, \sin^{-1}, \tan, \tan^{-1}$ can be replaced with their arguments

Ex:- $$\lim_{x \to 2} \frac{\tan^{-1}(x^2-4) \cdot \cos(2x\pi)}{\tan(4\pi x) \cdot \sin x}$$

$\cos(2x\pi) \to 1$, $\tan(4\pi x) \to 8\pi$, $\sin x \to \sin 2$

$$= \lim_{x \to 2} \frac{(x^2-4) \cdot 1}{-\tan(8\pi - 4\pi x) \cdot \sin 2}$$

$$= \lim_{x \to 2} \frac{(x-2)(x+2)}{-8\pi + 4\pi x \cdot \sin 2}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}\,(x+2)}{4\pi\cancel{(-2+x)}(\sin 2)}$$

$$= \frac{1}{\pi \sin 2}$$

Note :- If x do not tends to zero make it as tends to zero by using trigonometric formulas

Ex:- $$\lim_{x \to \pi} \cos x = \lim_{x \to \pi} \cos(\pi - x)$$

$$\lim_{x \to \pi/2} \cos x = \lim_{x \to \pi/2} \sin\left(\frac{\pi}{2} - x\right)$$

Skipped many shortcuts

Using Expansions

1) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$

when lim $x \to 0$

factorial $- 1!$ (or) $\underline{1|}$

If $x = -x$

$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} \dots \infty$

If $x = x\ln a$
$= \ln_e(a^x)$
$= a^x$

2) $a^x = 1 + \frac{x\ln a}{1!} + \frac{(x\ln a)^2}{2!} + \frac{(x\ln a)^3}{3!} + \dots \infty$

$\cancel{3)\ \sin x = 1 + \frac{x^3}{3!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \frac{x^7}{7!} \dots \infty}$

$\cancel{4)\ \cos x = 1 - \frac{x}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}}$

For $\sin x$ odd
$\cos x$ even
start alternate signs with $-$

3) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots \infty$

4) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \infty$

5) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \dots \infty$

continuous
start x &
$-ve$
and no factorial

6) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty$

(all $-ve$)

7) $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty$

Ex:-

1) $\underset{x \to 0}{\mathrm{lt}} \dfrac{1 - \frac{x^2}{2} - \cos x}{x^4}$

$= \underset{x \to 0}{\mathrm{lt}} \dfrac{1 - \frac{x^2}{2} - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right)}{x^4}$

$= \underset{x \to 0}{\mathrm{lt}} \dfrac{-\cancel{x^4}}{4!} \times \dfrac{1}{\cancel{x^4}}$

$= \dfrac{-1}{24}$

2) $\underset{x \to 0}{\mathrm{lt}} \dfrac{\sin x - \tan x}{x^3}$

$= \underset{x \to 0}{\mathrm{lt}} \dfrac{x - \frac{x^3}{3!} - \left(x + \frac{x^3}{3}\right)}{x^3}$

$= \underset{x \to 0}{\mathrm{lt}} \dfrac{-\frac{x^3}{3!} - \frac{x^3}{3}}{x^3}$

$= \underset{x \to 0}{\mathrm{lt}} -\dfrac{1}{6} - \dfrac{1}{3}$

$= \dfrac{-3}{6} = -\dfrac{1}{2}$

3) $\underset{x \to 0}{\mathrm{lt}} \dfrac{e^x - 1 - x}{x^2}$

$= \underset{x \to 0}{\mathrm{lt}} \dfrac{e^x - 1}{x^2} - \underset{x \to 0}{\mathrm{lt}} \dfrac{x}{x^2}$

$= \underset{x \to 0}{\mathrm{lt}} (1)\left(\dfrac{1}{x}\right) - \underset{x \to 0}{\mathrm{lt}} \left(\dfrac{1}{x}\right)$

