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MATHEMATICS-II

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FUNCTIONS AND LIMITS

I. Properties of functions

1. A relation f form a set A into B is said to be a function or mapping from A into B if for each $x \in A$ there exists a unique $y \in A$ there exists a unique $y \in B$ such that (x,y) $\in f$. If is denoted by $f: A \to B$
2. A relation f form a set A into a set B is said to be a function or mapping from A into B if
   i) $x \in A \Rightarrow f(x): x \in B$
   ii) $x_1, x_2 \in A, x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$
3. If $f: A \to B$ is a function, then A is called domain, B is called condomain and $f(A) = \{f(x): x \in A)$ is called range of f.
4. real numbers: The set of all real numvers is denoted by R.
   i) If $a \in R \, and \, a > 0$ then a is called a postitive real number
   ii) If $a \in R$ and $a \leqslant 0$ then a is called a -ve positive real number
5. The set of all positive real numbers is denoted by R^+. The set of all negative real numbers is denoted by R^-. $\therefore R = R^+ \cup R^- \cup \{0\}$
6. If $x \in R$, then the modulus or the absolute value of x is denoted by $|x|$ and is defined as follows.
   i) If $x \geq 0$, then $|x| = x$.
   ii) If x<0 , then $|x| = -x$

2. Properties of modulus

1. D_f (domain of function) = R $(-\infty, \infty)$
2. R_f (range of function) $[0, \infty) = R^+ \cup \{0\}$
3. $|x|$ is even function
4. $|x|$ is continuous $\forall x \in R$
5. $|x|$ is symmetrical about +ve Y-axis
6. $x \in R \Rightarrow |x| \geq 0$
7. $x \in R \Rightarrow |x| = \max\{x, -x\}$
8. If $x \in R$ then $|x| = 0 \Leftrightarrow x = 0$
9. $x \in R \Rightarrow |x| = |-x|$
10. $x \in R \Rightarrow |x|^2 = x^2$
11. $x, y \in R \Rightarrow |x+y| \leq |x| + [y)$
12. $x, y \in R \Rightarrow |x-y| \geq ||x| - |y||$
13. $x, y \in R \Rightarrow |x+y| \geq ||x| - |y||$
14. $x, y \in R \Rightarrow |x-y| \leq |x| + |y|$
15. $x, y \in R \Rightarrow |xy| = |x||y|$
16. $x, y \in R, y \neq 0 \Rightarrow \left|\frac{x}{y}\right| = \frac{|x|}{|y|}$.
17. If $x, \delta \in R, \delta > 0$ then $|x| < \delta \Leftrightarrow -\delta < x < \delta$
18. If $x, a, \delta \in R, \delta > 0$ then $|x-a| < \delta$
    $\Leftrightarrow a - \delta < x < a + \delta$
19. If $x, \delta \in R, \delta > 0$ then $|x| > \Leftrightarrow x > \delta$ or $x < -\delta$

3. Properties of step function (greatest interger function)

1. $y = f(x) = [x]$
2. $[x] = n; n \leq x < n+1 \quad \forall n \in z$
3. $D_f = R$
4. $R_f = z = \{0, \pm 1, \pm 2, \pm 3 .....\}$
5. $[x]$ is continuous function for $R - \{Z\}$
6. $[x]$ is discontinuous function for Z

7. $[x]+[-x]=0 \quad x\in z$
$\qquad = -1 \quad x\notin z$

Ex: $[9.17]=9$

$[-10.05]=-11 \quad [e]=2$

$[\Pi]=3$

$[0]=0$

$[3]=3$

$0\le x<1 \Rightarrow [x]=0$

$1\le x<2 \Rightarrow [x]=1$

4. **Fractional part of x:-**
1. $y=f(x)=\{x\}\Rightarrow 0\le f(x)<1$
2. $\{x\}=x-[x]$
3. $D_f=R$
4. $R_f=[0,1]$
5. period - 1
6. $\{x\}$ is continuous for R-Z
7. $\{x\}$ is discontinuous for Z

Ex: $\{7.5\}=0.5$
$\{9.01\}=0.01$
$\{2\}=0$
$\{-4.7\}=0.3$

5. **Properties of singnum function**
1. $D_f=R$
2. $R_f=\{-1,0,1\}$
1. $\frac{|x|}{x}$ is continuous
2. $\frac{|x|}{x}$ is odd function
3. Let $f:A\to R$ be a function. Then
i) f is said to be monotonically increasing on A if $x_1,x_2\in A, x_1<x_2\Rightarrow f(x_1)\le f(x_2)$
ii) f is said to be strictly increasing on A if $x_1<x_2\Rightarrow (x_1)<f(x_2)$
iii) f is said to be mononically decreasing on A if $x_1,x_2\in A, x_1<x_2\Rightarrow f(x_1)\ge f(x_2)$
iv) f is said to be monotonic on A if is either monotonically increasing or monotonically decrasing on A.
4. A function $f:A\to R$ is said to be even function if $f(-x)=f(x),\forall x\in A$.
5. A function $f:A\to R$ is said to be an odd function if $f(-x)=-f(x),\forall x\in A$.
6. The function $f:R\to R$ defined as $f(x)=n$ where $n\in Z$ such that $n\le x<n+1,\forall x\in R$ is called step function or greatest interger function. It is denoted by $f(x)=[x]$.
7. i) If f,g are even fg is even
ii) If f,g are odd then fg is even
iii) If f is even function and g is an odd function then fg an odd function

Standard time saving results
1. i) $(x-\alpha)(x-\beta)>0, \alpha<\beta \Rightarrow x<\alpha$ or $x>\beta$
ii) $(x-\alpha)(x-\beta)<0, \alpha<\beta\Rightarrow \alpha<x<\beta$
iii) $x^2-\alpha^2<0\Rightarrow -\alpha<x<\alpha$
2. i) The domain of $\sqrt{x^2-a^2}$ is $[-a,a]$
ii) Domain $\sqrt{x^2-a^2}$ is $(-\infty,a]\cup[a,\infty)$
iii) Domain of $\sqrt{(x-a)(b-x)}$ when $a<b$ is $[a,b]$
iv) Domain of $\sqrt{(x-a)(x-b)}$ when a<b is $(-\infty,a]\cup[b,\infty)$
3. i) The domain of $\frac{1}{\sqrt{a^2-a^2}}$ is $(-a,a)$
ii) Domain of $\frac{1}{\sqrt{x^2-a^2}}$ is $(-\infty,a)\cup(a,\infty)$
iii) Domain of $\frac{1}{\sqrt{(x-a)(a-x)}}$ when a < b is (a,b)
iv) Domain of $\frac{1}{\sqrt{(x-a)(x-b)}}$ when $a<$b is (a,b)
4. i) Domain of $\sqrt{\left[\frac{x-a}{x-b}\right]}$ when $a<b$ is $(-\infty,a]\cup(b,\infty)$
ii) Domain of $\sqrt{\left[\frac{x-a}{x-b}\right]}$ when $a>b$ is $(-\infty,b)\cup[a,\infty)$
iii) Domain of $\sqrt{\left[\frac{a-x}{b-x}\right]}$ when $a<b$ is $[a,b)$.
iv) Domain of $\sqrt{\left[\frac{a-x}{b-x}\right]}$ when $a>b$ is $(b,a]$
5. i) Range of $f(x)=\sqrt{a^2-x^2}$ is $[0,a]$
ii) Range of $f(x)=a\cos x+b\sin x+c$ is $\left[c-\sqrt{a^2+b^2}, c+\sqrt{a^2+b^2}\right]$.

I. **Properties of Limits :**
1. If $f(x)=k$ (constant), then $\underset{x\to a}{Lt} f(x)=k$
2. If $\underset{x\to a}{Lt} f(x)=\ell$, then $\underset{x\to a}{Lt}|f(x)|=|\ell|$. But the converse of the above result is not true. Can be observed from the following example.
$f(x)=1$, if x is rational
$\quad =-1$, if x is irrational
Then for any real number 'a' $\underset{x\to a}{Lt} f(x)$ does not exists. But $|f(x)|=|f(x)|=1\,\forall x\in R$ and
$\underset{x\to a}{Lt} f|(x)|=\underset{x\to a}{Lt}|f(x)|=\underset{x\to a}{Lt} 1=1$
3. $\underset{x\to a}{Lt}(f\pm g)(x)=\underset{x\to a}{Lt} f(x)\pm \underset{x\to a}{Lt} g(x)$
4. $\underset{x\to a}{Lt}(fg)(x)=\underset{x\to a}{Lt} f(x).\ \underset{x\to a}{Lt} g(x)$
5. $\underset{x\to a}{Lt}(f/g)(x)=\underset{x\to a}{Lt} f(x)/\underset{x\to a}{Lt} g(x)$, when $\underset{x\to a}{Lt} g(x)\ne 0$
6. $\underset{x\to a}{Lt} kf(x)=k\ \underset{x\to a}{Lt} f(x)$, for any real 'k'
7. $\underset{x\to a}{Lt}(f(x))^k=\left[\underset{x\to a}{Lt} f(x)\right]^k$
8. If $f(x)\le g(x)$, then $\underset{x\to a}{Lt} f(x)\le \underset{x\to a}{Lt} g(x)$
9. If $f(x)\le h(x)\le g(x)$ on a deleted neighbourhood of 'a' and $\underset{x\to a}{Lt} f(x)=\underset{x\to a}{Lt} g(x)=\ell$, then $\underset{x\to a}{Lt} h(x)=\ell$
10. $\underset{x\to a}{Lt} e^{f(x)}=e^{\underset{x\to a}{Lt} f(x)}$ if $\underset{x\to a}{Lt} f(x)$ exists
11. $\underset{x\to a}{Lt}\log[f(x)]=\log \underset{x\to a}{Lt} f(x)$, if $\underset{x\to a}{Lt} f(x)>0$

Indeterminate form :
If a function is such that for a certain arranged value of the variable involved, its value cannot be found by simply substituting the value fo the variable, the function is said to take indeterminate form

Then forms $\frac{0}{0},\frac{\infty}{\infty}, 0x\infty, \infty x\infty, \infty-\infty, 1^{\infty}, 0^{\infty}, 0^0$ are called indeterminate forms. The limiting form of the indeterminate form is called its true value.

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FORMULA - 1

1. If n is real number, then

$$\underset{x \to a}{Lt} \frac{x^n - a^n}{x - a} = n.a^{n-1}$$

2. if m,n are reals, then

$$\underset{x \to a}{Lt} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n}.a^{m-n}$$

3. $$\underset{x \to 0}{Lt} \frac{(1+x)^m - 1}{x} = m$$

4. $$\underset{x \to 0}{Lt} \frac{(1+x)^m - 1}{(1+x)^n - 1} = \frac{m}{n}$$

FORMULA - 2

1. $$\Sigma n = 1+2+3+....+n = \frac{n(n+1)}{2}$$

2. $$\Sigma n^2 = 1^2+2^2+3^2+...+n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. $$\Sigma n^3 = \frac{n^2(n+1)^2}{4}$$

4. $$\Sigma n^4 = \frac{n(n+1)(2n+1)}{6}.\frac{(3n^2+3n-1)}{5}$$

FORMULA - 3

1. $$\underset{x \to 0}{Lt} \frac{e^x - 1}{x} = 1$$

2. $$\underset{x \to 0}{Lt} \frac{e^{ax} - 1}{x} = a$$

3. $$\underset{x \to 0}{Lt} \frac{e^{ax} - e^{bx}}{x} = (a-b)$$

4. $$\underset{x \to 0}{Lt} \frac{a^x - 1}{x} = \log a$$

5. $$\underset{x \to 0}{Lt} \frac{a^x - b^x}{x} = \log(a/b)$$

6. $$\underset{x \to \infty}{Lt} x\left(a^{1/x} - 1\right) = \log a$$

7. $$\underset{x \to \infty}{Lt} \left(\frac{a^{1/x} - 1}{b^{1/x} - 1}\right) = \log_b a$$

8. $$\underset{x \to \infty}{Lt} x\left(a^{1/x} - b^{1/x}\right) = \log \frac{a}{b}$$

FORMULA - 4

1. $$\underset{\theta \to 0}{Lt} \frac{\sin\theta}{\theta} = 1$$

2. $$\underset{\theta \to 0}{Lt} \frac{\sin a\theta}{\theta} = a$$

3. $$\underset{\theta \to 0}{Lt} \frac{Tan\, a\theta}{\theta} = a$$

4. $$\underset{x \to 0}{Lt} \frac{\sin^{-1} x}{x} = 1$$

5. $$\underset{x \to 0}{Lt} \frac{\tan^{-1} x}{x} = 1$$

6. $$\underset{x \to 0}{Lt} \frac{\sin hx}{x} = 1$$

7. $$\underset{x \to 0}{Lt} \frac{\tanh x}{x} = \underset{x \to 0}{Lt} \frac{\tan h^{-1} x}{x} = \underset{x \to 0}{Lt} \frac{\sinh^{-1} x}{x} = 1$$

8. $$\underset{x \to 0}{Lt} \frac{\sin ax \pm \sin bx}{\sin cx \pm \sin dx} = \frac{a \pm b}{c \pm d}$$

9. $$\underset{x \to 0}{Lt} \frac{\tan ax \pm \tan bx}{\tan cx \pm \tan dx} = \frac{a \pm b}{c \pm d}$$

10. $$\underset{x \to 0}{Lt} \frac{1 - \cos mx}{1 - \cos nx} = \left(\frac{m}{n}\right)^2$$

FORMULA : 5

1. $$\underset{x \to 0}{Lt} \frac{\sin x^0}{x} = 1^0 = \frac{\pi}{180}$$

2. $$\underset{x \to 0}{Lt} \frac{\sin x^0}{x^0} = 1$$

3. $$\underset{x \to 0}{Lt} \frac{\tan x^0}{x} = \frac{\pi}{180}$$

4. $$\underset{x \to 0}{Lt} \frac{\sin x^g}{x} = \frac{\pi}{200}$$

FORMULA : 6

1. $$\underset{x \to 0}{Lt} (1+x)^{1/x} = e$$

2. $$\underset{x \to 0}{Lt} (1+ax)^{1/x} = e^a$$

3. $$\underset{x \to 0}{Lt} (1+x)^{a/x} = e^a$$

4. $$\underset{x \to \infty}{Lt} \left(1+\frac{1}{x}\right)^x = e$$

5. $$\underset{x \to \infty}{Lt} \left(1+\frac{a}{x}\right)^x = e^a$$

6. $$\underset{x \to \infty}{Lt} \left(1+\frac{1}{kx}\right)^x = e^{1/k}$$

7. $$\underset{x \to \infty}{Lt} \left(1+\frac{1}{a+bx}\right)^{c+dx} = e^{d/b}$$

8. $$\underset{x \to 0}{Lt} (\cos x + a\sin bx)^{1/x} = e^{ab}$$

9. $$\underset{x \to 0}{Lt} (\sec x + a\tan bx)^{1/x} = e^{ab}$$

FORMULA : 7

1. $$\underset{x \to \infty}{Lt} \left[\frac{x+a}{x+b}\right]^x = e^{a-b}$$

2. $$\underset{x \to \infty}{Lt} \left[\frac{x+a}{x-b}\right]^x = e^{a+b}$$

3. $$\underset{x \to \infty}{Lt} \left[\frac{x^2+bx+c}{x^2+kx+d}\right]^x = e^{b-k}$$

4. $$\underset{x \to \infty}{Lt} \left[\frac{ax^2+bx+c}{ax^2+dx+e}\right]^{px} = e^{\frac{p(b-d)}{a}}$$

5. $$\underset{x \to \infty}{Lt} \left[1+\frac{1}{a+bx}\right]^{dx+c} = e^{\frac{d}{b}}$$

6. $$\underset{x \to \infty}{Lt} \left[\frac{a_1x^2+b_1x+c_1}{a_2x^2+b_2x+c_2}\right]^{\frac{d_1x+c_1}{d_2x+c_2}} = \left(\frac{a_1}{a_2}\right)^{\frac{d_1}{d_2}}$$

Ex 1: $$\underset{x \to \infty}{Lt} \left[\frac{x^2+4x-3}{x^2-3x+5}\right]^x = e^{4+3} = e^7$$

Ex 2: $$\underset{x \to \infty}{Lt} \left[\frac{x+5}{x-3}\right]^x = e^{5+3} = e^8$$

FORMULA : 8

1. $$\underset{x \to 0}{Lt} \frac{x^k.\tan^m ax}{\sin^n bx} = \frac{a^m}{b^n}, \text{ if } k+m=n$$

2. $$\underset{x \to 0}{Lt} \frac{x^k.\sin^m ax}{\tan^n bx} = \frac{a^m}{b^n}, \text{ if } k+m=n$$

Ex 3: $$\underset{x \to 0}{Lt} \frac{x^5.\tan^3 3x}{\sin^8 2x} = \frac{3^3}{2^8} = \frac{27}{2^8}$$

Ex 4: $$\underset{x \to 0}{Lt} \frac{x^2 \sin 4x}{\tan^3 2x} = \frac{4}{2^3} = \frac{1}{2}$$

FORMULA : 9

1. $$\underset{x \to \infty}{Lt} \left[\sqrt{x^2+ax+b} - x\right] = \frac{a}{2}$$

2. $$\underset{x \to 0}{Lt} \frac{\sqrt[n]{a+x} - \sqrt[n]{a}}{x} = \frac{1}{n}a^{1/n-1}$$

3. $$\underset{x \to 0}{Lt} \frac{\sqrt[n]{a+bx} - \sqrt[n]{a}}{x} = \frac{b}{n}a^{1/n-1}$$

4. $$\underset{x \to 0}{Lt} \frac{\sqrt[n]{a+x+x^2+....x^m} - \sqrt[n]{a}}{x} = \frac{1}{n}a^{1/n-1}$$

FORMULA : 10

1. $\underset{x\to0}{Lt}\dfrac{\sqrt[n]{1+\sin x}-1}{x}=\dfrac{1}{n}$

2. $\underset{x\to0}{Lt}\dfrac{\sqrt[n]{1+\sin x}-\sqrt[n]{1-\sin x}}{x}=\dfrac{2}{n}$

3. $\underset{x\to0}{Lt}\dfrac{\sqrt[n]{1+a\sin x}-1}{x}=\dfrac{a}{n}$

FORMULA - 11

1. $\underset{x\to0}{Lt}\left(\dfrac{a^x+b^x}{2}\right)^{1/x}=\sqrt{ab}$

2. $\underset{x\to\infty}{Lt}\left(\dfrac{a^{\frac{1}{x}}+b^{\frac{1}{x}}}{2}\right)^{x}=\sqrt{ab}$

3. $\underset{x\to0}{Lt}\left(\dfrac{a^x+b^x}{2}\right)^{n/x}=\left(\sqrt{ab}\right)^n$

4. $\underset{x\to0}{Lt}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{1/x}=\sqrt[3]{abc}$

5. $\underset{x\to\infty}{Lt}\left(\dfrac{a^{\frac{1}{x}}+b^{\frac{1}{x}}+c^{\frac{1}{x}}}{3}\right)^{x}=\sqrt[3]{abc}$

6. $\underset{x\to0}{Lt}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{n/x}=\left(\sqrt[3]{abc}\right)^n$

7. $\underset{x\to0}{Lt}\left(\dfrac{a_1^x+a_2^x+...+a_n^x}{n}\right)^{1/x}=\sqrt[n]{a_1a_2....a_n}$

8. $\underset{x\to\infty}{Lt}\left[\dfrac{a_1^{1/x}+a_2^{1/x}+...a_n^{1/x}}{n}\right]^{x}=\sqrt[n]{a_1a_2...a_n}$

9. $\underset{x\to0}{Lt}\left(\dfrac{a_1^x+a_2^x+....+a_n^x}{n}\right)^{k/x}=\left(\sqrt[n]{a_1a_2....a_n}\right)^k$

FORMULA : 12

1. $\underset{x\to a}{Lt}\dfrac{|x-a|}{x-a}=$ does not exist

2. $\underset{x\to a+}{Lt}\dfrac{|x-a|}{x-a}=1$

3. $\underset{x\to a-}{Lt}\dfrac{|x-a|}{x-a}=-1$

FORMULA : 13

1. $\underset{x\to a}{Lt}[x]=$ does not exist, if 'a' is any integer $=[a]$, if 'a' is not integer

2. $\underset{x\to a}{Lt}\{x\}=$ does not exist, if 'a' is any integer $=\{a\}$, if 'a' is not integer

   Ex 5 : $\underset{x\to\sqrt{2}}{Lt}[x]=\lfloor\sqrt{2}\rfloor=1$

   Ex 6 : $\underset{x\to7/2}{Lt}[x]=$ does not exist

FORMULA : 14

1. $\underset{x\to\infty}{Lt}\dfrac{[ax+b]}{x}=a$ where $a>0$

2. $\underset{x\to\infty}{Lt}\dfrac{[x]}{x}=1$

3. $\underset{n\to\infty}{Lt}\dfrac{[1x]+[2x]+[3x]+....+[nx]}{n^2}=\dfrac{x}{2}$

4. $\underset{n\to\infty}{Lt}\dfrac{[1^2x]+[2^2x]+[3^2x]+....+[n^2x]}{n^3}=\dfrac{x}{3}$

FORMULA : 15

1. $\underset{n\to\infty}{Lt}\Sigma r^n=0$ if $|r|<1=\infty$ if $|r|>1$

2. $\underset{n\to\infty}{Lt}\left[a+ar+ar^2+.....+ar^{n-1}\right]=\dfrac{a}{1-r}, if$ $0<r<1=\infty$ if $r\geq1$

   Ex 8 : $\underset{n\to\infty}{Lt}\lfloor 2+2^2+2^3+....+2^n\rfloor=\infty$

   Let, $\phi=\{x,\sin x,\tan x,Tan^{-1}x,\sin^{-1}x,$
   $\sinh x,\tanh x,\sinh^{-1}x,\tanh^{-1}x\}$

   Then

FORMAT-1

$$\underset{x\to0}{Lt}\dfrac{f_1(ax)\pm f_2(bx)}{g_1(cx)\pm g_2(dx)}=\dfrac{a\pm b}{c\pm d}$$

Where $f_1,f_2,g_1,g_2\in\phi$ and f_1,f_2g_1,g_2 are first degree function.

FORMAT-2

$$\underset{x\to0}{Lt}\dfrac{f_1^m(ax).g_1^n(bx)}{f_2^p(cx).g_2^q(dx)}$$

where $f_1f_2,g_1,g_2\in\phi$

If m+n=p+q then $\dfrac{a^mb^n}{c^pd^q}$

If m+n>p+q then 0

If m+n<p+q then ∞ or Doesnot exist

FORMAT-3

$$\underset{x\to a}{Lt}\dfrac{f(x)}{x}=\dfrac{0}{0}$$

where f(x) $\in$ exponential functions

then Answer $=\log\left[\dfrac{\text{product of +ve term bases}}{\text{product of-ve term bases}}\right]$

Ex:

1. $\underset{x\to0}{Lt}\dfrac{a^x-1}{x}=\log\left(\dfrac{a}{1}\right)=\log a$

2. $\underset{x\to0}{Lt}\dfrac{3^x+2^x-5^x-1}{x}=\dfrac{1+1-1}{0}=\dfrac{0}{0}$

   $=\log\left(\dfrac{3\times2}{5}\right)$

   $=\log\left(\dfrac{6}{5}\right)$

3. $\underset{x\to0}{Lt}\left(\dfrac{a^x-1}{b^x-1}\right)$

   $=\underset{x\to0}{Lt}\dfrac{\left(\dfrac{a^x-1}{x}\right)}{\left(\dfrac{b^x-1}{x}\right)}=\dfrac{\log(a)}{\log(b)}=\log_b^a$

4. $\underset{x\to0}{Lt}\dfrac{(ab)^x-a^x-b^x+1}{x^2}=\log a\log b$

Ex:

$$\underset{x\to0}{Lt}\dfrac{6^x-3^x-2^x+1}{x^2}=\log3\log2$$

INDETERMINANT FORMS:-

$$\left[\dfrac{0}{0},\pm\dfrac{\infty}{\infty},0\times\pm\infty,\infty-\infty,0^0,\infty^0,1^{\infty}\right]$$

FORMAT-1

METHOD-1

L Hospital rule can be used to the form $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$.

If $\dfrac{f(a)}{g(a)}=\dfrac{0}{0}or\dfrac{\infty}{\infty}$, then

$$\underset{x\to a}{Lt}\dfrac{f(x)}{g(x)}=\underset{x\to a}{Lt}\dfrac{f^1(x)}{g^1(x)}$$

METHOD : 2 for $\pm\dfrac{\infty}{\infty}\Rightarrow$

$$\dfrac{\text{Take common of highest degree term in numerator}}{\text{Take common of highest degree term in denominator}}$$

$$\underset{x\to\infty}{Lt}\dfrac{a_0x^m+a_1x^{m-1}+a_2x^{m-2}+........a_m}{b_0x^n+b_1x^{n-1}+b_2x^{n-2}+.....b_n}(a_0,b_0\neq0)$$

$=\dfrac{a_0}{b_0};m=n$

$=0;$ m < n

$=\infty;a_0b_0>0\,\&\,m>n$

$=-\infty;a_0b_0<0\,\&\,m>n$

FORMAT:2 $(0 \times \pm\infty)$

Ex: $\underset{x \to 1}{Lt} (1-x)\tan\left(\frac{\Pi x}{2}\right) = 0 \times \infty \Rightarrow$

$\underset{x \to 1}{Lt} \frac{1-x}{\cot\left(\frac{\Pi x}{2}\right)} = \frac{0}{0}$ *then apply L hospital rule*

FORMAT:3 $(\infty - \infty)$

Ex:

$\underset{x \to 0}{Lt}\left[\frac{1}{x} - \frac{1}{\sin x}\right] = \frac{1}{0} - \frac{1}{0} = \infty - \infty$

Take LCM and rationalising

$\underset{x \to 0}{Lt}\left[\frac{\sin x - x}{x \sin x}\right] = \frac{0-0}{0 \times 0} = \frac{0}{0}$

then apply L hosptial rule.

FORMAT:4 $(0^0, \infty^0)$

$\underset{x \to a}{Lt} (f(x))^{g(x)} = e^{\underset{x \to a}{Lt}\, g(x)\log f(x)}$

FORMAT:5 $(1^{\pm\infty})$

$\underset{x \to a}{Lt} [f(x)]^{g(x)} \Rightarrow 1^{\pm\infty}$ *then answer* $= e^{\underset{x \to a}{Lt}[g(x)](f(x)-1)}$

THERE IS NO SUBSTITUTE TO HARDWORK

PRACTICE SET - I

01. $\underset{x \to 2}{Lt} \frac{x^3\sqrt{x} - 8\sqrt{2}}{x-2} =$

(1) $14\sqrt{2}$ (2) $12\sqrt{2}$ (3) $8\sqrt{2}$ (4) $4\sqrt{2}$

02. $\underset{x \to -2}{Lt} \frac{7x^3 + 11x - 6}{3x^3 + x - 10} =$

(1) 11 (2) $\frac{11}{17}$ (3) $\frac{17}{11}$ (4) $\frac{17}{13}$

03. $\underset{x \to 1}{Lt} \frac{x^{-3/4} - 1}{x^{4/3} - 1} =$

(1) -1 (2) 1 (3) $\frac{-9}{16}$ (4) $\frac{-16}{9}$

04. $\underset{x \to 0}{Lt} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{x^2}$

(1) 0 (2) -1 (3) 1 (4) 2

05. $\underset{x \to 0}{Lt} \frac{x}{\sqrt{9-x} - \sqrt{9+x}}$

(1) -3 (2) -6 (3) -9 (4) 0

06. $\underset{x \to 2^-}{Lt} \frac{x^2 - 4}{|x-2|}$

(1) -4 (2) 4 (3) 0 (4) None

07. $\underset{x \to 0}{Lt} \frac{\tan x^2}{|x|}$

(1) 0 (2) 1 (3) does not exist (4) None

08. $\underset{x \to 0}{Lt} \frac{\sqrt[3]{1+x^2} - \sqrt[3]{1-x^2}}{x^2} =$

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) 1 (4) does not exist

09. $\underset{x \to \infty}{Lt}\, x\left(a^{\frac{1}{x}} - b^{\frac{1}{x}}\right)$

(1) 0 (2) $\log(a+b)\log(a-b)$

(3) $\log\frac{b}{a}$ (4) $\log\frac{a}{b}$

10. $\underset{x \to -1}{Lt} \frac{x+1}{\sqrt{x^2+3} - 2}$

(1) 2 (2) $\frac{1}{2}$ (3) $-\frac{1}{2}$ (4) -2

11. $\underset{x \to 0}{Lt} \sqrt{x}\sin\left(\frac{\theta}{x}\right)$

(1) 0 (2) 1 (3) α (4) None

12. $\underset{x \to 0}{Lt} \frac{x - \sin x}{x - \tan x}$

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) 2 (4) -2

13. $\underset{x \to \infty}{Lt}\left(\frac{x}{x+2}\right)^x$

(1) e (2) e^2 (3) e^{-1} (4) e^{-2}

14. $\underset{x \to 0}{Lt} \frac{e^{\sin x} - e^x}{\sin x - x}$

(1) e (2) $\frac{1}{e}$ (3) 1 (4) 0.

15. $\underset{x \to 1^-}{Lt} \frac{\sqrt{1-x}}{\sqrt{1-x^2}}$

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) $\sqrt{2}$ (4) 2

16. $\underset{x \to \infty}{Lt} \frac{2.3^{n+1} - 3.5^{n+1}}{5.3^n - 4.5^n}$

(1) $\frac{6}{5}$ (2) $\frac{15}{4}$ (3) $\frac{-3}{5}$ (4) $\frac{-3}{5}$

17. $\underset{x \to \frac{\pi}{4}}{Lt} \frac{1 - \tan x}{\left(\frac{\pi}{4} - x\right)\sec x}$

(1) 2 (2) -2 (3) $\sqrt{2}$ (4) $-\sqrt{2}$

18. $\underset{x \to 0}{Lt} \frac{\sqrt[5]{1+5ax} - \sqrt[3]{1-5ax}}{x}$

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) $\frac{3}{5}$ (4) 5

19. $\underset{x \to \infty}{Lt}\left(\frac{x+3}{x-1}\right)^{x+1}$

(1) e (2) e^{-1} (3) e^3 (4) e^4

20. $\underset{x \to 0}{Lt} \frac{\int_0^x \tan^2 x \cdot \sec^2 x \, dx}{x^3}$

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 1 (4) None

21. $\underset{x \to 0}{Lt} \frac{\int_0^x \sin^{-1} x \, dx}{x^2}$

(1) 1 (2) 0.5 (3) 0.25 (4) None

22. $\underset{x \to 0}{Lt} \frac{e^x - 1}{1 - \sqrt{1-x}}$

(1) 1 (2) $\frac{1}{2}$ (3) 2 (4) None

23. $\underset{x \to 0}{Lt} \frac{2\sin x - \sin 2x}{x^3}$

(1) $\frac{1}{2}$ (2) 2 (3) 1 (4) None

24. $\underset{x \to 0}{Lt}\, x(\operatorname{cosec} x + \cot x)$

(1) 1 (2) 2 (3) 3 (4) $\frac{1}{2}$

25. $\underset{x \to 0}{Lt} \dfrac{\cos 9x - \cos 7x}{\cos 3x - \cos 7x}$

(1) $\frac{4}{5}$ (2) $\frac{5}{4}$ (3) $\frac{-4}{5}$ (4) $\frac{-5}{4}$

26. $\underset{x \to 0}{Lt} \dfrac{\tan^{-1} x - \sin^{-1} x}{x^3}$

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) 2 (4) -2

27. $\underset{x \to 0}{Lt} \dfrac{e^{x^2} - \cos x}{x^2}$

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) $\frac{2}{3}$ (4) $\frac{1}{3}$

28. $\underset{x \to \infty}{Lt} x^{5/2}\left(\sqrt{x^5+1} - \sqrt{x^5-1}\right) =$

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) 1 (4) -1

29. $\underset{x \to 0}{Lt} \dfrac{x + x^2 + x^3 + \ldots + x^n - n}{x-1} =$

(1) $\frac{n}{2}$ (2) $\frac{n+1}{2}$

(3) $\frac{n(n-1)}{2}$ (4) $\frac{n(n+1)}{2}$

30. $\underset{x \to 0}{Lt} \dfrac{1}{x} \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) =$

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) 2 (4) 1

31. $\underset{x \to a}{Lt} \left(\dfrac{\sin x}{\sin a}\right)^{\frac{1}{x-a}} =$

(1) $e^{\sin a}$ (2) $e^{\cos a}$ (3) $e^{\tan a}$ (4) $e^{\cot a}$

32. $\underset{x \to 1}{Lt} (1 + \sin \pi x)^{\cot \pi x} =$

(1) e (2) $\frac{1}{e}$ (3) 1 (4) 0

33. $\underset{x \to 1}{Lt} \dfrac{x^n - 1}{x \log x} =$

(1) log2 (2) e (3) 1 (4) n

34. $\underset{x \to 0}{Lt} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 - 1}} =$

(1) 1 (2) -1 (3) -2 (4) -4

35. $\underset{x \to 0}{Lt} \dfrac{1 - \cos x}{x \cos x} =$

(1) $\sqrt{2}$ (2) 0 (3) 2 (4) $\frac{1}{2}$

36. $\underset{x \to a}{Lt} \dfrac{\sin x - \sin a}{\tan x - \tan a}$

(1) $\sin^3 a$ (2) $\operatorname{cosec}^2 a$

(3) $\cos^3 a$ (4) $\sec^3 a$

37. If $\underset{x \to 0}{Lt} \dfrac{a \cos x - b}{x^2} = 1$ then $a+b$

(1) 0 (2) 4 (3) -4 (4) -2

38. $\underset{x \to 0}{Lt} \left(\dfrac{1 + \sin x}{1 - \tan x}\right)^{\operatorname{cosec} x} =$

(1) e (2) e^2 (3) e^3 (4) 1

39. $\underset{x \to 0}{Lt} \dfrac{\log\left(x + \sqrt{x^2 - 1}\right)^2}{x}$

(1) 1 (2) 2 (3) -1 (4) 1/2

40. $\underset{x \to 0}{Lt} \left(\dfrac{e^x + e^{-x}}{2}\right)^{\frac{1}{x}}$

(1) e (2) e^2 (3) e^3 (4) 1

41. $\underset{x \to 0}{Lt} \left(\operatorname{cosec}^2 x - \operatorname{cosec} h^2 x\right)$

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{-2}{3}$ (4) 0

42. $\underset{x \to 0}{Lt} \cot x \log \cos x$

(1) 0 (2) 1 (3) α (4) $-\alpha$

43. $\underset{n \to \infty}{Lt} \dfrac{1}{n^3}\left[(1 + 5 + 9 + 13 + \ldots + (4n-3)\right]$

(1) 4 (2) 3 (3) 2 (4) 1

44. $\underset{x \to \infty}{Lt} \dfrac{1}{n^3}\left[(1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1)\right]$

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) 3

45. $\underset{n \to \infty}{Lt} \dfrac{1}{n^4}(1.2.3 + 2.3.4 + 3.4.5 + \ldots \text{up to n terms})$

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

46. $\underset{x \to -\infty}{Lt} \dfrac{5|x| + 2x}{5|x| + x}$

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{7}{4}$ (4) $\frac{3}{4}$

47. $\underset{n \to \infty}{Lt} \dfrac{(n+1)!}{(n+1)! - n!}$

(1) 0 (2) 1 (3) α (4) -1

48. $\underset{x \to 1.5}{Lt} [x^2]$

(1) 2 (2) -2

(3) 3 (4) does not exist

49. $\underset{x \to \sqrt{2}}{Lt} [x^3]$

(1) 2 (2) 1

(3) 3 (4) does not exist

50. $\underset{x \to 0}{Lt} \dfrac{(8^x - 4^x - 2^x + 1)}{1 - \cos 2x}$

(1) log 2 (2) log 4 (3) log 8 (4) $(\log 2)^2$

51. $\underset{x \to \infty}{Lt} x\left[\log(x+3) - \log(x+1)\right]$

(1) 0 (2) 3 (3) 1 (4) 2

52. If $|x|$ then $\underset{n \to \infty}{Lt} (1+x)(1+x^2)(1+x^4)\ldots(1+x^{2n}) =$

(1) $\frac{1}{x-1}$ (2) $\frac{1}{1-x}$ (3) 0 (4) $(1-x)$

53. $\underset{x \to 0}{Lt} \dfrac{1}{x} \sec^{-1}\left(\dfrac{1+x^2}{1-x^2}\right)$

(1) 2 (2) 1

(3) 0 (4) does not exist

54. $\underset{x \to 0}{Lt} \dfrac{\tan^4 x - \sin^4 x}{x^6}$

(1) 2 (2) $\frac{2}{3}$ (3) $\frac{1}{30}$ (4) 0

55. $\underset{x \to \tan^{-1} 2}{Lt} \dfrac{\tan^2 x + \tan x - 6}{\tan^2 x - 5 \tan x + 6}$

(1) 5 (2) -5 (3) $\frac{1}{3}$ (4) $-\frac{1}{5}$

56. $\underset{x \to 0}{Lt} \dfrac{1 - \cos 3x - \cos 5x + \cos 3x \cos 5x}{x^4}$

(1) $\frac{15}{2}$ (2) $\frac{15}{4}$ (3) $\frac{285}{2}$ (4) $\frac{225}{4}$

57. $\underset{x \to \frac{\pi}{4}}{Lt} \dfrac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$

(1) 0 (2) 2 (3) 3 (4) 4

58. $\underset{x \to 0}{Lt} \dfrac{3^x - \cos 3x}{9^x - \cos 9x}$

(1) 2 (2) $\frac{1}{2}$ (3) log 3 (4) $\frac{1}{\log 3}$

59. $\underset{x \to \frac{\pi}{2}}{Lt} \cos 3x \sec 5x$

(1) $\frac{3}{5}$ (2) $\frac{5}{3}$ (3) $-\frac{3}{5}$ (4) $-\frac{5}{3}$

60. $\underset{x \to 0}{Lt} \sin 3x \operatorname{cosec} 5x$

(1) $\frac{3}{5}$ (2) $\frac{5}{3}$ (3) $\frac{-3}{5}$ (4) $\frac{-5}{3}$

61. $\underset{x \to e}{Lt} [x]$

(1) e (2) 1 (3) 2 (4) 0

62. $\underset{x\to 0}{Lt}\left(\operatorname{cosec}x-\frac{1}{x}\right)$

(1) 0 (2) 1 (3) 2 (4) 0

63. $\underset{x\to 0}{Lt}\frac{\tan x-\sin x}{x^3}$

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{1}{6}$

64. $\underset{x\to 0}{Lt}\frac{\sin 3x-\sin 5x}{\tan 5x-\tan 9x}$

(1) $\frac{1}{2}$ (2) $\frac{7}{2}$ (3) 2 (4) $\frac{2}{7}$

65. $\underset{x\to \frac{\pi}{2}}{Lt}\frac{\cot x-\cos x}{\left(\frac{\pi}{2}-x\right)^3}$

(1) 0 (2) 1 (3) 2 (4) -2

66. $\underset{x\to \frac{1}{2}}{Lt}\frac{\cot^2 \pi x}{1-\operatorname{cosec}\pi x}$

(1) -1 (2) 1 (3) 2 (4) -2

67. $\underset{x\to 0}{Lt}\frac{\sqrt[3]{1+\sin x}-\sqrt[3]{1-\sin x}}{x}$

(1) 0 (2) 1 (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

68. $\underset{x\to 0}{Lt}\frac{\log(1+ax)-\log(1-bx)}{x}=$

(1) $a-b$ (2) $a+b$ (3) $\frac{a}{b}$ (4) $\frac{b}{a}$

69. $\underset{x\to 0}{Lt}\frac{x\cos x-\log(1+x)}{x^2}=$

(1) 0 (2) 1
(3) 1/2 (4) None of these

70. If $f(x)=\sqrt{17-x^2}$ then $\underset{x\to 1}{Lt}\frac{f(x)-f(1)}{x-1}$

(1) $\frac{1}{4}$ (2) $-\frac{1}{4}$ (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{-1}{3\sqrt{2}}$

71. $\underset{x\to 0}{Lt}\frac{x\cos x-\sin x}{x^2\sin x}$

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

72. $\underset{x\to \infty}{Lt}(x)^{\frac{1}{x}}$

(1) 1 (2) e (3) $\frac{1}{e}$ (4) None

73. $\underset{x\to 0}{Lt}\frac{\sqrt{1+x}-\sqrt[3]{1-x}}{x}=$

(1) $\frac{1}{6}$ (2) $\frac{1}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

74. $\underset{x\to \frac{\pi}{6}}{Lt}\frac{\cot^3 x-3\cot x}{\sin\left(x-\frac{\pi}{6}\right)}=$

(1) 0 (2) ∞ (3) 24 (4) -24

75. $\underset{x\to 0}{Lt}\frac{\log\cos x}{x^2}=$

(1) 0 (2) ∞ (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

76. $\underset{x\to 0}{Lt}\frac{a^x-1}{\sqrt{1+x}-1}=$

(1) $\log_e^a$ (2) $\frac{1}{2}\log_e^a$ (3) $a\log_e^2$ (4) $a\log_2^e$

77. $\underset{x\to 0}{Lt}\left[\frac{1+\tan x}{1+\sin x}\right]^{\operatorname{cosec}x}=$

(1) e (2) $\frac{1}{e}$ (3) 1 (4) None

78. $\underset{x\to 1}{Lt}\frac{x^x-1}{x\log x}=$

(1) 0 (2) 1
(3) -1 (4) does not exist

79. $\underset{x\to 2^-}{Lt}\frac{|x-1|}{x-1}=$

(1) 0 (2) -1
(3) 1 (4) does not exist

80. $\underset{x\to \infty}{Lt}\left(\frac{2^{\frac{1}{x}}+4^{\frac{1}{x}}+8^{\frac{1}{x}}+16^{\frac{1}{x}}}{4}\right)^x=$

(1) $\sqrt{32}$ (2) 32 (3) 16 (4) 8

81. $\underset{x\to \infty}{Lt}\left(\frac{x-2}{x+1}\right)^{x+3}=$

(1) $\frac{1}{e}$ (2) $\frac{1}{e^2}$ (3) $\frac{1}{e^3}$ (4) $\frac{1}{e}$

82. $\underset{x\to 0}{Lt}\frac{1-\cos 3x}{\sin^{-1}2x.\sin^{-1}6x}=$

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$ (3) $\frac{3}{8}$ (4) $\frac{1}{4}$

83. $\underset{x\to 0}{Lt}\frac{1-\cos^4 2x}{\sin^{-1}(3x)\tan^{-1}(2x)}$

(1) $\frac{2}{3}$ (2) $\frac{8}{3}$ (3) $\frac{4}{3}$ (4) $\frac{1}{2}$

84. $\underset{x\to 0}{Lt}\frac{\sqrt{4+x^3}-2}{\left(\tan^{-1}x\right)^3}$

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{6}$

85. $\underset{x\to \infty}{Lt}\left(\frac{x^3-1}{x^3+1}\right)^{x^3+3}$

(1) 1^1 (2) e^{-1} (3) e^{-2} (4) e^{-3}

86. $\underset{x\to 0}{Lt}\left[\cot\left(\frac{\pi}{4}+x\right)\right]^{\frac{1}{x}}$

(1) e (2) $\frac{1}{e}$ (3) e^2 (4) $\frac{1}{e^2}$

87. $\underset{n\to \infty}{Lt}\left(\frac{1}{n^2}+\frac{n+1}{n^2}+\frac{n+2}{n^2}+....+\frac{3n}{n^2}\right)$

(1) 2 (2) 3 (3) 4 (4) 0

88. $\underset{x\to \infty}{Lt}\left(\frac{3x+2\sin x}{6x-5\cos x}\right)^{\frac{1}{2}}=$

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4}$ (4) 1

89. $\underset{x\to 0}{Lt}(\cos x+2\sin 3x)^{\frac{1}{2x}}=$

(1) e^2 (2) e^3 (3) e^{12} (4) e

90. $\underset{x\to 0}{Lt}\cot x\left(3^{\sin x}-1\right)$

(1) 0 (2) 1 (3) log 2 (4) log 3

91. $\underset{a\to b}{Lt}\frac{a^b-b^a}{a^a-b^b}$

(1) $\log_{\frac{b}{e}}(be)$ (2) $\log_{ae}\left(\frac{e}{a}\right)$

(3) $\log_{be}\left(\frac{e}{b}\right)$ (4) 0

92. $\underset{x\to 1}{Lt}\frac{1-\sqrt{x}}{\left(\cos^{-1}x\right)^2}$

(1) $\frac{1}{6}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) 1

93. $\underset{x\to \frac{1}{\sqrt{2}}}{Lt}\frac{x\cos\left(2\sin^{-1}x\right)}{1-\tan\left(\sin^{-1}x\right)}$

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) 1 (4) $\sqrt{2}$

94. $\underset{x\to \pi}{Lt}\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{-1}{4}$ (4) $\frac{1}{4}$

95. $\underset{x\to\frac{\pi}{2}}{Lt}\left[x\tan x-\frac{\pi}{2}\sec x\right]$

(1) 0 (2) α (3) 1 (4) -1

96. $\underset{x\to 0}{Lt}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$

(1) e (2) $\frac{1}{2}$ (3) $e^{\frac{1}{2}}$ (4) 0

97. $\underset{x\to\frac{\pi}{4}}{Lt}\frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$

(1) $\frac{1}{2}$ (2) 1
(3) 2 (4) None of these

98. $\underset{x\to\infty}{Lt}\left(\frac{2^{\frac{1}{x}}+3^{\frac{1}{x}}+4^{\frac{1}{x}}+5^{\frac{1}{x}}}{4}\right)^{4x}$

(1) 30 (2) $\sqrt{30}$ (3) 120 (4) $\sqrt[4]{120}$

99. $\underset{x\to -1^+}{Lt}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}=$

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{2\pi}}$ (3) $\frac{1}{\sqrt{\pi}}$ (4) $\frac{1}{2\sqrt{\pi}}$

100. $\underset{x\to 0}{Lt}\frac{\tan x-x}{x^2\tan x}$

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{6}$

PRACTICE SET-I KEY

1) 1	2) 3	3) 3	4) 3	5) 1
6) 1	7) 1	8) 2	9) 4	10) 4
11) 1	12) 2	13) 4	14) 1	15) 1
16) 2	17) 3	18) 2	19) 4	20) 2
21) 2	22) 3	23) 3	24) 2	25) 3
26) 2	27) 1	28) 3	29) 4	30) 3
31) 4	32) 2	33) 4	34) 4	35) 2
36) 3	37) 3	38) 2	39) 2	40) 4
41) 1	42) 1	43) 2	44) 3	45) 4
46) 4	47) 2	48) 1	49) 1	50) 4
51) 4	52) 2	53) 1	54) 1	55) 2
56) 4	57) 2	58) 2	59) 3	60) 1
61) 3	62) 1	63) 1	64) 1	65) 4
66) 4	67) 3	68) 2	69) 3	70) 2
71) 4	72) 2	73) 1	74) 4	75) 3
76) 1	77) 1	78) 2	79) 2	80) 1
81) 3	82) 3	83) 3	84) 3	85) 3
86) 4	87) 3	88) 2	89) 2	90) 4
91) 2	92) 2	93) 2	94) 4	95) 4
96) 3	97) 1	98) 4	99) 2	100) 2

ALL POWER IS WITHIN YOU

YOU CAN DO ANYTHING AND EVERYTHING

PRACTICE SET - II

01. $f(x)=\frac{|x+3|}{x+3}$, at $x=-3$ $f(x)$ is

(1) Continuous (2) discontinuous
(3) differentiable (4) None

02. The value of K when $f(x)=\frac{1-\sin 2x}{(\pi-4x)^2}$ for $x\neq\frac{\pi}{4}$ and $f\left(\frac{\pi}{4}\right)=K$ is continuous at $x=\frac{\pi}{4}$

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{8}$ (4) $\frac{1}{16}$

03. If $f(x)=a[x+n]+b[x-n]$, where $[x]$ is the integral part function is continuous at $x=n, n\in Z$. then'

(1) $a+b=0$ (2) $a-b$
(3) $2a-b=0$ (4) $a-2b=0$

04. If $f(x)=(1-x)^{\frac{3}{x}}$ is continuous at $x=0$ then $f(0)$

(1) e^3 (2) e^{-3} (3) e^2 (4) e^{-3}

05. If $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2} & for\ x<0\\ k & for\ x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & for\ x>0\end{cases}$ is continuous at $x=0$ then K =

(1) -16 (2) -8 (3) 16 (4) 8

06. If $f(x)=\frac{1}{x^{1-x}}$ for $x\neq 1$ and $f(x)$ is continuous at $x=1$ Then $f(1)$ is

(1) 0 (2) 1 (3) e (4) $\frac{1}{e}$

07. If the function $f(x)=\begin{cases}\left(\frac{Tan x}{x}\right)^{\frac{1}{x^2}} & x\neq 0\\ a & x=0\end{cases}$ is continuous at $x=0$. Then the value of a is

(1) 0 (2) e (3) $e^{\frac{1}{2}}$ (4) $e^{\frac{1}{3}}$

08. The value of $f(0)$, So that the function $f(x)=\frac{3-(8-2x)^{\frac{1}{4}}}{(5x+27)^{\frac{1}{3}}-3}$ (for $x\neq 0$) continuous every where is given by

(1) $\frac{1}{2}$ (2) $\frac{1}{5}$ (3) $\frac{1}{10}$ (4) $\frac{1}{7}$

09. If $f(x)=\left[\frac{\sqrt{1+ax}-\sqrt{1-ax}}{x}\right.$ $-1\le x<0$ and $\frac{3x-2}{5x-1}$ $0\le x\le 1$ is continuous on $[-1,1]$. Then $a=$

(1) $\frac{1}{2}$ (2) 2 (3) $-\frac{1}{2}$ (4) -2

10. If $f(x)=\frac{1+3x^2-\cos 2x}{x^2}, x\neq 0$ is continuous at $x=0$ then $f(0)=$

(1) 2 (2) 5 (3) 6 (4) 0

11. If $f:|R\to|R$ is continuous such that $f(x+y)=f(x)+f(y)$ $x,y\in|R$ and $f(1)=3$ Then $f(60)$

(1) 30 (2) 60 (3) 120 (4) 180

12. If $f(x)=\frac{\sec^2 ax-1}{x^2}, x\neq 0$ and $f(0)=9$ is continuous at $x=0$ then the value of 'a'

(1) 3 (2) -3 (3) 9 (4) ± 3

13. The value of $f(0)$ so that $f(x)=\dfrac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ is continuous at $x=0$ is

(1) 0 (2) $\sqrt{a}$ (3) $-a\sqrt{a}$ (4) $-\sqrt{a}$

14. The number of points at which $f(x)=\dfrac{1}{\log|x|}$ is discontinuous

(1) 1 (2) 2 (3) 1 (4) None

15. If $f(x)=\dfrac{4x}{x-3}$ if $x<2$ and $\dfrac{x+1}{3}, x\geq 2$ and $=5$ if $x=1$ is continuous on

(1) $\mathbb{R}$ (2) $\mathbb{R}-\{3\}$

(3) $\mathbb{R}-\{2,3\}$ (4) N

16. If $f(x)=\dfrac{x^2}{|x|^2}; x\neq 0$ $f(0)=0$, Then

(1) $f(x)$ at continuous $x\in\mathbb{R}$

(2) $f(x)$ is continuous $x\in\mathbb{R}-\{0\}$

(3) $f(x)$ is continuous $x\in\mathbb{R}-\{1\}$

(4) $f(x)$ is continuous $x\in\mathbb{R}-\{0,1\}$

17. Given that the functions f defined by $f(x)=f(x)=\begin{cases}2x+1 & x>3\\ a & x=3\\ x-2 & x<3\end{cases}$ is continuous every where. Then $a=$

(1) 3 (2) 2 (3) -1 (4) 7

18. If $f(x)=\begin{cases}3x+K & x<2\\ \ell-x & x\geq 2\end{cases}$ is such that $f(x)$ is continuous at $x=2$ Then

(1) $\ell+K=8$ (2) $K-\ell=8$

(3) $\ell-K=8$ (4) $\ell-K=4$

19. $f(x)=\text{Cosec}x$ is continuous on

(1) $[0,\pi]$ (2) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(3) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (4) $(0,\pi)$

20. $f(x)=\sec^{-1}x$ is continuous on

(1) $\mathbb{R}$ (2) $\mathbb{R}-\{-1,1\}$

(3) $\mathbb{R}-(-1,1)$ (4) $\mathbb{R}-[-1,1]$

21. $f(x)=\cos h^{-1}(x)$ is continuous on

(1) $(0,\infty)$ (2) $[0,\infty]$ (3) $(1,\infty)$ (4) $[1,\infty]$

22. $f(x)=\dfrac{\tan^3 2x-\sin^3 2x}{\sin^5 x}$ for $x\neq 0$ if $f(x)$ is continuous at $x=0$ Then $f(0)=$

(1) 24 (2) 48 (3) 96 (4) 8

23. $f(x)=\dfrac{1-\cos 2x}{\sin 3x\tan 2x}$ for $x\neq 0$ $f(x)$ is continuous and $f(0)=\dfrac{1}{K}$, Then K =

(1) 1 (2) 2 (3) 3 (4) 4

24. If $f(x)=\dfrac{x-2}{|x-2|}+a; x<2; f(2)=a+b;$ $f(x)=\dfrac{|x-2|}{(x-2)}+b, x>2$, $f(x)$ continuous at $x=2$ Then $(a,b)=$

(1) $(-1,-1)$ (2) $(1,1)$

(3) $(1,-1)$ (4) $(-1,1)$

25. If $f(x)=\dfrac{2^x-2^{-x}}{x}, x\neq 0$ and $f(0)=K$ is continuous at $x=0$ Then $K=$

(1) $\log_e 2$ (2) $\log_e 3$ (3) $\log_e 4$ (4) 2

26. If $f(x)=\dfrac{x(2^x-1)}{\sin x^2}$, for $x\neq 0$ is continuous at $x=0$. Then $f(0)$

(1) $\log_e 2$ (2) $\log_e 3$ (3) $\log_e 4$ (4) 0

27. The value of $f(0)$ so that $f(x)=\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ is every where continuous is

(1) e (2) $\frac{1}{2}$ (3) $e^{\frac{1}{2}}$ (4) 0

28. If the function $f(x)=\begin{cases}\dfrac{Tan5x}{x} & \text{for } x\neq 0\\ \dfrac{a}{2} & \text{for } x=0\end{cases}$ is continuous at $x=0$, Then $a=$

(1) $\frac{5}{2}$ (2) 5 (3) 25 (4) 10

29. $f(x)=\sqrt{|x|}$ is continuous on

(1) TR (2) TR^+ (3) TR^- (4) at $x=0$

30. If $f(x)=\dfrac{x^3+x^2-16x+20)}{(x-2)^2}$ for $x\neq 2$ and $f(x)=K$ for $x=2$ is continuous at for all x then K =

(1) 7 (2) -7 (3) $\frac{1}{7}$ (4) $-\frac{1}{7}$

31. The function defined by $f(x)=f(x)=\begin{cases}x\sin\frac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$ at $x=0$; $f(x)$ is

(1) continuous (2) Not continuous

(3) Not defined (4) None of these

PRACTICE SET-II KEY

1) 2	2) 3	3) 1	4) 2	5) 4
6) 4	7) 4	8) 3	9) 2	10) 2
11) 4	12) 4	13) 4	14) 3	15) 3
16) 2	17) 4	18) 3	19) 4	20) 3
21) 4	22) 2	23) 3	24) 3	25) 3
26) 1	27) 3	28) 4	29) 4	30) 4
31) 1				

SELF TEST

01. $\lim\limits_{x\to 0} x^2\sin\dfrac{\pi}{x}=$

1) 1 2) 0

3) does not exist 4) ∞

02. If $f:R\to R$ defined by

$f(x)=\begin{cases}\dfrac{x-2}{x^2-3x+2} & \text{if } x\in R-\{1,2\}\\ 2 & \text{if } x=1\\ 1 & \text{if } x=2\end{cases}$

03. then $\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}=$

1) 0 2) -1 3) 1 4) $-\frac{1}{2}$

04. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{3\sin x-\sqrt{3}\cos x}{6x-\pi}=$

1) $\sqrt{3}$ 2) $\frac{1}{\sqrt{3}}$ 3) $-\sqrt{3}$ 4) $-\frac{1}{\sqrt{3}}$

05. If a > 0 and $\lim\limits_{x\to a}\dfrac{a^x-x^a}{x^x-a^a}=-1$ then a =

1) 0 2) 1 3) e 4) 2e

06. $\lim\limits_{x\to 0}\dfrac{4^x-9^x}{x(4^x+9^x)}=$

1) $\log\left(\frac{3}{2}\right)$ 2) $\log\left(\frac{2}{3}\right)$ 3) $\log\left(\frac{4}{3}\right)$ 4) $\log 2$

07. The quadratic equation whose roots l,m where $l=\lim_{\theta\to 0}\frac{3\sin\theta-4\sin^3\theta}{\theta}$, $m=\lim_{\theta\to 0}\frac{2\tan\theta}{(1-\tan^2\theta)}$ is

1) $x^2-5x+6=0$ 2) $x^2+5x+6=0$
3) $x^2-x+6=0$ 4) $x^2-x-6=0$

08. If $f(x)=\begin{cases}2x+b & (x<a)\\ x+d & (x\geq a)\end{cases}$ is such that $\lim_{x\to a} f(x)=l$, then $l=$

1) 2d-b 2) 2b-d 3) 2d+b 4) b-2d

09. $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b}$

1) 1 2) e^{b-a} 3) e^{a-b} 4) e^{b}

10. $\lim_{x\to 0}\frac{x.10^x-x}{1-\cos x}$

1) log 10 2) 2log10 3) 3log 10 4) 4log 10

11. If $f(x)=\frac{x^2-10x+25}{x^2-7x+10}$ for $x\neq 5$ and f is continuous at x=5, then f(5) =

1) 0 2) 5 3) 10 4) 25

12. $\lim_{\theta\to\frac{\pi}{2}}\frac{1-\sin\theta}{\cos\theta\left(\frac{\pi}{2}-\theta\right)}=$

1) 1 2) -1 3) -(1/2) 4) 1/2

13. $\lim_{x\to 0}\frac{\log_e(1+x)}{3^x-1}=$

1) $\log_e 3$ 2) 0 3) 1 4) $\log_3 e$

14. If the function $f(x)=\begin{bmatrix}\frac{\sin 3x}{x} & (x\neq 0)\\ \frac{k}{2} & (x=0)\end{bmatrix}$ is continuous at x = 0, then k =

1) 3 2) 6 3) 9 4) 12

15. $\lim_{x\to\infty}\frac{2x+7\sin x}{4x+3\cos x}=$

1) 1 2) -1 3) $\frac{1}{2}$ 4) $-\frac{1}{2}$

16. $\lim_{x\to 0}\log\left|\frac{\log(1+x)}{x}\right|=$

1) 0 2) 1 3) e 4) 1/e

17. $\lim_{x\to\infty}\left(\frac{a^{1/x}+b^{1/x}+c^{1/x}}{3}\right)^x$, where a,b,c are real and non-zero =

1) 0 2) $(abc)^{1/3}$ 3) $(abc)^{-1/3}$ 4) 1

18. $\lim_{x\to\infty}\left(\sin\sqrt{x+1}-\sin\sqrt{x}\right)=$

1) 2 2) -2 3) 0 4) 1

19. $\lim_{x\to 0}\left(\frac{1}{\sin^2x}-\frac{1}{\sinh^2x}\right)=$

1) $\frac{2}{3}$ 2) 0 3) $\frac{1}{3}$ 4) $-\frac{2}{3}$

20. $\lim_{x\to\infty}\frac{|3x^2+1|}{2x^2+1}=$

1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) $-\frac{3}{2}$ 4) $-\frac{2}{3}$

21. $\lim_{x\to 0}\frac{x^2}{\int_0^x\tan^{-1}x\,dx}=$

1) 2 2) 1 3) 3 4) -1

22. $\lim_{n\to\infty}\frac{3.2^{n+1}-4.5^{n+1}}{5.2^n+7.5^n}=$

1) $-\frac{20}{7}$ 2) +20/7 3) 10/7 4) -10/7

23. $\lim_{x\to 0}\frac{x}{\sqrt{x+4}-2}=$

1) 4 2) $\sqrt{2}$ 3) $2\sqrt{2}$ 4) $1/\sqrt{2}$

24. $\lim_{x\to 0}\left(\frac{\int_0^x\sin^3x\cos x\,dx}{x^4}\right)=$

1) 0.25 2) 2.5 3) 5.2 4) 0.52

25. $\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}=$

1) 1 2) e 3) e^{-1} 4) 0

26. $\lim_{x\to 1^+}\frac{\sqrt{x-1}+\sqrt{x-1}}{\sqrt{x^2-1}}=$

1) $\frac{1}{2}$ 2) $\sqrt{2}$ 3) 1 4) $\frac{1}{\sqrt{2}}$

27. $\lim_{x\to 0}\frac{a^x-b^x}{x}=$

1) $xa^{x-1}-xb^{x-1}$ 2) log a/b 3) log b/a 4) log ab

28. $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x=$

1) 1 2) e 3) 1/e 4) ∞

29. $\lim_{n\to\infty}\frac{1}{n^3}\left\{1+3+6+\dots\dots+\frac{n(n+1)}{2}\right\}=$

1) 0 2) 2 3) $\frac{1}{6}$ 4) $\frac{1}{3}$

30. If a,b,c,d are positive real numbers then $\lim_{n\to\infty}\left(1+\frac{1}{a+bn}\right)^{c+dn}=$

1) $e^{d/b}$ 2) $e^{c/a}$ 3) $e^{(c+d)/(a+b)}$ 4) $e^{(a+b)/(c+d)}$

31. $\lim_{x\to\infty}\frac{x-\sin x}{x+\cos^2x}=$

1) 0 2) 1 3) -1 4) 2

32. $\lim_{n\to\infty}\frac{\sin n\theta}{\sqrt{n}}=$

1) 0 2) ∞ 3) 1 4) 2

33. If $f:R\to R$ is defined by $f(x)=x^2+3x+2$, then $f(x-1)=$

1) x^2+x 2) x^2-3x+2
3) x^2+2^x 4) x^2-x

34. If $f:R\to R$ is defined by $f(x)=x^2-3x+2$, then $f(x^2-3x-2)=$

1) x^4+1 2) x^4-2x^2+2
3) $x^4-6x^3+2x^2+21x+12$
4) x^4+2x+2

35. If $f(x)=2x-1$, if $x>1$; x^2+1, if $-1\leq x\leq 1$, then $\frac{f(1)+f(3)+f(0)}{f(2)+f(-1)+(1/2)}=$

1) 32/5 2) 32/25 3) 5/32 4) 25/32

36. If $f(x)=\frac{\cos^2x+\sin^4x}{\sin^2x+\cos^4x}$ for $\in R$ then $f(2002)=$

1) 1 2) 2 3) 3 4) 4

37. If $f(x)=\sin[\pi^2]x+\sin[-\pi^2]x$ when $[x]$ is the step function, then $f(\pi/6)=$

1) $-1+\sqrt{3}/2$ 2) $1+\sqrt{3}/2$
3) $1-\sqrt{3}/2$ 4) 1

38. If $f:R\to(0,1]$ is defined by $f(x)=\frac{1}{x^2+1}$, then f is

1) a function 2) one one 3) onto 4) one one onto

39. $f(x)=\frac{2^{x^4}}{\sin x}$ is

1) even 2) odd
3) neither even nor odd
4) one-one on R-{x/sinx=0}

40. If $f(x)=\frac{a^{2x}-a^{-2x}}{a^{2x}+a^{-2x}}$, then $f(x)$ is

1) even 2) odd 3) none of these
4) cannot be determined

41. $x\left(\frac{a^{x}+1}{a^{x}-1}\right)$

1) is an even function 2) is an odd function
3) is neither even nor odd 4) noe

42. $x^{2}\left(\frac{a^{x^2}+1}{a^{2x}-1}\right)$

1) is an even function 2) is an odd function
3) is neither even nor odd
4) cannot be determined

43. The function $f(x)=\log(x+\sqrt{x^2+1})$ is

1) an even function 2) an odd function
3) perodic function
4) neither even nor odd function

44. $\frac{\log(x+\sqrt{x^2+1})}{x}$ is

1) an even function 2) an odd function
3) neither even nor odd 4) none

45. Let $f(x)=\frac{x}{e^x-1}+\frac{x}{2}+1$ then f is

1) an odd function 2) an even function
3) periodic function 4) none of these

46. If $f(x)=x^2, g(x)=x^2-5x+6$ then

$\frac{g(2)+g(3)+g(0)}{f(0)+f(1)+f(-2)}=$

1) 2 2) 1 3) 5/6 4) 6/5

47. $\underset{x\to 2}{Lt}\frac{x^3-8}{\sqrt{x}-\sqrt{2}}=$

1) $24\sqrt{2}$ 2) $6\sqrt{2}$ 3) 0 4) none

48. $\underset{x\to 0}{Lt}\frac{\cos ax-\cos bx}{x^2}$

1) $\frac{b^2+a^2}{2}$ 2) $\frac{a^2+b^2}{2}$
3) $\frac{b^2-a^2}{2}$ 4) $\frac{a^2-b^2}{2}$

49. $\underset{x\to 0}{Lt}\frac{1-\cos\theta}{\theta^2}$

1) 0 2) 1 3) -1 4) none

50. $\underset{\theta - 0}{Lt}\frac{1-\cos\theta}{\theta^2}$

1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) 1 4) $\frac{1}{4}$

51. $\underset{n\to\infty}{Lt}\frac{\cos(n^5x)+\sin(n^5x)}{n^{5/2}}$

1) 0 2) 1/2 3) 5 4) 10

52. $\underset{x\to 3}{Lt}\frac{1}{x-3}\int_3^x e^x dx=$

1) 0 2) 1 3) e 4) e^3

53. $\underset{x\to 0}{Lt}\frac{x-\cos(x^2)}{x^4}$

1) 0 2) 1 3) 1/2 4) 2

54. $\underset{x\to 0}{Lt}2\cot x\log(1+\tan x)=$

1) 1/2 2) $1/e^2$ 3) 2 4) e^2

55. $\underset{x\to 0}{Lt}\frac{x(1+a\cos x)-b\sin x}{x^3}=1 then a+b=$

1) 2 2) - 2 3) 4 4) - 4

56. $\lim_{x\to 1}\frac{\cos\left(\frac{\pi}{2}x\right)}{1-\sqrt{x}}=$

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

57. $\underset{x\to 0}{Lt}\frac{\tan^4 x-\sin^4 x}{x^6}$

1) 1 2) 2 3) 3 4) $\frac{1}{2}$

58. $\underset{x\to 2}{Lt}\left[\frac{x^3\sqrt{x}-8\sqrt{2}}{x-2}\right]$

1) $7\sqrt{2}$ 2) $\sqrt{2}$ 3) $14\sqrt{2}$ 4) $2\sqrt{2}$

59. $\underset{x\to 0}{Lt}2\cot x\log(1+\tan x)=$

1) 2 2) 0 3) e^2 4) 1

60. $\underset{x\to 0}{Lt}\frac{a^x-b^{2x}}{2x}$

1) $\log\left(\frac{a}{b}\right)$ 2) $2\log\left(\frac{a}{b}\right)$
3) $\log\left(\frac{2a}{b}\right)$ 4) $\log\left(\frac{a}{2b}\right)$

61. $\underset{x\to\infty}{Lt}\left[\frac{x+1}{x-2}\right]^{2x+1}$

1) e^6 2) e^{-1} 3) -1 4) log 2

SELF TEST KEY

1) 2	2) 2	3) 3	4) 2	5) 1
6) 2	7) 1	8) 1	9) 3	10) 2
11) 1	12) 4	13) 4	14) 2	15) 3
16) 1	17) 2	18) 3	19) 1	20) 1
21) 1	22) 1	23) 1	24) 1	25) 1
26) 2	27) 2	28) 3	29) 3	30) 1
31) 1	32) 1	33) 1	34) 3	35) 2
36) 1	37) 1	38) 3	39) 2	40) 2
41) 1	42) 2	43) 2	44) 1	45) 2
46) 4	47) 1	48) 3	49) 2	50) 1
51) 1	52) 4	53) 3	54) 3	55) 4
56) 1	57) 3	58) 3	59) 1	60) 1
61) 1				

PREVIOUS ECET QUESTIONS

2008

01. $\underset{x\to\infty}{Lt}\left[\frac{x+1}{x-2}\right]^{2x+1}=$

(1) e^6 (2) e^{-1} (3) -1 (4) log 2

2009

02 $\underset{x\to 0}{Lt}\left(\frac{\operatorname{cosec}x-\cot x}{x}\right)=$

(1) 4 (2) 0 (3) $-\frac{5}{7}$ (4) $\frac{1}{2}$

2010

03 $\underset{x\to 0}{Lt}\frac{\cos ax-\cos bx}{x^2}=$

(1) 0 (2) 1 (3) $\frac{a^2-b^2}{2}$ (4) $\frac{b^2-a^2}{2}$

2011

04. If $f(x)=x\sin\frac{1}{x};\ x\neq 0\ f(x)=0, x=0$ then

$\lim_{x\to 0}f(x)=$

(1) 1 (2) 0 (3) -1 (4) 2

05 $\lim_{x\to\infty}\frac{\sqrt{x^2-1}}{2x+1}=$

(1) 1 (2) 0 (3) -1 (4) $\frac{1}{2}$

06 $\lim_{x\to 0}\frac{\log(\cos x)}{x}=$

(1) 0 (2) ∞ (3) 1 (4) $-\infty$

07 $\underset{n\to\infty}{Lt}\frac{1^3+2^3+3^3+....+n^3}{n^4}=$

(1) 1 (2) $1/2$ (3) $1/3$ (4) $1/4$

2012

08. $\underset{x\to 0}{Lt}\frac{\sqrt{5x-4}-\sqrt{x}}{x-1}$ is

1) 3 2) 2 3) 4 4) 1

2013

09. $\underset{x\to 0}{Lt}\frac{x^2\sin\left(\frac{1}{x}\right)}{\sin x}=$

1) 1 2) -1 3) 2 4) 0

10. A function f defined in (0,3) given by

$f(x)=x^2$ when $0<x<1$

$=x$ when $1\le x<2$

$=x^3/4$ when $2\le x<2$

1) is continuous at x = 1 only
2) is continuous at x = 2 only
3) is continuous at x = 1 and x = 2
4) is discontinuous in (0,3)

2014

11. $\lim_{x\to\infty}\frac{3^{x+1}+4}{3^{x+2}+4}=$

1) 1 2) 0 3) $\frac{3}{4}$ 4) $\frac{1}{3}$

T.S ECET - 2015

12. $\underset{x\to\infty}{Lt}\left(1+\frac{1}{x}\right)^x=$

1) 0 2) 1 3) e 4) x

13. $\underset{x\to 0}{Lt}\left(\frac{e^x-1}{x}\right)=$

1) 0 2) 1 3) e 4) x

A.P ECET-2015

14. If f(x) =

$\begin{cases} ax^2-b, & |x|<1 \\ \frac{1}{|x|}, & |x|\ge 1\end{cases}$ is differentiable at x = 1, then

1) a = 1/2, b = -1/2 2) a = -1/2, b = -3/2
3) a = b = 1/2 4) a = b = -1/2

T.S ECET - 2016

15. $\lim_{x\to 0}\left[\frac{e^{2x}-1-2x}{x^2}\right]=$

1) 1 2) 2 3) 3 4) $\frac{1}{2}$

16. $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]^{\frac{1}{x^2}}=$

1) 0 2) 1 3) e^{-1} 4) $e^{-1/6}$

A.P ECET-2016

17. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2+r^2}=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{8}$

18. $\lim_{x\to 1}\frac{x^2-1}{|x-1|}=$

1) 1 2) -1 3) 2 4) -2

19. $\lim_{x\to 1}\frac{\log(x+2)}{2^x-1}=$

1) $\log_e 4$ 2) $\log_2 e$ 3) $\log_e 2$ 4) $\log_4 e$

T.S ECET - 2017

20. $\lim_{x\to 0}\frac{8^x-2^x}{x}=$

1) log 2 2) 0
3) log 4 4) 1

A.P ECET-2017

21. $\lim_{x\to 2^+}\frac{x|x-2|}{x-2}=$ ______

1) 1 2) -1 3) 2 4) -2

22. If $f(x)=(1+x)^{\frac{2}{x}}$ is continuous at x = 0 then $f(0)=$

1) e 2) e^2 3) e^3 4) e^4

23. For the function $f(x)=\log(x^2+y^2)$, which of the following is true

1) $f_x+f_y=0$ 2) $f_{xx}+f_{yy}=0$
3) $f_x-f_y=0$ 4) $f_{xx}-f_{yy}=0$

T.S ECET - 2018

24. The function $y=|x|-\infty<x<\infty$ is ______

1) differentiable at x=0
2) not continuous at x=0
3) continuous and differentiable at $x\ne 0$
4) continuous but not differentiable at $x\ne 0$

25. $\lim_{x\to 0}\left(\frac{\sqrt{1-\cos 2x}}{x}\right)$

1) does not exist 2) 1 3) -1 4) 0

26. If $f(x)=|x^2-3x+2|$ then $\frac{df}{dx}=$

1) 2x-3 when x>2 2) 3-2x, when x<1
3) 3-2x when x>2 4) 2x+3, when 1<x<2

A.P ECET-2018

27. The value of $\lim_{x\to 1}\frac{x^3-1}{x-1}$ is

1) 3 2) -3 3) 2 4) 1

28. The value of $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{n+n}\right]$ is

1) log 2 2) log 3 3) log 2 4) log n

29. The range of x for which the function $x^3-3x^2-45x+2$ is increasing with x is

1) (3, -5) 2) (-3, -5) 3) (3,5) 4) (-3,5)

30. The maximum value of the function $2x^3-12x^2+18x+5$ is

1) 13 2) 12 3) 10 4) 15

PREVIOUS ECET QS KEY

1) 1	2) 4	3) 4	4) 4	5) 2
6) 4	7) 1	8) 2	9) 4	10) 3
11) 4	12) 3	13) 2	14) 4	15) 2
16) 4	17) 3	18) 4	19) 4	20) 3
21) 3	22) 2	23) 2	24) 3	25) 1
26) 1	27) 1	28) 1	29) 4	30) 1

PUT YOUR FULL EFFORTS
DON'T WORRY ABOUT RESULTS
THEY ARE BOUND TO COME TO YOU

ACE FOR IMPORTANT NOTES

SPACE FOR IMPORTANT NOTES

DERIVATIVES

FORMULAE : 1

01. $y=c \Rightarrow \frac{dy}{dx}=0$

02. $y=x^n \Rightarrow \frac{dy}{dx}=n.x^{n-1}$

03. $y=\sqrt{x} \Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{x}}$

04. $y=e^x \Rightarrow \frac{dy}{dx}=e^x$

05. $y=\frac{1}{x} \Rightarrow \frac{dy}{dx}=\frac{-1}{x^2}$

06. $y=\frac{1}{\sqrt{x}} \Rightarrow \frac{dy}{dx}=\frac{-1}{2x\sqrt{x}}$

07. $y=a^x \Rightarrow \frac{dy}{dx}=a^x.\log a$

08. $y=\log x \Rightarrow \frac{dy}{dx}=\frac{1}{x}$

9. $y=\log_a^x \Rightarrow \frac{dy}{dx}=\frac{1}{x\log a}$

FORMULAE : 2

01. $y=\sin x \Rightarrow \frac{dy}{dx}=\cos x$

02. $y=\cos x \Rightarrow \frac{dy}{dx}=-\sin x$

03. $y=\tan x \Rightarrow \frac{dy}{dx}=\sec^2 x$

04. $y=\cot x \Rightarrow \frac{dy}{dx}=-\operatorname{cosec}^2 x$

05. $y=\sec x \Rightarrow \frac{dy}{dx}=\sec x.\tan x$

06. $y=\operatorname{cosec} x \Rightarrow \frac{dy}{dx}=-\operatorname{cosec} x.\cot x$

FORMULAE : 3

01. $y=\sin^{-1}x \Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$

02. $y=\cos^{-1}x \Rightarrow \frac{dy}{dx}=\frac{-1}{\sqrt{1-x^2}}$

03. $y=\tan^{-1}x \Rightarrow \frac{dy}{dx}=\frac{1}{1+x^2}$

04. $y=\cot^{-1}x \Rightarrow \frac{dy}{dx}=\frac{-1}{1+x^2}$

05. $y=\sec^{-1}x \Rightarrow \frac{dy}{dx}=\frac{1}{|x|\sqrt{x^2-1}}$

06. $y=\operatorname{cosec}^{-1}x \Rightarrow \frac{dy}{dx}=\frac{-1}{|x|\sqrt{x^2-1}}$

FORMULAE : 4

01. $y=\sin hx \Rightarrow \frac{dy}{dx}=\cos hx$

02. $y=\cos hx \Rightarrow \frac{dy}{dx}=\sin hx$

03. $y=\tan hx \Rightarrow \frac{dy}{dx}=\sec h^2x$

04. $y=\cot hx \Rightarrow \frac{dy}{dx}=-\operatorname{cosec} h^2x$

05. $y=\sec hx \Rightarrow \frac{dy}{dx}=-\sec hx.\tan hx$

06. $y=\operatorname{cosec} hx \Rightarrow \frac{dy}{dx}=-\operatorname{cosec} hx.\cot hx$

FORMULAE : 5

01. $y = \sin h^{-1} x = \frac{1}{\sqrt{x^2+1}}$

02. $y = \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$

03. $y = \tan h^{-1} x = \frac{1}{1-x^2}, \ |x|<1$

04. $y = \cot h^{-1} x = \frac{1}{x^2-1}, \ x>1$

05. $y = \sec h^{-1} x = \frac{-1}{x\sqrt{1-x^2}}$

06. $y = \operatorname{cosec} h^{-1} x = \frac{-1}{x\sqrt{x^2+1}}$

FORMULAE : 6

01. $y = x^x \Rightarrow \frac{dy}{dx} = x^x(1+\log x)$

02. $y = x^{1/x} \Rightarrow \frac{dy}{dx} = x^{1/x}\left(\frac{1-\log x}{x^2}\right)$

03. $y = |x| \Rightarrow \frac{dy}{dx} = \frac{|x|}{x} = \frac{x}{|x|}; x \neq 0$

FORMULAE : 7

01. If $y = \sin^n x.\cos nx$.

$$\Rightarrow \frac{dy}{dx} = n.\sin^{n-1} x.\cos(n+1)x$$

02. If $y = \sin^n x.\sin nx$.

$\Rightarrow$

$$\frac{dy}{dx} = n.\sin^{n-1} x.\sin(n+1)x$$

03. If $y = \cos^n x.\cos nx$.

$$\Rightarrow \frac{dy}{dx} = -n \cos^{n-1} x.\sin(n+1)x$$

04. If $y = \cos^n x \sin nx$.

$$\Rightarrow \frac{dy}{dx} = n \cos^{n-1} x.\cos(n+1)x$$

FORMULAE : 8

01. If $y = f(x)^y \Rightarrow \frac{dy}{dx} = \frac{y^2.f^1(x)}{f(x)[1-y\log f(x)]}$

02. If $y = \sqrt{f(x)+y} \Rightarrow \frac{dy}{dx} = \frac{f^1(x)}{2y-1}$

03. If $y = \sqrt{f(x).y}. \Rightarrow \frac{dy}{dx} = f^1(x)$

FORMULAE : 9

01. $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = n \Rightarrow \frac{dy}{dx} = \frac{\left(\frac{n^2-2}{2}\right)y - x}{y - \left(\frac{n^2-2}{2}\right)x}$

02. $\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = n \frac{dy}{dx} = \frac{\left(\frac{n^2+2}{2}\right).y - x}{y - \left(\frac{n^2+2}{2}\right)x}$

FORMULAE : 10

01. If $x^m.y^n = (x+y)^{m+n} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$

02. If $x^m.y^n = a^{m+n} \Rightarrow \frac{dy}{dx} = \frac{-my}{nx}$

03. $e^x + e^y = e^{x+y} \Rightarrow dy/dx = -e^{y-x}$

04. $e^x + e^{-y} = e^{x-y} \Rightarrow dy/dx = -e^{-x-y}$

05. $a^x + a^y = a^{x+y} \Rightarrow dy/dx = -a^{y-x}$

FORMULAE : 11

01. If $y = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sin h^{-1}\left(\frac{x}{a}\right)$

$$\Rightarrow \frac{dy}{dx} = \sqrt{a^2+x^2}$$

02. If $y = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

$$\Rightarrow \frac{dy}{dx} = \sqrt{a^2-x^2}$$

03. If $y = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cos h^{-1}\left(\frac{x}{a}\right)$

$$\Rightarrow \Rightarrow \frac{dy}{dx} = \sqrt{x^2-a^2}$$

FORMULAE : 12

1. $\frac{d}{dx}(\sin^m nx.\cos^n mx) = mn\sin^{m-1}\cos^{n-1} mx$

$\cos(m+n)x$

02. $\frac{d}{dx}(\cos^m nx.\sin^n mx) = mn\cos^{m-1} nx.$

$\sin^{n-1} mx\cos(m+n)x$

03. $\frac{d}{dx}(\sin^m nx.\sin^n mx) = nm\sin^{m-1} nx$

$\sin^{n-1} mx\sin(m+n)x$

04. $\frac{d}{dx}(\cos^m nx.\cos^n mx) = -nm\cos^{m-1} nx$

$\cos^{n-1} mx\sin(m+n)x$

FORMULAE : 13

01. $\frac{d}{dx}\log\sqrt{\frac{1+\sin nx}{1-\sin nx}} = n\sec nx$

02. $\frac{d}{dx}\log\sqrt{\frac{1-\sin nx}{1+\sin nx}} = -n\sec nx$

03. $\frac{d}{dx}\log\sqrt{\frac{1+\cos nx}{1-\cos nx}} = -n\operatorname{cosec} nx$

04. $\frac{d}{dx}\log\sqrt{\frac{1-\cos nx}{1+\cos nx}} = n\operatorname{cosec} nx$

FORMULAE : 14

01. If $y = \tan^{-1}\left[\frac{a\cos f(x)+b\sin f(x)}{b\cos f(x)-a\sin f(x)}\right]$ then

$\frac{dy}{dx} = f^1(x)$

02. If $y = \tan^{-1}\left[\frac{a\cos f(x)-b\sin f(x)}{b\cos f(x)+a\sin f(x)}\right]$ then

$\frac{dy}{dx} = -f^1(x)$

03. If $y = \cot^{-1}\left[\frac{a\cos f(x)+b\sin f(x)}{b\cos f(x)-a\sin f(x)}\right]$

then $\frac{dy}{dx} = -f^1(x)$

04. If $y = \cot^{-1}\left[\frac{a\cos f(x)-b\sin f(x)}{b\cos f(x)+a\sin f(x)}\right]$ then

$\frac{dy}{dx} = f^1(x)$

FORMULAE : 15

01. If f is even then $f^1(x)$ is odd

02. If f is odd then $f^1(x)$ is even

FUNDAMENTAL THEOREMS

Let u,v,w be the functions of 'x'. Whose derivative exist

01. $\frac{d(K)}{dx} = 0$ where K is constant and

$\frac{d(Ku)}{dx} = K\frac{du}{dx}$

02. $\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}.$

we write $u^1 = \frac{du}{dx}$, $v^1 = \frac{dv}{dx}$

03. $(uv)^1 = u^1v + vu^1$

04. $(uvw)^1 = u^1vw + uv^1w + uvw^1$

05. $\left(\frac{u}{v}\right)^1 = \frac{u^1v - vu^1}{v^2}$

06. If $x = f(t), y = g(t)$ then.

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g^1(t)}{f^1(t)}$ is differentiation of the functions in the parametric form

07. $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ if $\frac{dy}{dx} \neq 0$

08. If $y = g(x), u = f(x)$

Then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

or $y = g[f(x)]$

$\Rightarrow \frac{dy}{dx} = g^1[f(x)].f^1(x)$

09. Logarithmic differentiation

a) If $Y = [f(x)]^{g(x)}$

$\Rightarrow$ where $f(x) > 0$

$f(x), g(x)$ are differentiable functions at 'x'

Then

$$\frac{dy}{dx} = [f(x)]^{g(x)}.\left[g^1(x).\log f(x) + g(x)\frac{f^1(x)}{f(x)}\right]$$

b) If $Y = f(x)^{f(x)}$

Then $\frac{dy}{dx} = [f(x)]^{f(x)}.f^1(x)[1 + \log f(x)]$

10. Implicit differentiation :

If $f(x,y) = 0, \frac{dy}{dx} = -\left[\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}\right]$ where

$\frac{\partial f}{\partial x}$ is the partial derivative of $f(x)$ w.r.t. x

$\frac{\partial f}{\partial y}$ is the partial derivative of $f(x)$ w.r.t y

11. Derivative of a functions w.r.t to another functions

Let $u = f(x)$. and $y = g(x)$ be differentiable functions at x and $g^1(x) \neq 0$

Then $\frac{du}{dv} = \frac{f^1(x)}{g^1(x)} = \frac{du/dx}{dv/dx}$

12. $f(x) = g(ax) \Rightarrow f^1(x) = a.g^1(ax)$

13. $\frac{d[e^x.f(x)]}{dx} = e^x.[f(x) + f^1(x)]$

SOLVED EXAMPLES

01. If $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$ then $\frac{dy}{dx} =$

1) $\frac{x}{y}$ 2) $\frac{y}{x}$ 3) $-\frac{x}{y}$ 4) $-\frac{y}{x}$

Sol: $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} \Rightarrow \sin^{-1}\sqrt{1-x^2} + \cos^{-1}y = \frac{\pi}{2}$

$\therefore y = \sqrt{1-x^2}$ ($\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x^2}}(-2x) = -\frac{x}{y}$

Ans : 3

02. If 'a' is a constant, then $\frac{d}{dx}\left|\log(ax)^x\right| =$

1) 1 2) 1/a 3) $\log ax$ 4) $\log(ax)+1$

Sol: Let $y = \log(ax)^x = x\log(ax)$.

