⇒ **Formula:-** To find the roots of Q. eq (quadratic)

$ax^2+bx+c=0$ we have

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1Q) Find the roots of the quadratic equation

$x^2-7x+12=0$

Sol:- $x^2-7x+12=0$

$\Rightarrow x^2-3x-4x+12=0$

$\Rightarrow x(x-3)-4(x-3)=0$

$\Rightarrow x-3=0$ (or) $x-4=0$

$x=3$ (or) $x=4$

Sol:- G.T

$x^2-7x+12=0$

It is in the form of

$ax^2+bx+c=0$ where

$a=1 \quad b=-7 \quad c=12$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(1)(12)}}{2(1)}$$

$$=\frac{7\pm\sqrt{49-48}}{2}=\frac{7\pm\sqrt{1}}{2}$$

$$\frac{7+1}{2} \quad (\text{or}) \quad \frac{7-1}{2}$$

$$\Rightarrow\frac{8}{2} \quad (\text{or}) \Rightarrow \frac{6}{2}$$

$\Rightarrow (x=4)$ (or) $x=3$

⇒ **Note:-** (formula

→ If α, β a

ax^2+bx+

i) sum of

ii) produ

→ If α, β

equation

$(x-\alpha$

c

x^2-

Q) Find

are

Sol:- Let

$\alpha=$

$\alpha+\beta$

$\alpha\beta$

⇒

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⇒ Note:- (formula) :-

→ If α, β are the roots of the equation $ax^2+bx+c=0$

i) sum of roots $\Rightarrow \alpha+\beta = -b/a$

ii) product of roots $\Rightarrow \alpha\beta = c/a$.

→ If α, β are the roots then the required equation is

$$(x-\alpha)(x-\beta)=0$$

(or)

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

Q) Find the quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

Sol:- Let

$\alpha = 2+\sqrt{3}$

$\alpha+\beta = 2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}$

$= 4$

$\alpha\beta = (2+\sqrt{3})(2-\sqrt{3})$

$= (2)^2-(\sqrt{3})^2$

$= 4-3 = 1$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow x^2-4x+1=0$ //

⇒ **Discriminent:** The value b^2-4ac is called discriminant of both quadratic expression ax^2+bx+c and quadration equation ax^2+bx

→ It is denoted by 'Δ'

where $\Delta = b^2-4ac$

⇒ **Nature of the roots of quadratic equation:**

⇒ Let α, β be the roots of quadratic equation $ax^2+bx+c=0$ where a, b, c are real numbers.

(i) If $\Delta \overset{>}{\cancel{<}} 0$, then the roots are real & distinct.

(ii) If $\Delta = 0$, then the roots are real and equal.

(iii) If $\Delta < 0$, then the roots are conjugate complex numbers $(\alpha \pm i\beta)$

⇒ If a, b, c is are rational numbers, and α, β are the roots of quadratic equation $ax^2+bx+c=0$

$\Delta = 0$, roots are rational and ~~distinct~~ equal.

$\Delta > 0$ & Δ is square of ~~po~~ non-zero rational no.s then the roots are rational and distinct.

(iii) $\Delta < 0$ then the roots are complex conjugate no.s or Img ($\alpha \pm i\beta$)

(iv) $\Delta > 0$ & Δ is not a square no then the roots are conjugate surds.

Ex:- 3(a)

I.

1) Find the roots of the following equations

(i) $x^2 - 7x + 12 = 0$

Sol:- $x^2 - 4x - 3x + 12 = 0$

$= x(x-4) - 3(x-4)$

$= (x-4) = 0 \; (x-3) = 0$

$= x = 3$ (or) $x = 4$

(iv) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Sol:- It is in the form of $ax^2 + bx + c = 0$, where

$a = \sqrt{3}$, $b = 10$, $c = -8\sqrt{3}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(\sqrt{3})(-8\sqrt{3})}}{2(\sqrt{3})}$$

$x = \frac{-10+14}{2\sqrt{3}}$ (or) $x = \frac{-10-14}{2\sqrt{3}}$

$x = \frac{4}{2\sqrt{3}}$ $x = \frac{-24}{2\sqrt{3}}$

$x = 2/\sqrt{3}$ // $x = \frac{-12}{\sqrt{3}}$ //

H.W

(ii) $-x^2+x+2=0$

(iii) $2x^2+3x+2=0$

v) $\sqrt{5}x^2-9x-3\sqrt{5}=0$

(ii) $-x^2+x+2=0$

Sol:- $-x^2+2x-1x+2=0$

$= -x(x-2)-1(x-2)$

$\Rightarrow -x-1=0$ $x-2=0$

$\Rightarrow x=1$ (or) $\Rightarrow x=2$ //

2) Find the quadratic equation whose roots are

(ii) $\frac{m}{n}, \frac{-n}{m}$ $(m\neq 0, n\neq 0)$

Sol:- Let $\alpha = \frac{m}{n}$, $\beta = \frac{-n}{m}$

sum of roots $\alpha+\beta = \frac{m}{n}-\frac{n}{m} = \frac{m^2-n^2}{mn}$

product of roots $\alpha\beta = \left(\frac{\cancel{m}}{\cancel{n}}\right)\left(\frac{-\cancel{n}}{\cancel{m}}\right) = -1$

If α, β are the roots then the required eqn is $x^2-(\alpha+\beta)x+\alpha\beta=0$

$= x^2-\left(\frac{m^2-n^2}{mn}\right)x+(-1)=0$

$= mnx^2-(m^2-n^2)x-mn=0$ //

(iv) $7\pm2\sqrt{5}$

Sol:- $\alpha = 7+2\sqrt{5}$

$\beta = 7-2\sqrt{5}$

$\alpha+\beta = 7+\cancel{2\sqrt{5}}+7-\cancel{2\sqrt{5}}$

$= 14$

$\alpha\beta = (7+2\sqrt{5})(7-2\sqrt{5})$

$= 7^2-(2\sqrt{5})^2$

$= 49-20 = 29$

$= x^2-(\alpha+\beta)x+\alpha\beta = 0$

$= x^2-14x+29=0$

v) $-3\pm5i$

Sol:- $\alpha = -3+5i$

$\beta = -3-5i$

$\alpha+\beta = -3+5i-3-5i = -6$

$[\because i^2=-1]$

$\alpha\beta = (-3+5i)(-3-5i)$

$= (-3)^2-(5i)^2 = 9+25 = 34$ //

H.W

(iii) $\frac{p-q}{p+q}, -\left(\frac{p+q}{p-q}\right)$

(i) 2, 5

Sol:- $\alpha = 2$, $\beta = 5$

$\alpha + \beta = 7$

$\alpha\beta = 10$

$= x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$= x^2 - (7)x + 10 = 0$

$= x^2 - 7x + 10 = 0$

3) Find the nature of roots of the followin equations without finding the roots:-

(i) $2x^2 - 8x + 3 = 0$

Sol:- It is in the form of $ax^2 + bx + c = 0$

$a = 2$ $b = -8$ $c = 3$

$\Delta = b^2 - 4ac$

$\Delta = (-8)^2 - 4(2)(3)$

$= 64 - 24$

$= 40 > 0$

∴ The roots are real and distinct.

(iv) $2x^2 - 7x + 10 = 0$

Sol:- $a = 2$, $b = -7$, $c = 10$.

$\Delta = b^2 - 4ac$

$= (-7)^2 - 4(2)(10)$

$= 49 - 80$

$= -31 < 0$

∴ The roots are conjugate and complex

H.W

(ii) $9x^2-30x+25=0$

(iii) $x^2-12x+32=0$

(ii) $9x^2-30x+25=0$

Sol:- $\Delta = b^2-4ac$

$= (-30)^2-4(9)(25)$

$= 900-900$

$= 0$

$\Delta=0$

$\therefore$ The roots are real and equal.

Eg:- Find the ~~number~~ nature of roots of

$3x^2+7x+2=0$

Sol:- $\Delta = b^2-4ac$

$= (7)^2-4(3)(2)$

$= 49-24$

$= 25>0$

$= (5)^2$

$\therefore$ The roots are rational and unequal.

4) If α, β are the roots of the $ax^2+bx+c=0$ find the following expression interms of a, b, c

i) $\alpha^2+\beta^2$ (ii) $\alpha^3+\beta^3$

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$ (iv) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$\alpha^4\beta^7+\alpha^7\beta^4$

) $\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)^2, \ c\neq 0$

i) $\frac{\alpha^2+\beta^2}{\alpha^{-2}+\beta^{-2}}$ if $c\neq 0$

<u>ol:-</u> G.T

α, β are the roots of the equation ax^2+bx

then $\alpha+\beta=\frac{-b}{a}, \ \alpha\beta=\frac{c}{a}$

(i) $\alpha^2+\beta^2$

WKT

$$=(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$$

$$=\left(-\frac{b}{a}\right)^2=\alpha^2+\beta^2+2\left(\frac{c}{a}\right)$$

$$=(\alpha^2+\beta^2)=\frac{b^2}{a^2}-\frac{2c}{a}=\frac{b^2-2ac}{a^2}$$

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$

$$=\frac{-b/a}{c/a}=-b/c$$

(ii) $\alpha^3+\beta^3$

$$(\alpha^{\beta}+\beta)^3=\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)$$

$$=\left(\frac{-b}{a}\right)^3=\alpha^3+\beta^3-3(c/a)(-b/a)$$

$$=\alpha^3+\beta^3=\frac{-b^3}{a^3}+\frac{3bc}{a^2}$$

$$=\frac{3abc-b^3}{a^3}$$

(iv) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$= \frac{\beta^2+\alpha^2}{\alpha^2\beta^2} = \cancel{\left[\frac{\left(-b/a\right)^2}{\left(c/a\right)^2} = \frac{\frac{b^2}{a^2}}{\frac{c^2}{a^2}}\right]}$$

$$= \frac{\frac{b^2-2ac}{a^2}}{\frac{c^2}{a^2}} = \frac{b^2-2ac}{c^2}$$

(v) $\alpha^4\beta^7+\alpha^7\beta^4$

sol: $\alpha^4\beta^7+\alpha^7\beta^4 = \alpha^4\beta^4\left(\beta^3+\alpha^3\right)$

$$= (\alpha\beta)^3\left(\alpha^3+\beta^3\right)$$

$$= \left(\frac{c}{a}\right)^3\left[\frac{3abc-b^3}{a^3}\right]$$

$$= \frac{c^3\left[3abc-b^3\right]}{a^6}$$

(vi) $\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)^2,\ c\neq 0$

sol: $= \left[\frac{\alpha^2-\beta^2}{\alpha\beta}\right] = \left[\frac{(\alpha+\beta)(\alpha-\beta)}{\alpha\beta}\right] = \frac{(\alpha+\beta)^2(\alpha-\beta)^2}{(\alpha\beta)^2}$

$$= \frac{(\alpha+\beta)^2\left[(\alpha+\beta)^2-4\alpha\beta\right]}{(\alpha\beta)^2} \Rightarrow \frac{\frac{b^2}{\cancel{a^2}}\left[\frac{b^2}{a^2}-\frac{4c}{a}\right]}{c^2/\cancel{a^2}}$$

$$= \frac{b^2\left[\frac{b^2-4ac}{a^2}\right]}{c^2} = \frac{b^2-4ab^2c}{a^2c^2} \quad //$$

(vii) $\frac{\alpha^2+\beta^2}{\alpha^{-2}+\beta^{-2}}$

Sol:- $\dfrac{\alpha^2+\beta^2}{\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}} \Rightarrow \dfrac{\cancel{\alpha^2+\beta^2}}{\dfrac{\cancel{\beta^2+\alpha^2}}{\alpha^2\beta^2}} = (\alpha\beta)^2$

$$= \frac{c^2}{a^2}$$

1) (iii) $2x^2+3x+2=0$

Sol:- $2x^2+3x+2=0$

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2-4(2)(2)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9-16}}{4}$$

$$= \frac{-3 \pm \sqrt{7}}{4}$$

$$= \frac{-3+\sqrt{7}}{4} \text{ (or) } = \frac{-3-\sqrt{7}}{4} \text{ //}$$

v) $6\sqrt{5}x^2 - 9x - 3\sqrt{5} = 0$

ol: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(6\sqrt{5})(-3\sqrt{5})}}{2(6\sqrt{5})}$$

$$= \frac{9 \pm \sqrt{81 + 4(18(5))}}{12\sqrt{5}}$$

$$= \frac{9 \pm \sqrt{81 + 360}}{12\sqrt{5}}$$

$$= \frac{9 \pm \sqrt{\cancel{279}\,441}}{12\sqrt{5}}$$

$$= \frac{9 + \sqrt{441}}{12\sqrt{5}} \quad \text{(or)} \quad = \frac{9 - \sqrt{441}}{12\sqrt{5}}$$

2(iii) $\frac{p-q}{p+q}$, $-\left(\frac{p+q}{p-q}\right)$

sol:- $\alpha = \frac{p-q}{p+q}$, $\beta = -\left(\frac{p+q}{p-q}\right)$

$$\alpha + \beta = \frac{p-q}{p+q} + (-)\left(\frac{p+q}{p-q}\right)$$

$$= \frac{p-q}{p+q} - \frac{p+q}{p-q}$$

$$= \frac{(p-q)(p-q) - (p+q)(p+q)}{(p+q)(p-q)}$$

$$= \frac{(p-q)(p-q) - (p+q)(p+q)}{(p+q)(p-q)}$$

$$= \frac{(p^2+q^2)-(p^2+q^2)}{(p+q)(p-q)}$$

$$= (p^2+q^2)($$

$$\alpha\beta = \left(\frac{p-q}{p+q}\right)\left(\frac{-p+q}{p-q}\right)$$

$$= \frac{-(p^2\cancel{-q^2})}{p^2\cancel{-q^2}} = -1$$

3(iii) $x^2-12x+32=0$

Sol: $\Delta = b^2-4ac$

$= (12)^2-4(1)(32)$

$= 144-128$

$= 16 > 0$

If $\Delta > 0$

$\therefore$ The roots are real and distinct.

19:- For what values of 'm' the equations $x^2-2(1+3m)x+7(3+2m)=0$ will have equal roots?

Sol:- G.T

$x^2-2(1+3m)x+7(3+2m)=0$ —— ①

It is in the form of $ax^2+bx+c=0$

where $a=1$, $b=-2(1+3m)$

$c=7(3+2m)$

G.T equ ① has equal roots i.e, $\Delta=0$

$= b^2-4ac=0$

$\Rightarrow [-2(1+3m)]^2-4(1)7(3+2m)=0$

$- 4(1+3m)^2-4(21+14m)=0$

$= 4(1+9m^2+6m-21-14m)=0$

$= 9m^2-8m-20$

$= 9m^2-18m+10m-20=0$

$= 9m(m-2)+10(m-2)=0$

$= (9m+10)(m-2)=0$

$\Rightarrow 9m+10=0$ $\quad m-2=0$

$m=\frac{-10}{9}$ $\quad m=2$

$m\in\left\{\frac{-10}{9},2\right\}$ //

5) Find the values of 'm', for which of the fol
equation has equal roots:-

(ii) $(m+1)x^2+2(m+3)x+(m+8)=0$ —①

Sol:- It is in the form of $ax^2+bx+c=0$

$a=(m+1)$, $b=2(m+3)$, $c=(m+8)$

G.T eq① has equal roots

$\Delta=0$

$\Delta=b^2-4ac=0$

$=(2(m+3))^2-4((m+1)(m+8))=0$

$=4(m+3)^2-4[(m+1)(m+8)]$

$=4[(m+3)^2-(m^2+8m+n+8)]=0$

$=4[m^2+9+6m-(m^2+9m+8)]=0$

$=\cancel{m^2}+9+6m-\cancel{m^2}-9m-8=0$

$=-3m+1=0$

$=3m=1$

$=m=1/3$//

iv) $(3m+1)x^2+2(m+1)x+m=0$ —①

Sol:- It is in the form of $ax^2+bx+c=0$

$a=(3m+1)$, $b=(2(m+1))$, $c=m$

G.T eq① has equal roots

$\Delta=0$

$\Delta=b^2-4ac$

$=(2(m+1))^2-4(3m+1)(m)=0$

$= 4(m+1)^2 - 4(3m^2+m) = 0$

$= 4(m^2+1+2m-3m^2-m) = 0$

$= -2m^2+m+1 = 0$

$= 2m^2-m-1 = 0$

$= 2m^2-2m+m-1 = 0$

$= 2m(m-1)+1(m-1) = 0$

$= (2m+1) = 0 \quad (m-1) = 0$

$\Rightarrow 2m+1 = 0$ (or) $m-1 = 0$

$\therefore 2m = -1 \qquad m = 1$

$\Rightarrow m = -\frac{1}{2}$ (or) $m = 1$

HW

(i) $x^2 - 15 - m(2x-8) = 0$

(iii) $x^2 + (m+3)x + (m+6) = 0$

(v) $(2m+1)x^2 + 2(m+3)x + (m+5) = 0$

6) If α, β are the roots of $x^2+px+q=0$, form a quadratic equation, whose roots are $(\alpha-\beta)^2$ & $(\alpha+\beta)^2$

sol: G.T

α, β are the roots of $x^2+px+q=0$

$\alpha+\beta = -p \quad , \alpha\beta = q$

Let $A = (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= (-p)^2 - 4q$

$A = p^2 - 4q$

$B = (\alpha+\beta)^2 = (-p)^2$

$= p^2$

(i) sum of roots

$$A+B = p^2-4q+p^2$$
$$= 2p^2-4q$$

(ii) product of roots

$$AB = (p^2-4q)p^2$$
$$= p^4 \; 4p^2q$$

If A, B are the roots, then the required equ

$$= x^2 - (\cancel{\alpha+\beta})(A+B)x + AB = 0$$
$$= x^2-(2p^2-4q)x+p^4-4p^2q = 0$$

II

2) If α, β are the roots of quadratic equ $ax^2+bx+c=0$, $c \neq 0$, then find the Q. equ. whose roots are $\alpha^2+\beta^2$ & $\alpha^{-2}+\beta^{-2}$.

Sol:- G.T

α, β are the roots of $ax^2+bx+c=0$

$$\alpha+\beta = -b/a \quad , \quad \alpha\beta = c/a$$

Let

$$A = \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$
$$= \left(-b/a\right)^2 - 2c/a$$
$$= \frac{b^2}{a^2} - \frac{2c}{a}$$
$$A = \frac{b^2-2ac}{a^2}$$

$$B = \alpha^{-2}+\beta^{-2}$$
$$= \frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$$

$$B = \frac{\frac{b^2-2ac}{a^2}}{c^2/a^2} = \frac{b^2-2ac}{c^2}$$

If A, B, are the roots, then required eq is

$$= x^2 - (A+B)x + AB = 0$$

$$= x^2 - \left[\frac{b^2-2ac}{a^2} + \frac{b^2-2ac}{c^2}\right]x + \left(\frac{b^2-2ac}{a^2}\right)\left(\frac{b^2-2ac}{c^2}\right) = 0$$

$$= a^2c^2x^2 - (b^2-2ac)(a^2+c^2)x + (b^2-2ac)^2 = 0 //$$

ij If $x^2+bx+c=0$, $x^2+cx+b=0$ $(b \neq c)$ have a common root, then S.T $b+c+1=0$

sol:- G.T

$= x^2+bx+c=0$ —①

$= x^2+cx+b=0$ —②

Let α be the common root of above two given equs,

then put $x=\alpha$

$\alpha^2+b\alpha+c=0$ —③

$\alpha^2+c\alpha+b=0$ —④

Solving eq③ & eq④, we get

$$\begin{array}{l} = \alpha^2 + b\alpha + c = 0 \\ \underline{-\alpha^2 \overset{-}{+} c\alpha \overset{-}{+} b = 0} \\ b\alpha - c\alpha + c - b = 0 \end{array}$$

$= b\alpha - c\alpha = b - c$

$= \alpha(b-c) = b-c$

$$\alpha = \frac{b-c}{b-c} = 1$$

put $\alpha=1$ in eq③, we get

$= \alpha^2+b\alpha+c=0$

$= 1+b+c=0$

$= b+c+1=0$ //

8) P.T, the roots of $(x-a)(x-b)=b^2$ are alway real.

Sol:- G.T

$= (x-a)(x-b)=b^2$

$= x^2-bx-ax+ab-b^2=0$

$= x^2 - x(a+b)+(ab-b^2)=0$

It is in the form of $Ax^2+Bx+c=0$

$\Delta = B^2-4ac$

$= (-(a+b))^2-4(1)(ab-b^2)$

$= (a+b)^2-4(ab-b^2) \geq 0$

$\therefore$ The given eqn is always real.

9) Find the condition that are roots of the Q. eq $ax^2+bx+c=0$ shall be 'n' times the other, where 'n' is a +ve integer.

Sol:- Let 'α', $n\alpha$ be the roots of $ax^2+bx+c=0$

$\alpha+n\alpha = -b/a$ $\qquad$ $\alpha . n\alpha = c/a$

$= \alpha(1+n) = -b/a$ $\qquad$ $\alpha^2 = \frac{c}{na}$

$= \alpha = \frac{-b}{(n+1)a}$

$\therefore \alpha^2 = \frac{b^2}{(n+1)^2a^2}$

$\Rightarrow \frac{b^2}{(n+1)^2a^2} = \frac{c}{na}$

$\Rightarrow nb^2 = c(n+1)^2 a$

$= nb^2 = (n+1)^2 ac$ //

10) Find two consecutive +ve even integers, the sum of whose squares is 340.

Sol- Let the two consecutive +ve even integers be

$2n, 2n+2$

G.T

$= (2n)^2 + (2n+2)^2 = 340$

$= 4n^2 + 4n^2 + 4 + 8n - 340 = 0$

$= 8n^2 + 8n - 336 = 0$

$= 8[n^2 + n - 42) = 0$

$= n^2 + n - 42 = 0$

$= n^2 + 7n - 6n - 42 = 0$

$= n(n+7] - 6(n+7] = 0$

$= (n-6)(n+7) = 0$

$= n - 6 = 0$ (or) $n + 7 = 0$

$= n = 6$ (or) $n = -7$

does not exist

$\boxed{n = 6}$

∴ The two consecutive +ve even integers be

$2n = 2(6) = 12$

$2n + 2 = 2(6) + 2$

$= 14$ //

II.