$= \underset{x \to 0}{\mathrm{lt}} \left(\dfrac{1}{x} - \dfrac{1}{x}\right)$

$= 0.$

$\infty - \infty$ ←
To add no indeterminate forms should be there

3) $\lim\limits_{x\to 0} \dfrac{e^x-1-x}{x^2}$

$= \lim\limits_{x\to 0} \dfrac{\left(\cancel{1}+\cancel{\frac{x}{1!}}+\frac{x^2}{2!}-\cancel{1}-\cancel{x}\right)}{x^2}$

$= \dfrac{1}{2!} = \dfrac{1}{2}$

4) $\lim\limits_{x\to 0} \dfrac{x-\sin x}{x^3}$

$-(-1)$
$=+$

$= \lim\limits_{x\to 0} \dfrac{x - x + \frac{x^3}{3!}}{x^3}$

$= \dfrac{+1}{3!} = \dfrac{+1}{6}$

5) $\lim\limits_{x\to 0} \dfrac{\tan x - x}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{x+\frac{x^3}{3}-x}{x^3}$

$= \dfrac{1}{3}$

6) $\lim\limits_{x\to 0} \dfrac{e^x+e^{-x}-2-x^2}{x^4}$

$= \lim\limits_{x\to 0} \dfrac{1+\cancel{\frac{x}{1!}}+\frac{x^3}{3!}+1-\cancel{\frac{x}{1!}}-2-x^2}{x^4}$

$= \lim\limits_{x\to 0} \dfrac{2+2-x^2}{x^4}$

$= \lim\limits_{x\to 0} \dfrac{-1}{}$

8) $\underset{x\to 3}{Lt} \dfrac{e^{x^2}-e^9}{\sin\pi x}$

$= \underset{x\to 3}{Lt} \dfrac{e^9\left(e^{x^2-9}-1\right)}{\sin\pi x}$

$= \underset{x\to 3}{Lt} \dfrac{e^9\left((x-3)(x+3)\right)}{\sin(3\pi-\pi x)}$

$= \underset{x\to 3}{Lt} \dfrac{e^9\left((x-3)(x+3)\right)}{3\pi-\pi x}$

$= \underset{x\to 3}{Lt} \dfrac{e^9\cancel{(x-3)}(x+3)}{-\pi\cancel{(x-3)}}$

$= \dfrac{-6e^9}{\pi}$

L Hospital Rule (or) L'opital rule

If $x\to a$ & $f(x), g(x) \to 0$ (or) $f(x), g(x) \to \infty$

then they are differentiable

$$\underset{x\to a}{Lt} \frac{f(x)}{g(x)} = \underset{x\to a}{Lt} \frac{f'(x)}{g'(x)}$$

All standard rules of $\sin x$, $\cos x$, $\tan x$ are derived from this

$$\underset{x\to 0}{Lt} \frac{\sin x}{x} = 1$$

It also can be done by factorisation But too lengthy has 12 terms

Ex:-1) $\underset{x\to 1}{Lt} \dfrac{x^9+7x^5-8}{x^{12}+11x^7-12}$

$= \underset{x\to 1}{Lt} \dfrac{9x^8+35x^4}{12x^{11}+77x^6}$

$= \dfrac{9+35}{12+77} = \dfrac{44}{89}$

2) $\lim\limits_{x\to\infty} \frac{\ln(x)}{x}$

$x \to \infty$

$\frac{1}{x} = 0$

$= \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{x}$

$= 0$

$$\boxed{\lim_{x\to\infty} \frac{\ln(x)}{x} = 0}$$

3) $\lim\limits_{x\to 0} \frac{\tan^{-1}(x) - \sin^{-1}(x)}{x^3}$

$= \lim\limits_{x\to 0} \frac{\frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}}{x^3}$