Then $\frac{dy}{dx} = x.\frac{1}{ax}.a + \log(ax).1 + 1 + \log(ax)$

Ans : 4

03. If $f(x) = \log x$, then $f'(\log x) =$

1) $\frac{x}{\log x}$ 2) $\frac{\log x}{x}$ 3) $\frac{1}{x\log x}$ 4) none

Sol: $f(\log x) = \log(\log x)$

$f'(\log x) = \frac{1}{\log x}.\frac{d}{dx}(\log x) = \frac{1}{\log x}$

Ans : 3

04. If $y = 7^{x^3+3x}(x > 0)$ then $\frac{dy}{dx} =$

1) $3(x^2-1)7^{x^3+3x}$ 2) $3(x^2-1)7^{x^3+3x}.\log 7$

3) $3(x^2+1)7x^{3+3x}$ 4) $3(x^2+1)7^{x^3+3x}\log 7$

Sol : Given $y = 7^{x^3+3x}$

$\therefore \frac{dy}{dx} = 7^{x^3+3x}.\log 7.\frac{d}{dx}(x^3+3x)$

$= 3(x^2+1)7^{x^3+3x}.\log 7$

Ans : 4

05. If $x = y\log xy$ then $\frac{dy}{dx} =$

1) $\frac{x-y}{1+\log xy}$ 2) $\frac{x-y}{x(1+\log xy)}$

3) $\frac{x+y}{x(\log + xy)}$ 4) none

Sol: We have

$x = y(\log x + \log y) = y\log x + y\log y$

Differentiating w.r.t 'x', we get,

$1 = \left(y.\frac{1}{x} + \log x.\frac{dy}{dx}\right) + \left(y.\frac{1}{y}\frac{dy}{dx} + \log y.\frac{dy}{dx}\right)$

$\Rightarrow 1 - \frac{y}{x} = (\log x + 1 + \log y)\frac{dy}{dx} = (1 + \log xy)\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{x-y}{x(1+\log xy)}$

Ans : 2

06. If $x = \cos\theta, y = \sin^3\theta$ then $\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}$ at $\theta = \pi/2$ is

1) 1 2) 2 3) -2 4) -3

Sol : Here $\frac{dx}{d\theta} = -\sin\theta$ and $\frac{dy}{d\theta} = 3\sin^2\theta\cos\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\sin^2\theta\cos\theta}{\sin\theta}$

$= -3\sin\theta\cos\theta = -\frac{3}{2}\sin 2\theta$

Now $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{x}\right).\frac{d\theta}{dx}$

$= -\frac{3}{2}(2\cos 2\theta).\left(-\frac{1}{\sin\theta}\right) = \frac{3\cos 2\theta}{\sin\theta}$

$\therefore \left(\frac{dy}{dx}\right)^2 + y.\frac{d^2y}{dx^2} = \frac{9}{4}\sin^2 2\theta + \sin^3\theta.\frac{3\cos 2\theta}{\sin\theta}$

$= \frac{9}{4}\sin^2 2\theta + 3\sin^2\theta\cos 2\theta$

At $\theta = \frac{\pi}{2}, \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = \frac{9}{4}(0) + 3(1)^2(-1) = -3$

Ans : 4

07. If $x=\cos\theta+\theta\sin\theta, y=\sin\theta-\theta\cos\theta$ then $y_2=$

1) $\frac{\sec^2\theta}{\theta}$ 2) $\frac{\theta}{\sec^2\theta}$ 3) $\frac{\theta}{\cos^3\theta}$ 4) $\sec^2\theta$

Sol : $\frac{dx}{d\theta}=-\sin\theta+\theta.\cos\theta+\sin\theta=\theta\cos\theta$

$\frac{dy}{d\theta}=\cos\theta-(-\theta\sin\theta+\cos\theta)=\theta\sin\theta$

$\therefore\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\theta\sin\theta}{\theta\cos\theta}=\tan\theta$

$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$

$=\sec^2\theta.\frac{1}{\theta\cos\theta}=\frac{\sec^2\theta}{\theta}$

Ans : 1

08. If $y=\cos\left(3\cos^{-1}x\right)$ then $y_3=$

1) 0 2) 24 3) 24x 4) $24x^2$

Sol : $y=\cos\left(3\cos^{-1}x\right)=\cos\left[\cos^{-1}\left(4x^3-3x\right)\right]$

$=4x^3-3x$

$\therefore\frac{dy}{dx}=y_1=12x^2-3\Rightarrow y_2=24x\Rightarrow y_3=24$

Ans : 2

09. If $y=\cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$ then $f^1(e)=$

1) does not exist 2) 2/e 3) 1/e 4) 1

Sol: Put $\log x=\tan\theta$. Then

$f(x)=\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$=\cos^{-1}(\cos 2\theta)=2\theta=2\tan^{-1}(\log x)$

$=f^1(x)=\frac{2}{1+(\log x)^2}.\frac{1}{x}$

Then $f^1(e)=\frac{2}{1+(\log e)^2}.\frac{1}{e}=\frac{1}{e}$

Ans : 3

10. If $3\sin(xy)+4\cos(xy)=13$ then $dy/dx=$

1) $\frac{x}{y}$ 2) $-\frac{x}{y}$ 3) $\frac{y}{x}$ 4) $-\frac{y}{x}$

Sol: Let $f(xy)=3\sin(xy)+4\cos(xy)$.

Then $\frac{\partial f}{\partial x}=3\cos(xy)y-4\sin(xy).y$

$=y[3\cos(xy)-4\sin(xy)]$

and $\frac{\partial f}{\partial y}=3\cos(xy).x-4\sin(xy)$.

$x=x[3\cos(xy)-4\sin(xy)]$. $\frac{dy}{dx}=-\frac{\partial f/\partial x}{\partial f/\partial y}=-\frac{y}{x}$

Ans : 4

THERE IS NO SUBSTITUTE TO HARDWORK

PRACTICE SET - I

01. $\frac{d\left(e^{-\log\tan x}\right)}{dx}=$

1) $\tan x$ 2) $\cot x$ 3) $\sec^2 x$ 4) $-\operatorname{cosec}^2 x$

02. $\frac{d\left(e^{\frac{1}{2}\log(1+\cot^2 x)}\right)}{dx}=$

1) $\operatorname{cosec} x$ 2) $\sec x$
3) $-\operatorname{cosec} x\cot x$ 4) $\sec x\tan x$

03. $\frac{d\left(e^{x+k\log x}\right)}{dx}$

1) $e^{x+k\log x}$ 2) $e^x x^{k-1}(x+k)$
3) $e^x x^k$ 4) None

04. $\frac{d}{dx}\left(\frac{\cot x+\tan x}{\cot x-\tan x}\right)=$

1) $2\sin 2x$ 2) $-2\sec 2x\tan 2x$
3) $2\sec 2x\tan 2x$ 4) $2\sec 2x$

05. If $Y=\sin^4 x\cos 4x$, then dy/dx

1) $-4\sin^3 x\cos 5x$
2) $4\sin^3 x\cos 5x$
3) $-4\sin^3 x\sin 5x$ 4) None

06. If $y=\frac{1+\sin x-\cos x}{1+\sin x+\cos x}$ Then $\frac{dy}{dx}$

1) $\sec^2 x/2$ 2) $\frac{1}{2}\sec^2 x/2$
3) $\sec^2 x/2 \tan x/2$ 4) None

07. $\frac{d}{dx}(\log_{10}x)=$

1) $\frac{1}{x}$ 2) $\frac{1}{x}\log 10$
3) $\frac{1}{x\log 10}$ 4) $-\frac{\log 10}{x(\log x)^2}$

08. $\frac{d}{dx}(\log_x 10)=$

1) $\frac{\log 10}{x}$ 2) $x\log 10$
3) $\frac{-\log 10}{x\log x}$ 4) $\frac{-\log 10}{x(\log x)^2}$

09. $\frac{d}{dx}\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)=$

1) $\frac{1}{a^2+x^2}$ 2) $\frac{3}{a^2+x^2}$ 3) $\frac{3a}{a^2+x^2}$ 4) None

10. $y=\cot^{-1}\frac{x}{\sqrt{1-x^2}}+\operatorname{cosec}^{-1}\frac{1}{\sqrt{1-x^2}}$ then $\frac{dy}{dx}=$

1) $\frac{1}{\sqrt{1-x^2}}$ 2) $\frac{-1}{\sqrt{1-x^2}}$
3) $\frac{2}{\sqrt{1-x^2}}$ 4) $\frac{-2}{\sqrt{1-x^2}}$

11. If $Y=\cot^{-1}\left(\frac{1-2x^2}{2x\sqrt{1-x^2}}\right)$ Then $\frac{dy}{dx}$

1) $\frac{1}{\sqrt{1-x^2}}$ 2) $\frac{2}{\sqrt{1-x^2}}$
3) $\frac{-1}{\sqrt{1-x^2}}$ 4) $\frac{-2}{\sqrt{1-x^2}}$

12. $\frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x\sqrt{x}}\right)=$

1) $\frac{1}{2\sqrt{x}(1+x)}-\frac{1}{1+x^2}$ 2) $\frac{1}{2\sqrt{x}(1+x)}+\frac{1}{1+x^2}$

3) $\frac{1}{1+x^2}-\frac{1}{2\sqrt{x}(1+x)}$ 4) $\frac{1}{1+x^2}-\frac{1}{1+x}$

13. $y=\cos h^{-1}(\sec x)$ then dy/dx

1) $\log\sec x$ 2) $\sec x$ 3) $\sec x\tan x$ 4) $\tan x$

14. $\frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{x}-\sqrt{a}}{1+\sqrt{ax}}\right)=$

1) $\frac{1}{\sqrt{x}(1+x)}$ 2) $\frac{1}{x(1+\sqrt{x})}$

3) $\frac{1}{2\sqrt{x}(1-x)}$ 4) $\frac{1}{2x(1+\sqrt{x})}$

15. If $y=\left(x+\sqrt{x^2-1}\right)^n$ Then $(x^2-1)\left(\frac{dy}{dx}\right)^2=$

1) ny 2) $\frac{n}{y}$ 3) n^2y^2 4) $\frac{n^2}{y^2}$

16. $\frac{d}{dx}\left[\cos^{-1}\left(\frac{x^2-x+1}{x^2+x+1}\right)+\operatorname{cosec}^{-1}\left(\frac{x^2+x+1}{x^2-x+1}\right)\right]=$

1) 0 2) 1 3) $\frac{2x-1}{2x+1}$ 4) $\frac{2x+1}{2x-1}$

17. If $y=(\cosh x+\sinh x)^n$ Then $\frac{dy}{dx}$

1) $n(\cosh x+\sinh x)^{n-1}$ then $\frac{dy}{dx}=$

2) $n(\cosh x+\sinh x)^n$

3) $n(\cosh x-\sinh x)^{n-1}$

4) None

18. $\frac{d}{dx}\left(\sin^{-1}\frac{x}{a}\right)=$

1) $\frac{1}{\sqrt{a^2+x^2}}$ 2) $\frac{1}{\sqrt{a^2-x^2}}$

3) $\frac{-1}{\sqrt{a^2-x^2}}$ 4) $\frac{1}{\sqrt{x^2-a^2}}$

19. $\frac{d}{dx}\left(4\cos^3 x^0-3\cos x^0\right)=$

1) $\frac{-\pi}{60}\sin 3x^0$ 2) $\cos 3x^0$

3) $\tan 3x^0$ 4) $\frac{\pi}{60}\sin 3x^0$

20. If $y=\cos^3 x\cos 3x$; $\frac{dy}{dx}=$

1) $3\cos^2 x\cos 4x$ 2) $3\cos^4 x\cos 2x$

3) $3\cos^2 x\sin 4x$ 4) $3\cos^4 x\sin 2x$

21. $\frac{d}{dx}\left[\tan^{-1}(\sin x)\right]=$

1) $\frac{\sin x}{1+\sin^2 x}$ 2) $\frac{-\cos x}{1+\sin^2 x}$

3) $\frac{\cos x}{1+\sin^2 x}$ 4) none

22. $\frac{d}{dx}\left(\frac{2x+1}{3x+2}\right)=$

1) $\frac{1}{(3x+2)^2}$ 2) $\frac{-1}{(3x+2)^2}$

3) $\frac{2}{(2x+1)^2}$ 4) $\frac{-1}{(2x+1)^2}$

23. If $y=\log\left(\frac{x+\sqrt{x^2+a^2}}{x-\sqrt{x^2+a^2}}\right)$ then $\frac{dy}{dx}=$

1) $\frac{1}{\sqrt{x^2+a^2}}$ 2) $\frac{2}{\sqrt{x^2+a^2}}$

3) $\log\frac{x}{\sqrt{x^2+a^2}}$ 4) none

24. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{1+x^2}\right)+\tan^{-1}\left(\frac{1+x^2}{x}\right)\right]=$

1) 0 2) 1 3) $1+x^2$ 4) $\frac{1}{1+x^2}$

25. $\frac{d}{dx}\left[\cos^{-1}\left(\frac{1-9x^2}{1+9x^2}\right)\right]=$

1) $\frac{6}{1+x^2}$ 2) $\frac{6}{1-9x^2}$ 3) $\frac{6}{1+9x^2}$ 4) none

26. $\frac{d}{dx}\left[\cot^{-1}(\operatorname{cosec}x+\cot x)\right]=$

1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) 0 4) 1

27. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\right]=$

1) $\frac{2}{1+x^2}$ 2) $\frac{3}{1+x^2}$ 3) $\frac{1}{1+x^2}$ 4) $\frac{3}{1+9x^2}$

28. If $y=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ then $\frac{dy}{dx}=$

1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) -1 4) 2

29. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)\right]=$

1) $\frac{1}{1+4x^2}$ 2) $\frac{3}{1+4x^2}$

3) $\frac{3}{1+6x^2}$ 4) $\frac{6}{1+4x^2}$

30. $\frac{d}{dx}\left[\cot^{-1}\left(\frac{1+ax}{a-x}\right)\right]=$

1) $\frac{1}{1+x^2}$ 2) $\frac{-1}{1+x^2}$ 3) $\frac{-1}{1-x^2}$ 4) none

31. If $y=\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ then $\frac{dy}{dx}=$

1) 1 2) -1 3) 1/2 4) 1/4

32. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a\sin x+b\cos x}{a\cos x-b\sin x}\right)\right]=$

1) -1 2) 0 3) 1 4) none

33. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a\cos x-b\sin x}{a\sin x+b\cos x}\right)\right]=$

1) -1 2) $\frac{-1}{2}$ 3) $\frac{1}{2}$ 4) 0

34. If $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx}=$

1) $\frac{-2}{1+x^2}$ 2) $\frac{2}{1+x^2}$ 3) $\frac{1}{2+x^2}$ 4) $\frac{2}{2-x^2}$

35. $\frac{d}{dx}\left(\tan^{-1}e^{ax}\right)=$

1) $\frac{1+e^{ax}}{e^{ax}}$ 2) $\frac{a}{1+e^{2ax}}$ 3) $\frac{a.e^{ax}}{1+e^{2ax}}$ 4) none

36. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)\right]=$

1) -1 2) 1 3) 0 4) 2

37. If $y=\left(x+\sqrt{x^2+a^2}\right)^n$ then $\frac{dy}{dx}=$

1) xy 2) $\frac{ny}{\sqrt{x^2+a^2}}$

3) $\frac{n}{\sqrt{x^2+a^2}}$ 4) none

38. $\frac{d}{dx}\left[\log\left(\sqrt{x-a}+\sqrt{x-b}\right)\right]=$

1) $\frac{1}{\sqrt{(x-a)(x-b)}}$ 2) $\frac{1}{2\sqrt{(x-a)(x-b)}}$

3) $\frac{1}{2\sqrt{(x+a)(x+b)}}$ 4) none

39. $\frac{d}{dx}\left[\cos^{-1}\sqrt{1-x^2}-\cos^{-1}\left(4x^3-3x\right)\right]=$

1) $\frac{4}{\sqrt{1-x^2}}$ 2) $\frac{-4}{\sqrt{1-x^2}}$

3) $\frac{-2}{\sqrt{1-x^2}}$ 4) none

40. $\frac{d}{dx}\left[\tan^{-1}\frac{x}{\sqrt{1-x^2}}+\sec^{-1}\frac{1}{\sqrt{1-x^2}}\right]$

1) $\frac{2}{\sqrt{1-x^2}}$ 2) $\frac{x}{\sqrt{1-x^2}}$

3) $\frac{1}{\sqrt{1-x^2}}$ 4) 0

PRACTICE SET - I KEY

01) 4	02) 3	03) 2	04) 3	05) 2
06) 1	07) 3	08) 4	09) 3	10) 4
11) 2	12) 1	13) 2	14) 3	15) 3
16) 1	17) 2	18) 2	19) 1	20) 1
21) 3	22) 1	23) 2	24) 1	25) 3
26) 1	27) 2	28) 2	29) 4	30) 2
31) 3	32) 3	33) 1	34) 1	35) 3
36) 1	37) 2	38) 2	39) 1	40) 1

PRACTICE SET - II

01. If $x^2+4xy+3y^2-6x-8y+7=0$ then $\frac{dy}{dx}$

1) $-\left(\frac{x+2y-3}{2x+3y-4}\right)$ 2) $\frac{x+2y-3}{2x+3y-4}$

3) $-\left(\frac{2x+3y-4}{x+2y-3}\right)$ 4) $\frac{2x+3y-4}{x+2y-3}$

02. If $x^2-y^2=2xy$ Then $\frac{dy}{dx}$

1) $\frac{2x+y}{x+2y}$ 2) $\frac{x+2y}{2x+y}$

3) $\frac{2(x-y)}{(x+2y)}$ 4) $\frac{x+2y}{2x-y}$

03. If $e^x+e^{-y}=e^{x-y}$ Then $\frac{dy}{dx}$

1) e^{-x-y} 2) $-e^{-x-y}$ 3) e^{x+y} 4) $-e\,e^{x+y}$

04. If $\cos^{-1}x+\cos^{-1}y=\frac{\pi}{2}$ then $\frac{dy}{dx}$

1) $\frac{x}{y}$ 2) $\frac{y}{x}$ 3) $-\frac{x}{y}$ 4) $-\frac{y}{x}$

05. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{2}$ then $\frac{dy}{dx}=$

1) $\frac{1}{x^2}$ 2) $-\frac{1}{x^2}$ 3) $\frac{1}{y}$ 4) $\frac{1}{y^2}$

06. If $y=e^{x+e^{x+e^{x}\dots}}$ ——

1) $\frac{y}{1-y}$ 2) $\frac{y}{y-1}$ 3) $\frac{x}{x-1}$ 4) $\frac{x}{1-x}$

07. If $Y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x}+\dots}}$ Then $\frac{dy}{dx}=$

1) $\frac{1}{x(2y-1)}$ 2) $\frac{1}{x(2y+1)}$

3) $\frac{x}{(2y-1)}$ 4) $\frac{x}{(2y+1)}$

08. If $x=\frac{t^2-1}{t^2+1}, y=\frac{2t}{1+t^2}$ Then $\frac{dy}{dx}=$

1) $2t\left(1-t^2\right)$ 2) $\frac{1-t^2}{2t}$

3) $\frac{1\left(1+t^2\right)}{2t}$ 4) $2t\left(1+t^2\right)$

09. If $x=a(\theta+\sin\theta), y=a(1-\cos\theta)\frac{dy}{dx}=$

1) $\tan\theta/2$ 2) $\cot\theta/2$ 3) $\sec\theta/2$ 4) $\operatorname{cosec}\theta/2$

10. If $x=3\cos\theta-2\cos^3\theta,\ y=3\sin\theta-2\sin^3\theta$, $\frac{dy}{dx}=$

1) $\tan\theta$ 2) $\cos\theta$ 3) $\sec\theta$ 4) $\operatorname{cosec}\theta$

11. If $x=3\cos\theta+\cos 3\theta, y=3\sin\theta-\sin 3\theta, \frac{dy}{dx}=$

1) $\tan\theta$ 2) $-\tan\theta$ 3) $\cot\theta$ 4) $-\cot\theta$

12. If $x^y=y^x$ Then $\frac{dy}{dx}=$

1) $y(x\log y-y)$ 2) $x(y\log x-x)$

3) $\frac{y(x\log y-y)}{x(y\log x-x)}$ 4) $\frac{x(y\log x-x)}{y(x\log y-y)}$

13. If $y=\sqrt{\cos x+y}$ then $\frac{dy}{dx}=$

1) $\frac{\sin x}{2y-1}$ 2) $\frac{\sin x}{1-2y}$ 3) $\frac{\cos x}{2y-1}$ 4) $\frac{\cos x}{1-2y}$

14. If $y=\sin\left(2\sin^{-1}x\right)$ Then $\frac{dy}{dx}$

1) $1\sqrt{\frac{1-y^2}{1-x^2}}$ 2) $2\sqrt{\frac{1-y^2}{1-x^2}}$

3) $2\sqrt{\frac{1-x^2}{1-y^2}}$ 4) $\sqrt{\frac{1-x^2}{1-y^2}}$

15. If $x=a\cos^3 t, y=a\sin^3 t$, Then $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$

1) 1 2) -1 3) -2 4) 2

16. $\frac{d}{dx}\{(x-a)(x+a)\left(x^2+a^2\right)\left(x^4+a^4\right)\left(x^8+a^8\right)\}$

1) $16x^{16}$ 2) $15x^{16}$ 3) $16x^{15}$ 4) $15x^{15}$

17. $\frac{d}{dx}(\cos x)^{\sec x}=(\cos x)^{\sec x}f(x)$. Then $f(x)=$

1) $\sec x\tan x\log\left(\frac{e}{\cos x}\right)$

2) $\sec x\tan x\log\left(\frac{\cos x}{e}\right)$

3) $-\sec x\tan x\log(\cos x)$ 4) None

18. $\frac{d}{dx}\left\{\log\frac{\sin(x-a)}{\sin(x-b)}\right\} =$

1) $\frac{\cos(a-b)}{\sin(x-a)\sin(x-b)}$

2) $\frac{\sin(a-b)}{\sin(x-a)\sin(x-b)}$

3) $\frac{\cos(b-a)}{\sin(x-a)(x-b)}$ 4) None

19. If $y=\frac{1-t^2}{1+t^2}, x=\frac{2t}{1+t^2}$ Then $\frac{dy}{dx}=$

1) $-xy$ 2) $\frac{x}{y}$ 3) xy 4) $\frac{-x}{y}$

20. If $x=a\left(t+\frac{1}{t}\right), x=a\left(t-\frac{1}{t}\right)$ Then $\frac{dy}{dx}=$

1) $\frac{x}{y}$ 2) $-\frac{x}{y}$ 3) $\frac{y}{x}$ 4) $-\frac{y}{x}$

21. Derivative of $\cos^{-1}\left(\frac{1}{1+t^2}\right)$ w.r to $\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$ is

1) 0 2) 1 3) t 4) None

22. Derivative of $e^{\cos^{-1}x}$ w.r. to $\cos^{-1}x$

1) $e^{\cos^{-1}x}$ 2) $-e^{\cos^{-1}x}$

3) $\frac{-e^{\cos^{-1}x}}{\sqrt{1-x^2}}$ 4) $\frac{e^{\cos^{-1}x}}{\sqrt{1-x^2}}$

23. Derivative of $\frac{\sin^{-1}x}{1+\sin^{-1}x}$ w.r. to $\sin^{-1}x$

1) $\frac{1}{1+\sin^{-1}x}$ 2) $\frac{1}{\left(1+\sin^{-1}x\right)^2}$

3) $\frac{-1}{\left(1+\sin^{-1}x\right)^2}$ 4) None

24. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ $x\neq y$ Then $\frac{dy}{dx}$

1) $\frac{1}{(1+x)^2}$ 2) $\frac{-1}{(1+x^2)}$

3) $\frac{1}{(1+x^2)}$ 4) $\frac{-1}{(1+x^2)}$

25. If $\log\sqrt{x^2+y^2}=\tan^{-1}\frac{y}{x}$ Then $\frac{dy}{dx}$

1) $\frac{x-y}{x+y}$ 2) $\frac{y-x}{x+y}$ 3) $\frac{x+y}{x-y}$ 4) $\frac{x+y}{y-x}$

26. $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=4$, Then $\frac{dy}{dx}=$

1) $\frac{7y-x}{y-7x}$ 2) $\frac{y-7x}{7x-y}$ 3) $\frac{7x+y}{x-7y}$ 4) $\frac{y+7x}{7y-x}$

27. Derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t $\sqrt{1-x^2}$

1) 0 2) x 3) $\frac{x}{2}$ 4) $\frac{2}{x}$

28. $y=\tan^{-1}\left(\frac{4x}{1+5x^2}\right)+\tan^{-1}\left(\frac{2+3x}{3-2x}\right)$ then $\frac{dy}{dx}$

1) $\frac{1}{1+25x^2}$ 2) $\frac{5}{1+25x^2}$

3) $\frac{5}{1+5x^2}$ 4) $\frac{2}{1+5x^2}$

29. If $x=\frac{1+t}{t^3}, y=\frac{3+4t}{2t^2}$ then $x\left(\frac{dy}{dx}\right)^3=$

1) $1+\frac{dy}{dx}$ 2) $1-\frac{dy}{dx}$ 3) $\frac{dy}{dx}-1$ 4) $\frac{dy}{dx}$

30. Derivative of $\sin^{-1}\left(\frac{1-x}{1+x}\right)$ w.r to $\sqrt{x}$

1) $\frac{-1}{1+x}$ 2) $\frac{-2}{1+x}$ 3) $\frac{1}{1+x}$ 4) $\frac{2}{1+x}$

31. If $y=\left\lfloor(\tan x)^{\tan x}\right\rfloor$ Then $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$

1) 0 2) -1 3) 1 4) 2

32. If $y=\sqrt{\log x+y}$ Then $\frac{dy}{dx}$

1) $\frac{\log x}{2y-1}$ 2) $\frac{\log x}{2y+1}$ 3) $\frac{1}{x(2y-1)}$ 4) $\frac{1}{x(2y+1)}$

33. If $y=e^{xy}$ Then $\frac{dy}{dx}=$

1) $\frac{y^2}{1-y\log x}$ 2) $\frac{y^2}{1-xy}$

3) $\frac{y^2}{1+y\log x}$ 4) $\frac{y^2}{1+xy}$

34. $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$. Then $\frac{dy}{dx}$ at $x=a, y=b$

1) $\frac{a}{b}$ 2) $\frac{b}{a}$ 3) $-\frac{a}{b}$ 4) $\frac{-b}{a}$

35. If $\sqrt{x}+\sqrt{y}=\sqrt{xy}$ Then $\frac{dy}{dx}=$

1) $\frac{-y\sqrt{y}}{x\sqrt{x}}$ 2) $\frac{y\sqrt{y}}{x\sqrt{x}}$

3) $-\frac{x\sqrt{x}}{y\sqrt{y}}$ 4) $-\frac{y\sqrt{x}}{x\sqrt{y}}$

36. $x=a(\sec\theta+\tan\theta), y=b(\sec\theta-\tan\theta); \frac{dy}{dx}$

1) $\frac{x}{y}$ 2) $-\frac{x}{y}$ 3) $\frac{y}{x}$ 4) $-\frac{y}{x}$

37. If $Y=x+\cfrac{1}{x+\cfrac{1}{x+\frac{1}{x}\ldots\ldots\infty}}$ Then $\frac{dy}{dx}=$

1) $\frac{y}{2y-x}$ 2) $\frac{y}{x+2y}$ 3) $\frac{y}{x-2y}$ 4) y

38. Derivative of $\tan^{-1}\left(\frac{x}{\sqrt{1+x^2}-1}\right)$ w.r. to $\tan^{-1}x$

1) 0 2) 1 3) 1/2 4) -1/2

39. If $x^y=\log x$ Then $\frac{dy}{dx}$ at $x=e$

1) 0 2) 1 3) e 4) 1/e

40. If $y=\tan^{-1}x+\frac{1}{y}$ Then $\frac{dy}{dx}=$

1) $\frac{y^2\tan^{-1}x}{(1+y^2)}$ 2) $\frac{y}{(1+x^2)(1+y^2)}$

3) $\frac{y^2}{(1+x^2)(1+y^2)}$ 4) None

41. $y=\log\left(\sqrt{\operatorname{cosec}x+1}-\sqrt{\operatorname{cosec}x-1}\right)$ Then $\frac{dy}{dx}=$

1) $\operatorname{cosec}x$ 2) $2\operatorname{cosec}x$

3) $\frac{1}{2}\operatorname{cosec}x$ 4) $\frac{1}{2}\cot x$

42. If $x=\cos\theta\sin 2\theta, y=\sin\theta\cos 2\theta$ Then $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$

1) -2 2) 2 3) -1/2 4) 1/2

43. If $x=\left(\cos ec\theta-\sin\theta\right)$

$y=\left(\cos ec^3\theta-\sin^3\theta\right)$, $\frac{dy}{dx}$

1) $3\left(\sec^2\theta-\cos^2\theta\right)$ 2) $3\left(\cos ec^2\theta-\cos^2\theta\right)$

3) $3\left(\cos ec^2\theta-\sin^2\theta\right)$ 4) $3\left(\sec^2\theta-\sin^2\theta\right)$

44. If $x=\tan\theta+\cot\theta, y=\tan\theta-\cot\theta$, Then $\frac{dy}{dx}=$

1) $-\frac{x}{y}$ 2) $-\frac{y}{x}$ 3) $\frac{x}{y}$ 4) $\frac{y}{x}$

45. If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$ Then $x^3y\frac{dy}{dx}=$

1) -1 2) 2 3) -2 4) 1

46. If $x^m y^n=(x+y)^{m+n}$ Then $\frac{dy}{dx}=$

1) $\frac{y}{x}$ 2) $-\frac{y}{x}$ 3) $\frac{m}{n}\frac{x}{y}$ 4) $-\frac{m}{n}\frac{x}{y}$

47. If $\sin^{-1}x-\sin^{-1}y=\pi$ Then $\frac{dy}{dx}$

1) $\frac{x}{y}$ 2) $\frac{y}{x}$ 3) -1 4) 0

48. If $y=\cos^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$ Then $\frac{dy}{dx}=$

1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) $\frac{-x}{\sqrt{1-x^2}}$ 4) $\frac{x}{\sqrt{1-x^2}}$

49. If $x^3-xy^2+2y^2+2=0$ then $\frac{dy}{dx}=$

1) $\frac{y^2-3x^2}{2y(2-x)}$ 2) $\frac{3x^2-y^2}{2y(3-x)}$

3) $\frac{2y(3-x)}{y^2-3x^2}$ 4) $\frac{2y(3-x)}{3x^2-y^2}$

50. If $y=x^y$ then $\frac{dy}{dx}=$

1) $\frac{y^2}{x(1-y\log x)}$ 2) $\frac{y^2}{1-x\log y}$

3) $\frac{y^2}{1+x\log y}$ 4) none

51. If $xy=e^y$ then $\frac{dy}{dx}=$

1) $\frac{y}{x(y+1)}$ 2) $\frac{-y}{x(y+1)}$

3) $\frac{-y}{x(y-1)}$ 4) $\frac{y}{x(y-1)}$

52. If $\sin y=x\sin(a+y)$ then $\frac{dy}{dx}=$

1) $\frac{\sin^2(a-y)}{a}$ 2) $\frac{\sin^2(a+y)}{\cos a}$

3) $\frac{\sin^2(a+y)}{\sin a}$ 4) $\frac{\cos^2(a+y)}{\sin a}$

53. If $2^x+2^y=2^{x+y}$ then $\frac{dy}{dx}=$

1) $\frac{2^{x+y}\left(2^y-1\right)}{\left(2^x-1\right)}$ 2) $\frac{-2^{y-x}\left(2^y-1\right)}{\left(2^x-1\right)}$

3) $\frac{-2^{x-y}\left(2^y-1\right)}{\left(2^x-1\right)}$ 4) none

54. If $e^x+e^y=e^{x+y}$ then $\frac{dy}{dx}=$

1) $-e^{x-y}$ 2) $-e^{y-x}$ 3) e^{x-y} 4) none

55. If $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ then $\frac{dy}{dx}=$

1) $\cot\frac{\theta}{2}$ 2) $2\cot\frac{\theta}{2}$ 3) $\tan\frac{\theta}{2}$ 4) $-\tan\frac{\theta}{2}$

56. $\frac{d}{dx}\left(\frac{e^{4x}-e^{-4x}}{e^{4x}+e^{-4x}}\right)=$

1) $4\coth^2 x$ 2) $4\sec h^2(4x)$

3) $4\cos ech^2 x$ 4) none

57. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ then $\frac{dy}{dx}=$

1) $-(1+x)^{-2}$ 2) $(1+x)^{-2}$

3) $-(1+x)^2$ 4) $-(1-x)^{-2}$

58. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ then $\frac{dy}{dx}=$

1) $-\frac{ax+hy+g}{hx+by+f}$ 2) $\frac{ax+hy+g}{hx+by+f}$

3) $-\frac{hx+by+f}{ax+hy+g}$ 4) $\left(\frac{hx+by+f}{ax+hy+g}\right)$

59. If $x^y=e^{x-y}$ then $\frac{dy}{dx}=$

1) $\frac{1}{(1+\log x)^2}$ 2) $\frac{\log x}{(1+\log x)^2}$

3) $\frac{\log x}{1+\log x}$ 4) $\frac{1}{1+\log x}$

60. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then $\frac{dy}{dx}=$

1) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ 2) $2\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

3) $2\frac{\sqrt{1+y^2}}{\sqrt{1+x^2}}$ 4) none

PRACTICE SET - II KEY

01) 1	02) 3	03) 1	04) 3	05) 2
06) 1	07) 1	08) 2	09) 1	10) 2
11) 2	12) 3	13) 2	14) 2	15) 2
16) 3	17) 2	18) 2	19) 4	20) 1
21) 2	22) 1	23) 2	24) 2	25) 3
26) 1	27) 4	28) 2	29) 1	30) 2
31) 4	32) 3	33) 2	34) 4	35) 1
36) 4	37) 1	38) 4	39) 4	40) 2
41) 4	42) 2	43) 2	44) 3	45) 4
46) 1	47) 3	48) 1	49) 1	50) 1
51) 4	52) 3	53) 3	54) 2	55) 3
56) 2	57) 1	58) 1	59) 2	60) 1

PRACTICE SET - III

01. The derivative of $\cos^{-1}\left(2x^2-1\right)$ with respect to $\cos^{-1}x$ is

1) 2 2) $\frac{2}{x}$ 3) $\frac{1}{2\sqrt{1-x^2}}$ 4) $\sqrt{1-x^2}$

02. If $y=\sin^{-1}(x\sin(\alpha+y))$ then $\frac{dy}{dx}=$

1) $\frac{\sin\alpha}{\sin^2(\alpha+y)}$ 2) $\frac{\sin^2(\alpha+y)}{\sin\alpha}$

3) $\sin\alpha\sin^2(\alpha+y)$ 4) $\frac{\sin^2(\alpha+y)}{\sin\alpha}$

42. If $x=\cos\theta\sin 2\theta, y=\sin\theta\cos 2\theta$ Then $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$

1) -2 2) 2 3) -1/2 4) 1/2

43. If $x=(\operatorname{cosec}\theta-\sin\theta)$

$y=(\operatorname{cosec}^3\theta-\sin^3\theta)$, $\frac{dy}{dx}$

1) $3(\sec^2\theta-\cos^2\theta)$ 2) $3(\operatorname{cosec}^2\theta-\cos^2\theta)$

3) $3(\operatorname{cosec}^2\theta-\sin^2\theta)$ 4) $3(\sec^2\theta-\sin^2\theta)$

44. If $x=\tan\theta+\cot\theta, y=\tan\theta-\cot\theta$, Then $\frac{dy}{dx}=$

1) $-\frac{x}{y}$ 2) $-\frac{y}{x}$ 3) $\frac{x}{y}$ 4) $\frac{y}{x}$

45. If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$ Then $x^3y\frac{dy}{dx}=$

1) -1 2) 2 3) -2 4) 1

46. If $x^m y^n=(x+y)^{m+n}$ Then $\frac{dy}{dx}=$

1) $\frac{y}{x}$ 2) $-\frac{y}{x}$ 3) $\frac{m}{n}\frac{x}{y}$ 4) $-\frac{m}{n}\frac{x}{y}$

47. If $\sin^{-1}x-\sin^{-1}y=\pi$ Then $\frac{dy}{dx}$

1) $\frac{x}{y}$ 2) $\frac{y}{x}$ 3) -1 4) 0

48. If $y=\cos^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$ Then $\frac{dy}{dx}=$

1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) $\frac{-x}{\sqrt{1-x^2}}$ 4) $\frac{x}{\sqrt{1-x^2}}$

49. If $x^3-xy^2+2y^2+2=0$ then $\frac{dy}{dx}=$

1) $\frac{y^2-3x^2}{2y(2-x)}$ 2) $\frac{3x^2-y^2}{2y(3-x)}$

3) $\frac{2y(3-x)}{y^2-3x^2}$ 4) $\frac{2y(3-x)}{3x^2-y^2}$

50. If $y=x^y$ then $\frac{dy}{dx}=$

1) $\frac{y^2}{x(1-y\log x)}$ 2) $\frac{y^2}{1-x\log y}$

3) $\frac{y^2}{1+x\log y}$ 4) none

51. If $xy=e^y$ then $\frac{dy}{dx}=$

1) $\frac{y}{x(y+1)}$ 2) $\frac{-y}{x(y+1)}$

3) $\frac{-y}{x(y-1)}$ 4) $\frac{y}{x(y-1)}$

52. If $\sin y=x\sin(a+y)$ then $\frac{dy}{dx}=$

1) $\frac{\sin^2(a-y)}{a}$ 2) $\frac{\sin^2(a+y)}{\cos a}$

3) $\frac{\sin^2(a+y)}{\sin a}$ 4) $\frac{\cos^2(a+y)}{\sin a}$

53. If $2^x+2^y=2^{x+y}$ then $\frac{dy}{dx}=$

1) $\frac{2^{x+y}(2^y-1)}{(2^x-1)}$ 2) $\frac{-2^{y-x}(2^y-1)}{(2^x-1)}$

3) $\frac{-2^{x-y}(2^y-1)}{(2^x-1)}$ 4) none

54. If $e^x+e^y=e^{x+y}$ then $\frac{dy}{dx}=$

1) $-e^{x-y}$ 2) $-e^{y-x}$ 3) e^{x-y} 4) none

55. If $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ then $\frac{dy}{dx}=$

1) $\cot\frac{\theta}{2}$ 2) $2\cot\frac{\theta}{2}$ 3) $\tan\frac{\theta}{2}$ 4) $-\tan\frac{\theta}{2}$

56. $\frac{d}{dx}\left(\frac{e^{4x}-e^{-4x}}{e^{4x}+e^{-4x}}\right)=$

1) $4\coth^2 x$ 2) $4\operatorname{sech}^2(4x)$

3) $4\operatorname{cosech}^2 x$ 4) none

57. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ then $\frac{dy}{dx}=$

1) $-(1+x)^{-2}$ 2) $(1+x)^{-2}$

3) $-(1+x)^2$ 4) $-(1-x)^{-2}$

58. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ then $\frac{dy}{dx}=$

1) $-\frac{ax+hy+g}{hx+by+f}$ 2) $\frac{ax+hy+g}{hx+by+f}$

3) $-\frac{hx+by+f}{ax+hy+g}$ 4) $\left(\frac{hx+by+f}{ax+hy+g}\right)$

59. If $x^y=e^{x-y}$ then $\frac{dy}{dx}=$

1) $\frac{1}{(1+\log x)^2}$ 2) $\frac{\log x}{(1+\log x)^2}$

3) $\frac{\log x}{1+\log x}$ 4) $\frac{1}{1+\log x}$

60. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then $\frac{dy}{dx}=$

1) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ 2) $2\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

3) $2\frac{\sqrt{1+y^2}}{\sqrt{1+x^2}}$ 4) none

PRACTICE SET - II KEY

01) 1	02) 3	03) 1	04) 3	05) 2
06) 1	07) 1	08) 2	09) 1	10) 2
11) 2	12) 3	13) 2	14) 2	15) 2
16) 3	17) 2	18) 2	19) 4	20) 1
21) 2	22) 1	23) 2	24) 2	25) 3
26) 1	27) 4	28) 2	29) 1	30) 2
31) 4	32) 3	33) 2	34) 4	35) 1
36) 4	37) 1	38) 4	39) 4	40) 2
41) 4	42) 2	43) 2	44) 3	45) 4
46) 1	47) 3	48) 1	49) 1	50) 1
51) 4	52) 3	53) 3	54) 2	55) 3
56) 2	57) 1	58) 1	59) 2	60) 1

PRACTICE SET - III

01. The derivative of $\cos^{-1}(2x^2-1)$ with respect to $\cos^{-1}x$ is

1) 2 2) $\frac{2}{x}$ 3) $\frac{1}{2\sqrt{1-x^2}}$ 4) $\sqrt{1-x^2}$

02. If $y=\sin^{-1}(x\sin(\alpha+y))$ then $\frac{dy}{dx}=$

1) $\frac{\sin\alpha}{\sin^2(\alpha+y)}$ 2) $\frac{\sin^2(\alpha+y)}{\sin\alpha}$

3) $\sin\alpha\sin^2(\alpha+y)$ 4) $\frac{\sin^2(\alpha+y)}{\sin\alpha}$

03. If $\sqrt{\tan y} = e^{\cos 2x}\sin x$ then $\frac{dy}{dx} =$

1) $\sin 2y(\cot x - 2\sin 2x)$
2) $\sin 2x(\cot y - \sin y)$
3) $\sin 2y \sin 2x$ 4) $\sin 2y \cos 2x$

04. $\frac{d}{dx}\left(\sin^2 \cot^{-1}\sqrt{\frac{1+x}{1-x}}\right) =$

1) 0 2) $\frac{1}{2}$ 3) $-\frac{1}{2}$ 4) -1

05. $\frac{d}{dx}\sin^{-1}\left(\frac{3x}{2} - \frac{x^3}{2}\right) =$

1) $\frac{3}{\sqrt{4-x^2}}$ 2) $\frac{-3}{\sqrt{4-x^2}}$
3) $\frac{1}{\sqrt{4-x^2}}$ 4) $\frac{-1}{\sqrt{4-x^2}}$

06. If $y\sin x = x + y$ then $\frac{dy}{dx}$ at $x=0$ is

1) 1 2) -1 3) 0 4) 2

07. If $y = 2^{ax}$ and $\frac{dy}{dx} = \log 256$ at $x = 0$ then $a =$

1) 0 2) 1 3) 2 4) 3

08. If $x^y = \log_e x$ then the value of $\frac{dy}{dx}$ at $x = e$ is

1) 0 2) 1 3) e 4) 1/e

09. If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x \ldots\ldots \infty}}}$ then dy/dx

1) $\frac{\cos^2 x}{2y-1}$ 2) $\frac{\sec^2 x}{2y-1}$ 3) $\frac{\tan x}{2y-1}$ 4) $\frac{\cot x}{2y-1}$

10. $\frac{d}{dx}\left\{\cos^{-1}\left(\frac{4x^3}{27} - x\right)\right\}$

1) $\frac{3}{\sqrt{9-x^2}}$ 2) $\frac{1}{\sqrt{9-x^2}}$ 3) $\frac{-3}{\sqrt{9-x^2}}$ 4) $\frac{-1}{\sqrt{9-x^2}}$

11. If $x^y = e^{x-y}$ then $\frac{dy}{dx} =$

1) $\frac{\log_e x}{(1+\log_e x)^2}$ 2) $\frac{1-x}{y + x\log_e x}$
3) $\frac{x-y}{x\log_e ex}$ 4) $\frac{-\log_e x}{(1+\log_e x)^2}$

12. If $y = \sqrt{\cos 2x}$ then $y\frac{d^2y}{dx^2} + 2y^2 =$

1) 0 2) $-\left(\frac{dy}{dx}\right)^2$ 3) $\left(\frac{dy}{dx}\right)^2$ 4) $y\frac{dy}{dx}$

13. If $f(x) = \begin{vmatrix} 2\cos x & 1 & 0 \\ x - \frac{\pi}{2} & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

1) 2 2) $\pi/2$ 3) 1 4) 0

14. $\frac{d}{dx}(\cos x^0) =$

1) $-\sin x^0$ 2) $-\frac{\pi}{180}\sin x^0$
3) $\frac{\pi}{180}\sin x^0$ 4) $\frac{2\pi}{180}\sin x^0$

15. Derivative of $\sin^2 x$ with respect to $(\log_e x)^2$ is

1) $\frac{x\sin x\cos x}{\log_e x}$ 2) $\frac{2\sin x\cos x}{(\log_e x)^2}$
3) $\frac{\sin^2 x}{2\log_e x}$ 4) $x\log_e x$

16. $\frac{d}{dx}\left(\cos^{-1} x + \sin^{-1}\sqrt{1-x^2}\right) =$

1) 0 2) 1 3) $\frac{2}{\sqrt{1-x^2}}$ 4) $\frac{-2}{\sqrt{1-x^2}}$

17. The derivative of $\sin^{-1} x$ with respect to $\cos^{-1}\sqrt{1-x^2}$ is

1) $\frac{1}{\sqrt{1-x^2}}$ 2) $\cos^{-1} x$ 3) 1 4) 0

18. If $y = \cot^{-1}\left(\tan\left(\frac{\pi}{2} - x\right)\right)$ then $\frac{dy}{dx} =$

1) x 2) 1 3) $\frac{1}{1+x^2}$ 4) $\frac{-1}{1+x^2}$

19. If $y = 2^{2^x}$ then $\frac{dy}{dx} =$

1) $y(\log_{10} 2)^2$ 2) $y(\log_e 2)^2$
3) $y^{2^x}(\log_e 2)^2$ 4) $y(\log_e 2)^2 2^x$

20. If $xy = (x+y)^n$ and $\frac{dy}{dx} = \frac{y}{x}$ then $n =$

1) 1 2) 2 3) 3 4) 4

21. Let $f(x) = e^x, g(x) = \sin^{-1} x$ $h(x) = f(g(x))$ then $\frac{h^1(x)}{h(x)} =$

1) $\sin^{-1} x$ 2) $\frac{1}{\sqrt{1-x^2}}$
3) $\frac{1}{\sqrt{1+x^2}}$ 4) $e^{\sin^{-1} x}$

22. If $f(x) = \sqrt{ax} = \frac{a^2}{\sqrt{ax}}$ then $f^1(a) =$

1) a 2) 0 3) 1 4) -1

23. If $f(x) = \frac{x}{1+|x|}$ for $x \in R$ then $f^1(0) =$

1) 0 2) 1 3) 2 4) 3

24. $x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow \frac{dy}{dx} =$

1) $\frac{1}{(1+x)^2}$ 2) $\frac{-1}{(1+x)^2}$ 3) $\frac{1}{1+x^2}$ 4) $\frac{1}{1-x^2}$

25. $x^y = y^x \Rightarrow x(x - y\log x)\frac{dy}{dx} =$

1) $y(y - x\log y)$ 2) $y(y + x\log y)$
3) $x(x + y\log x)$ 4) $x(y - x\log y)$

26. If $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$ then $\frac{dy}{dx} =$

1) $\frac{3y-4x-1}{2y-3x+2}$ 2) $\frac{3y+4x+1}{2y+3x+2}$
3) $\frac{3y-4x+1}{2y-3x-2}$ 4) $\frac{3y-4x+1}{2y+3x+2}$

27. If $y = \log\left\{\left(\frac{1+x}{1-x}\right)^{1/4}\right\} - \frac{1}{2}\tan^{-1}(x)$, then $\frac{dy}{dx} =$

1) $\frac{x}{1-x^2}$ 2) $\frac{x^2}{1-x^4}$ 3) $\frac{x}{1+x^4}$ 4) $\frac{x}{1-x^4}$

28. If $x = a\left\{\cos\theta + \log\tan\left(\frac{\theta}{2}\right)\right\}$ and $y = a\sin\theta$ then $\frac{dy}{dx} =$

1) $\cot\theta$ 2) $\tan\theta$ 3) $\sin\theta$ 4) $\cos\theta$

29. If $y = \sin(\log_e x)$ then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} =$

1) $\sin(\log_e x)$ 2) $\cos(\log_e x)$
3) y^2 4) $-y$

PRACTICE SET - III KEY

01) 1	02) 2	03) 1	04) 3	05) 1
06) 2	07) 3	08) 4	09) 2	10) 3
11) 1,3	12) 2	13) 1	14) 2	15) 1
16) 4	17) 3	18) 2	19) 3	20) 2
21) 2	22) 2	23) 2	24) 2	25) 1
26) 1	27) 2	28) 2	29) 4	

SELF TEST

01. If $f(x)=\frac{e^x+e^{-x}}{e^x-e^{-x}}$ then $f'(x)=$

1) $\frac{2e^x}{\left(e^x-e^{-x}\right)^2}$ 2) $\frac{-2e^x}{\left(e^x-e^{-x}\right)^2}$

3) $\frac{4e^x}{\left(e^x-e^{-x}\right)^2}$ 4) $\frac{-4e^x}{\left(e^x-e^{-x}\right)^2}$

02. $\frac{d}{dx}\left[\log\left(\frac{\sin^m x}{\cos^n x}\right)\right]=$

1) $m\tan x+n\cot x$ 2) $m\cot x+n\tan x$
3) $m\sin x+n\cos x$ 4) $m\cos x+n\sin x$

03. If $y=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots\infty$, then $\frac{dy}{dx}=$

1) e^x 2) x 3) 0 4) 1

04. If $y=1-x+x^2-x^3+$ for $|x|<$ then $dy/dx=$

1) $\frac{1}{1+x}$ 2) $\frac{1}{(1+x)^2}$

3) $-\frac{1}{(1+x)^2}$ 4) none of these

05. If $y=x-x^2+x^3-x^4+\ldots$ then $dy/dx=$

1) $\frac{1}{(1+x)^2}$ 2) $\frac{2}{(1+x)^2}$

3) $\frac{-1}{(1+x)^2}$ 4) none of these

06. If $y=\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$ then $dy/dx=$

1) $-\frac{1}{2}$ 2) $\frac{1}{2}$ 3) 1 4) -1

07. If $y=\sin^n x\cos nx$ then $dy/dx=$

1) $n\sin^{n-1}x\cos(n+1)x$
2) $-n\sin^{n-1}x\cos(n+1)x$
3) $n\sin^{n-1}x\sin(n+1)x$
4) none of these

08. If $y=\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$ then $\frac{dy}{dx}=$

1) 0 2) 1 3) -1 4) $\pi/2$

09. $\frac{d}{dx}\left(\cos x^0\right)=$

1) $-\sin x^0$ 2) $-\frac{\pi}{180}\sin x^0$

3) $\frac{\pi}{180}\sin x^0$ 4) $\sin x^0$

10. $\frac{d}{dx}\left[\sec\left(x^0+30^0\right)\right]$

1) $\sec\left(x^0+30^0\right)\tan\left(x^0+30^0\right)$

2) $\frac{180}{\pi}\sec\left(x^0+30^0\right)\tan\left(x^0+30^0\right)$

3) $\sec x\tan x$

4) $\frac{\pi}{180}\sec\left(x^0+30^0\right)\tan\left(x^0+30^0\right)$

11. If $y=\cot^{-1}\left(\frac{\sqrt{1-\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ then $\frac{dy}{dx}=$

1) 1 2) -1 3) 1/2 4) -(1/2)

12. If $y=\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ then $\frac{dy}{dx}=$

1) $\frac{\pi}{2}$ 2) $\frac{x}{\sqrt{1-x^2}}$ 3) -1 4) 0

13. If $y=e^{2\log(\sin x+\cos x)}$ then $\frac{dy}{dx}=$

1) $e^{2\log(\sin x+\cos x)}$ 2) $\log(\sin x+\cos x)$
3) $2\sin 2x$ 4) $2\cos 2x$

14. $\frac{d}{dx}\left[\sec^{-1}\frac{1}{\sqrt{1-x^2}}\right]=$

1) $\frac{1}{1+x^2}$ 2) $\frac{1}{\sqrt{1+x^2}}$

3) $\frac{1}{\sqrt{1-x^2}}$ 4) $\frac{2}{\sqrt{1-x^2}}$

15. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)\right]=$

1) $\frac{1}{a^2+x^2}$ 2) $\frac{3}{a^2+x^2}$

3) $\frac{3a}{a^2+x^2}$ 4) $\frac{3a}{a^2-x^2}$

16. $\frac{d}{dx}\left[\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=$

1) $\frac{1}{1+x^2}$ 2) $\frac{2}{1+x^2}$ 3) $-\frac{2}{1+x^2}$ 4) 0

17. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)\right]=$

1) $\frac{1}{a^2+x^2}$ 2) $\frac{3}{a^2+x^2}$ 3) $\frac{3a}{a^2+x^2}$ 4) $\frac{3a}{a^2-x^2}$

18. $\frac{d}{dx}\left[\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=$

1) $\frac{1}{1+x^2}$ 2) $\frac{2}{1+x^2}$ 3) $-\frac{2}{1+x^2}$ 4) 0

19. If $y=\tan^{-1}\left(\frac{x-\sqrt{a^2-x^2}}{x+\sqrt{a^2-x^2}}\right)$ then $dy/dx=$

1) $\frac{1}{\sqrt{a^2+x^2}}$ 2) $\frac{1}{\sqrt{a^2-x^2}}$

3) $\frac{1}{\sqrt{a^2-x^2}}$ 4) none of these

20. If $y=\tan^{-1}\left(\frac{\sqrt{2ax-x^2}}{a-x}\right)$ then $dy/dx=$

1) $\frac{1}{\sqrt{2ax-x^2}}$ 2) $\frac{-1}{\sqrt{2ax-x^2}}$
3) $-\sqrt{2ax-x^2}$ 4) $\sqrt{2ax-x^2}$

21. If $y=\sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)$ then $dy/dx=$

1) $\frac{1}{\sqrt{1-x^2}}$ 2) $\sqrt{1-x^2}$ 3) $-\sqrt{1-x^2}$ 4) $\frac{-1}{\sqrt{1-x^2}}$

22. If $\sinh^{-1}x+\sinh^{-1}y=1$ then $dy/dx=$

1) $\sqrt{\frac{1+y^2}{1+x^2}}$ 2) $\sqrt{\frac{1-x^2}{1-y^2}}$

3) $\sqrt{\frac{1-y^2}{1-x^2}}$ 4) $-\sqrt{\frac{1+y^2}{1+x^2}}$

23. If $3x^2+4xy+2y^2+x-8=0$ then $\left(\frac{dy}{dx}\right)_{(-1,3)}$ is equal to
1) 3/8 2) 5/8 3) -5/8 4) -7/8

24. If $e^x+e^y=e^{x+y}$ then $dy/dx=$
1) $-e^{x+y}$ 2) $-e^{x-y}$
3) $-e^{y-x}$ 4) none of these

25. If $\sqrt{x}+\sqrt{y}=\sqrt{xy}$ then $dy/dx=$
1) $\frac{y\sqrt{y}}{x\sqrt{x}}$ 2) $\frac{-y\sqrt{y}}{x\sqrt{x}}$ 3) $\frac{x\sqrt{x}}{y\sqrt{y}}$ 4) $\frac{-y\sqrt{x}}{x\sqrt{y}}$

26. If $x=3\cos\theta-2\cos^3\theta,\ y=3\sin\theta-2\sin^3\theta$ then $dy/dx=$
1) $\cot\theta$ 2) $-\cot\theta$ 3) $2\cot\theta$ 4) $-\frac{1}{2}\tan\theta$

27. If $x=a(\cos\theta+\theta\sin\theta),\ y=a(\sin\theta-\theta\cos\theta)$ then $dy/dx=$
1) $\tan\theta$ 2) $-\tan\theta$ 3) $-\cot\theta$ 4) $\cot\theta$

28. If $x=\frac{2at}{1+t^2},\ y=\frac{b(1-t^2)}{1+t^2}$ then $dy/dx=$
1) $\frac{2bt}{a(1-t^2)}$ 2) $\frac{-2bt}{a(1-t^2)}$
3) $\frac{-2at}{a(1-t^2)}$ 4) $\frac{2bt}{a(1+t^2)}$

29. If $x=\frac{3at}{1+t^3},\ y=\frac{3at^2}{1+t^3}$ then $dy/dx=$
1) $\frac{t(2+t^3)}{1-2t^3}$ 2) $\frac{t(2-t^3)}{1-2t^3}$
3) $\frac{2-t^3}{t(1-2t^3)}$ 4) none of these

30. If $x=\sec\theta-\cos\theta,\ y=\sec^n\theta-\cos^n\theta$ then $(dy/dx)^2=$
1) $\frac{x^2+4}{y^2+4}$ 2) $\frac{y^2+4}{x^2+4}$
3) $\frac{n^2(x^2+4)}{y^2+4}$ 4) $\frac{n^2(y^2+4)}{x^2+4}$

31. The derivative of $\tan^{-1}\left(\frac{\sqrt{1-x^2}-1}{x}\right)$ w.r.t $\tan^{-1}x$ is
1) 1 2) 1/2 3) 2/3 4) 3/2

32. Derivative of $x^{\sin^{-1}x}$ w.r.t $\sin^{-1}x$ is
1) $x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x}+\frac{\log x}{\sqrt{1-x^2}}\right)$
2) $\frac{x^{\sin^{-1}x}}{\sqrt{1-x^2}}\left(\frac{\sin^{-1}x}{x}-\frac{\log x}{\sqrt{1-x^2}}\right)$
3) $\sqrt{1-x^2}.x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x}+\frac{\log x}{\sqrt{1-x^2}}\right)$
4) none of these

33. Derivative of $\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$ w.r.t $\cos^{-1}x$ is
1) 1 2) 1/2 3) -(1/2) 4) -2

34. If $y=e^{\sin^{-1}x}$ and $e^{-\cos^{-1}x}$ then dy/dx=
1) 0 2) 1 3) $e^{\pi/2}$ 4) $e^{-\pi/2}$

35. If $y=x^{x^{\cdots\infty}}$ then dy / dx =
1) $\frac{y}{x(1-y\log x)}$ 2) $\frac{y}{x^2(1-y\log x)}$
3) $\frac{y^2}{x(1-y\log x)}$ 4) $\frac{y^2\cot x}{1-y\log\sin x}$

36. If $y=\left(\sqrt{\sec x}\right)^{(\sqrt{\sec x})^{(\sqrt{\sec x})\cdots\infty}}$ then $\frac{dy}{dx}=$
1) $\frac{y^2\tan x}{2-y\log\sec x}$ 2) $\frac{y^2\tan x}{1-y\log\sec x}$
3) $\frac{y^2\cot x}{1-y\log\sec x}$ 4) $\frac{y^2\cot x}{2-y\log\sec x}$

37. If $y=\sin x\cos x$ then $\frac{d^2y}{dx^2}=$
1) 2y 2) $-2y$ 3) $4y$ 4) $-4y$

38. If $y=x^2\log x$ then $\frac{d^2y}{dx^2}$ at $x=e$ is
1) 1 2) 3 3) 5 4) 0

39. If $y=\cos x.\cos 2x.\cos 3x$ then $y_2=$
1) $\cos 2x+4\cos 4x+9\cos 6x$
2) $-(\cos 2x+4\cos 4x+9\cos 6x)$
3) $-(\cos 2x+16\cos 4x+36\cos 6x)$
4) $\cos 2x+4\cos 4x-9\cos 6x$

40. The second derivative of $e^{\sin^{-1}x}$ w.r.t $\sin^{-1}x$ is
1) $e^{\sin^{-1}x}$ 2) $(1-x^2)e^{\sin^{-1}x}$
3) $\sqrt{1-x^2}e^{\sin^{-1}x}$ 4) $\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

SELF TEST KEY

01) 4	02) 2	03) 1	04) 3	05) 1
06) 4	07) 2	08) 3	09) 1	10) 1
11) 2	12) 4	13) 3	14) 4	15) 4
16) 4	17) 3	18) 3	19) 2	20) 1
21) 2	22) 4	23) 4	24) 2	25) 1
26) 1	27) 2	28) 2	29) 2	30) 4
31) 2	32) 3	33) 3	34) 3	35) 3
36) 1	37) 4	38) 3	39) 2	40) 4

SPACE FOR IMPORTANT NOTES

SPACE FOR IMPORTANT NOTES

SECOND ORDER DERIVATIVES

SYNOPSIS

01. If y is a function of x, then its derivative dy/dx will be, in general, a function of x which can be further differentiated. The derivative of dy/dx is called the **second derivative** of y and is denoted by $\frac{d^2y}{dx^2}$.

Similarly the derivative of $\frac{d^2y}{dx^2}$ is called the **third derivative** of y and is denoted by $\frac{d^3y}{dx^3}$. In general, the nth derivative of y is denoted by $\frac{d^ny}{dx^n}$.

Thus $\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right), \frac{d^3y}{dx^3}=\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right),$

$\ldots\ldots, \frac{d^ny}{dx^n}=\frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)$

If y = f(x), the successive derivatives are also denoted by

$y_1, y_2, y_3, \ldots\ldots y_n$ or $y', y'', y''', \ldots\ldots, y^n$

or Dy, D^2y, D^3y,.....D^ny where D stands for $\frac{d}{dx}$,

D^2 stands for $\frac{d^2}{dx^2}$, etc.

or $f'(x), f''(x), f'''(x), \ldots\ldots, f^n(x)$

02. (i) If x = f(t), y = g(t) then

$$\frac{d^2y}{dx^2}=\frac{1}{(dx/dt)^3}\left[\frac{dx}{dt}\cdot\frac{d^2y}{dt^2}-\frac{d^2x}{dt^2}\cdot\frac{dy}{dx}\right]$$

$$=\frac{g''(t)f'(t)-g'(t)f''(t)}{[f'(t)]^3}$$

ii. If f(x, y) = c then

$$y_2=\frac{\left[f_{xx}f_y^2-2f_{xy}f_xf_y+f_{yy}\cdot f_x^2\right]}{f_y^3}$$

where $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$ are partial derivatives.