** 1) If x_1, x_2 are the roots of the Q. equ $ax^2+bx+c=0$ and $c \neq 0$, find the value of $(ax_1+b)^{-2}+(ax_2+b)^{-2}$ in terms of a, b,

Sol:- G.T

$x_1 x_2$ are the roots of $ax^2+bx+c=0$

$x_1+x_2 = -b/a$; $x_1 x_2 = c/a$

If x_1 is the roots of $ax^2+bx+c=0$, the put $x=x_1$, we get

$\Rightarrow ax_1^2+bx_1+c=0$

$\Rightarrow ax_1^2+bx_1=-c$

$\Rightarrow x_1(ax_1+b)=-c$

$\Rightarrow ax_1+b = \frac{-c}{x_1}$

$\Rightarrow (ax_1+b)^{-2} = \left(\frac{-c}{x_1}\right)^{-2} = \frac{x_1^2}{c^2}$

lly

$\Rightarrow (ax_2+b)^{-2} = \frac{x^2}{c^2}$

$= (ax_1+b)^{-2}+(ax_2+b)^{-2} = \frac{x_1^2}{c^2}+\frac{x_2^2}{c^2}$

$= \frac{x_1^2+x_2^2}{c^2}$

$= \frac{(x_1+x_2)^2-2x_1x_2}{c^2}$

$= \frac{\left(\frac{-b}{a}\right)^2-\frac{2c}{a}}{c^2} = \frac{\frac{b^2-2ac}{a^2}}{c^2}$

$= \frac{b^2-2ac}{a^2c^2}$ //

** 3) solve

$2x^4+x^3-$

Sol:- $= \frac{2x^4+x^3}{}$

$= \frac{2x^4}{x^2} +$

$= 2(x^2$

put $x+$

S

$= x^2 + \frac{1}{x}$

$= x^2$

from

$= 2(y^2$

$= 2y^2-$

$= 2y^2 +$

$= 2y^2 +$

$= 2y($

$= (2y$

case (i)

$2y-5 =$

$y = 5/2$

But y

**3) solve

$2x^4+x^3-11x^2+x+2=0$

Sol:- $= \dfrac{2x^4+x^3-11x^2+x+2}{x^2} = 0$

$= \dfrac{2x^4}{x^2} + \dfrac{x^3}{x^2} - \dfrac{11x^2}{x^2} + \dfrac{x}{x^2} + \dfrac{2}{x^2} = 0$

$= 2\left(x^2+\dfrac{1}{x^2}\right) + \left(x+\dfrac{1}{x}\right) - 11 = 0$ —— ①

put $x+\dfrac{1}{x} = y$, we get

SOBS we get

$= x^2 + \dfrac{1}{x^2} + 2x\cdot\dfrac{1}{x} = y^2$

$= x^2 + \dfrac{1}{x^2} = y^2 - 2$

from eq ①

$= 2(y^2-2) + y - 11 = 0$

$= 2y^2 - 4 + y - 11 = 0$

$= 2y^2 + y - 15 = 0$

$= 2y^2 + 6y - 5y - 15 = 0$

$= 2y(y+3) - 5(y+3) = 0$

$= (2y-5)(y+3) = 0$

case(i)

$2y - 5 = 0$

$y = 5/2$

But $y = x + 1/x$

$\Rightarrow x + \frac{1}{x} = 5/2 \Rightarrow \frac{x^2+1}{x} = \frac{5}{2}$

$\Rightarrow 2x^2 + 2 = 5x$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x-2) - 1(x-2) = 0$

$\Rightarrow (2x-1)(x-2) = 0$

$\Rightarrow 2x - 1 = 0$ $\quad$ $x - 2 = 0$

$\Rightarrow x = 1/2$ $\quad$ $x = 2$

case (ii)

$\Rightarrow y + 3 = 0$

$\Rightarrow y = -3$

$\Rightarrow x + 1/x = -3$

$\Rightarrow \frac{x^2+1}{x} = -3$

$\Rightarrow x^2 + 1 = -3x$

$\Rightarrow x^2 + 3x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-3 \pm \sqrt{9 - 4(1)(1)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{5}}{2}$

∴ The roots of given equ. are $\frac{-3 \pm \sqrt{5}}{2}, 2,$

4) solve

$3^{1+x} + 3^{1-x}$

Sol:- G.T

$\Rightarrow 3^{1+x} + 3^{1-x} =$

$\Rightarrow 3 \cdot 3^x + \frac{3}{3^x}$

$\Rightarrow$ put 3^x

$\Rightarrow 3 \cdot y + 3/$

$\Rightarrow \frac{3y^2+3}{y}$

$\Rightarrow 3y^2 + 3$

$\Rightarrow 3y^2 - 10$

$\Rightarrow 3y^2 - 9$

$\Rightarrow 3y(y -$

$\Rightarrow (y-3)($

case (i)

$= y - 3 = 0$

$= y = 3$

$\Rightarrow 3^x = 3^1$ [

$\Rightarrow x = 1$

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4) solve

$3^{1+x}+3^{1-x}=10$

Sol:- G.T

$\Rightarrow 3^{1+x}+3^{1-x}=10$

$\Rightarrow 3\cdot3^{x}+\frac{3}{3^{x}}=10$

$\Rightarrow$ put $3^{x}=y$, we get

$\Rightarrow 3\cdot y+3/y=10$

$\Rightarrow \frac{3y^2+3}{y}=10$

$\Rightarrow 3y^2+3=10y$

$\Rightarrow 3y^2-10y+3=0$

$\Rightarrow 3y^2-9y-1y+3=0$

$\Rightarrow 3y(y-3)-1(y-3)=0$

$\Rightarrow (y-3)(3y-1)=0$

case(i)

$= y-3=0$

$= y=3$

$\Rightarrow 3^{x}=3^{1}\ [\because y=3^{x}]$

$\Rightarrow x=1$

case(ii)

$3y-1=0$

$y=1/3$

$3^{x}=3^{-1}$

$x=-1$

$\therefore x=1$ (or) $x=-1$ //

7) solve

$$\sqrt{\frac{3x}{x+1}} + \sqrt{\frac{x+1}{3x}} = 2$$

Sol:- G.T

$$= \sqrt{\frac{3x}{x+1}} + \frac{1}{\sqrt{\frac{3x}{x+1}}} = 2$$

= put $\sqrt{\frac{3x}{x+1}} = y$, we get

$\Rightarrow y + \frac{1}{y} = 2$

$\Rightarrow \frac{y^2+1}{y} = 2$

$\Rightarrow y^2 - 2y + 1 = 0$

$\Rightarrow (y-1)^2 = 0 \Rightarrow y - 1 = 0$

$\Rightarrow y = 1$

$\Rightarrow \sqrt{\frac{3x}{x+1}} = 1$

SOBS, we get

$\Rightarrow \frac{3x}{x+1} = 1$

$\Rightarrow 3x = x + 1$

$\Rightarrow 3x - x = 1$

$\Rightarrow 2x = 1$

$\Rightarrow x = 1/2$ //

H.W

6) solve

$$\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = 5/2$$

8) solve

$$2\left(x+\frac{1}{x}\right)^2-7\left(x+\frac{1}{x}\right)+5=0$$

Sol:- put $x+\frac{1}{x}=y$, we get

$\Rightarrow 2y^2+7y+5=0$

$\Rightarrow 2y^2-2y-5y+5=0$

$\Rightarrow 2y(y-1)-5(y-1)=0$

$\Rightarrow (2y-5)(y-1)=0$

case (i)

$\Rightarrow 2y-5=0$

$\Rightarrow y=5/2$

$\Rightarrow x+\frac{1}{x}=5/2$

$\Rightarrow \frac{x^2+1}{2}=5/2$

$\Rightarrow 2x^2+2=5x$

$\Rightarrow 2x^2-5x+2=0$

$\Rightarrow 2x^2-4x-x+2=0$

$\Rightarrow 2x(x-2)-1(x-2)=0$

$\Rightarrow (2x-1)(x-2)=0$

$\Rightarrow 2x-1=0 \qquad x-2=0$

$\Rightarrow x=1/2 \qquad x=2$

case (ii)

$\Rightarrow y-1=0$

$\Rightarrow y=1$

$\Rightarrow x+\frac{1}{x}=1$

$\Rightarrow \frac{x^2+1}{x}=1$

$\Rightarrow x^2-x+1=0$

$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$=\frac{1\pm\sqrt{1-4}}{2}$

$=\frac{1\pm\sqrt{-3}}{2}$

$=\frac{1\pm i\sqrt{3}}{2} \quad \because [i^2=-1]$

10) Find the Q. equ for which the sum of the roots is 7 & the sum of the squares of the roots is 25.

Sol:- Let

α, β be the roots

G.T

$\alpha+\beta = 7$, $\alpha^2+\beta^2 = 25$

$\Rightarrow \alpha^2+\beta^2 = 25$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = \alpha^2+\beta^2$

$\Rightarrow (7)^2 - 2\alpha\beta = 25$

$\Rightarrow 49 - 25 = 2\alpha\beta$

$\Rightarrow 24 = 2\alpha\beta$

$\alpha\beta = \frac{24}{2} = 12$

$\therefore$ If α, β are the roots then the required Q. equ is

$\Rightarrow x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 7x + 12 = 0$ //

$\Rightarrow$ common root:- A necessary and sufficient condition for the quadratic equations $a_1x^2+b_1x+c_1=0$ & $a_2x^2+b_2x+c_2=0$ to have a common root is

$$(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$$

⇒ **Note:-**

Let $f(x) = ax^2+bx+c=0$ be a Q. equ. If α, β the roots, the

→ If $c \neq 0$, $\alpha\beta \neq 0$ & $f\left(\frac{1}{x}\right)=0$ is an equation roots are $\frac{1}{\alpha}$ & $\frac{1}{\beta}$.

→ $\alpha + K$ & $\beta + K$ are the roots of the equ. $f(x-K)=0$

→ $\alpha - K$ & $\beta - K$ are the roots of the equ $f(x+K)=0$

→ $-\alpha, -\beta$ are the roots of the equ $f(-x)=0$

→ $K\alpha$ & $K\beta$ are the roots of the equ $f\left(\frac{x}{K}\right)$.

⇒ **Sign of quadratic expression:-**

→ Let $a, b, c \in R$, and) If a, b are nonzero real no.s then we can say that a, b have same sign if both of a & b are +ve (or) both of them are −ve.

→ Let $a, b, c \in R$ and $a \neq 0$. then the roots of $ax^2+bx+c=0$ are non-real complex no.s.
⇔ ax^2+bx+c and 'a' have the same sign $\forall x \in R$.

→ Let $a, b, c \in R$ and $a \neq 0$. if the equation $ax^2+bx+c=0$ has equal roots, then ax^2+bx+c and 'a' have same

$x = \frac{-b}{2a}$

⇒ **Theorem:-** Let $a,$
equ. $ax^2+bx+c=$
$\alpha < \beta$ then

(i) for $\alpha < x < \beta$, ax
sign.

(ii) for $x < \alpha$ (or) $x > \beta$
sign.

Proof:- α, β are
have

$ax^2+bx+c = a(x$

$\Rightarrow \frac{ax^2+bx+c}{a} =$

(i) when $\alpha < x < \beta$,

∴ from equ ①

$\therefore \frac{ax^2+bx+c}{a}$

Hence ax^2+bx+

(ii) when $x < \alpha$

we have $x-\alpha$
(−)

from eq ①

and 'a' have same sign $\forall x \in R$ except for

$x = \frac{-b}{2a}$

⇒ Theorem:- Let $a, b, c \in R$ & $a \neq 0$ such that the equ. $ax^2+bx+c=0$ has real roots α & β with $\alpha < \beta$ then

(i) for $\alpha < x < \beta$, ax^2+bx+c & 'a' have opposite sign.

(ii) for $x < \alpha$ (or) $x > \beta$, ax^2+bx+c & 'a' have the same sign.

Proof:- α, β are the roots of $ax^2+bx+c=0$, we have

$$ax^2+bx+c = a(x-\alpha)(x-\beta)$$

$$\Rightarrow \frac{ax^2+bx+c}{a} = (x-\alpha)(x-\beta) \text{ — ①}$$

(i) when $\alpha < x < \beta$, we have $x-\alpha > 0$ & $x-\beta < 0$

$(+) \times (-) = -(<)$

∴ from equ ①

$$\therefore \frac{ax^2+bx+c}{a} < 0$$

Hence ax^2+bx+c & 'a' have opposite sign.

(ii) when $x < \alpha$

we have $x-\alpha < 0$ & $x-\beta < 0$

$(-) \times (-) = +(>)$

from eqr ①

$$= \frac{ax^2+bx+c}{a} > 0$$

⇒ when $x > \beta$, we have

$$x-\beta > 0 \ \& \ x-\alpha > 0$$

$$(+) \times (+) = +(>)$$

∴ From eq ①

$$\frac{ax^2+bx+c}{a} > 0$$

∴ For $x < \alpha$ (or) $x > \beta$, ax^2+bx+c & (a) have same sign.

⇒ **Maximum & minimum values:-**

The extreme values of a quadratic expression are maximum minimum values with real coefficient depends on sign of the x^2.

⇒ **Note [Formula]:-**

⇒ If $a, b, c \in R$ where $[a \neq 0]$ and $f(x) = ax^2 + bx + c$.

(i) If $\underline{a > 0}$, then $f(x)$ has absolute minimum at $x = \frac{-b}{2a}$ and

$$\text{minimum value} = \frac{4ac-b^2}{4a}$$

If $\underline{a < 0}$ then $f(x)$ has absolute maximum at $x = -\frac{b}{2a}$ and minimum value $= \dfrac{4ac-b^2}{4a}$

Ex:- 3(b)

I.

1) If the quadratic equation $ax^2+2bx+c=0$ & $ax^2+2cx+b=0$ $(b\neq c)$ have a common roots then S.T $a+4b+4c=0$

Sol:- G.T

$$ax^2+2bx+c=0 \quad \text{——} ①$$

$$ax^2+2cx+b=0 \quad \text{——} ②$$

Let 'α' be the common roots of given eqs' then

$$a\alpha^2+2b\alpha+c=0$$

$$a\alpha^2+2c\alpha+b=0$$

solving the above two equ's we get

$$\cancel{a\alpha^2}+2b\alpha+c=0$$

$$\underline{\underset{-}{\cancel{a\alpha^2}}+\underset{-}{2c\alpha}+\underset{-}{b}=0}$$

$$2b\alpha-2c\alpha+c-b=0$$

$$\Rightarrow 2\alpha(b-c)=(b-c)$$

$$2\alpha=\frac{b-c}{b-c}=1$$

$$\alpha=1/2$$

Sub $\alpha=1/2$ in eq ③, we get

$$\Rightarrow a\alpha^2+2b\alpha+c=0$$

$$\Rightarrow a\left(\tfrac{1}{2}\right)^2+\cancel{2}b\left(\tfrac{1}{\cancel{2}}\right)+c=0$$

$$\Rightarrow \frac{a}{4}+b+c=0 \Rightarrow a+4b+4c=0 //$$

2) If $x^2-6x+6=0$ & $x^2-12x+p=0$ have a common root, then find 'p'.

Sol:- G.T

$= x^2-6x+5=0$

$= x^2-12x+p=0$

Let 'α' be the common root for given eq's then

$\alpha^2-6\alpha+5=0$ —— ①

$\alpha^2-12\alpha+p=0$ —— ②

from equ ①

$\alpha^2-6\alpha+5=0$

$\alpha^2-5\alpha-\alpha+5=0$

$\Rightarrow \alpha(\alpha-5)-1(\alpha-5)=0$

$\Rightarrow (\alpha-1)(\alpha-5)=0$

$\alpha-1=0 \qquad \alpha-5=0$

$x=1 \qquad \alpha=5$

i) sub. $\alpha=1$ in equ ②, we get

$\Rightarrow (1)^2-12(1)+p=0$

$\Rightarrow -11+p=0 \Rightarrow p=11$

ii) sut $\alpha=5$ in equ ①, we get

$= (5)^2-12(5)+p=0$

$= 25-60+p=0$

$= p=35$

$p=11$ (or) 35 //

4) If the equ x^2

have a common

equal roots, th

Sol: Let α be the

x^2+ax+b

x^2+cx+d

G.T

α, α be the

x^2+ax+b

$\alpha+\alpha=a$

$2\alpha=-a$

$\alpha=-a/2$

∴ 'α' is a

$\alpha^2+c\alpha+$

$\Rightarrow b+c\left(-\frac{a}{2}\right)$

$\Rightarrow \frac{2b-ac+2}{2}$

$\Rightarrow 2b+2d=a$

$\Rightarrow 2(b+d)$

5) Discuss the

-ratic exp's wh

(i) x^2-5x+4

Sol:- G.T

$\Rightarrow x^2-5x+4$

$\Rightarrow x^2-4x-1x+4$

$\Rightarrow x(x-4)-1($

***
4) If the equ $x^2+ax+b=0$ & $x^2+cx+d=0$ have a common roots & the first eq. has equal roots, then P.T $2[b+d]=ac$.

Sol: Let α be the common root for given equs

$$x^2+ax+b=0 \quad \text{—①}$$

$$x^2+cx+d=0 \quad \text{—②}$$

G.T

α,α be the roots of eq①

$$x^2+ax+b=0$$

$\alpha+\alpha=a$ $\qquad$ $\alpha\cdot\alpha=b$

$2\alpha=-a$ $\qquad$ $\alpha^2=b$

$\alpha=-a/2$

∴ 'α' is a root of $x^2+cx+d=0$, then

$\alpha^2+c\alpha+d=0$.

$\Rightarrow b+c(-a/2)+d=0$

$\Rightarrow \dfrac{2b-ac+2d}{2}=0$

$\Rightarrow 2b+2d=ac$

$\Rightarrow 2(b+d)=ac$ //

**
5) Discuss the signs of the following quad-ratic exp's when 'x' is ~~root~~ real.

(i) x^2-5x+4

Sol:- G.T

$\Rightarrow x^2-5x+4$

$\Rightarrow x^2-4x-1x+4$

$\Rightarrow x(x-4)-1(x-4)$

⇒ $(x-1)(x-4)$

∴ $a=1>0$

∴ The exp x^2-5x+4 is +ve if $x<1$ (or) $x>4$ & -ve if $1<x<4$//

6) For what values of 'x', the following expressions are positive?

sol (i) x^2-5x+6

Sol ⇒ x^2-5x+6

⇒ $x^2-2x-3x+6$

⇒ $x(x-2)-3(x-2)$

⇒ $(x-3)(x-2)$

$x-2=0$ $\quad$ $x-3=0$

$x=2$ $\quad$ $x=3$

∴ The values 2, 3 are real. Here $a=1>0$

the exp x^2-5x+6 is positive if $x<2$ (or) $x>3$//

(iii) $4x-5x^2+2$

sol G.T

⇒ $-5x^2+4x+2$

$ax^2+bx+c=0$, where

$a=-5$, $b=4$, $c=2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{16-4(-5)2}}{2(-5)}$$

$$x=\frac{-4\pm\sqrt{16+40}}{-10} \Rightarrow x=\frac{-4\pm\sqrt{56}}{-10}$$

$= \frac{-4\pm2\sqrt{14}}{-10}$

$x = \frac{+2\pm\sqrt{14}}{5} \in R$

∴ The exp $4x-5x^2+2$ is positive when

$\frac{2-\sqrt{14}}{5} < x < \frac{2+\sqrt{14}}{5}$ [∵ $a=-5<0$]

H.W

(ii) $3x^2+4x+4$

(iv) $x^2-5x+14$

7) For what values of 'x' the following expressions are negative.

(i) $x^2-7x+10$

sol:- Here $a=1>0$

G.T

$\Rightarrow x^2-7x+10$

$\Rightarrow x^2-5x-2x+10$

$\Rightarrow x(x-5)-2(x-5)$

$\Rightarrow (x-2)(x-5)$

$x-2=0$ $x-5=0$

$x=2$ $x=5$

The value 2,5 are real

Here $a=1>0$. then the expression $x^2-7x+10$ is -ve if $2<x<5$ ✓

(ii) $15+14x-3x^2$

sol:- $-3x^2+4x+15$

It is in the form of ax^2+bx+c

where $a=-3<0$

Let

$-3x^2+4x+15=0$

$x = \frac{-4 \pm \sqrt{16-4(-3)(15)}}{2(-3)}$

$= \frac{-4 \pm \sqrt{16+180}}{-6}$

$= \frac{-4 \pm \sqrt{196}}{-6}$

$= \frac{-4 \pm 14}{-6}$

$x = \frac{-4+14}{-6}$ $\qquad$ $x = \frac{-4-14}{-6}$

$x = \frac{-10}{6}$ $\qquad$ $x = \frac{-18}{-16}$

$x = -5/3$ $\qquad$ $x = 3$

∴ The exp. $15+4x-3x^2$ is −ve if $x < -5/3$ (or) $x > 3$ [∵ $a<0$]//

8) Find the changes in the sign of the following exp.s & find their extreme values

i) x^2-5x+6

Sol: Let

⇒ $x^2-5x+6=0$

⇒ $x^2-2x-3x+6=0$

⇒ $x(x-2)-3(x-2)=0$

⇒ $(x-2)(x-3)=0$

$x-2=0$ $\qquad$ $x-3=0$

$x=2$ $\qquad$ $x=3$

∴ the value 2,

Here $a=1>0$, th

(i) The given exp

$x<2$ (or) $x>3$.

(ii) The given exp

$2<x<3$

Here $a=1>0$, th

absolute min at

$x=$

min value

9) Find the mi

following exp.s

(i) x^2-x+7

Sol: G.T

x^2-x+7, it is

$a=1$, $b=-1$, $c=7$

Here

$a=1>0$

∴ the absolute

$\therefore$ the value 2, 3 are real.

Here $a = 1 > 0$, then

(i) The given exp. $x^2 - 5x + 6$ is +ve when $x < 2$ (or) $x > 3$.

(ii) The given exp. $x^2 - 5x + 6$ is -ve when.

$2 < x < 3$

Here $a = 1 > 0$, then

absolute min at $x = \frac{-b}{2a}$

$$x = \frac{-(-5)}{2(1)} = \frac{5}{2}$$

min value $= \frac{4ac - b^2}{4a}$

$$= \frac{4(1)(6) - (-5)^2}{4(1)}$$

$$= \frac{24 - 25}{4} = -\frac{1}{4}$$ //

9) Find the minimum (or) maximum of the following exp's as 'x' varies over R.

(i) $x^2 - x + 7$

<u>Sol:-</u> G.T

$x^2 - x + 7$, it is in the form of $ax^2 + bx + c$, where

$a = 1$, $b = -1$, $c = 7$

Here

$a = 1 > 0$

$\therefore$ The absolute min. at $x = -b/2a$

$$x = \frac{-(-1)}{2(1)} = \frac{1}{2}$$

minimum value $= \frac{4ac-b^2}{4a} = \frac{4(1)(7)-1}{4(1)}$

$= \frac{28-1}{4} = \frac{27}{4}$

(iii) $2x+5-3x^2$

Sol:- G.T

$-3x^2+2x+5$, which is in the form of

ax^2+bx+c, where

$a=-3,\ b=2,\ c=5$

Here $a=-3<0$

∴ The absolute max at

$x = \frac{-b}{2a} = \frac{-2}{2(-3)} = 1/3$

∴ max value $= \frac{4ac-b^2}{4a}$

$= \frac{4(-3)(5)-4}{4(-3)}$

$= \frac{-60-4}{-12} = \frac{-64}{-12} = \frac{16}{3}$

II

1) Determine the range of the following exp's

(i) $\frac{x^2+x+1}{x^2-x+1}$

Sol:- Let

$y = \frac{x^2+x+1}{x^2-x+1} \in R$

$\Rightarrow y(x^2-x+1) = x^2+x+1$

$\Rightarrow yx^2-yx+y-x^2-x-1=0$

$\Rightarrow (y-1)x^2+(-y$

It is in the form

where

$a=y-1,\ b$

Here $x\in R$

$\Delta \geq 0$

$\Rightarrow b^2-4ac \geq$

$\Rightarrow [-(y+1)]^2-$

$\Rightarrow [y^2+1+2y$

$\Rightarrow [y^2+1+2y-$

$\Rightarrow [y^2+1+2y$

$\Rightarrow [-3y^2+10y$

$\Rightarrow -(3y^2-10y$

$\Rightarrow 3y^2-10y+$

$\Rightarrow 3y^2-9y-$

$\Rightarrow 3y(y-3)$

$\Rightarrow (3y-1)(y$

$y \in [1/3, 3]$

∴ The range

(ii) $\frac{x+2}{2x^2+3x+6} \in R$

Sol:- Let

$y = \frac{x+2}{2x^2+3x+6}$

$\Rightarrow (y-1)x^2+(y-1)x+(y-1)=0$

It is in the form of $ax^2+bx+c=0$

where
$a=y-1$, $b=-(y+1)$, $c=y-1$

Here $x \in R$

$\Delta \geq 0$

$\Rightarrow b^2-4ac \geq 0$

$\Rightarrow [-(y+1)]^2-4(y-1)(y-1)] \geq 0$

$\Rightarrow [y^2+1+2y-4(y-1)^2] \geq 0$

$\Rightarrow [y^2+1+2y-4(y^2+1-2y)] \geq 0$

$\Rightarrow [y^2+1+2y-4y^2-4+8y] \geq 0$

$\Rightarrow [-3y^2+10y-3] \geq 0$

$\Rightarrow -(3y^2-10y+3] \geq 0$

$\Rightarrow 3y^2-10y+3 \leq 0$

$\Rightarrow 3y^2-9y-y+3 \leq 0$

$\Rightarrow 3y(y-3)-1(y-3) \leq 0$

$\Rightarrow (3y-1)(y-3) \leq 0$

$\therefore 3y-1=0$
$3y=1 \Rightarrow y=\frac{1}{3}$
$\therefore y-3=0$
$y=3$

$y \in [\frac{1}{3}, 3]$

$\therefore$ The range of given exp is $[\frac{1}{3}, 3]$//

(ii) $\frac{x+2}{2x^2+3x+6} \in R$

sol- Let

$y=\frac{x+2}{2x^2+3x+6}$

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$\Rightarrow y(2x^2+3x+6) = x+2$

$\Rightarrow 2yx^2+3yx+6y = x+2$

$\Rightarrow 2yx^2+3yx+6y-x-2=0$

$\Rightarrow 2y(x^2)+(3y-1)x+(6y-2)=0$

$\Rightarrow$ It is in the form of $ax^2+bx+c=0$

where $a=2y$, $b=3y-1$, $c=6y-2$

'x' is real

$\Delta \geqslant 0$

$\Rightarrow b^2-4ac \geqslant 0$

$\Rightarrow [(3y-1)^2-4(2y)(6y-2)] \geqslant 0$

$\Rightarrow [9y^2+1-6y-8y(6y-2)] \geqslant 0$

$\Rightarrow [9y^2+1-6y-48y^2+16y] \geqslant 0$

$\Rightarrow [-39y^2+10y+1] \geqslant 0$

$\Rightarrow -(39y^2-10y-1] \geq 0$

$\Rightarrow (39y^2-10y-1) \leq 0$

$\Rightarrow 39y^2-13y+13y-1 \leq 0$

$\Rightarrow 13y(3y-1)+(3y-1) \leq 0$

$\Rightarrow (13y+1)(3y-1) \leq 0$

$y \in [-\frac{1}{13}, \frac{1}{3}]$

$\therefore$ The range is $[-\frac{1}{13}, \frac{1}{3}]$

**

2) P.T $\frac{1}{3x+1} + \frac{1}{x-1} - \frac{1}{(3x+1)(x+1)}$ does not lie between 1 & 4, if x is real.