$= \lim\limits_{x\to 0} \frac{(1+x^2)^{-1} - (1+x^2)^{-\frac{1}{2}}}{x^3}$

$(1+x)^n = 1+nx + \frac{n(n-1)}{2!} \cdots \infty$

$= \lim\limits_{x\to 0} \frac{1+(-1)(x^2) - 1+(-\frac{1}{2})x^2}{3x^2}$

$= \lim\limits_{x\to 0} \frac{-x^2 - \frac{1}{2}x^2}{3x^2}$

$= \lim\limits_{x\to 0} \frac{-\frac{3}{2}x^2}{3x^2}$

$= \frac{-1}{2}$

4) $\lim\limits_{x\to \pi/2} \frac{\tan x}{\sec x}$

$= \lim\limits_{x\to \frac{\pi}{2}} \frac{\sec^2 x}{\sec x \tan x}$

$= \lim\limits_{x\to \pi/2} \frac{\sec x}{\tan x}$

order changed but if we apply L-H rule it get reversed & remain same

$\lim\limits_{x\to \frac{\pi}{2}} \frac{\sin x}{\cos x} \cdot \frac{\frac{d}{dx}\cos x}{\cos x}$

$= \sin(\frac{\pi}{2}) = 1$

le

tan x are

5) $\lim_{x\to 0} \dfrac{e^{-1/x^2}}{x}$

LH $= \dfrac{\left(e^{-1/x^2}\right)\left(\frac{2}{x^3}\right)}{1}$

$= \dfrac{2e^{-1/x^2}}{x^3}$

LH $= \dfrac{2e^{-1/x^2}\cdot\left(\frac{2}{x^3}\right)}{3x^2}$

$= \dfrac{4}{3}\dfrac{e^{-1/x^2}}{x^5}$

So, we cant apply LH rule since it is continuous

$= \lim_{x\to 0} \dfrac{1/x}{e^{1/x^2}}$

$= \lim_{x\to 0} \dfrac{-\frac{1}{x^2}}{e^{1/x^2}\cdot\frac{-2}{x^3}}$

$= \dfrac{x}{2e^{1/x^2}}$ $\left(\frac{0}{\infty}\right)$

$= 0$

Memorize $\boxed{\lim_{x\to\infty} \dfrac{x^n}{e^x} = \dfrac{n!}{e^x} = 0}$ $n\in R$

If L.H.S rule is not working then try to change $\frac{0}{0} \rightleftarrows \frac{\infty}{\infty}$

0^∞ (or) ∞^0 forms

as $x\to a$; $f \to 0$ (or) ∞

$g \to 0$

$(f(x))^{g(x)} \to 0^\infty$ (or) ∞^0

Ex: $\lim_{x\to 0^+} x^x$

$= \lim_{x\to 0^+} e^{\ln x^x}$

$= \lim_{x\to 0^+} e^{\frac{\ln x}{1/x}}$

LHR $= \lim_{x\to 0^+} \dfrac{1/x}{e^{-1/x^2}} = \lim_{x\to 0^+} e^{(-x)} = e^0 = 1$

e^{1/x^2}

$\frac{-1}{x^2}$

$\frac{-2}{x^3}$

$\left(\frac{0}{\infty}\right)$

Memorize $\boxed{\lim_{x\to 0^+} x^x = 1}$

Ex: 1) $\lim_{x\to 0^+} x^{\sin x}$

$= \lim_{x\to 0} \left(x^x\right)^{\frac{\sin x}{x}}$

$= 1^{\frac{\sin x}{x}} = 1$

2) $\lim_{x\to 0^+} \left(\frac{1}{x}\right)^x$

$= \lim_{x\to 0^+} \frac{1^x}{x^x} = \lim_{x\to 0^+} \frac{1}{x^x} = 1/1 = 1$

$\lim_{x\to 0^+} x^{x^x} = \lim_{x\to 0^+} (x)^{x^2} = \lim_{x\to 0^+} x^{(x^x)}$

$\neq \lim_{x\to 0^+} (x^x)^x$

$= 0^1 = 0$

ng

$\frac{\infty}{\infty}$

Memorize $\boxed{\lim_{x\to 0^+} x^{x^x} = 0}$

$0 = 1$