03. **Some Important Results :**

i. If $y^2=4ax$, then $\frac{d^2y}{dx^2}=\frac{-4a^2}{y^3}$

ii. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then $\frac{d^2y}{dx^2}=\frac{-b^4}{a^2y^3}$

iii. If $ax^2+2hxy+by^2=1$, then $y_2=\frac{h^2-ab}{(hx+by)^3}$

iv. If $x^3+y^3=3axy$, then $y_2=\frac{-2a^3xy}{(y^2-ax)^3}$

v. If $y=a\cos mx+b\sin mx$, then $\frac{d^2y}{dx^2}+m^2y=0$

vi. If $y=ae^{mx}+be^{-mx}$, then $y_2=m^2y$

vii. If $y=\sin^{-1}x$, then $(1-x^2)y_2-xy_1=0$

viii. If $y=\tan^{-1}x$, then $(1+x^2)y_2+2xy_1=0$

ix. If $x=at^2, y=2at$, then $\frac{d^2y}{dx^2}=\frac{-1}{2at^3}$

x. If $x=a\cos\theta, y=b\sin\theta$ then

$\frac{d^2y}{dx^2}=\frac{-b}{a^2}\operatorname{cosec}^3\theta$

xi. If $x=a\cos^3\theta, y=b\sin^3\theta$, then $\frac{d^2y}{dx^2}=\frac{b}{3a^2}$

$\operatorname{cosec}\theta, \sec^4\theta$

xii. If $ax^2+2hxy+by^2+2gx+2fy+c=0$, then

$$\frac{d^2y}{dx^2}=\frac{abc+2fgh-af^2-bg^2-ch^2}{(hx+by+f)^3}$$

xiii. If $ky=\sin(x+y)$, where k is constant then

$y_2=-y(1+y_1)^3$

xiv. If $y=e^{ax}$ then $y_n=a^n e^{ax}$

xv. If $y=a^{mx}$ then $y_n=m^n a^{mx}(\log a)^n$

xvi. If $y=\frac{1}{ax+b}$ then $y_n=\frac{(-1)^n a^n n!}{(ax+b)^{n+1}}$

PRACTICE SET- I

01. If $y=x+x^{-1}$ then $y_2=$

1) 2 2) $1-1/x^2$ 3) $-2x^{-3}$ 4) $2x^{-3}$

02. If $y=\tan^{-1}x$ then $y_2=$

1) $\frac{1}{1+x^2}$ 2) $\frac{-1}{1+x^2}$

3) $\frac{-2x}{(1+x^2)^2}$ 4) $\frac{2x}{(1+x^2)^2}$

03. If $y=x^2+\frac{2}{x}$, then $\frac{d^2y}{dx^2}=$

1) $2x-2/x^2$ 2) $2x+4x^3$

3) $2+4/x^3$ 4) $2+4/x^2$

04. $D^2(2^{3x+4})=$

1) 2^{3x+4} 2) $(3\log 2)^2.2^{3x+4}$

3) $\log 2.2^{3x+4}$ 4) 0

05. If $y=\frac{x-3}{x+4}$ then $D^2y=$

1) $\frac{14}{(x+4)^3}$ 2) $\frac{-14}{(x+4)^3}$

3) $\frac{-7}{(x+4)^3}$ 4) none

06. If $y=\sin(ax+b)$ then $D^2y=$

1) $-y$ 2) y 3) a^2y 4) $-a^2y$

07. If $y=x\log x$ then $y_2=$

1) 0 2) 1 3) 1/x 4) -1/x

08. If $y=x^2e^x$ then $y_2=$

1) $(x^2+2x+2)e^x$ 2) $(x^2+4x+2)e^x$

3) $(x^2+4x+4)e^x$ 4) none

09. $D^2(x^3\log x)=$

1) $x(5+6\log x)$ 2) $x(2+3x\log x)$

3) $x(5+6x\log x)$ 4) $5x+6x\log x$

10. If $y=x\cos x$, then $y_2=$

1) $x\cos x+2\sin x$ 2) $-(x\cos x+2\sin x)$

3) $-(x\sin x+2\cos x)$ 4) $x\cos x-2\sin x$

11. If $y=\sin^3 x$ then $y_2=$

1) $\frac{3}{4}(3\sin 3x-\sin x)$ 2) $\frac{3}{4}(9\sin 3x-\sin x)$

3) $\frac{3}{4}(3\cos 3x-\sin x)$ 4) $\frac{3}{4}(\sin x-3\sin 3x)$

12. If $y=\cos^4 x$ then $y_2=$

1) $\cos 2x+\cos 4x$ 2) $2(\cos 2x+\cos 4x)$

3) $-2(\cos 2x+\cos 4x)$ 4) $2(\cos 2x-\cos 4x)$

13. If $y=\sin^4 x$ then $y_2=$

1) $\sin 2x+\sin 4x$ 2) $2(\sin 2x+\sin 4x)$

3) $-2(\sin 2x+\sin 4x)$ 4) $2(\cos 2x-\cos 4x)$

14. If $y=\cos 2x\ \cos 4x$ then $\frac{d^2y}{dx^2}=$

1) $2(\cos 2x+9\cos 6x)$

2) $-2(\cos 2x+9\cos 6x)$

3) $2(\cos 2x-9\cos 6x)$

4) none

15. If $y=\sin 6x\cos 4x$ then $y_2=$

1) $25\sin 10x+\sin 2x$

2) $2(25\sin 10x+\sin 2x)$

3) $-2(25\sin 10x+\sin 2x)$ 4) none

16. If $y=\coth 3x$ then $\frac{d^2y}{dx^2}=$

1) $18\operatorname{cosech}^2 3x\coth 3x$

2) $3\operatorname{cosech}^2 3x$

3) $6\operatorname{cosech}^2 3x\coth 3x$ 4) none

17. If $y=e^x\sin x$ then $y_2=$

1) $2e^x\sin x$ 2) $2e^x\cosh$

3) $-2e^x\sin x$ 4) none

18. If $y=\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)$ then $\frac{d^2y}{dx^2}=$

1) $\frac{1}{4x^2+9}$ 2) $\frac{8x}{4x^2+9}$

3) $\frac{8x}{(4x^2+9)^2}$ 4) $\frac{-8x}{(4x^2+9)^2}$

19. If $y=e^{\log\sin 4x}$ then $y_2=$

1) $e^{\log\sin 4x}(4\sin 4x-4\cos 4x)$

2) $16\sin 4x$ 3) $-16\sin 4x$ 4) $\frac{1}{16}\sin 4x$

20. If $y=\log\sec x$, then $y_2=$

1) $\sec^2 x$ 2) $\sec x$ 3) $\sec x\tan x$ 4) $\tan x$

21. If $f(x)=\log\sin x$, then $f''\left(\frac{\pi}{2}\right)=$

1) 1 2) -1 3) 2 4) -2

22. If $y=\log(\cosh x)$ then $y_2=$

1) $\tanh x$ 2) $\sec hx$ 3) $\sec h^2x$ 4) $\tanh^2 x$

23. If $y=\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$ then $y_2=$

1) $\sin x$ 2) $\cos x$ 3) x 4) 0

24. If $y=x^2\log x$ then $\frac{d^2y}{dx^2}$ at $x=e$ is

1) 5 2) 3 3) 1 4) none

25. If $y=\log(x^x)$ then $y_2=$

1) 1/x 2) $1/x^2$ 3) $x^x(1+\log x)$ 4) $1+\log x$

26. If $y=a^x$ then $\frac{d^2y}{dx^2}=$

1) $a^x\log a$ 2) $a^x(\log a)^2$

3) $2a^x\log a$ 4) none

27. If $f(x)=e^{2x}+e^{3x}$ then $f''(0)=$

1) 2 2) 5 3) 13 4) 1/13

28. If $y=a\sin 2x-b\cos 2x$ then $y_2+4y=$

1) 1 2) -1 3) 0 4) 1/2

29. If $y=ax^4+\frac{b}{x^3}$ then $y_2=$

1) $\frac{12x^2}{y}$ 2) $\frac{12y}{x^2}$ 3) $12x^2y$ 4) none

30. If $x=e^t\cos 2t$ then $\frac{d^2x}{dt^2}$ at $t=\frac{\pi}{2}$ is

1) $-3e^{\pi/2}$ 2) $3e^{\pi/2}$ 3) $4e^{\pi/2}$ 4) $e^{\pi/2}$

31. If $y=\sinh^2 x-\cosh^2 x$ then $y_2=$

1) e 2) 2e 3) 3e 4) 0

32. If $s=a\cos nt+b\sin nt$ then $\frac{d^2s}{dt^2}=$

1) ns 2) n^2s 3) ns^2 4) $-n^2s$

33. If $y=\log(6x^2-7x+1)$ then $y_2=$

1) $\frac{6}{6x-1}+\frac{1}{(x-1)^2}$

2) $\left(\frac{6}{6x-1}\right)^2+\frac{1}{(x-1)^2}$

3) $-\left[\left(\frac{6}{6x-1}\right)^2+\frac{1}{(x-1)^2}\right]$

4) $\left(\frac{6}{6x-1}\right)^2-\frac{1}{(x-1)^2}$

34. If $y=\sin^2 x\cos^2 x$ then $y_2=$

1) $\frac{1}{2}\cos 4x$ 2) $\sin 4x$
3) $2\sin 4x$ 4) $2\cos 4x$

35. If $y=\tan^{-1}\left(\frac{\log(e/x^2)}{\log ex^2}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$ then $y_2=$

1) 1/2 2) 2 3) -1 4) 0

36. If $y=\log\left(x+\sqrt{1+x^2}\right)$ then $y''(0)=$

1) 1 2) -1 3) 0 4) e

37. If $e^y+xy=e$ then $y''(0)=$

1) e 2) e^2 3) 1/e 4) $1/e^2$

38. If $y=e^{\log\sin^{-1}x}+e^{\log\cos^{-1}x}, 0<x<1$ then

1) $y_1=5x$ 2) $y_2=5$
3) y_1 does not exist 4) $y_1=y_2$

39. If $y=\tan^{-1}\left(\frac{a\sin x+b\cos x}{a\cos x-b\sin x}\right)$ then $y_1=$

1) $\frac{2ab\sin x\cos x}{(a\cos x+b\sin x)^2}$ 2) 1
3) x 4) 0

40. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $y_2(a)=$

1) a 2) 1/a 3) 1/2a 4) a^2

41. If $e^{\sqrt{x}}+e^{-\sqrt{x}}$ then $xy_2=\frac{1}{2}y_1=$

1) y 2) y/2 3) y/4 4) 4y

42. If $y=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ then $y_2=$

1) $\frac{x}{(1-x^2)^{3/2}}$ 2) $\frac{-x}{(1-x^2)^{3/2}}$
3) $x(1-x^2)^{3/2}$ 4) none

43. If $f(x)=\frac{1}{4-x}$ then $(4-x)^3 f''(x)=$

1) 8 2) 2 3) x 3) 0

44. If $y=3x+\sin x$ then $y_2+y=$

1) 0 2) 3 3) 3x 4) none

45. If $y=\cos x$ then $\frac{d^2y}{dx^2}+y=$

1) 0 2) 1 3) sin x 4) $-\cos x$

46. If $y=\sin mx$ then $y_2=$

1) m^2y 2) $-m^2y$ 3) 0 4) none

47. If $x^2+xy+3y^2=1$, then $y_2=$

1) $\frac{-22}{(x+6y)^3}$ 2) $\frac{-11}{(x+6y)^3}$
3) $\frac{22}{(x+6y)^2}$ 4) $\frac{11}{(x+6y)^2}$

48. If $y=\cos nt+\sin nt$ then $\frac{d^2y}{dt^2}+n^2y=$

1) 1 2) -1 3) 0 4) none

49. If $y=a\cos(\log x)$ then $x^2y_2+xy_1=$

1) y 2) -y 3) 0 4) xy

50. If $y=\sin(m\sin^{-1}x)$ then $(1-x^2)y_2-xy_1=$

1) m^2y 2) $-m^2y$ 3) my 4) $-my$

PRACTICE SET - I KEY

01–4	02–3	03–3	04–2	05–2
06–4	07–3	08–2	09–1	10–2
11–1	12–3	13–4	14–2	15–3
16–1	17–2	18–4	19–3	20–1
21–2	22–3	23–4	24–1	25–1
26–2	27–3	28–3	29–2	30–2
31–4	32–4	33–3	34–4	35–4
36–3	37–4	38–4	39–4	40–3
41–3	42–1	43–2	44–3	45–1
46–2	47–1	48–3	49–2	50–2

PRACTICE SET - II

01. If $\frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}$ then $(1+x^2)y_1+xy=$

1) y 2) 0 3) -y 4) 1

02. If $\cos^{-1}\frac{y}{b}=n\log\left(\frac{x}{n}\right)$ then $x^2y_2+xy_1=$

1) n^2y 2) n^2y^2 3) $-n^2y$ 4) none

03. If $y=a\cos(\log x)+b\sin(\log x)$, then $x^2y_2=xy_1=$

1) y 2) -y 3) -1 4) 0

04. If $y=\sin(\log x)$ then $x^2y^2+xy_1=$

1) y 2) -y 3) 1/y 4) 0

05. If $y=e^{a\cos^{-1}x}$ then $(1-x^2)y_2-xy_1=$

1) a^2y 2) $-a^2y$ 3) y 4) $-y$

06. If $y=a\sin x+b\cos x$ then $y_2+y=$

1) 0 2) a 3) b 4) a^2b^2

07. If $y=ae^x-be^{-x}$ then $y_2-y=$

1) 0 2) a 3) b 4) 1

08. If $y=(\sin^{-1}x)^2$, then $(1-x^2)y_2-xy_1=$

1) 0 2) 2 3) -2 4) y

09. If $y=\log\left(x+\sqrt{x^2+1}\right)$ then $(1+x^2)y_2+xy_1=$

1) y 2) -y 3) 0 4) 2y

10. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ then $\left(1-x^2\right)y_2-3x\,y_1=$

1) y 2) $-y$ 3) 0 4) 2y

11. If $y=\cos(\cos x)$ then $y_2-\cot x\,y_1+y\sin^2 x=$

1) sin x 2) cos x 3) 1 4) 0

12. If $y=\sin(\sin x)$ then $y_2+\tan x\,y_1+y\cos^2 x=$

1) cot x 2) cos x 3) 0 4) sin x

13. If $y=\tan^{-1}(\sinh x)$ then $y_2+\tan y.\,y_1^2=$

1) 0 2) y 3) 2y 4) -y

14. If $y=x^x$ then $\frac{d^2y}{dx^2}$ at x = 1

1) 0 2) 1 3) 2 4) 1/2

15. If $y=(1/x)^x$ then $y_2(1)=$

1) 0 2) 1 3) 2 4) e

16. The value of y_2 for the curve $y^2+3x=25$ at the point $(3,4)$ is

1) $\frac{9}{256}$ 2) $\frac{-9}{256}$ 3) $\frac{3}{256}$ 4) $\frac{1}{256}$

17. If $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$, then $\frac{d^2y}{dx^2}=$

1) $\sec^4\frac{\theta}{2}$ 2) $4a\sec^4\frac{\theta}{2}$
3) $\frac{1}{4a}\sec^4\frac{\theta}{2}$ 4) $\frac{-1}{4a}\sec^4\frac{\theta}{2}$

18. If $a(\theta-\sin\theta), y=a(1-\cos\theta)$, then $y_2=$

1) $\operatorname{cosec}^4\frac{\theta}{2}$ 2) $4a\operatorname{cosec}^4\frac{\theta}{2}$
3) $\frac{1}{4a}\operatorname{cosec}^4\frac{\theta}{2}$ 4) none

19. If $x=\cos\theta+\theta\sin\theta$, $y=\sin\theta-\theta\cos\theta$, then $y_2=$

1) $\frac{\sec^3\theta}{\theta}$ 2) $\frac{\theta}{\sec^3\theta}$ 3) $\frac{\theta}{\cos^3\theta}$ 4) $\sec^2\theta$

20. If $x = f(t), y = g(t)$ then $\frac{d^2y}{dx^2} =$

1) $\left(\frac{d^2y}{dt^2}\cdot\frac{dx}{dt}-\frac{dy}{dt}\cdot\frac{d^2x}{dt^2}\right)\Big/\left(\frac{dx}{dt}\right)^3$

2) $\left(\frac{dx}{dt}\cdot\frac{d^2y}{dt^2}-\frac{d^2x}{dt^2}\cdot\frac{dy}{dt}\right)\Big/\left(\frac{dx}{dt}\right)^3$

3) $\frac{d^2y}{dt^2}\Big/\frac{d^2y}{dt^2}$ 4) none

21. If $x=\frac{1-t}{1+t}, y=\frac{2t}{1+t}$ then $\frac{d^2y}{dx^2}=$

1) 0 2) $\frac{1}{1+t}$

3) $\frac{1}{(1+t)^2}$ 4) none

22. If $x=a(t-\sin t), y=a(1+\cos t)$ then the value of y_2 at $t=\pi/2$ is

1) 0 2) a 3) 1/a 4) 1

23. If $x=2\sin t-\sin 2t, y=2\cos t-\cos 2t$ then the value of $\frac{d^2y}{dx^2}$ at $t=\pi/2$ is

1) 0 2) 2/3 3) 3/2 4) -3/2

24. If $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$ then $y_2 =$

1) $\frac{\sec^3\theta}{a\theta}$ 2) $\frac{a\theta}{\sec^3\theta}$ 3) $\frac{a\theta}{\cos^3\theta}$ 4) none

25. If $x=a\cos^3\theta, y=a\sin^3\theta$ then $\frac{d^2y}{dx^2}$ at $\theta=\pi/4$ is

1) $\frac{8\sqrt{2a}}{3}$ 2) $\frac{4\sqrt{2a}}{3}$ 3) $\frac{4\sqrt{2}}{3a}$ 4) $\frac{8\sqrt{2}}{3a}$

26. If $x=a\left(\cos t+\log\tan\frac{t}{2}\right), y=a\sin t$ then $y_2 =$

1) $\frac{\sin t}{\cos^4 t}$ 2) $\frac{1}{a}\sin t\cos^2 t$

3) $\frac{a\sin t}{\cos^4 t}$ 4) $\frac{\sin t}{a\cos^4 t}$

27. If $x=\sec\theta, y=\log(\sec\theta+\tan\theta)$ then $y_2 =$

1) $\cot\theta\,\mathrm{cosec}\,\theta$ 2) $-\cot^2\theta\,\mathrm{cosec}\,\theta$

3) $\cot^2\theta\,\mathrm{cosec}\,\theta$ 4) $\cot\theta\,\mathrm{cosec}^2\theta$

28. If $\log y=\tan^{-1}x$ then $(1+x^2)y_2-(2x-1)y_1+4=0$

1) 0 2) 2 log y 3) 4 4) 1

29. If $x=t^2, y=t^3$ then $y_2 =$

1) $3/2t$ 2) $3/4t$ 3) $3t/2$ 4) 3/2

30. If $x=t^4-5, y=t^3+6$ then $y_2\left(\frac{1}{2}\right)=$

1) 21/8 2) 21/32 3) 8/21 4) 21/3

31. If $y=b\cos[n\log(x/n)]$ then $x^2y_2+xy_1=$

1) n^2y 2) $-n^2y$ 3) ny 4) $-ny$

32. If $e^{y/x}=\frac{x}{a+bx}$ then $\frac{y_2}{(xy_1-y)^2}=$

1) x^2 2) x^3 3) $1/x^2$ 4) $1/x^3$

33. If $y=\left[\log\left(x+\sqrt{x^2+a^2}\right)\right]^2$ $(a^2+x^2)y_2+xy_1=$

1) 1 2) 2 3) -1 4) -2

34. If $y=\cos(a\sin^{-1}x)$ then $(1-x^2)y_2-xy_1=$

1) a^2y 2) $-a^2y$ 3) ay 4) none

35. If $y=\log(1-t^2)$ and $x=\sin^{-1}t$ then $y_2\left(\frac{1}{2}\right)=$

1) 8/3 2) 3/8 3) -3/8 4) none

36. If $x=\cos t$ and $y=\sin 4t$ then $(1-x^2)y_2-xy_1=$

1) 4y 2) $-4y$ 3) $-16y$ 4) $16y$

37. If $y=ax^{n+1}+bx^{-n}$ then $x^2y_2=$

1) $n(n-1)y$ 2) $n(n+1)y$

3) ny 4) n^2y

38. If $y=\sqrt{\cos 2x}$ then $y\frac{d^2y}{dx^2}+2y^2=$

1) 0 2) $\left(\frac{dy}{dx}\right)^2$ 3) $-\left(\frac{dy}{dx}\right)^2$ 4) $y\frac{dy}{dx}$

39. If $y=\log\left(x+\sqrt{x^2+1}\right)$ then $y''(1)=$

1) 0 2) 1 3) $-1/2\sqrt{2}$ 4) $-2/\sqrt{2}$

40. If $x^2-4y^2=4$ then $y_2=$

1) $-1/4y^3$ 2) $1/4y^3$

3) $4y^3$ 4) $1/2y^3$

41. If $y=\sqrt{\sec 2x}$ then $\frac{d^2y}{dx^2}+y=$

1) y^5 2) $2y^5$ 3) $3y^5$ 4) 0

42. If $y=\cos(n\sin^{-1}x)$ then $(1-x^2)y_2-xy_1=$

1) n^2y 2) $-n^2y$ 3) y 4) none

43. If $y=(\sin^{-1}x)^2$ then $(1-x^2)y_2-xy_1=$

1) y 2) 0 3) 2 4) 1

44. If $y=\frac{\sinh^{-1}x}{\sqrt{1+x^2}}$ then $(1+x^2)y_2+y=$

1) xy_1 2) $-3xy_1$ 3) $3xy_1$ 4) y_1

45. The second derivative of tanx with respect to sinx is

1) $\sec^2 x\tan x$ 2) $\sec^3 x$

3) $3\sec^3 x\tan x$ 4) $3\sec^4 x\tan x$

46. The second derivative of $\log_{10}$ w.r.t. x^2 is

1) $\frac{1}{x^2\log 10}$ 2) $\frac{1}{2x^2\log 10}$

3) $\frac{1}{2x^4\log 10}$ 4) $\frac{-1}{2x^4\log 10}$

47. The second derivative of $e^{\sin^{-1}x}$ w.r.t $\sin^{-1}$ is

1) $e^{\sin^{-1}x}$ 2) $(1-x^2)e^{\sin^{-1}x}$

3) $e^{\sin^{-1}x}/(1-x^2)$ 4) $\sqrt{1-x^2}\,e^{\sin^{-1}x}$

48. If $x=\frac{2t}{1+t^2}$ and $y=\frac{1-t^2}{1+t^2}$ then $\frac{d^2y}{dx^2}=$

1) $\left(\frac{1-t^2}{1+t^2}\right)^3$ 2) $\left(\frac{1+t^2}{1-t^2}\right)^3$

3) $-\left(\frac{1+t^2}{1-t^2}\right)^3$ 4) $-\left(\frac{1-t^2}{1+t^2}\right)^3$

49. If $x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$ then $y_2=$

1) $\frac{t^2-1}{2t}$ 2) $\frac{1-t^2}{2t}$ 3) $\frac{t^2+1}{2t}$ 4) $\frac{t^2-1}{t}$

50. If $x=\sin t$ and $y=\sin at$ then $(1-x^2)y_2-xy_1=$

1) a^2y 2) $-a^2y$ 3) ay 4) 0

PRACTICE SET - II KEY

01-4	02-3	03-1	04-2	05-1
06-1	07-1	08-2	09-3	10-1
11-4	12-3	13-1	14-3	15-1
16-2	17-3	18-3	19-1	20-2
21-1	22-2	23-4	24-1	25-3
26-4	27-2	28-3	29-4	30-1
31-2	32-4	33-2	34-2	35-4
36-3	37-2	38-3	39-3	40-1
41-3	42-2	43-3	44-2	45-4
46-4	47-1	48-3	49-1	50-2

SELF TEST

01. The second derivative of $\log(ax+b)$ is

1) $\frac{1}{ax+b}$ 2) $\frac{a}{(ax+b)^2}$

3) $\frac{a^2}{(ax+b)^2}$ 4) $\frac{-a^2}{(ax+b)^2}$

02. If $y=\cos(3\cos^{-1}x)$ then $\frac{d^2y}{dx^2}=$

1) $-9\sin(3\cos^{-1}x)$ 2) 24x

3) 24 4) none

03. If $x=t^2, y=t^3$ then $y_2=$

1) 3/2 2) 3/2t 3) 3t/2 4) 3/4t

04. If $y=\cos(m\sin^{-1}x)$ then $(1-x^2)y_2-xy_1=$

1) $-m^2y$ 2) y/m^2 3) m^2y 4) m^2/y

05. If $y=(1/x)^x$ then the value of $\frac{d^2y}{dx^2}$ at x = 1 is

1) 1/2 2) 1 3) -1 4) 0

06. If $x=e^{2t}\cos 3t$ then $\frac{d^2x}{dt^2}$ at $t=\frac{\pi}{2}$ is

1) $6e^{\pi}$ 2) $12e^{\pi}$ 3) $-12e^{\pi}$ 4) $e^{\pi}/12$

07. If $y=x^3+e^x$ then the least value of n so that $y_n=y_{n+1}$ is

1) 1 2) 2 3) 3 4) 4

08. If $f(x)=\frac{x^2}{a+x}$ then $f''(a)=$

1) 4a 2) $\frac{1}{8a}$ 3) $\frac{1}{4a}$ 4) $8a$

09. $f(x)=10\cos+(13+2x)\sin x \Rightarrow f''(x)+f(x)=$

1) cos x 2) 4 cos x 3) sin x 4) 4 sin x

10. If $y=\sin(\log x)$ then $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=$

1) $\sin(\log_e x)$ 2) $\cos(\log_e x)$

3) y^2 4) $-y$

SELF TEST KEY

01–4	02–2	03–4	04–1	05–4
06–2	07–3	08–3	09–2	10–4

ALL POWER IS WITHIN YOU
YOU CAN DO ANYTHING AND EVERYTHING

PREVIOUS ECET BITS

2007

01. The derivative of $y=5x^2\sin x$ is

1) $10x.\cos x$ 2) $10x.\sin x$

3) $5(x^2\cos x+2x\sin x)$

4) $5(x\cos x+x\sin x)$

02. Derivative of sin 2x is

1) cos 2x 2) 2 sin 2x

3) 2cos 2x 4) 2x sin 2x

03. Derivative of $\cot^{-1}(x)=$

1) $-1/(1+x^2)$ 2) $1/(1+x^2)$

3) $\sin^{-1}(x)$ 4) $\tan^{-1}(x)$

04. If $y=\sin^{-1}(\sqrt{x})$, then dy/dx is

1) $1/\sqrt{x}$ 2) $1/2\sqrt{(x-x^2)}$

3) $1/2(1-x^2)$ 4) $1/(1+\sqrt{x})$

05. If $2x^2-3xy+y^2+x+2y-8=0$, then dy/dx =

1) $\frac{3y-4x-1}{2+2y-3x}$

2) $(1+4x+3y)/(3x+2y+2)$

3) $(x+y+1)/(2x-y+2)$

4) $(2x+3y+1)/(x+4y-6)$

2008

06. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right]$, a being a constant is

1) $\frac{1}{a^2+x^2}$ 2) $\frac{1}{1+x^2}$ 3) $\frac{1}{2\sqrt{1-x^2}}$ 4) $\frac{a+x}{2\sqrt{x}}$

07. If $3^x-3^y=3^{x+y}$, then $\frac{dy}{dx}=$

1) 3^{y-x} 2) $3^{y/x}$ 3) $-3^{x/y}$ 4) -3^{x-y}

08. If $x=\theta-\frac{1}{\theta}$ and $y=\theta+\frac{1}{\theta}$, then $\frac{dy}{dx}=$

1) $-\frac{y}{x}$ 2) $-\frac{x}{y}$ 3) $\frac{y}{x}$ 4) $\frac{x}{y}$

2009

09. If $y=\log[\sin(\log x)]$ then $\frac{dy}{dx}=$

1) $\tan(\log x)$ 2) $\cot(\log x)$

3) $\frac{\tan(\log x)}{x}$ 4) $\frac{\cot(\log x)}{x}$

10. If $x=\tanh^2 y$, then $\frac{dy}{dx}=$

1) $\frac{e^x}{\sinh y}$ 2) $\frac{1}{x\sqrt{1-x^2}}$

3) $\frac{1}{2(1-x)\sqrt{x}}$ 4) $\sinh^{-1}\left(\frac{3x}{4}\right)$

11. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\ldots\ldots\ldots to\ \infty}}}$ then $\frac{dy}{dx}=$

1) $\frac{\sin x}{2y-1}$ 2) $\frac{\cos x}{2y-1}$ 3) $\frac{\cos x}{1-2y}$ 4) $\frac{\sin x}{1-2y}$

12. If $x=\cos\theta(1+\cos\theta)$ and $y=\sin\theta(1+\cos\theta)$ then $\frac{dy}{dx}$ at $\theta=\frac{\pi}{2}$ is

1) 0 2) -1 3) 1 4) $-\frac{1}{2}$

2010

13. $\frac{d}{dx}[x^n\log x]=$

1) $x^n(1+n\log x)$ 2) $x^{n-1}(1+n\log x)$

3) $x^n(1-n\log x)$ 4) $x^{n-1}(1-n\log x)$

2011

14. If $y = \sec + \tan x$, then $\frac{dy}{dx}$ is

1) $y \sin x$ 2) $y \cos x$
3) $y \sec x$ 4) $y \operatorname{cosec} x$

2012

15. $\frac{d}{dx}[\log_7 X] =$

1) $\frac{1}{x}$ 2) $X \log_7^e$ 3) $\frac{1}{x}\log_e^7$ 4) $\frac{1}{x}\log_7^e$

16. $\frac{d}{dx}[2\cosh x] =$

1) $\frac{e^x + e^{-x}}{2}$ 2) $\frac{e^x - e^{-x}}{2}$
3) $e^x + e^{-x}$ 4) $e^x - e^{-x}$

17. $\frac{d}{dx}\left[\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] =$

1) $\frac{1}{1+x^2}$ 2) $\frac{-1}{1+x^2}$ 3) $\frac{2}{1+x^2}$ 4) $\frac{-2}{1+x^2}$

18. If $x = at^2, y = 2at$, then $\frac{dy}{dx} =$

1) $\sqrt{\frac{y}{x}}$ 2) $\sqrt{\frac{x}{a}}$ 3) $\sqrt{\frac{a}{x}}$ 4) $\sqrt{\frac{x}{y}}$

19. The derivative of e^x with respect to $\sqrt{x}$ is

1) $\frac{2\sqrt{x}}{e^x}$ 2) $2\sqrt{x}e^x$ 3) $\frac{e^x}{2\sqrt{x}}$ 4) $\sqrt{x}e^x$

2013

20. The derivative of log (cot x) is

1) -sec x cosecx 2) -sinx cos x
3) sin x cos x 4) tan x

21. If $x^y = e^{x-y}$, then $dy/dx =$

1) $x^y\left(e^x + e^y\right)$ 2) $\log x(1+\log x)^2$
3) $\log x(1+\log x)^2$ 4) $(1+\log x)/2\log x$

22. The derivative of $\cos^{-1}\left[\left(1-x^2\right)/\left(1+x^2\right)\right]$ is

1) $1/\left(1-x^2\right)$ 2) $1/\left(1+x^2\right)$
3) $\sin\left(1+x^2\right)$ 4) $2/\left(1+x^2\right)$

2014

23. Derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with reference to x is

1) $\frac{2}{1+x^2}$ 2) $\frac{2}{1-x^2}$
3) 2x 4) $\sqrt{\left(1+x^2\right)}$

24. If $y = 3^x, (x>0)$ then $\frac{dy}{dx} =$

1) $3x^{3x-1}$ 2) $3x^{2x}$
3) $3y(1+\log x)$ 4) $\frac{3y}{\log x}$

25. If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ then $\frac{dy}{dx} =$

1) $\left(\frac{x}{y}\right)^{\frac{1}{3}}$ 2) $-\left(\frac{y}{x}\right)^{\frac{1}{3}}$
3) $-\left(\frac{x}{y}\right)^{\frac{1}{3}}$ 4) $\left(\frac{y}{x}\right)^{\frac{1}{3}}$

26. The derivative of log secx with respect to tan x is

1) secx. tanx 2) cosx.cot x
3) cosx. sinx 4) secx.cotx

A.P ECET - 2015

27. If f(x) =

$$\begin{cases} ax^2 - b, & |x| < 1 \\ \frac{1}{|x|}, & |x| \geq 1 \end{cases}$$ is differentiable at x=1, then

1) a=1/2, b = -1/2 2) a = -1/2, b= -3/2
3) a = b = 1/2 4) a = b = -1/2

28. If $y = |\sin x| + |\cos x|$, then $\frac{dy}{dx}$ at $x = \frac{4\pi}{3}$ is

1) 0 2) $\frac{-\sqrt{3}+1}{2}$ 3) $\frac{\sqrt{3}+1}{2}$ 4) $\frac{\sqrt{3}}{2}$

29. If y $y = x^x$, then $\frac{dy}{dx} =$ ______

1) $x^x(1+\log x)$ 2) $x^x(1-\log x)$
3) $x^x(\log x - 1)$ 4) $x^x \log x$

30. If $y = \tan^{-1}\frac{\cos x}{1+\sin x}$, then the value of $\frac{dy}{dx} =$

1) -1/2 2) 1/2 3) x/2 4) -x/2

31. If x=a $(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$,

then the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

1) 0 2) $\frac{1}{\sqrt{3}}$ 3) 1 4) $\sqrt{3}$

T.S ECET - 2015

32. If $y = (x)^x$, then $\frac{dy}{dx}$ is

1) x logx 2) $x^x \log x$
3) $x^x(1-\log x)$ 4) $x^x(1+\log x)$

33. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots\infty}}}$, then $\frac{dy}{dx}$ is

1) 0 2) $\frac{1}{2x-1}$ 3) $\frac{1}{2y-1}$ 4) 1

34. If y = log (sin(xosx)), then $\frac{dy}{dx}$ is

1) cosec(cosx) 2) sinx cot(cosx)
3) -sinx cot(cosx) 4) sec(cosx)

35. If y=A cos x + B sinx, then $\frac{d^2y}{dx^2}$ is

1) 0 2) 1 3) -y 4) y

36. If $x = at^2; y = 2$ at then $\frac{dy}{dx}$ is

1) 0 2) t 3) 1/t 4) 1

37. If u=$\log(e^x + e^y)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ is equal to

1) 0 2) 1 3) 2 4) 3

38. If u is a homogeneous function of order n, then :
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

1) 0 2) nu 3) xu 4) yu

39. If $u = \frac{x^4 + y^4}{x+y}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

1) 0 2) 1u 3) 2 u 4) 3 u

A.P.ECET - 2016

40. If $u = \log\left(\frac{x^2}{y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) 2u 2) 3u 3) u 4) 1

41. If $x = t^2, y = t^3$ then $\frac{d^2y}{dx^2} =$

1) $\frac{3}{2}$ 2) $\frac{3t}{4}$ 3) $\frac{3}{4t}$ 4) $\frac{3}{2t}$

42. If $x^3 + y^3 = 3axy$ then $\frac{dy}{dx} =$

1) $\frac{x^2 - ay}{ax - y^2}$ 2) $\frac{x^2 + ay}{ay - x^2}$
3) $\frac{y^2 - ax}{x^2 - ay^2}$ 4) $\frac{x^2 + ay}{ax + y^2}$

43. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx} =$

1) $-\frac{2}{1+x^2}$ 2) $\frac{2}{1+x^2}$
3) $\frac{1}{1+x^2}$ 4) $-\frac{1}{1+x^2}$

T.S ECET - 2016

44. If $x^y = e^{x-y}$, then $\frac{dy}{dx} =$

1) $\frac{\log x}{(1+\log x)^2}$ 2) $\frac{1}{1+\log x}$

3) $\left(\frac{\log x}{1+\log x}\right)^2$ 4) $\frac{e^x}{1+e^x}$

45. If $f(x)=(x^2+2x+1)^{10}$, then $f'(x)=$

1) $20(x+1)^{19}$ 2) $20(x+1)^{10}$

3) $20(1+x)^{21}$ 4) $20(1+x)^{11}$

46. If $f(x)=7^{x^3+3x};(x>0)$, then $f'(x)=$

1) $(x^2+1)7^{x^3+3x}$

2) $3(x^2+1)7^{x^3+3x}.\log 7$

3) $(x^2+1).7^{x^3+3x}\log 7$ 4) $(x^2+1)(27)^{x^3+3x}$

47. If $f(x)=|\sin x-\cos x|$, then $f'\left(\frac{\pi}{2}\right)=$

1) -1 2) 0 3) $\frac{1}{\sqrt{2}}$ 4) 1

48. If $y=\frac{x}{x^2+1}$, then $\frac{d^4y(0)}{dx^4}=$

1) 12 2) 24 3) 6 4) 1

49. If $u=\log(\tan x+\tan y)$, then $(\sin 2x)\frac{\partial u}{\partial x}+(\sin 2y)\frac{\partial u}{\partial y}=$

1) 1 2) 0 3) 2 4) $\frac{1}{2}$

50. If $u=e^{x+y}+f(x)+g(y)$, then $\frac{\partial^2 u}{\partial x\partial y}=$

1) e^{x-y} 2) e^{xy} 3) e^{x+y} 4) 0

A.P ECET - 2017

51. If $x=a\sec\theta, y=b\tan\theta$ then $\frac{dy}{dx}=$

1) $\frac{b}{a}\sec\theta$ 2) $\frac{b}{a}\operatorname{cosec}\theta$

3) $\frac{a}{b}\sec\theta$ 4) $\frac{a}{b}\operatorname{cosec}\theta$

52. If $x^y=e^{x-y}$ then $\frac{dy}{dx}=$

1) $\frac{\log x}{(1+\log x)^2}$ 2) $\frac{\log x}{(1-\log x)^2}$

3) $\frac{-\log x}{(1+\log x)^2}$ 4) $\frac{-1}{(1+\log x)^2}$

53. If $y=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ then $\frac{dy}{dx}=$ ______

1) $-\frac{1}{1+x^2}$ 2) $\frac{1}{1+x^2}$

3) $\frac{2x}{1+x^2}$ 4) $-\frac{2}{1+x^2}$

T.S ECET - 2017

54. If $y=\cos^{-1}(4x^3-3x)$, then $\frac{dy}{dx}=$

1) $\frac{-3}{\sqrt{1-x^2}}$ 2) $\frac{4}{\sqrt{1-x^2}}$

3) $\frac{1}{\sqrt{1+x^2}}$ 4) $\frac{-4}{3\sqrt{1-x^2}}$

55. $y=(\sin x)^{\log x}$, then $\frac{dy}{dx}=$

1) $(\sin x)^{\log x}\{\tan x.\log x+\log(\sin x)\}$

2) $\log x\{\cot x.\sin x+\frac{1}{x}\log(\sin x)\}$

3) $(\sin x)^{\log x}\{\cot x.\log x+\frac{1}{x}\log(\sin x)\}$

4) $(\cos x)^{\log x}\{\tan x.\log x+\frac{1}{x}\log(\cos x)\}$

56. If $y=\log\left(x+\sqrt{1+x^2}\right)$, then $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=$

1) 1 2) 0 3) x 4) $\frac{1}{\sqrt{1+x^2}}$

57. If $x^y=e^{x-y}$, then $\frac{dy}{dx}=$

1) $\frac{\log x}{(1+\log x)^2}$ 2) $\frac{1}{(1+\log x)^2}$

3) $\frac{\log x}{1+\log x}$ 4) $\frac{(\log x)^2}{(1+\log x)^2}$

A.P ECET - 2018

58. If $y=\sec x+\tan x$ then $\frac{dy}{dx}$ is

1) y cos x 2) y sec x
3) -y sin x 4) y tan x

59. If $y=\sqrt{\frac{1-\cos x}{1+\cos x}}$ then $\frac{dy}{dx}$ is

1) $\sec^2\left(\frac{x}{2}\right)$ 2) $\cos^2\left(\frac{x}{2}\right)$

3) $\frac{1}{2}\cos^2\left(\frac{x}{2}\right)$ 4) $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

60. If $y=x^3e^x$ then $\frac{dy}{dx}$ is

1) $(x-3)x^2e^x$ 2) $(x-2)x^2e^x$

3) $(x+3)x^2e^x$ 4) $(x-1)x^3e^x$

61. If $y=\frac{2+3\sinh x}{3+2\sinh x}$ then the derivative of y with respect to x is

1) $\frac{5\cosh x}{(3+2\sinh x)^2}$ 2) $\frac{5\sinh x}{(4+2\sinh x)^2}$

3) $\frac{5\sin x}{(3-2\cosh x)^2}$ 4) $\frac{\sinh^2 x}{(2-3\sinh x)^2}$

62. If $y=\frac{a+bx}{b-ax}$ then the derivative of y with respect to x is

1) $\frac{a^2+b^2}{(b-ax)^2}$ 2) $\frac{a^2+b^2}{(b+ax)^2}$

3) $\frac{a^2-b^2}{(b-ax)^2}$ 4) $\frac{a+b}{(b-ax)^2}$

63. If u is a homogeneous functioni of x and y with degree n then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) -nu 2) n^2u 3) nu 4) nu^2+u

T.S ECET - 2018

64. The derivative of $\sin x^2$ with respect to x^5 is ___

1) $\frac{\cos x^2}{5x^4}$ 2) $\frac{2\cos x^2}{5x^4}$ 3) $\frac{2\cos x^3}{5x^3}$ 4) $\frac{2\sin x^2}{5x^4}$

65. If $y=x^y$ then $\frac{dy}{dx}=$

1) $\frac{y}{x(1-y\log x)}$ 2) $\frac{y^2}{x(1-y\log x)}$

3) $\frac{y^2}{x(1+y\log x)}$ 4) $\frac{y}{(1-y\log x)}$

66. If $x=at^2$, $y=2at$, then $\frac{d^2y}{dx^2}=$

1) $-\frac{1}{t^2}$ 2) $-\frac{1}{2at}$ 3) $-\frac{1}{2at^2}$ 4) $-\frac{1}{2at^4}$

67. If $Z=\log\left(\frac{xy}{x+y}\right)$, then $x\frac{\partial Z}{\partial x}+\frac{\partial Z}{\partial y}=$

1) 0 2) 2Z 3) 1 4) $\frac{Z}{2}$

PREVIOUS ECET BITS KEY

01) 3	02) 3	03) 1	04) 2	05) 1
06) 2	07) 1	08) 4	09) 4	10) 3
11) 2	12) 1	13) 2	14) 3	15) 4
16) 3	17) 3	18) 1	19) 2	20) 1
21) 2	22) 4	23) 1	24) 3	25) 2
26) 3	27) 2	28) 2	29) 1	30) 1
31) 3	32) 2	33) 3	34) 4	35) 3
36) 3	37) 2	38) 2	39) 4	40) 4
41) 3	42) 1	43) 1	44) 1	45) 1
46) 2	47) 4	48) 2	49) 3	50) 3
51) 2	52) 1	53) 2	54) 1	55) 3
56) 2	57) 1	58) 2	59) 4	60) 3
61) 1	62) 1	63) 3	64) 3	65) 2
66) 3	67) 3			

SPACE FOR IMPORTANT NOTES

PARTIAL DIFFERENATIATION

SYNOPSIS

01. Let u be a function of two independent variables x and y and let us assume the functional relation as $u = f(x, y)$. Then the derivatives of u with respect to x when x varies and y remains constant is called the *partial derivative* of u with respect to x and is denoted by the symbol $\frac{\partial u}{\partial x}$ or u_x

$$\therefore \frac{\partial u}{\partial x} = \underset{\Delta x \to 0}{Lt} \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x}$$

Thus $\frac{\partial u}{\partial x}$ is nothing but the ordinary derivative of u w.r.t. x treating y as constant. Similary, when x remains constant and y varies, the partial derivative of u with respect to y is denoted by the symbol $\frac{\partial u}{\partial y}$ or u_y

$$\therefore \frac{\partial u}{\partial y} = \underset{\Delta y \to 0}{Lt} \frac{f(x, y+\Delta y) - f(x, y)}{\Delta y}$$

Hence $\frac{\partial u}{\partial y}$ = ordinary derivative of u w.r.t y treating x as constant.

02. If $u = f(x, y)$ then the first order partial derivatives may be differentiated again partially with respect to either of the independent variables giving rise to second order partial derivatives denoted as follows

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial^2 u}{\partial x^2} = u_{xx}; \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial^2 u}{\partial y^2} = u_{yy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial^2 u}{\partial x \partial y} = u_{xy}; \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial^2 u}{\partial y \partial x} = u_{yx}$$

It may be noted that, in general $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

03. If $z = f(u)$ and $u = g(x, y)$ then

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} \text{ and } \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y}$$

04. If $f(x, y) = c$ where c is a constant, then

$$\frac{dy}{dx} = \frac{-\partial f / \partial x}{\partial f / \partial y}$$

05. $f(x, y)$ is said to be a homogeneous function of degree n if and only if

$f(\lambda x, \lambda y) = \lambda^n f(x, y)$ for all values of λ or

$$f(x, y) = x^n F\left(\frac{y}{x}\right) \text{ or } y^n F\left(\frac{x}{y}\right)$$

06. **Euler's theorem :**

If $u = f(x, y)$ is a homogeneous function of degree n, then

i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

ii) $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x}$

iii) $x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial y}$

iv) $x^2\frac{\partial^2 u}{\partial x^2} = 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$

v) If $f(u) = \phi(x, y)$ is a homogeneous function of degree n in x and y then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{n f(u)}{f'(u)}$$

Note : In general if $f(x_1, x_2, \ldots\ldots, x_n)$ is a homogeneous function of degree n, then

$$x_1\frac{\partial f}{\partial x_1}+x_2\frac{\partial f}{\partial x_2}+\ldots\ldots+x_n\frac{\partial f}{\partial x_n}=nf$$

07. If $u=f(x,y)$ where x and y are functions of t, then

$$\frac{du}{dt}=\frac{\partial u}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{dt}$$

08. If $u=f(x,y,z)$ and x,y, z are all functions of t, then

$$dy=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy+\frac{\partial u}{\partial z}dz$$

09. **Some Standard Results :**

i) If $u = f(x-y, y-z, z-x)$ is a homogeneous function, then $\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}=0$

ii) If $u=(x-y)(y-z)(z-x)$ then

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$

iii) if $u=\log\left(x^3+y^3+z^3-3xyz\right)$ then

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$$

iv) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$$

v) if $u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\cot u$$

vi) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

vii) If $u=f\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$

STANDARDS

1. $u=f(r), r^2=x^2+y^2$

$$\Rightarrow\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$$

2. $u=f(r); r^2=x^2+y^2+z^2$

$$\Rightarrow\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=$$

3. $z=f(a+y)+g(-ax+y)$ then $\frac{\partial^2 z}{\partial x^2}=a^2\frac{\partial^2 z}{\partial y^2}$

4. $z=f(x+by)+g(x-by)$ then $\frac{\partial^2 z}{\partial y^2}=a^2\frac{\partial^2 z}{\partial x^2}$

PRACTICE SET -I

01. If $u=x^y$ then $\frac{\partial u}{\partial x}=$

1) 0 2) yx^{y-1} 3) $x^y\log x$ 4) none

02. If $u=x^y$ then $\frac{\partial u}{\partial y}=$

1) 0 2) yx^{y-1} 3) $x^y\log x$ 4) none

03. If $u=\sin(ax+by)$ then $\frac{\partial u}{\partial y}=$

1) $\cos(ax+by)$ 2) $a\cos(ax+by)$
3) $b\cos(ax+by)$ 4) $-b\cos(ax+by)$

04. If $u=x^2+y^2$ then $\frac{\partial^2 u}{\partial x\partial y}=$

1) 2 2) 0 3) $2(x+y)$ 4) none

05. If $u=e^{xy}$ then $x\frac{\partial u}{\partial x}=$

1) u 2) ue^x 3) log u 4) u log u

06. If $u=\log\left(\frac{x^2+y^2}{xy}\right)$ then $\frac{\partial^2 u}{\partial x\partial y}=$

1) $\frac{xy}{(x^2+y^2)^2}$ 2) $\frac{2xy}{(x^2+y^2)^2}$
3) $\frac{4xy}{(x^2+y^2)^2}$ 4) $\frac{-4xy}{(x^2+y^2)^2}$

07. If $u=x^y$ then $\frac{\partial^2 u}{\partial y\partial x}=$

1) $x^y(1+y\log x)$ 2) $x^{y-1}(1+y\log x)$
3) $x^{y-1}\left(1+\frac{1}{y}\log x\right)$ 4) $x^{y-1}(1+x\log x)$

08. If $z=\log(x^2+y^2)$ then $\frac{\partial z}{\partial y}=$

1) $\frac{1}{x^2+y^2}$ 2) $\frac{2x}{x^2+y^2}$
3) $\frac{2y}{x^2+y^2}$ 4) $\frac{2y}{(x^2+y^2)^2}$

09. If $z=x^3+y^3-3axy$ then $\frac{\partial^2 z}{\partial x^2}=$

1) 6x 2) 2x 3) 6ax 4) none

10. If $z=e^x\log y$ then $\frac{\partial z}{\partial x}=$

1) e^x/y 2) z 3) -z 4) none

11. If $r=\tan^{-1}(\log xy)$ then $\frac{\partial r}{\partial x}=$

1) $\frac{1}{x\left[1+(\log xy)^2\right]}$ 2) $\frac{1}{y\left[1+(\log xy)^2\right]}$
3) $\frac{1}{y\left[1-(\log xy)^2\right]}$ 4) $\frac{1}{x\left[1+(\log xy)^2\right]}$

12. If $u=x^y+y^x$ then $\frac{\partial u}{\partial x}=$

1) $x^y\log x+xy^{x-1}$
2) $yx^{y-1}\log x+y^x\log y$
3) $x^y\log x+xy^{x-1}\log y$
4) $yx^{y-1}+xy^{x-1}$

13. If $f(x,y)=\log(e^x+e^y)$ then $\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}=$

1) 0 2) 1 3) f 4) 1/f

14. If $z=\log\left(\frac{x+\sqrt{x^2-y^2}}{x-\sqrt{x^2-y^2}}\right)$ then $\frac{\partial z}{\partial x}=$

1) $\frac{2}{\sqrt{x^2-y^2}}$ 2) $\frac{2x}{y\sqrt{y^2-x^2}}$
3) $\frac{2x}{y\sqrt{x^2-y^2}}$ 4) none

15. If $x=\sin^{-1}(z+y^2)$ then $\frac{\partial z}{\partial x}=$

1) $\cos x$ 2) 2y 3) $-2y$ 4) $-\cos x$

16. If $u=\frac{x+y}{x-y}$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=$

1) $\frac{1}{x-y}$ 2) $\frac{2}{x-y}$
3) $\frac{2}{y-x}$ 4) $\frac{2y}{(x-y)^2}$

17. If $z=\log\sqrt{x^2+y^2}$ then $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=$

1) $\frac{2(y^2-x^2)}{(x^2+y^2)^2}$ 2) $\frac{2}{x^2+y^2}$
3) 1 4) 0

18. If $u=\sin^{-1}(y/x)$ then $\frac{\partial^2 u}{\partial x\partial y}=$

1) $(x^2-y^2)^{-3/2}$ 2) $x(x^2-y^2)^{-3/2}$
3) $-x(x^2-y^2)^{-3/2}$ 4) none

19. If $u=x\sin y+y\sin x$ then $\frac{\partial^2 u}{\partial y\partial x}=$

1) $\sin x+\sin y$ 2) $\cos x+\cos y$
3) $(\cos x+\cos y)$ 4) none

20. If $u=x^3+y^3-3xy$ then $\frac{\partial^2 u}{\partial x\partial y}=$

1) 0 2) 1 3) 3 4) -3

21. If $z=\tan(\tan^{-1}x+\tan^{-1}y)$ then $z_x+z_y=$

1) $(1+x^2)+(1+y^2)$ 2) 1
3) $(1+y^2)+(1+x^2)$ 4) $(1-y^2)+(1-x^2)$

22. If $z=x^2y-x\sin xy$ then $\frac{\partial z}{\partial x}=$

1) 0 2) $xy(2-\cos xy)+\sin xy$
3) $xy(2+\cos xy)-\sin xy$
4) $xy(2-\cos xy)-\sin xy$

23. If $xy+yz+zx=1$ then $\frac{\partial z}{\partial x}=$

1) $\frac{1+y^2}{(x+y)^2}$ 2) $\frac{1-y^2}{(x+y)^2}$

3) $\frac{-(1-y^2)^2}{(x+y)^2}$ 4) $\frac{2y^2}{(x+y)^2}$

24. If $u=3x^2y-x\sin(xy)$ then $\frac{\partial u}{\partial y}=$

1) $x^2(3-\cos xy)$ 2) $x^2(3+\cos xy)$
3) $x(3-\cos xy)$ 4) none

25. If $u=x^3-3xy^2$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$

1) 0 2) 6x 3) 12x 4) $-6xy$

26. If $u=x^2y+y^2z+z^2x$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=$

1) 0 2) x + y + z
3) $(x+y+z)^2$ 4) $2(x+y+z)^2$

27. If $z=(ax+by)^2-(x^2+y^2)$ and $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$ then $a^2+b^2=$

1) 0 2) 2 3) -2 4) 4

28. If $u=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ then $\frac{\partial^2 u}{\partial x\partial y}=$

1) 0 2) $\frac{1}{1+x^2}$ 3) $\frac{1}{1+y^2}$ 4) none

29. If $u=\log\left(\frac{x^2+y^2}{xy}\right)$ then $\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial^2 u}{\partial y\partial x}=$

1) 0 2) 1 3) -1 4) 2

30. If $u=x+y$ and $v=x^2-y^2$ then $\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix}=$

1) u 2) v 3) u + v 4) - 2u

31. If $u=\left(x^2+y^2+z^2\right)^{-1/2}$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=$

1) $\left(x^2+y^2+z^2\right)^{1/2}$ 2) $x^2+y^2+z^2$
3) $\left(x^2+y^2+z^2\right)^{-3/2}$ 4) 0

32. If $u=\tan^{-1}\left(\frac{y}{x}\right)$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$

1) 0 2) $\frac{4xy}{(x^2+y^2)^2}$ 3) $\frac{4xy}{x^2+y^2}$ 4) $\frac{-4xy}{(x^2+y^2)^2}$

33. If $z=\tan(y+ax)+(y-ax)^{3/2}$ then $\frac{\partial^2 z}{\partial x^2}-a^2\frac{\partial^2 z}{\partial y^2}=$

1) $\frac{3a^2}{4}(y-ax)^{-1/2}$ 2) $\frac{3a^2}{4}(y+ax)^{-1/2}$
3) 0 4) none

34. If $z=f(x^2+y^2)$ then $x\frac{\partial z}{\partial y}-y\frac{\partial z}{\partial x}=$

1) 0 2) $2xy\,f'(x^2+y^2)$
3) $4xy\,f'(x^2+y^2)$ 4) none

35. If $z=\sin(x-y)+\log(x+y)$ then $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=$

1) 0 2) $2\sin(x-y)$ 3) $\frac{2}{(x+y)^2}$ 4) $\frac{-2}{(x+y)^2}$

36. If $u=\log(x^2+y^2+z^2)$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{k}{x^2+y^2+z^2}$, then k =

1) 1 2) 2 3) -2 4) 3

37. If $z=e^x(x\cos y-y\sin y)$ then $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=$

1) $4e^x\cos y$ 2) $2e^x(y\sin y+2\cos y)$
3) $4e^x\sin y$ 4) 0

38. If $u=(y-z)(z-x)(x-y)$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=$

1) 0 2) 4 xy 3) 4 yz 4) 4 zx

39. If $u=f(x-y,y-z,z-x)$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=$

1) 3 2) -3
3) $\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}$ 4) 0

40. If $u=(x-y)^4+(y-z)^4+(z-x)^4$ then $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=$

1) 0 2) $8(x-y)^3$ 3) $8(y-z)^3$ 4) none

41. If $e^{\frac{z}{x^2-y^2}}=x-y$ then $y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=$

1) x^2+y^2 2) x^2-y^2
3) $x^2-2xy-y^2$ 4) none

42. If $z=e^{ax+by}f(ax-by)$ then $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=$

1) 0 2) ab 3) abz 4) 2abz

43. If $u=(x^2+y^2+z^2)^{1/2}$ then $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=$

1) $3u^{4/3}$ 2) $9u^{4/3}$ 3) $9u^{2/3}$ 4) $u^{4/3}$

44. If $z=\sin\left(\frac{x}{y}\right)+\cos\left(\frac{x}{y}\right)$ then $x=\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) 0 2) z 3) 2z 4) none

45. If $u=\frac{x}{y}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) x + y 2) x - y 3) 0 4) none

46. If $u=\tan^{-1}(y/x)$, the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is

1) 0 2) sin u 3) cos u 4) tan u

47. If $u=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) $\frac{1}{x^2+y^2}$
3) $\frac{2}{y^2-x^2}$ 4) none

48. If $u=x^3+y^3+z^3+3xyz$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=$

1) 0 2) u 3) 2u 4) 3u

49. If $u=x^3-2x^2y+3xy^2+y^3$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) u 3) 2u 4) 3u

50. If $z=xy\,f\left(\frac{x}{y}\right)$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) 0 2) z 3) 2z 4) none

PRACTICE SET - I KEY

01) 2	02) 3	03) 3	04) 2	05) 4
06) 4	07) 2	08) 3	09) 1	10) 2
11) 1	12) 2	13) 2	14) 1	15) 1
16) 2	17) 4	18) 3	19) 2	20) 4
21) 3	22) 4	23) 3	24) 1	25) 1
26) 3	27) 2	28) 1	29) 1	30) 4
31) 4	32) 1	33) 3	34) 1	35) 1
36) 2	37) 4	38) 1	39) 4	40) 1
41) 2	42) 4	43) 2	44) 1	45) 3
46) 1	47) 1	48) 4	49) 4	50) 3

PRACTICE SET - II

01. If f(u) = g(x, y) and g(x, y) is a homogeneous function of degree n, then

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=k\frac{f(u)}{f'(u)}$ then k=

1) 1 2) n 3) n - 1 4) n (n-1)

02. If $u=\sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) cvos u 2) tan u 3) cot u 4) $\frac{1}{2}\tan u$

03. If $u=\tan^{-1}\left(\frac{x-y}{x+y}\right)^{1/2}$ then $\frac{\partial u}{\partial x}=$

1) $\frac{\partial u}{\partial y}$ 2) $\frac{y}{x}\frac{\partial u}{\partial y}$ 3) $\frac{x}{y}\frac{\partial u}{\partial y}$ 4) $\frac{-y}{x}\frac{\partial u}{\partial y}$

04. If $u=\tan^{-1}\frac{x^3+y^3}{x-y}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) sin u 2) sin 2u 3) tan u 4) 0

05. If $u=\sin^{-1}\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ then $\frac{\partial u}{\partial x}=$

1) $\frac{\partial u}{\partial y}$ 2) $\frac{x}{y}\frac{\partial u}{\partial y}$ 3) $\frac{y}{x}\frac{\partial u}{\partial y}$ 4) $\frac{-y}{x}\frac{\partial u}{\partial y}$

06. If $u=\frac{x^2+y^2}{x+y}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial u}=$

1) u 2) [illegible]u 3) 2u 4) none

07. If $u=\sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) tan u 2) 2 tan u 3) cot u 4) 2 cot u

08. If $u=\log\left(\frac{x^4+y^4}{x-y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) u 2) 2u 3) 3u 4) 4u

09. If $u=\cos\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=$

1) 0 2) u 3) -u 4) sin 2u

10. If $f(x,y)=\frac{1}{x^2}+\frac{1}{xy}+\frac{\log x-\log y}{x^2+y^2}$ then

$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+2f(x,y)=$

1) 0 2) 1 3) -1 4) 2

11. If $u=e^{\frac{x^2+y^2}{2xy}}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) u 3) u/2 4) log u

12. If $z=\frac{x^2-y^2}{x^2+y^2}$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) 0 2) z 3) -z 4) 2z

13. If $z=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) tan z 2) cot z

3) $\frac{1}{2}\tan z$ 4) $\frac{-1}{2}\tan z$

14. If $z=\frac{x^{1/25}-y^{1/25}}{x^{1/20}+y^{1/20}}$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$

1) $\frac{9z}{100}$ 2) $\frac{z}{100}$ 3) $\frac{-z}{100}$ 4) 0

15. If $u=\tan^{-1}(x+y)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) $\sin u$ 2) $\sin 2u$

3) $\frac{1}{2}\sin 2u$ 4) $2\sin 2u$

16. If $u=\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) $\frac{u}{20}$ 2) $\frac{-u}{20}$ 3) $\frac{9u}{20}$ 4) 0

17. If $u=\tan^{-1}\left(\frac{x^4+y^4}{x^2+y^2}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) sin u 2) cos u 3) sin 2u 4) 2 sin 2u

18. If $u=\log\left(\frac{x^3+y^3}{x-y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) 1 3) 2 4) none

19. If $u=\log\frac{x^2}{y}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) 1 3) u 4) 2u

20. If $u=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$ then $\Sigma x\frac{\partial u}{\partial x}=$

1) 0 2) u 3) 2u 4) 3u

21. If $u=\frac{1}{\sqrt{x^3+y^3}}$ then the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ at the point (1, 1) is

1) $\frac{1}{2\sqrt{2}}$ 2) $\frac{3}{2\sqrt{2}}$ 3) $\frac{-3}{2\sqrt{2}}$ 4) $\frac{-1}{2\sqrt{2}}$

22. If $u=\frac{x^2(x^2-y^2)^3}{(x^2+y^2)^2}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) u 3) 2u 4) 4u

23. If $u=\log(x^2+xy+y^2)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

1) 0 2) 1 3) 2 4) 3

24. If $u=\log(x^3+y^3+z^3-3xyz)$ then

$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 y=$

1) $\frac{1}{(x+y+z)^2}$ 2) $\frac{3}{(x+y+z)^2}$

3) $\frac{9}{(x+y+z)^2}$ 4) $\frac{-9}{(x+y+z)^2}$

25. If $x^x.y^y.z^z=k$ and z is a function of x and y then

$\frac{\partial z}{\partial x}=$

1) $\frac{-(1+\log x)}{1+\log z}$ 2) $\frac{-(1+\log z)}{1+\log x}$

3) $\frac{1-\log x}{1+\log z}$ 4) $\frac{1-\log z}{1+\log x}$

26. If u be a homogeneous function of degree n and

$x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x\partial y}=k\frac{\partial u}{\partial x}$ then k =

1) 1 2) n 3) n - 1 4) n (n-1)

27. If $u=x^3-3x^2y+y^3$ then $x\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial y}=$

1) $6(x^2-2xy)$ 2) $6(x^2+2xy)$

3) $3x(x-2y)$ 4) $3(x^2-2xy)$

28. If $z = x\left(e^{1/y} + \tan^{-1}\frac{y}{x}\right)$ then

$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} =$

1) 0 2) z 3) 2z 4) none

29. If $x^x y^y z^z = c$ then at $x = y = z$ the value of

$\frac{\partial^2 z}{\partial x \partial y} =$

1) $(x\log ex)^{-1}$ 2) $-(x\log ex)^{-1}$
3) $(y\log ey)^{-1}$ 4) none

30. If $f(x,y) = ax^2 + 2hxy + by^2$ then

$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} =$

1) 0 2) f 3) 2f 4) none

31. If $x = r\cos\theta,\ y = r\sin\theta$ then $\frac{\partial r}{\partial x} + \frac{\partial r}{\partial y} =$

1) r 2) r^2 3) 1/r 4) none

32. If $x = r\cos\theta, y = r\sin\theta$ then $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 =$

1) 0 2) 1 3) r^2 4) none

33. If $u = e^x \cos y, v = e^x \sin y$ then $\frac{\partial u}{\partial x}$

1) $\frac{\partial u}{\partial y}$ 2) $\frac{-\partial u}{\partial y}$ 3) $\frac{-\partial v}{\partial y}$ 4) $\frac{\partial v}{\partial y}$

34. If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) tan u 2) $\frac{1}{2}\tan u$ 3) $\frac{1}{2}\cot u$ 4) $\frac{-1}{2}\cot u$

35. If $u = \sqrt{x^2 - y^2}\sin^{-1}\frac{y}{x}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) 0 2) u 3) 2u 4) u/2

36. $f(x,y) = \frac{\cos(x-4y)}{\cos(x+4y)} \Rightarrow \left(\frac{\partial f}{\partial x}\right)_{y=\frac{\pi}{2}} =$

1) -1 2) 0 3) 1 4) 2

37. If z = sec (y - ax) + tan (y + ax), then

$\frac{\partial^2 z}{\partial x^2} - a^2\frac{\partial^2 z}{\partial y^2} =$

1) z 2) 2z 3) 0 4) -z

38. If z = log (tanx + tan y) then

$\sin 2x.z_x + \sin 2y.z_y =$

1) 1 2) 2 3) 3 4) 4

39. If $z = \sec^{-1}\left(\frac{x^4 + y^4 - 8x^2y^2}{x^2 + y^2}\right)$ then

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$

1) cot z 2) 2 cot z 3) 2 tan x 4) 2 sec z

40. If $z = \frac{y}{x}\left[\sin\frac{x}{y} + \cos\left(1 + \frac{y}{x}\right)\right]$ then $x\frac{\partial z}{\partial x} =$

1) $y\frac{\partial z}{\partial y}$ 2) $-y\frac{\partial z}{\partial y}$ 3) $2y\frac{\partial z}{\partial y}$ 4) $2y\frac{\partial z}{\partial x}$

PRACTICE SET - II KEY

01) 2	02) 4	03) 4	04) 2	05) 4
06) 1	07) 4	08) 3	09) 1	10) 1
11) 1	12) 1	13) 1	14) 3	15) 3
16) 1	17) 4	18) 3	19) 2	20) 1
21) 3	22) 3	23) 3	24) 4	25) 1
26) 3	27) 1	28) 1	29) 2	30) 3
31) 4	32) 2	33) 4	34) 4	35) 2
36) 2	37) 3	38) 2	39) 2	40) 2

SELF TEST

01. If $u = \frac{x+y}{x-y}$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} =$

1) $\frac{1}{x-y}$ 2) $\frac{2}{x-y}$ 3) $\frac{2}{y-x}$ 4) $\frac{2}{(x-y)^2}$

02. If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$ then $\frac{\partial^2 u}{\partial y \partial x} =$

1) $\frac{x^2 - y^2}{x^2 + y^2}$ 2) $\frac{y^2 - x^2}{x^2 + y^2}$
3) $\frac{x^2 - y^2}{(x^2 + y^2)^2}$ 4) none

03. If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{k}{x+y+z}$, then k =

1) 1 2) −3 3) 3 4) 9

04. If u be a homogeneous function of degree n, then

$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} =$

1) nu 2) $n\frac{\partial u}{\partial x}$
3) $(n-1)\frac{\partial u}{\partial x}$ 4) $n(n-1)\frac{\partial u}{\partial x}$

05. If $u = \frac{xy}{x+y}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) u 2) 2u 3) 0 4) none

06. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x+y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

1) u 2) sin u 3) tan u 4) none

07. If $z = \frac{x^2(x^2 - y^2)^3}{(x^2 + y^2)^2}$ then

$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} =$

1) 0 2) u 3) 2u 4) 12u

08. If $u = x^3 - 3x^2y + y^3$ then $x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} =$

1) $6(x^2 - y^2)$ 2) $6(x^2 + y^2)$
3) $6(y^2 - x^2)$ 4) $3(y^2 - x^2)$

09. If $u = \log_e(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} =$

1) 0 2) 2u 3) $\frac{1}{u}$ 4) u

10. If $u = e^{x^2 - y^2}$. Then

1) $xu_x = y.u_y$ 2) $y.u_x = x.u_y$
3) $y.u_x + xu_y = 0$ 4) $x^2u_y + y^2u_x = 0$

SELF TEST KEY

01) 2	02) 1	03) 3	04) 2	05) 1
06) 3	07) 4	08) 3	09) 1	10) 2

ALL POWER IS WITHIN YOU YOU CAN DO ANYTHING AND EVERYTHING

IMPORTANT QUESTIONS

01. If $\left(\sqrt{x}+\sqrt{y}\right)y=\sqrt{x^2+y^2}$, the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is

1) 1 2) u 3) 3u/2 4) none

02. If $V=f\left(\frac{x}{y},\frac{y}{z}\right)$ then $x\frac{\partial V}{\partial x}+y\frac{\partial V}{\partial y}+z\frac{\partial V}{\partial z}=$

1) V 2) $\sqrt{V}$ 3) 0 4) none

03. The maximum value of the function $f(x)=10+2x-x^2$ is

1) 12 2) 11 3) 10 4) 14

04. If $u=f(x,y,z)$ then du = ____

05. If $x=\sin^{-1}\left(z+y^2\right)$ then $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=$ ___

06. If a,b are constants and $y=a\cos(\log x)+b\sin(\log x)$ then, $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=y$ is

1) 0 2) $a+b$ 3) $\frac{ab}{a+b}$ 4) $\frac{a^2+b^2}{ab}$

07. If $x=t+\frac{1}{t}$ and $y=t-\frac{1}{t}$ then $\frac{d^2y}{dx^2}$

1) $\frac{x^2}{y^2}$ 2) $\frac{4}{y^3}$ 3) $\frac{-4}{y^3}$ 4) $\frac{4t^3}{\left(t^2-1\right)^3}$

08. $y=e^{\log x}+e^{-\log x}$ then $x^2\frac{d^2x}{dx^2}+x\frac{dy}{dx}-y=0$

1) 1 2) log x 3) -log x 4) 0

09. The solution of $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$ is

1) $y=Ae^{2x}+Be^{-2x}$ 2) $y=Ae^{2x}+Be^{-2x}$

3) $y=\left(A+Bx^2\right)e^{2x}$ 4) $y=(A+Bx)e^{2x}$

10. Solution of $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=0$ is

1) $e^{3x}\left(c_1\cos 3x+c_2\sin 3x\right)$

2) $c_1e^{3x}+c_2e^{2x}$

3) $e^{2x}\left(c_1\cos 3x+c_2\sin 3x\right)$ 4) None of the

IMPORTANT QUESTIONS KEY

01-2	02-3	03-2		
04- $du=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy+\frac{\partial f}{\partial z}dz$				
05-cos x - 2y				
06-1	07-3	08-4	09-4	10-3

ALL POWER IS WITHIN YOU YOU CAN DO ANYTHING AND EVERYTHING

APPLICATIONS OF DERIVATIVES

➪ Definition. Tangent : Let P and Q be two neighbouring points on the curve. Then as $Q\to P$ along the curve, then the secant PQ tends to a definite line is called the tangent to the curve at the point P.

➪ Geometrically the derivative of f(x) at 'C' represents the slope of the tangent to the curve $y=f(x)$ at the point $(c, f(c))$.

➪ i) The slope of the tangent to the curve $y=f(x)$ at $P(x_1,y_1)$ is $\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$ and it is generally denoted by 'm'. i.e., $m=\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$

ii) The slope of the tangent to the curve at a point is also called gradient to the curve at that point.

➪ Sope of the tangent to the curve

i) $f(x,y)=0$ at P (x_1,y_1) is $-\left(\frac{\partial f/\partial x}{\partial f/\partial y}\right)_{(x_1,y_1)}$

➪ Definition, Normal : Let P be a point on a curve $y=f(x)$. The straight line passing through P and perpendicular to the tangent to the curve at P is called the normal to the curve at P.

➪ Slope of the normal to the curve

i) $y=f(x)$ at $P(x_1,y_1)$ is $-\frac{1}{dy/dx}$

ii) $f(x,y)=0$ at $P(x_1,y_1)$ is $\left(\frac{\partial f/\partial y}{\partial f/\partial x}\right)_{(x_1,y_1)}$

iii) $x=f(t), y=g(t)$ at $P(t)$ is $-\frac{f'(t)}{g'(t)}$

➪ Equations of Tangent and Normal

i. Equation of the tangent to the curve $y=f(x)$ at the point (x_1,y_1) is

$y-y_1=m(x-x_1)$, where $m=\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$

ii. Equation of the normal to the curve $y=f(x)$ at (x_1,y_1) is

$y-y_1=-\frac{1}{m}(x-x_1)$, where $m=\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$

iii. If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)}=0$, then the tangent is parallel to x-axis or perpendicular to y-axis.
In this case, the curve $y=f(x)$ is said to have a horizontal tangent at the point (x_1,y_1)

iv. If $\left(\frac{dy}{dx}\right)_{(x_1,y_1)}=\infty$ or $-\infty$, or if $\frac{dx}{dy}=0$ then the tangent is parallel to y-axis or perpendicular to x-axis. In this case,t he curve $y=f(x)$ is aid to have a vertical tangent at the point (x_1,y_1).

v. If the tangent at (x_1, y_1) to the curve $y = f(x)$ makes an acute angle with the positive direction of the x-axis, then $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} > 0$, otherwise $\left(\frac{dy}{dx}\right)_{(x_1,y_1)} < 0$.

⇨ **Lengths of Tangent, Normal Subtangent and Subnormal.**

a. Let the tangent and normal at the point $P(x_1, y_1)$ on the curve $y = f(x)$ meet the x-axis at the points T and N respectively. Let PM be the ordinate of the point P. Then

i) Length of the tangent at P =

$$PT = \left|\frac{y}{dy/dx}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right| \text{ at } (x_1, y_1)$$

(or) $PT = \left|\frac{y_1}{m}\right|\sqrt{1+m^2}$ where $m = \left(\frac{dy}{dx}\right)_{(x_1,y_1)}$

ii. Length of the normal at P = PN = $|y_1|.\sqrt{1+m^2}$

iii. Length of the subtangent at P = TM = $\left|\frac{y_1}{m}\right|$

iv. Length of the subnormal at P = MN = $|y.m|$.

b. For the curve $x = f(t), y = g(t)$

i Length of the tangent at $P(t) = \left|\frac{y}{dy/dt}\right| . \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$.

ii. Length of the normal at P(t) = $\left|\frac{y}{dx/dt}\right| . \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

iii. Length of the sub tangent at P(t) = $\left|y.\frac{dx/dt}{dy/dt}\right|$

iv. Length of the sub normal at P (t) = $\left|y.\frac{dy/dt}{dx/dt}\right|$.

⇨ **Definition. Angle between two curves :**

The angle between any two intersecting curves is defined as the angle between the tangents to the two curves at their point of intersection.

i. Let θ be the angle between two curves $y = f(x)$ and $y = g(x)$ intersecting at the points $P(x_1, y_1)$.

Then $\pm \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}\left(\text{if } \theta \neq \frac{\pi}{2}\right)$

$= \frac{f'(x_1) - g'(x_1)}{1 + f'(x_1).g'(x_1)}$

where m1 and m2 are the slopes of the tangents to the given curves at $P(x_1, y_1)$

ii. Orthogonal curves : If the angle of intersection between two curves is a right angle then the two curves are said to be intersecting orthogonally. The two curves $y = f(x)$ and $y = g(x)$ cut each other orghogonally if $m_1 m_2 = f'(x_1).g'(x_1) = -1$

iii. The two curves $f(x, y) = 0$ and $g(x, y) = 0$ cut each other orthogonally if $\frac{\partial f}{\partial x}.\frac{\partial g}{\partial x} + \frac{\partial f}{\partial y}.\frac{\partial g}{\partial y} = 0$

iv. The two curves $y = f(x), y = g(x)$ touch each other if $\theta = 0$. i.e. if $m_1 = m_2$.

v. The two curves $f(x, y) = 0$, $g(x, y) = 0$ touch each other if $\frac{\partial f}{\partial x}.\frac{\partial g}{\partial y} = \frac{\partial f}{\partial y}.\frac{\partial g}{\partial x}$

⇨ **Some important Results :**

A. The angle between the curves $y^2 = 4ax$ and $x^2 = 4$ u by at origin is $\frac{\pi}{2}$ and at other point is $Tan^{-1}\left[\frac{3a^{1/3}b^{1/3}}{2(a^{2/3}+b^{2/3})}\right]$

B. The angle between the curves $y^2 = 4ax$ and $x^2 = 4ay$ at origin is $\frac{\pi}{2}$ and at the point $(4a, 4a)$ is $Tan^{-1}\left(\frac{3}{4}\right)$. The area between the two curves is $\frac{16a^2}{3}$.

C. The angle between the curves $y^2 = ax$ and $x^2 + y^2 = a^2$ is $Tan^{-1}(3)$.

D. The angle between the curves $y = a^x$ and $y = b^x\ (a \neq b > 0)$ is $Tan^{-1}\left(\frac{\log(a/b)}{1 + \log a . \log b}\right)$

E. The angle between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x^2 + y^2 = ab$ is $Tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

F. The angle between the curves $x^2 = 4ay$ and $y = \frac{8a^3}{x^2 + 4a^2}$ is $Tan^{-1}(3)$.

⇨ If the curves $xy = c^2$ and $y^2 = 4ax$ cut each other orthogonally, then $c^4 = 32a^4$.

⇨ If the curves $a_1x^2 + b_1y^2 = 1$ and $a_2x^2 + b_2y^2 = 1$ are intersecting orthogonally, then $\frac{1}{a_1} - \frac{1}{a_2} = \frac{1}{b_1} - \frac{1}{b_2}$.

⇨ If the curves $\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1$ and $\frac{x^2}{a_2^2} + \frac{y^2}{b_2^2} = 1$ are intersecting orthogonally, then $a_1^2 - b_1^2 = a_2^2 - b_2^2$.

⇨ If the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ intersect orthogonally, then $a^2 = b^2$.

⇨ If the curves $y^2 = x$ and $y = \frac{k}{x}$ cut each other orthogonally, then $8k^2 - 1 = 0$

⇨ A tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ or $x = a\cos^4\theta, y = a\sin^4\theta$ cuts the coordinate axes in A and B then OA + OB = a.

⇨ A tangent to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ or $x = a\cos^3\theta, y = a\sin^3\theta$ cuts the axes in A and B, then AB = a.

⇨ Equations of the tangent and normal to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ at (a, b) are $\frac{x}{a} + \frac{y}{b} = 2$ and $ax - by = a^2 - b^2$ respectively.

⇨ The subtangent at any point of the curve $x^m y^n = a^{m+n}$ varies as its abscissa.

⇨ For the curve $x^{m+n} = a^{m-n} y^{2n}$, the mth power of the subtangent varies as the nth power of the subnormal.

⇨ The subtangent to the curve $y = a^x$ at any point is of tangent length and the subnormal is equal to $y^2 \log a$.

⇨ For n=2 in the curve $y^n = a^{n-1}x$, then subtangent at any point is constant.

⇨ The length of the portion of the tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ intercepted between the coordinate axes is constant.

⇨ For the catenary $y = c\cosh\left(\frac{x}{c}\right)$.

a. length of normal is y^2/c.

b. length of the subtangent is c cot h $\left(\frac{x}{c}\right)$.

c. length of the subnormal is $\frac{c}{2}\sin h\left(\frac{2x}{c}\right)$.

- The subnormal at any point of the curve $xy = c^2$ varies as cube of the ordinate.
- The subnormal at any point of the curve $y = be^{x/a}$ varies as square of the ordinate.
- For the curve $y = x^n$ if the subnormal is constant then $n = \frac{1}{2}$.
- At any point on the curve $y = f(x)$ if the subnormal is constant, then the curve is a parabola.
- For $n = \frac{1}{2}$ in the curve $y = x^n$, the subnormal is constant.
- Length of the perpendicular from the origin to the tangent at (a, a) on the curve $x^m . y^n = a^{m+n}$ is $\frac{(m+n)a}{\sqrt{m^2+n^2}}$.
- Area of the triangle formed by the coordinate axes with the tangent to the curve $xy = a^2$ at the point (x_1, y_1) is $2a^2$. sq. units.
- Area of the triangle formed by the tangent at $P(x_1, y_1)$, normal at $P(x_1, y_1)$ on $y = f(x)$ and

a. The x-axis is $\frac{y_1^2.(1+m^2)}{2|m|}$ sq.units

b. The y-axis is $\frac{x_1^2.(1+m^2)}{2|m|}$ sq. units

29. Area of the triangle formed by the

a. tangent at (x_1, y_1) on $y = f(x)$ with coordinate axes is $\frac{(y_1 - mx_1)^2}{2|m|}$ sq. units.

b. normal at (x_1, y_1) on $y = f(x)$ with coordinate axes is $\frac{(x_1 + my_1)^2}{2|m|}$ sq.units.

MAXIMA - MINIMA

- Increasing and Decreasing Functions :
- **Definition 1 :** Let D be the domain of a real valued function f. Then f is said to be.

a) increasing function in D if
$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in D$

b) strictly increasing in D if
$x_1 < x_2 \Rightarrow f(x_1) < f(x)$ for all $x_1, x_2 \in D$

c) decreasing function in D if
$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$, for all $x_1, x_2 \in D$

d) strictly decreasing in D if
$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in D$

- **Definition 2 :** Let f be the real valued function defined in a neighbourhood of the point 'a' which is in the domain of f.

A function f is said to be increasing at 'a', if there exists $\delta > 0$ such that

$a - \delta < x < a \Rightarrow f(x) < f(a)$

$a < x < a + \delta \Rightarrow f(x) > f(a)$

A function f is said to be decreasing at 'a', if there exists $\delta > 0$ such that

$a - \delta < x < a \Rightarrow f(x) > f(a)$

$a < x < a + \delta \Rightarrow f(x) < f(a)$

- First derivative test for increasing and decreasing functions : Let f be continuous on [a, b] and differentiable on the open interval (a, b). Then

I) f is (strictly) increasing in [a, b] if $f'(x) > 0$ for each $x \in (a,b)$

b) f is strictly decreasing in [a, b] if $f'(x) < 0$ for each $x \in (a,b)$

c) f is a constant function in [a, b] if $f'(x) = 0$ for each $x \in (a,b)$

- If $f'(a) > 0$ then f is the increasing at x = a and if $f'(a) < 0$ then f is decreasing at x = a.

Note : The converse of the above statement is not true. If f is increasing or decreasing at 'a' then $f'(a)$ may be zero.

Thus a function is an increasing function at a point if its derivative at that point is positive.

Similarly, a function is a decreasing function at a point of its derivative at that point is negative

- Monotonicity : A function f(x) is said to be monotonic if it is either increasing or decreasing in its domain.

Note : If $f'(a) = 0$, then f may or may not be monotonic at 'a'.

- f(x) is increasing at x = a $\Leftrightarrow$ -f(x) is decreasing at x = a.
- Stationary Point : If f is differential and $f'(a) = 0$ then the function f(x) is said to be stationary at x = a and f(a) is called the stationary value of f at x = a. The point (a, f(a)) is called stationary point of f.
- **Maxima and Minima**

Let f(x) be a function defined in the domain D. Then

i) F(x) is said to have a maximum value in D, if there exists a point x_0 in D such that $f(x_0) \geq f(x)$, for all x in D. That is, the value of f at the point x_0 is not exceeded by the value of f at any other point x of the domain D of f.
The value $f(x_0)$ is called the maximum value of f in D and the point x_0 is called a point of maximum.

ii) f is said to have a minimum value in D, if there exists a point x_0 in D such that $f(x_0) \leq f(x)$, for all $x \in D$.
The value value $f(x_0)$ is called the minimum value of f in D and the point x_0 is called a point of minimum.

Note :

1) The maximum and minimum values of a function taken together are called its extreme values and the points at chich the function attains the extreme values are called the turning points of the function.

2) Any function y = f(x) need not have a unique maximum point or a unique minimum point. There can be any number of maximum and minimum points.

3. The maximum value of the function is not necessarily the greatest value of the function and the minimum value of the function need not be the smallest value of the function in any finite interval. In fact, there may be several maximum and minimum values of a function in an interval and a minimum value may be even greater than a maximum value.

- **Critical points :** A necessary condition for the existence of an extremum at the point x_0 of the function f(x) is that where $f'(x_0) = 0$, or $f'(x_0)$ does not exist. The points at which $f'(x) = 0$, or where $f'(x)$ does not exist are called critical points of the function.
- **Working Rule for finding maxima and minima :**

Let $y = f(x)$ be the given function

Step 1 : Find $\frac{dy}{dx} = f'(x)$ and $\frac{d^2y}{dx^2} = f''(x)$

Step 2 : Solve $f'(x) = 0$. Let a,b,c,..... be the real roots of $f'(x) = 0$

Step 3 : i) For x = a, if $f'(a) < 0$, then f(x) is maximum at x = a and f(a) is the maximum value of f(x).

ii) For x = a, if $f''(a) > 0$, then $f(x)$ is minimum at x = a and f(a) is the minimum value of f(x).