<u>Sol</u>:- Let

$$y = \frac{1}{3x+1} + \frac{1}{x+1} - \frac{1}{(3x+1)(x+1)}$$

$\Rightarrow y = \frac{x+1+3}{(3x+1}$

$\Rightarrow y = \frac{4x+1}{3x^2+}$

$\Rightarrow y(3x^2+4x$

$\Rightarrow 3yx^2+4x$

$\Rightarrow (3y)x^2+($

Now it is in

$a=3y$, $b=4$

a.T 'x' is

i.e, $\Delta \geqslant$

$\Rightarrow b^2-4ac \geqslant$

$\Rightarrow [(4y-4]^2 -$

$\Rightarrow [16y^2+16-$

$\Rightarrow [4y^2-20y$

$\Rightarrow 4[y^2-5y$

$\Rightarrow y^2-5y+$

$\Rightarrow y^2-4y-$

$\Rightarrow y(y-4)$

$\Rightarrow (y-1)($

$y \leq 1$

$\therefore$ The give

$$\Rightarrow y = \frac{x+1+3x+1-1}{(3x+1)(x+1)}$$

$$\Rightarrow y = \frac{4x+1}{3x^2+3x+x+1} \Rightarrow \frac{4x+1}{3x^2+4x+1}$$

$$\Rightarrow y(3x^2+4x+1) = 4x+1$$

$$\rightarrow 3yx^2+4xy+y-4x-1=0$$

$$\Rightarrow (3y)x^2+(4y-4)x+(y-1)=0$$

Now it is in the form of $ax^2+bx+c=0$, where

$a=3y$, $b=4y-4$, $c=y-1$

G.T 'x' is real

i.e, $\Delta \geq 0$

$$\Rightarrow b^2-4ac \geq 0$$

$$\Rightarrow [[4y-4]^2 - 4(3y)(y-1)] \geq 0$$

$$\Rightarrow [16y^2+16-32y-12y^2+12y] \geq 0$$

$$\Rightarrow [4y^2-20y+16] \geq 0$$

$$\Rightarrow 4[y^2-5y+4] \geq 0$$

$$\Rightarrow y^2-5y+4 \geq 0$$

$$\Rightarrow y^2-4y-y+4 \geq 0$$

$$\Rightarrow y(y-4)-1(y-4) \geq 0$$

$$\Rightarrow (y-1)(y-4) \geq 0$$

$$y \leq 1 \text{ (or) } y \geq 4$$

$\therefore$ The given exp. doesnot lie between 1 & 4 //

Eg :- If 'x' is real show exp.

$\frac{x^2+34x-71}{x^2+2x-7}$ doesnot lie b/w 5 & 9.

Sol:- Let

$$\Rightarrow y = \frac{x^2+34x-71}{x^2+2x-7}$$

$$\Rightarrow y(x^2+2x-7) = x^2+34x-71$$

$$\Rightarrow x^2y+2xy-7y-x^2-34x+71=0$$

$$\Rightarrow (y-1)x^2+(2y-34)x+(71-7y)=0$$ where

Now, it is in the form of $ax^2+bx+c=0$

$a=y-1$, $b=2y-34$, $c=71-7y$

Ex. $r_{(x)}$ is real

i.e $\Delta \geq 0$

$$\Rightarrow b^2-4ac \geq 0$$

$$\Rightarrow [(2y-34)^2-4(y-1)(71-7y)] \geq 0$$

$$\Rightarrow [4(y-17)^2-4(71y-7y^2-71+7y)] \geq 0$$

$$\Rightarrow 4[y^2+289-34y-(-7y^2+78y-71)] \geq 0$$

$$\Rightarrow [y^2+289-34y+7y^2-78y+71] \geq 0$$

$$\Rightarrow [8y^2-112y+360] \geq 0$$

$$\Rightarrow 8[y^2-14y+45] \geq 0$$

$$\Rightarrow (y^2-14y+45) \geq 0$$

$$\Rightarrow (y^2-9y-5y+45) \geq 0$$

$$\Rightarrow y(y-9)-5(y-9) \geq 0$$

$$\Rightarrow (y-5)(y-9) \geq 0$$

$\therefore y \leq 5$ (or) $y \geq 9$

3) If 'x' is real prove that $\frac{x}{x^2-5x+9}$ lies b/w $\frac{-1}{11}$ & 1.

Sol: Let

$\Rightarrow y = \frac{x}{x^2-5x+9}$

$\Rightarrow y(x^2-5x+9) = x$

$\Rightarrow yx^2 - 5yx + 9y - x = 0$

$\Rightarrow yx^2 + -(5y-1)x + 9y = 0$

Now it is in the form of $ax^2+bx+c=0$

where $a = y$, $b = -5y-1$, $c = 9y$

G.T

'x' is real then $\Delta \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow [(-(5y+1))^2 - 4(y)(9y)] \geq 0$

$\Rightarrow [25y^2 + 1 - 10y - 36y^2] \geq 0$

$\Rightarrow [-11y^2 + 10y + 1] \geq 0$

$\Rightarrow -[11y^2 - 10y - 1] \geq 0$

$\Rightarrow [11y^2 - 10y - 1] \leq 0$

$\Rightarrow (11y^2 - 11y + y - 1) \leq 0$

$\Rightarrow 11y(y-1) + 1(y-1) \leq 0$

$\Rightarrow (11y+1)(y-1) \leq 0$

$\Rightarrow \frac{-1}{11} \leq y \leq 1$

$\therefore$ The given exp lie b/w $-\frac{1}{11}$ & 1//

**) 4) If the exp $\frac{x-p}{x^2 \pm 3x+2}$ takes all real values for $x \in R$, then find the limits (bounds) for p.

Sol:- Let

$\Rightarrow y = \frac{x-p}{x^2-3x+2}$

$\Rightarrow y(x^2-3x+2) = x-p$

$\Rightarrow x^2y - 3xy + 2y - x + p = 0$

$\Rightarrow yx^2 + (-3y-1)x + (2y+p) = 0$

Now it is in the form of $ax^2+bx+c=0$

$a = y$; $b = -(+3y+1)$, $c = 2y+p$

G.T $x \in R$

ie $\Delta \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow [(-(3y+1))^2 - 4(y)(2y+p)] \geq 0$

$\Rightarrow [9y^2+1+6y-8y^2-4py] \geq 0$

$\Rightarrow [y^2+y(6-4p)+1] \geq 0$

$\Rightarrow$ $y^2+y(6-4p)+1$ > 0 then

$\Rightarrow \forall x \in R$, sign of the exp. > 0 then

G.T coff of $x^2 > 0$

then $a = 1$, $b = 6-4p$, $c = 1$

$\Rightarrow b^2 - 4ac < 0$

$\Rightarrow (6-4p)^2 - 4(1)(1) < 0$

$\Rightarrow 36 + 16p^2 - 48p - 4 < 0$

$\Rightarrow 16p^2 - 48p + 32 < 0$

$\Rightarrow 16(p^2-3p+2) < 0$

$\Rightarrow p^2 - 3p + 2 < 0$

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$\Rightarrow p^2-2p-p+2<0$

$\Rightarrow p(p-2)-1(p-2)<0$

$\Rightarrow (p-1)(p-2)<0$

$\Rightarrow 1<p<2$

(or)

$p\in(1,2)$

∴ The limits for 'p' are 1 & 2 //

5) If $c^2\neq ab$ & the roots of $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ are equal then S.T $a^3+b^3+c^3=3abc$ (or) $a=0$

Sol:- G.T

$\Rightarrow (c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)$

It is in the form of $Ax^2+Bx+C=0$ where

$A=c^2-ab$, $B=-2(a^2-bc)$, $C=b^2-ac$

G.T eq① has equal roots, then

$\Delta=0$

$\Rightarrow B^2-4AC=0$

$\Rightarrow [(-2(a^2-bc))]^2-4(c^2-ab)(b^2-ac))]=0$

$\Rightarrow 4(a^4+b^2c^2-2a^2bc-b^2c^2+ac^3+ab^3-a^2bc)=0$

$\Rightarrow a^4-3a^2bc+ac^3+ab^3=0$

$\Rightarrow a[a^3+b^3+c^3-3abc]=0$

∴ $a=0$ (or)

$a^3+b^3+c^3-3abc=0$

∴ $a^3+b^3+c^3=3abc$ //

⇒ **Qudratic Inequation:-** If a, b, c are real numbers, where $a \neq 0$ then the inequation which are in the form $ax^2+bx+c>0$, $ax^2+bx+c<0$, $ax^2+bx+c \geq 0$; $ax^2+bx+c \leq 0$ are called "quadratic inequation".

The values of 'x' which satisfy the given inequation are called solution of inequation.

⇒ **Methods of solving inequations:-** 1) Algebraic method 2) Graphical method, These are two methods of solving inequations these are:-

⇒ **Algebraic Method:-**

Note:- In this method, we can find the solution by fractorizing this quadratic expression and observing the changes in the sign of the quadratic expression.

⇒ **Graphical method:-** In this method, we can find the solution from graph of the inequation.

Eg:- Solve $x^2-10x+21<$ graphical method.

Sol:- **Algebraic method:-**

$x^2-10x+21<0$

Let $x^2-10x+21 = x$

$=$

$=$

coff of $x^2 \Rightarrow$

Let $x^2-10x+21=0$

then $x-3=0$

$x=3$

∴ The coff of

$x^2-10x+2$ is -ve

∴ The solution set

Graphical method:-

Let $y = x^2-10x+$

$= x^2-7x$

$= (x-3)$

Let $y=0 \Rightarrow$

x	0	1	2	3
y	21	12	5	

Eg: solve $x^2-10x+21<0$ by algebraic method & graphical method.

Sol: **Algebraic method:-**

$x^2-10x+21<0$

Let $x^2-10x+21 = x^2-7x-3x+21$

$= x(x-7)-3(x-7)$

$= (x-3)(x-7)$

coff of $x^2 \Rightarrow a=1>0$, which is the +ve

Let $x^2-10x+21=0$

then $x-3=0$ $\quad x-7=0$

$x=3$ $\quad x=7$

∴ the coff of x^2 is +ve then the exp

$x^2-10x+2$ is -ve $3<x<7$;

∴ The solution set is $\{x\in R: 3<x<7\}$//

Graphical method:-

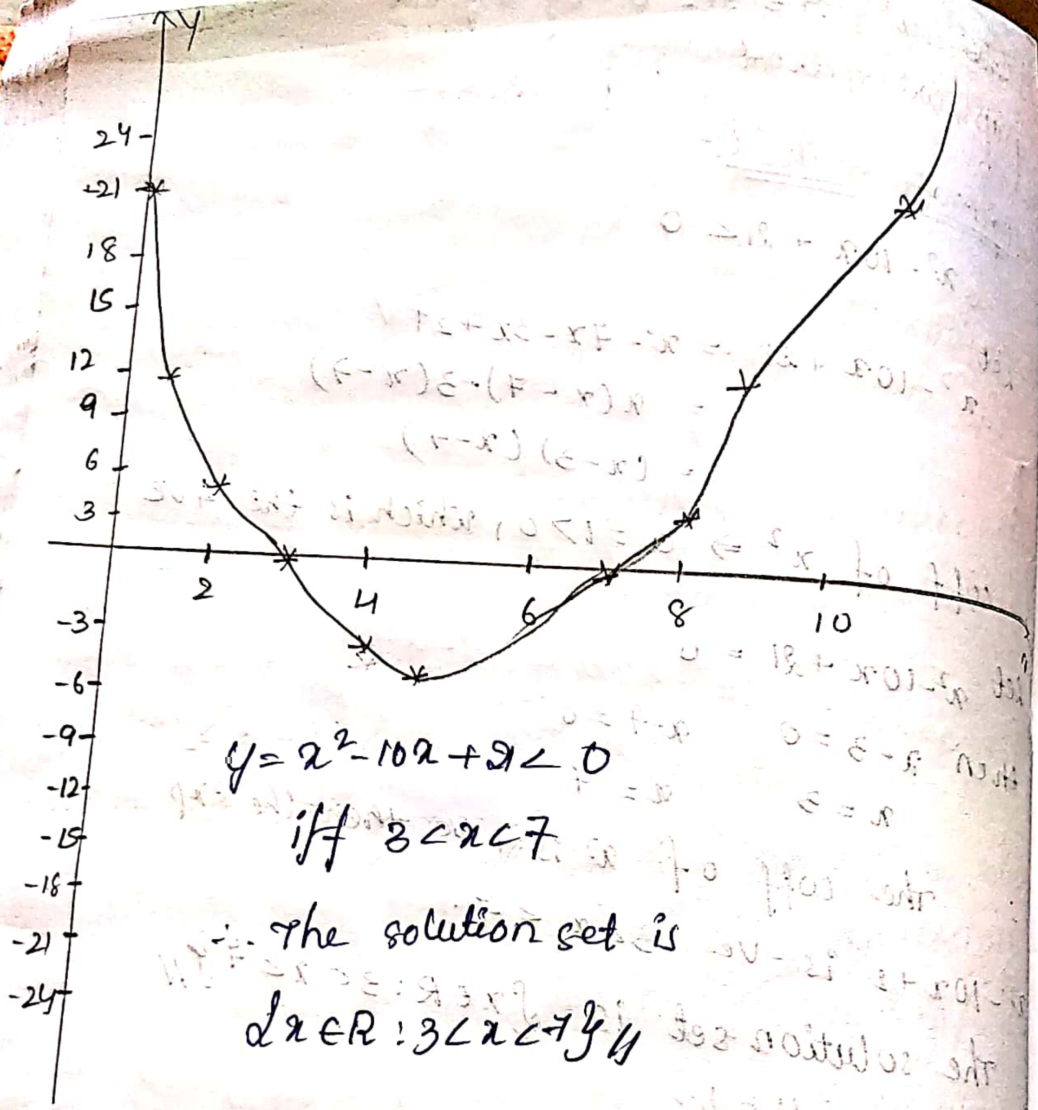
Let $y=x^2-10x+21$

$= x^2-7x \cancel{+} -3x+21$

$= (x-3)(x-7)$

Let $y=0 \Rightarrow x=3$ & $x=7$

x	0	1	2	3	4	5	6	7	8	9	10
y	21	12	5	0	-3	-4	-3	0	5	12	21



$y = x^2 - 10x + 21 < 0$

iff $3 < x < 7$

$\therefore$ The solution set is

$\{x \in R : 3 < x < 7\}$

Ex

I) 1) solve the follow

method:- and gra

(i) $15x^2 + 4x - 4 \leq 0$

Sol:- G.T

$\Rightarrow 15x^2 + 4x - 4 \leq 0$

$\Rightarrow 15x^2 - 6x + 10x -$

$\Rightarrow 3x(5x-2) + 2(5x$

$\Rightarrow (5x-2)(3x+2$

Let

$5x - 2 = 0$ 3x

$5x = 2$

$x = 2/5$

the coff of x^2

Given eqn is

$\Rightarrow x$ lies betwe

$\Rightarrow$ Graphical me

Let $y = 15x^2 +$

x	-1	-
y	715	

Ex:-3(C)

I) Solve the following inequation by algebraic method:- and graphical method.

(i) $15x^2+4x-4 \le 0$

Sol:- G.T

$\Rightarrow 15x^2+4x-4 \le 0$

$\Rightarrow 15x^2-6x+10x-4 \le 0$

$\Rightarrow 3x(5x-2)+2(5x-2) \le 0$

$\Rightarrow (5x-2)(3x+2) \le 0$

Let

$5x-2=0$ $\quad$ $3x+2=0$

$5x=2$ $\quad$ $3x=-2$

$x=2/5$ $\quad$ $x=-2/3$

the coff of x^2 is $a=15>0$, which is +ve

Given eqn is ≤ 0

$\Rightarrow x$ lies between $-2/3$, $2/5$ ie $-2/3 \le x \le 2/5$

$\Rightarrow$ Graphical method:-

Let $y = 15x^2+4x-4 \le 0$

x	-1	$-2/3$	0	$2/5$	1
y	7	0	-4	0	15

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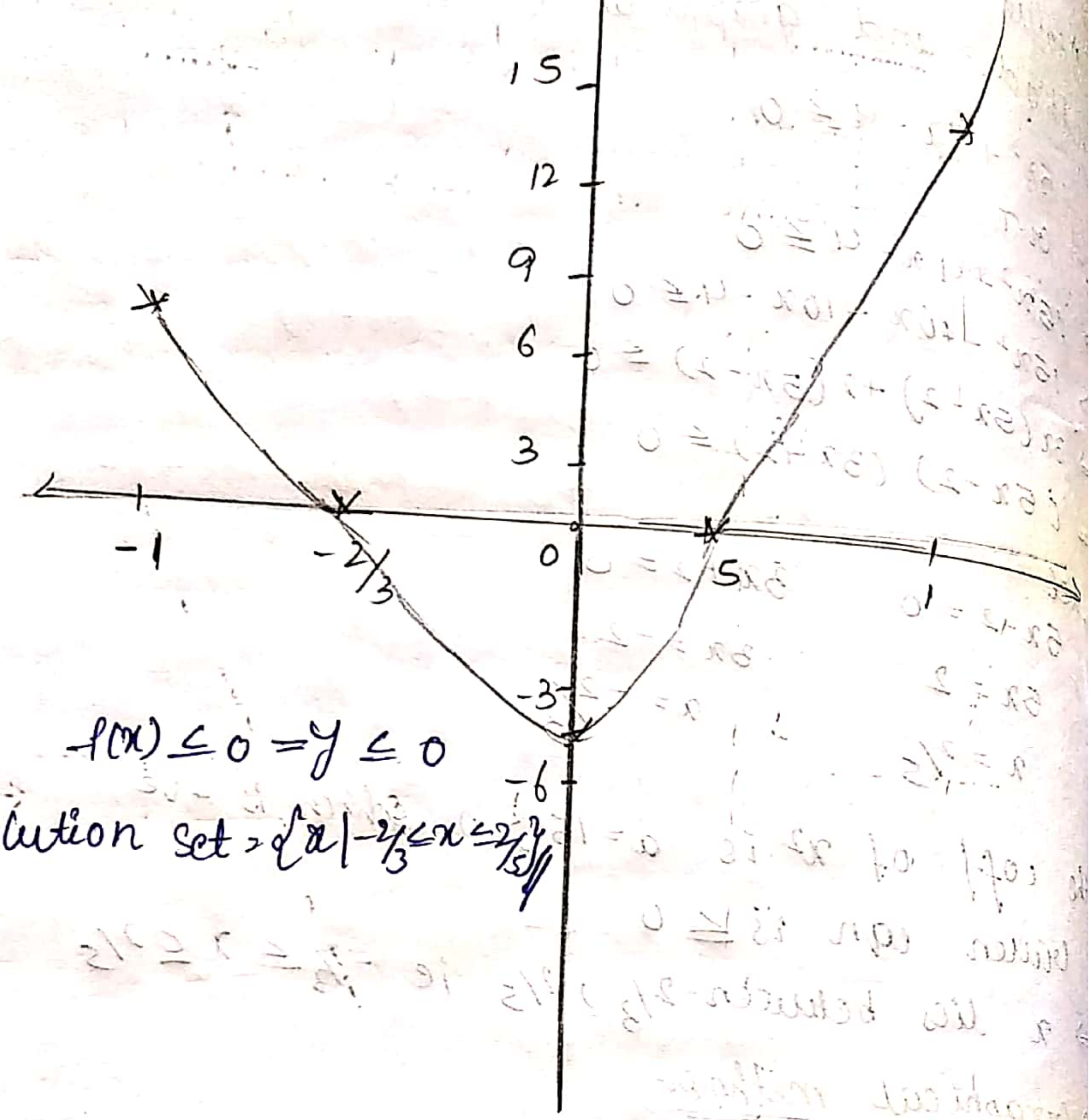


$f(x) \le 0 \Rightarrow y \le 0$

solution set $= \{x \mid -2/3 \le x \le 2/5\}$ //

(iv) $x^2 - 4x - 21 \geq 0$

soln $\Rightarrow x^2 - 7x + 3x - 21 \geq 0$

$\Rightarrow x(x-7) + 3(x-7) \geq 0$

$\Rightarrow (x+3)(x-7) \geq 0$

coff of x^2 is $a = 1 > 0$

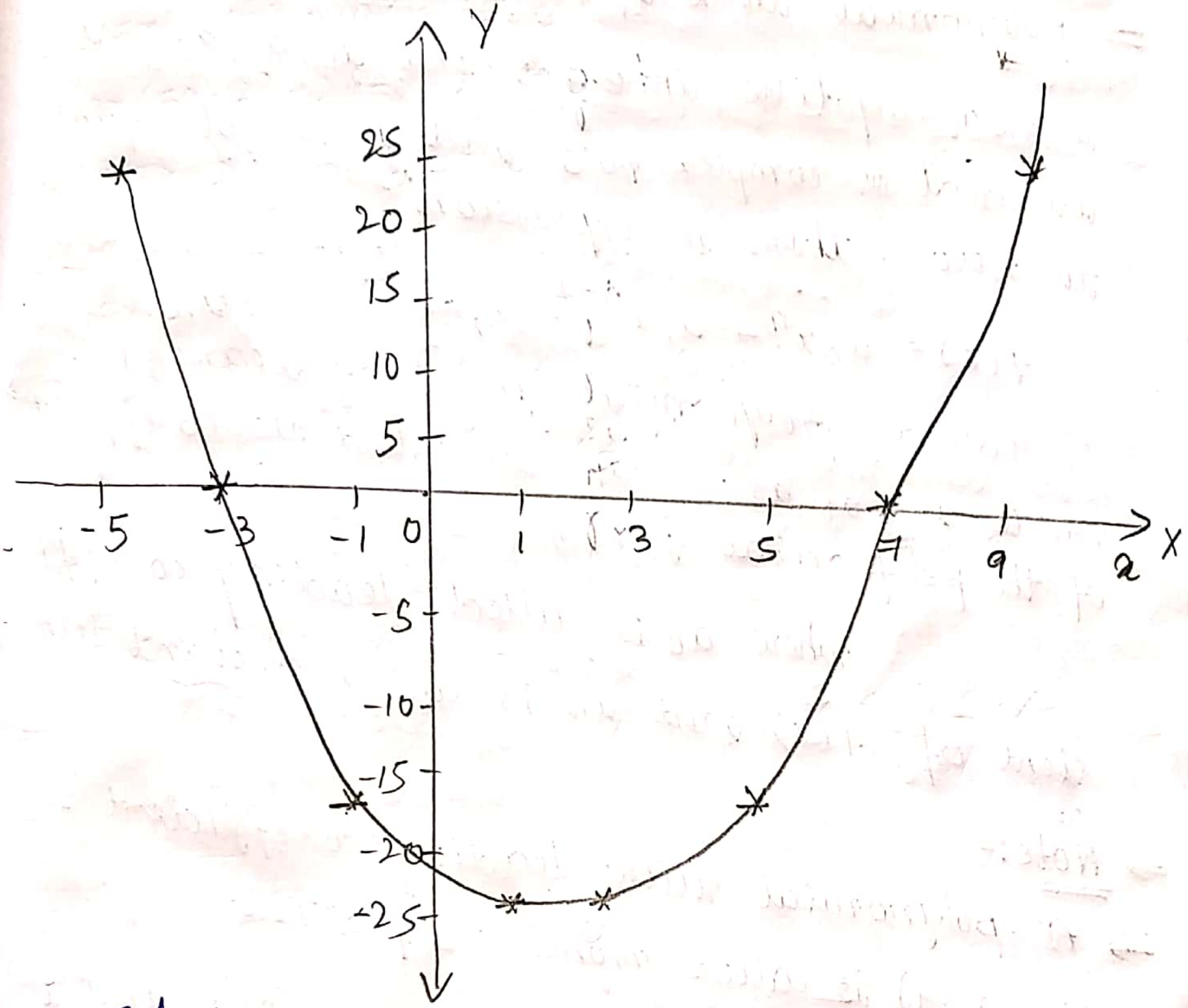
Given exp is ≥ 0

'x' doesnot lie b/w -3 & 7 solution set is $\{x \mid x \in (-\infty, -3) \cup (7, \infty)\}$ //

Graphical method

Let $y = x^2 - 4x - 21$

x	-5	-3	-1	1	3	5	7	9
y	24	0	-16	-24	-24	-16	0	24



solution set is

$$\{x \mid x \in (-\infty, -3] \cup [7, \infty)\}$$ //

2/4/20

CH-4

Theory of equations :-

$1 \times 2 = 2m$

$1 \times 7 = 7m$

$9m$

⇒ **polynomial in 'x' of degree 'n'**:- If 'n' is a non-negative integer and $a_0, a_1, a_2 \ldots a_n$ are real or complex no.s and a_0 is not equal to zero, then an expression

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots a_n$$

is called polynomial in 'x' of degree 'n'. where $a_0, a_1, a_2 \ldots a_n$ are called the co-efficients of the polynomial $f(x)$,

where a_0 is called leading co-efficient of $f(x)$ and a_n is called constant term.

⇒ **Note**:-

→ A polynomial with leading co-efficient '1' (i.e $a_0 = 1$) is called monic polynomial.

⇒ **An algebraic equation of degree 'n'**:- If $f(x)$ is a polynomial of degree $n \geq 0$, then the equation $f(x) = 0$

i.e, $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n = 0$

is called a polynomial equation of degree 'n'. (or) an Algebraic equation of degree 'n'.

⇒ **Note**:-

⇒ (i) A real (or) complex no

of polynomial f(x) (o

$f(x) = 0$ iff $f(\alpha) = 0$

⇒ (ii) **Remainder Theorem**:

of degree 'n' ≥ 0 Let

a polynomial q(x) o

$f(x) = (x-a)$ q

(iii) Let f(x) be a polyn

Let $a \in C$. Then we

of f(x), if there

such that

$f(x) = (x-$

(iv) Let f(x) be a polyn

$(x-a)$ is a factor

(v) If $\alpha_1, \alpha_2, \alpha_3 \ldots$ α

required polynomial

$f(x) = a(x-\alpha_1)(x-\alpha$

where a is

(vi) Suppose 'n' is a po

& $b_0, b_1, b_2 \ldots b_n$

$a_0\alpha^n + a_1\alpha^{n-1} + \ldots$

for more than 'n'

Then $a_K = b_K$ for

(vii) If f(x) & g(x) a

for infinatly man

**

⇒ Note:-

⇒ (i) A real (or) complex no. 'α' is set to be at zero of polynomial $f(x)$ (or) root of the equation $f(x)=0$ iff $f(\alpha)=0$.

⇒ (ii) Remainder Theorem:- Let $f(x)$ be a polynomial of degree 'n' ≥ 0 Let $a \in C$. Then there exist a polynomial $q(x)$ of degree $(n-1)$ such that

$$f(x) = (x-a)\,q(x) + f(a)$$

(iii) Let $f(x)$ be a polynomial of degree $n > 0$. Let $a \in C$. Then we say that $(x-a)$ is a factor of $f(x)$, if there exist a polynomial $q(x)$ such that

$$f(x) = (x-a)\,q(x)$$

(iv) Let $f(x)$ be a polynomial of degree $n > 0$. then $(x-a)$ is a factor of $f(x)=0$ iff $f(a)=0$. ←

** (v) If $\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n$ be the roots then the required polynomial expression is ~~$f(x) = (x-\alpha_1)$~~

$$f(x) = a(x-\alpha_1)(x-\alpha_2)(x-\alpha_3) \ldots (x-\alpha_n).$$

where a is leading coff. of $f(x)$.

(vi) Suppose 'n' is a positive integer $a_0, a_1, a_2 \ldots a_n$ & $b_0, b_1, b_2 \ldots b_n$ are complex no.s such that

$$a_0\lambda^n + a_1\lambda^{n-1} + \ldots + a_n = b_0\lambda^n + b_1\lambda^{n-1} + \ldots + b_n.$$

for more than 'n' distinct elements $\lambda \in R$. Then $a_k = b_k$ for $0 \leq k \leq n$.

(vii) If $f(x)$ & $g(x)$ are polynomial such that $f(\alpha) = g(\alpha)$ for infinatly many no. 'α'. then $f(x) = g(x)$.

⇒ Relation b/w the roots & the cofficient of n^{th} degree polynomial equation :-

Consider the n^{th} degree polynomial equation

$$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \cdots + P_{n-1} x + P_n = 0$$

let $\alpha_1, \alpha_2 \cdots \alpha_n$ be its roots

Then

$$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \cdots + P_{n-1} x + P_n = (x-\alpha_1)\cdot(x-\alpha_2)\cdots\cdots(x-\alpha_n)$$

$$= x^n - (\alpha_1 + \alpha_2 + \cdots \cdot \alpha_n)x^{n-1} + (\alpha_1\alpha_2 + \alpha_2\alpha_3 + \cdots\cdots \alpha_{n-1}\alpha_n)x^{n-2} + \cdots + (-1)^n(\alpha_1, \alpha_2, \alpha_3 \cdots \alpha$$

Now comparing co-fficients of both sides we get

$$\to S_1 = \frac{-P_1}{P_0} = \sum_{i=1}^{n} \alpha_i$$ (sum of roots) $(\alpha_1 + \alpha_2 + \cdots + \alpha_n)$

$$\to S_2 = \frac{+P_2}{P_0} \sum_{1 \le i < j \le n} \alpha_i \alpha_j$$ (sum of the product of the roots taken two at a time) $(\alpha_1\alpha_2 + \alpha_2\alpha_3 + \cdots\cdots)$

$$\to S_3 = \frac{-P_3}{P_0} = \sum_{1 \le i \le j < k \le n} \alpha_i \alpha_j \alpha_k$$ (sum of product of roots taken three at a time) $(\alpha_1\alpha_2\alpha_3 + \alpha_2\alpha_3\alpha_4 + \cdots)$

$$\vdots \qquad \vdots$$

$$\to S_n = \frac{(-1)^n P_n}{P_0} = \alpha_1 \alpha_2 \cdots \alpha_n$$ (product of roots)

⇒ **Quadratic equation**:- If α, β are roots of quadratic equation $ax^2+bx+c=0$ then

(i) sum of roots ⇒ $\alpha+\beta = -b/a$

(ii) product of roots ⇒ $\alpha\beta = c/a$

⇒ **cubic equation**:- If α, β, γ are the roots of cubic equation $P_0x^3+P_1x^2+P_2x+P_3=0$, then

$S_1 \Rightarrow \alpha+\beta+\gamma = \frac{-P_1}{P_0}$

$S_2 \Rightarrow \alpha\beta+\beta\gamma+\dots = \frac{P_2}{P_0}$

$S_3 \Rightarrow \alpha\beta\gamma = \frac{-P_3}{P_0}$

⇒ **Bi-quadratic equation**:- If $\alpha, \beta, \gamma, \Delta$ are the roots of bi-quadrodic equ $P_0x^4+P_1x^3+P_2x^2+P_3x+P_4=0$, then

$S_1 \Rightarrow \alpha+\beta+\gamma+\Delta = \frac{-P_1}{P_0}$

$S_2 \Rightarrow \alpha\beta+\beta\gamma+\gamma\Delta+\dots = \frac{P_2}{P_0}$

$S_3 \Rightarrow \alpha\beta\gamma+\beta\gamma\Delta+\dots = \frac{-P_3}{P_0}$

$S_4 \Rightarrow \alpha\beta\gamma\Delta = \frac{P_4}{P_0}$ //

⇒ Formula:-

1) $\Sigma \alpha^2\beta^2 = \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$
$= S_2^2 - 2S_1S_3$

2) $\Sigma \alpha\beta(\alpha+\beta) = \alpha^2\beta + \beta^2\gamma + \gamma^2\alpha + \alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2$
$= S_1S_2 - 3S_3$

3) $\Sigma \frac{1}{\alpha^2\beta^2} = \frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$

$= \frac{S_1^2 - 2S_2}{S_3^2}$

4) $\Sigma \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{\alpha^2+\beta^2}{\alpha\beta} + \frac{\beta^2+\gamma^2}{\beta\gamma} + \frac{\alpha^2+\gamma^2}{\alpha\gamma}$
$= \frac{S_1S_2 - 3S_3}{S_3}$

Ex:- 4(a)

I.

1) Form a polynomial equations of the lowest degree, with the roots are as given below.

(i) 1, -1, 3

Sol: Let $\alpha = 1$, $\beta = -1$, $\gamma = 3$

If α, β, γ are the roots then required polynomial eq is

$\Rightarrow (x-\alpha)(x-\beta)(x-\gamma) = 0$

$\Rightarrow (x-1)(x+1)(x-3) = 0$

$\Rightarrow (x^2-1)(x-3) = 0$

$\Rightarrow x^3 - 3x^2 - x + 3 = 0$ //

(iii) $2 \pm \sqrt{3}$, $1 \pm 2i$

Sol: Let

$\alpha = 2+\sqrt{3}$, $\beta = 2-\sqrt{3}$, $\gamma = 1+2i$

$\delta = 1-2i$

If $\alpha, \beta, \gamma, \delta$ are the roots then required polynomial eq is

$\Rightarrow (x-\alpha)(x-\beta)(x-\gamma)(x-\delta)=0$

$\Rightarrow (x-2-\sqrt{3})(x-2+\sqrt{3})[x-1-2i][x-1+2i]=0$

$\Rightarrow [(x-2)^2-(\sqrt{3})^2][(x-1)^2-(2i)^2]=0$

$\Rightarrow [x^2+4 \mp 2x-3][x^2+1-2x-4i^2]=0$

$\Rightarrow$

(iv) 0, 0, 2, 2, −2, −2

Sol- Let

$\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 2, \alpha_4 = 2, \alpha_5 = -2, \alpha_6 = -2$

∴ The required polynomial equ is

$\Rightarrow (x-\alpha_1)(x-\alpha_2)(x-\alpha_3) \cdots (x-\alpha_6)=0$

$\Rightarrow$ ~~(x)~~ $(x-0)(x-0)(x-2)(x-2)(x+2)(x+2)=0$

$\Rightarrow (x)(x)[x+2)(x-2)][(x+2)(x-2)]=0$

$\Rightarrow x^2[x^2-4][x^2-4]=0$

$\Rightarrow x^2[x^2-4]^2=0$

$\Rightarrow x^2[x^4+16-8x^2]=0$

$\Rightarrow x^2(x^4+16-8x^2)=0$

$\Rightarrow x^6+16x^2-8x^4=0$ //

2) If α, β, γ are the roots of $4x^3-6x^2+7x+3=0$, then find the value of.
$\alpha\beta+\beta\gamma+\gamma\alpha$.

Sol G.T
$4x^3-6x^2-7x+3=0$
It is in the form of $P_0x^3+P_1x^2+P_2x+P_3=0$
$P_0=4$, $P_1=-6$, $P_2=7$, $P_3=3$
where $\alpha\beta+\beta\gamma+\gamma\alpha=S_2$
$\therefore \alpha\beta+\beta\gamma+\gamma\alpha = \frac{P_2}{P_0} = \frac{7}{4}$ //

3) If 1, 1, α are the roots of $x^3-6x^2+9x-4=0$, find α.

Sol: let
$\alpha_1=1$, $\alpha_2=1$, $\alpha_3=\alpha$
G.T
$\Rightarrow x^3-6x^2+9x-4=0$
$\Rightarrow P_0x^3+P_1x^2+P_2x+P_3=0$
$\Rightarrow S_3 \Rightarrow \alpha_1\cdot\alpha_2\cdot\alpha_3 = \frac{-P_3}{P_0}$
$\Rightarrow (1)(1)(\alpha) = \frac{-(-4)}{1}$
$\Rightarrow \alpha = 4$ //

4) $-1, 2, \alpha$
$2x^3+x^2-7x+6=0$

5) If $1, -2$ & 3 are
then find 'a'.

Sol: Let
$\alpha=1$, $\beta=-2$, $\gamma=$

G.T
$\Rightarrow x^3-2x^2+ax+6=$
It is in the form
$\Rightarrow P_0x^3+P_1x^2+P_2x+$
$S_2 \Rightarrow \alpha\beta+\beta\gamma+\gamma\alpha$

$\Rightarrow -2-6+3 = \frac{a}{1}$
$\Rightarrow -8+3 = a$
$\Rightarrow a = -5$ //

6) If the product
is 9, then fin

Sol:- G.T
$\Rightarrow 4x^3+16x^2-9x-$
Let α, β, γ be
$S_3 \Rightarrow \alpha\beta\gamma$
$\Rightarrow S_3 = \frac{-P_3}{P_0}$
$\Rightarrow 9 = \frac{-(-a)}{4}$
$\Rightarrow a = 36$ //

4) $-1, 2, \alpha$

$2x^3+x^2-7x+6=0$

5) If $1, -2$ & 3 are the roots of $x^3-2x^2+ax+6=0$, then find 'a'.

Sol: Let

$\alpha=1, \beta=-2, \gamma=3$

G.T

$\Rightarrow x^3-2x^2+ax+6=0$

It is in the form of

$\Rightarrow P_0x^3+P_1x^2+P_2x+P_3=0$

$S_2 \Rightarrow \alpha\beta+\beta\gamma+\gamma\alpha=\frac{P_2}{P_0}$

$\Rightarrow -2-6+3=\frac{a}{1}$

$\Rightarrow -8+3=a$

$\Rightarrow a=-5$ //

6) If the product of roots of $4x^3+16x^2-9x-a=0$ is 9, then find 'a'.

Sol:- G.T

$\Rightarrow 4x^3+16x^2-9x-a=0$

Let α, β, γ be the req roots.

$S_3 \Rightarrow \alpha\beta\gamma=9$

where

$P_0=4$

$P_1=16$

$P_2=-9$

$P_3=-a$

$\Rightarrow S_3=\frac{-P_3}{P_0}$

$\Rightarrow 9=\frac{-(-a)}{4}$

$\Rightarrow a=36$ //

7) Find S_1, S_2, S_3 & S_4 for each of the foll

equations. G.T

i) $x^4 - 16x^3 + 86x^2 - 176x + 105 = 0$

Sol:- It is in the form of

$\Rightarrow P_0x^4 + P_1x^3 + P_2x^2 + P_3x + P_4 = 0$

where $P_0 = 1$, $P_1 = -16$, $P_2 = 86$, $P_3 = -176$, $P_4 = 10$

$$S_1 = \frac{-P_1}{P_0} = \frac{-(-16)}{1} = 16$$

$$S_2 = \frac{P_2}{P_0} = \frac{86}{1} = 86$$

$$S_3 = \frac{-P_3}{P_0} = \frac{-(-176)}{1} = 176$$

$$S_4 = \frac{P_4}{P_0} = \frac{105}{1} = 105 \,//$$

II.

1) If α, β and γ are the roots of equation $x^3 - 2x^2 - 5x + 6 = 0$, then find α and β.

Sol:- G.T

α, β and γ are the roots of the equation $x^3 - 2x^2 - 5x + 6 = 0$, which is in the form of $P_0x^3 + P_1x^2 + P_2x + P_3 = 0$, where

$P_0 = 1$, $P_1 = -2$, $P_2 = -5$, $P_3 = 6$

$$\therefore S_1 = \alpha + \beta + 1 = \frac{-P_1}{P_0}$$

$$\Rightarrow \alpha + \beta + 1 = \frac{-(-2)}{1} = \alpha + \beta + 1 = 2$$

$$\alpha + \beta = 2 - 1, \quad \alpha + \beta = 1$$

$$S_3 = \alpha\beta\gamma = \frac{-P_3}{P_0}$$

$= \alpha\beta(1) = \frac{-(6)}{1}$

$\alpha\beta = -6$

W.K.T $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= (1)^2 - 4(-6)$

$= 1+24 = 25$

$= (\alpha-\beta)^2 = 25$

$= \alpha-\beta = \pm 5$

1) $\alpha+\beta = 1$

$\alpha-\beta = 5$

$2\alpha = 6$

$\alpha = 3$

$3+\beta = 1$

$\beta = 1-3$

$\beta = -2$

2) $\alpha+\beta = 1$

$\alpha-\beta = -5$

$2\alpha = -4$

$\alpha = -2$

$-2+\beta = 1$

$\beta = 1+2$

$\beta = 3$

$\alpha = 3$ (or) -2 //

2) If α, β and γ are the roots of equation $x^3-2x^2+3x-4=0$, then find

(i) $\Sigma\alpha^2\beta^2$ (ii) $\Sigma\alpha\beta(\alpha+\beta)$

sol:- G.T

α, β, γ are the roots of eq $x^3-2x^2+3x-4=0$

which is in form of $P_0x^3+P_1x^2+P_2x+P_3=0$

where $P_0 = 1$ $P_1 = 2$, $P_2 = 3$, $P_3 = -4$

$s_1 = \alpha+\beta+\gamma = \frac{-P_1}{P_0} = \frac{-(-2)}{1} = 2$

$s_2 = \alpha\beta+\beta\gamma+\gamma\alpha = \frac{P_2}{P_0} = 3/1 = 3$

$s_3 = \alpha\beta\gamma = \frac{-P_3}{P_0} = \frac{-(-4)}{1} = 4$

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(i) $\Sigma\alpha^2\beta^2$

$\Sigma\alpha^2\beta^2 = \alpha^2\beta^2 + \beta^2\alpha^2 + \gamma^2\alpha^2 = S_2^2 - 2S_1S_3$

$= (3)^2 - 2(2)(4)$

$= 9 - 16 = -7$

(ii) $\Sigma\alpha\beta(\alpha+\beta) = \alpha^2\beta + \beta^2\gamma + \gamma^2\alpha + \alpha\beta^2 + \beta\gamma^2 + \alpha^2\gamma$

$= S_1S_2 - 3S_3$

$= 2(3) - 3(4) = 6 - 12 = -6$ //

III.

1) If α, β, γ are the roots of eq $x^3 - 6x^2 + 11x - 6 = 0$ then find the equ whose roots are

(i) $\alpha^2+\beta^2$ (ii) $\beta^2+\gamma^2$ (iii) $\gamma^2+\alpha^2$

Sol:- G.T

If α, β, γ are the roots of the equ

$x^3 - 6x^2 + 11x - 6 = 0$

Let $f(x) = x^3 - 6x^2 + 11x - 6$, put $x = 1$

$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 12 - 12$

$= 0$

$\therefore x = 1$ is a factor of $f(x)$ then by synthetic division, WKT

$$\begin{array}{c|cccc} x=1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$\therefore f(x) = (x-1)(x^2 - 5x + 6)$

$f(x) = 0$

$(x-1) = 0$ | $x^2 - 5x + 6 = 0$

$x = 1$ | $x^2 - 2x - 3x + 6 = 0$

$x(x-2) - 3(x-2) = 0$

$(x-2) = 0$ $(x-3) = 0$

$x = 2$ $x = 3$

$\alpha = 1, \beta = 2, \gamma = 3$

Let $A = \alpha^2 + \beta^2 = (1)^2 + (2)$

$B = \beta^2 + \gamma^2 = (2)^2 + (3)^2$

$C = \gamma^2 + \alpha^2 = (3)^2 + (1)$

$\therefore$ The required poly e

$\Rightarrow (x-A)(x-B)(x$

$\Rightarrow (x-5)(x-13)($

$\Rightarrow (x^2 - 13x - 5x + 6$

$\Rightarrow x^3 - 13x^2 - 5x^2 +$

$\Rightarrow x^3 - 28x^2 + 245x$

III.

2) If α, β, γ are th

then find the equ

$(\gamma - \alpha)^2$.

Sol:- G.T

α, β, γ are the roo

Let $f(x) = x^3 - 7x +$

put $x = 1$, we get

$f(1) = 1 - 7 + 6 = 7$

$\therefore x = 1$ is a facto

division, we get

$$\begin{array}{c|cc} x=1 & 1 & 0 \\ & 0 & 1 \\ \hline & 1 & 1 \end{array}$$

$f(x) = (x-1)(x^2$

$f(x) = 0$

$x - 1 = 0$ | $x^2 -$

$x = 1$ | $x^2 -$

$\alpha = 1,\ \beta = 2,\ \gamma = 3$

Let $A = \alpha^2 + \beta^2 = (1)^2 + (2)^2 = 1 + 4 = 5$

$B = \beta^2 + \gamma^2 = (2)^2 + (3)^2 = 4 + 9 = 13$

$C = \gamma^2 + \alpha^2 = (3)^2 + (1)^2 = 9 + 1 = 10$

∴ The required poly equ with roots

$\Rightarrow (x-A)(x-B)(x-C) = 0$

$\Rightarrow (x-5)(x-13)(x-10) = 0$

$\Rightarrow (x^2 - 13x - 5x + 65)(x-10) = 0$

$\Rightarrow x^3 - 13x^2 - 5x^2 + 65x - 10x^2 + 130x + 50x - 650 = 0$

$\Rightarrow x^3 - 28x^2 + 245x - 650 = 0$ //

III.

2) If α, β, γ are the roots of $x^3 - 7x + 6 = 0$, then find the equ whose roots are $(\alpha-\beta)^2, (\beta-\gamma)^2, (\gamma-\alpha)^2$.