Similarly the nature of the function at x = b,c,...can be determined.

➩ Point of Inflection :

A point $x = c$ is said to be a point of inflection for a curve $y = f(x)$ if $f''(c) = 0$ or is not defined and $f'''(c) \neq 0$.

➩ Some Important Results :

➩ If f(x) is increasing function in [a, b], then

a) minimum value of f(x) is f(a).
b) maximum value of f(x) is f(b).

➩ If f(x) is decreasing function in [a, b], then

a) minimum value of f(x) is f(b)
b) maximum value of f(x) is f(a).

➩ If $f(x) = \frac{a\cos x + b\sin x}{c\cos x + d\sin x}$, then the condition for $f(x)$ to be

i. an increasing function is bc > ad.
ii. a decreasing function is bc < ad.

➩ If $x + y = k$ then $x^m y^n$ is maximum at $x = \frac{am}{m+n}, y = \frac{an}{m+n}$.

➩ If $mx + ny = k$, then xy is maximum when x : y = n : m.

➩ If the sum of three positive numbers is a constant (k), then their product will be maximum and its maximum value is $\frac{k^3}{27}$.

➩ If $f(x) = (x-a)(x-b)$, then minimum value of $f(x)$ is $-\frac{(a-b)^2}{4}$f

➩ If a, b, x are positive numbers and $f(x) = ax + \frac{b}{x}$, the least value of $f(x)$ is $2\sqrt{ab}$.

➩ a. If $f(x) = a\sin x + b\cos x + c$, then

i) maximum value of $f(x)$ is $c + \sqrt{a^2 + b^2}$
ii) minimum value of f(x) is $c - \sqrt{a^2 + b^2}$

b. Let $f(x) = ax^2 + bx + c$. Then f(x) has

i) minimum of $a > 0$ and minimum value = $\frac{4ac - b^2}{4a}$
ii) maximum if $a < 0$ and maximum value = $\frac{4ac - b^2}{4a}$

➩ The minimum value of $f(x) = a\tan x + b\cot x$ is $2\sqrt{ab}$ at $Tan^{-1}\sqrt{\frac{b}{a}}$.

➩ The minimum value of $f(x) = a\sec x - b\tan x \, (a, b > 0)$ is $\sqrt{a^2 - b^2}$.

➩ $f(x) = (x - a_1)^2 + (x - a_2)^2 + \dots + (x - a_n)^2$ has minimum when $x = \frac{a_1 + a_2 + \dots + a_n}{n}$

➩ If $f(x) = (x+a)(x+b)(x+c)$ where a, b, c are in A.P. with common difference 'd', then

i) Minimum value of $f(x) = \frac{-2d^3}{3\sqrt{3}}$

ii) Maximum value of $f(x) = \frac{2d^3}{3\sqrt{3}}$

➩ The minimum value of

i) $|x-a| + |x-b|$ is $|a-b|$
ii) $\sqrt{e^{x^2} - 1}$ is 0

➩ The maximum value of
i) $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$
ii) $x^{1/x}$ is $e^{1/e}$
iii) $\frac{\log x}{x}$ is $\frac{1}{e}$

➩ i) The minimum value of x^x is $e^{-1/e}$.

ii) For $0 < a < x$, the minimum value of $\log a^x + \log_x a$ is 2.

➩ If $f(x) = \sqrt{a^2\cos^2 x + b^2\sin^2 x} + \sqrt{a^2\sin^2 x + b^2\cos^2 x}$ then

i) Minimum value of $f(x) = a + b$.
ii) Maximum value of $f(x) = \sqrt{2(a^2 + b^2)}$.

➩ i) The maximum rectangle inscribed in a circle is a square.
ii) Of all the rectangles inscribed in a given fixed circle, the square has the maximum area.
iii) The maximum area of a rectangle in a circle of radius r is $2r^2$ and sides are $\sqrt{2}r, \frac{r}{\sqrt{2}}$.
iv) The maximum area of a rectangle inscribed in a circle of radius r is $2r^2$ sq. units.

➩ The maximum area of a triangle in a circle of radius r is $\frac{3\sqrt{3}}{4} \cdot r^2$.

➩ The semi-vertical angle of a cone of maximum volume with given

i. slant height is $Tan^{-1}\sqrt{2}$
ii. surface area is $Sin^{-1}\left(\frac{1}{3}\right)$.
iii. curved surface area is $Sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

➩ The maximum volume of a cone in sphere of radius R is $\frac{32}{81}\pi R^3$.

➩ The volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ times the volume of the sphere.

➩ The height of a cone maximum volume inscribed in a sphere of radius R is $\frac{4R}{3}$ and base radius is $\frac{2\sqrt{2}}{3}R$

➩ i. The volume of a right circular cylinder inscribed in a sphere of radius R is maximum when its height is $\frac{2R}{\sqrt{3}}$ and its base radius is $\frac{\sqrt{2}R}{\sqrt{3}}$

ii. The volume of the greatest cylinder that can be inscribed in a sphere of radius R is $\frac{4\pi R^3}{3\sqrt{3}}$.

iii. The volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

iv. The height of a cylinder of maximum curved surface area inscribed in a sphere of radius R is $\sqrt{2}R$. Its base radius is $\frac{R}{\sqrt{2}}$.

➩ The area of a rectangle of given fixed perimeter is maximum when the rectangle is a square.

➩ For a given perimeter, if area of a triangle is maximum then it is equilateral.

➩ Given two sides of a triangle and area of the triangle is maximum. Then the angle between the sides is $\frac{\pi}{2}$.

➩ The area of the greatest rectangle inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is 2ab sq. units and the sides of the rectangle are $a\sqrt{2}$ and $b\sqrt{2}$.

➩ The minimum distance from the origin to a point on the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ is a+b.

⇨ The sides of a rectangle with maximum perimeter inscribed in a semi-circle of radius R are $\frac{4R}{\sqrt{5}}, \frac{4R}{\sqrt{5}}$.

⇨ A window is in the form of a rectangle surmounted by a semi-circle. If the perimeter of the window is of fixed length 'l' then the maximum area of the window is $\frac{l^2}{2\pi+8}$

⇨ A wire of length 'a' is cut into two parts which are bent respectively in the form of a square and a circle. Then the least value of the sum of the areas so formed is $\frac{a^2}{4(\pi+4)}$

⇨ i. An open box is to be made out of the square sheet of side 'a', by cutting off equal squares at each corner and turning up the sides. If the volume of the box so formed is to be maximum, the side of the square removed is $\frac{a}{6}$. The sides of the box are $\frac{2a}{3}, \frac{2a}{3}, \frac{a}{6}$ and maximum volume is $\frac{2a^3}{27}$ cubic units.

ii. An open box made by a sheet area 'a^2' units whose base is a square. It has maximum volume when the sides are $\frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}, \frac{a}{2\sqrt{3}}$ and maximum volume is $\frac{a^3}{6\sqrt{3}}$ cubic units.

RATE MEASURE

01. Let $y=f(x)$ be a function defined on an interval I. Let Δy be the change in y corresponding to a change Δx in x. Then, $\frac{\Delta y}{\Delta x}$ is called the average rate of change of y in the interval I. If $\frac{\Delta y}{\Delta x}$ is constant then we see that $y=f(x)$ changes at a uniform rate with respect to x. If $\frac{\Delta y}{\Delta x}$ is not constant, then we say that y = f(x) changes at a variable rate w.r.t x. If $\underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta y}\left(i.e., \frac{dy}{dx}\right)$ exists then the limit is called the rate changes of y at x.

02. If a particle move long a straight line by the relation $s=f(t)$ then $\frac{ds}{dt}$ is the rate of change of distance s and is called velocity of the particles at time t.

Denoting $\frac{ds}{dt}$ by v. we have $v=\frac{ds}{dt}$

Note:

i) If v > 0 then 's' increasing and the particle is moving from left to right.

ii) If v < 0 then 's' is decreasing and the particle is moving from right to left.

iii) If v = 0 then the particle moving on a straight line comes to rest.

03. The rate of change of velocity is called the acceleration and is denoted by 'a'.

$$\therefore a=\frac{dv}{dt}=\frac{d}{dt}\left(\frac{ds}{dt}\right)=\frac{d^2s}{dt^2}=\frac{dv}{ds}.\frac{ds}{dt}=v.\frac{dv}{ds}$$

04. A particle moving on a straight line

i) Comes to rest if $\frac{ds}{dt}=0$ and $\frac{d^2s}{dt^2}=0$

ii) is at rest momentarily if $\frac{ds}{dt}=0$ and $\frac{d^2s}{dt^2}\neq 0$

5. A particle projected vertically upwards, attains the maximum height when $\frac{ds}{dt}=0$

⇨ A particle acquires maximum velocity if $\frac{dv}{dt}=0$

⇨ A particle changes it's direction if $\frac{ds}{dt}=0$ and $\frac{d^2s}{dt^2}\neq 0$.

⇨ If the equation of motion of a particle P (x,y) on a curve are x = f(t) and y = g (t) then the velocity of the particle is given by $\frac{ds}{dt}=\sqrt{[f'(t)^2+[g'(t)]^2}$

⇨ Negative acceleration is called Retardation.

⇨ **Some Important Results :**

i. The side of an equilateral triangle is 'a' cm and it increases at the rate of k cm/sec. Then the rate of increase of its area is $\frac{\sqrt{3}}{2}$ ak. sq. cm/sec.

ii. A variable triangle is inscribed in a circle of radius R. If the rate of change of side is R times the rate of change of the opposite angle then the angle is $\frac{\pi}{3}$.

iii. An inverted conical vessel of semi-vertical angle $\frac{\pi}{4}$ is being filled with water at the rate of k. cm³/sec. The rate of change in water level when it is 'h' cm is $\frac{k}{\pi h^2}$ cm/sec.

iv. A source of light is hung 'h' meters directly above a straight horizontal path on which a boy 'a' meters in height is walking. If a boy walks at a rate of b meters / second from the light then the rate at which his shadow increases is $\frac{ab}{h-a}$ mt/sec.

ERRORS ANDAPPROXIMATION

⇨ **Infinitesimals :** There are quantities of very small magnitude with reference to the subject under consideration. The product of two or more infinitesimals is very very small and hence such products may be called infinitesimals of higher order.

⇨ Let $y=f(x)$ be a function differentiable at a point x in the interval [a, b]. If Δx is any change in x, then the corresponding change in y is Δy. It is given by $\Delta y=f(x+\Delta x)-f(x)$

⇨ The value $f'(x)\,\Delta x$ is called the differential of the function f(x) and is denoted by the symbol dy or df.

$\therefore dy=f'(x)\,\Delta x=\frac{dy}{dx}\Delta x$

⇨ Δy is approximately equal to dy. i.e., $\Delta y=dy$.

⇨ $dy=f'(x)\,\Delta x$ is called the approximate change in corresponding to the change of Δx in x.

⇨ **Definition :** Let y = f(x) be a function defined on an interval [a, b] and $x\in[a,b]$. Let Δx be the change

i) Δy is called error in y or absolute error in y

ii) $\frac{\Delta y}{y}$ is called the relative error in y and

iii) $\frac{\Delta y}{y}\times 100$ is called the percentage error in y.

⇨ Finding the differential of a function is similar to that of finding the derivative.

1) $d(u+v)=du\pm dv$

2) $d(uv)=udv+vdu$

3) $d\left(\frac{u}{v}\right)=\frac{vdu-udv}{v^2}$

⇨ If $y=kx^n$, where k is a constant, then the relative error in y is n times the relative error in x.

➪ Some Important Results :

1. **Circle :** If r is the radius of the circle, x is the diameter then

i) $x = 2r$

ii) Area of the circle, $A = 2\pi r^2$ or $A = \frac{\pi x^2}{4}$

iii) Circumference of the circle, $C = 2\pi r$ or $C = \pi x$

2. **Sector :** If r is the radius, ℓ is the length of the arc and θ is the angle, P is the perimeter and A is the area of a sector. then

i) $\ell = r\theta$

ii) $P = l + 2r$ or $P = r\theta + 2r - r(\theta + 2)$

iii) $A = \frac{1}{2} lr$ or $A = \frac{1}{2} r^2 \theta$

3. **Equilateral Triangle :** If x is the side. P is the perimeter. A is the area and H is the height of an equilateral triangle, then

i) $P = 3x$

ii) $H = \frac{\sqrt{3}x}{2}$

iii) $A = \frac{\sqrt{3}}{4} x^2$

4. **Triangle :** If a, b, c are the sides of a triangle ABC, then Area of ABC is

$A = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C$

5. **Rectangle :** If ℓ is the length, b is the breadth, P is the perimeter and A is the area of a rectangle. then

i) $P = 2(\ell + b)$

ii) $A = \ell b$

6. **Square :** If x is the side, P is the perimeter and A is the area of a square, then

i) $P = 4x$

ii) $A = x^2$

7. **Cube :** If x is the side, S is the surface area and V is the volume of a sphere, then

i) $S = 6x^2$

ii) $V = x^3$

8. **Sphere :** If r is the radius, S is the surface area and V is the volume of a sphere, then

i) $S = 4\pi^2 \to 4\pi r^2$

ii) $V = x^3 \to \frac{4}{3}\pi r^3$

9. **(Right Circular) Cylinder :** If r is the radius (of cross section), h is the height L is the lateral (curved) surface area, S is the total surface area and V is the volume of a (right circular) cylinder. then

i) $L = 2\pi h$

ii) $S = 2\pi rh + 2\pi r^2$

iii) $V = \pi r^2 h$

10. **(Right Circular) Cone :** If r is the base radius, h is the height, ℓ is the slant height, θ is the semivertical angle, α is the vertical angle, L is the lateral surface area, S is the total surface area and V is the volume of a cone, then

i) $\ell^2 = r^2 + h^2$

ii) $\theta = Tan^{-1}\left(\frac{r}{h}\right)$

iii) $\alpha = 2\theta$

iv) $L = \pi rl = \pi r\sqrt{r^2 + h^2}$

v) $S = \pi rl + \pi r^2$

vi) $V = \frac{1}{3}\pi r^2 h$

11. **Simple Pendulum :** If l is the length, T is the period of oscillation of a simple pendulum and g the acceleration due to gravity, then $T = 2\pi\sqrt{\frac{l}{g}}$

PRACTICE SET - I

01. The slope of the tangent at (2, 8) on the curve $y = x^3$ is
1) 3 2) 6 3) 12 4) none

02. Which of the following is not correct ?
In a curve $y = f(x)$
1) dy/dx is the slope of the tangent at any point (x, y) on the curve
2) dy/dx = 0 implies that the tangent is parallel to x-axis
3) dy/dx = ∞ implies the tangent is parallel to y-axis
4) dy/dx = 1 implies that the tangent is parallel to the line y = 1

03. The slope of the normal to the curve $y = \frac{6x}{x^2 - 1}$ at (2, 4) is
1) 3/10 2) −10/3 3) 10/3 4) −3/10

04. The slope of the tangent to the curve $x = \frac{t}{t+1}, y = \frac{t^2}{t+1}$ at any point t is
1) t 2) t + 2 3) t(t + 2) 4) none

05. The slope of the normal to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, −1) is
1) 6/7 2) −6/7 3) −7/6 4) 7/6

06. The slope of the curve $y = x^2 + 2x + 2$ at the point whose ordinate is 3 and abscissa is 2 is
1) 8 2) 6 3) 2 4) none

07. Slope of the normal at any point 'θ' to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ is
1) $\tan\theta$ 2) $-\tan\theta$ 3) $-\cot\theta$ 4) $\cot\theta$

08. If the slope of the tangent to the curve $xy + ax + by = 0$ at (1, 1) is 2 then (a, b) = [or if the tangent to the curve $xy + ax + by = 0$ at (1, 1) is inclined at an angle $\tan^{-1} 2$ with x-axis then (a, b) =]
1) (−2, 1) 2) (1, −2)
3) (−1, 2) 4) (1, 2)

09. Slope of the normal at the point (a, b) on the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ is
1) − b/a 2) b/a 3) −a/b 4) a/b

10. The equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = 5$ at (9, 4) is
1) $2x + 3y - 30 = 0$ 2) $3x - 2y + 30 = 0$
3) $3x - 4y + 91 = 0$ 4) none

11. Equation of the tangent line to the curve $y(x^2 + 1) = x + 3$ at (2, 1) is
1) $3x + 5y - 11 = 0$ 2) $3x + 5y - 1 = 0$
3) $3x - 4y + 91 = 0$ 4) none

12. For the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ at (a,b) the equation of the tangent is
1) $\frac{x}{a} + \frac{y}{b} = 1$ 2) $\frac{x}{a} - \frac{y}{b} = 1$
3) $\frac{x}{a} + \frac{y}{b} = 2$ 4) $\frac{x}{a} - \frac{y}{b} = 2$

13. Equation of the normal line at (1, 1) on the curve $3x^2 - xy + y^2 = 3$ is
1) $5x + y - 6 = 0$ 2) $5x + y + 4 = 0$
3) $x - 5y + 4 = 0$ 4) $x + 5y - 4 = 0$

14. Equation of the tangent to the curve $y = 1 - e^{x/2}$ at the point of intersection with y-axis is
1) $x + y = 0$ 2) $x - y = 0$
3) $x + 2y = 0$ 4) $2x + y = 0$

15. Equation of the normal at $\theta = \pi/4$ to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ is
1) $y = x$ 2) $y = -x$ 3) $x + y = a/\sqrt{2}$ 4) none

16. The tangent of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at $\left(\frac{a}{\sqrt{8}}, \frac{a}{\sqrt{8}}\right)$ is parallel to the line
1) $y = x$ 2) $y = -x$ 3) $y = 2x$ 4) none

17. If $\frac{x}{a}+\frac{y}{b}=p$ touches the curve $\left(\frac{x}{a}\right)^4+\left(\frac{y}{b}\right)^4=2$ at (a,b), then p=
1) 0 2) 1 3) 2 4) none

18. The point on the curve $y=3x^2+2x+5$ at which the line $x+2y=10$ is normal to the curve is
1) $(0,5)$ 2) $(0,-5)$ 3) $(5,0)$ 4) $(-5,0)$

19. The equation of the tangent to the curve $4y=x^3$ which is parallel to the line $3x-y-12=0$ is
1) $3x-y-4=0$ 2) $3x-y+8=0$
3) $6x-2y+7=0$ 4) none

20. Equation of the tangent line to the curve $y^2=3x+2$ which is parallel to the line $3x-4y+1=0$ is
1) $3x-4y+2=0$ 2) $3x-4y+6=0$
3) $3x+4y+6=0$ 4) $3x-4y-6=0$

21. Equation of the tangent line to the curve $y=5x^3-2x+7$ where the tangent is perpendicular to $x+13y=0$ is
1) $x+13y=51$ 2) $x+13y=131$
3) $x+13y+131=0$ 4) $x+13y+51=0$

22. The tangent at $(9,-3)$ to the curve $x^2-3y^2=54$ cuts the coordinate axes at A and B. The area of the ΔOAB is
1) 18 2) 36 3) 48 4) 54

23. The tangent to the curve $y=5-2x^2$ at the point $x=1$ cuts the coordinate axes at A and B. The area of the ΔOAB is
1) 49 2) 49/2 3) 49/4 4) 49/8

24. Area of the triangle formed by the tangent, normal at $(1,1)$ on the curve $\sqrt{x}+\sqrt{y}=2$ and the x-axis is
1) 1 2) 2 3) 1/2 4) 4

25. The angle of intersection of the curves $y^2=x$ and $x^2=y$ at $(1,1)$ is
1) $\tan^{-1}\frac{3}{4}$ 2) $\tan^{-1}\frac{4}{3}$
3) $\tan^{-1}\frac{1}{2}$ 4) none

26. The angle between the curves $y^2=4ax$ and $ay=2x^2$ at (a, 2a) is
1) $\tan^{-1}\frac{3}{4}$ 2) $\tan^{-1}\frac{3}{5}$
3) $\tan^{-1}\frac{4}{3}$ 4) none

27. The angle of intersection of the two curves $y^2=4x$ and $x^2=4y$ at the point other than the origin is
1) $\tan^{-1}\frac{3}{2}$ 2) $\tan^{-1}\frac{2}{3}$ 3) $\tan^{-1}\frac{3}{4}$ 4) $\tan^{-1}\frac{5}{2}$

28. Angle between the curves $y^2=x$ and $x^2=y$ at $(1,1)$ is
1) $\pi/2$ 2) $\tan^{-1}\frac{4}{3}$ 3) $\tan^{-1}\frac{3}{4}$ 4) $\tan^{-1}\frac{1}{2}$

29. Angle between two parabolas $y^2=4ax$ and $ay=2x^2$ at the point other than the origin is
1) $\tan^{-1}\frac{3}{4}$ 2) $\tan^{-1}\frac{3}{5}$
3) $\tan^{-1}\frac{4}{3}$ 4) none

30. Acute angle between the curves $xy=2$ and $y^2=4x$ is
1) $\tan^{-1}\frac{1}{3}$ 2) $\tan^{-1}3$
3) $\tan^{-1}\frac{1}{2}$ 4) $\tan^{-1}\frac{2}{3}$

31. Angle between $y=\sin x$ and $y=\cos x$ at a common point is
1) $\pi/4$ 2) $\pi/2$
3) $\tan^{-1}\sqrt{2}$ 4) $\tan^{-1}2\sqrt{2}$

32. The angle between the curves $x^2=3y$ and $x^2+y^2=4$ is
1) $\tan^{-1}\frac{3}{4}$ 2) $\tan^{-1}\frac{\sqrt{5}}{3}$
3) $\tan^{-1}\frac{5}{3}$ 4) $\tan^{-1}\frac{5}{\sqrt{3}}$

33. The curves $y=x^2$ and $6y=7-x^3$
1) touch each other at $(1,1)$
2) cut orthogonally, at $(1,1)$
3) intersecting an angle $\pi/4$ at $(1,1)$
4) none

34. If the curves $y^2=x$ and $y=\frac{k}{x}$ cut each other at right angles then $k^2=$
1) 1/8 2) 8 3) $-1/8$ 4) 1/2

35. If the curves $y^2=16x$ and $9x^2+ky^2=16$ cut each other orthogonally, then k=
1) 2 2) 4
3) 9/2 4) none

36. The curves $x^3-3xy^2=4$ and $3x^2y-y^3=4$
1) touch each other at (x_1,y_1)
2) cut orthogonally at (x_1,y_1)
3) intersect at an angle $\pi/4$ at (x_1,y_1)
4) none

37. The curves $y^2=4(x+1)$, $y^2=36(9-x)$
1) cut orthogonally
2) touch each other
3) intersect at an angle $\pi/4$ 4) $\pi/3$

38. The two curves $y^2=4ax, xy=c^2$ cut at right angles if
1) $c^4=32a^4$ 2) $c^2=32a^4$
3) $c^4=16a^2$ 4) $c^4=16a^4$

39. If the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a_1^2}+\frac{y^2}{b_1^2}=1$ cut each other at right angles then
1) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{a_1^2}+\frac{1}{b_1^2}$ 2) $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{a_1^2}-\frac{1}{b_1^2}$
3) $a^2+b^2=a_1^2+b_1^2$ 4) $a^2-b^2=a_1^2-b_1^2$

40. If the curves $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{a'}+\frac{y^2}{b'}=1$ cut orthogonally, then
1) $a+b=a'+b'$ 2) $a-b=a'-b'$
3) $\frac{1}{a}+\frac{1}{b}=\frac{1}{a'}+\frac{1}{b'}$ 4) $\frac{1}{a}-\frac{1}{b}=\frac{1}{a'}-\frac{1}{b'}$

41. If the curves $ax^2+by^2=1, a_1x^2+b_1y^2=1$ cut orthogonally, then
1) $\frac{1}{a}+\frac{1}{b}=\frac{1}{a_1}+\frac{1}{b_1}$ 2) $a+b=a_1+b_1$
3) $\frac{1}{a}-\frac{1}{b}=\frac{1}{a'}-\frac{1}{b'}$ 4) $a-b=a_1-b_1$

42. The length of tangent at $\sqrt{x}+\sqrt{y}=5$ at (9,4) is
1) $2\sqrt{13}$ 2) $\sqrt{13}$ 3) $4\sqrt{13}$ 4) none

43. For the catenary $y=a\cos h\frac{x}{c}$, the length of the normal is
1) $\frac{y}{c}$ 2) $\frac{y}{c\sinh\frac{x}{c}}$ 3) $\frac{y^2}{c}$ 4) none

44. Length of the normal to the curve $y=x^3-2x^2+4$ at $(2,4)$ is
1) $\sqrt{17}$ 2) $2\sqrt{17}$ 3) $4\sqrt{17}$ 4) $4/\sqrt{17}$

45. Length of the normal to the curve $y^2=\frac{x^3}{2a-x}$ at (a, a) is
1) $\sqrt{5}a$ 2) 5a 3) $\frac{\sqrt{5}}{2}a$ 4) $2\sqrt{5}a$

46. The length of the subnormal to the curve $y^2 = 2px$ is

1) p 2) py 3) p^2y 4) 1/p

47. The length of the subnormal at (-1,4) on $y = 4x^2$ is

1) 4 2) 12 3) 16 4) 32

48. Equation of the tangent to the curve $6y = 7 - x^2$ at (1,1) is

1) $2x + y = 3$ 2) $x + 2y = 3$
3) $x + y + 1 = 0$ 4) $x + y = 2$

49. At any point on the curve $y = f(x)$, product of length of subtangent and length of subnormal is

1) square of abscissa 2) square of ordinate
3) constant 4) none

50. The subnormal to the curve $xy = c^2$ varies as

1) cube of the abscissa 2) cube of ordinate
3) constant 4) none

PRACTICE SET - I KEY

01-3	02-4	03-1	04-3	05-3
06-2	07-3	08-2	09-4	10-1
11-1	12-3	13-3	14-3	15-1
16-2	17-3	18-1	19-1	20-2
21-2	22-1	23-1	24-1	25-1
26-2	27-4	28-3	29-2	30-2
31-4	32-4	33-2	34-2	35-3
36-2	37-1	38-1	39-4	40-2
41-3	42-1	43-3	44-3	45-1
46-1	47-4	48-2	49-2	50-2

ALL POWER IS WITHIN YOU YOU CAN DO ANYTHING AND EVERYTHING

PRACTICE SET - II

01. The subnormal at any point on the curve $y^n = a^{n+1}$ is constant when n =

1) 1 2) −1 3) 2 4) −2

02. The portion of the tangent intercepted between the curve $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, $y = a\sin t$ and $x-axis$ is

1) a^2 2) a 3) 2a 4) none

03. For the curve $ay^2 = (x+b)^3$, the subnormal at any point is

1) equal to subtangent
2) double the subtangent
3) varies as square of subtangent 4) none

04. If the relation between subnormal SN and subtangent ST at any point S on the curve $by^2 = (x+a)^3$ is p (SN) = q (ST)^2 then $\frac{p}{q}$ =

1) $\frac{8b}{27}$ 2) $\frac{4b}{27}$ 3) $\frac{5b}{27}$ 4) $\frac{6b}{27}$

05. The points on the curve $x^2 + y^2 - 2x - 3 = 0$, the tangents at which parallel to x - axis are

1) (1,2), (3, −2) 2) (1,2), (1, −2)
3) (2,5), (2, −3) 4) none

06. If the subnormal of the curve $xy^n = a^{n+1}$ is constant then n =

1) 1 2) −1 3) −2 4) 5

07. The tangent to the curve is perpendicular to x-axis is

1) $\frac{dy}{dx} = 0$ 2) $\frac{dy}{dx} = 1$ 3) $\frac{dx}{dy} = 0$ 4) $\frac{dx}{dy} = 1$

08. The subtangent, ordinate and subnormal to the parabola $y^2 = 4ax$ at a point (different from the origin) are in

1) AP 2) GP 3) HP 4) none

09. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then

1) $a > 0, b > 0$ 2) $a > 0, b < 0$
3) $a < 0, b < 0$ 4) none

10. For the curve $x^m y^n = a^{m+n}$, the subtangent at any point is proportinal to

1) abscissa 2) ordinate 3) (abscissa)^2 4) none

11. A particle moves according to the law's $s = 12t^2 - 3t + 5$ (s in cms and t insecs). The velocity after 2 seconds is

1) 24 cm/sec 2) 45 cm/sec
3) 51 cm/sec 4) none

12. A particle moves according to the law $s = t^3 - 6t^2 + 9t + 8$ (s in cms and t in secs). The initial velocity of the particle is

1) 8 cm/sec 2) 17 cm/sec 3) 9 cm/sec 4) none

13. If the velocity V = 3t^2 + t, the acceleration when V = 2 is

1) −5 2) 5 3) 7 4) none

14. If $s = \sqrt{t^2 + 1}$ then acceleration is

1) $2/s^3$ 2) $1/s^3$ 3) $-1/s^3$ 4) none

15. If $s = t^3 - 6t + 1$ find the time when the velocity is zero

1) 2 2) 1 3) $\sqrt{2}$ 4) none

16. If $S = ae^{nt} + be^{-nt}$ then acceleration is

1) S 2) nS 3) S/n 4) n^{2}S

17. A particle is projected vertically upwards. Its length h at a time t is given by $h = 60 - t - 16t^2$. Then the velocity with which it hits the ground is

1) 30 2) 60 3) 90 4) 120

18. An object moves along the x-axis so that its abscissa obeys the law $x = 3t^2 + 8t + 1$. The time when its velocity and acceleration are equal to

1) 1 sec 2) 2 sec 3) 3/4 sec 4) 2/3 sec

19. For a particle moving on a straight line it is observed that the distance at time t is given by $S = 6t - \frac{t^2}{2}$. The maximum velocity during the motion is

1) 3 2) 6 3) 9 4) 12

20. The acceleration of a particle moving on a straight line is given by a (t) = 2t + 1. It starts at t = 0 from the origin with velocity 3m/sec. At the end of 1 second it will be at a distance of

1) 4m 2) 23/3m 3) 23/6m 4) none

21. If the displacement in time t of a particle is given by $s = ae^t + be^{-t}$, then the acceleration is equal to

1) velocity 2) displacement
3) initial velocity 4) none

22. If the rate of decrease $x^2 - 2x + 9$ is twice the rate of decrease of x then x =

1) 1 2) 2 3) 3 4) 4

23. If the rate of increase $x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of x, then the values of x are

1) ±3 2) 3, 1/3 3) −3, −1/3 4) 3, −1/3

24. The side of an equilateral triangle expands at the rate of 2 cm/sec. The rate of increase of its area when each side is 10 cm is

1) 10 sq. cm 2) $10\sqrt{3}$ sq.cm
3) $10/\sqrt{3}$ sq.cm 4) 5 sq.cm

25. The side of a cube is equal to the radius of a sphere. If the side and the radius increases at the same rate then the ratio of the increases in surface areas of the cube and sphere is

1) $2\pi : 3$ 2) $3 : 2\pi$ 3) $6 : \pi$ 4) none

26. A square plate expands uniformly, the side increasing at the rate of 0.2 inches per second. The rate at which area is increasing when the side is 10 cm is

1) 0.75 π sq.cm/sec 2) 0.75 sq.cm/sec
3) 4 sq.cm/sec 4) none

27. The rate of change of volume of a sphere is equal to the rate of change of its radius. Then its radius is

1) $1/2\pi$ 2) $2\sqrt{\pi}$ 3) $\sqrt{2\pi}$ 4) $1/2\sqrt{\pi}$

28. At what point of the parabola $y^2 = 4x$, the abscissa and the ordinate increases at the same rate

1) (1, 2) 2) (2, 1)
3) (1, −2) 4) (3, 1)

29. The point on the ellipse $16x^2+9y^2=400$ where the ordinate decrease at the same rate at which the abscissa increases is

1) (9, 16) 2) $\left(3,\frac{16}{3}\right)$ 3) $\left(\frac{16}{3},3\right)$ 4) $\left(-3,\frac{16}{3}\right)$

30. The radius of a circular plate is increasing in length at 0.01 inch/sec. The rate at which the area is increasing when the radius is 12 inches is

1) 0.12π. sq. inch/sec 2) 0.24π sq. inch/sec
3) 0.24 inch/sec 4) 24π sq.inch/sec

31. A ladder 20 feet long has one end on the ground and the other in contact with vertical wall. The lower end slips along the ground. When the foot of the ladder is 16 feet away from the wall, the upper end is moving

1) 3/4 2) 4/3 3) 1/3 4) 16/3

32. A man 6 ft high walks at a uniform speed of 90 ft/sec away from a lamp 15 ft high. The length of his shadow is increasing at the rate of

1) 6 ft/sec 2) 9 ft/sec
3) 12 ft/sec 4) 15 ft/sec

33. The area of a rectangle whose length is twice its breadth is increasing at the rate of 8 cm^2/sec. The rate at which length is increasing when the breadth is 5 cm is

1) $\frac{4}{5}cm/sec$ 2) 5 cm/sec
3) $\frac{5}{4}$ cm/sec 4) $\frac{2}{5}$ cm/sec

34. A balloon which always remains spherical is being inflated by pumping in 10 cu. inches of gas per minute. The rate at which the radius of the balloon is increasing when the radius is 15 inches is

1) $\frac{1}{30\pi}$ inch/min. 2) $\frac{1}{60\pi}$ inch/min.
3) $\frac{1}{90\pi}$ inch/min 4) $\frac{\pi}{90}$ inch/min

35. A spherical balloon is pumped at the rate of 10 c.u. inches / min. The rate of increase of surface area when the radius is 10 cm is

1) 200 sq.in/min 2) 2 sq.inch/min
3) 20 sq.inch/min 4) none

36. Gas is escaping from a spherical balloon at the rate of 2 cu.cm/sec. The rate at which the surface area shrinking when the radius is 16 cm is

1) $\frac{1}{8}$ sq.cm/sec 2) $\frac{1}{4}$ sq.cm/sec
3) $\frac{1}{2}$ sq.cm/sec 4) none

37. Water is poured into a conical vessel at the rate of 5 c.c. per second. If the semi-vertical angle of the cone is 30o, the rate at which the level of water is rising when the height of the water is 6 cm is

1) $5/6\pi\ cm/sec$ 2) 0.5 cm/sec
3) $5/12\pi$ cm/sec 4) none

38. A man is walking at the rate of 8 kmph towards the foot of a tower 60 m high. The rate at which he is approaching the top when he is 80 m from the foot of the tower is

1) 3.2 kmph 2) 6 kmph
3) 6.4 kmph 4) none

39. The function $y=2x^2-6x+7$ increases in

1) (2, 4) 2) $(3,\infty)$
3) $\left(\frac{3}{2},\infty\right)$ 4) none

40. $f(x)=\sqrt{9-x^2}$ is decreasing in

1) (−3, 0) 2) (0, 3)
3) (−3, 3) 4) none

41. $f(x)=\sqrt{25-9x^2}$ is increasing in

1) $\left(\frac{-5}{3},0\right)$ 2) $\left(0,\frac{5}{3}\right)$ 3) $\left(\frac{5}{3},0\right)$ 4) $\left(\frac{-5}{3},\frac{5}{3}\right)$

42. The function $x^3-9x^2+15x+5$ is decreasing in

1) IR 2) (1, 5) 3) (−1, 5) 4) none

43. The function $f(x)=2x^3-9x^2+12x+4$ is decreasing function in

1) (1, 2) 2) (−1,2) 3) (1, −2) 4) none

44. The function $f(x)=\frac{\log x}{x}$ increases in

1) (0, e) 2) (0, 1/e)
3) (1/e, 2e) 4) (0, 1)

45. The function $f(x)=\log(1+x)-\frac{2x}{2+x}, x>0$ is

1) increasing function 2) decreasing function
3) neither increasing nor decreasing
4) none

46. $f(x)=\left(\frac{1}{x}\right)^x$ is increasing in the interval

1) (0, e) 2) (0, 1/e)
3) (e,∞) 4) $\left(\frac{1}{e},\infty\right)$

47. $f(x)=x^x$ is decreasing in the interval

1) (0, e) 2) (0, 1/e) 3) (0, 2/e) 4) (0, 1)

48. The maximum value of $7-2x^2-3x$ is

1) 65 2) 65/4 3) 65/8 4) none

49. The minimum value of $f(x)=4x^2-8x+5$ is

1) 1 2) 2 3) −3 4) none

50. The maximum and minimum values of $y=2x^3-9x^2+12x+7$ are attained at x =

1) 1, 2 2) 2, 3 3) -1, 2 4) 1, -2

PRACTICE SET - II KEY

01–3	02–2	03–3	04–1	05–2
06–3	07–3	08–2	09–2	10–1
11–2	12–3	13–2	14–2	15–3
16–4	17–2	18–4	19–2	20–3
21–2	22–1	23–2	24–2	25–2
26–1	27–4	28–1	29–2	30–2
31–2	32–1	33–1	34–3	35–2
36–2	37–3	38–3	39–3	40–2
41–1	42–2	43–1	44–1	45–1
46–2	47–2	48–3	49–1	50–1

PRACTICE SET - III

01. Least value of $x^2-8x+17, x\in R$ is

1) 1 2) 2 3) -2 4) -1

02. The function $y=2x^3-3x^2-36x+10$ attains minimum at x =

1) 1 2) 2 3) -2 4) 3

03. Minimum value of x log x is

1) e 2) 1/e 3) $-1/e$ 4) $e^{1/e}$

04. The minimum value of $\sin x+\cos x$ is

1) 0 2) $\sqrt{2}$ 3) $-\sqrt{2}$ 4) $-1/\sqrt{2}$

05. Maximum value of $f(x)=a\sin x+b\cos x+c$ is

1) $\frac{a+b}{2}$ 2) $\sqrt{a^2+b^2}$
3) $a^2+b^2+c^2$ 4) $\sqrt{a^2+b^2}+c$

06. ax^2+bx+c has minimum when

1) $a<0$ 2) $a>0$
3) $b^2=4ac$ 4) $x=b/2a$

07. The maximum value of $5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$ is

1) 2 2) 5 3) 5/2 4) 10

08. If x and y are real such that $x>0, xy=1$ then minimum of $x+y$ is

1) 2 2) $\sqrt{2}$ 3) 3 4) $\sqrt{3}$

09. The maximum value of x^{-x} is

1) 1/e 2) $e^{1/e}$ 3) $e^{-1/e}$ 4) none

10. The maximum value of $\sin^3 x+\cos^3 x$ is

1) 1 2) 2 3) $\sqrt{2}$ 4) $2\sqrt{2}$

11. The minimum value of $64\sec\theta+27\operatorname{cosec}\theta$ where θ lies in $(0,\pi/2)$ is

1) 64 2) 27 3) 125 4) 3/4

12. If $f(x)=a\log|x|+bx^2+x$ has extreme values at $x=-1$ and $x=2$ then (a, b) =

1) (2, −1) 2) (2, −1/2)
3) (−2, 1/2) 4) none

13. The minimum value of $a^2\sec^2 x+b^2\operatorname{cosec}^2 x$ is
1) $(a-b)^2$ 2) a^2+b^2
3) $(a+b)^2$ 4) a^2-b^2

14. Extreme values of $3\sin x+4\cos x$ are
1) 5, -5 2) 5, 5
3) $\frac{1}{5},\frac{-1}{5}$ 4) $-5,-5$

15. The extreme values of $5\cos^2 x+4\sin^2 x$ are
1) $\pm\sqrt{41}$ 2) 5, 4
3) $-5,4$ 4) $5,-4$

16. The maximum value of $(x-1)(x-2)(x-3)$ is
1) $\frac{2}{3}$ 2) $\frac{2}{\sqrt{3}}$ 3) $\frac{1}{3\sqrt{3}}$ 4) $\frac{2}{3\sqrt{3}}$

17. Maximum value of xe^{-x} is
1) e 2) $1/e$ 3) $2/e$ 4) none

18. The greatest value of x^3-3x in $[0,2]$ is
1) 1 2) -1 3) 2 4) -2

19. Minimum value of $a\cot x+b\tan x$ is
1) ab 2) 2ab 3) $\sqrt{ab}$ 4) $2\sqrt{ab}$

20. Minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is
1) 1 2) 3 3) 1/3 4) none

21. The stationary point of $y=x^2+\frac{250}{x}$ is
1) (1, 5) 2) (5, 1)
3) (5, 25) 4) (5, 75)

22. x^x has a stationary at x =
1) 1 2) e 3) 1/e 4) $\sqrt{e}$

23. The maximum value of $\frac{\log x}{x}$ in $(0,\infty)$ is
1) e 2) 1/e 3) $-1/e$ 4) $2/e$

24. The point on the parabola $x^2=y$ which is closest to the point $(3,0)$ is
1) $(1,-1)$ 2) $(-1,1)$
3) $(-1,-1)$ 4) $(1,1)$

25. The sum of two numbers is 20. If the product of the square of one number and cube of the other is maximum, then the numbers are
1) 10, 10 2) 12, 8
3) 5, 15 4) 20, 0

26. The sum of two non-zero numbers is 6. The minimum value of the sum of their reciprocals is
1) 3/4 2) 6/5 3) 2/3 4) none

27. Two positive numbers whose sum is 64 and sum of whose cubes is minimum are given by
1) 32, 32 2) 48, 16
3) 40, 24 4) none

28. Two positive numbers whose sum is 24 and sum of whose squares is minimum are given by
1) 12, 12 2) no such numbers exist
3) 0, 24 4) none

29. Divide 15 into two parts such that the square of one multiplied with the cube or the other is maximum. The parts are
1) 7, 8 2) 6, 9 3) 10, 5 4) none

30. The sum of two positive numbers is 6. If the product of the first and square of the second is maximum, then the two numbers are
1) 3, 3 2) 2, 4
3) no such numbers exist
4) none

31. The positive number x such that sum of x and its reciprocal is minimum is
1) 1 2) 2 3) 1/2 4) none

32. The sum of two numbers is 20. Their product is maximum if the two numbers are
1) 20, 0 2) 15, 5 3) 10, 10 4) 8, 12

33. The maximum area of the rectangle that can be inscribed in a circle of radius r is
1) r^2 2) $2r^2$ 3) πr^2 4) $\pi r^2/4$

34. The semi-vertical angle of a cone of maximum volume and given slant height is
1) $\tan^{-1}2$ 2) $\tan^{-1}\sqrt{2}$
3) $\pi/4$ 4) none

35. If $y=x^n$ then the ratio of the relative error in y and x is
1) 1 : n 2) n : 1 3) 1 : 1 4) 1 : 2

36. If there is a possible error of 0.01 cm in the measurement of the side of a cube, the possible percentage error in its surface area when the side is 10 cm is
1) 0.1% 2) 0.2% 3) 2% 4) 1%

37. If the radius of the sphere increases from 10cm to 10.04 then the increase in its volume is
1) 4π c. cms 2) 8π c.cms
3) 16π c.cms 4) $16\pi/3$ c.cms

38. If there is a possible error of 0.02 cm in the measurement of the diameter of a sphere, the possible percentage error in its volume, when the radius is 10 cm is
1) 0.2% 2) 2% 3) 3% 4) 0.3%

39. If an error of 2% is made in measuring the side of a triangle, then percentage error in calculating area is
1) 1 2) 4 3) 8 4) 2

40. The side of a square of side 6 m is incorrectly measured as 6.11 m. The resulting error in the calculation of the area of the square is
1) 0.12 m^2 2) 1.12 m^2
3) 0.32 m^2 4) 1.32 m^2

41. If there is an error of 1% in measuring the side of a square plate, the percentage error in its area is
1) 2% 2) 0.5% 3) 1% 4) none

42. If $T=2\pi\sqrt{\frac{l}{g}}$ then the ratio of the relative error in T to relative error in l is
1) 1 2) 2 3) 1/2 4) none

43. The length of a simple pendulum is increased by 4%. The percentage increase in its period of oscillation is
1) 1 2) $\sqrt{3}$ 3) 2 4) 4

44. If the radius of a spherical balloon is measured within 1%, the percentage error in the volume is
1) 2% 2) 3% 3) 4% 4) $\frac{1}{2}$%

45. If there is a relative error of 2 units in measuring the volume of a liquid with pressure P then the relative error in P when PV is constant is
1) 1 2) 2 3) -2 4) -1

46. The pressure p and volume v of a gas are connected by the relation $pv^{1.4}$ = constant. The percentage increase in the pressure corresponding to a diminution of 0.5% in the volume is
1) 0.1 2) 2.8 3) 0.7 4) 0.25

47. The diameter of a sphere can be measured to an accuracy of 0.01 cm. The percentage error in calculating the surface area of sphere when the radius is 5 cm is
1) 0.2 2) 0.6 3) 0.05 4) $\frac{\pi}{10}$

48. The percentage error in the side of a cube is 0.5. Then the percentage error in its volume is
1) $\frac{1}{2}$ 2) 1 3) $\frac{3}{2}$ 4) 12

49. The appropriate value of $(1.0002)^{3000}$ is
1) 1.2 2) 1.4 3) 1.6 4) 1.8

50. The circumference of a circle is measured as 56 cm with an error of 0.02 cm. The percentage error in its area is
1) $\frac{1}{7}$ 2) $\frac{1}{28}$ 3) $\frac{1}{14}$ 4) $\frac{1}{56}$

PRACTICE SET - III KEY

01-1	02-4	03-3	04-3	05-4
06-2	07-4	08-1	09-2	10-1
11-3	12-2	13-3	14-1	15-2
16-4	17-2	18-3	19-4	20-3
21-4	22-3	23-2	24-4	25-2
26-3	27-1	28-1	29-2	30-2
31-1	32-3	33-2	34-2	35-2
36-2	37-3	38-4	39-2	40-4
41-1	42-3	43-3	44-2	45-3
46-3	47-1	48-3	49-3	50-3

SELF TEST

01. The slope of the tangent to the curve $xy^2 + yx^2 = 2$ at $(1, -2)$ is
1) 5/8 2) 8/3 3) -8/3 4) 0

02. Equation of the tangent (1,1) on the curve $2y = 3 - x^2$ is
1) $x + y = 2$ 2) $x + y + 1 = 0$
3) $x - y + 1 = 0$ 4) $x - y = 0$

03. Equation of the normal to the curve $x = 3t$, $y = 4/t$ at $t = 2$ is
1) $x + 3y = 12$ 2) $x + 3y + 12 = 0$
3) $3x - y = 16$ 4) $3x - y + 16 = 0$

04. If the gradient to the curve $y = k\ (2x-1)^2$ at $\left(1, \frac{1}{4}\right)$ is 1 then k=
1) 1 2) 1/2 3) 1/3 4) 1/4

05. The slope of the normal to the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at $\theta = \pi/2$ is
1) 1 2) -1 3) 2 4) none

06. The length of subtangent of the rectangular hyperbola $x^2 - y^2 = a^2$ at the point $(a, \sqrt{2}a)$ is
1) $\sqrt{2}\, a$ 2) $2a$ 3) $1/2\, a$ 4) none

07. The angle of intersection of the curves $y^2 = x$ and $x^2 = y$ at (1,1) is
1) $\tan^{-1}\left(\frac{3}{4}\right)$ 2) $\tan^{-1}\left(\frac{4}{3}\right)$
3) $\tan^{-1}\left(\frac{1}{2}\right)$ 4) none

08. For the curve $xy = c^2$ the length of subnormal varies as
1) cube of the abscissa
2) square of the abscissa
3) square of the ordinate
4) cube of the ordinate

09. If $y = 4x - 5$ touches $y^2 = ax^3 + b$ at (2,3) then (a, b) =
1) (2, 7) 2) (2, -7) 3) (-2, 7) 4) (-2,-7)

10. Angle between the curves $y = \sin x$ and $y = \cos x$ is
1) $\pi/4$ 2) $\pi/2$
3) $\tan^{-1}\sqrt{2}$ 4) $\tan^{-1} 2\sqrt{2}$

11. If the curve $y = x^2 + px + q$ touches the line $y = x$ at (1, 1) then (p, q) =
1) (-1, 1) 2) (-1, 2) 3) (2, 1) 4) (1, 1)

12. Area of the triangle formed by a tangent to the curve $2xy = a^2$ and coordinate axes is
1) a^2 2) $2a^2$ 3) $3a^2$ 4) $4a^2$

13. If a particle is moving on a straight line according to the low $s = 4t^3 - 6t^2 + t - 7$ then acceleration of the particle after 2 secs. is
1) 0 2) 18 3) 24 4) 36

14. A particle moves so that the space s described in time t is square root of a quadratic function of t. Then the acceleration of the particle varies as
1) $1/s^2$ 2) $1/s^3$ 3) s^3 4) none

15. A stone is dropped into a quite lake and waves moves in a circle at a speed of 6 cm/sec. At the instant when the radius of the circular wave is 16 cm, the enclosed area increases at the rate
1) $100\ \pi\ cm^2 / sec$ 2) $32\ \pi\ cm^2 / sec$
3) $192\ \pi\ cm / sec$ 4) none

16. A ladder 13 feet long, moves with its ends A and B on two perpendicular lines OX and OY respectively. When A is 5 feet from O, it is moving at the rate of 2 feet/sec. At this instant, B is moving at the rate
1) 5/6 ft / sec upwards
2) 5/6 ft/sec downwards
3) 6/5 ft/sec downwards 4) none

17. A cube is expanding in such a way that is edge is changing at a rate of 5 cm/sec. If its edge is 4 cm long, then the rate of change of its volume is
1) $100\ cm^3 / sec$ 2) $120\ cm^3 / sec$
3) $180\ cm^3 / sec$ 4) $240\ cm^3 / sec$

18. If the rate of change of volume of a sphere is equal to the rate of change of its radius then the radius is
1) 1 2) 1/2 3) $1/\sqrt{2\pi}$ 4) $1/\sqrt{2\pi}$

19. A car starts from rest and attains the speed of 10 km/hr respectively at the end of 1st and 2nd minutes. If the car moves on a straight road, distance travelled in 2 minutes is
1) 1/4 km 2) 1/3 km 3) 15 km 4) 20 km

20. If $f(x) = x^3 - 3x^2 - 9x + 22$, then $f(x)$ increases with x for the range of values
1) $x > 1$ and $x < 3$ 2) $x < -1$ and $x > 3$
3) $x < 1$ and $x > 3$ 4) none

21. The increasing function in $\left(0, \frac{\pi}{2}\right)$ is
1) $\cos x + \sin x$ 2) $\cos x - \sin x$
3) $\frac{\sin x}{x}$ 4) $\frac{x}{\sin x}$

22. Maximum value of $(x-1)(x-2)(x-3)$ is
1) $\frac{1}{\sqrt{3}}$ 2) $\frac{2}{\sqrt{3}}$ 3) $\frac{1}{3\sqrt{3}}$ 4) $\frac{2}{3\sqrt{3}}$

23. The minimum of $2x^3 - 3x^2 - 12x + 8$ occurs at $x =$
1) -1 2) 2 3) $\sqrt{6}$ 4) $-\sqrt{6}$

24. In a ΔABC, the maximum value of $\cos A + \cos B + \cos C$ is
1) 1/2 2) 2/3 3) 3/2 4) 0

25. The function $y = x - \sin x$ has
1) maximum value 2) minimum value
3) no extreme value 4) none

26. The maximum value of the expression $\sin\theta + \cos\theta$ is
1) 0 2) 1 3) 2 4) $\sqrt{2}$

27. The point on the curve $y^2 = 4x$ which is nearest to the point (2, 1) is
1) (1, 2) 2) (1, -2) 3) (4, 4) 4) (4, 6)

28. The point on the curve $y = x^2 + 7x + 2$ closest to the line $y = 3x - 7$ is
1) (2, 8) 2) (4, 6) 3) (-2, 8) 4) (-2, -8)

29. The period of oscillation of a simple pendulum is directly proportional to the square root of its length. If there is an error of 2% in measuring its length the percentage error in the period will be
1) 1 2) $\sqrt{2}$ 3) 2 4) 4

30. If there is a possible error of 0.02 cm in the measurement of the diameter of a sphere, the possible percentage error in its volume, when the radius is 10 cm is
1) 3% 2) 0.3% 3) 0.03% 4) 0.1%

31. Equation of the tangent line at the point (a,b) on the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ is
1) $x + y = a$ 2) $x + y = b$
3) $\frac{x}{a} + \frac{y}{b} = 1$ 4) $\frac{x}{a} + \frac{y}{b} = 2$

32. The length of the subtangent at (2,2) to the curve $x^5 = 2y^4$ is
1) 5/2 2) 8/5 3) 2/5 4) 5/8

33. The equation of the tangent to the curve $6y = 7 - x^3$ at (1,1) is
1) $2x + y = 3$ 2) $x + 2y = 3$
3) $x + y = -1$ 4) $x + y + z = 0$

34. The equation to the normal to the curve $y^4 = ax^3$ at (a, a) is
1) $x + 2y = 3a$ 2) $3x - 4y + a = 0$
3) $4x + 3y = 7a$ 4) $4x - 3y = 0$

35. The tangent to the curve $y = e^{2x}$ at the point (0,1) meets the x-axis is
1) (0,0) 2) (2,0)
3) $\left(\frac{-1}{2}, 0\right)$ 4) $\left(0, \frac{-1}{2}\right)$

36. The points on the curve $y = 2x^3 - 3x^2 + 5$ where the tangent line is parallel to x-axis is
1) (0,5) (1, 4) 2) (0, 5), (3, 4)
3) (0, 5) (4, 1) 4) none

37. If θ is the angle between the curves $xy = 2$ and $x^2 + 4y = 0$ then $\tan\theta =$
1) 1 2) -1 3) 2 4) 3

38. The angle between the curves $y^2 = 4x$, $x^2 = 4y$ at (4, 4) is
1) $\tan^{-1}\frac{1}{x}$ 2) $\tan^{-1}\frac{3}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

39. The angle between the curves $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ is

1) 30° 2) 45° 3) 60° 4) 90°

40. The two curves $x = y^2, xy = a^3$ cut orthogonally at a point. Then $a^2 =$

1) 1/3 2) 1/2 3) 2 4) 3

41. The distance moved by a particle in time 't' is given by $s = t^3 - 12t^2 + 6t + 8$. At the instant, when its acceleration is zero, the velocity is

1) 42 2) -42 3) 48 4) -48

42. A particle moves along the curve $y = x^2 + 2x$. Then the point on the curve such that x and y coordinates of the particle change with the same rate

1) (1,3) 2) $\left(\frac{1}{2}, \frac{5}{2}\right)$ 3) $\left(\frac{-1}{2}, \frac{-3}{4}\right)$ 4) $(-1,-1)$

43. The radius of a circular plate is increasing at the rate of 0.01 cm/sec, when the radius is 12 cm. Then the rate at which the area increases is

1) 0.24 π sec.cm/sec. 2) 60 π sq.cm/sec
3) 24 π sq.cm/sec. 4) 1.2 π sq.cm/sec.

44. The real number x when added to its inverse gives the minimum value of the sum at x = 1) -2

2) 2 3) 1 4) -1

45. If p,q are positive numbers such that $p^2 + q^2 = 1$ then the maximum value of $p + q$ is

1) 2 2) 1/2 3) $1/\sqrt{2}$ 4) $\sqrt{2}$

46. The maximum value of xy subject to $x + y = 7$ is

1) 12 2) 10 3) 49/4 4) 55/4

47. If $\log(1+x) - \frac{2x}{2+x}$ is increasing then

1) $0 < x < \infty$ 2) $-\infty < x < 0$
3) $-\infty < x < \infty$ 4) $1 < x < 2$

48. The condition for $f(x) = x^3 + px^2 + qx + r (x \in R)$ to have no extreme value is

1) $p^2 < 3q$ 2) $2p^2 < q$
3) $4p^2 < q$ 4) $p^2 > 3q$

49. If m and M respectively denote the minimum and maximum of $f(x) = (x-1)^2 + 3$ for $x \in [-3,1]$ then the ordered pair $m(m, M) =$

1) (-3, 19) 2) (3, 19)
3) (-19, 3) 4) (-19, -3)

SELF TEST KEY

01-4	02-1	03-3	04-4	05-2
06-2	07-1	08-4	09-2	10-4
11-1	12-1	13-4	14-2	15-4
16-2	17-4	18-2	19-2	20-2
21-4	22-4	23-2	24-3	25-3
26-4	27-1	28-4	29-1	30-2
31-4	32-2	33-2	34-3	35-3
36-1	37-4	38-2	39-4	40-2
41-2	42-3	43-1	44-3	45-4
46-3	47-1	48-1	49-2	

IMPORTANT QUESTIONS

01. The tangent to the curve $xy = 1$ at the point $(t, 1/t)$ meets the coordinate axes at A and B. The area of ΔAOB is

1) 1 2) 2 3) t 4) t/2

02. The radius of a spherical baloon is increasing at a rate of 2 cm per second. The rate at which its surface is increasing when the radius is 5 cm is

1) $80\pi\ sq.cm/sec$ 2) $40\pi\ sq.cm/sec$
3) $60\pi\ sq.cm/sec$ 4) $20\pi\ sq.cm/sec$

03. If the distance described by a particle in t seconds is given by $s = ae^t + be^{-t}$, then the acceleration is

1) $ae^t + be^{-t}$ 2) $ae^{-t} + be^{-t}$
3) $ae^t + be^t$ 4) $ae^{-t} + be^t$

04. The point on the curve $y = \log \sec x \left(0 < x < \frac{\pi}{2}\right)$ at which the tangent is parallel to the line $y = x$ is ___

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

05. The volume of a sphere is increasing at a rate of 1200 cu.cm/sec. The rate of increase of its surface area at the instant when the radius is 10 cm is

1) 240 sq.cm/sec 2) 220 sq.cm/sec
3) 210 sq.cm/sec 4) 200 sq.cm/sec

06. A particle moves along a straight line according to the law $s = t^3 - 9t^2 + 24t$. The velocity of the particle will be increasing when t is greater than ___

1) $t > 2$ 2) $t > 1$ 3) $t > 3$ 4) $t < 3$

07. The maximum value of the function $f(x) = 10 + 2x - x^2$ is

1) 12 2) 11 3) 10 4) 14

08. The geometric mean between the subnormal and subtangent at any point of a plane curve is

1) the abscissa of the point
2) the ordinate of the point
3) the G.M. between the abscissa and the ordinate of the point
4) none of these

09. The equation of the normal to the curve $y^2 = 6x$ at the point (6,6) on it is ___

1) $2x - y + 18 = 0$ 2) $2x + y - 18 = 0$
3) $3x + y - 18 = 0$ 4) $3x - y + 18 = 0$

10. The maximum value of x^{-x} is

1) $e^{\frac{1}{e}}$ 2) $e^{\frac{2}{e}}$ 3) e 4) $\frac{1}{e}$

11. The slope of the normal at the point 't' on the curve $x = 1/t, y = t$ is

1) t^2 2) $-1/t^2$ 3) $1/t^2$ 4) $-t^2$

12. The subtangent and subnormal at the point (x_1, y_1) on the curve $y^n = a^{n-1}x$ are ___ and ___

1) $nx_1, \frac{y_1^2}{nx_1}$ 2) $n\,y_1, \frac{y_1}{nx_1}$

3) $ny_1^2, \frac{ny_1}{nx_1}$ 4) none

13. The surface area of a sphere is increasing at the rate of 1 sq. cm/sec. The rate of increase of volume when the radius is 3 cm is

1) $\frac{3}{2}$ cm^3/sec 2) $\frac{2}{3}$ cm^3/sec

3) 3 cm^3/sec 4) 2 cm^3/sec

14. The sum of the lengths of the subtangent and subnormal at the point $\theta = \pi/3$ on the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ is

1) $\frac{2}{\sqrt{3}}$ 2) $\frac{2a}{\sqrt{3}}$ 3) $\frac{a\sqrt{3}}{2}$ 4) $\frac{a}{2\sqrt{3}}$

15. The length of the subnormal to the curve $y^2 = x^3$ at the point (9,27) is

1) 27 2) 243 3) 243/2 4) 729

16. The period of oscillation of a simple pendulum is directly proportional to the square root of its length. If there is an error of 2% in measuring its length, then the percentage error in the estimation of the period will be

1) 1 2) -4 3) -1 4) 4

17. If the subnormal at any point on the curve $xy^n = k$ is a constant, then n =

1) -1 2) 1 3) -2 4) 2

18. The length of the tangent at any point θ on the curve $x = a\cos^3\theta, y = a\sin^3\theta$ is

1) $a\cos^2\theta$ 2) $a\cos\theta$ 3) $a\sin^2\theta$ 4) $a\sin\theta$

19. The sum of two non-zero numbers is 6. The minimum value of the sum of their reciprocals is

1) 2/3 2) 3/2 3) 6/5 4) 5/6

20. The value of n so that the subnormal at any point on the curve $xy^n = a^{n+1}$ is constant equal to

1) 1 2) 2 3) -1 4) -2

21. The angle between the curves $y = x^3$ and $6y = 7 - x^2$ is equal to

1) $\pi/4$ 2) $\pi/3$ 3) $\pi/6$ 4) $\pi/2$

22. Length of the normal between curve $y = c\cosh\frac{x}{c}$ and the x-axis varies as

1) y^2 2) y^3 3) y 4) $1/y^2$

23. The length of the subnormal at any point on $y^2 = 4ax$ is

1) a/2 2) a/3 3) a 4) 2a

24. The length of the subtangent to the curve $\sqrt{x} + \sqrt{y} = 3$ at the point (4,1) is

1) 1/2 2) 2 3) $1/\sqrt{2}$ 4) $\sqrt{2}$

25. If an error of 0.02 cm is made while measuring the radius 1 cm of a circle, then the percentage error made while calculating its area approximately is
1) 2 2) 4 4) 5 5) 6

26. The velocity v of a point moving along a straight line, when at a distance of x from the origin is given by the relation $a+bv^2=x^2$. The acceleration of the point is
1) $\frac{x}{b}$ 2) bx 3) $\frac{x}{a}$ 4) ax

27. The angle between the curves $y=2x$ and $x^3+y^3=6xy$ at $P\left(\frac{4}{3},\frac{8}{3}\right)$ is
1) $\tan^{-1}2$ 2) $\tan^{-1}\left(\frac{4}{5}\right)$
3) $\tan^{-1}\left(\frac{6}{13}\right)$ 4) $\tan^{-1}\left(\frac{1}{2}\right)$

28. The rate of decrease of $\frac{x^3}{3}-\frac{3}{2}x^2+5x+4$ is three times the rate of decrease of x. Then x =
1) 1, –2 2) 1, 2 3) –1, 2 4) –1, –2

29. The equation of the tangent of the curve $y(x+2)=6$ at (1,2) is
1) $3x+2y-8=0$ 2) $2x+3y+8=0$
3) $3x+2y+8=0$ 4) $2x+3y-8=0$

30. The values of x for which the rate of increase of x^3-5x^2+5x+8 is twice the rate of increase of x are:
1) 3 and 1/3 2) 2 and 1/3
3) 3 and 1/2 4) none of these

31. If there is an error of 0.05 sq. cm in the surface area of the sphere, then the error in the volume when the radius is 40 cm is
1) 0.25 c.c 2) 0.5 c.c
3) 1 c.c. 4) 0.75 c.c

32. A particle moves along the curve $y^2=8x$. The point at which the abscissa and the ordinate increase at the same rate is
1) (2,–4) 2) (2,4) 3) (4,–2) 4) (2,2)

33. The maximum value of the function $2x^3-3x^2-12x+5$ is
1) 9 2) 6 3) 10 4) 12

34. If the length of a simple pendulum is decreased by 4%, the percentage error in its period is
1) 3% 2) 1% 3) 4% 4) 2%

35. The ordinate of the point on the curve $y^2=4x$ which is nearest to the point (1,1) satisfies
1) $y^3+4y-8=0$ 2) $y=2$
3) $y^3-4y+8=0$ 4) $y^3-4y-8=0$

36. The height of the cylinder of maximum volume that can be inscribed in a sphere of radius $3\sqrt{3}$ is
1) 3 2) 6 3) 12 4) 8

37. The minimum value of $2x^3-3x^2-36x+10$ is
1) 71 2) -12 3) -71 4) 11

38. The maximum value of $x^{1/x}$ is
1) e 2) 1/e 3) $e^{1/e}$ 4) $\sqrt{e}$

IMPORTANT QUESTIONS KEY

01–2	02–1	03–1	04–3	05–1
06–3	07–2	08–2	09–2	10–1
11–3	12–1	13–1	14–2	15–3
16–1	17–3	18–3	19–1	20–4
21–4	22–1	23–4	24–2	25–2
26–1	27–3	28–2	29–4	30–1
31–3	32–2	33–4	34–4	35–1
36–2	37–3	38–3		

PREVIOUS ECET BITS

2007

01. A particle moving along a straight line has the relations $S=t^2+2t+3$ connecting the distances S described by the particle in time t, the acceleration at $t=3$ is
1) 2 units/sec^2 2) 3 units/sec^2
3) 4 units/sec^2 4) 8 units/sec^2

02. Find the point in which the local maxima of local minima for the function $f(x)=x^3-3x$
1) Min at $x=1$ and max $x=-1$
2) Min at $x=2$ and max $x=-2$
3) Min at $x=1$ and max $x=1$
4) Min at $x=3$ and max at $x=-3$

03. If an error of 3% in the side of the value, the percentage error in its volume is:
1) 3 2) 10 3) 6 4) 9

04. If the increase in the side of a square is 1%, find the % of change in the area of the square:
1) 2% 2) 3% 3) 1% 4) 0%

05. The slope of the tangent to the following curve $y=1/(x-1)$ at $x=3$ is:
1) 1/4 2) -1/4 3) 1/2 4) 1/3

06. The equations of the tangent and the normal to the curve $y=5x^4$ at the point (1,5) are:
1) $y=20x-15$ and $20y=101-x$
2) $y=x-1$ and $y=2+x$
3) $y=x-15$ and $y=100+x$
4) $x=y+1$ and $x=y-1$

2008

07. The slope of the tangent to the curve $x^{2/3}+y^{2/3}=a^{2/3}$ at the point where it meets the x-axis is
1) $\frac{a}{3}$ 2) 1 3) –1 4) 0

08. The conditiion for the two curves $x=y^2$ and $xy=K$, K being constant, to cut orthogonally is
1) $2K^2=1$ 2) $8K^2=1$
3) $8K^3=1$ 4) $2K^3=1$

09. The curve $y=xe^x$ has a
1) Minimum value at $x=1$
2) Minimum value at $x=0$
3) Maximum value at $x=-1$
4) Maximum value at $x=0$

2009

10. The point on the curve $y^3-3xy+2=0$ where the tangent is parallel to y-axis is
1) (0,0) 2) (0,1) 3) (–1,–1) 4) (1,1)

2010

11. The length of the normal to the curve $y=x^3+1$ at (1,2) is
1) $\sqrt{10}$ 2) $2\sqrt{10}$ 3) $2/3$ 4) 6

12. The maximum value of x^{-x} is
(1) e^e 2) e^{-e} 3) $e^{-1/e}$ 4) $e^{1/e}$

2011

13. The maximum possible area that can be enclosed in a curve of length 20 cm by bending it into the form of a sector in square centimeters is
1) 10 2) 25 3) 30 4) 45

14. The maximum value of $\frac{\log x}{x}$ is
1) 1 2) 2/e 3) e 4) 1/e

15. The set of all points where the function $f(x)=\frac{x}{1+|x|}$ is differentiable is
1) $(-\infty,\infty)$ 2) $[0,\infty]$ 3) $(-\infty,0)\cup(0,\infty)$ 4) $(0,\infty)$

2012

16. The equation of the normal to the curve $y=5x^4$ at the point (1,5) is
1) $x+20y=99$ 2) $x+20y=101$
3) $x-20y=99$ 4) $x-20y=101$

17. The angle between the curves $y^2=4x$ and $x^2+y^2=5$ is
1) $\frac{\pi}{4}$ 2) $\tan^{-1}(2)$
3) $\tan^{-1}(3)$ 4) $\tan^{-1}(4)$

2013

18. The angle between the curves $y^2=2x$ and $x+y=8$ at their point of intersection (2,2) is
1) $\tan^{-1}3$ 2) $\tan^{-1}2$ 3) 45° 4) 60°

19. If the sum of two positive numbers is 48, then the numbers such that the sum of their square is minimum are

1) 16, 32 2) 20, 28 3) 24, 24 4) 6, 42

20. A sphere of radius 10 cm shrinks to 9.8 cm. The approximate decrease in volume in cubic centimeters is

1) 80π 2) 20π 3) 60π 4) 40π

2014

21. The coordinates of the point P (x, y) on the curve of $y = x^2 - 4x + 5$ such that the tangent at P is parallel to $y = 2x + 4$ are

1) (3, 2) 2) (1, 2) 3) (2, 1) 4) (5, 4)

22. The function $f(x) = x\log^2 x$ has

1) Maximum value occurs when $x = \frac{1}{e}$

2) Maximum value occurs when $x = e$

3) Maximum value occurs when $x = e^{-2}$ 4) Maximum value occurs when $x = e^2$

23. In a cube the percentage increase in side is 2 units. The percentage increases in the volume of the cube is

1) 3 2) 6 3) 8 4) 16

24. The curves $x = y^2$ and $xy = m$ cut at right angle if

1) $m = 0$ 2) $m^2 = 8$ 3) $8m^2 = 1$ 4) $m = -1$

A.P.ECET 2015

25. Given the function $f(x) = x^2e^{-2x}, x > 0$. Then f(x) has the maximum value equal to

1) e^{-2} 2) $(2e)^{-1}$ 3) e^{-1} 4) none of these

26. If the curves $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at (1,1). Then a =

1) 1 2) -6 3) 6 4) 0

27. The maximum possible area thea can be enclosed by a wire of length 20 cm by bending it into the form of a sectro in sq. cm is

1) 20 sq. cm 2) 25 sq. cm
3) 30 sq. cm 4) 15 sq. cm

T.S ECET 2015

28. The maximum value of the function $y = 2x^3 - 6x^2 - 18x + 21$ is

1) 21 2) 31 3) -1 4) 3

A.P.ECET 2016

29. The rate of change of area of a circle with respect to radius when r=5cm is

1) $2\pi\, sq.cm/sec$ 2) $10\pi\, sq.cm/sec$
3) $100\pi\, sq.cm/sec$ 4) $20\pi\, sq.cm/sec$

30. The function $\frac{\log x}{x}$ attains its maximum value at

1) 0 2) $\sqrt{e}$ 3) e 4) $\frac{1}{e}$

31. If the increase in the side of a square is 2%, then the approximate percentage increase in the area of the square is

1) 2 2) 4 3) 6 4) 8

32. The slope of the normal to the curve $xy^2 = 4at(1,-2)$ is

1) 2 2) -1 3) $-\frac{1}{2}$ 4) 1

T.S ECET 2016

33. The normal to the curve $x = a(1+\cos\theta); y = a\sin\theta$ at 'θ' always passes through thye point

1) (0, 0) 2) (a, 0) 3) (0, a) 4) (a, a)

34. The maximum and minimum values of $f(x) = \sin^2 x + \cos^4 x$ are

1) 1,0 2) $\frac{1}{2},\frac{1}{2}$ 3) $1,\frac{1}{2}$ 4) $1,\frac{3}{4}$

A.P.ECET 2017

35. The slope of the normal to the curve $x = a\sec\theta, y = a\tan\theta$ at $\theta = \frac{\pi}{6}$ is______

1) 2 2) 0 3) $-\frac{1}{2}$ 4) 1

36. The rate of change of area of a circle with respect to radius when r=5cm is

1) $2\pi\, sq.cm/sec$ 2) $10\pi\, sq.cm/sec$
3) $100\pi\, sq.cm/sec$ 4) $20\pi\, sq.cm/sec$

37. Which of the following function has maxima or minima

1) e^x 2) logx
3) $x^3 + x^2 + x + 1$ 4) sinx

38. If the increase in the side of square is 2% then the approximate percentage increase in the area of the square is

1) 2 2) 4 3) 6 4) 8

T.S ECET 2017

39. At $\theta = \frac{\pi}{4}$, the slope of the normal to the curve $x = a\cos^3\theta;\ y = a\sin^3\theta$ is

1) -1 2) -2 3) 2 4) 1

40. Equation of the tangent to the curve $y = 5x^4$ at the point (1, 5) is

1) y = 15 (x-1) 2) y = 20x-5
3) x = 15y -20 4) y = 20 (x-1)

T.S ECET 2018

41. Two cars with equal speed V started from a place are such that one is moving towards East and the other is moving towards North. The rate at which they are separated from each other when they travel same distance is___

1) $V\sqrt{2}$ 2) $\frac{V}{\sqrt{2}}$ 3) $\frac{\sqrt{2}}{V}$ 4) $2V^2$

PREVIOUS ECET BITS KEY

01-1	02-3	03-4	04-2	05-2
06-1	07-4	08-2	09-1	10-4
11-2	12-4	13-2	14-4	15-1
16-2	17-3	18-1	19-3	20-1
21-2	22-3	23-2	24-3	25-1
26-3	27-2	28-2	29-2	30-3
31-2	32-2	33-2	34-4	35-1
36-2	37-4	38-2	39-4	40-2
41-1				

SPACE FOR IMPORTANT NOTES

SPACE FOR IMPORTANT NOTES

INDEFINITE INTEGRALS

I. Let f be a function defined on an interval I. If there is a function F on I such that $F'(x)' = f(x)$ for all $x \in I$, then we say that F is an antiderivative of f or a Primitive of f. In this case, we say that f has an integral on I and for any real constant, F + c is called Indefinite integral of f over I. We write.

$\int f(x)dx = F(x) + c$

Here f is called Integrand, c is called constant of integration and x is called the 'Variable of integration'

II. $\frac{d}{dx}\left(\int f(x)dx\right) = f(x)$

III. $\int \frac{d}{dx}(f(x))dx = f(x)+c$, where c is constant.

IV. **Important Properties of Integration**

If f, g have integrals on an interval I and a is a real number, then

1. $\int (f \pm g)(x)dx = \int f(x)dx \pm \int g(x)dx$
2. $\int (af)(x)dx = a\int f(x)dx$
3. If u and v have integrals, then write $U = \int u \ dx$ and $V = \int v \ dx$.

   Now $(UV)' = Uv + uV \quad \therefore UV = \int Uv + \int uV$

   That is $UV = UV - \int uV$

   $\therefore \int UdV = UV - \int V\,dU$
4. $\int f(x)\,g($.