Sol:- G.T

α, β, γ are the roots of equ $x^3 - 7x + 6 = 0$

Let $f(x) = x^3 - 7x + 6$

put $x = 1$, we get

$f(1) = 1 - 7 + 6 = 7 - 7 = 0$

∴ $x = 1$ is a factor of $f(x)$ then by synthetic division, we get

$$\begin{array}{c|cccc} x=1 & 1 & 0 & -7 & 6 \\ & 0 & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$f(x) = (x-1)(x^2 + x - 6)$

$f(x) = 0$

$x - 1 = 0 \quad | \quad x^2 + x - 6 = 0$

$x = 1 \quad | \quad x^2 + 3x - 2x - 6 = 0$

$\Rightarrow x(x+3)-2(x+2)=0$

$x+3=0 \quad x-2=0$

$x=-3 \quad x=2$

$\alpha=1,\ \beta=2,\ \gamma=-3$

put $A=(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta$

$=(1)^2+(2)^2-2(1)(2)$

$=1+4-4=1$

put $B=(\beta-\gamma)^2 \Rightarrow (2+3)^2=25$

put $C=(\gamma-\alpha)^2 \Rightarrow (-3-1)^2=16$

∴ The req poly is

$(x-1)(x-25)(x-16)=0 \Rightarrow (x^2-25x-x+25)(x-16)=0$

$\Rightarrow (x^3-26x+25)(x-16)=0$

$\Rightarrow x^3-26x^2+25x-16x^2+416x-400=0$

$= x^3-42x^2+441x-400=0$ //

3) If α,β,γ are the roots of equation $x^3-3ax+b=0$ then P.T $\Sigma(\alpha-\beta)(\alpha-\gamma)=9a$

Sol: G.T, α,β,γ are the roots of eq $x^3-3ax+b=0$ which is in form of $P_0x^3+P_1x^2+P_2x+P_3=0$

$P_0=1,\ P_1=0,\ P_2=-3a,\ P_3=b$

$S_1=\alpha+\beta+\gamma=\frac{-P_1}{P_0}=0,\quad S_2=\alpha\beta+\beta\gamma+\gamma\alpha=\frac{P_2}{P_0}=\frac{-3a}{1}$

$S_3=\alpha\beta\gamma=\frac{-P_3}{P_0}=-b/1=b$

$\Rightarrow \Sigma(\alpha-\beta)(\alpha-\gamma)$

$\Rightarrow (\alpha-\beta)(\alpha-\gamma)+(\beta-\gamma)(\beta-\gamma)+(\gamma-\alpha)(\gamma-\beta)$

$\Rightarrow 2[(\alpha^2+\beta^2+\gamma^2)+(\alpha\beta+\beta\gamma+\gamma\alpha)]$

$\Rightarrow 2[(\alpha^2+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)]+(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\Rightarrow 2(\alpha+\beta+\gamma)^2-4(\alpha\beta+\beta\gamma+\gamma\alpha)+(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\Rightarrow 2(\alpha+\beta+\gamma)^2-3(\alpha\beta+\beta\gamma+\gamma\alpha)$

$2(0)^2-3(-3a)$

$9a$ // R.H.S //

$\Rightarrow$ **Sythetic Division:-**

This method consists o

(i) finding the coefficient an $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\dots+$ by $(x-a)$

(ii) Finding the coefficient and $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\dots$ by (x^2-px-q)

i) Finding the quotient & t $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\dots$ by $(x-a)$

Sol: Let $x-a=0 \Rightarrow x=a$

$x=a$	a_0	a_1
	0	ab_0
	$a_0=b_0$	b_1

∴ The Values $b_0, b_1,$

quotients 'R' is rem

Eg:- Find the quotient

$x^4-6x^3+3x^2+26x-2$

Sol:- Let $x-4=0$

$x=4$	1	-6
	0	4
	1	-2

⇒ sythetic Division:-

This method consists of two parts. They are

(i) finding the coefficient and the remainder, when $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\dots+a_n\ (a_0\neq0)$ is divided by $(x-a)$

(ii) finding the coefficient and remainder, when $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\dots+a_n\ (a_0\neq0)$ is divided by (x^2-px-q)

i) finding the quotient & the remainder, when $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\dots+a_n\ (a_0\neq0)$ is divided by $(x-a)$

Sol: Let $x-a=0 \Rightarrow x=a$

$$\begin{array}{c|cccccc}
x=a & a_0 & a_1 & a_2 & \dots & a_{n-1} & a_n \\
 & 0 & ab_0 & ab_1 & \dots & ab_{n-2} & a\cdot b_{n-1} \\
\hline
 & a_0=b_0 & b_1 & b_2 & & b_{n-1} & R
\end{array}$$

∴ The values $b_0, b_1, b_2 \dots b_{n-1}$ coff's of quotients 'R' is remainder.

Eg:- Find the quotient & remainder, when $x^4-6x^3+3x^2+26x-24$ is divided by $x-4$.

Sol:- Let $x-4=0$

$$\begin{array}{c|ccccc}
x=4 & 1 & -6 & 3 & 26 & -24 \\
 & 0 & 4 & -8 & -20 & 24 \\
\hline
 & 1 & -2 & -5 & 6 & 0
\end{array}$$

$Q(x) = x^3 - 2x^2 - 5x + 6$

$R(x) = 0$

Eg:- Find the quotient & remainder, when $3x^4 - x^3 + 2x^2 - 2x - 4$ is divided by $x+2$.

Sol:- Let $x+2=0$

$x = -2$

$$\begin{array}{c|ccccc} x=-2 & 3 & -1 & 2 & -2 & -4 \\ & 0 & -6 & 14 & -32 & 68 \\ \hline & 3 & -7 & 16 & -34 & 64 \end{array}$$

$Q(x) = 3x^3 - 7x^2 + 16x - 34$

$R(x) = 64$ //

Eg:- Find the quotient & remainder, when $a_0x^n + a_1x^{n-1} + \ldots + a_n$ is divided by $x^2 - px - q$.

Sol:-

$$\begin{array}{c|cccccccc} & a_0 & a_1 & a_2 & a_3 & \cdots & a_{n-2} & a_{n-1} & a_n \\ p & 0 & pb_0 & pb_1 & pb_2 & & pb_{n-3} & pb_{n-2} & 0 \\ q & 0 & 0 & qb_1 & qb_1 & & qb_{n-4} & qb_{n-3} & qb_{n-1} \\ \hline & a_0=b_0 & b_1 & b_2 & b_3 & & b_{n-2} & R_1 & R_2 \end{array}$$

$Q(x) = b_0x^{n-2} + b_1x^{n-3} + \ldots b_{n-2}$

$R(x) = R_1x + R_2$.

g:- Find the quotient & remainder, when $2x^5-3x^4+5x^2-3x^2+7x-9$ is divided by x^2-x-3.

Sol:- G.T

$x^2-x-3 \Rightarrow x^2-px-q$

	2	-3	5	-3	7	-9
$p=1$	0	2	-1	10	4	0
$q=3$	0	0	6	-3	30	12
	2	-1	10	4	41	3

$Q(x) = 2x^3-x^2+10x+4$

$R(x) = 41x+3$ //

Eg:- Find the quotient & remainder, when $x^4-11x^3+44x^2-76x+48$ is divided by $x^2-7x+12$

Sol:- G.T

$x^2-7x+12 \Rightarrow x^2-px-q$

	1	-11	44	-76	48
$p=7$	0	7	-28	28	0
$q=-12$	0	0	-12	48	-48

⇒ **Trial and Error method**:- To find a root of $f(x)=0$, we have to find out the value of 'x' for which $f(x)=0$.

Eg:- $x^2-7x+6=0$

sol:- $f(x)=x^2-7x+6$

put $x=1 \Rightarrow f(1)=1-7+6$
$= 0$

⇒ **Multiple roots (or) repeated roots**:-

Let $f(x)$ degree $n>0$. Let $\alpha_1, \alpha_2 \ldots \alpha_n$ be the roots of $f(x)=0$, so ~~the~~ that $f(x)=a_0(x-\alpha_1)(x-\alpha_2)\ldots(x-\alpha_n)$.

A complex no. 'α' is said to be a roots of $f(x)=0$ multiple by 'm' if $\alpha=\alpha_k$ for exactly 'm' values of k among $1,2,\ldots n$.

Roots of multiplicity by $m>1$ are called multiple roots (or) repeated roots. ||-

Note:- Roots of multiplicity 1 ($m=1$) are called simple roots.

⇒ **Procedure to find multiple roots**:-

Let $f(x)$ be a polynomial. first we have to find $f'(x)$ and by using trial and error method substitute any value both in $f(x)$ and $f'(x)$, then that root is called multiple root.

$f(x)=x^3-3x^2+4x+1$ $x=-1$
$f'(x)=3x^2-6x+4$ $f(-1)=0$ $x=-1$
$f(-1)=0$

Ex:- 4(b)

I.
1) solve x^3-3x^2-16x+
two roots is zero.

sol:- Let α, β, γ be the
x^3-3x^2-16x
which is in the
$P_0x^3+P_1x^2+P_2$

G.T $\alpha+\beta=0$
$S_1 \Rightarrow \alpha+\beta+\gamma=$

$\Rightarrow 0+\gamma =$
$\gamma = 3$

Let
$x=\gamma=3$
$f(x)=x^3-3x^2-$
$=(3)^3-3(3$
$=27-27$
$=0$
$\therefore x=\gamma=3$ is

synthetic met

$x=\gamma=3$ 1
0
1

$f(x)=(x-3$
Let $f(x)=0$

Ex:- 4(b)

I. 1) solve $x^3-3x^2-16x+48=0$, G.T the sum of two roots is zero.

Sol:- Let α, β, γ be the roots of given equ

$$x^3-3x^2-16x+48=0 \quad ①$$

which is in the form of

$$P_0x^3+P_1x^2+P_2x+P_3=0$$

G.T $\alpha+\beta=0$

$$S_1 \Rightarrow \alpha+\beta+\gamma = \frac{-P_1}{P_0}$$

$$\Rightarrow 0+\gamma = 3/1$$

$$\gamma = 3$$

Let

$x=\gamma=3$

$$f(x) = x^3-3x^2-16x+48$$

$$= (3)^3-3(3)^2-16(3)+48$$

$$= 27-27-48+48$$

$$= 0$$

$\therefore$ $x=\gamma=3$ is root of $f(x)$ then by synthetic method

$$\begin{array}{c|cccc} x=\gamma=3 & 1 & -3 & -16 & 48 \\ & 0 & 3 & 0 & -48 \\ \hline & 1 & 0 & -16 & 0 \end{array}$$

$$f(x) = (x-3)(x^2-16)$$

Let $f(x)=0 \Rightarrow x-3=0$

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$x = 3$

$\Rightarrow x^2 - 16 = 0$

$x = \pm 4$

$\therefore$ the roots of given eq are $x = -4, 4 \& 3$ //

3) G.T the roots of $x^3 + 3px^2 + 3qx + r = 0$ are in

(i) A.P, S.T $2p^3 - 3pq + r = 0$

(ii) G.P, S.T $p^3 r = q^3$

(iii) H.P, S.T $2q^3 = r(3pq - r)$

Sol:- G.T

$x^3 + 3px^2 + 3qx + r = 0$ — ①

It is in the form of

$P_0 x^3 + P_1 x^2 + P_2 x + P_2 = 0$

where $P_0 = 1$, $P_1 = 3p$, $P_2 = 3q$, $P_3 = r$

A.P

Let $a-d, a, a+d$ be the roots in A.P of eq① then

$S_1 = a - \cancel{d} + a + a + \cancel{d} = \frac{-P_1}{P_0}$

$S_1 = 3a = -3p$

$3a = -3p \quad \boxed{a = -p}$

Sub $x = a \Rightarrow -p$ in eq①

$\Rightarrow (-p)^3 + 3p(-p)^2 + 3q(-p) + r = 0$

$\Rightarrow -p^3 + 3p^3 - 3pq + r = 0$

$\Rightarrow 2p^3 - 3pq + r = 0$ //

Let $\frac{a}{r}, a, ar$ be the roots in G.P of eq① then $S_3 = \frac{a}{r} \times a \times ar = \frac{-P_3}{P_0} = \frac{-r}{1}$

$\Rightarrow$

sub $x - a$

$\Rightarrow [(-r)$

$\Rightarrow -\cancel{p} +$

$\Rightarrow \cancel{3}pr^2$

combing

$\Rightarrow (p$

$\Rightarrow$

$\Rightarrow$

H.P $x^3 + 3p$

put $x =$

$\Rightarrow \left(\frac{1}{y}\right)$

$\Rightarrow \frac{1}{y^3}$

$\Rightarrow$

$\Rightarrow$

It is

$P_0 = r_1$

Let $a-$

then S_1

sub

$\Rightarrow a^3 = -r$

$a = (-r)^{1/3}$

sub $x - a = (-r)^{1/3}$ in eq ①, we get

$\Rightarrow [(-r)^{1/3}]^3 + 3p(-r^{1/3})^2 + 3q(-r)^{1/3} + r = 0$

$\Rightarrow \cancel{-r} + 3p \cdot r^{2/3} - 3q r^{1/3} \cancel{+r} = 0$

$\Rightarrow \cancel{3}pr^{2/3} = \cancel{3}qr^{1/3}$

c~~om~~ubing on both sides

$\Rightarrow (pr^{2/\cancel{3}})^{\cancel{3}} = (qr^{1/\cancel{3}})^{\cancel{3}}$

$\Rightarrow p^3 r^2 = q^3 r$

$\Rightarrow p^3 r = q^3$ //

H.P $x^3 + 3px^2 + 3qx + r = 0$ —— ①

put $x = 1/y$ in equ ①, we get $HP = \frac{1}{AP}$

$\Rightarrow \left(\frac{1}{y}\right)^3 + 3p\left(\frac{1}{y}\right)^2 + 3q\left(\frac{1}{y}\right) + r = 0$

$\Rightarrow \frac{1}{y^3} + \frac{3p}{y^2} + \frac{3q}{y} + r = 0$

$\Rightarrow \frac{1 + 3py + 3qy^2 + ry^3}{y^3} = 0$

$\Rightarrow \cancel{y^3}\; ry^3 + 3qy^2 + 3py + 1 = 0$ (A.P)

It is in the form of $P_0y^3 + P_1y^2 + P_2y + P_3 = 0$

$P_0 = r$, $P_1 = 3q$, $P_2 = 3p$, $P_3 = 1$

Let $a-d, a, a+d$ be the roots in A.P of equ ②

then $S_1 \Rightarrow a - d + a + a + d = -\frac{P_1}{P_0}$

$\Rightarrow \cancel{3}a = \frac{-\cancel{3}q}{r} \Rightarrow a = \frac{-q}{r}$

sub $y = a = -\frac{q}{r}$ in eq ②, we get

$\Rightarrow r\left(\frac{-q}{r}\right)^3 + 3q\left(\frac{-q}{r}\right)^2 + 3p\left(\frac{-q}{r}\right) + 1 = 0$

$\Rightarrow \frac{-q^3 r}{r^3} + \frac{3q^3}{r^2} - \frac{3pq}{r} + 1 = 0$

$\Rightarrow \frac{2q^3}{r^2} - \frac{3pq}{r} + 1 = 0$

$\Rightarrow \frac{2q^3 - 3pqr + r^2}{r^2} = 0$

$\Rightarrow 2q^3 = 3pqr - r^2$

$\Rightarrow 2q^3 = r(3pq - r)$ //

11. solve $9x^3 - 15x^2 - 7x - 1 = 0$, given that two of its roots are equal.

Sol:- Let α, β, γ be the roots of given eq which is in form

$9x^3 - 15x^2 + 7x - 1 = 0$ —— ①

of $P_0x^3 + P_1x^2 + P_2x + P_3 = 0$

$S_1 = \alpha + \beta + \gamma = \frac{-P_1}{P_0} = \frac{15}{9} = \frac{5}{3}$

$S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{P_2}{P_0} = \frac{7}{9}$

$S_3 = \alpha\beta\gamma = \frac{-P_3}{P_0} = 1/9$

G.T

$\alpha = \beta$

$S_1 = \alpha + \alpha + \gamma = 5/3$

$= 2\alpha + \gamma = 5/3$

$\Rightarrow \gamma = 5/3 - 2\alpha$

$S_2 \Rightarrow \alpha\cdot\alpha +$

$\Rightarrow \alpha^2 + 2\alpha$

$\Rightarrow \alpha^2 + 2$

$\Rightarrow \alpha^2 +$

$\Rightarrow -3\alpha^2$

$\Rightarrow -9\alpha^2$

$\Rightarrow -2$

$\Rightarrow -2$

$\Rightarrow 2$

$\Rightarrow 27$

$\Rightarrow 9\alpha$

$\Rightarrow (9$

$\Rightarrow (i)(3$

G.T

β

γ

γ

2) G.T One o

is double

equation.

Sol:- Let α, β, γ

$2x^3 + 3x^2$

$P_0x^3 + P_1x^2 +$

$S_2 \Rightarrow \alpha\cdot\alpha + \alpha\cdot\gamma + \gamma\cdot\alpha = 7/9$

$\Rightarrow \alpha^2 + 2\alpha\gamma = 7/9$

$\Rightarrow \alpha^2 + 2\alpha(5/3 - 2\alpha) = 7/9$

$\Rightarrow \alpha^2 + \frac{10\alpha}{3} - 4\alpha^2 = 7/9$

$\Rightarrow -3\alpha^2 + \frac{10\alpha}{3} = \frac{7}{9}$

$\Rightarrow \frac{-9\alpha^2 + 10\alpha}{\cancel{3}} = 7/\cancel{9}\,3$

$\Rightarrow -27\alpha^2 + 30\alpha = 7$

$\Rightarrow -27\alpha + 30\alpha - 7 = 0$ //

$\Rightarrow 27\alpha^2 - 30\alpha + 7 = 0$

$\Rightarrow 27\alpha^2 - 9\alpha - 21\alpha + 7 = 0$

$\Rightarrow 9\alpha(3\alpha - 1) - 7(3\alpha - 1) = 0$

$\Rightarrow (9\alpha - 7)(3\alpha - 1) = 0$

$\Rightarrow$ (i) $(3\alpha - 1) = 0$ (ii) $9\alpha - 7 = 0$

$\alpha = 1/3$ $\alpha = 7/9$

G.T $\alpha = \beta$ does not exist

$\beta = 1/3$ The roots are

$\gamma = 5/3 - 2\alpha$ $1/3, 1/3, 1$

$\gamma = 5/3 - 2/3 = 3/3 = 1$

$\gamma = 1$

2) G.T One of the root of $2x^3 + 3x^2 - 8x + 3 = 0$ is double the other root, find the roots of the equation.

Sol:- Let α, β, γ be the roots of the eqn $2x^3 + 3x^2 - 8x + 3 = 0$, which is in the form of

$P_0x^3 + P_1x^2 + P_2x + P_3 = 0$

$S_1 \Rightarrow \alpha+\beta+\gamma = -\frac{P_1}{P_0} = -3/2$

$S_2 \Rightarrow \alpha\beta+\beta\gamma+\gamma\alpha = \frac{P_2}{P_0} = -8/2 = -4$

$S_3 \Rightarrow \alpha\beta\gamma = -\frac{P_3}{P_0} = -3/2$

$S_1 \Rightarrow 2\beta+\beta+\gamma = -3/2$

$\Rightarrow \gamma = -3/2 - 3\beta$

$S_2 \Rightarrow (2\beta)\beta + \beta\gamma + \gamma(2\beta) = -4$

$\Rightarrow 2\beta^2 + 3\beta\gamma = -4$

$\Rightarrow 2\beta^2 + 3\beta(-3/2 - 3\beta) = -4$

$\Rightarrow 2\beta^2 - \frac{9\beta}{2} - 9\beta^2 = -4$

$\Rightarrow -7\beta^2 - \frac{9\beta}{2} = -4$

$\Rightarrow \frac{-14\beta^2 - 9\beta}{2} = -4$

$\Rightarrow 14\beta^2 + 9\beta - 8 = 0$

$\Rightarrow 14\beta^2 + 16\beta - 7\beta - 8 = 0$

$\Rightarrow 2\beta(7\beta+8) - 1(7\beta+8) = 0$

$\Rightarrow \beta = 1/2$ $\quad \beta = -8/7$ doesnot exist,

$\alpha = 2\beta$

$= 2 \times 1/2$

$= 1$

$\gamma = -3/2 - 3\beta$

$= -3/2 - 3/2$

$= -6/2 = -3$

$\therefore$ The roots of given eqn are $1, 1/2, -3$

3) solve $x^3 - 9x^2 + 14x + 2$
are in the ratio 3:2

Sol: Let
α, β, γ be the roo

$\Rightarrow x^3 - 9x^2 + 14x + 24 =$

$\Rightarrow P_0x^3 + P_1x^2 + P_2x +$

$S_1 = \alpha+\beta+\gamma = -P$

$S_2 = \alpha\beta + \beta\gamma + \gamma\alpha =$

$S_3 = \alpha\beta\gamma = \frac{-P_3}{P_0}$

G.T $\alpha/\beta = 3/2 \Rightarrow$

$S_1 = \alpha + \frac{2\alpha}{3} + \gamma =$

$\Rightarrow \frac{5\alpha}{3} + \gamma = 9$

$S_2 \Rightarrow \alpha\left(\frac{2\alpha}{3}\right) + ($

$\Rightarrow \frac{2\alpha^2}{3} + \frac{5\alpha}{3}$

$\Rightarrow \frac{2\alpha^2}{3} + \frac{145}{}$

$\Rightarrow \frac{6\alpha^2 + 135\alpha}{9}$

$\Rightarrow -19\alpha^2 +$

$\Rightarrow 19\alpha^2 -$

$\Rightarrow 19\alpha^2(\alpha$

$\Rightarrow (\alpha - 6)$

$\Rightarrow \alpha = 6$

$\beta = \frac{2\alpha}{3} = \frac{2(}{3}$

3) solve $x^3-9x^2+14x+24=0$, G.T two of its roots are in the ratio 3:2.

Sol: let α, β, γ be the roots of given equ.

$\Rightarrow x^3-9x^2+14x+24=0$

$\Rightarrow P_0x^3+P_1x^2+P_2x+P_3=0$

$S_1=\alpha+\beta+\gamma = \frac{-P_1}{P_0} = \frac{-(-9)}{1} = 9$

$S_2=\alpha\beta+\beta\gamma+\gamma\alpha = \frac{P_2}{P_0} = 14$

$S_3=\alpha\beta\gamma = \frac{-P_3}{P_0} = -24$

G.T $\alpha/\beta = 3/2 \Rightarrow \beta = \frac{2\alpha}{3}$

$S_1 = \alpha + \frac{2\alpha}{3} + \gamma = 9$

$\Rightarrow \frac{5\alpha}{3} + \gamma = 9 \Rightarrow \gamma = 9 - 5\alpha/3$

$S_2 \Rightarrow \alpha\left(\frac{2\alpha}{3}\right) + \left(\frac{2\alpha}{3}\right)\gamma + \gamma\alpha = 14$

$\Rightarrow \frac{2\alpha^2}{3} + \frac{5\alpha\gamma}{3} = 14 \Rightarrow \frac{2\alpha^2}{3} + \frac{5\alpha}{3}\left(9 - \frac{5\alpha}{3}\right) = 14$

$\Rightarrow \frac{2\alpha^2}{3} + \frac{145\alpha}{3} - \frac{25\alpha^2}{9} = 14$

$\Rightarrow \frac{6\alpha^2 + 135\alpha - 25\alpha^2}{9} = 14$

$\Rightarrow -19\alpha^2 + 135\alpha - 126 = 0$

$\Rightarrow 19\alpha^2 - 114\alpha - 21\alpha + 126 = 0$

$\Rightarrow 19\alpha(\alpha-6) - 21(\alpha-6) = 0$

$\Rightarrow (\alpha-6)(19\alpha-21) = 0$

$\Rightarrow \alpha = 6$ $\quad 19\alpha - 21 = 0$

$\beta = \frac{2\alpha}{3} = \frac{2(6)}{3}$ $\quad \alpha = 21/19$ doesn't exist

$\boxed{\beta = 4}$

$\gamma = \frac{9-5\beta}{3} \Rightarrow \frac{9-5(6)^2}{3}$

$\gamma = 9 - 10 = -1$

$\gamma = -1$

∴ The roots of given equ are 6, 4, -1.

4) solve the following eq, G.T four roots are in A.P.

(i) $8x^3 - 36x^2 - 18x + 81 = 0$

Sol:- Let $a-d, a, a+d$ be the roots of are in A.P of given equ $8x^3 - 36x^2 - 18x + 81 = 0$ —① which is in form of $P_0x^3 + P_1x^2 + P_2x + P_3 = 0$

$S_1 \Rightarrow a - d + a + a + d = \frac{-P_1}{P_0}$

$\Rightarrow 3a = \frac{36}{8}$

$\Rightarrow a = 3/2$

sub $x = a = 3/2$ in eq ① we get

$f(x) = 8x^3 - 36x^2 - 18x + 81$

$f(3/2) = 8(3/2)^3 - 36(3/2)^2 - 18(3/2) + 81$

$\Rightarrow \cancel{8}\left(\frac{27}{\cancel{8}}\right) - \overset{9}{\cancel{36}}\left(\frac{9}{\cancel{4}}\right) - 27 + 81$

$\Rightarrow 27 - 81 - 27 + 81 = 0$

∴ $x = a = 3/2$ is a factor of f(x) then by synthetic division, we get, $x = a = 3/2$ is a factor of f

$x = a = 3/2$	8	-36	-18	81
	0	12	-36	-81
	8	-24	-54	0

$f(x) = (x - 3/$

Let $f(x) = 0$

$x - 3/2 = 0$ $\quad x$

$\Rightarrow 8x^3 - 24x$

$\Rightarrow 2(4x^3 - 12$

$\Rightarrow 4x^3 - 16$

$\Rightarrow 2x(2x -$

$\Rightarrow (2x+3)$

$2x + 3 = 0$

$x = -3/2$

∴ The req

5) solve the fo in G.P

(ii) $3x^3 - 26x^2 -$

Sol: Let a/r, a

given eq $3x$

form of

$P_0x^3 + P_1x$

$S_3 = \frac{a}{r} \times a$

Let $f(x) =$

sub $x =$

$f(x) = 3($

$= 3($

$=$

$f(x) = (x - 3/2)(6x^2 - 24x - 54)$

Let $f(x) = 0$

$x - 3/2 = 0 \quad x = 3/2$

$\Rightarrow 6x^3 - 24x - 54 = 0$

$\Rightarrow 2(4x^3 - 12x - 27) = 0$

$\Rightarrow 4x^3 - 18x - 6x - 27 = 0$

$\Rightarrow 2x(2x - 9) + 3(2x - 9) = 0$

$\Rightarrow (2x + 3)(2x - 9) = 0$

$2x + 3 = 0 \qquad 2x - 9 = 0$

$x = -3/2 \qquad x = 9/2$

$\therefore$ The req roots are $3/2, -3/2, 9/2$

5) solve the following eq's G.T the roots are in G.P

(ii) $3x^3 - 26x^2 - 52x - 24 = 0$

Sol: Let $a/r, a, ar$ be the roots in G.P of $3x^3 - 26x^2 + 52x - 24 = 0$ which is in given eq $3x^3 - 26x^2$ form of

$P_0x^3 + P_1x^2 + P_2x + P_3 = 0$

$S_3 = \frac{a}{r} \times a \times ar = \frac{-P_3}{P_0} = 24/3$

$\alpha^3 = 8 = (2)^3$

$\alpha = 2$

Let $f(x) = 3x^3 - 26x^2 + 52x - 24$

sub $x = \alpha = 2$ in $f(x)$, we get

$f(x) = 3(2)^3 - 26(2)^2 + 52(2) \cancel{-24} - 24 = 0$

$= 3(8) - 26(4) + 124 - 24 = 0$

$= 24 - 104 + 104 - 24 = 0$

$= 0$

$x = a = 2$ is a factor of $f(x)$ by synthetic dilision, we get

$x = 2$	3	-26	52	-24
	0	6	-40	24
	3	-20	12	0

$f(x) = (x-2)(3x^2 - 20x + 12)$

Let $f(x) = 0$ $\quad x - 2 = 0 \quad x = 2$

$\Rightarrow 3x^2 - \cancel{18x}\ 20x + 12 = 0$

$\Rightarrow 3x^2 - 18x - 2x + 12 = 0$

$\Rightarrow 3x(x-6) - 2(x-6) = 0$

$\Rightarrow (x-6)(3x-2) = 0$

$x - 6 = 0 \qquad 3x - 2 = 0$

$x = 6 \qquad x = 2/3$

$\therefore$ The roots of given eq are $2, 2/3, 6$.

6) solve the following eq's given that roots are in H.P.

Sol:- (i) $6x^3 - 13x^2 + 6x - 1 = 0$

put $x = 1/y$ $\quad [H.P = 1/A.P]$

$\Rightarrow 6\left(\frac{1}{y}\right)^3 - 11\left(\frac{1}{y}\right)^2 + 6\left(\frac{1}{y}\right) - 1 = 0$

$\Rightarrow \frac{6}{y^3} - \frac{11}{y^2} + \frac{6}{y} - 1 = 0$

$\Rightarrow \frac{6 - 11y + 6y^2 - y^3}{y^2} = 0$

$\Rightarrow -y^3 + 6y^2 - 11y + 6 = 0$

$\Rightarrow y^3 - 6y^2 + 11y - 6 = 0$ which is in A.P

Let $a-d,\ a,\ a+$

$y^3 - 6y^2 + 11y -$

of $P_0y^3 + P_1y^2 +$

$S_1 \Rightarrow a - d + a$

$3a$

$a =$

Let $f(y) = y^3$

Sub $y = a =$

$f(2) = 8 - 6(4$

$= 8 - 24 +$

$= 30 - 3$

$y = 2$ is a

sythetic dilis

$y = 2 \quad 1$

0

1

$f(y) = (y$

Let $f(y)$

$\Rightarrow y^2 - 4y$

$\Rightarrow y(y-$

$\Rightarrow (y-3$

$y = 1$

$\therefore$ The roots

$1,$

let $a-d$, a, $a+d$ be roots in A.P of eq $y^3-6y^2+11y-6=0$ which is in the form of $P_0y^3+P_1y^2+P_2y+P_3=0$

$S_1 \Rightarrow a-d+a+a+d = \frac{-P_1}{P_0} = \frac{6}{1}$

$3a = 6$

$a = 2$

let $f(y) = y^3-6y^2+11y-6$

sub $y=a=2$ in $f(y)$, we get

$f(2) = 8-6(4)+11(2)-6$

$= 8-24+22-6$

$= 30-30 = 0$

$y=2$ is a factor of $f(y)$ then

sythetic division, we get

$$\begin{array}{c|cccc} y=2 & 1 & -6 & 11 & -6 \\ & 0 & 2 & -8 & 6 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

$f(y) = (y-2)(y^2-4y+3)$

let $f(y)=0 \Rightarrow y-2=0$ $\boxed{y=2}$

$\Rightarrow y^2-4y+3=0$

$\Rightarrow y(y-3)-1(y-3)=0$

$\Rightarrow (y-3)(y-1)=0$

$y=1$ and $y=3$

$\therefore$ The roots of eq are 2, 1, 3 AP

$1, \frac{1}{2}, \frac{1}{3}$ are in (H.P)

7) solve the following eq. given that they have multiple roots

(i) $x^4 - 6x^3 + 13x^2 - 24x + 36 = 0$

sol:- Let

$f(x) = x^4 - 6x^3 + 13x^2 - 24x + 36$

$f'(x) = 4x^3 - 18x^2 + 26x - 24$

put

$x = 1$, we get

$f(1) = 1 - 6 + 13 - 24 + 36 \neq 0$

$f(2) \neq 0$

$f(3) = (3)^4 - 6(3)^3 + 13(3)^2 - 24(3) + 36$

$= 81 - 6(27) + 13(9) - 72 + 36$

$= 81 - 162 + 117 - 72 + 36$

$= 0$

$f'(3) = 4(3)^3 - 18(3)^2 + 26(3) - 24$

$= 4(27) - 18(9) + 78 - 24$

$= 108 - 162 + 78 - 24$

$= 0$

$\therefore$ $x = 3$ is a factor of $f(x)$ & $f'(x)$ then synthetic division, we get

$x = 3$	1	-6	13	-24	36
	0	3	-9	12	36
$x = 3$	1	-3	4	-12	0
	0	3	0	12	
	1	0	4	0	

$\Rightarrow f(x) =$

$\Rightarrow f(x) =$

$\Rightarrow x^2 -$

$\Rightarrow x^2 = -$

$\Rightarrow x^2 =$

$\Rightarrow x =$

$\therefore$ The roots

III

i) solve the eq

G.T the equ

sol:- Let

$f(x) = 8x^3$

$f'(x) = 24$

Let $f'(x) =$

$\Rightarrow 24x^2 -$

$\Rightarrow 2[12x$

$\Rightarrow 12x^2 - 2$

$\Rightarrow 12x^2 - 2$

$\Rightarrow 2x(6x$

$\Rightarrow (2x - 3$

$\Rightarrow 2x - 3 =$

$x = 3/$

$f(3/2) =$

$=$

$=$

$\Rightarrow f(x) = (x-3)(x-3)(x^2+4)$

$\Rightarrow f(x) = 0, x=3, x=3$

$\Rightarrow x^2+4=0$

$\Rightarrow x^2 = -4$

$\Rightarrow x^2 = (2i)^2 \quad [\because i^2 = -1]$

$\Rightarrow x = \pm 2i$

$\therefore$ The roots of the given eq. are $3, 3, \pm 2i$ //

III

i) solve the equation $8x^3 - 20x^2 + 6x + 9 = 0$, G.T the equation has multiple roots.

sol:- Let

$f(x) = 8x^3 - 20x^2 + 6x + 9$

$f'(x) = 24x^2 - 40x + 6$

Let $f'(x) = 0$

$\Rightarrow 24x^2 - 40x + 6 = 0$

$\Rightarrow 2(12x^2 - 20x + 3) = 0$

$\Rightarrow 12x^2 - 20x + 3 = 0$

$\Rightarrow 12x^2 - 2x - 18x + 3 = 0$

$\Rightarrow 2x(6x-1) - 3(6x-1) = 0$

$\Rightarrow (2x-3)(6x-1) = 0$

$\Rightarrow 2x - 3 = 0$

$x = 3/2$

$f(3/2) = 8(3/2)^3 - 20(3/2)^2 + 6(3/2) + 9$

$= 8\left(\frac{27}{8}\right) - 20(9/4) + 9 + 9$

$= 27 - 45 + 18 = 45 - 45 = 0$

$f'(3/2) = 24(3/2)^2 - \overset{20}{\cancel{40}}(3/\cancel{2}) + 6$

$= 24\left(\frac{9}{4}\right) - 60 + 6$

$= 54 + 6 - 60$

$= 60 - 60$

$\therefore$ $x = 3/2$ is a factor of $f(x)$ & $f'(x)$

then by synthetic division.

$x = 3/2$	8	−20	6	9
	0	12	−12	−9
$x = 3/2$	8	−8	−6	0
	0	12	6	
	8	4	0	

$f(x) = (x - 3/2)(x - 3/2)(8x + 4)$

$f(x) = 0$

$\Rightarrow x - 3/2 = 0,\ x - 3/2 = 0,\ 8x + 4 = 0$

$x = 3/2,\ x = 3/2,\ x = -4/8 = -1/2$

$\therefore$ the roots are $3/2, 3/2, -1/2$

5) i) S.T $x^5 - 5x^3 + 5x^2 - 1 = 0$ has three equal roots and find this roots

Sol:- Let

$f(x) = x^5 - 5x^3 + 5x^2 - 1$

$f'(x) = 5x^4 - 15x^2 + 10x$

$f(1) = 1 - 5 + 5 - 1 = 0$

$f'(1) = 5 - 15 + 10 = 0$

$x = 1$ is a

$\therefore$ by synthet

$x = 1$	1	0
	0	1
$x = 1$	1	
	0	
	1	

$f(x) = (x$

Let $f(x) =$

$x - 1 = 0$

$\Rightarrow x^3$

Let

$g(x) =$

$g(1)$

x

$\therefore$ g'

1) solve x of two roo

Sol:- Let α,

eq. $x^4 + x$

which is

W.T

$\alpha\beta =$

$S_4 \Rightarrow$

$\Rightarrow$

$\Rightarrow$

∴ $x=1$ is a factor of $f(x)$ & $f'(x)$ then by synthetic division we get

	1	0	-5	5	0	-1
$x=1$	0	1	1	-4	1	1
	1	1	-4	1	1	0
$x=1$	0	1	2	-2	-1	
	1	2	-2	-1	0	

$f(x) = (x-1)(x-1)(x^3+2x^2-2x-1) = 0$

let $f(x) = 0$

$x-1=0$ & $x-1=0 \Rightarrow x=1, x=1$

$\Rightarrow x^3+2x^2-2x-1=0$

Let $g(x) = x^3+2x^2-2x-1$

$g(1) = 1+2-2-1=0$

$x=1$

∴ '1' is the repeated root//

1) Solve $x^4+x^3-16x^2-4x+48=0$, G.T the product of the two roots is 6.

Sol:- Let $\alpha, \beta, \gamma, \delta$ be the roots of the given eq. $x^4+x^3-16x^2-4x+48=0$ — ①

which is in the form of $P_0x^4+P_1x^3+P_2x^2+P_3x+P_4=$

G.T

$\alpha\beta = 6$

$S_4 \Rightarrow \alpha\beta\gamma\delta = \frac{P_4}{P_0} = \frac{48}{1}$

$\Rightarrow 6(\gamma\delta) = 48$

Let $\alpha+\beta=p$, $\gamma+\delta=q$

If α & β are the roots, then req. eq is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow x^2-px+6=0$ —— ②

Illy

$\Rightarrow x^2+(\gamma+\delta)x+\gamma\delta=0$

$\Rightarrow x^2-qx+8=0$ —— ③

Now eq ① = eq ② × eq ③ , we get

$\Rightarrow x^4+x^3-16x^2-4x+48=(x^2-px+6)(x^2-qx+8)$

$\Rightarrow x^4-qx^3+8x^2-px^3+pqx^2-8px+6x^2-6qx+48$

$\Rightarrow x^4+x^3(-p-q)-x^2(-pq+14)-x(8p+6q)+48$

comparing 'x^3' & 'x' coff on b.s , we get

x^3

$1=p-q$

$p+q=-1$

$4p+4q=-4$

x

$4=8p+6q$

$4p+3q=2$

$$\Rightarrow \begin{array}{r} \cancel{4p}+3q=+2 \\ \underline{\overset{-}{\cancel{4q}}+\overset{-}{4q}=\overset{+}{-4}} \\ -q=6 \end{array}$$

$$\boxed{q=-6}$$

$p+q=-1$

$-6+p=-1$

$p=6-1$

$p=5$

$\Rightarrow x^2-px+6=$

$\Rightarrow x^2-5x+6=$

$\Rightarrow x^2-3x-2x+$

$\Rightarrow x(x-3)-$

$\Rightarrow (x-2)\ (x$

$x-2=0.$

$x=2$

$\therefore$ from ③

$\Rightarrow x^2-qx+8=0$

$\Rightarrow x^2+6x+8=$

$\Rightarrow x^2+2x+4x$

$\Rightarrow x(x+2)+4($

$\Rightarrow (x+2)\ (x+4$

$x=-2$ $\quad x$

$\therefore$ The roots

2) solve

$8x^4-2x^3-27x^2$

absolute value,

<u>Sol</u>:- Let $\alpha,\beta,\gamma,$

$\Rightarrow 8x^4-2x^3-2$

$\Rightarrow x^4-\frac{2x^3}{8}-$

$\Rightarrow x^4-\frac{x^3}{4}-\frac{27}{8}$

G.T

$\beta=-\alpha$ $\quad\Rightarrow$

$S_1\Rightarrow\alpha+\beta+\gamma+$

$\Rightarrow 0+\gamma+$

$\Rightarrow x^2 - px + 6 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x-3) - 2(x-3) = 0$

$\Rightarrow (x-2)(x-3) = 0$

$x - 2 = 0$ (or) $x - 3 = 0$

$x = 2$ $\quad x = 3$

$\therefore$ from ③

$\Rightarrow x^2 - qx + 8 = 0$

$\Rightarrow x^2 + 6x + 8 = 0$

$\Rightarrow x^2 + 2x + 4x + 8 = 0$

$\Rightarrow x(x+2) + 4(x+2) = 0$

$\Rightarrow (x+2)(x+4) = 0$

$x = -2$ $\quad x = -4$

$\therefore$ The roots of the given equ are $2, 3, -2$ & -4.

2) solve $8x^4 - 2x^3 - 27x^2 + 6x + 9 = 0$, G.T two roots have same absolute value, but are in opposite sign.

<u>Sol</u>:- Let $\alpha, \beta, \gamma, \delta$ be the roots

$\Rightarrow 8x^4 - 2x^3 - 27x^2 + 6x + 9 = 0$

$\Rightarrow x^4 - \frac{2x^3}{8} - \frac{27}{8}x^2 + \frac{6x}{8} + \frac{9}{8} = 0$

$\Rightarrow x^4 - \frac{x^3}{4} - \frac{27x^2}{8} + \frac{3x}{4} + \frac{9}{8} = 0$ —— ①

G.T

$\beta = -\alpha \quad \Rightarrow \alpha + \beta = 0$

$S_1 \Rightarrow \alpha + \beta + \gamma + \delta = \frac{-P_1}{P_0} = \frac{-\left(-\frac{1}{4}\right)}{1}$

$\gamma+\delta=1/4$

Let $\alpha\beta=p$ & $\gamma\delta=q$

If α,β are the roots, then the required Q. Eq is

$\Rightarrow x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow x^2-(0)x+p=0$

$\Rightarrow x^2+p=0$ —②

If γ,δ are the roots then, the req. Q. equ is

$\Rightarrow x^2-(\gamma+\delta)x+\gamma\delta=0$

$\Rightarrow x^2-1/4\,x+q=0$ —③

∴ equ① = equ② × equ③

$\Rightarrow x^4-\frac{x^3}{4}-\frac{27}{3}x^3+\frac{3x}{4}+\frac{9}{8}=(x^2+p)(x^2-1/4\,x+q)$

$\Rightarrow x^4-\frac{1}{4}x^3+qx^2+px^2-\frac{p}{4}x+pq$

$\Rightarrow x^4-1/4\,x^3+(p+q)x^2-\frac{p}{4}x+pq$

comparing coff of 'x' & 'x²' on b.s, we get

'x'

$\Rightarrow 3/4=\frac{-p}{4}$

$\Rightarrow p=-3$

'x²'

$\Rightarrow -\frac{27}{8}=p+q$

$\Rightarrow -\frac{27}{8}=-3+q$

$\Rightarrow q=3-\frac{27}{8}$

$\Rightarrow q=\frac{24-27}{8}$

$\Rightarrow q=-3/8$

from eq ②

$\Rightarrow x^2+p=0$

$\Rightarrow x^2-3=0$

$\Rightarrow x=\pm\sqrt{3}$

from eq ③

$x^2 - \frac{1}{4}x + q = 0$

$\Rightarrow x^2 - \frac{1}{4}x - \frac{3}{8} = 0$

$8x^2 - 2x - 3 = 0$

$\Rightarrow 8x^2 + 4x - 6x - 3 = 0$

$\Rightarrow 4x(2x+1) - 3(2x+1) = 0$

$\Rightarrow (2x+1)(4x-3) = 0$

$2x+1 = 0 \qquad 4x-3 = 0$

$x = -1/2 \qquad x = 3/4$

3) solve $18x^3 + 81x^2 + 121x + 60 = 0$, given that are root is equal to half the sum of the remaining roots.

sol:- Let α, β, γ be the roots of the given eq, which is in the form $18x^3 + 81x^2 + 121x + 60 = 0$, of $P_0x^3 + P_1x^2 + P_2x + P_3 = 0$

$S_1 \Rightarrow \alpha + \beta + \gamma = \frac{-P_1}{P_0} = -\frac{81}{18} = -\frac{9}{2}$

$S_2 \Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = \frac{P_2}{P_0} = \frac{121}{8}$

$S_3 \Rightarrow \alpha\beta\gamma = \frac{-P_3}{P_0} = \frac{-60}{18} = \frac{-10}{3}$

G.T

$\alpha = \frac{\beta+\gamma}{2}$

$2\alpha = \beta + \gamma$

$S_1 \Rightarrow \alpha + 2\alpha = -\frac{9}{2}$

$\Rightarrow \cancel{3}\alpha = -\frac{\cancel{9}^{3}}{2}$

$\Rightarrow \alpha = -\frac{3}{2}$

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Let

$f(x) = 18x^3 + 81x^2 + 121x + 60$

$f(-3/2) = 18\left(\frac{-27}{8}\right) + 81\left(\frac{9}{4}\right) + 121\left(-\frac{3}{2}\right) + 60$

$= \frac{-243}{4} + \frac{729}{4} - \frac{363}{2} + 60$

$= 0$

$\therefore \alpha = x = -3/2$ is a root of $f(x)$, then by synthetic division, we get

$$\begin{array}{c|cccc} x=-3/2 & 18 & 81 & 121 & 60 \\ & 0 & -27 & -81 & -60 \\ \hline & 18 & 54 & 40 & 0 \end{array}$$

$f(x) = (x - 3/2)(18x^2 + 54x + 40)$

Let $f(x) = 0$

$x - 3/2 = 0 \Rightarrow x = 3/2$

$\Rightarrow 18x^2 + 54x + 40 = 0$

$\Rightarrow 2(9x^2 + 27x + 20) = 0$

$\Rightarrow 9x^2 + 27x + 20 = 0$

$\Rightarrow 9x^2 + 12x + 15x + 20 = 0$

$\Rightarrow 3x(3x+14) + 5(3x+4) = 0$

$\Rightarrow (3x+4)(3x+5) = 0$

$3x + 4 = 0$ $\qquad$ $3x + 5 = 0$

$3x = -4$ $\qquad$ $3x = -5$

$x = -4/3$ $\qquad$ $x = -5/3$

$\therefore$ The roots of the given equ are

$-3/2, -4/3, -5/3$ //

⇒ Equation with

⇒ Lemma (conditi

Let $f(x)$ be

Let $\alpha \in C$ then

$\overline{f(\alpha)} = \overline{f}$

⇒ Note:-

→ Let $f(x)$ be

with real co

$f(x) = 0$ of

root of $f($

→ Let $f(x)$ be

with real

that none

real, then

$\frac{f(\alpha)}{a_0} > 0$

→ Let $f(x)$ be

real coff's is

$f(x)$ then

(i) If the e

is even and

(ii) If 'n' is

atleast one

**

→ Let $f(x)$

with ratio

number b

⇒ Equation with real coefficients:-

⇒ Lemma (condition):-

Let $f(x)$ be a polynomial with real coff's —

Let $\alpha \in C$ then

$$f(\bar{\alpha}) = \overline{f(\alpha)}$$

⇒ Note:-

→ Let $f(x)$ be a polynomial of degree $n>0$, with real coff's. Let 'α' be a root of the equ $f(x)=0$ of multiplicity 'm'. Then $\bar{\alpha}$ is a root of $f(x)=0$ of multiplicity 'm'.