   $= f(x)\int g(x)dx - \int f'(x)\left(\int g(x)dx\right)dx$

   (integration by parts)
5. If $\int f(x)dx = g(x)+c$

   then $\int f(\phi(t))\,\phi'(t)dt = g(\phi(t))+c$
6. If $\int f(x)dx = g(x)+c$, then $\int f(ax+b)dx = \frac{g(ax+b)}{a}+c$

FORMULA - I

1. $\int K\,dx = Kx + c,\ K \in R$
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1}+c,\ \ n+1$
3. $\int \frac{1}{x}dx = \log|x|+c$
4. $\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c$
5. $\int e^x\,dx = e^x + c$
6. $\int a^x\,dx = \frac{a^x}{\log a}+c$
7. $\int \log x\,dx = x.[\log x - 1]+c$
8. $\int \sqrt{x}\ dx = \frac{2}{3}x\sqrt{x}+c = \frac{2}{3}.x^{3/2}+C$

FORMULA - 2

1. $\int \sin x\,dx = -\cos x + c$
2. $\int \cos x\,dx = \sin x + c$
3. $\int \sec^2 x\,dx = \tan x + c$
4. $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$
5. $\int \sec x.\tan x\ dx = \sec x + c$
6. $\int \operatorname{cosec} x.\cot x\ dx = -\operatorname{cosec} x + c$

FORMULA - 3

1. $\int \tan x\,dx = \log|\sec x| + c$ or $-\log|\cos x| + c$
2. $\int \cot x\,dx = \log|\sin x| + c$ or $-\log|\operatorname{cosec} x| + c$
3. $\int \sec x\,dx = \log|\sec x + \tan x| + c$

$$= \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$$

4. $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c$

(or) $\log\left|\tan \frac{x}{2}\right| + c$

FORMULA - 4

1. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$

$$= -\cos^{-1}x + c$$

2. $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$

$$= -\cot^{-1}x + c$$

3. $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}x + c$

$$= -\operatorname{cosec}^{-1}x + c$$

FORMULA - 5

1. $\int \sinh x\ dx \to \cosh x + c$
2. $\int \cos hx\ dx \to \sin hx + c$
3. $\int \tan hx\ dx \to \log|\cos hx| + c$
4. $\int \cot hx\ dx \to \log|\sin hx| + c$
5. $\int \operatorname{sech} x\ dx \to 2\tan^{-1}\left(e^{-x}\right) + c$
6. $\int \operatorname{cosec} hx\ dx \to \log\left|\tan h\frac{x}{2}\right| + c$

FORMULA - 6

1. $\int \sec h^2 x\ dx \to \tan hx + c$
2. $\int \operatorname{cosec} h^2 x\ dx \to -\cot hx + c$

3. $\int \sec hx.\tan hx\ dx \to -\sec hx + c$
4. $\int \operatorname{cosec} hx.\cot hx\ dx \to -\operatorname{cosec} hx + c$
5. $\int \frac{1}{\sqrt{1+x^2}}\,dx = \sin h^{-1}x + c$
6. $\int \frac{1}{\sqrt{x^2-1}}\,dx = \cos h^{-1}x + c$

FORMULA - 7

1. $\int \frac{1}{(x+a)(x+b)}\,dx = \frac{1}{b-a}\log\left|\frac{x+a}{x+b}\right| + c$

$$\left[\frac{1}{(x-a)(x^2+b)} = \frac{1}{a^2+b}\left[\frac{1}{x-a} - \frac{x+a}{x^2+b}\right]\right]$$

2. $\int \frac{1}{(ax+b)(cx+d)}\,dx = \frac{1}{ad-bc}\log\left|\frac{ax+b}{cx+d}\right| + c$
3. $\int \frac{1}{(x^2+a^2)(x^2+b^2)}\,dx =$

$$\frac{1}{b^2-a^2}\left[\frac{1}{a}\tan^{-1}\frac{x}{a} - \frac{1}{b}\tan^{-1}\frac{x}{b}\right] + c$$

4. $\int \frac{x}{(x^2+a^2)(x^2+b^2)}\,dx = \frac{1}{2(b^2-a^2)}\log\left|\frac{x^2+b^2}{x^2+a^2}\right| + c$
5. $\int \frac{1}{x(x^n+1)}\,dx = \frac{1}{n}\log\left|\frac{x^n}{1+x^n}\right| + c$
6. $\int \frac{1}{x(1-x^n)}\,dx = \frac{1}{n}\log\left|\frac{x^n}{1-x^n}\right| + c$

FORMULA - 8 (STANDARD INTEGRALS) :

1. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
2. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$
3. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$
4. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$
5. $\int \frac{dx}{\sqrt{a^2+x^2}} = \sin h^{-1}\left(\frac{x}{a}\right) + c$ (or)

$$\log\left|x + \sqrt{a^2+x^2}\right| + c$$

6. $\int \frac{dx}{\sqrt{x^2-a^2}} = \cos h^{-1}\left(\frac{x}{a}\right) + c$ (or)

$$\log\left|x + \sqrt{x^2-a^2}\right| + c$$

7. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$
8. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cos h^{-1}\left(\frac{x}{a}\right) + c$
9. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sin h^{-1}\left(\frac{x}{a}\right) + c$

NOTE :

1. $\int \frac{f^1(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\left[\frac{f(x)}{a}\right] + c$
2. $\int \frac{f^1(x)}{\sqrt{a^2+[f(x)]^2}}\,dx = \sin h^{-1}\left[\frac{f(x)}{a}\right] + c$
3. $\int \frac{f'(x)}{\sqrt{[f(x)]^2-a^2}}\,dx = \cos h^{-1}\left[\frac{f(x)}{a}\right] + c$

FORMULA - 9

1.

a. $\int \frac{1}{a^2\sin^2 x + b^2\cos^2 x}\,dx = \frac{1}{ab}\tan^{-1}\left(\frac{a\sin x}{b\cos x}\right) + c$

b. $\int \frac{1}{a+b\sin^2 x}\,dx,\ \int \frac{1}{a+b\cos^2 x}\,dx,$

$$\int \frac{1}{a+b\cos^2 x + c\sin^2 x}\,dx,\ \int \frac{1}{(a\sin x + b\cos x)^2}\,dx$$

Rule : Multiply Nr, Dr with $\sec^2 x$ **and use** $\tan x = t$

2. $\int \frac{1}{a\sin x + b\cos x}\,dx =$

$$\frac{1}{\sqrt{a^2+b^2}}\log\left|\tan\left(\frac{x}{2} + \frac{1}{2}\tan^{-1}\frac{b}{a}\right)\right| + c$$

3. $\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx =$

$$\frac{ac+bd}{c^2+d^2}x + \frac{ad-bc}{c^2+d^2}\log|c\cos x + d\sin x| + c$$

4. If u and v are function of x then

$$\int u\,dv = uv - \int v\,du$$

5. $\int f(x)g(x)\,dx = f(x)\int g(x)\,dx$

$$-\int\left[f^1(x)\int g(x)\right].dx$$

where $f(x) =$ first function

$g(x) =$ second function

6. Proper choice of first and second function.

a. The first function is the function which comes first in the word I LATE.

b. If one of the two functions is not directly integrable, then take this function as the first function.

c. If one of the function is not directly integrable, and there is not other function, then unity is taken as the second function.

d. u and v are easily derivable, integrable functions,

$$\int u\,dv = uv_1 - u^1v_2 + u^{11}v_3 \ldots\ldots$$

7. $\int x^n.\log x\,dx = \frac{x^{n+1}}{n+1}\left[\log x - \frac{1}{n+1}\right] + c$

$[\text{for } n \neq -1]$

8. $\int e^x\left[f(x)+f^1(x)\right]dx = e^x f(x)+c$

9. $\int e^{ax}\left[f(x)+\frac{f^1(x)}{a}\right]dx = \frac{e^{ax}f(x)}{a}+c$

10. $\int\left[xf^1(x)+f(x)\right]dx = xf(x)+c$

11. $\int e^{ax}\sin(bx+c)dx =$

$$\frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c)-b\cos(bx+c)\right]+k$$

12. $\int e^{ax}\cos(bx+c)dx =$

$$\frac{e^{ax}}{a^2+b^2}\left[a\cos(bx+c)+b\sin(bx+c)\right]+k$$

FORMULA - 10

Important Formulae to Remember :

1. $\int \sin^{-1}x\,dx = x\sin^{-1}x+\sqrt{1-x^2}+c$
2. $\int \cos^{-1}x\,dx = x\cos^{-1}x-\sqrt{1-x^2}+c$
3. $\int \tan^{-1}x\,dx = x\tan^{-1}x-\frac{1}{2}\log(1+x^2)+c$
4. $\int \cot^{-1}x.dx = x\cot^{-1}x+\frac{1}{2}\log(1+x^2)+c$
5. $\int \sec^{-1}x.dx = x\sec^{-1}x-\cos h^{-1}x+c$
6. $\int \operatorname{cosec}^{-1}x.dx = x\operatorname{cosec}^{-1}x+\cos h^{-1}x+c$

PRACTICE SET - I

01. $\int a^x e^x\,dx =$

1) $(ae)^x + c$ 2) $\frac{a^x e^x}{\log a+\log x}+c$

3) $\frac{(ae)^x}{1+\log a}+c$ 4) $\frac{(ax)^x}{\log ax}+c$

02. $\int\frac{1+\cos^2 x}{1+\cos 2x}dx=$

1) $\operatorname{Tan}x - x + c$ 2) $\cot x + x + c$

3) $\frac{Tanx}{2}+x+c$ 4) $\frac{1}{2}(Tan\,x+x)+c$

03. $\int\frac{\cos 2x}{\cos^2 x.Sin^2 x}dx=$

1) $-\operatorname{Tan}x-\cot x+c$ 2) $\operatorname{Tan}x+\operatorname{Sin}x+c$

3) $\operatorname{Tan}x+\cot x+c$ 4) $\operatorname{Tan}x+\operatorname{Cos}x+c$

04. $\int\frac{1}{1+\sin x}dx=$

1) $\operatorname{Tan}x+\operatorname{Sec}x+c$ 2) $\operatorname{Tan}x-\operatorname{Sec}x+c$

3) $\cot x-\operatorname{cosec}x+c$ 4) $-\cot x+\sec x+c$

05. $\int(\operatorname{Tan}x+\operatorname{Cot}x)^2dx=$

1) $\operatorname{Tan}x+\cot x+c$ 2) $\operatorname{Tan}x-\operatorname{Cot}x+c$

3) $\operatorname{Tan}x+\cot x-4x+c$

4) $\tan x+\cot x-4x+c$

06. $\int\frac{Sin^3x+\cos^3x}{Sin^2x.\cos^2x}dx=$

1) $\operatorname{Sec}x-\operatorname{Cosec}x+c$ 2) $\sec x+\operatorname{cosec}^x+c$

3) $\operatorname{Sin}x-\operatorname{Cos}x+c$ 4) $\sin x+\cos x+c$

07. $\int(\sin^{-1}x+\operatorname{Cos}^{-1}x)^2\,dx=$

1) 0 2) $\frac{\pi^2x}{4}+c$ 3) $\frac{\pi x}{4}+c$ 4) $\frac{\pi x}{2}+c$

08. $\int\frac{1}{Cos 2x+\sin^2 x}dx=$

1) $\tan x+c$ 2) $\cot x+c$

3) $\frac{\tan^2 x}{2}+c$ 4) $\frac{\cot^2 x}{2}+c$

09. $\int\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+........\right)dx=$

1) e^x+c 2) $-e^x+c$

3) $\frac{e^x}{2}+c$ 4) $-\frac{e^x}{2}+c$

10. $\int\frac{1-Cos 2x}{1+Cos 2x}dx=$

1) $\tan x+x+c$ 2) $\tan x-x+c$

3) $x\tan x+c$ 4) $x-\tan x+c$

11. $\int\frac{Sin^2x}{1-Cosx}dx=$

1) $x\operatorname{Sin}x+c$ 2) $x-\operatorname{Sin}x+c$

3) $x+\operatorname{Sin}x+c$ 4) $x^2\sin x+c$

12. $\int\frac{1-Sin\,x}{1+Sin\,x}dx=$

1) $2\tan x-2\operatorname{Sec}x-x+c$

2) $2\tan x-\operatorname{Sec}x-x+c$

3) $\tan x+2\operatorname{Sec}x+x+c$

4) $\tan x-2\operatorname{Sec}x+x+c$

13. $\int\frac{1+2x^2}{x^2(1+x^2)}dx=$

1) $Tan^{-1}x+\frac{1}{x}+c$ 2) $Tan^{-1}x-\frac{1}{x}+c$

3) $\frac{Tan^{-1}x}{x}+c$ 4) $\frac{Tan^{-1}x}{x^2}+c$

14. $\int\cos x^0 dx=$

1) $\sin x^0+c$ 2) $\frac{\pi}{180}\cos x^0+c$

3) $\frac{180}{\pi}\sin x^0+c$ 4) $180\sin x^0+c$

15. $\int Sec^2x.Cosec^2x\,dx=$

1) $\tan x-\cot x+c$ 2) $\tan x+\cot x+c$

3) $\tan x\cot x+c$ 4) $\operatorname{Sec}x\tan x+c$

16. $\int\sqrt{1+Sin2x}\,dx.\left[\forall x\in\left(0,\frac{\pi}{4}\right)\right]=$

1) $\operatorname{Sin}x-\operatorname{Cos}x+c$ 2) $\operatorname{Sin}x+\operatorname{Cos}x+c$

3) $\operatorname{Cos}x-\operatorname{Sin}x+c$ 4) $\frac{(Sin\,x+Cos\,x)^2}{2}+c$

17. $\int\frac{1}{1+Sin\,ax}dx=$

1) $\frac{1}{a}[Tan\,ax-Sec\,ax]+c$

2) $\frac{1}{a}[Cosec\,ax-Cot\,ax]+c$

3) $\frac{1}{a}[Sec\,ax-Cot\,ax]+c$ 4) $2Tan(\frac{ax}{2})+c$

18. $\int\frac{1}{1+Cos\,ax}dx$

1) $\frac{1}{a}(Cosec\,ax+\cot ax)+c)$

2) $Cot\frac{ax}{2}+c$ 3) $Tan\frac{ax}{2}+c$

4) $\frac{1}{a}(Cosec\,ax-Cot\,ax)+c$

19. $\int\frac{|x|}{x}dx=$

1) $x+c$ 2) $-x+c$

3) $|x|+c$ 4) x^2+c

20. $\int\left(\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}\right)dx$

1) $x+c$ 2) $3x^2+c$

3) $\frac{x^3}{3}+c$ 4) $\frac{x^3}{2}+c$

21. $\int Cot^2x\,dx$

1) $-\operatorname{Cot}x-x+c$ 2) $\operatorname{Tan}x-x+c$

3) $\operatorname{Cot}x-x+c$ 4) $\operatorname{Sin}x-x+c$

22. $\int(Tanx-Cotx)^2dx=$

1) $\operatorname{Tan}x+x+c$ 2) $\operatorname{Tan}x-x+c$

3) $\operatorname{Tan}x-\operatorname{Cot}x+c$ 4) $\operatorname{Tan}x-\operatorname{Cot}x-4x+c$

23. If $\int f(x)dx=f(x)$, then

1) $f(x)=x$ 2) $f(x)=constant$

3) $f(x)=0$ 4) $f(x)=e^x$

24. $Tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ can be integrated by substituting

1) $x=\tan\theta$ 2) $x=\cos\theta$

3) $x=\sin\theta$ 4) $x=\sec\theta$

25. $\int\frac{dx}{\sqrt{x+a}+\sqrt{x+b}}=$

1) $\frac{1}{(a-b)}\left[(x+a)\sqrt{x+a}-(x+b)\sqrt{x+b}\right]+c$

2) $\frac{2}{3(a-b)}\left[(x+a)\sqrt{x+a}+(x+b)\sqrt{x+b}\right]+c$

3) $\frac{2}{3(a-b)}\left[(x+a)\sqrt{x+a}-(x+b)\sqrt{x+b}\right]+c$

4) $\sqrt{x+a}-\sqrt{x+b}+c$

26. $\int\frac{Sinx}{1-Cosx}dx=$

1) $\log|1-Cos\,x|+c$ 2) $2\log\left|\sec\frac{x}{2}\right|+c$

3) $\frac{1}{2}\log|1-Cos\,x|+c$ 4) $-\log|1-Cos\,x|+c$

27. $\int\frac{x^3Tan^{-1}x^4}{1+x^8}dx=$

1) $\frac{Tan^{-1}x^4}{4}+c$ 2) $\frac{x^3Tan^{-1}x^4}{4}+c$

3) $\frac{1}{8}\left(Tan^{-1}x^4\right)^2+c$ 4) $\frac{\tan^{-1}x^4}{8}+c$

28. $\int\frac{Cosec\,x}{\log\left|\tan\frac{x}{2}\right|}dx=$

1) $\log\left|\log\left(\tan\frac{x}{2}\right)\right|+c$ 2) $-\log\left|\log\left(\tan\frac{x}{2}\right)\right|+c$

3) $\log\left|\log\left(\cot\frac{x}{2}\right)\right|+c$ 4) $-\log\left|\log\left(\cot\frac{x}{2}\right)\right|+c$

29. $\int\frac{e^{x-1}+x^{e-1}}{e^x+x^e}dx=$

1) $\log|e^x+x^e|+c$ 2) $\frac{1}{e}\log|e^x+x^e|+c$

3) $-\frac{1}{e}\log|e^x+x^e|+c$ 4) $-\log|e^x+x^e|+c$

30. $\int\frac{10x^9+10^x\log_e 10}{10^x+x^{10}}dx=$

1) $\log|10^x+x^{10}|+c$ 2) $-\log|10^x+x^{10}|+c$

3) $10^x+x^{10}+c$ 4) $\log|10^x+x^9|+c$

31. $\int Cosec\,x.\log\left|\tan\frac{x}{2}\right|dx=$

1) $\frac{1}{2}\log\left|\tan\frac{x}{2}\right|+c$ 2) $\left(\log\left|\tan\frac{x}{2}\right|\right)^2+c$

3) $\frac{1}{2}\left(\log\left|\tan\frac{x}{2}\right|\right)^2+c$ 4) $2\log\left|\tan\frac{x}{2}\right|+c$

32. $\int\frac{e^x}{\sqrt{a+be^x}}dx=$

1) $\frac{2}{b}\sqrt{a+be^x}+c$ 2) $\frac{1}{b}\sqrt{a+be^x}+c$

3) $2\sqrt{a+be^x}+c$ 4) $\sqrt{a+be^x}+c$

33. $\int\frac{(Sin^{-1}x)^3}{\sqrt{1-x^2}}dx=$

1) $\frac{(Sin^{-1}x)^3}{3}+c$ 2) $\frac{1}{4}(Sin^{-1}x)^4+c$

3) $(Sin^{-1}x)^4+c$ 4) $(Sin^{-1}x)^3+c$

34. $\int\sec^p x.Tan\,x\,dx=$

1) $\frac{\sec^p x}{p}+c$ 2) $\frac{\sec^{p-1}x}{p-1}+c$

3) $\frac{\sec^{p+1}x}{p+1}+c$ 4) $\frac{\sec^{p-1}x}{p+1}+c$

35. $\int\frac{\sin 2x}{a^2+b^2\sin^2 x}dx=$

1) $\frac{1}{b^2}\log|a^2+b^2Sin^2x|+c$

2) $\frac{1}{a^2-b^2}\log|a^2+b^2\sin^2 x|+c$

3) $\frac{1}{a^2}\log|a^2+b^2\sin^2 x|+c$

4) $\frac{1}{a^2+b^2}\log|a^2+b^2\sin^2 x|+c$

36. $\int\frac{x\sin^{-1}x^2}{\sqrt{1-x^4}}dx=$

1) $\frac{Sin^{-1}x^2}{4}+c$ 2) $\frac{x}{\sqrt{1-x^4}}+c$

3) $\frac{-\left[Sin(x^2)\right]^2}{4}+c$ 4) $\frac{\left[Sin^{-1}(x^2)\right]^2}{4}+c$

37. $\int\frac{1}{(1+e^x)(1+e^{-x})}dx=$

1) $\frac{1}{1+e^x}+c$ 2) $-\frac{1}{1+e^{-x}}+c$

3) $-\frac{1}{1+e^x}+c$ 4) $\frac{1}{1+e^{-x}}+c$

38. $\int\frac{Sec\,x}{(Sec\,x+Tan\,x)^2}dx=$

1) $\frac{1}{2(Sec\,x+Tan\,x)^2}+c$

2) $\frac{-1}{2(Sec\,x+Tan\,x)^2}+c$

3) $\frac{1}{2(Sec\,x+Tan\,x)}+c$

4) $\frac{-1}{2(Sec\,x+Tan\,x)}+c$

39. $\int\frac{dx}{(arc\sin x)^3\sqrt{1-x^2}}=$

1) $\frac{-1}{2(arc\,Sin\,x)^2}+c$ 2) $\frac{1}{2(arc\,Sin\,x)^2}+c$

3) $\frac{1}{3(arc\,Sin\,x)^3}+c$ 4) $\frac{1}{4(arc\,Sin\,x)^2}+c$

40. $\int\frac{\cos x}{\sqrt[3]{\sin^2 x}}dx=$

1) $3\sqrt[3]{\sin x}+c$ 2) $3\sqrt[3]{\sin^2 x}+c$

3) $\sqrt[3]{\sin x}+c$ 4) $\sqrt[3]{\sin^2 x}+c$

41. $\int\frac{x^2}{\sqrt{a^3-x^3}}dx=$

1) $2\sqrt{a^3-x^3}+c$ 2) $\frac{1}{\sqrt{a^2-x^2}}+c$

3) $-\frac{2}{3}\sqrt{a^3-x^3}+c$ 4) $\frac{1}{3}\sqrt{a^2-x^2}+c$

42. $\int\frac{e^{x-1}+x^{e-1}}{\sqrt{e^x+x^e}}dx=$

1) $\frac{2}{e}\sqrt{e^x+x^e}+c$ 2) $2\sqrt{e^x+x^e}+c$

3) $\frac{1}{2}\sqrt{e^x+x^e}+c$ 4) $\frac{1}{e}\sqrt{e^x+x^e}+c$

43. $\int\frac{1+Cos\,x}{(x+Sin\,x)^2}dx=$

1) $\frac{1}{x+Sin\,x}+c$ 2) $\frac{-1}{x+Sin\,x}+c$

3) $\frac{1}{(x+Sin\,x)^2}+c$ 4) $\log|x+Sin\,x|+c$

44. $\int \frac{Sec^2 x}{5+4\tan x}dx=$

1) $\log|5+4\tan x|+c$ 2) $\frac{-1}{5+4\tan x}+c$

3) $\frac{1}{4}\log|5+4\tan x|+c$

4) $-\frac{1}{4}\log|5+4\tan x|+c$

45. $\int \frac{2x+3}{\sqrt{x^2+3x-4}}dx=$

1) $\sqrt{x^2+3x-4}+c$ 2) $2\sqrt{x^2+3x-4}+c$

3) $2(x^2+3x-4)^{3/2}+c$

4) $-2(x^2+3x-4)^{1/2}+c$

46. $\int \sqrt[3]{\sin x}\cos x\,dx=$

1) $\frac{4}{3}Sin^{\frac{4}{3}}x+c$ 2) $\frac{3}{4}Sin^{\frac{4}{3}}x+c$

3) $-\frac{3}{4}Sin^{\frac{4}{3}}x+c$ 4) $\frac{-4}{3}Sin^{\frac{4}{3}}x+c$

47. $\int Tan^7 x\,dx+\int Tan^9 x\,dx=$

1) $\frac{-Tan^8 x}{7}+c$ 2) $Tan^7 x+c$

3) $\frac{Tan^{10}x}{10}+c$ 4) $\frac{Tan^8 x}{8}+c$

48. $\int \frac{1}{(2Sin x+3Cos x)^2}dx=$

1) $\frac{1}{2Tan x+3}+c$ 2) $\frac{1}{2(2Tan x+3)}+c$

3) $-\frac{1}{2(2Tan x+3)}+c$ 4) $-\frac{1}{2(2Tan x+3)}+c$

49. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})}dx=$

1) $\log|1+\sqrt{x}|+c$ 2) $2\log|1+\sqrt{x}|+c$

3) $-\log|1+\sqrt{x}|+c$ 4) $\frac{1}{(1+\sqrt{x})^2}+c$

50. $\int \frac{1-\tan x}{1+\tan x}dx=$

1) $\log|1+\tan x|+c$ 2) $\log|1-\tan x|+c$

3) $\log|\sin x+\cos x|+c$ 4) $\log|\sin x-\cos x|+c$

51. $\int e^x\sqrt{1+e^x}\,dx=$

1) $(1+e^x)^{\frac{3}{2}}+c$ 2) $\frac{2}{3}(1-e^x)^{3/2}+c$

3) $(1-e^x)^{3/2}+c$ 4) $\frac{2}{3}(1+e^x)^{3/2}+c$

52. $\int \frac{\sqrt{x}}{x+1}dx=$

1) $\sqrt{x}-Tan^{-1}\sqrt{x}+c$ 2) $\sqrt{x}+Tan^{-1}\sqrt{x}+c$

3) $2\left[\sqrt{x}-Tan^{-1}\sqrt{x}\right]+c$ 4) $2\left[\sqrt{x}+Tan^{-1}\sqrt{x}\right]+c$

53. $\int x^4\cdot Tan\,x^5\,dx=$

1) $\frac{1}{5}Tan x^5+c$ 2) $\frac{1}{5}\log|\sec x^5|+c$

3) $\frac{1}{5}\log|\sec x^5+\tan x^5|+c$

4) $\frac{1}{5}\log|\tan x^5|+c$

54. $\int \cos 7x.\cos 3x\,dx=$

1) $\frac{Sin10x}{4}-\frac{Sin8x}{6}+c$

2) $\frac{1}{40}[2Sin10x+5Sin4x]+c$

3) $\frac{1}{40}[2Cos10x+5Cos4x]+c$

4) $[2Cos10x+5Cos4x]+c$

55. $\int 2Sin5x\,Sin3x\,dx=$

1) $\frac{Sin2x}{4}-\frac{Sin8x}{16}+c$

2) $\frac{1}{8}[4\sin 2x-Sin8x]+c$

3) $\frac{1}{16}[4Cos2x-Cos8x]+c$

4) $\frac{1}{16}[4Cos2x+Cos8x]+c$

56. $\int 3^{2x-7}dx=$

1) $\frac{3^{2x-7}}{2\log 3}+c$ 2) $\frac{3^{2x-7}}{\log 3}+c$

3) $3^{2x-7}+c$ 4) $\frac{3^{2x-7}}{\log 2}+c$

57. $\int Sec^2(3x+5)dx=$

1) $\tan(3x+5)+c$ 2) $\tan^2(3x+5)+c$

3) $\frac{1}{3}\tan(3x+5)+c$ 4) $\frac{1}{3}\tan^2(3x+5)+c$

58. $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx=$

1) $e^{\tan^{-1}x}+c$ 2) $-e^{\tan^{-1}x}+c$

3) $\left(-e^{\tan^{-1}x}\right)^2+c$ 4) $\frac{e^{\tan^{-1}x}}{x}+c$

59. $\int \frac{Cos\sqrt{x}}{\sqrt{x}}dx=$

1) $2Sin\sqrt{x}+c$ 2) $Sin^2\sqrt{x}+c$

3) $Cos^2\sqrt{x}+c$ 4) $2Cos\sqrt{x}+c$

60. $\int \frac{x^3}{\sqrt{1-x^8}}dx=$

1) $\frac{1}{4}Sin^{-1}x^4+c$ 2) $\frac{1}{4}Sin^{-1}\frac{1}{x^4}+c$

3) $\frac{1}{4}Cos^{-1}x^4+c$ 4) $\frac{1}{4}Cos^{-1}\frac{1}{x^4}+c$

61. $\int \frac{x^2}{\sqrt{1-x^6}}dx=$

1) $\frac{1}{3}Sin^{-1}(x^3)+c$ 2) $\frac{1}{3}Cos^{-1}(x^3)+c$

3) $-\frac{1}{3}Sin^{-1}(x^3)+c$ 4) $\frac{1}{4}Cos^{-1}(x^3)+c$

62. $\int \frac{dx}{\sin(x-a)\sin(x-b)}=$

1) $\frac{1}{\sin(a-b)}\log\left[\frac{\sin(x-a)}{\sin(x-b)}\right]$

2) $\frac{-1}{\sin(a-b)}\log\left[\frac{\sin(x-a)}{\sin(x-b)}\right]$

3) $\log[\sin(x-a).\sin(x-b)]$

4) $\log\left[\frac{\sin(x-a)}{\sin(x-b)}\right]$

63. $\int \frac{\sin x.\cos x}{a^2\cos^2 x+b^2\sin^2 x}dx$

1) $\frac{1}{a^2-b^2}\log\left(a^2\cos^2 x+b^2\sin^2 x\right)$

2) $\frac{1}{2(a^2-b^2)}\log\left(a^2\cos^2 x+b^2\sin^2 x\right)$

3) $\tan^{-1}\left(\frac{b\tan x}{a}\right)$ 4) $\tan^{-1}\left(\frac{b\cot x}{a}\right)$

64. $\int \frac{(a-b)\sin x+(a+b)\cos x}{a\sin x+b\cos x}dx$

1) $x+\log(a\sin x+b\cos x)$

2) $\frac{1}{a^2+b^2}\log(a\sin x+b\cos x)$

3) $\frac{a^2+b^2}{a^2-b^2}x+\frac{1}{a^2+b^2}\log(a\sin x+b\cos x)$

4) none of these

65. $\int \frac{1}{1+\cot x}dx =$

1) $\frac{1}{2}x+\frac{1}{2}\log(\sin x+\cos x)$

2) $\frac{1}{2}x-\frac{1}{2}\log(\sin x+\cos x)$

3) $\log(1+\tan x)$ 4) $x+\log(\sin x+\cos x)$

66. $\int \frac{1}{\cos(x-a)\cos(x-b)}dx =$

1) $\frac{1}{\cos(a-b)}\log_e\left[\frac{\cos(x-a)}{\cos(x-b)}\right]+c$

2) $\frac{1}{\cos(a-b)}\log_e\left[\frac{\sin(x-a)}{\sin(x-b)}\right]+c$

3) $\frac{1}{\sin(a-b)}\log_e\left[\frac{\sin(x-a)}{\sin(x-b)}\right]+c$

4) $\frac{1}{\sin(a-b)}\log_e\left[\frac{\cos(x-a)}{\cos(x-b)}\right]+c$

67. $\int \frac{dx}{\sin^2 x\cos^2 x}=$

1) $-\cot x+\tan x$ 2) $-\cot x-\tan x$
3) $\cot x-\tan x$ 4) $\cot x+\tan x$

68. $\int \frac{\log x^2}{x}dx =$

1) $(\log x)^2$ 2) $\frac{1}{2}(\log x)^2$

3) $\log(x^2)$ 4) $2\log(x^2)$

69. $\int \frac{x^3+x^2+1}{x+1}dx$

1) $\frac{x^2}{2}+\log(x+1)$ 2) $\frac{x^3}{3}+\log(x+1)$

3) $\frac{x^4}{4}+\frac{x^3}{3}+\log x$ 4) none of these

70. $\int \frac{dx}{x+\sqrt{x}}=$

1) $\log(1+\sqrt{x})$ 2) $\frac{1}{2}\log(x+\sqrt{x})$

3) $2\log(1+\sqrt{x})$ 4) $\frac{x^2}{2}+\frac{2}{3}x^{3/2}$

71. $\int \frac{dx}{1-\cos x}=$

1) $\operatorname{cosec} x+\cot x$ 2) $-\cot\frac{x}{2}$

3) $-\tan\frac{x}{2}$ 4) $\operatorname{cosec} x-\cot x$

72. $\int \frac{dx}{\sqrt{1-x}}=$

1) $\sin^{-1}\sqrt{x}$ 2) $c-2\sqrt{1-x}$
3) $2\sqrt{1-x}$ 4) $-\sin\sqrt{x}$

73. $\int \frac{x^2-1}{x^2+1}dx =$

1) $\tan^{-1}x^2$ 2) $x+\tan^{-1}x$
3) $x-2\tan^{-1}x$ 4) $\log(1+x^4)$

74. $\int \frac{\sin 2x}{1+\cos^2 x}dx =$

1) $\frac{1}{2}\log(1+\cos 2x)+c$ 2) $c-\log(1+\cos^2 x)$

3) $-\frac{1}{2}\log(1+\cos^2 x)+c$ 4) $2\log(1+\cos^2 x)+c$

75. The value of $\int \frac{1}{1+\cos 8x}dx$ is

1) $\frac{\tan 8x}{8}+c$ 2) $\frac{\tan 2x}{x}+c$ 3) $\frac{\tan 4x}{8}+c$ 4) $\frac{\tan 4x}{4}+c$

76. $\int \sin^3 x.\cos^3 x\, dx =$

1) $\frac{1}{3}\sin^3 x-\frac{1}{5}\sin^5 x+c$

2) $\frac{1}{3}\cos^3 x-\frac{1}{5}\sin^5 x+c$

3) $\frac{1}{3}\sin^3 x-\frac{1}{5}\sin^5 x+c$

4) $\frac{1}{3}\tan^3 x-\frac{1}{5}\sin^5 x+c$

77. $\int \frac{x^5}{x^2+1}dx =$

1) $\frac{x^4}{4}+\frac{x^2}{2}+\tan^{-1}+c$ 2) $\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2}\log(x^2+1)+c$

3) $\frac{x^4}{4}+\frac{x^3}{3}+\tan^{-1}+c$ 4) $\frac{x^4}{4}-\frac{x^3}{3}-\tan^{-1}+c$

78. $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x.\cos^2 x}dx =$

1) $\sin x+\cos x+c$ 2) $\tan x+\cot x+c$
3) $\sec x-\operatorname{cosec} x+c$ 4) $\sin x-\cos x+c$

79. $\int \frac{dx}{x(1+\log x)^2}=$

1) $\frac{-1}{2(1+\log x)^2}+c$ 2) $\frac{1}{2(1+\log x)^2}+c$

3) $\frac{-1}{1+\log x}+c$ 4) $\frac{-1}{3(1+\log x)^3}+c$

80. $\int \frac{\sin^6 x}{\cos^8 x}dx =$

1) $\tan 7x+c$ 2) $\frac{\tan^7 x}{7}+c$

3) $\frac{\tan 7x}{7}+c$ 4) $\sec^7 x+c$

81. $\int \frac{dx}{(x+100)\sqrt{x+99}}=f(x)+c$ then $f(x)=$

1) $2(x+100)^{1/2}$ 2) $3(x+100)^{1/2}$
3) $2\tan^{-1}\sqrt{x+99}$ 4) $2\tan^{-1}\sqrt{x+100}$

82. $\int \frac{\sin x}{\sin(x-a)}dx =$

1) $x\cos a-\sin a.\log(x-a)+c$
2) $x\sin a+c$
3) $x\sin a+\sin a.\log\sin(x-a)+c$
4) $x\cos a+\sin a.\log\sin(x-a)+c$

83. $\int \frac{f'(x)}{f(x)\log[f(x)]}dx =$

1) $\frac{f(x)}{\log f(x)}+c$

2) $f(x).\log f(x)+c$

3) $\log[\log f(x)]+c$

4) $\frac{1}{\log[\log f(x)]}+c$

84. $\int \frac{dx}{\sin(x-a)\sin(x-b)}=$

1) $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$

2) $\frac{-1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$

3) $\log\sin(x-a).\sin(x-b)+c$

4) $\frac{1}{\sin(x-a)}\log\sin(x-a)\sin(x-b)+c$

85. $\int \frac{x^2}{1+x^6}dx =$

1) x^3+c 2) $\frac{1}{3}\tan^{-1}(x^3)+c$

3) $\log(1+x^3)$ 4) none of these

PRACTICE SET-I KEY

01) 3	02) 4	03) 1	04) 2	05) 2
06) 1	07) 2	08) 1	09) 1	10) 2
11) 3	12) 1	13) 2	14) 3	15) 1
16) 1	17) 1	18) 4	19) 3	20) 3
21) 1	22) 4	23) 4	24) 1	25) 2
26) 1	27) 3	28) 1	29) 2	30) 1
31) 1	32) 2	33) 1	34) 1	35) 1
36) 4	37) 3	38) 2	39) 1	40) 1
41) 3	42) 1	43) 2	44) 3	45) 2
46) 2	47) 4	48) 3	49) 2	50) 3
51) 4	52) 3	53) 2	54) 2	55) 2
56) 1	57) 3	58) 1	59) 1	60) 1
61) 1	62) 1	63) 2	64) 1	65) 2
66) 4	67) 1	68) 1	69) 2	70) 3
71) 2	72) 2	73) 3	74) 2	75) 3
76) 1	77) 2	78) 3	79) 3	80) 2
81) 3	82) 4	83) 3	84) 1	85) 2

PRACTICE SET - II

01. $\int \frac{1}{\sqrt{2+3x^2}}dx =$

1) $\frac{1}{\sqrt{3}}\sin h^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}}\right)+c$

2) $\frac{1}{\sqrt{3}}\sin h^{-1}\left(\frac{\sqrt{2}x}{\sqrt{3}}\right)+c$

3) $\frac{1}{\sqrt{2}}\cos h^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}}\right)+c$

4) $\frac{1}{\sqrt{2}}\cos h^{-1}\left(\frac{\sqrt{2}x}{\sqrt{3}}\right)+c$

02. $\int \frac{3^x}{\sqrt{9^x-1}}dx =$

1) $\frac{1}{\log 3}\log\left|3^x+\sqrt{9^x-1}\right|+c$

2) $\frac{1}{\log 3}\log\left|3^x-\sqrt{9^x-1}\right|+c$

3) $\frac{1}{\log 9}\log\left|3^x-\sqrt{9^x-1}\right|+c$

4) $\frac{1}{\log 9}\log\left|9^x+\sqrt{9^x-1}\right|+c$

03. $\int \frac{1}{9-x^2}dx =$

1) $\frac{1}{6}\log\left|\frac{3-x}{3+x}\right|+c$ 2) $\frac{1}{3}\log\left|\frac{3+x}{3-x}\right|+c$

3) $\frac{1}{6}\log\left|\frac{3+x}{3-x}\right|+c$ 4) $\frac{1}{3}\log\left|\frac{3-x}{3+x}\right|+c$

04. $\int \frac{dx}{x^2+2x+2} = f(x)+c$ then $f(x)=$

1) $\tan^{-1}(x+1)$ 2) $2\tan^{-1}(x+1)$

3) $-\tan^{-1}(x+1)$ 4) $3\tan^{-1}(x+1)$

05. $\int \frac{1}{9x^2+16}dx =$

1) $\tan^{-1}\left(\frac{3x}{4}\right)+c$ 2) $\frac{1}{12}\tan^{-1}\left(\frac{3x}{4}\right)+c$

3) $\frac{1}{2}\tan^{-1}\left(\frac{3x}{4}\right)+c$ 4) $\frac{1}{4}\tan^{-1}\left(\frac{3x}{4}\right)+c$

06. $\int \frac{\sin x}{\sqrt{3+\sin^2 x}}dx =$

1) $\sin^{-1}\left(\frac{\cos x}{2}\right)+c$ 2) $\frac{1}{2}\sin^{-1}\left(\frac{\cos x}{2}\right)+c$

3) $\frac{1}{2}\sin^{-1}\left(\frac{\cos x}{2}\right)+c$ 4) $-\sin^{-1}\left(\frac{\cos x}{2}\right)+c$

07. $\int \sqrt{x^2+4}\,dx =$

1) $\frac{x}{2}\sqrt{x^2+4}+2\sin h^{-1}\left(\frac{x}{2}\right)+c$

2) $\frac{x}{4}\sqrt{x^2+4}+2\sin h^{-1}\left(\frac{x}{4}\right)+c$

3) $\frac{x}{2}\sqrt{x^2+4}-2\sin h^{-1}\left(\frac{x}{2}\right)+c$

4) $\frac{x}{2}\sqrt{x^2+4}+2\sin h^{-1}\left(\frac{x}{4}\right)+c$

08. $\int \sqrt{x^2-25}\,dx =$

1) $\frac{x}{2}\sqrt{x^2-25}-\frac{25}{2}\cos h^{-1}\left(\frac{x}{5}\right)+c$

2) $\frac{x}{2}\sqrt{x^2-25}+\frac{25}{2}\sin h^{-1}\left(\frac{x}{5}\right)+c$

3) $\frac{x}{3}\sqrt{x^2-25}+\frac{25}{2}\cos h^{-1}\left(\frac{x}{5}\right)+c$

4) $\frac{x}{3}\sqrt{x^2-25}-\frac{25}{2}\cos h^{-1}\left(\frac{x}{5}\right)+c$

09. $\int \frac{1}{\sqrt{9x^2-25}}dx =$

1) $\frac{1}{2}\cos h^{-1}\left(\frac{5x}{3}\right)+c$ 2) $\frac{1}{3}\cos h^{-1}\left(\frac{3x}{5}\right)+c$

3) $\frac{1}{2}\cos h^{-1}\left(\frac{3x}{5}\right)+c$ 4) $\frac{1}{15}\cos h^{-1}\left(\frac{3x}{5}\right)+c$

10. $\int \frac{3^x}{\sqrt{25-9^x}}dx =$

1) $\frac{1}{\log 3}\sin^{-1}\left(\frac{3^x}{5}\right)+c$ 2) $\frac{1}{\log 3}\cos h^{-1}(3^x)+c$

3) $\frac{1}{\log 3}\sin^{-1}\left(\frac{3^x}{2}\right)+c$ 4) $\frac{1}{\log 3}\sin^{-1}\left(\frac{3^x}{\sqrt{5}}\right)+c$

11. $\int \frac{3^x}{\sqrt{16+9^x}}dx =$

1) $\frac{1}{\log 3}\sin^{-1}\left(\frac{3^x}{4}\right)+c$ 2) $\frac{1}{\log 3}\cosh^{-1}\left(\frac{3^x}{4}\right)+c$

3) $\frac{1}{\log 3}\sin^{-1}\left(\frac{3^x}{2}\right)+c$ 4) $\frac{1}{\log 3}\sin^{-1}\left(\frac{3^x}{4}\right)+c$

12. $\int \frac{1}{\sqrt{x^2-4x-5}}dx =$

1) $\cos h^{-1}\left(\frac{x+2}{3}\right)+c$ 2) $\sin h^{-1}\left(\frac{x-2}{3}\right)+c$

3) $\cos h^{-1}\left(\frac{x-2}{3}\right)+c$ 4) $\sin h^{-1}\left(\frac{x+2}{3}\right)+c$

13. $\int \frac{1}{9+x^2}dx =$

1) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+c$ 2) $\tan^{-1}\left(\frac{x}{3}\right)+c$

3) $\frac{2}{3}\tan^{-1}\left(\frac{x}{3}\right)+c$ 4) $\frac{1}{6}\tan^{-1}\left(\frac{x}{3}\right)+c$

14. If $\int \frac{2x}{1-4^x}dx = K\ \log\left|\frac{1+2^x}{1-2^x}\right|+c$ then K =

1) $\log 2$ 2) $\log 4$

3) $\frac{1}{\log 2}$ 4) $\frac{1}{\log 4}$

15. $\int \frac{1}{1+16x^2}dx =$

1) $\frac{1}{4}\tan^{-1}(4x)+c$ 2) $\frac{1}{2}\tan^{-1}(2x)+c$

3) $\frac{1}{2}\tan^{-1}(4x)+c$ 4) $\tan^{-1}(4x)+c$

16. $\int \frac{x^2-a^2}{x^2+a^2}dx =$

1) $x+2a\tan^{-1}(x/a)+c$

2) $2x-a\tan^{-1}(x/a)+c$

3) $x-2a\tan^{-1}(x/a)+c$

4) $x+1/2a\tan^{-1}(x/a)+c$

17. $\int \frac{1}{2x^2-3}dx =$

1) $\frac{1}{\sqrt{6}}\log\left|\frac{\sqrt{2}x-\sqrt{3}}{\sqrt{2}x+\sqrt{3}}\right|+c$

2) $\frac{1}{2\sqrt{6}}\log\left|\frac{\sqrt{2}x-\sqrt{3}}{\sqrt{2}x+\sqrt{3}}\right|+c$

3) $\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{2}x-\sqrt{3}}{\sqrt{2}x+\sqrt{3}}\right|+c$

4) $\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}x-\sqrt{3}}{\sqrt{2}x+\sqrt{3}}\right|+c$

18. $\int \sqrt{4-x^2}\,dx =$

1) $\frac{x}{2}\sqrt{4-x^2}-2\sin^{-1}\left(\frac{x}{2}\right)+c$

2) $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\left(\frac{x}{2}\right)+c$

3) $\frac{x}{2}\sqrt{4-x^2}+2\sin h^{-1}\left(\frac{x}{2}\right)+c$

4) $\frac{x}{2}\sqrt{4+x^2}+2\sin^{-1}\left(\frac{x}{2}\right)+c$

19. $\int \frac{dx}{5+4\cos x} =$

1) $\frac{3}{2}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$ 2) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$

3) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$ 4) $3\log\left(\frac{3-\tan x/2}{3+\tan x/2}\right)$

20. $\int \frac{dx}{4+5\cos x} =$

1) $\frac{3}{2}\tan^{-1}\frac{1}{3}\left(\tan\frac{x}{2}\right)$ 2) $\frac{2}{3}\log\left(\frac{3+\tan x/2}{3-\tan x/2}\right)$

3) $\frac{1}{3}\log\left(\frac{3-\tan x/2}{3+\tan x/2}\right)$

4) $\frac{1}{3}\log\left(\frac{3+\tan x/2}{3-\tan x/2}\right)$

21. $\int \frac{dx}{5+4\sin x} =$

1) $\frac{2}{3}\tan^{-1}\left(\frac{4\cos\frac{x}{2}+5\sin\frac{x}{2}}{3\cos\frac{x}{2}}\right)$

2) $\frac{2}{3}\tan^{-1}\left(\frac{5+4\tan\frac{x}{2}}{3}\right)$

3) $\frac{2}{3}\cot^{-1}\left(\frac{4+5\tan\frac{x}{2}}{3}\right)$ 4) none of these

22. $\int \frac{dx}{(x+3)(x-3)} =$

1) $\frac{1}{3}\log\left(\frac{x+3}{x-3}\right)$ 2) $\frac{1}{6}\log(3x)$

3) $\frac{1}{6}\log\left(\frac{x-3}{x}\right)$ 4) $\frac{1}{6}\log\left(\frac{x-3}{x+3}\right)$

23. $\int \frac{dx}{\sqrt{4x+2+x^2}} =$

1) $2\sqrt{x^2+4x+2}$ 2) $\cosh^{-1}(x+2)$

3) $\cos h^{-1}\left(\frac{x+2}{2}\right)$ 4) $\cos h^{-1}\left(\frac{x+2}{\sqrt{2}}\right)$

24. $\int \frac{dx}{\sqrt{2x^2+7x+3}} =$

1) $\cos h^{-1}\left(\frac{4x+3}{5}\right)$ 2) $\sin h^{-1}\left(\frac{4x+7}{5}\right)$

3) $\frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{4x+7}{5}\right)$

4) $\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{4x+7}{5}\right)$

25. $\int \frac{dx}{\sqrt{2x^2+3x+4}} =$

1) $\sin h^{-1}\left(\frac{4x+3}{\sqrt{23}}\right)$ 2) $\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{4x+3}{\sqrt{23}}\right)$

3) $\frac{1}{\sqrt{2}}\cos h^{-1}\left(\frac{4x+7}{5}\right)$

4) $\frac{1}{\sqrt{2}}\sin h^{-1}\left(\frac{4x+7}{5}\right)$

26. $\int \frac{dx}{\sqrt{2x^2+3x+4}} =$

1) $\sin h^{-1}\left(\frac{4x+3}{\sqrt{23}}\right)$ 2) $\frac{1}{\sqrt{2}}\sin h^{-1}\left(\frac{4x+3}{\sqrt{23}}\right)$

3) $\frac{1}{\sqrt{2}}\cos h^{-1}\left(\frac{4x+3}{\sqrt{23}}\right)$ 4) $\cos h^{-1}\left(\frac{4x+3}{\sqrt{23}}\right)$

27. $\int \frac{dx}{\sqrt{4+3x-2x^2}} =$

1) $\sin^{-1}\left(\frac{4x-3}{\sqrt{41}}\right)$ 2) $\frac{1}{\sqrt{2}}\sin h^{-1}\left(\frac{4x-3}{\sqrt{41}}\right)$

3) $\sqrt{2}\sin^{-1}\left(\frac{4x-3}{\sqrt{41}}\right)$ 4) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4x-3}{\sqrt{41}}\right)$

28. $\int \frac{x^2+1}{x^4+1}dx =$

1) $\frac{1}{\sqrt{2}}\tan^{-1}(x^2+1)$ 2) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)$

3) $\frac{1}{\sqrt{2}}\tan^{-1}(x^2-1)$ 3) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$

29. $\int \frac{dx}{\sin x+\cos x} =$

1) $\frac{1}{\sqrt{2}}\log\tan\left(x+\frac{\pi}{4}\right)$

2) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{2}+\frac{\pi}{8}\right)$ 3) $\log\tan\left(\frac{\pi}{2}+\frac{\pi}{8}\right)$

4) $\frac{1}{\sqrt{2}}\log\tan\frac{\sqrt{2}-1+\tan\frac{x}{2}}{\sqrt{2+1-\tan\frac{x}{2}}}$

30. The value of $\int \frac{x^2+1}{x^2-1}dx =$

1) $\log\left(\frac{x+1}{x-1}\right)+c$ 2) $\log\left(\frac{x-1}{x+1}\right)+c$

3) $\log(x^2-1)+c$ 4) $x+\log\left(\frac{x-1}{x+1}\right)+c$

31. $\int \frac{dx}{(x^2+4)(x^2+9)}$

1) $\frac{-1}{5}\left(\frac{1}{2}\tan{x/2}-\frac{1}{3}\tan^{-1}\frac{x}{3}\right)$

2) $\frac{1}{5}\left(\frac{1}{2}\tan^{-1}\frac{x}{2}-\frac{1}{3}\tan^{-1}\frac{x}{3}\right)$

3) $\tan^{-1}\frac{x}{2}-\tan^{-1}\frac{x}{3}$ 4) $\log\left(\frac{x^2+4}{x^2+9}\right)$

32. $\int \frac{x\,dx}{(x^2+9)(x^2+16)}$

1) $\frac{1}{7}\left(\frac{1}{3}\tan^{-1}\frac{x}{3}-\frac{1}{4}\tan^{-1}\frac{x}{4}\right)$

2) $\frac{1}{14}\log\left(\frac{x^2+16}{x^2+9}\right)$

3) $\frac{1}{14}\log\left((x^2+9)(x^2+16)\right)$

4) $\frac{1}{14}\log\left(\frac{x^2+9}{x^2+16}\right)$

33. If $\int \frac{\sin x}{\cos x(1+\cos x)}dx = f(x)+c \Rightarrow f(x)=$

1) $\log\left|\frac{1+\cos x}{\cos x}\right|$ 2) $\log\left|\frac{\cos x}{1+\cos x}\right|$

3) $\log\left|\frac{\sin x}{1+\sin x}\right|$ 4) $\log\left|\frac{1+\sin x}{\sin x}\right|$

34. $\int \frac{1}{(x+1)(x+2)}dx$

1) $\log\left(\frac{x+1}{x+2}\right)+c$ 2) $\frac{1}{2}\log\left(\frac{x+1}{x+2}\right)+c$

3) $\frac{1}{3}\log\left|\frac{2x-3}{3x-2}\right|+c$ 4) $\frac{2}{3}\left[\tan^{-1}\frac{-3}{2}\tan^{-1}\frac{x}{2}\right]+c$

35. $\int \frac{1}{(x-1)(x-2)}dx$

1) $\log\left(\frac{x-1}{x+2}\right)+c$ 2) $\log\left(\frac{x-2}{x-1}\right)+c$

3) $\log\left(\frac{x-2}{x+1}\right)+c$ 4) None

36. $\int \frac{1}{(2x-1)(3x+2)}dx$

1) $\log\left(\frac{2x-1}{3x+2}\right)+c$ 2) $\frac{1}{7}\log\left(\frac{2x-1}{3x+2}\right)+c$

3) $\frac{1}{7}\log\left(\frac{3x+2}{2x-1}\right)+c$ 4) none

37. $\int \frac{2x-5}{(x-1)(x+3)}dx$

1) $\frac{-3}{4}\log|x-1|+\frac{11}{4}\log|x+3|+c$

2) $\frac{-3}{4}\log|x-1|-\frac{11}{4}\log|x+3|+c$

3) $\frac{-3}{4}\log|x+1|+\frac{11}{4}\log|x+3|+c$

4) none

38. $\int \frac{\cos x}{\sin x(1+\sin x)}dx$

1) $\log\left|\frac{\sin x}{1+\sin x}\right|+c$ 2) $\log\left|\frac{\sin x}{1-\sin x}\right|+c$

3) $\log\left|\frac{1+\sin x}{\sin x}\right|+c$ 4) none

39. $\int \frac{e^x}{e^{2x}+4e^x+3}dx$

1) $\log\left|\frac{e^x+1}{e^x+3}\right|+c$ 2) $\log\left|\frac{e^x+3}{e^x+1}\right|+c$

3) $\log\left|\frac{e^x-1}{e^x-3}\right|+c$ 4) $\frac{1}{2}\log\left|\frac{e^x+1}{e^x+3}\right|+c$

40. $\int \frac{\sec^2 x}{(2\tan x+3)(4\tan x+1)}dx$

1) $\frac{1}{10}\log\frac{4\tan x+1}{2\tan x+4}$

2) $\frac{1}{5}\log\frac{4\tan x+1}{2\tan x+3}+c$

3) $\frac{1}{12}\log\frac{4\tan x+1}{2\tan x+3}+c$ 4) none

41. $\int \frac{1}{(x-1)(x+3)(x+5)}dx$

1) $\frac{1}{24}\log(x-1)+\frac{1}{8}(x+3)+\frac{1}{12}\log(x+5)+c$

2) $\frac{1}{24}\log(x-1)-\frac{1}{8}(x+3)+\frac{1}{12}\log(x+5)+c$

3) $\frac{1}{8}\log(x-1)+\frac{1}{24}(x+3)-\frac{1}{12}\log(x+5)+c$

4) none

42. $\int \frac{1}{(x^2+a^2)(x^2+b^2)}dx$

1) $\frac{1}{a^2-b^2}\left[\frac{1}{b}\tan^{-1}\frac{x}{b}-\frac{1}{a}\tan^{-1}\frac{x}{a}\right]+c$

2) $\frac{1}{a^2-b^2}\left[\tan^{-1}\frac{x}{b}-\frac{1}{a}\tan^{-1}\frac{x}{a}\right]+c$

3) $\frac{1}{a^2-b^2}\log\left(\frac{x^2+a^2}{x^2+b^2}\right)+c$ 4) none

43. $\int \frac{1}{(x^2+3)(x^2+5)}dx$

1) $\frac{1}{2}\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{3}+\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{5}\right]+c$

2) $\frac{1}{2}\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{5}\right]+c$

3) $\frac{1}{2}\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}+\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}}\right]+c$

4) none

44. $\int \frac{1}{(x^2+7)(x^2+3)}dx=$

1) $\frac{1}{4\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-\frac{1}{4\sqrt{7}}\tan^{-1}\frac{x}{\sqrt{7}}+c$

2) $\frac{1}{12}\tan^{-1}\frac{x}{3}-\frac{1}{28}\tan^{-1}\frac{x}{7}+c$

3) $\frac{1}{12}\tan^{-1}\frac{x}{3}-\frac{1}{28}\tan^{-1}\frac{x}{7}+c$ 4) none

45. $\int \frac{x}{(x^2+3)(x^2+5)}dx$

1) $\log\left(\frac{x^2+4}{x^2+5}\right)+c$ 2) $\frac{1}{2}\log\left(\frac{x^2+3}{x^2+5}\right)+c$

3) $\frac{-1}{4}\log\frac{x^2+5}{x^2+3}+c$ 4) none

46. $\int \frac{x}{(x^2+4)(x^2+5)}dx=$

1) $\log\left(\frac{x^2+4}{x^2+5}\right)+c$ 2) $\frac{1}{2}\log\left(\frac{x^2+4}{x^2+5}\right)+c$

3) $\frac{1}{2}\tan^{-1}\frac{x}{2}+\frac{1}{5}\tan^{-1}\frac{x}{5}+c$ 4) none

47. $\int \frac{1}{x(x^n+1)}dx$

1) $\log\left|\frac{x^n}{x^n+1}\right|+c$ 2) $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+c$

3) $\log\left|\frac{x^n+1}{x^n}\right|+c$ 4) $\frac{1}{n}\log\left|\frac{x^n+1}{x^n}\right|+c$

48. $\int \frac{1}{x(x^4+1)}dx$

1) $\frac{1}{3}\log\left|\frac{x^4}{x^4+1}\right|+c$ 2) $\frac{1}{4}\log\left|\frac{x^4}{x^4+1}\right|+c$

3) $\frac{1}{4}\log\left|\frac{x^4+1}{x^4}\right|+c$ 4) $\frac{-1}{4}\log\left|\frac{x^4}{x^4+1}\right|+c$

49. $\int \frac{1}{x(x^n-1)}dx$

1) $\frac{1}{n}\log\left|\frac{x^n}{x^n-1}\right|+c$ 2) $\frac{-1}{n}\log\left|\frac{x^n}{x^n-1}\right|+c$

3) $\frac{1}{n}\log\left|\frac{x^n-1}{x^n}\right|+c$ 4) $\frac{-1}{n}\log\left|\frac{x^n-1}{x^n}\right|+c$

50. $\int \frac{1}{x(x^5-1)}dx$

1) $\frac{1}{5}\log\left|\frac{x^5}{x^5-1}\right|+c$ 2) $\frac{-1}{5}\log\left|\frac{x^5}{x^5-1}\right|+c$

3) $\frac{1}{5}\log\left|\frac{x^5-1}{x^5}\right|+c$ 4) $\frac{-1}{5}\log\left|\frac{x^5-1}{x^5}\right|+c$

51. $\int \frac{1}{x(1-x^4)}dx$

1) $\frac{1}{4}\log\left|\frac{1-x^4}{x^4}\right|+c$ 2) $\frac{-1}{4}\log\left|\frac{1-x^4}{x^4}\right|+c$

3) $\frac{1}{4}\log\left|\frac{x^4}{1-x^4}\right|+c$ 4) $\frac{-1}{4}\log\left|\frac{x^4}{1-x^4}\right|+c$

52. $\int \frac{\cos x}{(3+\sin x)(4+\sin x)}dx=$

1) $\frac{3}{4}\log\left|\frac{\sin x+3}{\sin x+4}\right|+c$ 2) $\frac{4}{3}\log\left|\frac{3+\sin x}{4+\sin x}\right|+c$

3) $\log\left|\frac{3+\sin x}{4+\sin x}\right|+c$ 4) $\log\left|\frac{3-\sin x}{4-\sin x}\right|+c$

53. $\int \frac{ax+b}{cx+d}dx$

1) $\frac{a}{c}x-\frac{(ad+bc)}{c^2}\log(x+d)+c$

2) $\frac{a}{c}x-\frac{(ad-bc)}{c^2}\log(cx+d)+c$

3) $\frac{a}{c}x+\frac{ad-bc}{c^2}\log(cx+d)+c$ 4) none

54. $\int \frac{x^3}{1+x}dx$

1) $\frac{x^3}{3}-\frac{x^2}{2}+x-\log(1+x)+c$

2) $\frac{x^3}{3}+\frac{x^2}{2}-x-\log(1+x)+c$

3) $\frac{x^3}{3}-\frac{x^2}{2}-x-\log(1+x)+c$ 4) none

55. $\int \frac{x^4}{1+x}dx$

1) $\frac{x^4}{4}+\frac{x^3}{3}-\frac{x^2}{2}-x+\log(1+x)+c$

2) $\frac{x^4}{4}-\frac{x^3}{3}+\frac{x^2}{2}-x+\log(1+x)+c$

3) $\frac{x^4}{4}-\frac{x^3}{2}-\frac{x^2}{2}-x+\log(1+x)+c$ 4) none

56. $\int \frac{x^5}{1+x}dx$

1) $\frac{x^5}{5}-\frac{x^4}{4}+\frac{x^3}{3}-\frac{x^2}{2}+x-\log(1+x)+c$

2) $\frac{x^5}{5}-\frac{x^3}{3}+x-\log(1+x)+c$

3) $\frac{x^5}{5}-\frac{x^4}{4}+x-\log(1+x)+c$ 4) none

57. $\int \frac{\sec^2 x}{\tan x(1+\tan x)}dx$

1) $\log\left|\frac{1-\tan x}{\tan x}\right|+c$ 2) $\log\left|\frac{\tan x}{1+\tan x}\right|+c$

3) $\frac{1}{2}\log\left|\frac{\tan x}{1+\tan x}\right|+c$ 4) $\frac{1}{2}\log\left|\frac{\tan x}{1-\tan x}\right|+c$

58. $\int \frac{x^5}{x^2+1}dx$

1) $\frac{x^4}{4}+\frac{x^2}{2}+\tan^{-1}x+c$

2) $\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2}\log(x^2+1)+c$

3) $\frac{x^4}{4}+\frac{x^3}{3}+\tan^{-1}x+c$ 4) none

59. $\int xe^x\,dx$

1) xe^x+c 2) xe^x+x+c

3) xe^x-e^x+c 4) none

60. $\int x\sin x\,dx$

1) $x\cos x+\sin x+c$ 2) $-x\cos x+\sin x+c$

3) $x\cos x+\sin x-x+c$ 4) $x\cos x-\sin x+c$

61. $\int x\sec x\tan x\,dx$

1) $x\sec x+\log|\sec x+\tan x|+c$

2) $x\sec x+\log|\sec x-\tan x|+c$

3) $x\sec x-\log|\sec x-\tan x|+c$

4) $x\sec x-\log|\sec x+\tan x|+c$

62. $\int x\tan^2 x\,dx$

1) $x\tan x+\log|\cos x|-\frac{x^2}{2}+c$

2) $x\tan x-\log|\cos x|-\frac{x^2}{2}+c$

3) $x\cot x+\log|\sec x|+c$

4) $x\tan x+\log|\sin x|-\frac{x^2}{2}+c$

63. $\int x^2 e^{2x}\,dx$

1) $\frac{x^2e^{2x}}{2}+\frac{xe^{2x}}{2}+\frac{e^{2x}}{4}+c$

2) $\frac{x^2e^{2x}}{2}+\frac{xe^{2x}}{2}-\frac{e^{2x}}{2}+c$

3) $\frac{x^2e^{2x}}{2}-\frac{xe^{2x}}{2}+\frac{e^{2x}}{4}+c$ 4) None

64. $\int x^2\cos x\,dx$

1) $x^2\sin x+2x\cos x-2\sin x+c$

2) $x^2\sin x-2x\cos x+2\sin x+c$

3) $x^2\sin x+2x\cos x+2\sin x+c$

4) $x^2\sin x+(2x+1)\cos x+c$

65. $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

1) $e^x\tan x+c$ 2) $e^x\cot x+c$

3) $e^x\tan x/2+c$ 4) none

66. $\int \log x\,dx$

1) $x\log_{10}x+x+c$ 2) $x\log_{10}x-x+c$

3) $\log_{10}e(x\log x-x)+c$

4) $x\log x+x+c$

67. $\int (\log x)^2\,dx$

1) $\frac{x}{2}\left[(\log x)^2-2\log x+2\right]+c$

2) $x\left[(\log x)^2-2\log x+2\right]+c$

3) $x\left[(\log x)^2+2\log x+2\right]+c$

4) None

68. $\int \sin\sqrt{x}\,dx =$

1) $-\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}+c$

2) $\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}+c$

3) $2\left[-\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}\right]+c$ 4) none

69. $\int e^{\sqrt{x}}\,dx$

1) $e^{\sqrt{x}}\left(\sqrt{x}+1\right)+c$ 2) $2e^{\sqrt{x}}\left(\sqrt{x}-1\right)+c$

3) $e^{\sqrt{x}}+c$ 4) none

70. $\int x\tan^{-1}x\,dx$

1) $\frac{x^2+1}{2}\tan^{-1}x+c$ 2) $\tan^{-1}x-x+c$

3) $\frac{x^2+1}{2}\tan^{-1}x-\frac{1}{2}x+c$ 4) none

71. $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

1) $\sqrt{1-x^2}\sin^{-1}x+c$ 2) $-\sqrt{1-x^2}\sin^{-1}x+x+c$

3) $\sqrt{1-x^2}\cos^{-1}x+x+c$ 4) none

72. $\int \sin^{-1}\left(3x-4x^3\right)dx$

1) $3x\sin^{-1}x+3\sqrt{1-x^2}+c$

2) $3\sin^{-1}x+\sqrt{1-x^2}+c$

3) $3\left[x\sin^{-1}x-3\sqrt{1-x^2}+c\right]$

4) none

73. $\int \cos^{-1}\left(2x^2-1\right)dx$

1) $2x\cos^{-1}x-2\sqrt{1-x^2}+c$

2) $2x\cos^{-1}x+2\sqrt{1-x^2}+c$

3) $2x\cos^{-1}-2\sqrt{x^2-1}+c$

4) $2x\cos^{-1}x+2\sqrt{x^2-1}+c$

74. $\int \cos^{-1}\left(4x^3-3x\right)dx$

1) $3x\cos^{-1}x+3\sqrt{1-x^2}+c$

2) $3x\cos^{-1}x-3\sqrt{1-x^2}+c$

3) $3x\sin^{-1}x+3\sqrt{1-x^2}+c$

4) $3x\cos^{-1}x+3\sqrt{x^2-1}+c$

75. $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

1) $2x\tan^{-1}x+\log\left(1+x^2\right)+c$

2) $2x\tan^{-1}x-\log\left(1+x^2\right)+c$

3) $x\tan^{-1}x+\log\left(1+x^2\right)+c$

4) $x\tan^{-1}x+\log\left(1-x^2\right)+c$

76. $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

1) $3x\tan^{-1}x+\frac{3}{2}\log\left(1+x^2\right)$

2) $3x\tan^{-1}x-\frac{3}{2}\log\left(1+x^2\right)$

3) $3x\tan^{-1}x+\frac{3}{2}\log\left(1-x^2\right)$

4) $x\tan^{-1}x-\frac{3}{2}\log\left(1+x^2\right)$

77. $\int e^x\left(\frac{\cos x+\sin x}{1+\cos 2x}\right)dx$

1) $e^x\tan x+c$ 2) $e^x\cot x+c$

3) $e^x\sin x+c$ 4) $e^x\frac{\sec x}{2}+c$

78. $\int \sin h^{-1}x\,ds$

1) $x\sin h^{-1}x+\sqrt{1+x^2}+c$

2) $x\sin h^{-1}x-\sqrt{1+x^2}+c$

3) $x\cos h^{-1}x-\sqrt{1+x^2}+c$

4) $x\tan^{-1}x-\frac{3}{2}\log\left(1+x^2\right)$

79. $\int e^{-x}\left(\cos x+\sin x\right)dx$

1) $e^{-x}\cos x+c$ 2) $e^{-x}\sin x+c$

3) $-e^{-x}\cos x+c$ 4) none

80. $\int \tan h^{-1}x\,dx$

1) $x\tanh^{-1}x+\frac{1}{2}\log\left(1-x^2\right)$

2) $x\tan h^{-1}x-\frac{1}{2}\log\left(1-x^2\right)$

3) $\tan h^{-1}x+\frac{1}{2}x\log\left(1-x^2\right)$

4) $\tanh^{-1}x+\frac{1}{2}x\log\left(1+x^2\right)$

81. $\int x\sin^{-1}x\,dx$

1) $\frac{\left(2x^2-1\right)}{4}\sin^{-1}x+\frac{x}{4}\sqrt{1-x^2}+c$

2) $\frac{\left(2x^2-1\right)}{4}\sin^{-1}x+\frac{x}{4}\sqrt{1+x^2}+c$

3) $\frac{\left(2x^2-1\right)}{4}\cos^{-1}x+\frac{x}{4}\sqrt{1-x^2}+c$

4) $\frac{\left(2x^2+1\right)}{4}\sin^{-1}x-\frac{x}{4}\sqrt{1+x^2}+c$

82. $\int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx$

1) $e^x\tan x+c$ 2) $e^x\cos 2x+c$

3) $e^x\sin 2x+c$ 4) none

83. $\int e^x\sin x\,dx =$

1) $\frac{e^x}{2}\left[\cos x-\sin x\right]+c$ 2) $\frac{e^x}{2}\sin x+c$

3) $\frac{e^x}{2}\left[\sin x-\cos x\right]+c$ 4) none

84. $\int e^{-x}\cos x\,dx$

1) $\frac{e^{-x}}{2}\left(\sin x-\cos x\right)+c$ 2) $\frac{e^{-x}}{2}\sin x+c$

3) $\frac{e^{-x}}{2}\left(\sin x+\cos x\right)+c$ 4) none

85. $\int e^x\left(\tan x+\sec^2 x\right)dx =$

1) $e^x\sec^2 x+c$ 2) $e^x\sec x+c$

3) $-e^x\sec x+c$ 4) $e^x\tan x+c$

86. $\int e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx =$

1) $e^x\operatorname{cosec}^{-1}x+c$ 2) $e^x\tan^{-1}x+c$

3) $e^x\sin^{-1}x+c$ 4) $-e^x\tan^{-1}x+c$

87. $\int e^x\left(\frac{1}{x\sqrt{x^2-1}}+\sec^{-1}x\right)dx =$

1) $e^x\operatorname{cosec}^{-1}x+c$ 2) $e^x\sec^{-1}x+c$

3) $e^x\sin^{-1}x+c$ 4) $e^x\cos^{-1}x+c$

88. $\int e^x\sec x\left(1+\tan x\right)dx$

1) $e^x\tan x+c$ 2) $e^x\sec x+c$

3) $-e^x\tan x+c$ 4) none

89. $\int e^x\left(\tan x+\tan^2 x\right)dx$

1) $e^x\tan x+c$ 2) $e^x\left(\tan x-1\right)+c$

3) $e^x\sec x+c$ 4) none

90. $\int e^{-x}\left(\sec^2 x-\tan x\right)dx$

1) $e^{-x}\sec x+c$ 2) $-e^{-x}\sec x+c$

3) $e^{-x}\tan x+c$ 4 none

PRACTICE SET-II KEY

01) 1	02) 1	03) 3	04) 1	05) 2
06) 4	07) 1	08) 1	09) 2	10) 1
11) 4	12) 3	13) 1	14) 4	15) 1
16) 3	17) 2	18) 2	19) 3	20)
21) 1	22) 4	23) 4	24) 3	25) 4
26) 2	27) 4	28) 3	29) 4	30) 4
31) 2	32) 4	33) 1	34) 1	35) 2
36) 2	37) 1	38) 1	39) 4	40) 1
41) 2	42) 1	43) 2	44) 1	45) 3
46) 2	47) 2	48) 2	49) 3	50) 3
51) 3	52) 3	53) 2	54) 1	55) 2
56) 1	57) 2	58) 2	59) 3	60) 2
61) 4	62) 1	63) 3	64) 1	65) 3
66) 2	67) 2	68) 3	69) 2	70) 3
71) 2	72) 1	73) 1	74) 2	75) 2
76) 2	77) 4	78) 2	79) 3	80) 1
81) 1	82) 1	83) 3	84) 1	85) 4
86) 2	87) 2	88) 2	89) 2	90) 3

SELF TEST

01. $\int e^x \operatorname{cosec} x(1-\cot x)dx=$

1) $-e^x \cot x+c$ 2) $e^x \cot x+c$

3) $-e^x \operatorname{cosec} x+c$ 4) $e^x \operatorname{cosec} x+c$

02. $\int \sqrt{x} e^{\sqrt{x}} dx=$

1) $2e^{\sqrt{x}}(x-2\sqrt{x}+2)+c$

2) $2e^{\sqrt{x}}(x+2\sqrt{x}+2)+c$

3) $2e^{\sqrt{x}}(x-2\sqrt{x}-2)+c$ 4) $2e^{\sqrt{x}}+c$

03. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx=$

1) $-2\sin\sqrt{x}+c$ 2) $-2\cos\sqrt{x}+c$

3) $2\sin\sqrt{x}+c$ 4) $2\cos\sqrt{x}+c$

04. $\int \frac{dx}{e^x+e^{-x}}=$

1) $Tanh^{-1}e^x+c$ 2) $Tan^{-1}e^x+c$

3) $-Tanh^{-1}e^x+c$ 4) $-Tan^{-1}e^x+c$

05. $\int \frac{dx}{\sqrt{x}(x+9)}=$

1) $Tan^{-1}\left(\frac{\sqrt{x}}{3}+c\right)$ 2) $\frac{2}{3}Tan^{-1}(\sqrt{x})+c$

3) $\frac{2}{3}Tan^{-1}\left(\frac{\sqrt{x}}{3}\right)+c$ 4) $Tan^{-1}\sqrt{x}+c$

06. $\int \frac{x^5}{x^2+1}dx=$

1) $\frac{x^4}{4}+\frac{x^2}{2}+Tan^{-1}x+c$

2) $\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2}\log(1+x^2)+c$

3) $\frac{x^4}{4}+\frac{x^3}{3}+\frac{1}{2}Tan^{-1}x+c$

4) $\frac{x^4}{4}+\frac{x^3}{3}-Tan^{-1}x+c$

07. $\int \frac{e^x}{x}(1+x\log x)dx=$

1) $\frac{e^x \log x}{x}+c$ 2) $e^x(1+\log x)+c$

3) $e^x \log x+c$ 4) $xe^x \log x+c$

08. $\int \frac{dx}{x(1+\log x)^3}=$

1) $\frac{1}{2(1+\log x)^2}+c$ 2) $\frac{1}{(1+\log x)}+c$

3) $\frac{-1}{2(1+\log x)^2}+c$ 4) $\frac{1}{3(1+\log x)^3}+c$

09. $\int \frac{3}{2x^2-x-1}dx=$

1) $\log\left|\frac{x-1}{x+1}\right|+c$ 2) $\log\left|\frac{x+1}{2x+1}\right|+c$

3) $\log\left|\frac{x-1}{2x-1}\right|+c$ 4) $\log\left|\frac{x-1}{2x+1}\right|+c$

10. $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx=$

1) $\sin x-\cos x+c$ 2) $\sin x+\cos x+c$

3) $\sec x-\operatorname{cosec} x+c$ 4) $\sec x+\operatorname{cosec} x+c$

11. $\int e^x(1-\cot x+\cot^2 x)dx=$

1) $e^x \cot x+c$ 2) $-e^x \cot x+c$

3) $e^x \operatorname{cosec} x+c$ 4) $-e^x \operatorname{cosec} x+c$

12. $\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}=$

1) $Tan^{-1}\left(\frac{a\tan x}{b}\right)+c$ 2) $Tan^{-1}\left(\frac{b\tan x}{a}\right)+c$

3) $\frac{1}{ab}Tan^{-1}\left(\frac{a\tan x}{b}\right)+c$

4) $\frac{-1}{ab}Tan^{-1}\left(\frac{b\tan x}{a}\right)+c$

13. $\int \frac{dx}{1-\sin x-\cos x}=$

1) $\log\left|1+Tan\frac{x}{2}\right|+c$ 2) $\log\left|1-Tan\frac{x}{2}\right|+c$

3) $\log\left|1-\cot\frac{x}{2}\right|+c$ 4) $\log\left|1+\cot\frac{x}{2}\right|+c$

14. $\int \frac{dx}{(x+100)\sqrt{x+99}}=f(x)+c \Rightarrow f(x)=$

1) $2(x+100)^{1/2}$ 2) $3(x+100)^{1/2}$

3) $2\tan^{-1}\sqrt{x+99}$ 4) $2\tan^{-1}\sqrt{x+100}$

15. $\int \frac{\sin x}{\cos x(1+\cos x)}dx=f(x)+c \Rightarrow f(x)=$

1) $\log\left|\frac{1+\cos x}{\cos x}\right|$ 2) $\log\left|\frac{\cos x}{1+\cos x}\right|$

3) $\log\left|\frac{\sin x}{1+\sin x}\right|$ 4) $\log\left|\frac{1+\sin x}{\sin x}\right|$

16. $\int e^{-x}\left(\frac{1-\sin x}{1+\cos x}\right)dx$

1) $e^{-x}\cot x/2+c$ 2) $e^{-x}\tan x/2+c$

3) $e^{-x}\sec x/2+c$ 4) none

17. $\int e^{\sin x}[1+x\cos x]dx=$

1) $xe^{\sin x}+c$ 2) $\cos x, e^{\sin x}+c$

3) $\cos x, e^{\sin x}+c$ 4) none

18. $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$

1) $3x\tan^{-1}x+\frac{3}{2}\log(1+x^2)$

2) $3x\tan^{-1}x-\frac{3}{2}\log(1+x^2)$

3) $3x\tan^{-1}+\frac{3}{2}\log(1-x^2)$

4) $x\tan^{-1}x-\frac{3}{2}\log(1+x^2)$

19. $\int \cos h^{-1}x\,dx$

1) $x\cosh^{-1}x-\sqrt{1-x^2}+c$

2) $x\cos h^{-1}x+\sqrt{1-x^2}+c$

3) $x\cos h^{-1}x-\sqrt{x^2-1}+c$

4) $x\cosh^{-1}x+\sqrt{x^2-1}+c$

20. $\int x\log x\,dx$

1) $x\log x-\frac{x^2}{2}+c$ 2) $\frac{x^2}{2}\log x-\frac{x^2}{4}+c$

3) $x\log x+x+c$ 4) none

SELF TEST KEY

01) 4	02) 1	03) 3	04) 2	05) 3
06) 2	07) 3	08) 3	09) 3	10) 3
11) 2	12) 3	13) 3	14) 3	15) 1
16) 2	17) 1	18) 2	19) 3	20) 2

SPACE FOR IMPORTANT NOTES

SPACE FOR IMPORTANT NOTES

DEFINITE INTEGRALS

DEFINITIONS AND FORMULAE:

- Let f(x) be a function defined on [a, b]. If $\int f(x)dx = F(x)$, then F(b)-F(a) is called the definite integral of f(x) over [a, b]. It is denoted by $\int_a^b f(x)dx$. The real number a is called the lower limit and the real number b is called the upper limit.
- $\int f(x)dx = F(x)+c \Rightarrow \int_a^b f(x)dx = F(b)-F(a)$.
- $\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(y)dy = \dots\dots$
- If f(x) is an integrable function on [a, b] and g(x) is derivable on [a, b] then

$$\int_a^b (fog)(x)g^1(x)dx = \int_{g(a)}^{g(b)} f(x)dx$$

- $\int_a^b f(x)dx = -\int_b^a f(x)dx$.
- If a<c<b then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.
- $\int_{-a}^a f(x)dx = \int_0^a [f(x)+f(-x)]dx =$

$= 2\int_0^a f(x)\,dx$ if f is even

$= 0$ if f is odd

- $\int_0^{2a} f(x)dx = \int_0^a [f(x)+f(2a-x)]dx$

$= 2\int_0^a f(x)$ if $f(2a-x) = f(x)$

$= 0$ if $f(2a-x) = -f(x)$

- $\int_0^a f(x)dx = 2\int_0^{a/2} f(x)dx$ if f(a-x)=f(x).

$= 0$, if f(a-x)=-f(x).

- If f(x) is a periodic function with period a then

$\int_0^{na} f(x)dx = n\int_0^a f(x)dx$, where $n \in N$

- $\int_0^{\frac{\pi}{2}} \frac{f(\alpha)}{f(\alpha)+f(\beta)}dx = \frac{\pi}{4}$

$\alpha = \cos x$, $\beta = \sin x$

$\alpha = \tan x$, $\beta = \cot x$

$\alpha = \sec x$, $\beta = \operatorname{cosec} x$

WALLI'S FORMULAE

- If $I_n = \int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx$,

then $I_n = \frac{n-1}{n} I_{n-2}$, where $n \in Z+$

$\therefore I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)$

$\dots\dots\dots\dots \frac{1}{2}\cdot\frac{\pi}{2}$ in n is positive even integer.

$= \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\dots\dots\dots\left(\frac{2}{3}\right)1$ if n is odd positive integer.

- **INTEGRATION AS SUM OF LIMITS**

$$\int_0^1 f(x)dx = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r-1}{n}\right) = \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1} f\left(\frac{r}{n}\right)$$

STANDARD RESULTS

- $\int_0^{\pi/4}\log(1+\tan\theta)d\theta = \frac{\pi}{8}\log 2$
- $\int_0^{\pi/2}\log(\sin x)dx = \int_0^{\pi/2}\log(\cos x)dx = -\frac{\pi}{2}\log 2$
- $\int_0^{\pi/2}\frac{1}{a^2\sin^2 x + b^2\cos^2 x}dx = \frac{\pi}{2ab}$
- $\int_0^{\pi}\frac{x}{a^2\sin^2 x + b^2\cos^2 x}dx = \frac{\pi^2}{2ab}$
- If a > 0, $\int_0^{\infty} e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$
- If a > 0, $\int_0^{\infty} e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$
- $\int_1^{\infty}\frac{1}{(x+\sqrt{x^2-1})^n}dx = \int_0^{\pi/2}\frac{\sec^2 x}{(\sec x+\tan x)^n}dx = \frac{n}{n^2-1}$
- $\int_a^b \sqrt{(x-a)(b-x)}\,dx = \frac{\pi}{8}(b-a)^2$
- $\int_a^b \frac{1}{\sqrt{(x-a)(b-x)}}\,dx = \pi$
- $\int_a^b \frac{1}{x\sqrt{(x-a)(b-x)}}\,dx = \frac{\pi}{\sqrt{ab}}$
- $\int_a^b \sqrt{\frac{x-a}{b-x}}\,dx = \int_a^b \sqrt{\frac{b-x}{x-a}}\,dx = \frac{\pi}{2}(b-a)$

LEIBNITZ RULE

- $\frac{d}{dx}\left(\int_{\phi(x)}^{\psi(x)} f(t)dt\right) = \frac{d(\psi(x))}{dx}[f(\psi(x))] - \frac{d(\phi(x))}{dx}[f(\phi(x))]$

PRACTICE SET - I

01. $\int_0^2 (3x^2+4x+3)dx =$

1) 20 2) 22 3) 25 4) 30

02. $\int_0^1 e^x dx =$

1) e-1 2) 1 3) e 4) 2

03. $\int_0^1 (1+e^{-x})\,dx =$

1) -1 2) 2 3) 1+e^{-1} 4) $2-\frac{1}{e}$

04. $\int_0^{\pi/2}\sin x\,dx =$

1) 0 2) -1 3) 1 4) 1/2

05. $\int_0^4 x\sqrt{x}\,dx =$

1) 12.4 2) 8.4 3) 8.8 4) 12.8

06. $\int_{\pi/4}^{\pi/2} Cot x\,dx =$

1) 2log2 2) $\frac{\pi}{2}$log2 3) log$\sqrt{2}$ 4) log2

07. $\int_0^{\pi/2}\sqrt{1+Sin2x}\,dx =$

1) 1 2) -2 3) $\sqrt{2}$ 4) 2

08. $\int_0^1 \frac{1}{1+x^2}dx =$

1) π/4 2) π 3) $\pi/3$ 4) 0

09. $\int_{1/\sqrt{3}}^{k}\frac{1}{1+x^2}dx = \frac{\pi}{6}$ then upper limit k=

1) $\sqrt{3}$ 2) $\frac{1}{\sqrt{3}}$ 3) 1 4) $2+\sqrt{3}$

10. $\int_0^1 \frac{dx}{\sqrt{1-x^2}} =$

1) 0 2) -1 3) $\pi/2$ 4) $-\pi/2$

11. $\int_{1/2}^{1}\frac{1}{\sqrt{1-x^2}}dx =$

1) π 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/4$

12. $\int_1^2 \frac{1}{x\sqrt{x^2-1}}dx =$

1) π 2) $\pi/2$ 3) $\pi/4$ 4) $\pi/3$

13. $\int_0^{\pi}\frac{\sin\theta+\cos\theta}{\sqrt{1+\sin 2\theta}}d\theta =$

1) π 2) π+λ and λ>0
3) π/2 4) π/3

14. $\int_0^{\pi/4} Sec^2 x\,dx =$

1) e - 1 2) 1 3) e 4) 2

15. $\int_0^{\pi/4} Tan^2 x\,dx =$

1) $\frac{\pi}{4}$ 2) $1-\frac{\pi}{4}$ 3) $\pi/2$ 4) $1+\frac{\pi}{4}$

16. $\int_0^1 \frac{1}{1+x}dx =$

1) log 2 2) $\frac{1}{2}$log 2 3) 2 4) log 3

17. $\int_0^a \frac{1}{a^2+x^2}dx =$

1) π/2 2) π/3 3) π/4 4) π/4a

18. $\int_{a/2}^{a}\frac{1}{\sqrt{a^2-x^2}}dx =$

1) π/2 2) π/2a 3) π/2 - 1 4) π/3

19. $\int_0^a \sqrt{a^2-x^2}\,dx =$

1) $\frac{a^2}{4}$ 2) πa^2 3) $\frac{\pi a^2}{2}$ 4) $\frac{\pi a^2}{4}$

20. $\int_0^1 \sqrt{1-x^2}\,dx =$

1) $1-\frac{\pi}{4}$ 2) $1-\frac{\pi}{3}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

21. $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx =$

1) π 2) 2π 3) 4π 4) 3π

22. $\int_0^{\pi/2}\frac{Cos x}{1+Sin x}dx =$

1) log 2 2) log e 3) $\frac{1}{2}$log 3 4) 0

23. $\int_0^{\pi/3}\frac{Cos x}{3+4Sin x}dx =$

1) $\log\left(\frac{3+2\sqrt{3}}{3}\right)$ 2) $\frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$

3) $2\log\left(\frac{3+2\sqrt{3}}{3}\right)$ 4) $\frac{1}{2}\log\left(\frac{3+2\sqrt{3}}{2}\right)$

24. $\int_0^1 \frac{x}{1+x^2}dx =$

1) $\log 2$ 2) $\frac{1}{2}\log 2$ 3) 2 4) $\log 4$

25. $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx =$

1) $\frac{\pi^2}{4}$ 2) $\frac{\pi^2}{18}$ 3) $\frac{\pi^2}{32}$ 4) $\frac{\pi^2}{8}$

26. $\int_0^1 \frac{(\tan^{-1}x)^3}{1+x^2}dx =$

1) $\frac{\pi^4}{64}$ 2) $\frac{\pi^4}{256}$ 3) $\frac{\pi^4}{1024}$ 4) $\frac{\pi^4}{512}$

27. $\int_0^1 \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}dx =$

1) $\frac{\pi^3}{24}$ 2) π^2 3) $-\pi^2$ 4) 0

28. $\int_0^1 \frac{4x^3}{\sqrt{1-x^8}}dx =$

1) π 2) $-\pi$ 3) $\pi/2$ 4) $-\pi/2$

29. $\int_0^1 \frac{x^3}{1+x^8}dx =$

1) $\pi/16$ 2) $\pi/4$ 3) $\pi/2$ 4) $\pi/8$

30. $\int_0^1 \frac{x\,dx}{(x^2+1)^2} =$

1) 1/2 2) 1/3 3) 1/4 4) 0

31. $\int_{\sqrt{8}}^{\sqrt{15}} x\sqrt{1+x^2}\,dx =$

1) 15/8 2) 37/3 3) 37/6 4) $\frac{37}{9}$

32. $\int_1^e \frac{(\ln x)^3}{x}dx =$

1. $e^4/4$ 2) $1/4$ 3) $\frac{1}{4}(e^4-1)$ 4) e^4-1

33. $\int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}} =$

1) 2 2) $2\sqrt{2}$ 3) $\sqrt{2}$ 4) -2

34. $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x}dx =$

1) $e-1$ 2) $e^{-1}-1$ 3) $e^{-1}+1$ 4) $e^{-2}-1$

35. $\int_0^{\pi/4} \frac{\sin^9 x}{\cos^{11} x}dx =$

1) 10 2) 5 3) 1/10 4) 1/5

36. $\int_0^{\pi/2} e^{\sin^2 x}\sin 2x\,dx =$

1) e 2) $e+1$ 3) $e-1$ 4) $2e+1$

37. $\int_0^{\pi/4} (\tan^4 x+\tan^2 x)$

1) 1 2) 1/2 3) 1/3 4) 1/4

38. $\int_0^{\pi/2} \sqrt{1+\sin x}\,dx =$

1) 1 2) 1/2 3) 2 4) 3

39. $\int_{\pi/2}^{3\pi/2} \frac{1}{1+\cos x}dx =$

1) 2 2) -2 3) 1/2 4) -1/2

40. $\int_0^{\pi/2} \frac{Cos x}{1+Sin^2 x}dx =$

1) π 2) $\pi/3$ 3) $\pi/2$ 4) $\pi/4$

41. If $\int_0^k \frac{\cos x}{1+\sin^2 x}dx = \frac{\pi}{4}$ then k =

1. 1 2. $\pi/4$ 3. $\pi/2$ 4. $\pi/6$

42. $\int_0^{\pi/2} \cos^5 x.\sin 2x\,dx =$

1) 2/7 2) 1/7 3) -1/7 4) 3/7

43. $\int_0^{\pi} \frac{\cos x}{1+\sin^2 x}dx =$

1) $\frac{\pi}{4}$ 2) 0 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3}$

44. $\int_0^{\pi} \frac{Tan x}{Sec x+Cos x}dx =$

1) π 2) $\pi/2$ 3) $-\pi$ 4) 2π

45. $\int_0^{\pi/2} \frac{Sin^2 x}{(1+Cos x)^2}dx =$

1) $\pi/2$ 2) $2-\pi/2$ 3) $\pi/2-2$ 4) $2+\pi/2$

46. $\int_0^1 \frac{dx}{e^x+e^{-x}} =$

1) $Tan^{-1}e$ 2) $\frac{\pi}{4}$

3) $Tan^{-1}e-\frac{\pi}{4}$ 4) $Tan^{-1}e+\frac{\pi}{4}$

47. $\int_0^1 e^x(e^x+1)^3dx =$

1) $\frac{e^4}{4}-4$ 2) $\frac{(e+1)^4}{4}-4$

3) $\frac{(e+1)^4+16}{4}$ 4) $\frac{(e+1)^4}{4}+4$

48. $\int_0^1 e^x\sinh x\,dx =$

1) $\frac{e^2-3}{4}$ 2) $\frac{e^2+3}{4}$ 3) $4(e^2+3)$ 4) $\frac{e^2-5}{4}$

49. $\int_0^{\log 2} \cosh 2x\,dx =$

1) 15/16 2) -15 3) 16/17 4) 17/18

50. $\int_0^1 \tanh x\,dx =$

1) $\log(e+1/e)$ 2) $\log(e-1/e)$

3) $\log(e/2+1/2e)$ 4) $\log\left(\frac{e}{2}-\frac{1}{e}\right)$

51. $\int_0^{1/2} e^x\left[\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right]dx =$

1) $\frac{e^4}{4}$ 2) $\frac{\pi\sqrt{e}}{6}$ 3) $\frac{\sqrt{\pi e}}{4}$ 4) $\frac{\pi\sqrt{e}}{2}$

52. $\int_0^{\frac{\pi}{2}} (\cos x-\sin x)e^x dx =$

1) 0 2) 1 3) -1 4) 2

53. $\int_1^2 \left(\frac{1+x\log x}{x}\right)e^x dx =$

1) $e^2\log 2$ 2) $e\log 2$

3) $\frac{1}{2}\log 2$ 4) $\frac{e^2}{2}\log 2$

54. $\int_0^1 \frac{1-x}{1+x}dx =$

1) $\log 4$ 2) $\log(4/e)$ 3) 1 4) $\log(e/4)$

55. $\int_0^a \frac{x-a}{x+a}dx =$

1) $a+2a\log 2$ 2) $a-2a\log 2$

3) $2a\log -a$ 4) $2a\log 2$

56. $\int_0^1 \frac{x^2}{1+x^2}dx =$

1) $1-\frac{\pi}{4}$ 2) $1-\frac{\pi}{3}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

57. $\int_1^2 \frac{dx}{\sqrt{1+x^2}} =$

1) $\log_e\left(\frac{2+\sqrt{5}}{\sqrt{2}+1}\right)$ 2) $\log_e\left(\frac{\sqrt{2}+1}{2+\sqrt{5}}\right)$

3) $\log_e\left(\frac{2-\sqrt{5}}{\sqrt{2}-1}\right)$ 4) 0

58. $\int_0^1 \frac{1}{\sqrt{2+3x}}dx =$

1) $\frac{2}{3}(\sqrt{5}-\sqrt{2})$ 2) $\frac{2}{3}(\sqrt{5}+\sqrt{2})$

3. $\frac{3}{5}(\sqrt{5}-\sqrt{2})$ 4) $\frac{2}{3}(\sqrt{3}-\sqrt{2})$

59. $\int_{-1}^{1} xe^{x}dx =$

1) e 2) 1/e 3) e^2 4) 2/e

60. $\int_0^{\pi/4} x\sec^2 x\,dx =$

1) $\frac{\pi}{4}-\frac{1}{2}\log 2$ 2) $\frac{\pi}{4}-\frac{1}{4}\log 2$

3) $\frac{\pi}{4}+\frac{1}{2}\log 2$ 4) $\frac{\pi}{4}+\frac{1}{4}\log 2$

61. $\int_0^1 Tan^{-1}x\,dx =$

1) $\frac{\pi}{4}-\frac{1}{2}\log 2$ 2) $\frac{\pi}{4}-\frac{1}{4}\log 2$

3) $\frac{\pi}{4}+\frac{1}{2}\log 2$ 4) $\frac{\pi}{4}+\frac{1}{4}\log 2$

62. $\int_0^{\infty} xe^{-x^2}dx =$

1) 1 2) -1/2 3) 1/2 4) 0

63. $\int_0^{\infty}\left(a^{-x}-b^{-x}\right)dx =$ (a > 1 , b > 1)

1) $\frac{1}{\log a}-\frac{1}{\log b}$ 2) log a - log b

3) log a + log b 4) $\frac{1}{\log a}+\frac{1}{\log b}$

64. $\int_0^{\pi/2}\frac{f(\sin x)}{f(\sin x)+f(\cos x)}dx =$

1) π 2) 2π 3) $\pi/2$ 4) $\pi/4$

65. $\int_0^{\pi/2}\frac{\operatorname{cosec}^{2/3}x}{\operatorname{cosec}^{2/3}x+\sec^{2/3}x}dx =$

1) π 2) $-\pi$ 3) $\pi/4$ 4) 0

66. $\int_0^{\pi/2}\frac{a\cos x+b\sin x}{\cos x+\sin x}dx =$

1) π(a+b) 2) $\frac{\pi}{2}$(a+b) 3) $\frac{\pi}{4}$(a+b) 4) πab

67. $\int_0^{\pi/2}\frac{5\tan x-3\cot x}{\tan x+\cot x}dx =$

1) π 2) $\pi/3$ 3) $\pi/2$ 4) $\frac{\pi}{6}$

68. $\int_0^{a}\frac{dx}{x+\sqrt{a^2-x^2}} =$

1) π 2) $\pi/3$ 3) $-\pi$ 4) $\pi/4$

69. $\int_0^{\pi/2}\frac{1}{1+Cotx}dx =$

1) π 2) $\pi/2$ 3) $\pi/4$ 4) $\frac{\pi}{6}$

70. $\int_{\pi/6}^{\pi/3}\frac{\cos^3 x\,dx}{\sin^3 x+\cos^3 x}dx =$

1) $\frac{\pi}{3}$ 2) $\frac{-\pi}{2}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{12}$

71. $\int_0^{\infty}\frac{x}{(1+x)(1+x^2)}dx =$

1) $\pi/8$ 2) $\pi/4$ 3) $\pi/2$ 4) $\pi/6$

72. $\int_0^{\pi/2n}\frac{dx}{1+\cot^n nx} =$

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{8}$ 4) $\frac{\pi}{12}$

73. $\int_0^{\pi/2}\frac{e^{\sin x}}{e^{\sin x}+e^{\cos x}}dx =$

1) $\pi/4$ 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/8$

74. $\int_0^{\pi/2}\frac{Cos^{3/2}x}{Cos^{3/2}x+Sin^{3/2}x}dx =$

1) $\pi/3$ 2) $\pi/2$ 3) $\pi/4$ 4) $\pi/8$

75. $\int_{\pi/8}^{3\pi/8}\frac{\cos x}{\cos x+\sin x}dx =$

1) $\pi/6$ 2) $\frac{\pi}{8}$ 3) $\frac{\pi}{12}$ 4) $\frac{\pi}{24}$

76. $\int_0^{\pi/2}\frac{Secx}{Secx+Co\sec x}dx =$

1) $\pi/3$ 2) $\pi/2$ 3) $\pi/4$ 4) $\pi/8$

77. $\int_0^{\pi/2}\frac{Cotx}{Tanx+Cotx}dx =$

1) $\pi/4$ 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/8$

78. $\int_0^{\pi/2}\frac{\sqrt{Cotx}}{\sqrt{Tanx}+\sqrt{Cotx}}dx =$

1) π 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/4$

79. $\int_0^{\pi/2}\frac{3^{\sin^2 x}}{3^{\sin^2 x}+3^{\cos^2 x}}dx =$

1) 0 2) $\pi/2$ 3) $\pi/4$ 4) π

80. $\int_0^{\pi/2}\frac{2Sinx+3Cosx}{Sinx+Cosx}dx =$

1) $\frac{5\pi}{4}$ 2) $\frac{5\pi}{2}$ 3) $\frac{5\pi}{3}$ 4) $\frac{5\pi}{5}$

81. $\int_0^{\pi/2}\frac{3Secx+5Co\sec x}{Secx+Co\sec x}dx =$

1) π 2) 2π 3) 3π 4) $\pi/2$

82. $\int_0^{a} f(a-x)dx =$

1) $\int_0^{a} f(x)dx$ 2) $\int_0^{a} f(-x)dx$

3) $\int_0^{a} f(a)dx$ 4) 0

83. $\int_0^{\pi/2}\frac{1}{1+\sqrt[4]{Tanx}}dx =$

1) $\pi/4$ 2) $\pi/2$ 3) 0 4) $\pi/3$

84. $\int_0^{\pi/2}\frac{aSecx+bCosecx}{Secx+Cosecx}dx =$

1) $\pi/2$ 2) $\pi/4$

3) (a+b) $\pi/2$ 4) (a+b) $\pi/4$

85. $\int_0^{a}[f(a+x)+f(a-x)]dx =$

1) $\int_0^{a} f(x)dx$ 2) $\int_0^{2a} f(x)dx$

3) $\int_0^{a} f(x)dx$ 4) $\int_{-a}^{a} f(x)dx$

86. $\int_0^{\pi/2}\frac{dx}{4\cos^2 x+9\sin^2 x} =$

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{9}$ 4) $\frac{\pi}{6}$

87. $\int_0^{5} x(5-x)^{10}dx =$

1) $\frac{5^{12}}{132}$ 2) $\frac{5^{10}}{132}$

3) 132.5^{12} 4) $\frac{132.5^{12}}{3}$

88. $\int_0^{2a} \frac{f(x)dx}{f(x)+f(2a-x)} =$

1) a 2) a/2 3) 0 4) 2a

89. If $\int_0^a x^m(a-x)^n dx = k$ then $\int_0^a x^n(a-x)^m dx =$

1) k 2) -k 3) k/2 4) k/3

90. If $\int_0^{\pi/2} \log(\sin x)dx = k$ then $\int_0^{\pi/2} \log(\cos x)dx =$

1) k/2 2) 2k 3) -3k 4) k

91. $\int_0^1 x(1-x)^4 dx =$

1) 1/15 2) 1/30 3) -1/15 4) $\frac{1}{60}$

92. $\int_0^a \sqrt{ax-x^2}\,dx =$

1) $\frac{\pi a^2}{8}$ 2) $\frac{\pi a^2}{4}$ 3) $-\pi a^2$ 4) π

93. $\int_1^3 \frac{dx}{\sqrt{(x-1)(3-x)}} =$

1) π 2) $-\pi$ 3) $\pi/2$ 4) 0

94. $\int_0^{\pi/2} \frac{\sec^2 x\,dx}{(\sec x+\tan x)^n} =$ $(n>2)$

1) $\frac{1}{n^2-1}$ 2) $\frac{n}{n^2-1}$ 3) $\frac{n}{n^2+1}$ 4) $\frac{2}{n^2-1}$

95. $\int_0^\infty \frac{dx}{\left(x+\sqrt{x^2+1}\right)^5} =$

1) 1/24 2) 1/5 3) 5/24 4) 5/36

96. $\int_8^9 \sqrt{\frac{x-8}{9-x}}\,dx =$

1) π 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{6\sqrt{2}}$

97. $\int_0^\infty e^{-4x}.\cos 3x\,dx =$

1) $\frac{3}{25}$ 2) $\frac{4}{25}$ 3) $\frac{-1}{25}$ 4) $\frac{7}{25}$

98. $\int_0^\infty e^{-2x}.\sin 5x\,dx =$

1) $\frac{-2}{29}$ 2) $\frac{2}{29}$ 3) $\frac{5}{29}$ 4) $\frac{7}{25}$

99. $\int_0^\infty e^{-2x}(\cos 4x+\sin 4x)dx =$

1) 4/25 2) 3/10 3) 1/5 4) 7/25

100. $\int_0^{\pi/2} \frac{\cos x-\sin x}{1+\cos x.\sin x}dx =$

1) 0 2) $-\pi$ 3) 1/2 4) $\pi/4$

101. $\int_0^{\pi/2} \log(\tan x)dx =$

1) 0 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/4$

102. $\int_0^{\pi/2} \frac{f(\sec x)-f(\operatorname{cosec} x)}{1+f(\sec x)f(\operatorname{cosec} x)}dx$

1) 0 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/4$

103. $\int_0^{\pi/2} \log\left(\frac{a\cos x+b\sin x}{a\sin x+b\cos x}\right)dx$

1) 0 2) $\pi/4$ 3) e^{π} 4) $\frac{e^{\pi}}{4}$

104. $\int_0^{\pi/2} \frac{e^{\sin x}-e^{\cos x}}{e^{\sin x}+e^{\cos x}}dx$

1) 0 2) 1 3) $\pi/4$ 4) $\frac{4}{e^{\pi}}$

105. $\int_0^{\pi/2} \sin 2x \log(\tan x)dx =$

1) 0 2) -1/2 3) 1/3 4) 1/4

106. If $f(x) = \begin{cases} x^2 \text{ for } 0\le x\le 1 \\ \sqrt{x} \text{ for } 1\le x\le 2 \end{cases}$ then $\int_0^2 f(x)dx =$

1) $(1/3)(4\sqrt{2}+1)$ 2) $(1/3)(4\sqrt{2}-1)$
3) $(1/3)(2\sqrt{2}-1)$ 4) $(1/2)(3\sqrt{2}-1)$

107. $\int_1^4 f(x)dx$ where $f(x)=x^2$; $1\le x<2$ and $f(x)=3x-4$; $2\le x<4$ is

1) 7/3 2) 10 3) 23/3 4) 37/3

108. If $\int_{-1}^4 f(x)dx=4$ and $\int_2^4[3-f(x)]dx=7$ then $\int_{-1}^2 f(x)dx=$

1) -2 2) 3 3) 5 4) 8

109. $\int_0^2 |x-2|\,dx =$

1) 1/2 2) 3 3) 2 4) 3/4

110. $\int_{-a}^a x|x|\,dx =$

1) $\frac{a}{3}$ 2) $\frac{a^2}{3}$ 3) $\frac{a^2}{2}$ 4) 0

111. $\int_0^2 (|x|+|x-1|)dx =$

1) 1 2) -1 3) 2 4) 3

112. $\int_{-1}^2 |x-1|\,dx =$

1) 3/2 2) 2/3 3) 5/2 4) 0

113. $\int_0^4 |2-x|\,dx =$

1) 0 2) 4 3) 1/2 4) 3/2

114. $\int_2^5 (|x-2|+|x-5|)dx$

1) 0 2) 3 3) 9/2 4) 9

115. $\int_0^2 |2x-x^2|\,dx =$

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) $\frac{4}{3}$ 4) 1

116. $\int_{-1}^1 |1-x^2|\,dx =$

1) 4/3 2) 1 3) -1/3 4) -4/3

117. $\int_0^2 |x^2+2x-3|\,dx =$

1) -1 2) 4 3) 2 4) 0

118. $\int_2^3 |x^2-5x+6|\,dx =$

1) 1/6 2) 2/3 3) 4/7 4) 3/7

119. $\int_{-1}^1 |x^2-3x+2|\,dx =$

1) 2 2) 1 3) 14/3 4) 3/7

120. $\int_{e^{-1}}^{e^2} \left|\frac{\log x}{x}\right|dx =$

1) 3/2 2) 5/2 3) 3 4) 5

121. $\int_{-1}^1 x|x|\,dx =$

1) 2 2) 1 3) 0 4) 1/2

122. If $a<0<b$, then $\int_a^b \frac{|x|}{x}dx =$

1) b-a 2) a-b 3) a+b 4) 0

123. If a<b<0, then $\int_a^b \frac{|x|}{x}dx =$

1) a-b 2) a+b 3) 0 4) b-a

124. If 0<a<b, then $\int_a^b \frac{|x|}{x}dx =$

1) a-b 2) a+b 3) 0 4) b-a

125. $\int_{-1}^{1} \frac{|x|}{x}dx =$

1) 0 2) 1/2 3) 1/3 4) -1

126. $\int_0^{\pi} |\cos x|\, dx =$

1) 0 2) $2-\sqrt{2}$ 3) $2+\sqrt{2}$ 4) 2

127. $\int_0^{\pi/2} \sqrt{1-Cos2x}\,dx$

1) 1 2) -2 3) $\sqrt{2}$ 4) 2

128. $\int_{-\pi/2}^{\pi/2} \sin|x|\,dx =$

1) 1 2) 2 3) -1 4) 1/2

129. $\int_{\pi/4}^{3\pi/4} |\cos x|\,dx =$

1) $2-\sqrt{2}$ 2) $2\sqrt{2}$ 3) $\sqrt{2}-1$ 4) $\sqrt{2}+1$

130. $\int_0^2 \left|Cos\frac{\pi x}{2}\right| dx =$

1) $\frac{1}{\pi}$ 2) $\frac{2}{\pi}$ 3) $\frac{3}{\pi}$ 4) $\frac{4}{\pi}$

131. $\int_0^{\pi} |\cos x - \sin x|\, dx =$

1) $4\sqrt{2}$ 2) $2\sqrt{2}$ 3) $4\sqrt{3}$ 4) $3\sqrt{2}$

132. $\int_{-\pi/2}^{\pi/2} \sqrt{Cosx - Cos^3x}\,dx =$

1) 1 2) 4/3 3) -1/3 4) 0

133. $\int_0^{\pi/2} |\cos x - \sin x|\, dx =$

1) $2(\sqrt{2}-1)$ 2) $2(\sqrt{2}+1)$

3) $2\sqrt{2}$ 4) -1

134. $\int_0^{\pi} (\cos x + |\cos x|)dx =$

1) 1 2) 1/2 3) 2 4) -1

135. $\int_0^{\frac{\pi}{3}} |Tanx - 1|dx =$

1) $\frac{\pi}{4}+\frac{1}{2}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

136. $\int_0^{2} [x]dx =$

1) 1 2) -1 3) 0 4) 2

137. $\int_0^3 [x]dx =$

1) 1 2) 2 3) 3 4) 3/2

138. $\int_1^{\infty} \left[\frac{1}{1+x^2}\right] dx =$

1) 0 2) 1 3) 2 4) 3

139. $\int_1^4 \log[x]dx =$

1) log 4 2) log 5 3) log 6 4) zero

140. $\int_1^2 x^2[x]dx =$

1) 7/3 2) 8/3 3) 4/3 4) 5/3

141. $\int_0^4 [2x+3]\,dx =$

1) 12 2) 24 3) 26 4) 0

142. $\int_{-1}^{1} [x]dx =$

1) -1 2) 2 3) 3 4) 0

143. $\int_{2.1}^{3.5} [x]dx =$

1) 1 2) 0 3) 3.4 4) 3.3

144. $\int_0^{50} (x-[x])dx =$

1) 25 2) 20 3) 15 4) 10

145. $\int_0^{1000} e^{x-[x]}dx =$

1) $\frac{e^{1000}-1}{1000}$ 2) $\frac{e^{1000}-1}{e-1}$

3) 1000(e-1) 4) $\frac{e-1}{1000}$

146. $\int_0^{100\pi} \sqrt{1-Cos2x}\,dx =$

1) $150\sqrt{2}$ 2) $100\sqrt{2}$ 3) $200\sqrt{2}$ 4) $50\sqrt{2}$

147. $\int_0^{88\pi} \sqrt{1-Cos2x}\,dx =$

1) $176\sqrt{2}$ 2) $88\sqrt{2}$ 3) $44\sqrt{2}$ 4) $22\sqrt{2}$

148. $\underset{x\to 0}{Lt}\left(\frac{\int_0^x Sin\, t^3\, dt}{x^4}\right) =$

1) 0.25 2) 2.5 3) 5.2 4) 0.52

149. $\underset{x\to 0}{Lt}\left(\frac{x^2}{\int_0^x Tan^{-1}t\,dt}\right) =$

1) 1 2) 2 3) 0 4) 4

150. $\underset{x\to 0}{Lt}\left(\frac{\int_0^x Cos\, t^2\, dt}{x}\right) =$

1) 0 2) 1 3) -1 4) 2

151. $\underset{x\to 0}{Lt}\left(\frac{x^4}{\int_0^{x^2} \tan^{-1}t\,dt}\right) =$

1) 1 2) 2 3) 0 4) 3

152. If $\phi(x) = \int_{1/x}^{\sqrt{x}} Sin t^2\, dt$ then $\phi^1(1) =$

1) Sin 1 2) 2 Sin 1

3) $\frac{3}{2}$ Sin 1 4) $\frac{1}{2}$sin1

153. If $y = \int_0^x \sin x\,dx$ then $\left(\frac{dy}{dx}\right)$ at $x=\frac{\pi}{4}$ is

1) $\frac{-1}{\sqrt{2}}$ 2) $\frac{1}{\sqrt{2}}$ 3) $1-\frac{1}{\sqrt{2}}$ 4) $1+\frac{1}{\sqrt{2}}$

154. $\int_{-2}^{2} (x^{11}\cos x + e^x)dx =$

1) sin h2 2) 2 sin h2

3) $\frac{3}{2}$ sin h2 4) $\frac{\sinh 2}{2}$

155. $\int_{-\pi}^{\pi} x^3 \cos x\,dx =$

1) π 2) $\pi/2$ 3) $-\pi/3$ 4) 0

156. $\int_{-2\pi}^{2\pi} \sin^5 x\,dx =$

1) $\frac{\pi^2}{2}$ 2) $\pi/15$ 3) $3\pi/17$ 4) 0

157. $\int_{-\pi/3}^{\pi/3} \frac{x^3.Cosx}{Sin^2x}dx =$

1) 0 2) 1 3) -1 4) $\pi/3$

158. $\int_{-1}^{1}\frac{1}{1+x^2}dx=$

1) $\pi/2$ 2) $\pi/3$ 3) $\pi/4$ 4) $\pi/5$

159. $\int_{-\pi}^{\pi}x^4\sin x\,dx=$

1) 1 2) 0 3) 2 4) 2π

160. $\int_{-a}^{a}\{f(x)+f(-x)\}dx=$

1) $2\int_0^a f(x)dx$ 2) 0

3) $\int_0^a\{f(x)+f(-x)\}dx$ 4) $\int_0^a f(-x)dx$

161. $\int_{-1}^{+1}x^3(1+x^2)dx=$

1) 0 2) 5/12 3) 1/3 4) 1

162. If f(x) and g(x) are any two continuous functions $\forall x\in R$ then $\int_{-a}^{a}\frac{g(x)-g(-x)}{f(x)+f(-x)}dx$

1) 0 2) 4 3) 6 4) 3

163. $\int_0^{2\pi}\sin^5 x\cos^7 x\,dx$

1) π 2) $\pi/2$ 3) $-\pi$ 4) 0

164. $\int_{-1}^{1}\left(\sqrt{1-x+x^2}-\sqrt{1+x+x^2}\right)dx=$

1) 1/2 2) -1 3) 0 4) 2

165. If f(x) is even function in [-a, a] then $\int_{-a}^{+a}f(x)dx=$

1) 0

2) $\int_0^{2a}[f(x)+f(a-x)]dx$

3) $2\int_0^a f(x)dx$ 4) 2

166. The value of $\int_{-\pi/2}^{\pi/2}(x^3+xCosx+Tan^5x+1)dx=$

1) 0 2) 2 3) π 4) 2π

167. $\int_0^{2\pi}\cos^3 x\,dx=$

1) $\frac{\pi^2}{2}$ 2) $\frac{2\pi}{15}$ 3) $\frac{\pi^2}{8}$ 4) 0

168. $\int_0^{\pi/2}\sin^4 x\,dx=$

1) $\frac{\pi}{12}$ 2) $\frac{3\pi}{7}$ 3) $\frac{3\pi}{16}$ 4) 0

169. $\int_0^{\pi/2}\cos^5 x\,dx=$

1) 8/15 2) 7/15 3) 1/15 4) 0

170. $\int_0^{\pi/8}\cos^3 4x\,dx=$

1) 1/6 2) 1/5 3) -1/3 4) 1/8

171. $\int_0^{\pi/2}\sin^4 x.\cos^2 x\,dx=$

1) $\frac{\pi}{32}$ 2) $\frac{\pi^2}{16}$ 3) $\frac{\pi}{15}$ 4) $\frac{\pi}{64}$

172. $\int_0^{\pi/2}\sin^3 x.\cos^3 x\,dx=$

1) 1/12 2) $\frac{\pi}{24}$ 3) $\frac{\pi}{12}$ 4) $\frac{1}{24}$

173. The values of θ $(0\le\theta\le\pi)$ satiesfying $\int_0^{\theta}\cos x\,dx=\sin 2\theta$ are

1) $0,\frac{\pi}{3}$ 2) $\frac{\pi}{4},\frac{3\pi}{4}$ 3) $\frac{\pi}{2},\pi$ 4) $\frac{\pi}{6},\frac{5\pi}{6}$

174. $\int_0^{\pi/2}\sin^6 x\cos^5 x\,dx=$

1) $\frac{8}{693}$ 2) $\frac{5}{693}$ 3) $\frac{4}{693}$ 4) $\frac{10}{693}$

175. $\int_0^{\pi/2}\sin^9 x\,dx=$

1) $\frac{128\pi}{315}$ 2) $\frac{128}{315}$ 3) $\frac{\pi}{315}$ 4) $\frac{1}{315}$

176. $\int_0^{2\pi}\sin^m x\cos^n x\,dx=4\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ $(m,n\in N)$, when

1) m is odd n is even 2) m, n both odd

3) m,n both even 4) m is even n is odd

177. $\int_0^1\frac{x^6dx}{\sqrt{1-x^2}}=$

1) $\frac{\pi}{32}$ 2) $\frac{2\pi}{5}$

3) $\frac{5\pi}{32}$ 4) $\frac{7\pi}{32}$

178. $\int_{-2\pi}^{2\pi}\cos^6 x\,dx=$

1) $\frac{4\pi}{5}$ 2) $\frac{5\pi}{4}$ 3) 15π 4) $\frac{\pi^2}{4}$

179. $\int_0^a\frac{x^5dx}{\sqrt{a^2-x^2}}=$

1) $\frac{a^5}{15}$ 2) $\frac{8a^5}{15}$ 3) $\frac{8a}{15}$ 4) $\frac{11a^2}{15}$

180. $\int_0^{2\pi}\cos^4 x\,dx=$

1) $\frac{3\pi}{4}$ 2) -3π 3) $\frac{4\pi}{3}$ 4) 0

181. $\int_0^{\pi/4}\sin^4 x.\cos^4 x\,dx=$

1) $\frac{3\pi}{512}$ 2) $\frac{\pi}{512}$ 3) $\frac{-\pi^2}{512}$ 4) $\frac{7\pi}{512}$

182. $\int_{-\pi/2}^{\pi/2}\sin^2 x.\cos^2 x(\sin x+\cos x)dx=$

1) $\frac{1}{15}$ 2) $\frac{2}{15}$ 3) $\frac{4}{15}$ 4) 0

183. $\int_0^{\infty}\frac{x^2dx}{(1+x^2)^{7/2}}=$

1) 1/15 2) 2/15 3) -1/15 4) $\frac{4}{15}$

184. $\int_0^{\infty}\frac{dx}{(1+x^2)^4}=$

1) $\frac{\pi}{32}$ 2) $\frac{3\pi}{32}$ 3) $\frac{5\pi}{32}$ 4) $\frac{7\pi}{32}$

185. $\int_{-a}^{a}\frac{x^4dx}{\sqrt{a^2-x^2}}=$

1) $\frac{3\pi a^4}{8}$ 2) $\frac{\pi a^4}{8}$ 3) $\frac{-\pi a^4}{8}$ 4) $\frac{5\pi a^4}{8}$

186. $\int_{0}^{3}(9-x^2)^{3/2}dx=$

1) $\frac{243\pi}{16}$ 2) $\frac{\pi}{16}$ 3) $\frac{-243\pi^2}{15}$ 4) $\frac{81\pi}{16}$

187. $\underset{n\to\infty}{Lt}\sum_{r=0}^{n-1}\frac{n}{n^2+r^2}$

1) 1 2) π 3) $\pi/2$ 4) $\pi/4$

188. $\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\frac{1}{\sqrt{4n^2-r^2}}$

1) $\pi/2$ 2) $\pi/3$ 3) $\pi/6$ 4) $\pi/5$

189. $\underset{n\to\infty}{Lt}\sum_{r=0}^{n-1}\frac{1}{\sqrt{n^2-r^2}}$

1) π 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/6$

190. $\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\frac{1}{n}\sqrt{\frac{n+r}{n-r}}$

1) $\frac{\pi}{2}$ 2) 2π 3) $\frac{\pi}{2}-1$ 4) $\frac{\pi}{2}+1$

191. $\underset{n\to\infty}{Lt}\left\{\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{2n}\right\}=$

1) log 2 2) log 3 3) 4 4) $\pi/2$

192. $\underset{n\to\infty}{Lt}\sum_{r=0}^{n-1}\frac{1}{n+r}$

1) log 2 2) 2 log 2 3) 1/2 log 2 4) 1/4 log 2

193. $\underset{n\to\infty}{Lt}\left\{\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{3n}\right\}=$

1) $\pi/4$ 2) $\pi/6$ 3) log 2 4) log 3

194. $\underset{n\to\infty}{Lt}\left\{\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{6n}\right\}=$

1) log 2 2) log 3 3) log 5 4) log 6

195. $\underset{n\to\infty}{Lt}\left\{\frac{1}{2n+1}+\frac{1}{2n+2}+\frac{1}{2n+3}....+\frac{1}{2n+n}\right\}$

1) $\log_e\left(\frac{1}{3}\right)$ 2) $\log_e\left(\frac{2}{3}\right)$

3) $\log_e\left(\frac{3}{2}\right)$ 4) $\log_e\left(\frac{4}{3}\right)$

196. $\underset{n\to\infty}{Lt}\left[\frac{1}{3n+1}+\frac{1}{3n+2}+....+\frac{1}{3n+n}\right]$

1) log (2/3) 2) log(3/2) 3) log(4/3) 4) log(3/4)

197. $\underset{n\to\infty}{Lt}\left[\frac{1}{1-n^2}+\frac{1}{1-n^2}+....+\frac{n}{1-n^2}\right]=$

1) 0 2) -1/2 3) 1/2 4) 1

198. $\underset{n\to\infty}{Lt}\left[\frac{1}{n^2+1^2}+\frac{2}{n^2+2^2}+....+\frac{n}{n^2+n^2}\right]$

1) log 2 2) 2 log 2 3) $\frac{\log 2}{2}$ 4) $\frac{\log 2}{3}$

199. $\underset{n\to\infty}{Lt}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+...+\frac{n+n}{n^2+n^2}\right]=$

1) $\frac{\pi}{4}+\frac{1}{2}\log 2$ 2) $\frac{\pi}{4}-\frac{1}{2}\log 2$

3) $\frac{\pi}{2}+\frac{1}{2}\log 2$ 4) $\frac{\pi}{4}+\frac{1}{4}\log 2$

200. $\underset{n\to\infty}{Lt}\left[\frac{1}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+....+\frac{n^2}{n^3}\right]$

1) 0 2) 1/2 3) 1/3 4) 1

201. $\underset{n\to\infty}{Lt}\frac{\left[1+4+9+...+n^2\right]}{n^3}=$

1) 2 2) 3 3) 1/3 4) 1

202. $\underset{n\to\infty}{Lt}\frac{1^9+2^9+3^9+....+n^9}{n^{10}}=$

1) 1/2 2) 1/5 3) 1/10 4) 1/15

203. $\underset{n\to\infty}{Lt}\frac{1}{n}\left[e^{1/n}+e^{2/n}+....e\right]=$

1) e+1 2) e-1 3) e 4) 2e

204. $\underset{n\to\infty}{Lt}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+....+\frac{1}{8n}\right]=$

1) 2/5 2) 3/5 3) 3/8 4) 11/7

205. $\underset{n\to\infty}{Lt}\left[\frac{1}{1+n^3}+\frac{4}{8+n^3}+\frac{9}{27+n^3}+....+\frac{1}{2n}\right]=$

1) $\frac{1}{2}\log 2$ 2) $\frac{1}{3}\log 3$

3) $\frac{1}{3}\log 2$ 4) $\frac{1}{2}\log 3$

206. $\underset{n\to\infty}{Lt}\left[\frac{1^3}{n^4+1^4}+\frac{2^3}{n^4+2^4}+....+\frac{1}{2n}\right]=$

1) $\frac{1}{4}\log 4$ 2) $\frac{1}{2}\log 2$

3) $\frac{1}{4}\log 3$ 4) $\frac{1}{4}\log 2$

PUT YOUR FULL EFFORTS
DON'T WORRY ABOUT
RESULTS
THEY ARE BOUND TO
COME TO YOU

PRACTICE SET-I KEY

01) 2	02) 1	03) 4	04) 3	05) 4
06) 3	07) 4	08) 1	09) 1	10) 3
11) 3	12) 4	13) 1	14) 2	15) 2
16) 1	17) 4	18) 4	19) 4	20) 4
21) 2	22) 1	23) 2	24) 2	25) 3
26) 3	27) 1	28) 3	29) 1	30) 3
31) 2	32) 2	33) 1	34) 1	35) 3
36) 3	37) 3	38) 3	39) 2	40) 4
41) 3	42) 1	43) 2	44) 2	45) 2
46) 3	47) 2	48) 1	49) 1	50) 3
51) 2	52) 3	53) 1	54) 2	55) 2
56) 1	57) 1	58) 1	59) 4	60) 1
61) 1	62) 3	63) 1	64) 4	65) 3
66) 3	67) 3	68) 4	69) 3	70) 4
71) 2	72) 2	73) 1	74) 3	75) 2
76) 3	77) 1	78) 4	79) 3	80) 1
81) 2	82) 1	83) 1	84) 4	85) 2
86) 1	87) 1	88) 1	89) 1	90) 4
91) 2	92) 1	93) 1	94) 2	95) 3
96) 3	97) 2	98) 3	99) 2	100) 1
101) 1	102) 1	103) 1	104) 1	105) 1
106) 2	107) 4	108) 3	109) 3	110) 4
111) 4	112) 3	113) 2	114) 4	115) 3
116) 1	117) 2	118) 1	119) 3	120) 2
121) 3	122) 3	123) 1	124) 4	125) 1
126) 4	127) 3	128) 2	129) 1	130) 4
131) 2	132) 2	133) 1	134) 3	135) 2
136) 1	137) 3	138) 1	139) 3	140) 1
141) 3	142) 1	143) 4	144) 1	145) 3
146) 3	147) 1	148) 1	149) 2	150) 2
151) 2	152) 3	153) 2	154) 2	155) 4
156) 4	157) 1	158) 1	159) 2	160) 3
161) 1	162) 1	163) 4	164) 3	165) 3
166) 3	167) 4	168) 3	169) 1	170) 1
171) 1	172) 1	173) 1	174) 1	175) 2
176) 3	177) 3	178) 2	179) 2	180) 1
181) 1	182) 3	183) 2	184) 3	185) 1
186) 1	187) 4	188) 3	189) 2	190) 4
191) 1	192) 1	193) 4	194) 4	195) 3
196) 3	197) 2	198) 3	199) 1	200) 3
201) 3	202) 3	203) 2	204) 3	205) 3
206) 4				

PRACTICE SET - II

01. $\int_0^{\pi/2} \sqrt{\cos x}\,\sin^3 x\,dx =$

1) $\frac{34}{231}$ 2) $\frac{64}{231}$ 3) $\frac{30}{321}$ 4) $\frac{128}{231}$

02. $\int_1^2 \log x\,dx =$

1) 2 log 2 -1 2) log 2-1
3) 2 log 2+1 4) 2 log 2 -2

03. $\int_0^{\pi/2} \left(2\tan\frac{x}{2} + x\sec^2\frac{x}{2}\right)dx =$

1) π 2) $\pi/2$ 3) $2\pi/3$ 4) $\pi/6$

04. $\int_0^1 \frac{xe^x}{(x+1)^2}dx =$

1) $\frac{e}{2}$ 2) $\frac{e-1}{2}$ 3) $\frac{e}{2}-1$ 4) $\frac{e-3}{2}$

05. $\int_0^{\pi} e^x \sin x\,dx =$

1) $\frac{1}{2}e^{\pi}$ 2) $e^{\pi}+1$

3) $\frac{1}{2}\left(e^{\pi}-1\right)$ 4) $\frac{1}{2}\left(e^{\pi}+1\right)$

06. $\int_{\log 2}^{t} \frac{dx}{\sqrt{e^x-1}} = \frac{\pi}{6}$, then t=

1) 4 2) log 8 3) log 4 4) log2

07. $\int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right]dx =$

1) e - 2 2) $e + 2\log_2 e$
3) $e - 2\log_2 e$ 4) $\log_2 e$

08. $\int_0^{\pi/2} e^x\left(\frac{1+Sinx}{1+Cosx}\right)dx =$

1) $\pi/4$ 2) 0 3) $e^{\pi/2}$ 4) $e^{\pi/2} - 1$

09. $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}dx =$

1) 3 2) 0 3) 6 4) 4

10. If f(x) = f(a-x) and g(x) + g(a-x)=2 then $\int_0^a f(x).g(x)dx =$

1) 0 2) a 3) $\int_0^a f(x)dx$ 4) a^2

11. $\int_0^{\pi/4} \log(1+\tan x)dx =$

1) $\pi \log 2$ 2) $\frac{\pi}{8}\log 2$

3) $\frac{\pi}{4}\log 2$ 4) $-\pi \log 2$

12. $\int_0^1 \frac{\log(1+x)}{1+x^2}dx =$

1) $\pi \log 2$ 2) $\frac{\pi}{8}\log 2$

3) $\frac{\pi}{4}\log 2$ 4) $-\pi \log 2$

13. $\int_0^1 \frac{\log x}{\sqrt{1-x^2}}dx =$

1) $\pi \log 2$ 2) $-\pi \log 2$

3) $2^{\log 2}$ 4) $-\frac{\pi}{2}\log 2$

14. $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2}dx =$

1) $\pi \log 2$ 2) $-\pi \log 2$

3) $-\frac{\pi}{2}\log 2$ 4) $\frac{\pi}{2}\log 2$

15. $\int_0^{\pi/2} \log(\tan x + \cot x)dx =$

1) $\pi \log 2$ 2) $-\pi \log 2$

3) $-\frac{\pi}{2}\log 2$ 4) $\frac{\pi}{2}\log 2$

16. $\int_0^{\pi} \log(\sin x)dx =$

1) $\pi \log 2$ 2) $-\frac{\pi}{3}\log 2$

3) $-\pi \log 2$ 4) $-\frac{\pi}{2}\log 2$

17. $\int_0^1 \log\left(\cos\frac{\pi x}{2}\right)dx =$

1) -log 2 2) $\frac{1}{2}\log 2$ 3) $\log\sqrt{3}$ 4) log 8

18. $\int_0^a \sqrt{\frac{a+x}{a-x}}dx =$

1) $\frac{a}{2}(\pi+2)$ 2) $\frac{a}{2}(\pi-2)$

3) $\frac{a}{3}(\pi+2)$ 4) $\frac{a}{2}(\pi+3)$

19. $\int_2^5 \sqrt{\frac{5-x}{x-2}}dx =$

1) π 2) $\pi/2$ 3) $3\pi/2$ 4) $\pi/4$

20. $\int_0^1 \sqrt{x(1-x)}\,dx =$

1) $\pi/2$ 2. $\pi/4$ 3) $\pi/6$ 4) $\pi/8$

21. $\int_1^2 \sqrt{(x-1)(2-x)}\,dx =$

1) $\pi/8$ 2. $\pi/4$ 3) 1/8 4) 1/4

22. $\int_0^{\pi} \frac{xTanx}{Secx+Tanx}dx =$

1) $\frac{\pi-2}{2}$ 2) $\frac{\pi(\pi-2)}{2}$

3) $\frac{\pi+2}{2}$ 4) $\frac{\pi(\pi+2)}{2}$

23. $\int_0^1 \log Sin\left(\frac{\pi x}{2}\right)dx =$

1) log 2 2) - log 2

3) $\frac{\pi}{2}\log 2$ 4) $-\frac{\pi}{2}\log 2$

24. $\int_0^{\pi} xSin^3x\,dx =$

1) $\frac{2\pi}{3}$ 2) $\frac{\pi}{3}$ 3) $\frac{-2\pi}{3}$ 4) 2π

25. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx =$

1) a-b 2) b-a 3) $\frac{b-a}{2}$ 4) $\frac{a-b}{2}$

26. $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx =$

1) 1/2 2) 3/2 3) 5/2 4) 0

27. $\int_{-a}^{b} \frac{\sqrt[4]{x+a}}{\sqrt[4]{x+a}+\sqrt[4]{b-x}}dx =$

1) a-b 2) a+b 3) $\frac{a+b}{2}$ 4) b-a

28. If $\int_0^{\infty} \frac{x^2dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$ then

$\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)} =$

1) $\frac{\pi}{60}$ 2) $\frac{\pi}{20}$ 3) $\frac{\pi}{40}$ 4) $\frac{\pi}{80}$

29. $\int_0^{\pi} x f(\sin x)dx =$

1) 0 2) $\pi\int_0^{\pi} f(\sin x)dx$

3) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ 4) $\frac{\pi}{3}\int_0^{\pi} f(\sin x)dx$

30. If f(a+b-x)=f(x) then $\int_a^b x f(x)dx =$

1) $\frac{a+b}{2}\int_a^b f(b-x)dx$ 2) $\frac{a+b}{2}\int_a^b f(x)dx$

3) $\frac{b-a}{2}\int_a^b f(x)dx$ 4) $(a+b)\int_a^b f(x)dx$

31. $\int_0^{2\pi} Cos\,mx\,Sin\,nx\,dx$ where m, n are integers =

1) 0 2) π 3) $\pi/2$ 4) 2π

32. $\int_{1/2}^{1} \frac{1}{\sqrt{x-x^2}}dx =$

1) ½ 2) $\pi/4$ 3) $\pi/2$ 4) π

PRACTICE SET-II KEY

01) 2	02) 1	03) 1	04) 3	05) 4
06) 3	07) 3	08) 3	09) 3	10) 3
11) 2	12) 2	13) 4	14) 1	15) 1
16) 3	17) 1	18) 1	19) 3	20) 4
21) 1	22) 2	23) 2	24) 1	25) 3
26) 1	27) 3	28) 1	29) 3	30) 2
31) 1	32) 3			

SELF TEST

01. $\int_0^{\pi} \frac{\theta\sin\theta}{1+\cos^2\theta}d\theta =$

1) $\frac{\pi^2}{2}$ 2) $\frac{\pi^2}{3}$ 3) π^2 4) $\frac{\pi^2}{4}$

02. $\int_0^{\pi/2} \frac{200\sin x+100\cos x}{\sin x+\cos x}dx =$

1) 50π 2) 25π 3) 75π 4) 150π

03. $\int_0^2 \frac{2x-2}{2x-x^2}dx =$

1) 0 2) 2 3) 3 4) 4

04. $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta =$

1. 0 2. 1 3) 2 4) -1

05. $\int_0^2 [x]dx =$

1. 1 2. 2 3) 3 4) 4

06. $\int_0^1 \sin\left[2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right]dx =$

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) π

07. $\int_0^3 \frac{3x+1}{x^2+9}dx =$

1) $\log(2\sqrt{2})+\frac{\pi}{12}$ 2) $\log(2\sqrt{2})+\frac{\pi}{2}$

3) $\log(2\sqrt{2})+\frac{\pi}{6}$ 4) $\log(2\sqrt{2})+\frac{\pi}{3}$

08. $\int_2^3 \frac{dx}{x^2-x} =$

1) $\log\frac{2}{3}$ 2) $\log\frac{4}{3}$ 3) $\log\frac{8}{3}$ 4) $\log\frac{1}{4}$

09. $\int_{-\pi/2}^{\pi/2} \sin^4 x\cos^6 x\,dx =$

1) $\frac{3\pi}{128}$ 2) $\frac{3\pi}{256}$ 3) $\frac{3\pi}{572}$ 4) $\frac{3\pi}{64}$

10. $\int_0^{\pi/2} \sin^8 x\cos^2 x\,dx =$

1) $\frac{\pi}{512}$ 2) $\frac{3\pi}{512}$ 3) $\frac{5\pi}{512}$ 4) $\frac{7\pi}{512}$

11. $\int_{-1}^{1} (ax^3+bx)dx = 0$ for

1) $a, b \in R$ 2) $a, b \in R_+$
3) $a \in R_+$ and $b \in R_-$ 4) $a, b \in R_-$

12. If f(x) is integrable on [0, a] then

$\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$

1) 0 2) 1 3) a 4) $\frac{a}{2}$

13. $\left[\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}x\,dx\right]+\left[\sum_{n=1}^{-10}\int_{2n}^{2n+1}\sin^{27}x\,dx\right] =$

1) 27^2 2) -54 3) 54 4) 0

14. $\int_0^1 \frac{x}{(1-x)^{5/4}}dx =$

1) 16/3 2) 3/16 3) -3/16 4) -16/3

15. $\int_0^{\pi/4} (\tan^4 x+\tan^2 x)dx =$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{4}$

16. $\int_0^{\pi} \cos^3 x\,dx$

1) -1 2) 0 3) 1 4) $\frac{1}{\sqrt{2}}$

17. If $y(x) = e^{-x^2}\int_0^x e^{t^2}dt$ then $\frac{dy}{dx}+2xy =$

1) 0 2) 1 3) 2 4) -2

18. $\int_0^{\pi/2} \sin^6 x\cos^5 x\,dx =$

1) $\frac{8}{693}$ 2) $\frac{5}{693}$ 3) $\frac{4}{693}$ 4) $\frac{10}{693}$

19. $\int_0^a \sqrt{a^2-x^2}dx =$

1) πa^2 2) $\frac{\pi a^2}{2}$ 3) $\frac{\pi a^2}{3}$ 4) $\frac{\pi a^2}{4}$

20. $\int_{-1}^{1} \tan^{-1}x\,dx =$

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{-\pi}{4}$

21. $\int_0^1 \frac{dx}{x+\sqrt{x}}$

1) $2\log\frac{1}{2}$ 2) $\frac{1}{2}\log 2$ 3) $\log 2$ 4) $2\log 2$

22. $\int_0^{\pi/2} (\cos x-\sin x)e^x dx$

1) $\frac{\pi}{2}$ 2) 0 3) -1 4) 1

23. $\int_0^{\pi/4} \frac{\sin^9 x}{\cos^{11} x}dx =$

1) $\frac{\pi}{4}$ 2) $\frac{1}{10}$ 3) $\frac{-1}{10}$ 4) 1

24. $\int_0^{\pi/2}\left(2Tan\frac{x}{2}+x\sec^2\frac{x}{2}\right)dx =$

1) π 2) $\frac{\pi}{2}$ 3) 2 4) 0

25. $\int_0^1 \frac{1-x}{1+x}dx =$

1) 2 log 2-1 2) 1+log 4 3) log2-1 4) 2 log2

26. $\int_0^a \sqrt{a^2-x^2}dx =$

1) $\frac{\pi a}{4}$ 2) $\frac{\pi a^2}{4}$ 3) $\frac{3\pi a^2}{4}$ 4) $\frac{\pi a}{2}$

27. $\int_0^\pi x\sin^4 x\,dx =$

1) $\frac{3\pi}{16}$ 2) $\frac{3\pi^2}{16}$ 3) $\frac{16\pi}{3}$ 4) $\frac{16\pi^2}{3}$

29. $\underset{x\to 0}{Lt}\int_0^x \frac{\sin^3 t\cos t\,dt}{x^4} =$

1) 0.25 2) 2.5 3) 5.2 4) 0.52

30. $\int_0^\infty \left(a^{-x}-b^{-x}\right)dx\,(a>1, b>1) =$

1) $\frac{1}{\log a}-\frac{1}{\log b}$ 2) loga-logb

3) loga+logb 4) $\frac{1}{\log a}+\frac{1}{\log b}$

31. $\int_0^\pi \frac{x\sin x\,dx}{1+\cos^2 x} =$

1) $\frac{\pi^2}{8}$ 2) $\frac{\pi^2}{4}$ 3) $\frac{\pi^3}{8}$ 4) $\frac{\pi^4}{8}$

32. $\int_0^1 x\tan^{-1}x\,dx =$

1) $\frac{\pi}{4}-\frac{1}{2}$ 2) $\frac{\pi}{8}-\frac{1}{2}$

3) $\frac{\pi}{4}+\frac{1}{2}$ 3) $\frac{\pi}{8}+\frac{1}{2}$

33. $I_{m,n}=\int_0^1 x^m(\log x)^n dx =$

1) $\frac{n}{m+1}I_{m,n-1}$ 2) $\frac{-m}{n+1}I_{m,n-1}$

3) $\frac{-n}{m+1}I_{m,n-1}$ 4) $\frac{m}{m+1}I_{m,n-1}$

34. $\int_0^1 \sqrt{x(1-x)}\,dx =$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{8}$

35. $\int_0^{\pi/2} \frac{dx}{1+\tan x} =$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) 1 4) log 2

36. If $a<0<b$ $\int_a^b \frac{|x|}{x}dx =$

1) a-b 2) b-a 3) a+b 4) -a-b

37. $\int_{-1}^1 \frac{1}{\left(1+x^2\right)^2}dx =$

1) $\frac{\pi}{4}+\frac{1}{2}$ 2) $\frac{\pi}{4}-\frac{1}{2}$ 3) $\frac{\pi}{4}$ 4) 0

38. $\int_{\sqrt{2}/3}^{\sqrt{3}/3} \frac{dx}{\sqrt{4-9x^2}} =$

1) $\frac{\pi}{36}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{7\pi}{30}$

39. $\int_{-\pi/2}^{\pi/2} \sin^2 x\cos^2 x(\sin x+\cos x)dx$

1) $\frac{2}{15}$ 2) $\frac{4}{15}$ 3) $\frac{2}{5}$ 4) $\frac{8}{15}$

40. $\int_0^1 \frac{xe^x}{(1+x)^2}dx =$

1) $\frac{e}{2}$ 2) $e-\frac{1}{2}$ 3) $\frac{e}{2}-1$ 4) $e-\frac{3}{2}$

41. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}dx =$

1) 0 2) 1 3) $\frac{\pi}{4}$ 4) π

42. $\int_0^\infty \frac{1}{\left(x+\sqrt{x^2+1}\right)^3}dx =$

1) $\frac{-1}{8}$ 2) $\frac{1}{2}$ 3) $\frac{3}{8}$ 4) $\frac{8}{3}$

43. $\int_0^\pi \sin x\cos^4 dx =$

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{2}{5}$ 4) $\frac{1}{4}$

44. $\int_0^1 \frac{x\,dx}{(1-x)^{3/4}} =$

1) 4 2) 16/5 3) 4/5 4) -16/15

45. $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}dx =$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{3\pi}{2}$ 4) π

46. $\int_0^\pi \frac{x\tan x}{\sec x+\tan x}dx$

1) π^2-1 2) $\frac{\pi}{2}(\pi-1)$

3) $\pi\left(\frac{\pi}{2}-1\right)$ 4) $\pi\left(\frac{\pi}{2}+1\right)$

47. $\int_0^{\pi/2} e^{\sin^2 x}\sin 2x\,dx =$

1) e 2) e+1 3) e−1 4) 2e+1

48. $\int_0^{\pi/2} |\cos q-\sin q|dq =$

1) $\sqrt{2}-1$ 2) $\frac{1}{(\sqrt{2}-1)}$ 3) $2(\sqrt{2}-1)$ 4) $\sqrt{2}$

49. $\int_0^{\pi/2} \frac{\sin^{\frac{5}{2}}x}{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}dx =$

1) $\frac{\pi}{2}$ 2) 0 3) $\frac{\pi}{4}$ 4) $\frac{\pi^2}{4}$

50. $\int_0^{\pi/4} \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}dx$

1) log 2 2) $\log\sqrt{2}$ 3) 2 log 2 4) 3 $\log\sqrt{2}$

51. $\int_0^1 \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx =$

1) 1 2) 0 3) 2 4) 5

52. $\int_0^2 |1-x|\,dx =$

1) 2 2) -2 3) 1 4) 5

53. $\int_0^{\pi/2} \frac{\sqrt{\cot x}\,dx}{\sqrt{\tan x}+\sqrt{\cot x}} =$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) π 4) 0

54. $\int_0^{\pi/2} \frac{\sin^{3/2}x\,dx}{\sin^{3/2}x+\cos^{3/2}x} =$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) π 4) 0

SELF TEST KEY

01) 4.	02) 4	03) 1	04) 1	05) 3
06) 2	07) 1	08) 2	09) 2	10) 4
11) 1	12) 4	13) 4	14) 4	15) 2
16) 2	17) 1	18) 1	19) 4	20) 1
21) 4	22) 4	23) 2	24) 1	25) 1
26) 2	27) 2	28) 3	29) 1	30) 1
31) 2	32) 2	33) 1	34) 4	35) 1
36) 1	37) 4	38) 1	39) 2	40) 3
41) 2	42) 3	43) 3	44) 2	45) 3
46) 2	47) 3	48) 3	49) 3	50) 3
51) 1	52) 3	53) 2	54) 2	

INGEGRATION IMPORTANT QUESTIONS

01. $\int_0^{\pi/2} \frac{\cos\theta}{1+\cos^2\theta} d\theta =$

1) $\frac{1}{\sqrt{2}}\log(1+\sqrt{2})$ 2) $\frac{1}{2}\log(1+\sqrt{2})$

3) $\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)$ 4) $\frac{1}{2}\log(\sqrt{2}-1)$

02. $\int \sin(\tan^{-1}x)\,dx =$

1) $\frac{1}{\sqrt{1+x^2}}$ 2) $\frac{1}{\sqrt{1-x^2}}$ 3) $\sqrt{1+x^2}$ 4) none

03. $\int \frac{dx}{e^x+e^{-x}} =$ __

1) $\tan^{-1}(e^x)+c$ 2) $\tan^{1}(e^x)+c$

3) $\tan^{1}(e^{-x})+c$ 4) $\tan^{-1}(e^{-x})+c$

04. $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx =$ __

1) a 2) a/2 3) a/3 4) a/4

05. The value of $\int_{-2\pi}^{2\pi} \cos^6 x\,dx$ is

1) $4\pi/5$ 2) $5\pi/4$ 3) $2\pi/5$ 4) $3\pi/4$

06. $\int (x+1)e^x\,dx =$

1) $x+e^x$ 2) $x.e^x$ 3) $2x+e^x$ 4) $2x.e^x$

07. $\int e^x[f(x)+f'(x)]dx =$

1) $e^x.f^1(x)$ 2) $e^x+f^1(x)$

3) $e^x.f(x)$ 4) $e^x+f(x)$

08. $\int_0^{\pi/2} \sin^4 x\cos^3 x\,dx =$

1) $\frac{8}{320}$ 2) $\frac{8}{315}$ 3) $\frac{8}{325}$ 4) none

09. $\int_0^{\pi/2} \frac{\sqrt[3]{\cot x}}{\sqrt[3]{\tan x}+\sqrt[3]{\cot x}}dx =$

1) $\pi/6$ 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/2$

10. $\int \frac{\cot x}{\log\sin x}dx =$ __

1) $\log(\sin x)$ 2) $\log\log(\sin x)$

3) $\log\log(\cot x)$ 4) none

11. $\int_0^{\pi/4} e^{2\log\sec x}dx =$ ___

1) 0 2) 1 3) -1 4) 2

12. $\int_0^{\pi/2} \frac{dx}{5+4\cos x} =$

1) $\frac{2}{3}\tan^{-1}\frac{1}{3}$ 2) $\frac{3}{2}\tan^{-1}\frac{2}{3}$

3) $\frac{1}{2}\tan^{-1}\frac{1}{3}$ 4) none

13. $\int \frac{dx}{(1+e^x)(1+e^{-x})} =$ ____

1) $\frac{1}{1+e^x}+c$ 2) $\frac{-1}{1+e^x}+c$

3) $\frac{1}{1+e^{-x}}+c$ 4) $\frac{-1}{1+e^{-x}}+c$

14. $\int_0^2 |1-x|dx =$ ___

1) 0 2) -1 3) 1 4) 2

15. $\int \frac{dx}{x(x^n+1)} =$ __

1) $\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right)+c$ 2) $\log\left(\frac{x^n}{x^n+1}\right)+c$

3) $\frac{1}{n}\log\left(\frac{x^n+1}{x^n}\right)+c$ 4) none

16. $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx =$

1) $\pi/4$ 2) ∞ 3) -1 4) 1

17. $\int \frac{dx}{1+e^x} =$ _____ + constant

1) $\log(1+e^x)$ 2) $x\log(1+e^x)$

3) $x+\log(1+e^x)$ 4) $x-\log(1+e^x)$

18. $\int \cos\sqrt{x}\,dx =$ ___ + constant

1) $\cos\sqrt{x}+\sqrt{x}\sin\sqrt{x}$

2) $2(\cos\sqrt{x}+\sqrt{x}\sin\sqrt{x})$

3) $\cos\sqrt{x}-\sqrt{x}\sin\sqrt{x}$

4) $2(\cos\sqrt{x}-\sqrt{x}\sin\sqrt{x})$

19. $\int_0^{\sqrt{3}} \frac{(\tan^{-1}x)^3}{1+x^2}dx =$

1) $\frac{\pi^4}{324}$ 2) $\frac{\pi^4}{64}$ 3) $\frac{\pi^4}{256}$ 4) $\frac{\pi^4}{81}$

20. If $f(x)=\int_0^x t\,\exp(t)dt$, then $f'(-1)=$

1) 1/e 2) -1/e 3) e 4) 2/e

21. The simplified form of $\int \frac{dx}{x^2+x^4}$ is equal to one of the following plus a constant

1) $\frac{-1}{x}\tan^{-1}x$ 2) $\frac{1}{x}\tan^{-1}x$

3) $\frac{-1}{x+\cot^{-1}\frac{1}{x}}$ 4) $\frac{1}{x+\cot^{-1}\left(\frac{1}{x}\right)}$

22. $\int \frac{dx}{e^x-1}$ is equal to one of the following plus a constant

1) $\ln(e^x-1)$ 2) $\ln(e^{x/2}-e^{-x/2})$

3) $\ln(1-e^{-x})$ 4) $\ln(e^x+1)$

23. $\int_0^1 \tan^{-1} dx + \int_0^{\pi/4} \tan x\,dx =$

1) 1 2) $\pi/3$ 3) 4 4) $\pi/4$

24. $\int_0^{\pi/2} \frac{dx}{1+\tan x} =$

1) $\pi/2$ 2) $\pi/4$ 3) $\pi/8$ 4) $\pi/16$

25. $\int_0^{\pi} |\cos\theta-\sin\theta|d\theta =$

1) 2 2) $2\sqrt{2}$ 3) -2 4) -4

26. $\int_0^{2a} x^2\sqrt{2ax-x^2}\,dx =$

1) $\frac{5a^4}{4}$ 2) $\frac{5\pi a^4}{4}$ 3) $32a^4$ 4) $16a^4$

27. $\int \frac{x^2-1}{1+x^4}dx =$

1) $\frac{1}{2\sqrt{2}}\log\frac{x^2-x+1}{x^2+x+1}+c$

2) $\frac{1}{2\sqrt{2}}\log\frac{x^2-x+\sqrt{2}}{x^2+x+\sqrt{2}}+c$

3) $\frac{1}{2\sqrt{2}}\log\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}+c$

4) $\frac{1}{2\sqrt{2}}\log\frac{x^2+x-1}{x^2-x-1}+c$

28. $\int e^x\left(\frac{2-\sin 2x}{1-\cos 2x}\right)dx =$

1) $-e^x\cot x+c$ 2) $e^x\operatorname{cosec}x+c$

3) $-e^x\tan x+c$ 4) $e^x\cot x+c$

29. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx=$

1) $\sqrt{\tan x}-\sqrt{\cot x}$ 2) $\tan^{-1}\left[\frac{\sqrt{\tan x}-\sqrt{\cot x}}{\sqrt{2}}\right]$

3) $\sqrt{2}\left[\tan^{-1}\left\{\frac{\sqrt{\tan x}-\sqrt{\cot x}}{\sqrt{2}}\right\}\right]$

4) $\tan^{-1}\left[\frac{\tan x+1}{\sqrt{2}}\right]$

30. $\int_0^{\pi/2}\frac{\sin 4x}{\sin x}dx=$

1) $\frac{2}{3}$ 2) $\frac{4}{3}$ 3) $\frac{1}{3}$ 4) $\frac{5}{3}$

31. $\int\frac{2\log(\tan x)}{\sin 2x}dx=$

1) $\frac{1}{2}\left[\log(\tan x)\right]^2+c$ 2) $\frac{1}{2}\left[\log(\tan x)\right]+c$

3) $\log(\tan x)+c$ 4) none of these

32. $\int_0^{\pi/2}\sin^3 x\cos^4 x\,dx=$

1) $\frac{5}{63}$ 2) $\frac{1}{63}$ 3) $\frac{2}{63}$ 4) none

33. $\int\frac{dx}{\sqrt{2-3x+x^2}}=$

1) $\cos h^{-1}(2x-3)+c$ 2) $\cosh^{-1}(3x-2)+c$

3) $\cos^{-1}(2x-3)+c$ 4) none of these

34. $\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{n^2+r^2}=$

1) $\frac{\pi}{2}$ 2) π 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

35. $\int_0^{\infty}\frac{dx}{(1+e^x)(1+e^{-x})}=$

1) $\frac{1}{2}$ 2) $-\frac{1}{2}$

3) 1 4) none of these

36. $\int\tan^{-1}x\,dx=$

1) $x\tan^{-1}x+\log(1+x^2)+c$

2) $x\tan^{-1}x-\frac{1}{2}\log(1+x^2)+c$

3) $x\tan^{-1}x+\frac{1}{2}\log(1+x^2)+c$

4) none of these

37. $\int_0^1\frac{x^3}{1+x^8}dx=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{8}$ 4) $\frac{\pi}{16}$

38. $\int_0^{\pi/4}\tan x\,dx=$

1) $\log\left(\frac{\sqrt{3}}{2}\right)$ 2) 0 3) $\log\sqrt{2}$ 4) 1

39. $\int_1^2\frac{1}{x\sqrt{x^2-1}}dx=$

1) $\frac{\pi}{3}$ 2) π 3) $\frac{\pi}{4}$ 4) 1

40. $\int_0^2|1-x|dx=$

1) π 2) 2 3) 1 4) 0

41. $\int\frac{\tan^{-1}x}{1+x^2}dx=$

1) $\frac{1}{2}\tan^{-1}x$ 2) $\frac{1}{2}(\tan^{-1}x)^2$ + constant

3) $(\tan^{-1}x)^2$ + constant 4) $\tan^{-1}x$ + constant

42. $\int\frac{\sin x^{1/4}}{4x^{3/4}}dx=$

1) $\cos t+c$ 2) 3) $\cos x^{1/4}$ 4) $\sin\sqrt{x}$

43. $\int\frac{dx}{\sqrt{x^2+2x+5}}=$

1) $\sin^{-1}\left(\frac{x+1}{2}\right)+c$ 2) $\sin^{-1}\left(\frac{x}{2}\right)+c$

3) $\sinh^{-1}\left(\frac{x+1}{2}\right)+c$ 4) $\cosh^{-1}\left(\frac{x+1}{2}\right)+c$

44. $\int x^2e^x\,dx=$

1) $e^x(x^2-1)+c$ 2) $e^x(x^2+2x+1)+c$

3) $e^x(x^2-2x+2)+c$ 4) xe^x+x^2+c

IMPORTANT QUESTIONS KEY

01) 2	02) 3	03) 1	04) 2	05) 2
06) 2	07) 2	08) 2	09) 2	10) 2
11) 2	12) 1	13) 2	14) 3	15) 1
16) 1	17) 4	18) 2	19) 1	20) 2
21) 1	22) 3	23) 4	24) 2	25) 2
26) 2	27) 3	28) 1	29) 3	30) 2
31) 1	32) 1	33) 1	34) 2	35) 1
36) 2	37) 4	38) 3	39) 1	40) 3
41) 2	42) 2	43) 3	44) 3	

PREVIOUS ECET BITS

2007

01. $\int\left(2+4x+3\sin(x)+4e^x\right)dx=$

1) $2x+2x^2-3\cos(x)+4e^x+e$

2) $2+4x+3\sin(x)+4e^x+e$

3) $2x+2x^2+3\cos(x)-4e^x+e$

4) $2x+4x^2+3\cos(x)+4e^x+e$

02. $\int x^3e^{x^4}dx=$

1) $e^{x^4}+e$ 2) $\frac{1}{2}x^3e^{x^4}$

3) $x^3e^{x^4}+e$ 4) $\frac{1}{4}e^{x^4}+e$

03. $\int xe^x dx=$

1) xe^x+e 2) e^x+e

3) xe^x-e^x 4) $(1+x)e^x+e$

04. $\int_a^b C\,dx=$

1) $c(b-a)$ 2) $c(b+a)$

3) cab 4) $a+b+c$

05. $\int\cot(x)dx=$

1) $\ln|\cos x|+c$ 2) $\ln|\sin x|+c$

3) $\ln|\tan x|+c$ 4) $\ln|\sec x|+c$

06. $\int 1/(x+2)^4dx=$

1) $4\left[1/(x+2)^3\right]+e$ 2) $\left[4/(x+2)\right]+e$

3) $-\left[1/3(x+2)^3\right]+e$ 4) $\left[1/(x+2)\right]+e$

07. $\int_1^3(2x^2+2x+1)dx=$

1) 27 2) 28 3) $27\frac{1}{3}$ 4) 26

2008

08. $\int\cot^2 dx=$

1) $\cot x-x+c$ 2) $\cot x+x+c$

3) $-\cot x+x+c$ 4) $-\cot x-x+c$

09. $\int\frac{1}{e^x+e^{-x}}dx=$

1) $\log(e^x+e^{-x})+c$ 2) $\tan^{-1}(e^x)+c$

3) $\log(e^{2x}+1)+c$ 4) $\sin^{-1}(ex)$

10. The smallest interval $[a,b]$ such that $\int_0^1\frac{dx}{\sqrt{1+x^4}}\in[a,b]$ is given by

1) $\left[\frac{1}{\sqrt{2}},1\right]$ (2) $[0,1]$

(3) $\left[\frac{1}{2},1\right]$ (4) $\left[\frac{3}{4},1\right]$

11. $\int \frac{e^x(x\log x+1)}{x}dx=$

1) $xe^x\log x+c$
2) $\frac{e^x\log x}{x}+c$
3) $\frac{e^x}{\log x}+c$
4) $e^x\log x+c$

12. $\int_{-\pi}^{\pi}\sin^5\theta\cos^3\theta\,d\theta=$

1) $\frac{1}{80}$
2) $\frac{\pi}{80}$
3) 0
4) $\frac{\pi}{40}$

2009

13. The value of $\int_0^{\pi/2}\sin^2 x\,dx$ is

1) $-\frac{\pi}{2}$
2) 0
3) $\frac{\pi}{4}$
4) $\frac{\pi}{2}$

14. $\int x^3(\log x)^2dx=$

1) $\frac{1}{32}x^4\left[8(\log x)^2+4(\log x)-1\right]$
2) $\frac{1}{32}x^4\left[8(\log x)^2+4\log x+1\right]$
3) $\frac{1}{32}x^4\left[8(\log x)^2-4\log x-1\right]$
4) $\frac{1}{32}x^4\left[8(\log x)^2-4\log x+1\right]+c$

15. If $m\neq n$, then $\int_0^{\pi}\cos mx\cos nx\,dx$ is

1) 0
2) $\frac{\pi}{2}$
3) π
4) 2π

16. $\int(\tan x+\cot x)^2dx=$

1) $\tan x+\cot x+c$
2) $\cot x-\tan x+c$
3) $\tan x-\cot x+c$
4) $-\cot x-\tan x+c$

2010

17. $\int\frac{x^4}{x^2+1}dx=$

1) $\frac{x^3}{3}-x+\tan^{-1}x+c$
2) $\frac{x^3}{3}-x-\tan^{-1}x+c$
3) $\frac{x^3}{3}+x+\tan^{-1}x+c$
4) $x-\frac{x^3}{3}+\tan^{-1}x+c$

18. $\int\tan^{-1}dx=$

1) $\tan^{-1}x-\frac{1}{2}\log(1+x^2)+c$
2) $x\tan^{-1}x+\frac{1}{2}(1+x^2)+c$
3) $x\tan^{-1}x-\frac{1}{2}\log(1+x^2)+c$
4) $x\tan^{-1}x-\log(1+x^2)+c$