→ Let $f(x)$ be polynomial of degree $n>0$, with real coff's and leading coff a_0. Suppose that none of the roots of the eq $f(x)=0$ is real, then

$\frac{f(\alpha)}{a_0} > 0$ ∀ real α, and 'n' is an even integer.

→ Let $f(x)$ be a polynomial of degree $n>0$, with real coff's is Let a_0 be the leading coff of $f(x)$ then

(i) If the equ $f(x)=0$ has no real roots, then 'n' is even and $f(\alpha)$ & a_0 have the same sign $\forall \alpha \in R$

(ii) If 'n' is odd, then the eq. $f(x)=0$ has atleast one real root.

**

→ Let $f(x)$ be polynomial of degree $(n>0)$ with rational coff's. Let 'a' & 'b' be rational number $b>0$ & $\sqrt{b}$ irrational. then $a+\sqrt{b}$

is a root of $f(x)=0$ iff $a-\sqrt{b}$ is other roo

** → The rational coff of root $\sqrt{a}+i\sqrt{b}$ are $\sqrt{a}-i\sqrt{b}$, $-\sqrt{a}+\sqrt{b}$, $-\sqrt{a}-i\sqrt{b}$.

Ex:- 4(c)

I.

1) Find the polynomial equation whose roots are

(i) $2+3i$, $2-3i$, $1+i$, $1-i$

(ii) $3, 2, 1+i, 1-i$

(iii) $1+i$, $1-i$, $-1+i$, $-1-i$

(iv) $1+i$, $1-i$, $1+i$, $1-i$

(i)

Solt Let

$\alpha_1 = 2+3i$, $\alpha_2 = 2-3i$

$\alpha_3 = 1+i$, $\alpha_4 = 1-i$

If $\alpha_1, \alpha_2, \alpha_3$ & α_4 be the roots then the req. polynomial eq. is

$$\Rightarrow (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)=0$$

$$\Rightarrow [(x-2)-3i][(x-2)+3i][(x-1)-i][(x-1)+i]=0$$

$$\Rightarrow [(x-2)^2-(3i)^2][(x-1)^2-i^2]=0$$

$$\Rightarrow [x^2+4-4x+9][x^2+1-2x+1]=0$$

$[\because i^2=-1]$

$$\Rightarrow [x^2+4x+13](x^2-2x+2)=0$$

$$\Rightarrow x^4 6x^3+23x^2-34x+26=0$$ //

$\sqrt{a}-i\sqrt{b}$,

roots are

he

$i)+i]=0$

(iii)
Sol: Let
$\alpha_1 = 1+i$, $\alpha_2 = 1-i$
$\alpha_3 = -1+i$, $\alpha_4 = -1-i$

If α_1, α_2, α_3 & α_4 be the roots then the req. polynomial eq is

$\Rightarrow (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)=0$

$\Rightarrow [x-1-i][x-1+i][x+1-i][x+1+i]=0$

$\Rightarrow [(x-1)^2-i^2][(x+1)^2-i]=0$

$\Rightarrow [x^2+1-2x+1)(x^2+1+2x+1)=0$

$\because [i^2=-1]$

$\Rightarrow (x^2-2x+2)(x^2+2x+2)=0$

$\Rightarrow x^2+4=0$ //

2) Find the polynomial equational with rational coff's whose roots are

(i) $4\sqrt{3}$, $5+2i$
(ii) $1+5i$, $5-i$
(iii) $i-\sqrt{5}$
(iv) $-\sqrt{3}+i\sqrt{2}$

(ii)
Sol:- Given roots
$1+5i$, $5-i$

∴ The conjugate of given roots are $1-5i$ & $5+i$

Let $\alpha_1 = 1+5i$, $\alpha_2 = 1-5i$
$\alpha_3 = 5-\alpha$, $\alpha_4 = 5+i$

Req, polynomial eq is

$\Rightarrow (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)=0$

$\Rightarrow [x-1-5i][x-1+5i][x-5+i][x-5-i]=0$

$\Rightarrow [(x-1)^2-(5i)^2]((x-5)^2-i^2]=0$

$[\because i^2=-1]$

$\Rightarrow [x^2+1-2x+25][x^2+25-10x+1)=0$

$\Rightarrow [x^2-2x+26](x^2-10x+26)=0$

$\Rightarrow x^4-10x^3-19x^2-480x=1932=0$ //

(iv)

sol:- Given root

$-\sqrt{3}+i\sqrt{2}$

$\therefore$ The rational coffs of

$-\sqrt{3}+i\sqrt{2}$ are $-\sqrt{3}-i\sqrt{2}$, $\sqrt{3}+i\sqrt{2}$; $\sqrt{3}-i\sqrt{2}$

Let $\alpha_1=-\sqrt{3}+i\sqrt{2}$, $\alpha_2=-\sqrt{3}-i\sqrt{2}$

$\alpha_3=\sqrt{3}+i\sqrt{2}$, $\alpha_4=\sqrt{3}-i\sqrt{2}$

Req. polynomial eq is

$\Rightarrow (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)=0$

$\Rightarrow (x+\sqrt{3}-i\sqrt{2}][x+\sqrt{3}+i\sqrt{2}][x-\sqrt{3}-i\sqrt{2}][x-\sqrt{3}+i\sqrt{2}]=$

$\Rightarrow [(x+\sqrt{3})^2-(i\sqrt{2})^2][(x-\sqrt{3})^2-(i\sqrt{2})^2]=0$

$[\because i^2=-1]$

$\Rightarrow [x^2+3+2\sqrt{3}x+2](x^2+3-2\sqrt{3}x+2]=0$

$\Rightarrow [x^2+2\sqrt{3}x+5](x^2+2\sqrt{3}x+5)=0$

$\Rightarrow [(x^2+5)-2\sqrt{3}x)(x^2+5)-2\sqrt{3}x]=0$

$\Rightarrow (x^2+5^2)-(2\sqrt{3}x)^2=0$

$\Rightarrow x^4+25+10x^2-12x^2=0$

$\Rightarrow x^4 \pm 2x^2 + 25 = 0$ //

11) Solve the equation $x^4+2x^3-5x^2+6x+2=0$, G.T $1+i$ is one of its roots.

Sol: Given equ.

$\Rightarrow x^4+2x^3-5x^2+6x+2=0$

$\therefore$ The conjugate of $1+i$ is $1-i$

Let $\alpha = 1+i$, $\beta = 1-i$

$\Rightarrow (x-\alpha)(x-\beta)=0$

$\Rightarrow [x-1-i][x-1+i]=0$

$\Rightarrow [(x-1)^2-i^2]=0 \quad (\because i^2=-1)$

$\Rightarrow x^2+1-2x+1=0$

$\Rightarrow x^2-2x+2=0$

It is in the form of x^2-px-q by synthetic division, we get

	1	2	-5	6	2
p=2	0	2	8	2	0
q=-2	0	0	-2	-8	-2
	1	4	1	0	0

$g(x) = x^2+4x+1$

Let $g(x)=0$

$\Rightarrow x^2+4x+1=0$

$\Rightarrow x = \dfrac{-4 \pm \sqrt{16-4(1)(1)}}{2(1)}$

$\Rightarrow x = \dfrac{-4 \pm \sqrt{12}}{2}$

$$x = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$x = -2 \pm \sqrt{3}$$

$\therefore$ The roots of given eq. are $1 \pm i, -2 \pm \sqrt{3}$ //

Eq (2):-

solve the equation $6x^4 - 13x^3 - 35x^2 - x + 3 = 0$

G.T $2+\sqrt{3}$ is one of its roots.

Sol:- Given eq

$\Rightarrow 6x^4 - 13x^3 - 35x^2 - x + 3 = 0$

$\therefore$ The conjugate of $2+\sqrt{3}$ is $2-\sqrt{3}$

Let $\alpha = 2+\sqrt{3}$, $\beta = 2-\sqrt{3}$

$\Rightarrow (x-\alpha)(x-\beta) = 0$

$\Rightarrow (x-2-\sqrt{3})[x-2+\sqrt{3}] = 0$

$\Rightarrow [(x-2)^2 - (\sqrt{3})^2] = 0$

$\Rightarrow x^2 + 4 - 4x - 3 = 0$

$\Rightarrow x^2 - 4x + 1 = 0$

It is in the form of $x^2 - px - q$ by synthetic division, we get

$p = 4$	6	-13	-35	-1	3
	0	24	44	12	0
$q = -1$	0	0	-6	-11	-3
	6	11	3	0	0

$g(x) = 6x^2 + 11x + 3$

Let $g(x) = 0$

$\Rightarrow 6x^2 + 11x + 3 = 0$

$\Rightarrow 6x^2+2x+9x+3=0$

$\Rightarrow 2x(3x+1)+3(3x+1)=0$

$\Rightarrow (2x+3)(3x+1)=0$

$\Rightarrow x=-3/2$ (or) $x=-1/3$

∴ The roots of given are $2\pm\sqrt{3}, -3/2, -1/3$ //

Q) Solve the equation $x^4-4x^2+8x+35=0$ G.T $2+i\sqrt{3}$ is one of its roots.

Sol: Given eq

$\Rightarrow x^4-4x^2+8x+35=0$

∴ The conjugate of $2+i\sqrt{3}$ is $2-i\sqrt{3}$

Let $\alpha=2+i\sqrt{3}$, $\beta=2-i\sqrt{3}$

$\Rightarrow (x-\alpha)(x-\beta)=0$

$\Rightarrow (x-2-i\sqrt{3})(x-2+i\sqrt{3})=0$

$\Rightarrow [(x-2)^2-i\sqrt{3}]^2=0$ $[\because i^2=-1]$

$\Rightarrow x^2+4-4x+3$

$\Rightarrow x^2-4x+7=0$

It is in the form of x^2-px+q by synthetic division we get

	1	0	-4	8	35
p=4	0	4	16	20	0
q=-7	0	0	-7	-28	-35
	1	4	5	0	0

$q(x)=x^2+4x+5=0$

$x=\dfrac{-4\pm\sqrt{16-4(1)(5)}}{2(1)}$

$$x = \frac{-4 \pm \sqrt{16-20}}{2}$$

$$x = \frac{-4 \pm \sqrt{-4}}{2}$$

$$x = \frac{-4 \pm 2i}{2}$$

$$x = -2 \pm i \;//$$

5) G.T $-2+\sqrt{-7}$ is a root of the equ. $x^4+2x^2-16x+77=0$ solve it completely.

Sol:- Given eq

$x^4+2x^2-16x+77=0$

The conjugate of $-2+i\sqrt{7}$ is $-2-i\sqrt{7}$

Let $\alpha = 2+i\sqrt{7}$, $\beta = -2-i\sqrt{7}$

$\left[\because -2+\sqrt{i^2 7} = -2+i\sqrt{7}\right]$

$\Rightarrow (x-\alpha)(x-\beta)=0$

$\Rightarrow (x+2-i\sqrt{7})[x+2+i\sqrt{7}]=0$

$\Rightarrow [(x+2)^2-(i\sqrt{7})^2]$

$\Rightarrow x^2+4+4x+7=0$

$\Rightarrow x^2+4x+11=0$

It is in the form of x^2-px-q by synthetic division, we get

	1	0	2	-16	77
$p=-4$	0	-4	16	28	0
$q=-11$	0	0	-11	44	-77
	1	-4	7	0	0

$g(x) = x^2-4x+7$

Let $g(x) = 0 \Rightarrow x^2-4x+7=0$

$$x = \frac{4\pm\sqrt{16-4(7)(1)}}{2(1)}$$

$$x = \frac{4\pm\sqrt{16-28}}{2}$$

$$x = \frac{4\pm\sqrt{-12}}{2} \Rightarrow x = \frac{4\pm\sqrt{i^2\times4\times3}}{2}$$

$$x = \frac{4\pm i2\sqrt{3}}{2} = \frac{\cancel{2}(2\pm i\sqrt{3})}{\cancel{2}}$$

$$x = 2\pm i\sqrt{3}$$

∴ The roots of the given equ are $-2+i\sqrt{7}$, $-2-i\sqrt{7}$, $2+i\sqrt{3}$ & $2-i\sqrt{3}$ //

7) solve the equation $3x^5-4x^4-42x^3+56x^2+27x-36=0$ G.T $\sqrt{2}+\sqrt{5}$ is one of its roots.

Sol:- G.T

$\Rightarrow 3x^5-4x^4-42x^3+56x^2+27x-36=0$

the conjugate of $\sqrt{2}+\sqrt{5}$ is $\sqrt{2}-\sqrt{5}$

Let $\alpha=\sqrt{2}+\sqrt{5}$, $\beta=\sqrt{2}-\sqrt{5}$

$\Rightarrow (x-\alpha)(x-\beta)=0$

$\Rightarrow [x-\sqrt{2}-\sqrt{5})(x-\sqrt{2}+\sqrt{5}]=0$

$\Rightarrow [(x-\sqrt{2})^2-(\sqrt{5})^2]$

$\Rightarrow x^2+2-2\sqrt{2}x-5=0$

$\Rightarrow x^2-2\sqrt{2}x-3=0$

The eq. having roots $\pm\sqrt{2}\pm\sqrt{5}$ is

$\Rightarrow (x^2+2\sqrt{2}x-3)(x^2-2\sqrt{2}x-3)=0$

$\Rightarrow (x^2-3)^2-(2\sqrt{2}\,x)^2=0$

$\Rightarrow x^4+9-6x^2-8x^2=0$

$\Rightarrow x^4-14x^2+9=0$

$\Rightarrow 3x^5-4x^4-42x^3+56x^2+27x-36=0$

$\Rightarrow 3x(x^4-14x^2+9)-4(x^4-14x^2+9)=0$

$\Rightarrow (x^2-14x^2+9)(3x-4)=0$

$\Rightarrow 3x-4=0 \Rightarrow x=4/3$

$\therefore$ The roots of the given eq $\pm\sqrt{2}\pm\sqrt{5}$, $4/3$ //

8) solve the equation $x^4-9x^3+27x^2-29x+6=0$
G.T $2-\sqrt{3}$ is one of its roots.

Sol:- G.T

$\Rightarrow x^4-9x^3+27x^2-29x+6=0$

The conjugate of $2-\sqrt{3}$ is $2+\sqrt{3}$

Let $\alpha=2-\sqrt{3}$, $\beta=2+\sqrt{3}$

If α, β are the roots, then the req q. equ is $(x-\alpha)(x-\beta)=0$

$\Rightarrow [x-2+\sqrt{3}][x-2-\sqrt{3}]=0$

$\Rightarrow [(x-2)^2-\sqrt{3}]^2=0$

$\Rightarrow x^2+4-4x\cancel{x}-3=0$

$\Rightarrow x^2-4x+1=0$, it is in the form of x^2-px-q

By synthetic division, we get

	1	−9	27	−29	6
p= 4	0	4	−20	24	0
q=−1	0	0	−1	5	−6
	1	−5	6	0	0

$g(x) = x^2 - 5x + 6$

Let $g(x) = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 2x - 3x + 6 = 0$

$\Rightarrow x(x-2) - 3(x-2) = 0$

$\Rightarrow (x-2)(x-3) = 0$

$x - 2 = 0 \qquad x - 3 = 0$

$x = 2 \qquad x = 3$

∴ The roots of the given equ are $2 \pm \sqrt{3}, 2, 3$ //

tions:-

⇒ **Note (Formula):-**

→ If $\alpha_1, \alpha_2 \ldots \alpha_n$ are the roots of polynomial eq $f(x)=0$, then $\alpha_1-h, \alpha_2-h \ldots, \alpha_n-h$ are the roots of $f(x+h)=0$

→ If $\alpha_1, \alpha_2 \ldots, \alpha_n$ are the roots of polynomial eq $f(x)=0$, then $\alpha_1+h, \alpha_2+h, \ldots \alpha_n+h$ are the roots of $f(x-h)=0$

⇒ **Reciprocal roots:-**

Let $\alpha_1, \alpha_2 \ldots \alpha_n$ are the roots of polynomial eq. $f(x)=0$, then $\frac{1}{\alpha_1}, \frac{1}{\alpha_2} \ldots \frac{1}{\alpha_n}$ are the roots of eq $x^n f(\frac{1}{x})=0$.

→ If 'α' is a root of $f(x)=0$, then α^2 is a root of $f(\sqrt{x})=0$

→ If 'α' is a root of $f(x)=0$, then α^3 is the root of $f(\sqrt[3]{x})=0$ [cubes of the roots]

⇒ **Reciprocal equation:-** A polynomial $f(x)$ of degree $n>0$ is said to be reciprocal equation if $f(0) \neq 0$ and $f(x) = \frac{a_0}{f(0)} \cdot x^n \cdot f(\frac{1}{x})$ $\forall\, n \in \mathbb{R}\setminus\{0\}$ where a_0 is leading coeff of $f(x)$. If $f(x)$ is a reciprocal polynomial then $f(x)=0$ is said to be a reciprocal eq.

⇒ **Notes:-**

1) If $f(x)$ is a reciprocal polynomial & $\alpha \in C$ then 'α' is a root of $f(x)=0$ of multiplicity (m) iff $\frac{1}{\alpha}$ is a root of $f(x)=0$ of multiplicity

m.

2) Let $f(x) = a_0x^n + a_1x$

of degree $n>0$ the

$a_{n-k} = a_k$ for k

for $k=0, 1, 2,$

**3) A reciprocal polyn

leading coefficien

one (or) class two

a_0 (or) $-a_0$

4) If $f(x)$ is rec

equation $f(x)=0$

of class one (or)

reciprocal polyno

5) For an odd d

'-1' is a root

-cal eq of cl

6) For an even

class two of C

** I.

1) Find the al

3 times the roo

Sol:- $f(x) = x^3+$

Req equ is

$f(x/3) \Rightarrow (x/3$

$\Rightarrow \frac{x^3}{27}$

yrl.

2) Let $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_n$ is a polynomial of degree $n > 0$ then $f(x)$ is reciprocal iff $a_{n-k} = a_k$ for $k = 0, 1, 2 \dots n$ (or) $a_{n-k} = -a_{k_n}$ for $k = 0, 1, 2, \dots n$.

** 3) A reciprocal polynomial of degree 'n' with leading coefficient 'a_0' is said to be of class one (or) class two according as $f(0)$ is equal to 'a_0' (or) $-a_0$.

4) If $f(x)$ is reciprocal polynomial, then the equation $f(x)=0$ is said to be a reciprocal eqn of class one (or) class two according as $f(x)$ is a reciprocal polynomial of class one (or) class two.

5) For an odd degree reciprocal eqn of class one '-1' is a root and for an odd degree reciprocal eq of class two '1' is a root.

6) For an even degree reciprocal equation of class two of (1) and (-1) are the roots.

Exercise: 4(d)

I. 1) Find the algebraic equation whose roots are 3 times the roots of $x^3 + 2x^2 - 4x + 1 = 0$

Sol:- $f(x) = x^3 + 2x^2 - 4x + 1 = 0$

Req equ is

$$f(x/3) \Rightarrow (x/3)^3 + 2(x/3)^2 - 4(x/3) + 1 = 0$$

$$\Rightarrow \frac{x^3}{} + 2\left(\frac{x^2}{9}\right) - \frac{4x}{3} + 1 = 0$$

$$\Rightarrow \frac{x^3+6x^2-36x+27}{27} = 0$$

$$\Rightarrow x^3+6x^2-36x+27=0 //$$

2) Find the algebraic equation whose roots are 3 times the roots of $x^5-2x^4+3x^3-2x^2+4x+3$

Sol:- $f(x) = x^5-2x^4+3x^3-2x^2+4x+3=0$

req eqn is

$$f(\tfrac{x}{2}) = (\tfrac{x}{2})^5 - 2(\tfrac{x}{2})^4 + 3(\tfrac{x}{2})^3 - 2(\tfrac{x}{2})^2 + 4(\tfrac{x}{2}) + 3 = 0$$

$$\Rightarrow \left(\frac{x^5}{32}\right) - \frac{2x^4}{16} + \frac{3x^3}{8} - \frac{2x^2}{4} + \frac{4x}{2} + 3 = 0$$

$$\Rightarrow \frac{x^5}{32} - \frac{x^4}{8} + \frac{3x^3}{8} - \frac{x^2}{2} + 2x + 3 = 0$$

$$\Rightarrow \frac{x^5-4x^4+12x^3-16x^2+64x+96}{32} = 0$$

$$\Rightarrow x^5-4x^4+12x^3-16x^2+64x+96=0$$

4) Find the transformer eq whose roots are the -ve roots of $x^7+3x^5+x^3-x^2+7x+2=0$

Sol:- Let

$f(x) = x^7+3x^5+x^3-x^2+7x+2=0$

put $x=-x$, we get

$f(x) = (-x)^7 + 3(-x)^5 + (-x)^3 - (-x)^2 + 7(-x) + 2 = 0$

$= -x^7-3x^5-x^3-x^2-7x+2=0$

$= -[x^7+3x^5+x^3+x^2+7x-2]=0$

$\Rightarrow x^7+3x^5+x^3+x^2$

3) Find the transfor -ve of the roots

Sol: $f(x) = x^4+5x$

$f(-x) = (-x)^4$

$= x^4 -$

5) Find the poly are reciprocal of

$-2=0$.

Sol: Let

$f(x) = x^4-3x$

put $x = 1/x$

$f(\frac{1}{x}) = (\frac{1}{x})^4$

$\Rightarrow \frac{1}{x^4} - \frac{3}{x^3} +$

$\Rightarrow 1-3x+$

$\Rightarrow -2x^4+5$

$\Rightarrow 2x^4 - 5$

6) Find the pol the reciprocal x^5+11x^4+x

Sol:- Let

$f(x) = x^5$

put $x = 1/x$

roots

$x^2+4x+3=0$

$4\left(\frac{x}{2}\right)+3=0$

the

$=0$

$\Rightarrow x^7+3x^5+x^3+x^2+7x-2=0$ //

3) find the transformed eq whose roots are -ve of the roots of $x^4+5x^3+11x+3=0$

Sol: $f(x) = x^4+5x^3+11x+3=0$

$\Rightarrow f(-x) = (-x)^4+5(-x)^3+11(-x)+3=0$

$= x^4-5x^3-11x+3=0$ //

) Find the polynomial equation whose roots are reciprocal of the roots of $x^4-3x^3+7x^2+5x-2=0$.

Sol: Let

$f(x) = x^4-3x^3+7x^2+5x-2=0$

put $x = 1/x$

$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^4-3\left(\frac{1}{x}\right)^3+7\left(\frac{1}{x}\right)^2+5\left(\frac{1}{x}\right)-2=0$

$\Rightarrow \frac{1}{x^4}-\frac{3}{x^3}+\frac{7}{x^2}+\frac{5}{x}-2=0$

$\Rightarrow \frac{1-3x+7x^2+5x^3-2x^4}{x^4}=0$

$\Rightarrow -2x^4+5x^3+7x^2-3x+1=0$

$\Rightarrow 2x^4-5x^3-7x^2+3x-1=0$ //

6) Find the polynomial equation whose roots are the reciprocal of roots are $x^5+11x^4+x^3+4x^2-13x+6=0$.

Sol: Let

$f(x) = x^5+11x^4+x^3+4x^2-13x+6=0$

put $x = 1/x$

$f(\frac{1}{x}) = (\frac{1}{x})^5 + 11(\frac{1}{x})^4 + (\frac{1}{x})^3 + 4(\frac{1}{x})^2 - 13(\frac{1}{x}) + 6 = 0$

$\Rightarrow \frac{1}{x^5} + \frac{11}{x^4} + \frac{1}{x^3} + \frac{4}{x^2} - \frac{13}{x} + 6 = 0$

$\Rightarrow \frac{1 + 11x + x^2 + 4x^3 - 13x^4 + 6x^5 = 0}{x^5}$

$\Rightarrow 6x^5 - 13x^4 + 4x^3 + x^2 + 11x + 1 = 0$ //

**
II

1) Find the polynomial eq whose roots are squares of the roots of $x^4 + x^3 + 2x^2 + x + 1 = 0$

Sol:- Let

$f(x) = x^4 + x^3 + 2x^2 + x + 1 = 0$

put $x = \sqrt{x}$ in $f(x)$, we get

$f(\sqrt{x}) = (\sqrt{x})^4 + (\sqrt{x})^3 + 2(\sqrt{x})^2 + \sqrt{x} + 1 = 0$

$\Rightarrow (x^{1/2})^4 + (x^{1/2})^3 + 2(x^{1/2})^2 + x^{1/2} + 1 = 0$

$\Rightarrow x^2 + x^{3/2} + 2x + x^{1/2} + 1 = 0$

$\Rightarrow x^2 + 2x + 1 = -(x^{3/2} + x^{1/2})$

$\Rightarrow$ x^2. SOBS we get

$\Rightarrow (x^2 + 2x + 1)^2 = (-(x^{3/2} + x^{1/2}))^2$

$\Rightarrow x^4 + 4x^2 + 1 + 2(x^2)(2x) + 2(2x)(1) + 2(1)(x^2)$

$= (x^{3/2})^2 + (x^{1/2})^2 + 2(x^{2/3})(x^{1/2})$

$\Rightarrow x^4 + 4x^2 + 1 + 4x^3 + 4x + 2x^2 = x^3 + x + 2(x)^{\frac{3+1}{2}}$

$\Rightarrow x^4 + 4x^3 + 6x^2 + 4x + 1 - x^3 - x - 2x^2 = 0$

$x^4 + 3x^3 + 4x^2 + 3x + 1 = 0$ //

2) from the polynomial equation whose roots are the squares of the roots of $x^3+3x^2-7x+6=0$

Sol: Let

$f(x) = x^3+3x^2-7x+6=0$

put $x=\sqrt{x}$ in $f(x)$, we get

$f(\sqrt{x}) = (\sqrt{x})^3+3(\sqrt{x})^2-7(\sqrt{x})+6=0$

$\Rightarrow ((x)^{1/2})^3+3(x^{1/2})^2-7(x^{1/2})+6=0$

$\Rightarrow x^{3/2}+3x-7x^{1/2}+6=0$

$\Rightarrow 3x+6 = 7x^{1/2}-x^{3/2}$

SOBS, we get

$\Rightarrow (3x+6)^2 = (7x^{1/2}-x^{3/2})^2$

$\Rightarrow 9x^2+36+36x = 49(x^{1/2})^2+(x^{3/2})^2-2(7x^{1/2})(x^{3/2})$

$\Rightarrow 9x^2+36x+36 = 49x+x^3-14x^{\frac{1+3}{2}}$

$\Rightarrow 9x^2+36x+36 = x^3+49x-14x^2$

$\Rightarrow x^3+49x-14x^2-9x^2-36x-36=0$

$\Rightarrow x^3-23x^2+13x-36=0$ //

3) from the polynomial eqn whose roots are the cubes of the roots of $x^3+3x^2+2=0$

Sol: Let

$f(x) = x^3+3x^2+2=0$

put $x=\sqrt[3]{x} = (x)^{1/3}$ in $f(x)$ we get

$\Rightarrow f(x)^{1/3} = (x^{1/3})^3+3((x)^{1/3})^2+2=0$

$\Rightarrow x+2 = -3x^{2/3}$

cubing on b.s

$\Rightarrow (x+2)^3 = (-3x^{2/3})^3$

$\Rightarrow x^2+8+3(x)^2(2)+3(2)^2(x) = -27(x^{2/3})^3$

$\Rightarrow x^3+6x^2+12x+8 = -27x^2$

$\Rightarrow x^3+27x^2+6x^2+12x+8=0$

$\Rightarrow x^3+33x^2+12x+8=0$ //

III.

2) Find the polynomial equation whose roots are the translates of those of the equation.

$x^5-4x^4+3x^2-4x+6=0$ by -3

Sol:- $f(x) = x^5-4x^4+3x^2-4x+6=0$

$\Rightarrow x=-3$

req eq. $f(x+3)=0$

$f(x+h)=0$

$f(x+3) = (x+3)^5-4(x+3)^4+3(x+3)^2-4(x+3)+6=0$

$h=3$	1	-4	0	3	-4	6
	0	3	-3	-9	-18	-66
	1	-1	-3	-6	-22	-60 A_5
	0	3	6	9	9	
	1	2	3	3	-13 A_4	
	0	3	15	54		
	1	5	18	57 A_3		
	0	3	24			
	1	8	42 A_2			

$x\,^2/_3)^3$

ots are

	0	3
	1	11 A_1
	1 A_0	

The reqd polynomial eqn is $A_0x^5+A_1x^4+A_2x^3+A_3x^2+A_4x+A_5=0$

$\therefore x^5+11x^4+42x^3+57x^2-13x-60=0$ //

// Find the polynomial equation whose roots are the ~~translates~~ of those of the equation: $3x^5-5x^3+7=0$ by 4.

Sol:- Let

$f(x)=3x^5-5x^3+7=0$, $x=4 \Rightarrow x-4=0$

$f(x-4) \Rightarrow f(x+h) \Rightarrow h=-4$

$f(x-4)=3(x-4)^5-5(x-4)^3+7=0$

h=-4	3	0	-5	0	0	7
	0	-12	48	-172	688	-2752
	3	-12	43	-172	688	-2745 A_5
	0	-12	96	-556	2912	
	3	-24	139	-728	3600 A_4	
	0	-12	144	-1132		
	3	-36	283	-1860 A_3		
	0	-12	192			
	3	-48	475 A_2			
	0	-12				
	3	-60 A_1				
	0					
	3 A_0					

∴ The reqd polynomial eq. is $A_0x^5 + A_1x^4 + A_2x^3 + A_3x^2 + A_4x + A_5 = 0$

∴ $3x^5 - 60x^4 + 475x^3 - 1860x^2 + 3600x - 2745 = 0$ //

5) Transform each of the following eq into ones in which the coff-s of the second highe power of 'x' is zero and also find their transformed eqs.

i) $x^3 - 6x^2 + 10x - 3 = 0$

Sol: G.T

$\Rightarrow x^3 - 6x^2 + 10x - 3 = 0$, it is in the form of

$P_0x^3 + P_1x^2 + P_2x + P_3 = 0$

WKT

To remove the second term diminish the roots

by $h = \frac{P_1}{nP_0}$ ∴ $n = 3$

$h = \frac{-(-6)}{3(1)} = 2$

h = 2	1	-6	10	-3
	0	2	-8	4
	1	-4	2	1 A_3
	0	2	-4	
	~~0~~1	-2	-2 A_2	
	0	2		
	1	0 A_1		
	0			
	1 A_0			

The req polynomial eq is

$\therefore A_0x^3 + A_1x^2 + A_2x + A_3 = 0$

$x^3 - 2x + 1 = 0$ //

ii) $x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$

Sol: W.T

$\Rightarrow x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$ it is in the form of $P_0x^4 + P_1x^3 + P_2x^2 + P_3x + P_4 = 0$

W.K.T

To remove the second term diminish the roots by $h = \frac{-P_1}{nP_0}$. $n = 4$

$h = \frac{-4}{4(1)} = -1$

$h = -1$ 1 4 2 -4 -2

0 -1 -3 1 3

1 3 1 -3 1 A_4

0 -1 -2 3

1 2 -3 0 A_3

0 -1 -1

1 1 -4 A_2

0 -1

1 0 A_1

0

1 A_0

∴ The req. polynomial eq is

$A_0x^4+A_1x^3+A_2x^2+A_3x+A_4=0$

$\rightarrow x^4-4x^2+1=0$ //

6) Transform each of the following equation into ones in which the coeff of 3rd highest of n is zero

(i) $x^4+2x^3-12x^2+2x-1=0$

Sol:-

$\Rightarrow f''(h) = 12h^2+12h-24$

Let $f''(h)=0$

$\Rightarrow 12h^2+12h-24=0$

$\Rightarrow h^2+h-2=0$

$\Rightarrow h^2+2h-h-2=0$

$\Rightarrow h(h+2)-1(h+2)=0$

$\Rightarrow (h-1)(h+2)=0$

$h-1=0$ $\quad$ $h+2=0$

$h=1$ $\quad$ $h=-2$

$h=1$	1	2	-12	2	-1
	0	1	3	-9	-7
	1	3	-9	-7	$\lfloor -8$ A_4
	0	1	4	-5	
	1	4	-5	$\lfloor 12$ A_3	
	0	1	5		
	1	5	$\lfloor 0$ A_2		
	0	1			
	1	$\lfloor 6$ A_1			
	0				

1 A0

The required eq is $A_0x^4 + A_1x^3 + A_2x^2 + A_3x + A_4 = 0$

$x^4 + 6x^3 - 11x - 8 = 0$

1	2	-12	2	-1
0	-2	0	24	-52
1	0	-12	26	-53 A4
0	-2	4	16	
1	-2	-8	12 A3	
0	-2	8		
1	-4	0 A2		
0	-2			
1	-6 A1			
0				
1 A0				

i) solve the following equations

i) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

sol:- The given eqn is an even degree reciprocal eqn of class one. Then divide the eqn by x^2, we get

$\Rightarrow x^4 + 10x^3 + 26x^2 - 10x + 1 = 0$

$$\Rightarrow \frac{x^4 + 10x^3 + 26x^2 - 10x + 1}{x^2} = 0$$

$$\Rightarrow \frac{x^4}{x^2} + \frac{10x^3}{x^2} + \frac{26x^2}{x^2} - \frac{10x}{x^2} + \frac{1}{x^2} = 0$$

$$\Rightarrow x^2 + 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) - 10\left(\frac{1}{x} + x\right) + 26 = 0 \text{ --- } ①$$

Let $x + \frac{1}{x} = y$

SOBS, we get

$\Rightarrow x^2 + \frac{1}{x^2} + 2x\left(\frac{1}{x}\right) = y^2$

$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

from eq ①, we get

$\Rightarrow y^2 - 2 - 10y + 26 = 0$

$\Rightarrow y^2 - 10y + 24 = 0$

$\Rightarrow y^2 - 4y - 6y + 24 = 0$

$\Rightarrow y(y-4) - 6(y-4) = 0$

$\Rightarrow (y-6)(y-4) = 0$

$\Rightarrow y = 6$ (or) $y = 4$

<u>case (i)</u>

$y - 4 = 0 \Rightarrow y = 4$

But $y = x + \frac{1}{x}$

$\Rightarrow x + \frac{1}{x} = 4 \Rightarrow x^2 + 1 = 4x$

$\Rightarrow x^2 - 4x + 1 = 0$

$\Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{4 \pm \sqrt{16-4}}{2} \Rightarrow \frac{4 \pm \sqrt{12}}{2}$

$\Rightarrow x = \frac{4 + 2\sqrt{3}}{2}$

$\Rightarrow x = 2 \pm \sqrt{3}$

<u>case (ii)</u>

$y - 6 = 0 \Rightarrow y = 6$

$\Rightarrow$ But $y = x + \frac{1}{x}$

$\Rightarrow x + \frac{1}{x} = 6$

$\Rightarrow x^2 + 1 = 6x$

$\Rightarrow x^2 - 6x + 1 = 0$

$\Rightarrow x = \frac{6 \pm \sqrt{36-4}}{2}$

$\Rightarrow x = \frac{6 \pm \sqrt{32}}{2}$

$\Rightarrow x = \frac{6 \pm 4\sqrt{2}}{2}$

$\Rightarrow x = 3 \pm 2\sqrt{2}$

$\therefore$ the given roots of eqn are

$2 \pm \sqrt{3}, 3 \pm 2\sqrt{2}$ //

Example :-

Solve the eqn $6x^4-35x^3+62x^2-35x+6=0$.

Sol: The given eq is an even degree reciprocal eq of class one then divide the given eq by x^2 on b.s, we get

$$\Rightarrow \frac{6x^4-35x^3+62x^2-35x+6}{x^2}=0$$

$$\Rightarrow \frac{6x^4}{x^2}-\frac{35x^3}{x^2}+\frac{62x^2}{x^2}-\frac{35x}{x^2}+\frac{6}{x^2}=0$$

$$\Rightarrow 6x^2-35x+62-\frac{35}{x}+\frac{6}{x^2}=0$$

$$\Rightarrow 6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0 \text{ --- (1)}$$

Let $x+\frac{1}{x}=y$ SOBS on we get

$\Rightarrow x^2+\frac{1}{x^2}+2x\left(\frac{1}{x}\right)=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$

from eq (1)

$\Rightarrow 6(y^2-2)-35(y)+62=0$

$\Rightarrow 6y^2-12-35y+62=0 \Rightarrow 6y^2-35y+50=0$

$\Rightarrow 6y^2-15y-20y+50=0 \Rightarrow 3y(2y-5)-10(2y-5)=0$

$\Rightarrow (2y-5)(3y-10)=0$

$\Rightarrow y=5/2$ (or) $y=10/3$

Case (i)

$\Rightarrow y=5/2$

$\Rightarrow x+\frac{1}{x}=5/2 \Rightarrow \frac{x^2+1}{x}=5/2$

$\Rightarrow 2x^2+2=5x \Rightarrow 2x^2-5x+2=0$

$\Rightarrow 2x^2-4x-x+2=0 \Rightarrow 2x(x-2)-1(x-2)=0$

$\Rightarrow (2x-1)(x-2)=0$

$x-2=0$ $\quad 2x-1=0$

$x=2$ $\quad x=\frac{1}{2}$

case (ii):-

$\Rightarrow 3y-10=0 \Rightarrow y=\frac{10}{3}$

$\Rightarrow x+\frac{1}{x}=\frac{10}{3} \Rightarrow \frac{x^2+1}{x}=\frac{10}{3}$

$\Rightarrow 3x^2+3=10x \Rightarrow 3x^2-10x+3=0$

$\Rightarrow 3x^2-9x-x+3=0 \Rightarrow 3x(x-3)-1(x-3)=0$

$\Rightarrow (3x-1)(x-3)=0$

$\Rightarrow 3x-1=0 \quad x-3=0$

$x=\frac{1}{3} \quad x=3$

∴ The roots of given eqn are ~~x~~ $2, 3, \frac{1}{2}, \frac{1}{3}$ //

1) Find the polynomial eqn whose roots are translates of those of the eqn.

(i) $x^4-5x^3+7x^2-17x+11=0$ by -2.

Sol:- Given eqn

$f(x)=x^4-5x^3+7x^2-17x+11=0 \Rightarrow x=-2$

required eqn $f(x+2)=0 \Rightarrow f(x+h)=0$

$f(x+2)=(x+2)^4-5(x+2)^3+7(x+2)^2-17(x+2)+11=0$

∴ The required eqn is

$A_0x^4+A_1x^3+A_2x^2+A_3x+A_4=0$

$\Rightarrow x^4+3x^3+x^2-17x-19=0$ //

h=2	1	−5	9	−17	11
	0	2	−6	2	−30
	1	−3	1	−15	−19 A_4
	0	2	−2	−2	

1		−1
0	2	2
1	1	1
0	2	
1	3	A_1
0		
1	A_0	

4) solve

(ii) $2x^5+x^4-12x^3-1$

Sol:- The given eq

eqn of class -1

∴ −1 is the

$f(x)=2x^5+x^4-12$

$f(-1)=2(-1)^5+(-1$

$=-2+1+12-12$

$=0$

∴ $x=-1$ is a f

division we get

x=−1	2	1
	0	−2
	2	

∴ $f(x)=(x+$

let $f(x)=0$

$x+1=0 \Rightarrow x$

$\Rightarrow 2x^4-x^3-11x$

divide the

$\Rightarrow \frac{2x^4-x^3-11x^2}{x^2}$

1	-1	-1	-1+	A_3
0	2	2		
1	1	1	A_2	
0	2			
1	3	A_1		
0				
1	A_0			

solve $2x^5+x^4-12x^3-12x^2+x+2=0$

sol: The given eq is an odd degree reciprocal eqn of class -1

$\therefore$ -1 is the root.

$f(x) = 2x^5+x^4-12x^3-12x^2+x+2$

$f(-1) = 2(-1)^5+(-1)^4-12(-1)^3-12(-1)^2+(-1)+2$

$= -2+1+12-12-1+2$

$= 0$

$\therefore x=-1$ is a factor of $f(x)$, then by synthetic division we get

$x=-1$	2	1	-12	-12	1	2
	0	-2	1	11	1	-2
	2	-1	-11	-1	2	$\boxed{0}$

$\therefore f(x) = (x+1)(2x^4-x^3-11x^2-x-12)$

let $f(x)=0$

$x+1=0 \Rightarrow x=-1$

$\Rightarrow 2x^4-x^3-11x^2-x+2=0$

divide the above eqn by 'x^2' on both sides, we get

$\Rightarrow \dfrac{2x^4-x^3-11x^2-x+2}{x^2}=0$

$\Rightarrow 2x^2 - x - 11 - 1/x + 2/x^2 = 0$

$\Rightarrow 2\left(x^2 + \frac{1}{x^2}\right) - 1\left(x + 1/x\right) - 11 = 0$ —— ①

Let $x + 1/x = y$

SOBS, we get

$x^2 + \frac{1}{x^2} + 2x + 1/x = y^2$

$x^2 + 1/x^2 = y^2 - 2$

from eqn ①

$\Rightarrow 2(y^2-2) - y + 11 = 0$

$\Rightarrow 2y^2 - 4 - y - 11 = 0$

$\Rightarrow 2y^2 - y - 15 = 0$

$\Rightarrow 2y^2 - 6y + 5y - 15 = 0$

$\Rightarrow 2y(y-3) + 5(y-3) = 0$

$\Rightarrow (2y+5)(y-3) = 0$

case (i)

$y - 3 = 0$

$\Rightarrow y = 3$

but $y = x + 1/x$

$\Rightarrow x + 1/x = 3$

$\Rightarrow \frac{x^2+1}{x} = 3$

$\Rightarrow x^2 + 1 = 3x$

$\Rightarrow x^2 - 3x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{3 \pm \sqrt{9 - 4(1)(1)}}{2(1)}$

$x = \frac{3 \pm \sqrt{5}}{2}$

case (ii)

$2y + 5 = 0$

$\Rightarrow 2y = -5$

$\Rightarrow y = -5/2$

$\Rightarrow x + 1/x = -5/2$

$\Rightarrow \frac{x^2+1}{x} = -5/2$

$\Rightarrow 2x^2 + 2 = -5x$

$\Rightarrow 2x^2 + 5x + 2 = 0$

$\Rightarrow 2x^2 + 4x + x + 2 = 0$

$\Rightarrow 2x(x+2) + 1(x+2) = 0$

$\Rightarrow (x+2)(2x+1) = 0$

$x + 2 = 0$ $\quad 2x + 1 = 0$

$x = -2$ $\quad x = -1/2$

∴ The roots of given

Eg (3) Solve the eqn $x^5 - 5$

sol: The given eqn is a

eqn of class - two

∴ 1 is root. [∵

$f(1) = 1 - 5 + 9 - 9 + 5 - 1 =$

By synthetic division

$$\begin{array}{c|cccc} x=1 & 1 & -5 & 9 & \\ & 0 & 1 & -4 & \\ \hline & 1 & -4 & 5 & \end{array}$$

$f(x) = (x-1)(x^4 - 4x^3$

let $f(x) = 0$

$x - 1 = 0 \Rightarrow x = 1$

$\Rightarrow x^4 - 4x^3 + 5x^2 - 4x +$

divide the above

$\Rightarrow \frac{x^4 - 4x^3 + 5x^2 - 4x}{x^2}$

$\Rightarrow x^2 - 4x + 5 - 4/x$

$\Rightarrow (x^2 + 1/x^2) - 4($

Let $x + 1/x = y$

SOBS, we

$\Rightarrow x^2 + 1/x^2 + 2 \cdot x \cdot \frac{1}{x}$

$\Rightarrow x^2 + 1/x^2 = y^2 - 2$

from eq ①

∴ The roots of given eqn are $-1, 5, 1/2, -2$ & $3+\sqrt{5}/2$

Ex(3) Solve the eqn $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

Sol: The given eqn is an odd degree reciprocal eqn of class - two

∴ 1 is root. [∵ $a_0 = -a_n$]

$f(1) = 1 - 5 + 9 - 9 + 5 - 1 = 0$

By synthetic division, we get

$x = 1$	1	−5	9	−9	5	−1
	0	1	−4	5	−4	1
	1	−4	5	−4	1	0

$f(x) = (x-1)(x^4 - 4x^3 + 5x^2 - 4x + 1)$

let $f(x) = 0$

$x - 1 = 0 \Rightarrow x = 1$

$\Rightarrow x^4 - 4x^3 + 5x^2 - 4x + 1 = 0$

divide the above eqn by x^2 on b.s, we get

$$\Rightarrow \frac{x^4 - 4x^3 + 5x^2 - 4x + 1}{x^2} = 0$$

$$\Rightarrow x^2 - 4x + 5 - 4/x + 1/x^2 = 0$$

$$\Rightarrow (x^2 + 1/x^2) - 4(x + 1/x) + 5 = 0 \text{ — ①}$$

let $x + 1/x = y$

SOBS, we get

$$\Rightarrow x^2 + 1/x^2 + 2 \cdot x \cdot \frac{1}{x} = y^2$$

$$\Rightarrow x^2 + 1/x^2 = y^2 - 2$$

From eq ①

$\Rightarrow y-2-4y+5=0$

$\Rightarrow y^2-4y+3=0$

$\Rightarrow y^2-3y-y+3=0$

$\Rightarrow y(y-3)-1(y-3)=0$

$\Rightarrow (y-1)(y-3)=0$

case (i)

$\Rightarrow y-1=0$

$\Rightarrow y=1$

$\Rightarrow x+\frac{1}{x}=1$

$\Rightarrow \frac{x^2+1}{x}=1$

$\Rightarrow x^2+1=x$

$\Rightarrow x^2-x+1=0$

$ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{1\pm\sqrt{1-4(1)(1)}}{2(1)}$$

$$x=\frac{1\pm\sqrt{-3}}{2}$$

$$x=\frac{1\pm\sqrt{3(i^2)}}{2} \quad (i^2=-1)$$

$$x=\frac{1\pm i\sqrt{3}}{2}$$

case (ii)

$\Rightarrow y-3=0$

$\Rightarrow y=3$

$\Rightarrow x+\frac{1}{x}=3$

$\Rightarrow \frac{x^2+1}{x}=3$

$\Rightarrow x^2+1=3x$

$\Rightarrow x^2-3x+1=0$

$\Rightarrow x=\frac{3\pm\sqrt{9-4}}{2}$

$\Rightarrow x=\frac{3\pm\sqrt{5}}{2}$

. The roots of the given eqn are,

$\frac{1\pm i\sqrt{3}}{2}$ & $\frac{3+\sqrt{5}}{2}$ //

Example :- 5

Solve the equation $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$

Sol:- The given eq is an even degree reciprocal equation of class two $[a_0 = -a_n]$

$\therefore$ 1 & -1 are the roots, by synthetic division, we get

$x = 1$	6	-25	31	0	-31	25	-6
	0	6	-19	12	12	-19	6
$x = -1$	6	-19	12	12	-19	6	0
	0	-6	25	-37	25	-6	
	6	-25	37	-25	6	0	

$f(x) = (x-1)(x+1)(6x^4 - 25x^3 + 37x^2 - 25x + 6)$

Let $f(x) = 0$

$x - 1 = 0$ $\quad$ $x + 1 = 0$

$x = 1$ $\quad$ $x = -1$