19. $\int_0^a\sqrt{a^2-x^2}\,ax=$

1) $\frac{a^2}{4}$
2) πa^2
3) $\frac{\pi a^2}{2}$
4) $\frac{\pi a^2}{4}$

20. $\int e^x\sqrt{e^x+1}\,dx=$

1) $(1+e^x)^{3/2}+c$
2) $\frac{2}{3}(1-e^x)^{3/2}+c$
3) $(1-e^x)^{3/2}+c$
4) $\frac{2}{3}(1+e^x)^{3/2}+c$

21. $\int\frac{\cos\sqrt{x}}{\sqrt{x}}dx=$

1) $2\sin\sqrt{x}+c$
2) $\cos^2\sqrt{x}+c$
3) $\cos^2\sqrt{x}+c$
4) $2\cos\sqrt{x}+c$

22. $\int\frac{dx}{x^2-5x+6}=$

1) $\log\left|\frac{x+3}{x+2}\right|c$
2) $\log\left|\frac{x-3}{x-2}\right|c$
3) $\log\left|\frac{x-2}{x-3}\right|c$
4) $\log\left|\frac{x+2}{x+3}\right|c$

2011

23. $\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x}=$

1) $\frac{1}{2}\log(-1)$
2) $\log(-1)$
3) $\log 3$
4) $\log\sqrt{3}$

24. $\int_{-1}^{1}|1-x|\,dx=$

1) -2
2) 0
3) 2
4) 4

25. $\int\frac{dx}{e^x+e^{-x}}=$

1) $\log(e^x+e^{-x})$
2) $\tan^{-1}(e^x)$
3) $\log(e^{2x}+1)$
4) e^{3x}

26. $\int\frac{\log x^2}{x}dx=$

1) $\log(x)^2+c$
2) $\frac{1}{2}(\log x)^2+c$
3) $\log x^2+c$
4) $\log x+c$

27. $\int\log x\,dx=$

1) $x+x\log x$
2) $x\log x-x$
3) $x^2\log x$
4) $\frac{\log x}{x}+x$

28. $\int e^x[\phi(x)+\phi'(x)]dx$ is

1) $\int e^x\phi'(x)dx$
2) $e^x\phi(x)+c$
3) $e^x\phi'(x)+c$
4) $\phi'(x)e^{-x}$

29. If $(a-x)=f(x)$ then $\int_0^a xf(x)\,dx=$

1) $\frac{a}{2}\int_0^a f(x)\,dx$
2) $a\int_0^a f(x)\,dx$
3) $\frac{a^2}{2}\int_0^a f(x)\,dx$
4) $a^3\int_0^a f(x)\,dx$

2012

30. $\int\operatorname{cosec}x\,dx=$

1) $\log(\operatorname{cosec}x+\cot x)+C$
2) $\log(\cot x/2)+C$
3) $\log(\tan x/2)+C$
4) $-\operatorname{cosec}x.\cot x+C$

31. $\int_0^{\frac{\pi}{2}}\cos^{11}x\,dx=$

1) $\frac{256}{693}$
2) $\frac{256\pi}{693}$
3) $\frac{\pi}{4}$
4) $\frac{128}{693}$

32. $\int f^1(x).[f(x)]^n\,dx=$

1) $\frac{[f(x)]^{n-1}}{n-1}+C$
2) $\frac{[f(x)]^{n+1}}{n+1}+C$
3) $n[f(x)]^{n-1}+C$
4) $(n+1)[f(x)]^{n+1}+C$

33. $\int\frac{dx}{(x+7)\sqrt{x+6}}=$

1) $\tan^{-1}(\sqrt{x+6})+C$
2) $2\tan^{-1}(\sqrt{x+6})+C$
3) $\tan^{-1}(x+7)+C$
4) $2\tan^{-1}(x+7)+C$

34. $\int\tan^{-1}x\,dx=$

1) $x.\tan^{-1}+\frac{1}{2}\log(1+x^2)+C$
2) $\frac{1}{1+x^2}+C$
3) $x^2.\tan^{-1}+C$
4) $x.\tan^{-1}x-\log\sqrt{1+x^2}+C$

35. $\int\frac{dx}{1+e^{-x}}=$

1) $\log(1+e^{-x})+C$
2) $\log(1+e^x)+C$
3) $e^{-x}+C$
4) e^x+C

36. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin|x|\,dx=$

1) 0
2) 1
3) 2
4) -1

2013

37. $\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx=$

1) $\sin x-\cos x+c$ 2) $\cos 2x+c$
3) $\sin 2x+c$ 4) $x+c$

38. $\int_0^{\infty} \frac{dt}{t^2+2t+2}$

1) $\pi/2$ 2) $\pi/4$ 3) 0 4) ½

39. $\int e^x \frac{(1+x\log x)}{x}dx=$

1) $xe^x\log x+c$ 2) $x\log x+c$
3) $e^x\log x+c$ 4) $-x\log x+e^x+c$

40. $\int_{-2}^{3}(4x+3x^2+7x^3+12x^7)dx=$

1) 432 2) 516 3) 1132 4) 16

2014

41. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx=$

1) $\sqrt{x}\sin\sqrt{x}+c$ 2) $2\sin\sqrt{x}+c$
3) $\sqrt{\cos x}+c$ 4) $\frac{\sin\sqrt{x}}{\sqrt{x}}+c$

42. $\int\left(\frac{x+2}{x+1}\right)dx=$

1) $x\log(x+1)+c$
2) $x\log(x+1)+2\log(x+1)+c$
3) $x+\log(x+1)+c$
4) $\frac{1}{x}\log(x+1)+c$

43. $\int \frac{x^2}{\sqrt{1+x^6}}dx=$

1) $\frac{1}{2}\sin^{-1}(x^3)+c$ 2) $2\cos^{-1}(x^3)+c$
3) $\frac{1}{2}\cosh^{-1}(x^3)+c$ 4) $\frac{1}{3}\sinh^{-1}(x^3)+c$

44. $\int 8x^3e^{2x}dx=$

1) $(4x^3-6x^2+6x-3)e^{2x}+c$
2) $4x^3+6x^2+6x+3e^{2x}+c$
3) $\left(\frac{4x^2}{3}-\frac{2}{3}x+\frac{1}{3}\right)e^{2x}+c$ 4) $\left(\frac{4x^2}{3}+\frac{2}{3}x-\frac{1}{3}\right)e^{2x}+c$

45. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) 0 4) 2

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46. $\int \frac{\sin x-\cos x}{\sqrt{\sin 2x}}dx=$

1) $-\cosh^{-1}(\sin x+\cos x)$
2) $-\cosh^{-1}(\sin x-\cos x)$
3) $-\cosh^{-1}(\sin x+\cos x)$
4) $-\cosh^{-1}(\sin x-\cos x)$

47. $\int \frac{\sqrt{x-1}}{x}dx=$

1) $\frac{2}{5}\log 2$ 2) $\frac{1}{5}\log 2$
3) $\frac{5}{2}\log 5$ 4) $\frac{1}{2}\log 5$

48. $\int_0^1 \frac{\sin^{-1}}{x}dx=$

1) $\frac{\pi}{2}\log 2$ 2) $\frac{\pi}{4}\log 4$
3) $\frac{\pi}{6}\log 6$ 4) $\frac{\pi}{8}\log 8$

49. $\int_0^{\pi} \frac{1}{5+4\cos x}dx=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) π

TS ECET - 2015

50. $\int e^x dx$

1) e^x 2) e^x+c 3) e 4) log x

51. $\int \frac{1}{x}dx=$

1) e^x 2) logx+c 3) log x 4) 1/x

52. $\int e^{\log x}dx=$

1) $e^{\log x}$ 2) $e^{\log x}+c$ 3) $\frac{x^2}{2}+c$ 4) x

53. $\int \log e^x dx=$

1) e^{-x} 2) $e^{-x}+C$ 3) $\frac{x^2}{2}+c$ 4) x

54. $\int \sin x dx=$

1) 0 2) 1 3) 2 4) - cos x

55. $\int_0^{\pi/2} e^{\sin^2 x}\sin 2x dx=$

1) 0 2) e 3) e-1 4) 1

56. $\int_{-\pi/2}^{\pi/2} \cos x dx=$

1) 0 2) 1 3) 2 4) sin x

57. $\int_{-\pi/2}^{\pi/2} \log(\tan x)dx=$

1) 0 2) 1 3) 2 4) cot x

A.P ECET - 2016

58. $\int \operatorname{cosec}^5\theta\cot\theta d\theta=$

1) $\frac{\cot^2\theta}{2}$ 2) $\frac{-\operatorname{cosec}^5\theta}{5}$
3) $\frac{\operatorname{cosec}^6\theta}{6}$ 4) $\frac{-\operatorname{cosec}^6\theta}{6}$

59. $\int_2^3 \frac{dx}{x^2-x}=$

1) $\log\frac{2}{3}$ 2) $\log\frac{4}{3}$ 3) $\log\frac{8}{3}$ 4) $\log\frac{1}{4}$

60. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin|x|dx=$

1) 0 2) 2sinx 3) 2 4) 1

61. $\int_0^1 x\tan^{-1}x dx=$

1) $\frac{\pi}{4}-\frac{1}{2}$ 2) $\frac{\pi}{8}-\frac{1}{2}$ 3) $\frac{\pi}{4}+\frac{1}{2}$ 4) $\frac{\pi}{8}+\frac{1}{2}$

62. $\int_0^{\pi/4} \sec^4 x dx=$

1) $\frac{8}{3}$ 2) $\frac{28}{15}$ 3) $-\frac{28}{15}$ 4) $\frac{4}{5}$

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63. The value of $\int x^2\sqrt{1+x^3}dx=$

1) $\frac{1}{9}(1+x^3)^{\frac{2}{3}}+C$ 2) $\frac{2}{9}(1+x^3)^{\frac{3}{2}}+C$
3) $\frac{1}{3}(1+x^3)^{\frac{2}{3}}+C$ 4) $(1+x^3)^{\frac{3}{2}}+C$

64. $\int \frac{1}{(e^x+e^{-x})^2}dx=$

1) $\frac{-1}{2(e^{2x}+1)}+C$ 2) $\frac{1}{e^{2x}+1}+C$
3) $\frac{e^x}{1+e^{-x}}+C$ 4) e^x+C

2013

37. $\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx=$

1) $\sin x-\cos x+c$ 2) $\cos 2x+c$
3) $\sin 2x+c$ 4) $x+c$

38. $\int_0^{\infty}\frac{dt}{t^2+2t+2}$

1) $\pi/2$ 2) $\pi/4$ 3) 0 4) ½

39. $\int e^x\frac{(1+x\log x)}{x}dx=$

1) $xe^x\log x+c$ 2) $x\log x+c$
3) $e^x\log x+c$ 4) $-x\log x+e^x+c$

40. $\int_{-1}^{2}(4x+3x^2+7x^3+12x^7)dx=$

1) 432 2) 516 3) 1132 4) 16

2014

41. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx=$

1) $\sqrt{x}\sin\sqrt{x}+c$ 2) $2\sin\sqrt{x}+c$
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42. $\int\left(\frac{x+2}{x+1}\right)dx=$

1) $x\log(x+1)+c$
2) $x\log(x+1)+2\log(x+1)+c$
3) $x+\log(x+1)+c$
4) $\frac{1}{x}\log(x+1)+c$

43. $\int\frac{x^2}{\sqrt{1+x^6}}dx=$

1) $\frac{1}{2}\sin^{-1}(x^3)+c$ 2) $2\cos^{-1}(x^3)+c$
3) $\frac{1}{2}\cosh^{-1}(x^3)+c$ 4) $\frac{1}{3}\sinh^{-1}(x^3)+c$

44. $\int 8x^3e^{2x}dx=$

1) $(4x^3-6x^2+6x-3)e^{2x}+c$
2) $4x^3+6x^2+6x+3e^{2x}+c$
3) $\left(\frac{4x^2}{3}-\frac{2}{3}x+\frac{1}{3}\right)e^{2x}+c$ 4) $\left(\frac{4x^2}{3}+\frac{2}{3}x-\frac{1}{3}\right)e^{2x}+c$

45. $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) 0 4) 2

A.P ECET - 2015

46. $\int\frac{\sin x-\cos x}{\sqrt{\sin 2x}}dx=$

1) $-\cosh^{-1}(\sin x+\cos x)$
2) $-\cosh^{-1}(\sin x-\cos x)$
3) $-\cosh^{-1}(\sin x+\cos x)$
4) $-\cosh^{-1}(\sin x-\cos x)$

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1) $\frac{2}{5}\log 2$ 2) $\frac{1}{5}\log 2$
3) $\frac{5}{2}\log 5$ 4) $\frac{1}{2}\log 5$

48. $\int_0^1\frac{\sin^{-1}}{x}dx=$

1) $\frac{\pi}{2}\log 2$ 2) $\frac{\pi}{4}\log 4$
3) $\frac{\pi}{6}\log 6$ 4) $\frac{\pi}{8}\log 8$

49. $\int_0^{\pi}\frac{1}{5+4\cos x}dx=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) π

TS ECET - 2015

50. $\int e^x dx$

1) e^x 2) e^x+c 3) e 4) log x

51. $\int\frac{1}{x}dx=$

1) e^x 2) logx+c 3) log x 4) 1/x

52. $\int e^{\log x}dx=$

1) $e^{\log x}$ 2) $e^{\log x}+c$ 3) $\frac{x^2}{2}+c$ 4) x

53. $\int\log e^x dx=$

1) e^{-x} 2) $e^{-x}+C$ 3) $\frac{x^2}{2}+c$ 4) x

54. $\int\sin x dx=$

1) 0 2) 1 3) 2 4) - cos x

55. $\int_0^{\pi/2}e^{\sin^2 x}\sin 2x dx=$

1) 0 2) e 3) e-1 4) 1

56. $\int_{-\pi/2}^{\pi/2}\cos x dx=$

1) 0 2) 1 3) 2 4) sin x

57. $\int_{-\pi/2}^{\pi/2}\log(\tan x)dx=$

1) 0 2) 1 3) 2 4) cot x

A.P ECET - 2016

58. $\int \operatorname{cosec}^5\theta\cot\theta d\theta=$

1) $\frac{\cot^2\theta}{2}$ 2) $\frac{-\operatorname{cosec}^5\theta}{5}$
3) $\frac{\operatorname{cosec}^6\theta}{6}$ 4) $\frac{-\operatorname{cosec}^6\theta}{6}$

59. $\int_2^3\frac{dx}{x^2-x}=$

1) $\log\frac{2}{3}$ 2) $\log\frac{4}{3}$ 3) $\log\frac{8}{3}$ 4) $\log\frac{1}{4}$

60. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin|x|dx=$

1) 0 2) 2sinx 3) 2 4) 1

61. $\int_0^1 x\tan^{-1}x dx=$

1) $\frac{\pi}{4}-\frac{1}{2}$ 2) $\frac{\pi}{8}-\frac{1}{2}$ 3) $\frac{\pi}{4}+\frac{1}{2}$ 4) $\frac{\pi}{8}+\frac{1}{2}$

62. $\int_0^{\pi/4}\sec^4 x dx=$

1) $\frac{8}{3}$ 2) $\frac{28}{15}$ 3) $-\frac{28}{15}$ 4) $\frac{4}{5}$

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63. The value of $\int x^2\sqrt{1+x^3}dx=$

1) $\frac{1}{9}(1+x^3)^{\frac{2}{3}}+C$ 2) $\frac{2}{9}(1+x^3)^{\frac{3}{2}}+C$
3) $\frac{1}{3}(1+x^3)^{\frac{2}{3}}+C$ 4) $(1+x^3)^{\frac{3}{2}}+C$

64. $\int\frac{1}{(e^x+e^{-x})^2}dx=$

1) $\frac{-1}{2(e^{2x}+1)}+C$ 2) $\frac{1}{e^{2x}+1}+C$
3) $\frac{e^x}{1+e^{-x}}+C$ 4) e^x+C

65. $\int \frac{x^5}{1+x^{12}}dx =$

1) $\frac{1}{6}\tan^{-1}(x^6)+C$ 2) $\frac{1}{3}\tan^{-1}(x^3)+C$

3) $\frac{1}{6}\cot^{-1}(x^3)+C$ 4) $\frac{1}{9}\tan^{-1}(x^3)+C$

66. $\int_0^{\frac{\pi}{2}} \frac{\cos x\,dx}{(1+\sin x)(2+\sin x)} =$

1) $\log\left(\frac{1}{3}\right)$ 2) $\log\left(\frac{2}{3}\right)$

3) $\log\left(\frac{4}{3}\right)$ 4) $\log 2$

67. $\int_0^1 \frac{\log(1-x)}{x}dx =$

1) $\frac{\pi^2}{2}$ 2) $\frac{\pi^2}{3}$ 3) $\frac{\pi^2}{12}$ 4) $\frac{-\pi^2}{6}$

68. $\int \frac{xe^x}{(x+1)^2}dx =$

1) $\frac{e^x}{e^x+1}+C$ 2) $\frac{e^{-x}}{1+x^2}+C$

3) $\frac{e^x}{1+x}+C$ 4) $\frac{e^{-x}}{1+x}+C$

69. $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)d\theta$

1) $\frac{1}{8}\log 2$ 2) $\frac{\pi}{2}\log 2$

3) $\frac{\pi}{8}\log 2$ 4) $\frac{-\pi}{6}\log 2$

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70. $\int \operatorname{cosec}^5\theta \cot\theta\, d\theta =$ ______

1) $\frac{\cot^2\theta}{2}$ 2) $\frac{-\operatorname{cosec}^5\theta}{5}$

3) $\frac{\operatorname{cosec}^6\theta}{6}$ 4) $\frac{-\operatorname{cosec}^6\theta}{6}$

71. $\int_2^3 \frac{dx}{x^2-x} =$

1) $\log\frac{2}{3}$ 2) $\log\frac{4}{3}$ 3) $\log\frac{8}{3}$ 4) $\log\frac{1}{4}$

72. If $a<0<b$ then $\int_a^b \frac{|x|}{x}dx =$ ______

1) b-a 2) a-b 3) a+b 4) 0

73. $\int_0^1 x\tan^{-1}x\,dx =$ ______

1) $\frac{\pi}{4}-\frac{1}{2}$ 2) $\frac{\pi}{8}-\frac{1}{2}$ 3) $\frac{\pi}{4}+\frac{1}{2}$ 4) $\frac{\pi}{8}+\frac{1}{2}$

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74. $\int \frac{a}{b+ce^x}dx =$

1) $\frac{a}{b}\log\left(\frac{e^x}{b+ce^x}\right)+C$

2) $\frac{b}{a}\log\left(\frac{e^{-x}}{b+e^{-x}}\right)+C$

3) $\frac{a}{b}\log\left(\frac{1}{be^x+ce^{-x}}\right)+C$ 4) $\frac{b}{a}e^{(b+ce^x)}+C$

75. $\int \frac{1}{(1+x^2)\tan^{-1}x}dx =$

1) $\tan^{-1}x+C$ 2) $\cot^{-1}x+C$

3) log(secx)tanx+C 4) $\log(\tan^{-1}x)+C$

76. $\int \frac{\cos(\log x^2)}{x^4}dx =$

1) $\frac{1}{x^3}Cos\left[\log x^2+\tan^{-1}\left(\frac{3}{2}\right)\right]+C$

2) $\frac{x^3}{\sqrt{13}}Cos\left[\log x^2+\cot^{-1}\left(\frac{2}{3}\right)\right]+C$

3) $\frac{-1}{2x^3}Cos\left[\log x^2+\tan^{-1}\left(\frac{2}{3}\right)\right]+C$

4) $\frac{1}{x^3\sqrt{13}}Cos\left[\log x^2+\cot^{-1}\left(\frac{3}{2}\right)\right]+C$

77. $\int \frac{dx}{e^x-1} =$

1) $\log\left(\frac{1-e^x}{e^x}\right)+C$ 2) $\log(e^x-1)+C$

3) $\log\left(\frac{e^x-1}{e^x}\right)+C$ 4) $\log\left(\frac{e^{-x}-1}{e^{-x}}\right)+C$

78. $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx =$

1) sec x+cot x 2) cosecx- cotx

3) cosecx + tanx 4) sec x-cosecx

79. $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x}dx$

1) e-1 2) $e^{-1}-1$ 3) $e^{-1}+1$ 4) $e^{-2}-1$

80. $\int_0^{\pi} \sin^3 x(1-\cos x)^2 dx =$

1) 5/3 2) 8/5 3) 1 4) 0

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81. The value of $\int \frac{dx}{4x^2+4x+17}$ is

1) $\frac{1}{8}\tan^{-1}\left(\frac{2x+1}{4}\right)+C$

2) $\frac{1}{4}\cot^{-1}\left(\frac{2x+1}{4}\right)+C$

3) $\frac{1}{8}\sin^{-1}\left(\frac{2x+1}{4}\right)+C$

4) $\frac{1}{3}\tan^{-1}\left(\frac{2x+1}{4}\right)+C$

82. The value of $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$ is

1) $2\sin\sqrt{x}+c$ 2) $3\sin\sqrt{x}+c$

3) 2sinx+c 4) $\sin\sqrt{x}+c$

83. The value of $\int_0^{\pi/2} \sin^2 x\,dx$ is

1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{4}$

84. The value of $\int \log x\,dx$ is

1) $x\log x+x+c$ 2) $x^2\log x-x+c$

3) $x\log x-x+c$ 4) $x\log x-\frac{x^2}{2}+c$

85. The value of $\int \frac{dx}{\sqrt{a^2-x^2}}$ is

1) $\cos^{-1}\left(\frac{x}{a}\right)+c$ 2) $\sin^{-1}\left(\frac{x}{a}\right)+c$

3) $\sinh^{-1}\left(\frac{x}{a}\right)+c$ 4) $\sin^{-1}\left(\frac{a}{x}\right)+c$

86. The value of $\int_1^4 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$ is

1) $\frac{20}{3}$ 2) $-\frac{20}{3}$ 3) $\frac{10}{3}$ 4) $\frac{15}{3}$

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87. $I_1=\int_0^\infty e^{-x}x^n dx$, then $\int_0^\infty e^{-x^2}x^{2n+1}dx =$

1) 0 2) $\frac{I_1}{2}$ 3) $\frac{I_1}{3}$ 4) $2I_1$

88. If $\int \frac{\sin 2x}{\sin 5x\sin 3x}dx =$ A log sin 3x+Blog sin5x+C, then A+B=

1) 2/7 2) 1/3 3) -2/5 4) 2/15

89. If $0<x<\frac{\pi}{2}$, then $\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx =$

1) $\frac{1}{x}+c$ 2) x+c 3) 2x+c 4) $\frac{2}{x}+c$

90. $\int \frac{x^4+1}{x^2+1}dx =$

1) $\frac{x^3}{3}+x+2Tan^{-1}x+c$

2) $\frac{x^3}{3}+x+Tan^{-1}x+c$

3) $\frac{x^3}{3}-x+2Tan^{-1}x+c$

4) $\frac{x^3}{3}-x+Tan^{-1}x+c$

91. $\int \frac{e^x(1-x)}{x^2}dx =$

1) $-\frac{1}{xe^x}+C$ 2) $\frac{1}{xe^x}+C$

3) $-\frac{1}{x}e^x+C$ 4) xe^x+C

92. $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}dx =$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{8}$ 4) π

93. $\int_0^{\pi/2} \sin^4 x\cos^2 x\, dx=$

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{32}$ 3) $\frac{\pi}{42}$ 4) $\frac{\pi}{2}$

PREVIOUS ECET BITS

01) 1	02) 4	03) 4	04) 1	05) 2
06) 3	07) 3	08) 4	09) 2	10) 1
11) 4	12) 3	13) 3	14) 4	15) 1
16) 3	17) 1	18) 3	19) 4	20) 4
21) 1	22) 2	23) 4	24) 3	25) 2
26) 1	27) 2	28) 2	29) 1	30) 3
31) 2	32) 2	33) 2	34) 4	35) 2
36) 3	37) 4	38) 2	39) 3	40) 4
41) 2	42) 3	43) 4	44) 1	45) 2
46) 1	47) 4	48) 1	49) 2	50) 2
51) 2	52) 3	53) 3	54) 3	55) 3
56) 3	57) 1	58) 1	59) 2	60) 3
61) 3	62) 2	63) 2	64) 1	65) 1
66) 3	67) 4	68) 3	69) 3	70) 2
71) 2	72) 3	73) 3	74) 1	75) 4
76) 1	77) 3	78) 4	79) 1	80) 2
81) 1	82) 1	83) 4	84) 3	85) 2
86) 1	87) 2	88) 4	89) 2	90) 3
91) 3	92) 1	93) 2		

SPACE FOR IMPORTANT NOTES

APPLICATIONS OF THE DEFINITE INTEGRAL

SYNOPSIS

- The area bounded by the curve $y=f(x)$, the axis and the ordinates x = a, x = b is given by $\int_a^b y\,dx$.
- The area bounded by the curve x = f(y), the y-axis and the abscissae y = a, y = b is given by $\int_a^b x\,dx$.
- The area bounded by a curve, the x-axis and two ordinates is called the area under the curve.
- The process of finding the area bounded by a given curve is often called quadrature.
- i) The area $\int_a^b f(x)$ dx is positive if y = (x) > 0 over the range $a \le x \le b$.

  ii) The area $\int_a^b f(x)$ dx is negative if y = f(x) < 0 over the range $a \le x \le b$. Thus we consider the areas below the x-axis as negative.
- The area bounded by the curves y = f(x) and y = g(x) and the ordinates x = a, x = b (b>a) is given by

  $\int_a^b |f(x)-g(x)|\,dx$ *i.e.,* $\int_a^b$(y of upper curve - y of lower curve)dx
- i) The area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is π ab sq. units

  ii) The area of the circle $x^2+y^2=a^2 is \pi a^2$

  iii) The area of one arch of y = sin px and y = cos px is 2/p.
- The area bounded by one arch of the curve y = sin ax or y = cos ax and x-axis is $\frac{2}{a} sq\,units$.
- The area bounded by the parabola $y^2=4ax$ and its latus-rectum (x=a) is $\frac{8a^2}{3}$ sq. units.
- i) The area enclosed between the parabolas $y^2=4ax$ and $x^2=4by$ is $\frac{16ab}{3} sq\,units$.

  ii) The area included between the parabolas $y^2=4ax$ and $x^2=4ay is \frac{16a^2}{3}$ sq. units.
- The area common to the two ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 and \frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is 4ab $\tan^{-1}\frac{b}{a}$.

- The area between $y^2 = 4ax$ and y = mx is $\frac{8a^2}{3m^3}$ sq.units.
- The area enclosed between the parabolas $y^2 = 4a(x+a)$ and $y^2 = 4b(b-x)$ is $\frac{8}{3}\sqrt{ab}$ (a+b)
- The area common to the two curves $y^2 = ax$ and $x^2 + y^2 = 4ax$ is $a^2\left(3\sqrt{3} + \frac{4\pi}{3}\right)$
- The whole area of the astroid $x^{2/3} + y^{2/3}$ or x=a $\cos^3\theta, y = a\ \sin^3\theta$ is $\frac{3\pi a^2}{8}$ sq. units.
- The whole area of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ $[or\, x = a\cos^3\theta, y = b\sin^3\theta]$ is $\frac{3\pi a^2}{8}$.
- The area enclosed by the curve $xy^2 = a^2$ (a-x) and y-axis is πa^2.
- The area enclosed between one arch of the cycloid x=a $(\theta - \sin\theta)[or\, x = a(\theta + \sin\theta)]$, y=a $(1-\cos\theta)$ and its base is $3\pi a^2 sq.units$.
- The area lying above x-axis and included between the circle $x^2 + y^2 = 2ax$ and the parabola $y^2 = ax$ is $a^2\left(\frac{\pi}{4} - \frac{2}{3}\right)$
- i) Area bounded by $|x| + |y| = 1$ is 2 sq. units. ii) Area bounded by $|x| + |y| = 2$ is 8 sq. units.
- Area bounded by $\frac{|x|}{a} + \frac{|y|}{b} = 1$ where a, b > 0 is 2ab.
- The area of the region bounded by the curve y = sin ax and x - axes in $[0, n\pi]$ is 2n/a.
- The area of the region bunded by the curve y = cos ax and x-axes in $[0, n\pi]$ is 2n/a.
- The area of the region bounded by the parabola $y = ax^2 + bx + c$ and x-axis is $\frac{(b^2 - 4ac)^{3/2}}{6a^2}$
- The area of the region bounded by the parabola $x = ay^2 + by + c$ and y-axis is $\frac{(b^2 - 4ac)^{3/2}}{6a^2}$
- The area bounded between $y = \sin x; y = \cos x$ and y-axis is $\sqrt{2} - 1 Sq.units$
- The area bounded between any two point of intersection by $y = \sin x$ and $y = \cos x$ with x-axis is $2\sqrt{2}$ sq.units
- Area of one of the curve linear triangle formed between y = sinx y = cosx with x-axis is $2 - \sqrt{2}$ sq.units
- Area bounded by y = sinx ; y = cos x from x = 0 to ∏/2 is $2(\sqrt{2} - 1)$ sq. units
- Area bounded by y = sin x ; y = cos x from x = 0 to ∏ is 1 sq. units

Volumes of solids of Revolution:

- i) The volume of the solid generated by the revolution about the x-axis, of the area bounded by the curve y=f(x), the x-axis and the ordinates x=a, x=b is $\int_a^b \pi y^2\, dx$.

  ii) If, the axis of revolution is not the x-axis, but a line, say y=k parallel to x-axis then the volume will be $\pi\int_a^b (y-k)^2\, dx$.
- i) The volume of the solid generated by the revolution about y-axis of the area, bounded by the curve x=f(y) the y-axis and the abscissa y=a, y = b is $\int_a^b \pi x^2\, dy$.

  ii) If the axis of revolution is x = h, a line parallel to the y - axis, then volume will be $\pi\int_a^b (x-h)^2\, dy$.
- The volume of the solid generated by the revolution of the area bounded by the curves

  i) $y_1 = f(x), y_2 = g(x)$, the x-axis and the lines x = a, x = b is givne by $\pi\int_a^b (y_1^2 - y_2^2)dx$.

  ii) $x_1 = f(x), y_2 = g(y)$, the y-axis and the lines y = a, y = b is given by $\pi\int_a^b (x_1^2 - x_2^2)$ dy.
- The volume formed when the part of the parabola cut off by the latus-rectum is rotaed about the (i) latus-rectum is $\frac{32\pi a^3}{15}$ ii) about the axis is $4\pi a^3$.
- The volume of the solid generated gy the evolution about x-axis of the area, bounded by the parabola $y^2 = 4ax$ and its latus-rectum is $2\pi a^3$ cu units.
- The volume of the ellipsoid formed by the rotation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its

  i) major axis is $\frac{4}{3}\pi ab^2$ ii) minor axis is $\frac{4}{3}\pi a^2 b$ cu units.
- The volume of a sphere of radius a is $\frac{4}{3}\pi a^3$ cu units.
- The volume of a sphere of radis a is $\frac{4}{3}\pi a^3$ cu units.
- The volume generated by revolving the area of the parabola $y^2 = 4ax$ bounded by the ordinate x = h about its axis is $2\pi ah^2$.
- i) The volume of a right cirular cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$.

ii) The volume of a right cirular cone of height h and semi-vertical angle α is $\frac{\pi}{3}h^3\tan^2\alpha$.

- The volume of a spherical cap of height h cut off from a sphere of radius 'a' is $\pi h^2\left(a-\frac{h}{3}\right)$.
- The volume of the solid generated by the revolution of the area bounded by the parabola $y^2=4ax$ and $x^2=4ay$ about x-axis is $\frac{96\pi a^3}{5}$.

Mean and R.M.S. Values:

- Mean values of the function f(x) in [a, b] is given by $\frac{1}{b-a}\int_a^b f(x)dx$.
- Mean Square value of the function y = f(x) over the range x = a to x = b is $\frac{1}{b-a}\int_a^b [f(x)]^2dx$.
- Root mean square (R.M.S) value of the function y = f(x) in [a, b] is $\sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2dx}$ i.e, $\sqrt{\text{Mean Square Value}}$
- The mean value of sin (pt+α) $\sin(pt+\beta)$ as t varies from 0 to π/p is $\frac{1}{2}\cos(\alpha-\beta)$.
- The R. M. S value of f(t) = a sin pt + b cos pt as t varies from 0 to 2π, where p is an integer is $\sqrt{\frac{a^2+b^2}{2}}$
- R.M.S value of a + b sin θ is $\sqrt{\frac{2a^2+b^2}{2}}$
- R.M.S value of a cos θ over a half wave is a/ $\sqrt{2}$
- R.M.S value of a cos ωt over a period is a/ $\sqrt{2}$
- R.M.S value of a sin ωt over a complete wave is a/ $\sqrt{2}$.

Infinite faith and strength are only the conditions of success

PRACTICE SET - I

01. The area bounded by $y=x^2+2,\ x=1\ and\ x=2 is$

1) $\frac{16}{3}$ 2) $\frac{17}{3}$ 3) $\frac{13}{3}$ 4) $\frac{20}{3}$

02. The area (in square units) of the region bounded by the curves $2x=y^2-1$ and x=0 is

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) 1 4) 2

03. The area (in square units) of the region enclosed by the curves $y=x^2$ and $y=x^3$ is

1) 1 2) $\frac{1}{3}$ 3) $\frac{1}{6}$ 4) $\frac{1}{12}$

04. The area (in sq. units) of the region bounded by $y^2=4x\ and\ x^2=4y is$

1) $\frac{4}{3}$ 2) $\frac{16}{3}$ 3) $\frac{8}{3}$ 4) $\frac{10}{3}$

05. The area enclosed by $y^2=8x$ and y = 2x (in sq. units) is

1) $\frac{4}{3}$ 2) $\frac{3}{4}$ 3) $\frac{1}{4}$ 4) $\frac{1}{2}$

06. The area bounded by the curve $x^2=4y$, the line x=2 and the x-axis in sq. units.

1) 1 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) $\frac{8}{3}$

07. The volume formed when the area bounded by the parabola $y^2=4ax$, the x-axis and the ordinates at x=0 and x=h rotates about the x-axis is

1) $\frac{4}{3}a^2 cu\,units$ 2) $2\pi ah^2 cu\,units$

3) $4\pi ah^2 cu\,units$ 4) $\frac{\pi ah^2}{2} cu\,units$

08. The mean value of $\sin^3$ x over the range $0\le x\le 2\pi a$ is

1) $\frac{2}{\pi}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) 0

09. $\int_a^b f(x)dx$ represents

1) the area bounded by the curve and the x-axis
2) the area bounded by the curve and the ordinates x = a, x = b
3) the area bounded by the curve, the x-axis and the ordinates x=a and x = b
4) none

10. The area bounded by the curve $y=x^2$ and the ordinates x = 0, x = 3 is
1) 0 2) 9 3) 9/2 4) none

11. The area bounded by the curve $y=x^2+2x+3$, the x-axis and the ordinates x=1, x = 5 is

1) $\frac{232}{3}$ 2) $\frac{258}{3}$ 3) $\frac{116}{3}$ 4) none

12. The area bounded by xy=1, x-axis and x = 1, x = 2 is
1) log 2 2) 2 log 2 3) log 2-1 4) 2log 2-1

13. The area bounded by the curvey $y=4x-x^2$ and the x-axis is
1) 16/5 2) 32/5 3) 4/3 4) 32/3

14. The area bounded by the curve y = 2 cos x, the x-axis and the ordinates x = 0, $x=\pi/2$ is
1) 1 2) 2 3) 3 4) none

15. The area, in sq. units bounded by the x-axis and the curve $y=x^2-7x+10$ is
1) 2/3 3) 1/3 3) 2/9 4) 9/2

16. The area enclosed by the parabola $y^2=x^2-8x+15$ and the x-axis is
1) 4/3 2) 2/3 3) 1/3 4) 5/3

17. The area between the curve y=2 + cos x, the x-axis and the lines x = 0 and $x=\pi$ is
1) π 2) $\pi/2$ 3) 2π 4) 4π

18. The area bounded by the curve $y=e^x, y-axis$ and the lines y = 1, y = e is
1) -1 2) 1 3) 2 4) 0

19. The area in sq. units, bounded by the parabola $y^2=4ax$ and the line x = a is

1) $\frac{8a^2}{3}$ 2) $\frac{4a^2}{3}$ 3) $\frac{16a^2}{3}$ 4) none

20. The area of the region buounded by the curve $x^2=4y$, $y-axis$ and the lines y = 0, y = 1 is
1) 1/3 sq. units 2) 2/3 sq. units
3) 3/4 sq. units 4) 4/3 sq. units

21. The area of the region enclosed by the curve $x^2=4ay$, the x-axis and the ordinates x = 0 x = 4a is (in square units)
1) $\frac{2a^2}{3}$ 2) $\frac{4a^2}{3}$ 3) $\frac{8a^2}{3}$ 4) $\frac{16a^2}{3}$

22. Area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
1) πab 2) $\pi(a+b)$
3) $\frac{\pi ab}{4}$ 4) $\frac{\pi}{4}(a^2+b^2)$

23. The area of the region enclosed by the curve $y=e^{x/a}+e^{-x/a}$ the x-axis and the ordinates $x=\pm a$ is
1) $e-\frac{1}{e}$ 2) $2a\left(e-\frac{1}{e}\right)$
3) $\frac{a}{2}\left(e-\frac{1}{e}\right)$ 4) $a\left(e-\frac{1}{e}\right)$

24. The area of the region enclosed between y = cos x and y = sin x from x = 0 tgo $x=\pi/4$ is
1) $\sqrt{2}$ 2) $\sqrt{2}-1$ 3) $\sqrt{2}+1$ 4) $2\sqrt{2}$

25. The area, in square units, bounded by y = cos x, x-axis and lines x=0 and $x=2\pi$ is
1) 1 2) 2 3) 4 4) 1/2

26. The area bounded by the curve y = sin x, the x-axis and the lines x = 0, $x=2\pi$ is
1) 2 2) 1/2 3) 1/3 4) 4

27. The area bounded by the curve $y=4-x^2$ and the lines y = 0 and y = 3 is
1) $\frac{9}{2}$ 2) $\frac{14}{3}$ 3) $\frac{28}{3}$ 4) none

28. The area enclosed by the curve $|x|+|y|=1$ is
1) 2 2) π 3) π^2 4) none

29. The area bounded by the curve y = sin x and the x-axis from x = 0 to $x=\pi$ is
1) 2 2) π 3) π^2 4) none

30. The area of the region bounded by $y^2=x-1$ and $y=x-3$ is
1) 9/2 2) 2/9 3) 1/3 4) none

31. The area enclosed between the parabola $y=x^2$ and the line 2x-y+3=0 is
1) $\frac{16}{3}$ 2) $\frac{32}{3}$ 3) $\frac{3}{4}$ 4) $\frac{4}{3}$

32. Area of the region bounded by $y=e^{-x}$ and the line x = 1 is
1) $e-\frac{1}{e}$ 2) $e+\frac{1}{e}$
3) $e+\frac{1}{2}-2$ 4) $e+\frac{1}{e}+2$

33. The area bounded by $y^2=4ax$ and y = mx is $\frac{a^2}{3}sq.units$ then m =
1) 1 2) 2 3) - 2 4) 1/2

34. The area between the curve y = 1 - |x| and x-axis is
1) 1 2) 2 3) 1/2 4) 1/4

35. The area bounded by the line y = x and the parabola y = x (x - 1) is
1) $\frac{20}{3}sq.units$ 2) $\frac{4}{3}sq.units$
3) $\frac{2}{3}sq.units$ 4) none

36. The area included between the curves $y^2=-4x$ and the line x+y=0 is
1) 4 2) 8 3) 16 4) 32

37. The area ccommon to the two ellipses $a^2x^2+b^2y^2=1, b^2x^2+a^2y^2=1$ where 0 < a < b is
1) $\frac{1}{ab}\tan^{-1}\frac{a}{b}$ 2) $\frac{1}{ab}\tan^{-1}\frac{b}{a}$
3) $\frac{4}{ab}\tan^{-1}\frac{b}{a}$ 4) $\frac{4}{ab}\tan^{-1}\frac{a}{b}$

38. The area between $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$
1) $\frac{ab}{2}$ 2) $\frac{ab}{4}$ 3) $\frac{\pi ab}{2}$ 4) $\frac{\pi ab}{4}-\frac{ab}{2}$

39. The area, in square units, bounded by the x-axis, the curve $y=1+\frac{8}{x^2}$ and the line x = 2 and x = 4 is
1) 2 2) 3 3) 4 4) 5

40. The area enclosed by the curve $y^2=2x$ and the line y-x=0 is
1) 1/3 2) 2/3 3) 3/4 4) 4/3

41. Area bounded by the curves y = x and $y=x^3$ is
1) 1/2 2) 1/4 3) 1/6 4) 1/8

42. The area of the region bounded by the curve y = x is x and the x-axis between x = 0 and $x=2\pi$ is
1) 2π 2) 3π 3) 4π 4) $3\pi/4$

43. Area of the segment cut off from the parabola $x^2=8y$ by line x-2y+8=0 is
1) 12 2) 24 3) 36 4) 48

44. Area of the region bounded by the curve $y=x^2$ sin x and the x-axis between x = 0 and $x=2\pi$ is
1) π^2-4 2) $3\pi^2-8$
3) $5\pi^2-8$ 4) $6\pi^2-8$

45. The total area between the parabola $y^2=9x$ and the line y = x is
1) 8/3 2) 9/2 3) 27/2 4) none

46. The area enclosed by the parabola $ay=3(a^2-x^2)$ and x-axis is
1) a^2 2) $2a^2$ 3) $a^2/2$ 4) $4a^2$

47. The area of the segment cut off from the parabola $x^2=8y$ by the line $x-2y+8=0$ is
1) 18 2) 36 3) 27 4) 12

48. Area of the region bounded by the x-axis, the curve y = tan x $\left(-\frac{\pi}{3}\le x\le\frac{\pi}{3}\right)$ and y = cot $x\left(\frac{\pi}{6}\le x\le\frac{3\pi}{2}\right)$ is
1) log 2 2) $\log\sqrt{2}$ 3) 2 log 2 4) none

49. The area common to the two curves $y^2=2x$ and $x^2+y^2=4x$ is
1) $\frac{1}{3}(3\pi+8)$ 2) $\frac{1}{3}(3\pi-8)$
3) $\frac{3}{2}(2\pi+4)$ 4) none

50. The area enclosed by the curve $xy^2=4(2-x)$ and y-axis is
1) 2π 2) 3π 3) 4π 4) $\pi/4$

51. The volume generated when the area enclosed by $y^2=x^3$ and x = 4 revolves about x-axis is (in cubic units)
1) 16π 2) 32π 3) 64π 4) none

52. The volume generated by the rotaion of the area bounded by the curve $y^2=x^3$, the y-axis and the lines y = 0 y = 8 is
1) 192 cu.units 2) $\frac{384\pi}{7}cu.units$
3) $\frac{384\pi^2}{7}cu.units$ 4) $\frac{384\pi}{5}cu.units$

53. The volume of the solid generated when the area bounded by the curve y = x (1-x) and the x-axis is rotated about the x-axis is

1) 4π 2) $\frac{\pi}{8}$

3) $\frac{\pi}{15}$ 4) $\frac{\pi}{30}$

54. The volume of the solid of revolution formed when the curve y = tan x from x = 0 to x = $\frac{\pi}{4}$ is rotated about the x-axis is

1) $\frac{\pi^2}{4}$ 2) $\frac{\pi}{4}(4+\pi)$

3) $\frac{\pi}{4}(4-\pi)$ 4) $\frac{\pi}{2}(4-\pi)$

55. The volume formed when the region of the circle $x^2+y^2=16$ is revolved about a diameter is

1) $\frac{64\pi}{3}$ 2) $\frac{128\pi}{3}$ 3) $\frac{256\pi}{3}$ 4) $\frac{32\pi}{3}$

56. If the area enclosed by y=x, y=0 and x=a is revolved about x-axis, the volume generated is

1) πa^3 2) $2\pi a^3$ 3) $\frac{2\pi a^3}{3}$ 4) $\frac{\pi a^3}{3}$

57. If a circle in positive quadrant is rotated about y-axis then the volme generaed is

1) $\frac{4}{3}\pi a^3$ 2) $\frac{2}{3}\pi a^3$ 3) $4\pi a^3$ 4) πa^3

58. If the circle $x^2+y^2=a^2$ is rotated about x-axis, the volume generated is

1) πa^3 2) $2\pi a^3$ 3) $\frac{4}{3}\pi a^3$ 4) $\frac{2}{3}\pi a^3$

59. The volume generated by revolving the portion of the parabola $y^2=4ax$ cut off by its latus rectum about the axis of the parabola is

1) πa^3 2) $2\pi a^3$ 3) $\frac{4}{3}\pi a^3$ 4) $2\pi a^3$

60. The volume of the solid generated when the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is rotated about the minor axis is

1) $106\frac{1}{3}$ 2) $106\frac{2}{3}$ 3) $106\frac{3}{4}$ 4) none

61. The volume generated by revolving about y-axis the area cut off from the parabola $y^2=4x$ by the line y=8x-1 is

1) $\frac{27\pi}{512}$ 2) $\frac{72\pi}{512}$ 3) $\frac{36\pi}{512}$ 4) $\frac{144\pi}{512}$

62. The area between the parabolas $y^2=x$ and $x^2=y$ revolves about the x-axis. The volume of the solid so generated is

1) 0.1π 2) 0.3π 3) 0.5π 4) 2π

63. The volume of the solid generated by the revolution of the area bounded by $y^2=9x$ and y = 3x about the x-axis is

1) $\frac{2\pi}{3}$ 2) $\frac{3\pi}{2}$ 3) $\frac{\pi}{2}$ 4) $\frac{3\pi}{4}$

64. The volume of the solid generated by the revolution of the area bounded by the parabola $y^2=4x$ and the straight line $4x-3y+2=0$ about the y-axis is

1) $\frac{\pi}{5}$ 2) $\frac{\pi}{10}$ 3) $\frac{\pi}{15}$ 4) $\frac{\pi}{20}$

65. The curve y = f(x) is rotated about a line y = 1 between x = a and x = b. The volume generated is given by

1) $\pi\int_c^d (a-x)^2 dy$ 2) $\pi\int_a^b x^2 dy$

3) $\pi\int_a^b (y-1)^2 dx$ 4) $\pi\int_a^b (y+1)^2 dx$

66. Mean value of x^2 in [a, b] is

1) a^2-ab+b^2 2) a^2-b^2

3) $\frac{1}{3}(a^2-ab+b^2)$ 4) $\frac{1}{3}(a^2+ab+b^2)$

67. Mean value of $\frac{1}{1+x^2}$ on [-1, 1] is

1) 0 2) $\pi/2$ 3) $\pi/4$ 4) none

68. Mean value of f(t) = 2 sin 3t on $[0,\pi]$ is

1) $\frac{4}{3\pi}$ 2) $\frac{4\pi}{3}$ 3) $\frac{3\pi}{4}$ 4) $\frac{2}{3\pi}$

69. Mean value of f(x) = log x on [1, e] is

1) $\frac{1}{e}$ 2) $\frac{1}{\sqrt{e-1}}$ 3) $\frac{1}{e-1}$ 4) $\frac{1}{1-e}$

70. Mean value of f(x) = sin x+cos 2x on $\left[0,\frac{\pi}{2}\right]$ is

1) $\frac{2}{\pi}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{4}{\pi}$

71. Mean value of the first half cycle of i = sin θ is

1) $\frac{2}{\pi}$ 2) $\frac{\pi}{2}$ 3) $\frac{4}{\pi}$ 4) $\frac{\pi}{4}$

72. The mean value of $\sin^2\omega t$ from t = 0 to $t=\frac{2\pi}{\omega}$ is

1) 1 2) -1 3) -1/2 4) 1/2

73. The electric current I in a conductor at tiem t is give by the equation I = 4 sin 200 t. The meaqn value of I from t = 0 to $t=\frac{\pi}{100}$ sec is

1) -1 2) 1 3) $2/\pi$ 4) 0

74. The mean square value of $x^{3/2}$ in the interval (0, 1) is

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{-1}{4}$ 4) $\frac{1}{6}$

75. The R.M.S value of x^2 in the interval (0, 1) is

1) $\frac{1}{3}$ 2) $\frac{1}{\sqrt{3}}$ 3) $\frac{1}{\sqrt{5}}$ 4) $\frac{1}{5}$

76. The R.M.S value of $\sqrt{\log x}$ over the range x = 1 x = e is

1) $\sqrt{e-1}$ 2) $\frac{1}{e-1}$ 3) $\frac{1}{\sqrt{e-1}}$ 4) $\frac{1}{1-e}$

77. R.M.S value of the current $i=10\sin 24\pi$ is

1) $5\sqrt{2}$ 2) 25 3) 50 4) $2\sqrt{5}$

78. R.M.S value of a sin ωt over a complete wave is

1) a/2 2) $\sqrt{a/2}$ 3) $a/\sqrt{2}$ 4) $\sqrt{a}/2$

79. The mean value of $f(x)=\frac{1}{x^2+x}$ on the interval $\left[1,\frac{3}{2}\right]$ is

1) $\log\left(\frac{3}{5}\right)$ 2) $\log\left(\frac{6}{5}\right)$

3) $2\log\left(\frac{6}{5}\right)$ 4) $3\log\left(\frac{6}{5}\right)$

80. The mean value of the functioni $f(x)=\frac{2}{e^x+1}$ on the interval [0, 2] is

1) $\log\left(\frac{2}{e^2+1}\right)$ 2) $1+\log\left(\frac{2}{e^2+1}\right)$

3) $2+\log\left(\frac{2}{e^2+1}\right)$ 4) $2+\log(e^2+1)$

PRACTICE SET-1 KEY

01) 3	02) 2	03) 4	04) 2	05) 1
06) 2	07) 2	08) 4	09) 3	10) 2
11) 1	12) 1	13) 4	14) 2	15) 4
16) 1	17) 3	18) 2	19) 2	20) 4
21) 4	22) 1	23) 2	24) 2	25) 3
26) 4	27) 2	28) 2	29) 1	30) 1
31) 2	32) 3	33) 2	34) 1	35) 2
36) 3	37) 4	38) 4	39) 3	40) 2
41) 1	42) 3	43) 2	44) 4	45) 3
46) 4	47) 2	48) 1	49) 2	50) 4
51) 3	52) 2	53) 4	54) 3	55) 3
56) 4	57) 2	58) 3	59) 2	60) 2
61) 1	62) 2	63) 2	64) 4	65) 3
66) 4	67) 3	68) 1	69) 3	70) 1
71) 1	72) 4	73) 4	74) 4	75) 3
76) 3	77) 1	78) 3	79) 3	80) 2

PRACTICE SET - II

01. The area bounded by the curve $x = y^2 - 2y$, the y-axis and the ordinates y = 1, y = 2 is
1) 7 2) 3/2 3) 2/3 4) 1/3

02. The area bounded by the rectangular hyperbola $xy = c^2$, the x-axis and the ordinates x = c, x = 2c is
1) $c^2 \log 2$ 2) c log 2
3) 2c log 2 4) none

03. Area bounded by y = (x-1)(x-2)(x-3) between x = 0 and x = 3 is (in square units)
1) 7/4 2) 9/4 3) 11/4 4) 3/4

04. The area bounded by the curve $\sqrt{x} + \sqrt{y} = 2$, the x-axis and the lines x=0 and x=4 is
1) 4/3 2) 8/3 3) 16/3 4) 64/3

05. The area enclosed by the curve $4x^2 + 9y^2 = 36$ is
1) 6π 2) 36π 3) $6\pi^2$ 4) $36\pi^2$

06. The area bounded by the curve y = sin x and the x-axis from x = 0 to x = 2π is
1) 0 2) 1 3) 2 4) 4

07. The area bounded by the parabola $x = 4 - y^2$ and y-axis is (in square units)
1) $\frac{32}{3}$ 2) $\frac{3}{32}$ 3) $\frac{9}{32}$ 4) $\frac{4}{3}$

08. The area bounded by the curve $ay^2 = x^3$, the x-axis and the ordinate x = a is
1) $\frac{2a^2}{5}$ sq units 2) $\frac{3a^2}{5}$ sq units
3) $\frac{4a^2}{5}$ sq units 4) $\frac{8a^2}{5}$ sq units

09. The area of the region bounded by the curve y = sin x and the x-axis between $-\pi$ and π is
1) 1 2) 2 3) 4 4) 8

10. Area of the region bounded by y = tan x and tangent at $x = \frac{\pi}{4}$ and the x-axis is
1) $\sqrt{2} - \frac{\pi}{4}$ 2) $\log\sqrt{2}$
3) $\log\sqrt{2} - \frac{\pi}{4}$ 4) $\log\sqrt{2} + \frac{\pi^2}{16}$

11. The area of the region bounded by the curves $y = 2^x$ and $y = 2x - x^2$ between x = 0 and x = 2 is
1) $\frac{2}{\log 2} - \frac{4}{3}$ 2) $\frac{3}{\log 2} - \frac{4}{3}$
3) $\frac{3}{\log 2} - \frac{2}{3}$ 4) none

12. The area enclosed by the two curves $y = x^2$ and $y = x(2-x)$ is
1) 5/3 2) 2/3 3) 1/3 4) none

13. Area of the segment cut off from the parabola $y^2 = 2x$ by the stright line y = 4x - 1 is
1) 1/32 2) 3/32 3) 9/32 4) none

14. The area enclosed between the parabola $y^2 = 4ax$ and the straight lines x = a, x = 9a is
1) $8a^2$ 2) $\frac{8a^2}{3}$ 3) $\frac{208a^2}{3}$ 4) a^2

15. The area between the curve y = 1 − |x| and x-axis is
1) 1 2) 2 3) 1/2 4) 1/4

16. The area cut-off from the parabola $4y = 3x^2$ by the straight line 2y = 3x+12 is
1) 13 2) 20 3) 27 4) 27/2

17. The area bounded by the line y = x and the parabola y = x (x-1) is
1) $\frac{20}{3}$ sq units 2) $\frac{4}{3}$ sq unints
3) $\frac{2}{3}$ sq units 4) none

18. Thea area enclosed between the curve $y = x^2 - 4x$ and the line x + y = 0 is
1) 2/9 2) 9/2 3) 2/3 4) 9

19. The area included between the curves $y^2 = 4x$ and $x^2 = 12y$ is
1) 4 2) 8 3) 16 4) 32

20. The area between the parabolas $y^2 = x$ and $x^2 = y$ is
1) 2/3 2) 4/3 3) 1/4 4) 1/3

21. The area between the curves $y^2 = ax$ and $x^2 = ay$ is
1) a^2 2) $2a^2$ 3) $3a^2$ 4) $a^2/3$

22. The area enclosed between one arch of the curve y = sin 4x and the x-axis is
1) 1 2) 1/2 3) 2 4) 1/4

23. Area of the region bounded by $y = [x]$, the x-axis and the lines x = 1, x = 2 is
1) 1 2) 1/2 3) 2 4) 1/3

24. The area bounded by the x-axis and the curves $y = 4x - x^2 - 3$ is
1) 1/3 2) 2/3 3) 3/4 4) 4/3

25. The area bounded by the x-axis and the curves y = 2 cos x - sin x and x = 0, $x = \frac{\pi}{4}$ is
1) $1 - \frac{3}{\sqrt{2}}$ 2) $1 + \frac{3}{\sqrt{2}}$
3) $\frac{3}{\sqrt{2}} - 1$ 4) $\frac{2}{\sqrt{3}} - 1$

26. The area of the region bounded by the curve $y = \frac{x^2}{1+x^2}$, the x-axis and x = 0, x = 1 is
1) $\frac{\pi}{4}$ 2) $1 - \frac{\pi}{4}$ 3) $1 + \frac{\pi}{4}$ 4) $\frac{\pi}{4} - 1$

27. The area bounded by the parabola $y^2 = 4ax$ and its latus-rectum is
1) $\frac{2a^2}{3}$ 2) $\frac{4a^2}{3}$ 3) $\frac{8a^2}{3}$ 4) $\frac{16a^2}{3}$

28. AOB is the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in which OA = a, OB = b. Then the area between the chord AB and the arc AB of the ellipse is
1) πab 2) $(\pi - 2)ab$
3) $\frac{ab}{2}\left(\frac{\pi}{2} + 1\right)$ 4) $\frac{ab}{2}\left(\frac{\pi}{2} - 1\right)$

29. The area of the region bounded by $a^2y^2 = x^2(a^2 - x^2)$ is
1) $\frac{a^2}{3}$ 2) $\frac{2a^2}{3}$ 3) $\frac{4a^2}{3}$ 4) none

30. The area bounded by the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the lines $x = a$ and $x = 2a$ is
1) $ab\left[2\sqrt{2} - \log(2+\sqrt{3})\right]$
2) $ab\left[3\sqrt{2} - \log(2+\sqrt{3})\right]$
3) $2ab\left[2\sqrt{3} - \log(2+\sqrt{3})\right]$ 4) none

31. The area bounded by $y = x^2$ and $y = 3x$ is
1) 9/2 2) 7/2 3) 9 4) 10

32. The area between the two parabolas $y^2 = 20x$ and $x^2 = 16y$ is
1) $106\frac{1}{3}$ 2) $106\frac{2}{3}$ 3) $106\frac{1}{6}$ 4) $106\frac{5}{6}$

33. If A is the area under the curve $y = \sin x$ above x-axis in the interval $\left[0, \frac{\pi}{4}\right]$ then the area included between $y = \cos x$, and x-axis in the interval $\left[0, \frac{\pi}{4}\right]$ is given by
1) A 2) 2A 3) A/2 4) 1 + A

34. If A is the area under the curve $y = \sin x$, above x-axis such that $0 \le x \le \frac{\pi}{2}$ then the area under the curve $y = \sin 2x$ in the same interval is
1) A 2) 2A 3) A/2 4) 1 + A

35. If A_1, A_2 are the areas between the x-axis and the curve $y = \sin^n x$ and $y = \cos^n x$ in $\left[0, \frac{\pi}{2}\right]$ respectively, then

1) $A_1 = 1 - A$ 2) $A_2 = A_1$
3) $A_2 = 2A_1$ 4) none

36. The total area bounded by the curve $y = x^3 - 4x$ and the x-axis is
1) 2 2) 4 3) 8 4) 10

37. The area in the first quadrant enclosed by the x-axis, the line $x = y\sqrt{3}$ and the circle $x^2 + y^2 = 4$ is
1) π 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/4$

38. The area bounded by the curve $y = x(x-3)(x-5)$ and the x-axis is
1) $5\frac{1}{12}$ 2) $8\frac{1}{12}$ 3) $12\frac{1}{12}$ 4) $21\frac{1}{12}$

39. The area in the first quadrant bounded by the parabola $y = 4 - x^2, y = 0$ and $y = 3$ is
1) $4\frac{2}{3}$ 2) $9\frac{1}{3}$ 3) $9\frac{2}{3}$ 4) $6\frac{1}{3}$

40. Area of the segment cut off from the parabola $y = 4x - y^2$ by the straight line $y = x$ is
1) 9/2 2) 9/8 3) 9/4 4) 8/9

41. The area in the first quadrant bounded by $y = 4x^2, x = 0, y = 1$ and $y = 4$ is
1) 7/3 2) 3 3) 5/3 4) 2

42. Area of the region bounded by the x-axis and the curve $x = y^2 - 2y$ is
1) $6\frac{2}{3}$ 2) $6\frac{1}{3}$ 3) $1\frac{1}{3}$ 4) $4\frac{1}{3}$

43. Area of the region bounded by the x-axis, the curve $y = \sin^2 x$ and the ordinates $x = 0, x = \pi/2$ is
1) $\pi/2$ 2) $\pi/3$ 3) $\pi/4$ 4) $\pi/6$

44. Area bounded by the two curves $y = |x|$ and $y = 1 - |x|$ is
1) 1/2 2) 1 3) 1/4 4) 2

45. Area of the region bounded by $y = |x-1|, y = 0$ and $|x| = 2$ is
1) 4 2) 5 3) 8 4) none

46. The area common to the two curves $y^2 = 2x$ and $x^2 + y^2 = 4x$ is
1) $\frac{1}{3}(3\pi + 8)$ 2) $\frac{1}{3}(3\pi - 8)$
3) $\frac{3}{2}(2\pi + 4)$ 4) none

47. The volume generated when the area bounded by the paraboloa $y^2 = 4ax$, the x-axis and the ordinates, $x = 0$ and $x = h$ revolves about x-axis is
1) $\pi a h^2$ cu. units 2) $2\pi a h^2$ cu.units
3) $4\pi a h^2$ cu. units 4) $2\pi a h^2$ cu.units

48. The volume in cubic units, obtained when the area enclosed by the circle $x^2 + y^2 = 25$, the x-axis and the lines $x = 2, x = 3$ revolves about x-axis is
1) $15\frac{1}{3}\pi$ 2) $15\frac{2}{3}\pi$ 3) $18\frac{1}{3}\pi$ 4) $18\frac{2}{3}\pi$

49. The volume of the solid obtained by rotating one arch of the curve $y = \sin 3x$ about the x-axis is
1) $\frac{\pi^2}{6}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi^2}{4}$ 4) $\frac{\pi^3}{4}$

50. The volume generated by the roation of the area bounded by the curve $y = e^x \sin x$, the x-axis and the lines $x = 0, x = \pi$ about the x-axis is
1) $\frac{\pi}{6}(e^{2\pi} - 1)$ 2) $\frac{\pi}{8}(e^{2\pi} - 1)$
3) $\frac{\pi}{8}(e^{\pi} - 1)$ 4) $\frac{\pi}{8}(e^{\pi} + 1)$

51. The volume generated by the revolution of the area bounded by the x-axis, the catenary $y = c\cosh\frac{x}{c}$ and the ordinates $x = \pm c$ about x-axis is
1) $\pi c^3(1 + \sinh 1 \cosh 1)$
2) $\pi c^2(1 + \sinh 1 \cosh 1)$
3) $\pi c^2(1 - \sinh 1 \cosh 1)$
4) $\pi^2 c^3(1 + \sinh 1 \cosh 1)$

52. The volume formed when the area enclosed by the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is revolved about x-axis is
1) $\frac{32\pi a^2}{105}$ 2) $\frac{16\pi a^2}{105}$ 3) $\frac{32\pi a^3}{105}$ 4) $\frac{32\pi^2 a^3}{105}$

53. The volume in cubic units generated by revolving the area bounded by the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}, x = 0, y = 0$ about y-axis is
1) $\frac{\pi a^3}{15}$ 2) $\frac{2\pi a^3}{15}$ 3) $\frac{\pi a^3}{5}$ 4) $\frac{7\pi a^3}{15}$

54. The volume of the spheroid generated by rotation of the ellipse $4x^2 + 81y^2 = 324$ about the major axis is
1) 12π 2) 24π 3) 36π 4) 48π

55. The volume generated when the region bounded by the curves $y = x^3$ and $y = \sqrt{x}$ resolves about the x-axis is
1) $\frac{3\pi}{7}$ 2) $\frac{5\pi}{7}$ 3) $\frac{5\pi}{14}$ 4) $\frac{\pi}{4}$

56. The curve $y = f(x)$ is rotated about a line $x = a$ between $y = c$ and $y = d$. The volume generated is given by
1) $\pi\int_c^d (a-x)^2 dy$ 2) $\pi\int_c^d y^2 dx$
3) $\pi\int_c^d x^2 dy$ 4) none

57. Mean value of $f(t) = 2\sin 3t$ on $[0, \pi]$ is
1) 1 2) 1/2 3) 1/3 4) -1

58. Mean value of $f(\theta) = \cos^2\theta$ on $[0, \pi]$ is
1) 1 2) 1/2 3) 1/4 4) -1/2

59. Mean value of $x^2 - 4x + 3$ between the values of x where the expression vanishes is
1) 2/3 2) $-2/3$ 3) $-3/2$ 4) 4/3

60. If the mean value of $\sin x + k \sin 2x$ in the range $x = 0$ to $x = \frac{\pi}{2}$ is zero, then k,
1) 1 2) -1 3) 2 4) -2

61. The mean value of $x^2 e^x$ from $x = 1$ to $x = 3$ is
1) $\frac{5e}{2}(e+1)$ 2) $\frac{e}{2}(5e^2 - 1)$
3) $\frac{3e}{2}(e^2 - 1)$ 4) $\frac{2e}{5}(e^2 + 1)$

62. The mean height of the portion of the parabola $y = (x-1)(3-x)$ which lies above the x-axis is
1) 2/3 2) -2/3 3) 3/2 4) 9/2

63. The mean square value of tan x as x varies from 0 to $\pi/3$ is
1) $\frac{1}{\pi}\left(\sqrt{3} - \frac{\pi}{3}\right)$ 2) $\frac{1}{\pi}\left(\sqrt{3} + \frac{\pi}{3}\right)$
3) $\frac{1}{\pi}\left(3\sqrt{3} - \pi\right)$ 4) $\frac{\pi}{3}\left(\sqrt{3} - \frac{\pi}{3}\right)$

64. The R.M.S value of $\sqrt{8 - 4x^2}$ over the range $x = 0$ to $x = 2$ is
1) 8/3 2) $\sqrt{8/3}$ 3) 3/8 4) $\sqrt{3/8}$

65. The R.M.S value of $\sqrt{27 - 4x^2}$ from $x = 0$ to $x = 3$ is
1) $\sqrt{15}$ 2) 15 3) $\sqrt{5}$ 4) $\sqrt{10}$

66. The R.M.S value of log x from $x = 1$ to $x = e$ is
1) $\sqrt{\frac{e-2}{e-1}}$ 2) $\frac{e-2}{e-1}$
3) $\frac{e-1}{e-2}$ 4) $\sqrt{\frac{e-1}{e-2}}$

67. R.M.S value of a $\sin\omega t$ over a complete wave is
1) $a/\sqrt{2}$ 2) $\sqrt{a}/2$ 3) $\sqrt{a/2}$ 4) $a/2$

68. The equation of motion of a particle travelling in a straight line is $s = t^2 + t$. The average velocity on the interval from t = 0 to t = 3 is (assume s in metres and t in seconds)
1) 12 mt/sec. 2) 6 mt/sec.
3) 4 mt/sec. 4) 3 mt/sec.

69. The area bounded by the curve $x = 3 + \cos\theta, y = \sin\theta$ is (in square units)
1) 2π 2) 4π 3) 6π 4) $\pi/4$

70. The area of the region bounded by the cordinate axes of the curve $\sqrt{x}+\sqrt{y}=\sqrt{y}=\sqrt{a}(x,y\geq 0)$ is
1) a^2 2) $a^2/2$ 3) $a^2/3$ 4) $a^2/6$

71. The area between the x-axis and the curve y = $(x-1)^2-25$ is
1) $\frac{50}{3}sq\,units$ 2) $\frac{100}{3}sq\,units$
3) $\frac{200}{3}sq\,units$ 4) $\frac{500}{3}sq\,units$

72. The area bounded by the parabola $x=4-y^2$ and y-axis is
1) $\frac{32}{3}sq\,units$ 2) $\frac{3}{32}sq\,units$
3) $\frac{9}{32}sq\,units$ 4) $\frac{4}{3}sq\,units$

73. The volume of the solid genrated by rotating the area bounded by the curve $\sqrt{x}+\sqrt{y}=1$ and i9n the first quadrant about the y-axis is
1) $\frac{\pi}{5}$ 2) 5π 3) 15π 4) $\frac{\pi}{15}$

74. The mean value of the ordinate of a semi-circle of radius a taken along the diameter is
1) $\frac{a\pi}{2}$ 2) $2a\pi$ 3) $\frac{a\pi}{4}$ 4) $4a\pi$

75. The root mean square value of the function $\sqrt{\log x}$ from x = 1 to x = e is
1) 1 2) $\frac{1}{e-1}$ 3) $\frac{1}{\sqrt{e-1}}$ 4) $\frac{1}{\sqrt{e+1}}$

PRACTICE SET- II KEY

01) 3	02) 1	03) 1	04) 2	05) 1
06) 4	07) 1	08) 1	09) 3	10) 2
11) 2	12) 3	13) 3	14) 3	15) 1
16) 3	17) 2	18) 2	19) 3	20) 4
21) 4	22) 2	23) 1	24) 2	25) 3
26) 2	27) 3	28) 4	29) 3	30) 3
31) 1	32) 2	33) 3	34) 1	35) 2
36) 2	37) 3	38) 4	39) 1	40) 2
41) 1	42) 3	43) 3	44) 1	45) 2
46) 2	47) 2	48) 4	49) 1	50)2
51) 1	52) 3	53) 1	54) 4	55) 3
56) 1	57) 2	58) 2	59) 2	60) 2
61) 2	62) 1	63) 3	64) 2	65) 1
66) 1	67) 1	68) 3	69) 2	70) 4
71) 4	72) 1	73) 4	74) 3	75) 3

SELF TEST

01. The area bounded by the parabola $x=8+2y-y^2$, the y-axis and the lines, y = -1 and y = 3 is
1) $\frac{50}{3}sq\,units$ 2) $\frac{92}{3}sq\,units$
3) $\frac{200}{3}sq\,units$ 4) $\frac{500}{3}sq\,units$

02. The area bounded by x = 1, x = 2, $x^2y=2$ and x + y=4 is
1) 8 sq. units 2) 18 sq. units
3) $\frac{3}{2}$ sq. units 4) 24 sq. units

03. The part of the curve y = x (x-1) below the x-axis is rotated about that axis. The volume generated in a complete rotaiton is
1) $\frac{\pi}{20}$ 2) $\frac{\pi}{30}$ 3) $\frac{20}{\pi}$ 4) 30π

04. The area enclosed between the parabola $y^2=4ax$ and the straight lines x = a, x = 9 a is
1) $\frac{206}{6}$ 2) $\frac{104}{3}$ 3) $\frac{104}{6}$ 4) $\frac{203}{3}$

05. The area enclosed by the curve $2y=x^2$, the x-axis and the straight lines x = 1 and x = 3 in square units is
1) 26/3 2) 13/3 3) 4 4) 13

06. Volume of the solid formed by the revolution about the x-axis of the loop of the cruve $y^2=x(2x-1)^2$ is
1) $\pi/6$ 2) $\pi/12$ 3) $\pi/24$ 4) $\pi/48$

07. Volume of the solid obtained by revolving the ellipse $9x^2+16y^2=144$ about x-axis is
1) 48π 2) 192π 3) 16π 4) 44π

08. Area enclosed between one arch of the curve y=a sin x and x-axis is
1) a 2) $2a^2$ 3) 0 4) 2a

09. The mean value of the distances from each corner of the square to any point in the square of side 'a' is
1) $a\left[\sqrt{2}+\log(1+\sqrt{2})\right]$
2) $\frac{a}{3}\left[\sqrt{2}+\log(1+\sqrt{2})\right]$
3) $\frac{a}{3}\left[\sqrt{2}+\log\left(\sqrt{2}-1\right)\right]$
4) none of these

10. The area of the region bounded by the x-axis and the curve $y=4x-x^2$ is
1) $\frac{8}{3}$ 2) $\frac{16}{3}$ 3) $\frac{32}{3}$ 4) $\frac{64}{3}$

11. The area bounded between the curves $y=x^2$ and $y=x^3$ is
1) 1/6 2) 1/12 3) 1/4 4) 1/3

12. Area bounded by the curve $x^{2/3}+y^{2/3}=a^{2/3}$ and $\vec{O}X,\vec{O}Y$ is
1) $\frac{3\pi a^2}{16}$ 2) $\frac{3\pi a^2}{32}$
3) $\frac{\pi a^2}{32}$ 4) $\frac{\pi a^2}{16}$

13. The area bounded by the curve y = 1 $1+8/x^2$ with x-axis and the ordinates at x=2 and x = 4
1) 2 2) 3 3) 4 4) 5

14. The area bounded by the x-axis and the curve $y=4x-x^2-3$ is
1) 1/3 2) 2/3 3) 4/3 4) 8/3

15. The area bounded by the curve $y=\frac{1}{4}x^2+\frac{1}{4}x-\frac{1}{2}$ with x-axis in [0, 2] is
1) 1/5 2) 3/5 3) 3/4 4) 2/5

16. The area enclosed between the curve $y=\log_e(x+e)$ and the coordinate axes is
1) 1 2) 2 3) 3 4) 4

17. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x=\frac{3\pi}{2}$ is
1) $4\sqrt{2}-2$ 2) $4\sqrt{2}+2$
3) $4\sqrt{2}-1$ 4) $4\sqrt{2}+1$

18. The area bounded by y = cos x, y = x+1, y = 0 is
1) 3/2 2) 2/3 3) 1/2 4) 5/2

19. The area bounded by the parabola $x^2=4by$ and the double abscissa y = a si
1) $8\frac{a^{1/2}b^{3/2}}{3}$ 2) $8\frac{b^{1/2}a^{3/2}}{3}$
3) $4\frac{a^{1/2}b^{3/2}}{3}$ 4) $4\frac{b^{1/2}a^{3/2}}{3}$

20. The area contained between the curve $xy=a^2$, the vertical line x = a, x = $4a$ (a>0) and x-axis is
1) $a^2\log 2$ 2) $2a^2\log 2$
3) $a\log 2$ 4) $2a\log 2$

21. The area bounded by the curve $x^2=4ay$ and the line y = 2a is
1) $\frac{2a^2}{3}$ 2) $\frac{a^2}{3}$ 3) $\frac{8a^2}{3}$ 4) $\frac{16\sqrt{2}a^2}{3}$

22. The area (in square units) bounded by the curve $y^2=4x$ and $x^2=4y$ in the plane is
1) 8/3 2) 16/3 3) 32/3 4) 64/3

23. The area bounded by the curve y = x and $y=x^3$ is
1) 1/4 2) 1/6 3) 1/12 4) 1/2

24. The area, in square unit, bounded by the curves $y=x^3, y=x^2$ and the ordinates x = 1, x = 2 is
1) 17/12 2) 12/13 3) 2/7 4) 7/2

25. The area enclosed between the curve $y^2 = x$ and $y = |x|$ is

1) 2/3   2) 1   3) 1/6   4) 1/3

26. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is

1) 1/3   2) 2/3   3) 4/3   4) 5/3

27. The area bounded by the curve $x = y^2$ and $x = 3 - 2y^2$ is

1) 3   2) 4   3) 1   4) 2

28. The area bounded by the curves $y = \log x, y = 2^x$ and the lines $x = \frac{1}{2}, x = 2$ is

1) $\frac{1}{\log 2}(4-\sqrt{2}) - \frac{5}{2}\log 2 + \frac{3}{2}$

2) $\log 2(4-\sqrt{2}) + \frac{3}{2}$

3) $\frac{5}{2}\log 2 + \frac{3}{2}$ 4) none

29. The area of the region bounded by the curve $y = 16 - x^2$, the x-axes and the co-ordinates $x = 3, x = -3$ is: (2007)

1) 68   2) 42   3) 6   4) 78

30. The volume of the solid of revolution formed when the arch of the parabola $y^2 = 4ax$ between the co-ordinates (2007)

1) πa^3 2) $2\pi a^3$ 3) $6\pi ax^3$ 4) a^3

SELF TEST KEY

01) 2	02) 3	03) 2	04) 2	05) 2
06) 4	07) 1	08) 4	09) 2	10) 3
11) 2	12) 2	13) 3	14) 3	15) 3
16) 1	17) 1	18) 1	19) 2	20) 2
21) 1	22) 2	23) 4	24) 1	25) 4
26) 3	27) 2	28) 1	29) 1	30) 1

PREVIOUS ECET BITS

2008

01. Area enclosed between one arch of the curve v and x axis is

(1) a (2) $2a^2$ (3) 0   (4) $2a$

02. The volume of solid of revolution generated when the curve $y = x$ is revolved about x – axis between $(0,0)$ and $(4,0)$ is

(1) $\frac{16}{3}\pi$ (2) $\frac{64}{3}\pi$ (3) $\frac{8}{3}\pi$ (4) 16π

2009

03. Area lying between the parabola $y^2 = 4ax$ and its latus rectum is

1) $\frac{8a^2}{3}$ 2) $\frac{8a}{3}$ 3) $\frac{4a}{3}$ 4) $\frac{4a^2}{3}$

04. Area lying between the curves $y^2 = 4x$ and $y = 2x$ is equal to

1) $\frac{2}{3}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{2}$

05. The volume of the solid generated by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about y axis is

1) $\frac{2}{3}\pi a^2 b$ 2) $\frac{2}{3}\pi ab^2$

3) $\frac{4}{3}\pi ab^2$ 4) $\frac{4}{3}\pi a^2 b$

06. The volume of the solid generated by revolution of the part of the parabola $y^2 = 4ax$ off by the latus rectum about the tangent at the vertex is

1) $\frac{8\pi a^3}{5}$ 2) $\frac{16\pi a^3}{5}$ 3) $\frac{4\pi a^3}{5}$ 4) $\frac{2\pi a^3}{5}$

07. Find the volume of the solid generated by the revolution of $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about x axis

1) $\frac{32\pi a^3}{105}$ 2) $\frac{23\pi a^3}{105}$ 3) $\frac{16\pi a^3}{105}$ 4) $\frac{8\pi a^3}{105}$

2010

08. The area bounded by the curve $y = 7x - 10 - x^2$ with x-axis is

1) 9 sq.units   2) 3 sq.units

3) $\frac{9}{2}$ sq.units   4) $\frac{16\pi}{5}$

2011

09 The area enclosed by the curve $y^2 = 4x$ and the line y = x is

1) $\frac{2}{3}$ 2) $\frac{4}{3}$ 3) $\frac{1}{2}$ 4) $\frac{8}{3}$

10. The area of the figure bounded by the lines $x=0, x=\frac{\pi}{2}$, f(x) = sin x and g(x) = cos x is

1) $2(\sqrt{2}-1)$ 2) $\sqrt{3}-1$

3) $2(\sqrt{3}-1)$ 4) $2(\sqrt{2}+1)$

2012

11. Area under the curve $f(x) = \sin x$ in $[0, \pi]$ is

1) 4 sq. units   2) 2 sq. units

3) 6 sq. units   4) 8 sq. units

2013

12. The area included under $x + y = 2$ and the co-ordinate axis is

1) 8 units   2) 4 units

3) 2 units   4) 1 unit

13. The volume of solid of revolution in cubic units when y = 4 is rotated about x-axis between (0,0) and (0,4)

1) 64 π 2) 32 π

3) 16 π 4) 16

2014

14. The area of the region in the first quadrant enclosed by x-axis, y-axis, y = 3x - 2 and y = 4 is

1) 16   2) 8

3) $\frac{16}{3}$ 4) $\frac{8}{3}$

15. The root mean square (RMS) value of log x over the range x = 1 to x = e is

1) $\frac{\sqrt{(e+1)}}{\sqrt{(e-2)}}$ 2) $\frac{\sqrt{(e-2)}}{\sqrt{(e-1)}}$

3) $\frac{\sqrt{(e+2)}}{\sqrt{(e+1)}}$ 4) $\frac{\sqrt{(e+2)}}{\sqrt{(e-1)}}$

A.P ECET-2015

16. The area of the segment cut off from the parabola $x^2 = 8y$ by the line $x - 2y + 8 = 0$ is

1) 36   2) 34   3) 32   4) 38

17. The area bounded by the parabola $y = 4 - x^2, y = 0$ y = 3 is

1) $\frac{26}{3}$ 2) $-\frac{26}{3}$ 3) $-\frac{28}{3}$ 4) $\frac{28}{3}$

T.S ECET-2015

18. The area enclosed by the curbe $y = f(x)$, X-axis and ordinates x = a and x = b is

1) $\int_a^b \pi f(x)dx$ 2) $\int_a^b |f(x)dx|$

3) $\int |f(x)|dx$ 4) $\int \pi f(x)dx$

19. The bolume of the solid generated by the curve y = f(x) between x = a and x = b when it is revolved about the X-axis is given by

1) $\int_a^b \pi f(x)dx$ 2) $\int_a^b \pi^2 f(x)dx$

3) $\int_a^b \pi[f(x)]^2 dx$ 4) $\int_a^b \pi^2[f(x)]^2 dx$

20. The mean value of f(x) over [a,b] is

1) $\frac{1}{2}\int_a^b f(x)dx$ 2) $\frac{1}{b-a}\int_a^b f(x)dx$

3) $\frac{1}{a+b}\int_a^b f(x)dx$ 4) $\frac{1}{a-b}\int_a^b f(x)dx$

21. The root mean square value of f(x) over [a,b] is

1) $\sqrt{\frac{1}{2}\int_a^b f(x)dx}$ 2) $\sqrt{\frac{1}{b-a}\int_a^b f(x)dx}$

3) $\sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2dx}$ 4) $\sqrt{\frac{1}{2}\int_a^b [f(x)]^2dx}$

A.P ECET-2016

22. The area bounded by the y-axis and $x = 4-y^2$ is ______ square units

1) $\frac{3}{32}$ 2) $\frac{32}{3}$ 3) $\frac{33}{2}$ 4) $\frac{9}{2}$

23. The volume of the solid generated by rotating one arch of the curve y = sin 3x about the x-axis is-----

1) π^2 2) $\frac{\pi^2}{2}$ 3) $\frac{\pi^2}{4}$ 4) $\frac{\pi^2}{6}$

T.S ECET-2016

24. The area of the cardiod r=a $(1-\cos\theta)$ is

1) $\frac{3a^2\pi}{2}$ 2) $\frac{a^2\pi}{2}$ 3) $\frac{a\pi^2}{2}$ 4) $\frac{3a\pi^2}{2}$

25. The area bounded by the curve $y=7x-10-x^2$ and the x-axis is

1) $\frac{9}{2}sq units$ 2) $\frac{1}{3}sq units$

3) $\frac{2}{3}sq units$ 4) $\frac{3}{5}sq units$

26. The area bounded by the curve $x^2=4ay$ and the line y=2a is

1) $\frac{\sqrt{2}}{3}a^2 sq units$ 2) $\frac{8\sqrt{2}}{3}a^2 sq units$

3) $\frac{8}{3}a^2 sq units$ 4) $\frac{1}{\sqrt{3}}a^2 sq units$

27. The area of the ellipse x=a cost; y=b sin t is

1) $\frac{\pi}{2}ab$ 2) $\frac{\pi}{3}ab$ 3) πa^2b^2 4) πab

28. The length of the arc of the equiangular spiral $r=e^{\theta\cot\alpha}$, between the points for which the radil vectors are r_1 and r_2 is

1) $r_1r_2\sec\alpha$ 2) $r_1r_2\operatorname{cosec}\alpha$

3) $(r_2-r_1)\sec\alpha$ 4) $(r_1-r_2)\operatorname{cosec}\alpha$

A.P ECET-2017

29. The area bounded by the curve y = log x, x-axis and the straight line x-e=0 is__________ square units

1) e 2) $(e-1)$ 3) 0 4) $(1-e)$

30. The volume of the solid generated by rotating one arch of the curve y = sin3x about the x-axis is------

1) π^2 2) $\frac{\pi^2}{2}$ 3) $\frac{\pi^2}{4}$ 4) $\frac{\pi^2}{6}$

T.S ECET-2017

31. The volume generated by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about its major axis is

1) $4\pi ab^2$ 2) $\frac{4}{3}\pi ab^2$ 3) $\frac{4}{3}\pi a^2b$ 4) $\frac{8}{3}\pi a^2b^2$

A.P ECET-2018

32. The area enclosed between the curve $y^2=4ax$ and the line x = 2y is

1) $\frac{64}{5}sq units$ 2) $\frac{64}{3}sq units$

3) $\frac{65}{4}sq units$ 4) $\frac{63}{4}sq units$

T.S ECET-2018

33. The area of the region bounded by the curve $y=x^2-x$, x-axis and the line x=2 is ____.

1) 5/4 2) 5/3 3) 5/6 4) 5/2

PREVIOUS ECET BITS KEY

01) 4	02) 2	03) 1	04) 2	05) 4
06) 1	07) 1	08) 3	09) 4	10) 1
11) 2	12) 3	13) 1	14) 3	15) 2
16) 1	17) 4	18) 2	19) 3	20) 2
21) 3	22) 2	23) 4	24) 1	25) 1
26) 2	27) 4	28) 3	29) 3	30) 4
31) 2	32) 2	33) 3		

DIFFERENTIAL EQUATIONS

- **Definition :** An equation involving one or more dependent and independent variables, and the differential coefficients of the dependent variables with respect to the independent variables is called a Differential Equation (DE)

Definition : A different equation involving derivaties with respect to a single variable is called Ordinary Differential Equation (ODE) and that involving partial derivatives with respect to more than one independent variable is called Partial Differential Equation (PDE)

However, here we deal with only the ordinary differential equation involving one depenent and one independent variable. We simply call such an equation as differential equation.

- **Order :** The order of a DE is defined as the order of the highest derivative that appears in the DE.
- **Degree :** If a differential equation can be expressed as a polynomial equation in the derivatives occuring in it using algebraic operations such that the exponent of each of the derivatives is the least, then the largest exponent of the highest order derivative in the equation is called the degree of the differential equation. (Note that the exponent of x or y need not be an integer). Otherwise the degree is not defined for a differential equation.

In general a DE represents a family of curves usually having some property in common. The equations of all curves in such a family can be represented by a single equation involving the variables x,y and certain number of arbitrary constants which will be mentioned in the brackets.

- When the equation of a family of curves is given involving certain number of arbitrary constants, to form the DE representing the given family of curves we adopt the following 2 steps.

Step 1 : Differentiate the given equation with respect to the independent variable as many times as the number of arbitrary constants.

Step 2 : Eliminate all the arbitrary constants by using the given equation and the equations obtained in step 1.

Here we are concerned with first order and first degree DE's only. In general a DE of first order and first degree is of the form

$f(x,y)dy+g(x,y)dx=0$

where $f(x,y)$ and $g(x,y)$ are functions in the variables x, y. To solve such equations we use the following methods.

- **Variables separable :** When a first order and first degree DE of the form (1) is given we separate the variables x, y and bring all terms involving x alone to dx and the terms involving y to dy (if possible) to convert the given DE as

$\varphi(x)dx+\psi(y)dy=0$

Then we integrate to get the solution.

- **Homogeneous equations**

A differential equation of the form

$$f(x,y)dy+g(x,y)dx=0$$

is called a homogeneous equation if both $f(x,y)$ and $g(x,y)$ are homogeneous expressions in x, y of same degree (that is $f(\lambda x,\lambda y)=\lambda^n f(x,y)$ and $g(\lambda x,\lambda y)=\lambda^n g(x,y)$ for some non zero rational number n where λ is a constant)

The solve such an equation, we substitute $y=vx$ and write $\frac{dy}{dx}=v+x\frac{dv}{dx}$ or $dy=vdx+xdv$ to get a DE in the variables x and v then we solve it using variables separable method.

- **Non-homogeneous equation**

A differential equation of the form

$(ax+by+c)dy+(px+qy+r)dx=0$

is said to be non-homogeneous equation. We solve this equation as given below in two cases.

Case (i) : $\frac{a}{p}\neq\frac{b}{q}$

Put $x=X+h$ and $y=Y+k$ where

$h=\frac{br-cq}{aq-bp},\ k=\frac{cp-ar}{aq-bp}$

(here h, k are obtained by solving the equations $ah+bk+c=0$ and $ph+qk+r=0$)

Also note that $dY=dy$ and $dX=dx$. Now we get the equation in the form

$(aX+bY)dY+(pX+qY)dX=0$

This is homogeneous equation in X, Y we solve it using 6

Case (ii) : $\frac{a}{p}=\frac{b}{q}=k\text{(say)}$

Put $px+qy=z$, then $ax+by=k(px+qy)=kz$

and $\frac{dy}{dx}=\frac{1}{q}\left(\frac{dz}{dx}-p\right)$.

Now, the given DE coverts as

$(kz+c)\left(\frac{1}{q}\frac{dz}{dx}-p\right)+z+r=0$

This can be solved using variables separable method

Some differential equations become suitable for direct integration after multiplying by a factor (which is a function of x,y). Such a factor is called an integrating factor (IF) of the DE.

- **Linear Differential Equation (or Linear Equation)**

A differential equation of the form

$$\frac{dy}{dx}+P(x).y=Q(x)$$

where $P(x)$ and $Q(x)$ are functions of x alone, is called a linear equation in y. For this DE.

$e^{\int p(x)dx}$ is integrating factor (IF) and the solution is $ye^{\int p(x)dx}=\int q(x).e^{\int p(x)dx}.dx$

(Note that a DE of the form $\frac{dx}{dy}+f(y)x=g(y)$ is called linear in x)

- **Bernoulli's Equation**

A differential equation of the form

$$\frac{dy}{dx}+P(x).y=Q(x).y^n$$

is called a Bernoulli's equation in y.

This will be solved by converting it into linear equation. On dividing (*) both sides by y^n we get

$$y^{-n}\frac{dy}{dx}+P(x).y^{1-n}=Q(x)$$

Now put $y^{1-n}=z$. Then $(1-n)y^{-n}\frac{dy}{dx}=\frac{dz}{dx}$.

Hence given DE is

$\frac{dz}{dx}+(1-n)P(x).z=(1-n)Q(x)$

This is a linear equation in z. We solve this using 8

(Note : If P(y), Q(y) are functions of y, then a differential equation of the form

$\frac{dx}{dy}+P(y)x=Q(y).x^n$

is called Bernoulli's equation in x)

- **Some Standard Models**

A differential equation of the form $f(x,y)dy+g(x,y)dx=0$ is called an exact differential equation if $\frac{\partial f}{\partial y}\equiv\frac{\partial g}{\partial x}$.

The solution of such an equation is given by

$\int f(x,y)dy+\int(g(x,y)-\text{all terms involving y})dx=0$

(treating x as a constant)

The solution can also be given as

$\int(f(x,y)-\text{all terms involving }x)dy+\int g(x,y)dx=0$

(treating y as a constant)

11. In the following we give certain standard families of curves and the differential equations representing them (by eliminating the arbitrary constants as given in 4.

Sl. No Family of curves
The DE representing the family]

1. $y=mx+c,\ (c)$

$\frac{dy}{dx}=m$

2. $y=mx+c,\ (m)$

$y=x\frac{dy}{dx}+c$

3. $y=ax^2+bx+c,\ (a,b,c)$

$\frac{d^3y}{dx^3}=0$

4. $y=ae^{\alpha x}+be^{\beta x},(a,b)$

$\frac{d^2y}{dx^2}-(\alpha+\beta)\frac{dy}{dx}+\alpha\beta y=0$

5. $y=ae^{\alpha x}+be^{\beta x}+ce^{\gamma x},(a,b,c)$

$\frac{d^3y}{dx^3}-(\alpha+\beta+\gamma)\frac{d^2y}{dx^2}+$

$(\alpha\beta+\beta\gamma+\gamma\delta)\frac{dy}{dx}-\alpha\beta\gamma\ y=0$

6. $y=(a+bx)e^{\alpha x},\ (a,b)$

$\frac{d^2y}{dx^2}-2\alpha\frac{dy}{dx}+\alpha^2y=0$

7. $y=(a+bx+cx^2)e^{\alpha x},(a,b,c)$

$\frac{d^3y}{dx^3}-3\alpha\frac{d^2y}{dx^2}+3\alpha^2\frac{dy}{dx}-\alpha^3y=0$

8. $y=e^{\alpha x}(a\cos\beta x+b\sin\beta x),\ (a,b)$

$\frac{d^2y}{dx^2}-2\alpha\frac{dy}{dx}+(\alpha^2+\beta^2)y=0$

9. $y=(a+bx)e^{\alpha x}+c.e^{\beta x},\ (a,b,c)$

$\frac{d^3y}{dx^3}-(2\alpha+\beta)\frac{d^2y}{dx^2}+(\alpha^2+2\alpha\beta)\frac{dy}{dx}-\alpha^2\beta y=0$

10. $y=a\cos\alpha x+b\ \sin\alpha x,\ (a,b)$

$\frac{d^2y}{dx^2}+\alpha^2y=0$

- **Second Order Linear Homogeneous Differential Equation with constant coefficients**

A differential equation of the form

$$a\frac{d^2y}{dx^2}+b\frac{dy}{x}+cy=0 \quad ....(1)$$

where a,b,c are constants is called a second order homogeneous differential equation with constant coefficients.

Method of Solving the equation :

Step 1 : Write the equation in the operator form as

$(aD^2+bD+c)y=0$ ....(2)

Step 2 : Solve the Auxiliary Equation (A.E.)

$am^2+bm+c=0$

Case (i) : If the roots of A.E., are real and distinct say m_1 and m_2, then the general solution of (2) is

$y=c_1e^{m_1x}+c_2e^{m_2x}$ (where c_1,c_2 are constants)

Case (ii) : If the roots of A.E. are real and equal say m and m, then the general solution of (2) is

$y=(c_1+c_2x)e^{mx}$, where c_1 and c_2 are constants.

Case (iii) : If the roots A.E. are complex and conjugate, say $\alpha\pm i\beta$ then the general solution of (2) is

$y=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$

- Linear Differential Equations of Higher Order

An equation of the form

$(D^n+a_1D^{n-1}+a_2D^{n-2}+...a_n)y=X$...(1)

or $f(D)y=X$

where $a_1, a_2, \ldots a_n$ are constants and X is a function of x only is called Linear differential equation of order n.

If X = 0 [i.e., $f(D)y=0$] then (1) is said to be Homogeneous, otherwise (1) is said to be non-homogeneous linear equation.

(A) **Homogeneous Linear Equation :** Working rule to find the general solution of $f(D)y=0$

Step 1: Write the equation in operator forms as

$$\left(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \ldots + a_n\right)y = 0 \qquad \ldots(2)$$

Step 2 : Write the A.E. as

$$m^n + a_1 m^{n-1} + a_2 m^{n-2} + \ldots + a_n = 0 \qquad \ldots(3)$$

Step 3 : Solve the equation (3) for m. Let $m_1, m_2, \ldots m_n$ be its roots.

Step 4 : Write the general solution of (2) as follows.

(i) If all n roots are real and distinct, say $m_1, m_2, \ldots m_n$ then the general solution is

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \ldots + c_n e^{m_n x}$$

(ii) If two roots are real and equal and other roots are real and different then the general solution is

$$y = (c_1 + c_2 x)e^{m_1 x} + c_3 e^{m_3 x} + \ldots + c_n e^{m_n x}$$

(iii) Two roots are conjugate complex (i.e., $\alpha + i\beta, m_2 = \alpha - i\beta) + c_3 e^{m_3 x} + \ldots + c_n e^{m_n x}$

➡ **General Solution of Non-Homogeneous Linear Equation :** The general solution of non-homogeneous linear equation $f(D)y = X$ is $y = C.F + P.I.$

where C.F. (Complementary Function) is the general solution of $f(D)y=0$ and P.I. is the Particular Integral (or) solution of $f(D)y = X$ which contains no arbitrary constant.

➡ Short Methods of Finding Particular Integrals in certain cases

Case I : When $X = e^{ax}$

1. $P.I. = \dfrac{e^{ax}}{f(D)} = \dfrac{e^{ax}}{f(a)}$ if $f(a) \neq 0$

2. If $f(a) = 0$ then factorise $f(D) = (D-a)g(D)$ where $g(a) \neq 0$ then

$$P.I = \frac{e^{ax}}{(D-a)g(D)} = \frac{xe^{ax}}{g(a)}$$

3. If $f(D) = (D-a)^r\, g(D)$ then

$$P.I. = \frac{e^{ax}}{(D-a)^r\, g(D)} = \frac{x^r}{r!}\frac{e^{ax}}{g(a)}$$

4. $P.I. = \dfrac{e^{ax}}{(D-a)^r} = \dfrac{x^r}{r!}e^{ax}$

5. If $X = K$, a constant, then

$$P.I. = \frac{k}{f(D)} = \frac{ke^{0.x}}{f(D)} = \frac{K}{f(0)} \text{ if } f(0) \neq 0$$

Case II : When X = sin ax or cos ax

1. $P.I. = \dfrac{\cos ax}{f(D^2)} = \dfrac{\cos ax}{f(-a^2)}$ if $f(-a^2) \neq 0$

2. $P.I. = \dfrac{\sin ax}{f(D^2)} = \dfrac{\sin ax}{f(-a^2)}$ if $f(-a^2) \neq 0$

3. $\dfrac{\cos ax}{D^2+a^2} = \dfrac{x}{2a}\sin ax$, if $f(-a^2) = 0$

4. $\dfrac{\sin ax}{D^2+a^2} = \dfrac{-x}{2a}\cos ax$, if $(-a^2) = 0$

Case III : When $X = x^m$

$$P.I. = \frac{1}{f(D)}x^m$$

Working Rule :

➡ Write f(D) in the form $1 \pm \phi(D)$ by taking outside the lowest degree term.

➡ Bring $1 \pm \phi(D)$ into the numerator.

• Expand $\left[1 \pm \phi(D)\right]^{-1} x^m$ using the following expansions upto the term containing D^m.

(i) $(1-D)^{-1} = 1 + D + D^2 + D^3 + \ldots$

(ii) $(1+D)^{-1} = 1 - D + D^2 - D^3 + \ldots$

(iii) $(1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + \ldots$

(iv) $(1+D)^{-2} = 1 - 2D + 3D^2 - 4D^3 + \ldots$

(v) $(1-D)^{-3} = 1 + 3D + 6D^2 + 10D^2 + \ldots$

(vi) $(1+D)^{-3} = 1 - 3D + 6D^2 - 10D^3 + \ldots$

Case IV : When $X = e^{ax}V$

$$P.I = \frac{1}{f(D)}e^{ax}V = e^{ax}\frac{1}{f(D+a)}V$$

i.e., we have to take out e^{ax} and write $(D+a)$ for every D in f(D) so that f(D) changes to $f(D+a)$ and then operate $\dfrac{1}{f(D+a)}$ on V alone using the earlier methods.

Note: $\dfrac{X}{D} = \int X dx$. Here $\dfrac{1}{D}$ or D^{-1} is called the inverse operator of differential operator D.

Case V : When $X = x^m V$, where $V = \sin ax$ or $\cos ax$

1. $P.I. = \dfrac{1}{f(D)}x^m \sin ax$ = I.P of $\dfrac{1}{f(D)}x^m e^{iax}$

2. $P.I. = \dfrac{1}{f(D)}x^m \cos ax$ = R.P. of $\dfrac{1}{f(D)}x^m e^{iax}$

Now P.I. can be evaluated by the short method of case (iv).

Case VI : When $X = xV$

$$P.I. = \frac{1}{f(D)}xV = \left[x\frac{1}{f(D)} - \frac{f'(D)}{\left[f(D)\right]^2}\right]V$$

or $P.I. = \left[x\dfrac{1}{f(D)} + \dfrac{d}{dD}\dfrac{1}{f(D)}\right]V$

16. Important Results :

(i) The differential equation whose solution is $y = a\cos(mx+b)$ or $y = a\cos mx + b\sin mx$ where a,b are arbitrary constants is $y_2 + m^2 y = 0$

(ii) The differential equation whose solution is $y = ae^{\alpha x} + be^{\beta x}$ where a,b are arbitrary constants is $y_2 - (\alpha+\beta)y_1 + (\alpha\beta)y = 0$

SOLVED EXAMPLES

01. The degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^4 + \left(\dfrac{dy}{dx}\right)^7 + xy^8 = \sin x$ is

1) 7 2) 4 3) 8 4) does not exist

Sol : The index of the highest ordered derivative $\left(i.e., \dfrac{d^2y}{dx^2}\right)$ is 4.

Hence degree is 4.

Ans : 2

02. The degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^{3/2} = \left(\left(\dfrac{dy}{dx}\right)^2 + y\right)^{2/3}$ is___

1) 3/2 2) 3 3) 6 4) 9

Sol: To remove rational powers, we raise both sides to the power 6, to get

$$\left(\frac{d^2y}{dx^2}\right)^9 = \left(\left(\frac{dy}{dx}\right)^2 + y\right)^4$$

Hence the degree of the DE is 9

Ans : 4

03. The DE representing the family of curves $y = a\cos 5x + b\sin 5x\,(a,b)$ is

1) $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$ 2) $\frac{d^2y}{dx^2} - 25y = 0$

3) $\frac{d^2y}{dx^2} + 25y = 0$ 4) $\frac{d^2y}{dx^2} = y$

Sol: We can use 11 to get the solution directly. However, we derive the solution without using the formula. On differentiating given equation successively with respect to x, we get

$$\frac{dy}{dx} = -5a\sin 5x + 5b\cos 5x$$

and $\frac{d^2y}{dx^2} = -25a\cos 5x - 25b\sin 5x = -25y$

Ans : 3

04. The DE obtained by eliminating a,b from the equation $ax^2 + by^2 = 1$ is $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = $ __

1) $y\frac{dy}{dx}$ 2) $x\frac{dy}{dx}$ 3) $\frac{y}{x}\frac{dy}{dx}$ 4) $\frac{x}{y}\frac{dy}{dx}$

Sol : On differentiating the given equation with respect to x, we get $2ax + 2by\frac{dy}{dx} = 0 \Rightarrow \frac{y\frac{dy}{dx}}{x} = -\frac{a}{b}$

On differentiating again with respect to x, we get

$$x\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) - y\frac{dy}{dx} = 0$$

$$\Rightarrow xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx}$$

05. The solution of $(12x + 5y - 9)dx + (5x + 2y - 4)dy = 0$ is

1) $6x^2 + 5xy + y^2 + 9x + 4y = c$

2) $6x^2 + 5xy + y^2 - 9x - 4y = c$

3) $6x^2 - 5xy - y^2 - 9x - 4y = c$

4) $3x^2 + 5xy + 2y^2 - 9x - 4y = c$

Sol: Here $M = 12x + 5y - 9$ and $N = 5x + 2y - 4$

$$\therefore \frac{\partial M}{\partial y} = 5 = \frac{\partial N}{\partial x}$$

$\therefore$ The equation is exact.

Hence the general solution is

$$\int_{(y\ \text{constant})} (12x + 5y - 9)dx + \int (2y - 4)dy = c$$

i.e, $12\frac{x^2}{2} + 5xy - 9x + 2.\frac{y^2}{2} - 4y = c$ or

$6x^2 + 5xy + y^2 - 9x - 4y = c$

Ans : 2

06. The equaton of the curve passing through the origin and satisfying $\frac{dy}{dx} + y = e^x$ is

1) $2y = e^{2x} + 1$ 2) $2ye^x = e^x - 1$

3) $2ye^x = e^{2x} - 1$ 4) $2ye^{2x} = 2e^x + 1$

Sol : Given equation is linear in y

Here $P = 1, Q = e^x$

$$\text{I.F.} = e^{\int P dx} = e^{\int 1 dx} = e^x$$

$\therefore$ The solution is $y.(I.F.) = \int Q(I.F.)dx + c$

i.e., $ye^x = \int e^x.e^x\,dx + c = \int e^{2x}dx + c = \frac{1}{2}e^{2x} + c$

$\Rightarrow 2ye^x = e^{2x} + c$

If this curve passes through the origin, then $c = -1$

$\therefore$ Required curve is $2ye^x = e^{2x} - 1$

Ans : 3

PUT YOUR FULL EFFORTS
DON'T WORRY ABOUT RESULTS
THEY ARE BOUND TO COME TO YOU

PRACTICE SET - I

01. Order of the differential equation of the family of all concentric circles centred at (h,k) is

1) 1 2) 2 3) 3 4) 4

02. The order and degree of equation $\left(\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2}\right)^{3/2} = a\frac{d^3y}{dx^3}$ are m,n respectively then (m,n)

1) $(4,3)$ 2) $(3,4)$ 3) $(3,3)$ 4) $(4,2)$

03. The order of $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} - 3y = x$ is

1) 2 2) 3 3) 1 4) 4

04. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3\right)^{6/5} = 6y$ are __

1) $2, \frac{6}{5}$ 2) 1, 18 3) 2, 1 4) 6, 2

05. The differential equation of the family of curves whose equation is $y = A\sin 2x + B\cos 2x$ (A, B are arbitrary constants)

1) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ 2) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$

3) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ 4) $\frac{d^2y}{dx^2} + 4y = 0$

06. The order and degree of the differential equaton $y = \frac{\left[5 + \left(\frac{dy}{dx}\right)^2\right]^{4/3}}{\frac{d^2y}{dx^2}}$ are__

1) $(2,4)$ 2) $(1,2)$ 3) $(2,3)$ 4) $(2,4)$

07. The degree of the differential equation $7\left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{d^2y}{dx^2}\right)^2$ is....

1) 1 2) 2 3) not defined 4) 4

08. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of

1) first order and second degree
2) first order and first degree
3) second order and first degree
4) second order and second degree

09. The degree and order of $\left[\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]^{5/2} = \left(a\frac{d^2y}{dx^2}\right)^{2/3}$ are m, n respectively, then $(m,n) =$

1) $(15,2)$ 2) $(2,2)$ 3) $(5,2)$ 4) $(4,1)$

10. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$ is a parameter, is of

1) order 1, degree 2 2) order 1, degree 1
3) order 1, degree 3 4) order 2, degree 2

11. The differential equation of family of curves with equation $y = e^x(A\cos 2x + B\sin 2x)$ (A, B arbitrary constants) is

1) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ 2) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$

3) $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0$ 4) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$

12. The order of the D.E. whose solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c_5}$ where c_1, c_2, c_3, c_4, c_5 are arbitrary constants :

1) 5 2) 4 3) 3 4) 2

13. The order and degree of $\left(\frac{d^4y}{dx^4}+\frac{d^2y}{dx^2}\right)^{5/3}=a\frac{d^2y}{dx^2}$ are p,q respectively then $p+q=$

1) 9 2) 6 3) 7 4) 10

14. The degree and order of the differential equation $\left[1+2\left(\frac{dy}{dx}\right)^2\right]^{3/2}=5\frac{d^2y}{dx^2}$ are__

1) 1,2 2) 2, 2 3) 6, 1 4) 4, 3

15. The solution of the differential equation $\frac{dy}{dx}=(y+4x+5)^2$ is___

1) $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+5}{2}\right)=x+c$

2) $\frac{1}{2}\tan^{-1}(4x+y+5)=2x+c$

3) $\tan^{-1}\left(\frac{4x+y+5}{2}\right)=x+c$

4) $\tan^{-1}(4x+y+5)=2x+c$

16. The general solution of $\frac{dy}{dx}=(4x+y+1)^2$ is

1) $\tan^{-1}\left(\frac{4x+y+1}{2}\right)=x+c$

2) $\tan^{-1}\left(\frac{4x+y+1}{2}\right)=2x+c$

3) $\tan^{-1}\left(\frac{4x+y+1}{2}\right)=4x+c$ 4) None

17. The solution of $\frac{dy}{dx}+y=e^x$ is

1) $2y=e^{2x}+c$ 2) $2ye^x=e^x+c$

3) $2ye^x=e^{2x}+c$ 4) $2ye^{2x}=2e^x+c$

18. The general solution of $(D^2-D-6)y=0$ is

1) $y=c_1e^{2x}+c_2e^{3x}$ 2) $y=c_1e^{-2x}+c_2e^{3x}$

3) $y=c_1e^{-2x}+c_2e^{-3x}$ 4) $y=c_1e^{2x}+c_2e^{-3x}$

19. The general solution of $\frac{d^2y}{dx^2}=y$ is

1) $y=c_1e^x+c_2e^{2x}$ 2) $y=c_1e^x+c_2e^{-2x}$

3) $y=c_1e^x+c_2e^{-x}$ 4) $y=ce^x$

20. The general solution of $\frac{d^2y}{dx^2}+\frac{dy}{dx}-12y=0$ is

1) $y=c_1e^{3x}+c_2e^{4x}$ 2) $y=c_1e^{3x}+c_2e^{-4x}$

3) $y=c_1e^{-3x}+c_2e^{4x}$ 4) $y=c_1e^{-3x}+c_2e^{-4x}$

21. The general solution of $y^{11}+3y^1+2y=0$ is

1) $y=c_1e^x+c_2e^{2x}$ 2) $y=c_1e^{-x}+c_2e^{2x}$

3) $y=c_1e^x+c_2e^{-2x}$ 4) $y=c_1e^{-x}+c_2e^{-2x}$

22. The general solution of $(D^2+3D-54)y=0$ is

1) $y=c_1e^{3x}+c_2e^{18x}$ 2) $y=c_1e^{-3x}+c_2e^{18x}$

3) $y=c_1e^{6x}+c_2e^{-9x}$ 4) $y=c_1e^{-6x}+c_2e^{9x}$

23. The solution of $(D^2-2D+1)y=0$ is

1) $y=c_1e^x+c_2e^{-x}$ 2) $y=(c_1+c_2x)e^x$

3) $y=(c_1+c_2x)e^{-x}$ 4) none

24. The solution of $(D^2+4D+13)y=0$ is

1) $y=e^{2x}(c_1\cos 3x+c_2\sin 3x)$

2) $y=e^{-2x}(c_1\cos 3x+c_2\sin 3x)$

3) $y=e^{3x}(c_1\cos 2x+c_2\sin 2x)$

4) $y=e^{-3x}(c_1\cos 2x+c_2\sin 2x)$

25. The solution of $y^{11}+8y^1+25y=0$ is

1) $y=c_1e^{3x}+c_2e^{-4x}$

2) $y=c_1e^{-3x}+c_2e^{4x}$

3) $y=e^{4x}(c_1\cos 3x+c_2\sin 3x)$

4) $y=e^{-4x}(c_1\cos 3x+c_2\sin 3x)$

26. The particular solution of $(D^2-5D+6)y=e^{4x}$ is

1) $\frac{1}{2}e^{4x}$ 2) $\frac{-1}{2}e^{4x}$ 3) $\frac{1}{4}e^{4x}$ (4) $\frac{x}{2}e^{4x}$

27. The particular solution of $(D^2+D+1)y=e^{-x}$ is

1) $\frac{1}{3}e^{-x}$ 2) $\frac{1}{2}e^{-x}$ 3) e^{-x} 4) xe^{-x}

28. The particular solution of $(D^3-4D^2)y=5$ is

1) 5 2) $\frac{5}{2}x$ 3) $\frac{5}{8}x$ 4) $\frac{-5}{8}x^2$

29. The particular solution of $(D^2-3D+2)y=\cosh x$ is

1) $\frac{1}{2}\left(xe^x-\frac{e^{-x}}{3}\right)$ 2) $\frac{1}{2}\left(-xe^x+\frac{e^{-x}}{3}\right)$

3) $\frac{1}{2}\left(-xe^x+\frac{e^{-x}}{6}\right)$ 4) $\frac{1}{2}\left(xe^x-\frac{e^{-x}}{6}\right)$

30. The particular solution of $(D^2+D+1)y=e^{-x}$ is

1) e^{-x} 2) xe^{-x} 3) $\frac{1}{2}e^{-x}$ 4) $\frac{x}{2}e^{-x}$

31. The particular solution of $(D^2-6D+9)y=e^{2x}+e^{3x}$ is

1) $e^{2x}+e^{3x}$ 2) $\frac{1}{2}(e^{2x}+e^{3x})$

3) $e^{2x}+x^2e^{3x}$ 4) $e^{2x}+\frac{1}{2}x^2e^{3x}$

32. The particular solution of $(D^2+D+1)y=\sin 2x$ is

1) $\cos 2x+2\sin 2x$

2) $\frac{1}{7}(2\cos 2x+3\sin 2x)$

3) $\frac{1}{13}(2\cos 2x+3\sin 2x)$

4) $\frac{-1}{13}(2\cos 2x+3\sin 2x)$

33. The particular solution of $(D^2+9)y=\sin 3x$ is

1) $\frac{x}{2}\sin 3x$ 2) $\frac{x}{6}\cos 3x$

3) $\frac{-x}{6}\cos 3x$ 4) $\frac{x}{3}\cos 3x$

34. The particular solution of $(D^2+16)y=\cos 4x$ is

1) $x\sin 4x$ 2) $\frac{x}{4}\sin 4x$ 3) $\frac{x}{8}\sin 4x$ 4) $\frac{x}{16}\sin 4x$

35. The particular solution of $(D^2-4D+4)y=\cos 2x$ is

1) $\frac{1}{2}\sin 2x$ 2) $\frac{x^2}{2}\sin 2x$

3) $\frac{1}{8}\sin 2x$ 4) $\frac{-1}{8}\sin 2x$

36. The particular solution of $(D^2-3D+2)y=e^{-2x}+\sin x$ is

1) $e^{-2x}+\frac{1}{10}(\sin x-3\cos x)$

2) $xe^{-2x}+\frac{1}{10}(\sin x-3\cos x)$

3) $\frac{-e^{-2x}}{12}+\frac{1}{10}(\sin x-3\cos x)$

4) $-^{x}e^{-2x}+\frac{1}{10}(\sin x+3\cos x)$

37. The particular solution of $\left(D^2+D-6\right)y=x$ is

1) $\frac{x}{6}+\frac{1}{36}$ 2) $\frac{-x}{6}+\frac{1}{36}$

3) $\frac{x}{6}-\frac{1}{36}$ 4) $-\left(\frac{x}{6}+\frac{1}{36}\right)$

38. The particular solution of $\left(D^2-4D+3\right)y=x^2$ is

1) $\frac{x^2}{3}+\frac{8x}{9}+\frac{26}{27}$ 2) $\frac{x^2}{3}-\frac{8x}{9}+\frac{26}{27}$

3) $\frac{x^2}{3}+\frac{8x}{9}-\frac{26}{27}$ 4) $\frac{x^2}{3}-\frac{8x}{9}-\frac{26}{27}$

39. The particular integral of $\left(D^2-4\right)y=x^2$ is

1) $\frac{1}{2}\left(x^2+\frac{1}{2}\right)$ 2) $\frac{1}{4}\left(x^2-\frac{1}{2}\right)$

3) $\frac{1}{4}\left(x^2+\frac{1}{2}\right)$ 4) $-\frac{1}{4}\left(x^2+\frac{1}{2}\right)$

40. The particular integral of $\left(D^2+4\right)y=x+\sin x$ is

1) $\frac{x}{4}+\frac{1}{3}\sin x$ 2) $\frac{x}{4}-\frac{1}{3}\sin x$

3) $\frac{x}{3}-\frac{1}{4}\sin x$ 4) $\frac{x}{3}+\frac{1}{4}\cos x$

41. The particular integral of $\left(D^2-2D+4\right)y=e^x\cos x$ is

1) $e^x\cos x$ 2) $\frac{1}{2}e^x\cos x$

3) $\frac{1}{4}e^x\cos x$ 4) $\frac{1}{5}e^x\cos x$

42. The particular integral of $\left(D^2-4D+3\right)y=e^{2x}\sin 3x$ is

1) $\frac{1}{5}e^{2x}\sin 3x$ 2) $\frac{-1}{5}e^{2x}\sin 3x$

3) $\frac{1}{10}e^{2x}\sin 3x$ 4) $\frac{-1}{10}e^{2x}\sin 3x$

43. The particular solution of $\left(D^2-2D+1\right)y=x^2e^x$ is

1) $\frac{1}{2}e^x x^4$ 2) $\frac{1}{6}e^x x^4$

3) $\frac{1}{8}e^x x^4$ 4) $\frac{1}{12}e^x x^4$

44. The particular solution of $\left(D^2-2D+1\right)y=x^2e^{3x}$ is

1) $e^{3x}\left(2x^2-4x+3\right)$ 2) $\frac{e^{3x}}{2}\left(2x^2-4x+3\right)$

3) $\frac{e^{3x}}{4}\left(2x^2-4x+3\right)$ 4) $\frac{e^{3x}}{8}\left(2x^2-4x+3\right)$

45. The particular integral of $\left(D^2-5D+6\right)y=xe^{4x}$ is

1) $\frac{1}{2}e^{4x}(2x-3)$ 2) $\frac{1}{4}e^{4x}(2x-3)$

3) $\frac{1}{4}e^{4x}(2x+3)$ 4) $\frac{x}{4}e^{4x}(2x+3)$

46. The particular solution of $\left(D^2+4\right)y=x\sin x$ is

1) $\frac{x}{3}\sin x-\frac{2}{9}\cos x$ 2) $\frac{x}{3}\sin x+\frac{2}{9}\cos x$

3) $\frac{x}{3}\cos x-\frac{2}{9}\sin x$ 4) $\frac{-x}{3}\sin x+\frac{2}{9}\cos x$

47. The particular integral of $\left(D^2+4\right)y=x\cos x$ is

1) $\frac{x}{3}\cos x-\frac{2}{9}\sin x$ 2) $\frac{x}{3}\cos x+\frac{2}{9}\sin x$

3) $\frac{-x}{3}\cos x+\frac{2}{9}\sin x$ 4) $-\left(\frac{x}{3}\cos x+\frac{2}{9}\cos x\right)$

48. The solution of $\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$ is

1) $\tan y=\left(x^2-1\right)+ce^{x^2}$

2) $\tan y=\left(x^2-1\right)+ce^{-x^2}$

3) $2\tan y=\left(x^2-1\right)+ce^{-x^2}$

4) $2\tan y=\left(1-x^2\right)+ce^{-x^2}$

49. If c is a parameter then the differential equation whose solution is $y=c^2+\frac{c}{x}$ is

1) $y=x^4\left(\frac{dy}{dx}\right)-x\left(\frac{dy}{dx}\right)^2$

2) $y=x^4\left(\frac{dy}{dx}\right)^2+x\frac{dy}{dx}$

3) $y=x^4\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}$

4) $y=x^4\left(\frac{d^2y}{dx^2}\right)-x\frac{dy}{dx}$

PRACTICE SET-I KEY

01) 2	02) 1	03) 2	04) 3	05) 4
06) 3	07) 2	08) 3	09) 1	10) 3
11) 2	12) 3	13) 1	14) 2	15) 1
16) 2	17) 3	18) 2	19) 3	20) 2
21) 4	22) 3	23) 2	24) 2	25) 4
26) 1	27) 3	28) 4	29) 3	30) 1
31) 4	32) 4	33) 3	34) 3	35) 4
36) 3	37) 4	38) 1	39) 4	40) 1
41) 2	42) 4	43) 4	44) 4	45) 2
46) 1	47) 2	48) 3	49) 3	

PRACTICE SET - II

01. The general solution of $\left(1+e^x\right)y\,dy=e^x\,dx$ is

1) $y^2=\log\left[c^2\left(e^x+1\right)^2\right]$ 2) $y=\log c\left(e^x+1\right)$

3) $y^2=\log c\left(e^x+1\right)$ 4) $y=\frac{1}{2}\ln\left(e^x+1\right)$

02. The differential equation of the family of straight lines $y=mx+\frac{a}{m}$ where m is a parameter is

1) $x\frac{dy}{dx}=a$ 2) $(x-y)\frac{dy}{dx}=a$

3) $x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=a$ 4) $x\left(\frac{dy}{dx}\right)^2-y\frac{dx}{dy}=-a$

03. By eliminating the parameters A and α in $x=A\cos(pt+\alpha)$, the differential equation obtained is

1) $\frac{d^2x}{dt^2}=x$ 2) $\frac{d^2x}{dt^2}=p^2x$

3) $\frac{d^2x}{dt^2}+p^2x=0$ 4) $\frac{d^2x}{dt^2}=1$

04. The general solution $\frac{dy}{dx}=x+y$ is

1) $y=\log\left[c(1+x+y)\right]$ 2) $x=\ln\left[c(1+x+y)\right]$

3) $x^2+y^2=c$ 4) $xy=y+c$

05. The differential equation of family of curves given by $y=e^{3x}(Ax+B)$ (A, B arbitrary constants) is :

1) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$ 2) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}-9y=0$

3) $\frac{d^2y}{dx^2}-6\frac{dy}{dx}-9y=0$ 4) $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=0$

06. The general solution of $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

1) $\sin h^{-1}x+\sin h^{-1}y=c$

2) $\cos h^{-1}x+\cos h^{-1}y=c$

3) $\sin^{-1}x+\sin^{-1}y=c$

4) $\cot^{-1}x-\cot^{-1}y=c$

07. The general solution of $\frac{dy}{dx}=2^{x-y}$ is

1) $2^x+2^y=c$ 2) $2^x=2^y+c$

3) $2^y=c.2^x$ 4) $2^{x+y}=c$

08. The differential equation of hyperbolas with coordinate axes as asymptotes :

1) $x\frac{dy}{dx}-y=0$ 2) $x\frac{dy}{dx}+y=0$

3) $\frac{dy}{dx}=xy$ 4) $x\frac{dy}{dx}=y^2$

09. The differential equation of the family of parabolas having their vertices at the origin and foci on X-axis is

1) $y=2x\frac{dy}{dx}$ 2) $x=2y\frac{dy}{dx}$

3) $xy=\frac{dy}{dx}$ 4) $y=x\frac{dy}{dx}$

10. The differential equation of the system of curves given by $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$ (λ is arbitrary constant) is

1) $x^2-xy\frac{dy}{dx}=a^2$ 2) $x^2-\frac{xy}{\frac{dy}{dx}}=a^2$

3) $x^2+xy=a^2\frac{dy}{dx}$ 4) $x^2-xy=a^2\frac{dy}{dx}$

11. The differential equation obtained by eliminating arbitrary constants from the equation $y=Ae^{5x}+Be^{-2x}$ is

1) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-10y=0$ 2) $\frac{d^2y}{dx^2}-7\frac{dy}{dx}-10y=0$

3) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}-10y=0$ 4) $\frac{d^2y}{dx^2}+7\frac{dy}{dx}+10y=0$

12. The general solution of $(x-y)^2\frac{dy}{dx}=a^2$ is

1) $x+y+c=\log\left|\frac{x-y+a}{x+y+a}\right|$

2) $x+c=a\log\left|\frac{x-y+a}{x-y-a}\right|$

3) $2x+c=\log\left|\frac{x-y-a}{x-y+a}\right|$

4) $2y+c=a\log\left|\frac{x-y-a}{x-y+a}\right|$

13. The general solution $\frac{dy}{dx}=x\cos x+\sin x$ is

1) $xy+\sin x+c$ 2) $y=x\cos x+c$

3) $y=x+\sin x+c$ 4) $y=x\sin x+c$

14. The general solution of $dy/dx=4x-3xy-3y+4$ is

1) $\ln|4-3y|+3x+\frac{x^2}{2}+c=0$

2) $\ln|4-3y|+3x+\frac{3x^2}{2}+c$

3) $\ln|4-3y|-3x-\frac{3x^2}{2}+c$

4) $\ln|4-3y|+x+\frac{x^2}{2}+c$

15. The general solution of $y\frac{dy}{dx}=xe^{x^2-y^2}$ is

1) $e^{x^2}+e^{y^2}=c$ 2) $e^{x^2}.e^{y^2}=c$

3) $e^{x^2-y^2}=c$ 4) $e^{y^2}=e^{x^2}+c$

16. The general solution of $x\frac{dy}{dx}=y+xe^{y/x}$ is

1) $e^{\frac{y}{x}}=\log cx$ 2) $e^{\frac{y}{x}}+\log cx=0$

3) $e^{-\frac{y}{x}}+\log cx=0$ 4) $e^{\frac{y}{x}}\log cy=0$

17. The general solution of $\frac{dy}{dx}=\frac{(x+y)^2}{2x^2}$ is

1) $\tan^{-1}\left(\frac{y}{x}\right)+c=\ln x$ 2) $2\tan^{-1}\left(\frac{y}{x}\right)+c=\ln x$

3) $\tan^{-1}\left(\frac{y}{x}\right)=cx$ 4) $2\tan^{-1}\left(\frac{y}{x}\right)=c+x$

18. The solution of $e^{x-y}dx+e^{y-x}dy=0$ is

1) $e^{2x}-e^{2y}=c$ 2) $e^{2x}+e^{2y}=c$

3) $e^x+e^y=c$ 4) $e^x-e^y=c$

19. The solution of the differential equation $\frac{dy}{dx}-\frac{2xy}{1+x^2}=0$ is

1) $y=c(1+x^2)$ 2) $y=c\sqrt{1+x^2}$

3) $y=\frac{c}{1+x^2}$ 4) $y=\frac{c}{\sqrt{1+x^2}}$

20. The general solution of $(x+2y^3)\frac{dy}{dx}=y$ (for $y>0$) is

1) $x=\frac{y^3}{3}+cy$ 2) $3x+2y^3+cy$

3) $x=y^3+cy$ 4) $xy=\frac{2y^3}{3}+c$

21. The general solution of $x\ln x\frac{dy}{dx}+y=1$

1) $\ln x=\frac{c}{y-1}$ 2) $y\ln x=x+c$

3) $xy=\ln(\ln x)+c$ 4) $\frac{x}{y}\ln y+c$

22. The solution of $\frac{dy}{dx}=2xy-3y+2x-3$ is

1) $e^{x^2}+3x=c(y+1)$ 2) $e^{x^2-3x}=c(2y+1)$

3) $e^{x^2}-3x=c(y-1)$ 4) $e^{x^2-3x}=c(y+1)$

23. The general solution of $\left(\frac{x^5}{y}+x\right)\frac{dy}{dx}=y$ is

1) $xy=\frac{y^4}{4}+c$ 2) $\frac{x}{y}=\frac{y^6}{6}+c$

3) $xy=\frac{y^5}{5}+c$ 4) $\frac{x}{y}=\frac{y^4}{4}+c$

24. The solution of $\frac{dy}{dx}=e^{x+y}+x^2e^{x^3+y}$ is....

1) $e^x-e^{-y}+\frac{1}{3}e^{x^3}=c$ 2) $e^x+e^{-y}+\frac{1}{3}e^{x^3}=c$

3) $e^{-x}+e^{-y}+\frac{1}{3}e^{x^3}=c$ 4) $e^x+e^{-y}-\frac{1}{3}e^{x^3}=c$

25. The solution of $\frac{dy}{dx}=\frac{y^2}{xy-x^2}$ is

1) $e^{y/x}=kx$ 2) $e^{y/x}=ky$

3) $e^{-y/x}=kx$ 4) $e^{-y/x}=ky$

26. If $(1+y^2)dx=\left(e^{\tan^{-1}y}-x\right)dy$, then

1) $x=\frac{e^{\tan^{-1}y}}{2}+c.e^{-\tan^{-1}y}$

2) $xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c$

3) $x=2+ce^{-\tan^{-1}y}$ 4) $x=\frac{1}{2}+ce^{-\tan^{-1}y}$

27. Solution of $\frac{dy}{dx}=\frac{x}{y}$ is

1) $x^2+y^2=c$ 2) $xy=c$

3) $y^2=x^2+c$ 4) $x+y=c$

28. The solution of $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$ is

1) $(e^y+1)\cos x=c$ 2) $e^y\sin x=c$

3) $(e^y+1)\sin x=c$ 4) none

29. The solution of $(1+e^x)\frac{dy}{dx}=e^{x-y}$ is

1) $e^y=\log(1+e^x)+c$

2) $e^x=\log(1+e^y)+c$

3) $e^y=\log(1-e^x)+c$ 4) none

30. The solution of $e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$ is

1) $\tan y=c(1+e^x)$ 2) $\tan y=c(1-e^x)$

3) $\tan y=c(1-e^x)^2$ 4) none

31. The solution of $\frac{dy}{dx}=\frac{2x-y}{x-y}$ is

1) $2x^2-2xy+y^2=c$ (2) $2x^2+xy+y^2=c$

3) $x^2-2xy+2y^2=c$ (4) $x^2+xy-y^2=c$

32. The solution of $dx+x\,dy=e^{-y}\sec^2 y\,dy$ is

1) $xe^y=\tan y+c$ 2) $xe^{-y}=\tan y+c$

3) $xe^{-y}=\sec y+c$ 4) $xe^y=-\sec y+c$

33. The solution of $\frac{dy}{dx}+\frac{y}{x}=y^2x\sin x$ is

1) $y(\cos x+c)=1$ 2) $y(\cos x+c)=x$

3) $x(\cos x+c)=y$ 4) $xy(\cos x+c)=1$

34. The solution of $y''=0,\ y(0)=1,\ y'(0)=1$ is

1) $y=1+x$ 2) $y=1-x$

3) $y=1+2x$ 4) $y=1-2x$

35. The solution of

$y''-y'-2y=0,\ y(0)=0,\ y'(0)=4$ is

1) $y=\frac{4}{3}(e^{-x}+e^{2x})$ 2) $y=\frac{-4}{3}(e^{-x}+e^{2x})$

3) $y=\frac{4}{3}(-e^{-x}+e^{2x})$ 4) $y=\frac{4}{3}(e^{-x}-e^{2x})$

36. The solution of

$(D^2+1)y=0,\ y(0)=2,\ y\left(\frac{\pi}{2}\right)=-2$ is

1) $y=2(\sin x-\cos x)$

2) $y=2(\cos x-\sin x)$ 3) $y=2(\cos x+\sin x)$

4) $y=2(\cos x+\sin x)$

37. The solution of $(D^2-2D+10)y=0$ and $y(0)=4,\ y'(0)=1$ is

1) $y=e^x(4\cos 3x-\sin 3x)$

2) $y=e^{-x}(4\cos 3x-\sin 3x)$

3) $y=e^{-x}(4\cos 3x+\sin 3x)$

4) none

38. The solution of $(D^2+D-2)y=0,\ y(0)=0,\ y'(0)=3$ is

1) $y=e^x+2e^{-x}$ 2) $y=e^{-x}+2e^{2x}$

3) $y=2e^x+e^{-2x}$ 4) $y=2e^{-x}+e^{-2x}$

39. The particular solution of $(D^2+1)y=\sin x\sin 2x$ is

1) $\frac{1}{4}\sin x+\frac{1}{16}\cos 3x$ 2) $\frac{x}{4}\sin x+\frac{1}{16}\cos 3x$

3) $\frac{x}{4}\sin x-\frac{1}{16}\cos 3x$ 4) $\frac{x}{4}\sin x-\frac{1}{16}\cos 3x$

PRACTICE SET-II KEY

01) 1	02) 4	03) 3	04) 2	05) 4
06) 3	07) 2	08) 2	09) 1	10) 2
11) 3	12) 4	13) 4	14) 2	15) 4
16) 3	17) 2	18) 2	19) 1	20) 3
21) 1	22) 4	23) 4	24) 2	25) 2
26) 1	27) 3	28) 3	29) 1	30) 2
31) 1	32) 1	33) 4	34) 1	35) 3
36) 2	37) 1	38) 3	39) 2	

SELF TEST - I

01. A curve satisfies $(e^x+1)y\,dy+(y+1)dx=0$ and passes through (0,0) ; then its equation is

1) $x-y-\log(y+1)(e^x+1)+\log 2=0$

2) $(e^x+1)(y+1)=2e^{x+y}$

3) $x-y-\log(y+2)(e^x+1)+\log 4=0$

4) $(e^x-1)(y-1)=0$

02. The particular solution of $(x+x^2)\frac{dy}{dx}=1+2x$, given that $y=\ln 2$ when $x=1$ is

1) $y=\ln(x+x^2)$ 2) $y=\ln(x+x^3)$

3) $y\left(\frac{dy}{dx}\right)^2+y=2xy\frac{dy}{dx}$

4) $y\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}=-y$

03. General solution of the equation $\frac{dy}{dx}=\frac{x(2\ln x+1)}{\sin y+y\cos y}$ is

1) $y\sin y=x^2+c$ 2) $y+\sin y=x^2+\ln x+c$

3) $y^2=x^2+2xy\frac{dy}{dx}$ 4) $y^2=x^2-2xy\frac{dy}{dx}$

04. The equation of the curve passing through (0,1) which is a solution of the differential equation $(1+y^2)dx+(1+x^2)dy=0$ is given by

1) $\tan^{-1}x+\tan^{-1}y=0$

2) $\tan^{-1}x+\tan^{-1}y-\frac{\pi}{4}=0$

3) $\sin^{-1}x+\sin^{-1}y=0$

4) $\sin^{-1}x+\sin^{-1}y+\log(1+\sqrt{2})=0$

05. The equation of the curve passing through $\left(\frac{\pi}{6},0\right)$ and which is a solution of $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$ is

1) $4\sin x=1+e^y$ 2) $\sin x(1+e^y)=0$

3) $(1+e^y)\sin x=1$ 4) $2\sin x=e^y\cos x$

06. The equation of the curve passing through $\left(\frac{\pi^2}{4},1\right)$ which is a solution of the equation $y^2\cos\sqrt{x}\,dx-2\sqrt{x}e^{\frac{1}{y}}\,dy=0$ is ____

1) $\sin\sqrt{x}+e^{\frac{1}{y}}=e$ 2) $\sin\sqrt{x}-e^{\frac{1}{y}}=1-e$

3) $\sin\sqrt{x}=1+e+e^{\frac{1}{y}}$ 4) $\sin\sqrt{x}+e^{\frac{1}{y}}=1+e$

07. An integrating factor of the equation $\frac{dy}{dx}-\frac{2y}{x^2}=\frac{1}{x^3}$ is ___

1) $-\frac{1}{x}$ 2) x^2 3) $e^{\frac{2}{x^2}}$ 4) $e^{\frac{2}{x}}$

08. An integrating factor of the equation $(x^2+1)\frac{dy}{dx}+2xy=3x^2$ is

1) $\tan^{-1}x$ 2) $e^{\tan^{-1}x}$ 3) $1+x^2$ 4) $e^{\frac{2}{x}}$

09. The solution of $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$ is

1) $1+\tan\left\{\frac{(x+y)}{2}\right\}=ce^{x}$

2) $1-\tan\left\{\frac{(x+y)}{2}\right\}=ce^{x}$

3) $1+\tan\left\{\frac{(x-y)}{2}\right\}=ce^{x}$

4) $1-\tan\left\{\frac{(x-y)}{2}\right\}=ce^{x}$

10. In integrating factor of the solution $\frac{dy}{dx}-3y\cot x=\sin x$ is

1) $|\sin^3 x|$ 2) $\frac{1}{|\sin^3 x|}$ 3) $e^{|\sin^3 x|}$ 4) $e^{\log \sin^3 x}$

11. The equation of the curve through $(0,\pi/4)$ satisfying the differential equation $e^x\tan y\,dx+(1+e^x)\sec^2 y\,dy=0$ is given by

1) $(1+e^x)\tan y=2$ 2) $1+e^x=2\tan y$

3) $(1+e^x)=\sqrt{2}\sec y$ 4) $(1+e^x)\tan y=k$

12. The general solution of $(x^2+1)\frac{dy}{dx}+4xy=\frac{1}{1+x^2}$

1) $y=\frac{1}{1+x^2}+\frac{c}{(1+x^2)^2}$ 2) $y(1+x^2)^2=x+c$

3) $y(1+x^2)=\frac{x^3}{3}+x+c$ 4) $y(1+x^2)=x+c$

13. The solution of $\frac{dy}{dx}=(1+y^2)(1+x^2)^{-1}$ is__

1) $y-x=c(1+xy)$ 2) $x+y=c(1+xy)$

3) $y+x=c(1-xy)$ 4) $y-x=c(1-xy)$

14. The equation of the curve through the origin and satisfying the differential equation. $\frac{dy}{dx}=(x-y)^2$ is

1) $e^{2x}(1-x+y)=1+x-y$

2) $e^{2x}(1+x-y)=1-x+y$

3) $e^{2x}(1-x+y)=-(1+x+y)$

4) $e^{2x}(1+x+y)=1-x+y$

15. If $\frac{dy}{dx}=\frac{y+x\tan\frac{y}{x}}{x}\Rightarrow\sin\frac{y}{x}=$

1) cx^2 2) cx 3) cx^3 4) cx^4

16. The solution of $x\left(\frac{dy}{dx}\right)=y+xe^{y/x}$ is__

1) $e^{y/x}=\ln(cx)$ 2) $e^{y/x}+\ln(xy)=0$

3) $e^{y/x}+\ln(cx)=0$ 4) $e^{-y/x}+\ln(cx)=0$

17. Solution of $\frac{dy}{dx}+\frac{3x+2y-5}{2x+3y-5}=0$ is__

1) $3x^2+4xy+3y^2-10x-10y=c$

2) $3x^2+4xy-3y^2+10x+10y=c$

3) $3x^2-4xy+3y^2-10x-10y=c$

4) $3x^2-4xy+3y^2+10x+10y=c$

18. The equation of the curve passing through the point (3, 1) and the slope of whose tangent at any point (x,y) is $\frac{3x^2+2}{2y+3}$ is

1) $9y^2+3y=x^3+2x-21$

2) $y^2+3y+29=x^3+2x$

3) $x^3+2y-19=6y^2+8y$

4) $x^3+2x=4y^2+3y+26$

19. The solution of the differential equation $\ln\frac{dy}{dx}=4x-2y-1$. If $y=0$ when $x=1$ is

1) $e^{2y}=e^x+1-e$ 2) $e^{x^2}=2y+e$

3) $2e^{2y+1}=e^{4x}+2e-e^4$ 4) $2e^{2y}=e^{4x}+2-e^4$

20. If $y\sin 2x\,dx=(1+y^2+\cos^2 x)dy$, then

1) $3y\cos^2 x+y+y^3=c$

2) $y\cos^2 x+3y+y^3=c$

3) $3y\cos^2 x+3y+y^3=c$

4) $y\cos^2 x+y+y^3=c$

21. The differential equation representing the family of curves whose tangents make an angle $\pi/4$ with the hyperbola $xy=c^2$ is

1) $y=x^2 2c\tan^{-1}\frac{x}{c}+k$

2) $y^2=x^2-2c\tan^{-1}\frac{x}{c}+k$

3) $y^2=x-2c\tan^{-1}\frac{x}{c}+k$

4) $y=x-2c\tan^{-1}\frac{x}{c}+k$

22. The equation of the curve whose gradient at any point (x,y) is $1-\frac{1}{x^2}$ and which passes through the point (1, 1) is

1) $y=x-\frac{1}{x}+1$ 2) $y=x+\frac{1}{x}-1$

3) $y=x-\frac{1}{x^2}+1$ 4) $y=x+\frac{1}{x^2}-1$

23. The integrating factor of the linear equating $(1-x^2)\frac{dy}{dx}+2xy=\sin^{-1}x$ is

1) $1-x^2$ 2) $\frac{1}{1-x^2}$ 3) e^{1-x^2} 4) $e^{\frac{1}{1-x^2}}$

24. The equation of the curve satisfying the equation $(1+x^3)dy-x^2y\,dx=0$ and passing through the point $(0,1)$ is__

1) $y^2=1+x^3$ 2) $y^2=1+x^2$

3) $y^3=1+x^2$ 4) $y^3=1+x^3$

25. Solution of $\frac{dy}{dx}=\frac{x}{y}$ is

1) $x^2+y^2=c$ 2) $xy=c$

3) $y^2=x^2+c$ 4) $x+y=c$

26. The solution of $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$ is

1) $(e^y+1)\cos x=c$ 2) $e^y\sin x=c$

3) $(e^y+1)\sin x=c$ 4) none

27. The solution of $\frac{dy}{dx}=e^{2x+y}$ is

1) $e^{2x}+e^{-2y}=c$ 2) $e^{2x}+2e^{-y}=c$

3) $e^{2x}-2e^{-y}=c$ 4) none

28. The solution of $\cos^{-1}\left(\frac{dy}{dx}\right)=x+y$ is

1) $\tan\left(\frac{x+y}{2}\right)=x+c$ 2) $\sec\left(\frac{x+y}{2}\right)=x+c$

3) $\cos\left(\frac{x+y}{2}\right)=x+c$ 4) none

29. The solution of $\frac{dy}{dx}=\frac{y}{x}+\sin\left(\frac{y}{x}\right)$ is

1) $\tan\left(\frac{y}{2x}\right)=cx$ 2) $\sec\left(\frac{y}{2x}\right)=cx$

3) $\tan\left(\frac{y}{2x}\right)=cx$ 4) $\sec\left(\frac{x}{2y}\right)=cx$

30. The solution of $\frac{dy}{dx}=\frac{y}{x}+\tan\left(\frac{y}{x}\right)$ is

1) $\sin\left(\frac{x}{y}\right)=cx$ 2) $\sin\left(\frac{y}{x}\right)=cx$

3) $\operatorname{cosec}\left(\frac{y}{x}\right)=cx$ 4) $\cos\left(\frac{y}{x}\right)=cx$

31. The solution of $\frac{dy}{dx}=\frac{y}{x}-\frac{\tan(y/x)}{\sec^2(y/x)}$ is

1) $x\tan(y/x)=c$ 2) $y\tan(x/y)=c$

3) $x\tan(x/y)=c$ 4) $x\sec(y/x)=c$

32. The solution of $(x^3+3xy^2)dx+(3x^2y+y^3)dy=0$ is

1) $x^4-y^4+6x^2y^2=c$ 2) $x^4+y^4-6x^2y^2=c$

3) $x^4-y^4-6x^2y^2=c$ 4) $x^4+y^4+6x^2y^2=c$

33. The integrating factor of $\frac{dy}{dx}+3x^2y=x^5$ is

1) x^3 2) $\frac{1}{x^3}$ 3) $\log x^3$ 4) e^{x^3}

34. The integrating factor of $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$ is

1) x 2) $\log x$ 3) e^x 4) $\log(\log x)$

35. The integrating factor of $\frac{dy}{dx}-\frac{2}{x+1}y=(x+1)^3$ is

1) $\cos x$ 2) $\log\sec x$ 3) $\sec x$ 4) $(x+1)^{-2}$

36. The solution of $\frac{dy}{dx}+\frac{2y}{x}=\frac{1}{x^2}$ is

1) $xy^2=x+c$ 2) $x^2y=x+c$

3) $xy=x+c$ 4) none

37. The solution of $\frac{dy}{dx}+y\cos x=\sin x\cos x$ is

1) $y=(1+\sin x)+ce^{-\sin x}$

2) $y=(1-\sin x)+ce^{\sin x}$

3) $y=(1-\sin x)+ce^{\sin x}$

4) $y=(\sin x-1)+ce^{-\sin x}$

38. The solution of $x\log x\frac{dy}{dx}+y=2\log x$ is

1) $y=(\log x)^2+c$ 2) $y=2\log x+c$

3) $y\log x=(\log x)^2+c$ 4) $y\log x=2\log x+c$

39. The solution of $\frac{dy}{dx}+y\cot x=4x\operatorname{cosec} x$ given that $y=0, x=\frac{\pi}{2}$ is

1) $y\sin x=2x^2-\frac{\pi^2}{3}$ 2) $y\sin x=x^2-\frac{\pi^2}{2}$

3) $y\sin x=2x^2-\frac{\pi^2}{2}$ 4) $y\sin x=2x^2+\frac{\pi^2}{2}$

40. The integrating factor of the equation $x\frac{dy}{dx}+y\log x=e^x.x^{(\log x/2)}$ is

1) $x^{\log x}$ 2) $(\log x)^x$

3) $e^{\frac{(\log x)^2}{2}}$ 4) $(\log x)^{x/2}$

SELF TEST - I KEY

01) 1	02) 1	03) 2	04) 2	05) 3
06) 4	07) 4	08) 3	09) 1	10) 2
11) 1	12) 2	13) 1	14) 1	15) 2
16) 4	17) 1	18) 2	19) 3	20) 3
21) 4	22) 2	23) 2	24) 4	25) 3
26) 3	27) 2	28) 1	29) 1	30) 2
31) 1	32) 4	33) 4	34) 2	35) 4
36) 2	37) 4	38) 3	39) 3	40) 3

SELF TEST - II

01. The general solution of $\frac{dy}{dx}-\frac{2xy}{1+x^2}=0$

1) $y=C(1+x^2)$ 2) $y=C\sqrt{1+x^2}$

3) $y(1+x^2)=C$ 4) $y=\sqrt{1+x^2}=C$

02. The order of the differential equation $\left(\frac{dy}{dx}\right)^3+\left(\frac{dy}{dx}\right)^2+y^4=0$ is

1) 4 2) 3 3) 1 4) 2

03. If C is parameter, then the differential equation whose solution is $y=C^2+\frac{C}{x}$ is

1) $y=x^4\left(\frac{dy}{dx}\right)-x\left(\frac{dy}{dx}\right)^2$

2) $y=x^4\left(\frac{dy}{dx}\right)^2+x\frac{dy}{dx}$

3) $y=x^4\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}$

4) $y=x^4\left(\frac{d^2y}{dx^2}\right)^2-x\frac{dy}{dx}$

04. The equation of the curve passing through the origin and satisfying the differential equation $y_1=(x-y)^2$ is

1) $e^{2x}(1-x+y)=1+x-y$

2) $e^{2x}(1+y+y)=1-x+y$

3) $e^{2x}(1-x+y)=-(1+x+y)$

4) $e^{2x}(1+x+y)=1-x+y$

05. If $x^2+y^2=1$ then

1) $yy_1-2y_1+1=0$ 2) $yy_1+(y_1)^2+1=0$

3) $yy_1-(y_1)^2-1=0$

4) $y_{11}+2(y_1)^2+1=0$

06. The solution of $\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}+y=0$ is given that at $x=0, y=-4$ is

1) $y=2x+4$ 2) $y=x-4$

3) $y=2x-4$ 4) $y=-4$

07. The differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$ where c is +ve parameter is of

1) Order 1 2) Order 2

3) Degree 3 4) Both 1, 3

08. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt}-yt=1$ and $y(0)=-1$, then $y(1)=$

1) $-\frac{1}{2}$ 2) $e+\frac{1}{2}$ 3) $e-\frac{1}{2}$ (4) $\frac{1}{2}$

09. The differential equation of the family of parabolas with focus at the origin and x-axis is

1) $x\left(\frac{dy}{dx}\right)^2+4x\frac{dy}{dx}=4y$

2) $2x\left(\frac{dy}{dx}\right)-y+y\left(\frac{dy}{dx}\right)^2=0$

3) $2x\frac{dy}{dx}+y-y\left(\frac{dy}{dx}\right)^2=0$ 4) None

10. The solution of $\frac{dy}{dx}=\frac{2x\log x+x}{\sin y+y\cos y}$ is

1) $y\sin y=x^2\log_e x+C$

2) $y\sin y=x^2\log_e x$

3) $y\sin y=x^2+\log_e x+C$

4) $y\sin y=x\log_e x+C$

11. The general solution of $y^2dx+(x^2-xy+y^2)dy=0$ is

1) $2\tan^{-1}\left(\frac{x}{y}\right)+\log_e x+C=0$

2) $\log_e x+C=0$

3) $\tan^{-1}\left(\frac{x}{y}\right)+\log_e y+C=0$

4) $2\tan^{-1}\left(\frac{x}{y}\right)+\log_e x+C=0$

12. Order of differential equation of the family of all concentric circles centred at (h, k) is
1) 1 2) 2 3) 3 4) 4

13. The solution of $\frac{dy}{dx}+\frac{y}{3}=1$ is
1) $y=3+ce^{x/3}$ 2) $y=3+ce^{-x/3}$
3) $3y=c+e^{x/3}$ 4) $3y=c+e^{-x/3}$

14. The solution of $y_1=\left(\frac{y}{x}\right)^{\frac{1}{3}}$ is
1) $x^{2/3}+y^{2/3}=C$ 2) $x^{2/3}-y^{2/3}=C$
3) $x^{1/3}+y^{1/3}=C$ 4) $x^{1/3}-y^{1/3}=C$

15. The solution of $y+x^2=y_1$ is
1) $y+x^2+2x+2=ce^x$
2) $y-x^2-2x-2=ce^x$
3) $y+x^2-2x+2=ce^x$ 4) none

16. The solution of $xdx+ydy=x^2dy-xy^2dx$ is
1) $(x^2-1)=C(1+y^2)$ 2) $(x^2+1)=C(1-y^2)$
3) $x^3-1=C(1+y^3)$ 4) $x^3+1=C(1-y^3)$

17. The family of curves, in which the subtangent at any point to any curve is double the abscissa is given by
1) $x=Cy^2$ 2) $y=Cx^2$
3) $x^2=Cy^2$ 4) $y=Cx$

18. The solution of $x^2+y^2\frac{dy}{dx}=4$ is
1) $x^2+y^2=12x+C$ 2) $x^2+y^2=3x+C$
3) $x^3+y^3=3x+C$
4) $x^3+y^3=12x+C$

19. The solution of $\frac{dy}{dx}+y=e^x$ is
1) $2y=e^{2x}+C$ 2) $2e^x=e^x+C$
3) $2ye^x=e^{2x}+C$ 4) $2ye^{2x}=2e^x+C$

20. The solution of $\frac{d^2y}{dx^2}=e^{-2x}$ is y =
1) $\frac{1}{4}e^{-2x}$ 2) $\frac{1}{4}e^{-2x}+Cx+d$
3) $\frac{1}{4}e^{-2x}+Cx^2+d$ 4) $\frac{1}{4}e^{-2x}+C+d$

21. The order and degree of the differential equation $(1+3y_1)^{2/3}=4y_3$
1) 1,2/3 2) 3,1 3) 3, 3 4) 1, 2

22. The solution of $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)y_1=0$ is
1) $xe^{\tan^{-1}y}=\tan^{-1}y+k$
2) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$
3) $(x-2)=Ke^{-\tan^{-1}y}$
4) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

23. The order of the differential equation whose general solution is given by $y=(C_1+C_2)\cos(x+C_3)-C_4e^{x+C_5}$, where C_1, C_2, C_3, C_4, C_5 are arbitrary constants is
1) 5 2) 4 3) 3 4) 2

SELF TEST - II KEY

01) 1	02) 3	03) 3	04) 1	05) 2
06) 3	07) 4	08) 1	09) 2	10) 1
11) 3	12) 1	13) 2	14) 2	15) 1
16) 1	17) 1	18) 4	19) 3	20) 2
21) 3	22) 4	23) 3		

IMPORTANT QUESTIONS

01. The equation $(3x^2y+2xy^2)dx+(x^3+bx^2y)dy=0$ is exact then b =
1) 0 2) 2 3) 1 4) none

02. The solution of $(x+y-1)dy=(x+y-1)dx$ is
1) $\log(x+y)=c+x-y$
2) $\log(x-y)=c-x-y$
3) $\log(x-y)=c-x+y$
4) $\log(x+y)=c-x+y$

03. The solution of $3x(xy-2)dx(x^2+2y)dy=0$ is
1) $x^3y+3x^2+y^2=c$ 2) $xy^3+x^2-3y^2=c$
3) $3x^2y+2xy+y^2=c$ 4) $x^3y-3x^2+y^2=c$

04. The solution of the differential equation $\frac{dy}{dx}=e^{2x-y}+x^3e^{-y}$ is
1) $e^y=\frac{e^{2x}2}{2}+\frac{x^4}{4}+c$ 2) $e^y=\frac{e^{2x}2}{2}-\frac{x^4}{4}+c$
3) $e^y=\frac{e^{-2x}2}{2}-\frac{x^4}{4}+c$ 4) $e^y=\frac{e^{-2x}2}{2}+\frac{x^4}{4}+c$

05. The solution of $\frac{dy}{dx}+2xy=xe^{-x^2}$ is
1) $\int x\,dx+c=\frac{x^2}{2}+c$ 2) $\int x\,dx-c=\frac{x^2}{2}+c$
3) $\int x\,dx-c=\frac{x^2}{4}+c$ 4) none

06. The differential equation whose solution is $y=A\cos x+B\sin x$, A and B being arbitrary constants is ___
1) $\frac{dy^2}{dx^2}+y=0$ 2) $\frac{dy^2}{dx^2}-y=0$
3) $\frac{dy^2}{dx^2}+x=0$ 4) $\frac{dy^2}{dx^2}-x=0$

07. The differential equation of which $y=c_1e^x+c_2e^{-x}$ is the general solution is
1) $y_2+y=0$ 2) $y_2-2y=0$
3) $y_2-y=0$ 4) none

08. The solution of the differential equation $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$ is
1) $e^x=e^y+\frac{x^3}{3}$ 2) $e^y=e^x+\frac{x^3}{3}$
3) $e^x+e^{-y}=\frac{x^3}{3}$ 4) none

09. If $x\frac{dy}{dx}+y=x^3$ and $y=\frac{1}{4}$ when x = 1 then y =
1) $y=\frac{x^2}{4}$ 2) $y=\frac{x^3}{4}$
3) $y=\frac{x^3}{8}$ 4) none

10. The differential equation satisfied by all circles centered at the origin is ___
1) $y\frac{dy}{dx}=x$ 2) $y\frac{dy}{dx}=x$
3) $x\frac{dy}{dx}=y$ 4) $x\frac{dy}{dx}=-y$

11. The integral value, through the origin satisfying the differential equation $(x^2-ay)dx=(ax-y^2)dy$ is
1) $x^3+y^3+3axy=0$ 2) $x^3+y^3-3axy=0$
3) $x^3+y^3-2axy=0$ 4) none

12. If $y''(x)-y'(x)=0$, $y(0)=1$ and $y'(0)=0$ then $y(x)=$
1) 1 2) 2
3) 3 4) 0

13. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} = \sqrt{a}\frac{d^2y}{dx^2}$ are __ and __ respectively.

1) 2, 2
2) 2, 1
3) 1, 2
4) 2, 3

14. The general solution of $(y+x^2)dx+(x+y^2)dy=0$ is

1) $xy+\frac{x^3}{3}+\frac{y^3}{3}=c$
2) $xy-\frac{x^3}{3}-\frac{y^3}{3}=c$
3) $xy-\frac{x^3}{3}+\frac{y^3}{3}=c$
4) none

15. The integrating factor of the differential equation $x\frac{dy}{dx}+y\log x=e^x x^{(\log x)^2}$

1) $e^{\frac{(\log x)^2}{2}}$ 2) $e^{(\log x)^2}$ 3) $e^{\frac{(\log x)^2}{2}}$ 4) none

16. The differential equation obtained by eliminating the arbitrary constants A and B from $y=A+Bx^2$ is

1) $xy_2+y_1=0$
2) $xy_2-y_1=0$
3) $xy_2+2y=0$
4) $xy_2-y=0$

17. The differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\sin y=0$ is

1) a second order linear homogeneous equation
2) a second order non-linear homogeneous equation
3) of degree two and of order two
4) of degree two, of order two and non-linear

18. The general solution of the equation $x\frac{dy}{dx}+y+x^2y^2=0$ is

1) $y=x+\frac{c}{x}$
2) $y=\frac{x}{x+c}$
3) $y=x(x+c)$
4) $y=\frac{1}{x(x+c)}$

19. The solution of the equation $\left(x\frac{dy}{dx}+y\right)\left(y\frac{dy}{dx}-x\right)=0$ is

1) xy = constant
2) $x^2-y^2=$ constant
3) xy = constant or $x^2-y^2=$ constant
4) $xy(x^2-y^2)=$ constant

20. The curve that satisfies the differential equation $\frac{dy}{dx}=e^{x+y}$ and that passes through the point (0,0) is given by

1) $e^x+e^y=2$
2) $e^x+e^{-y}=2$
3) $e^{-x}+e^y=2$
4) $e^{-x}+e^{-y}=2$

21. The differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+e^y=0$ is

1) a second order linear differential equation
2) a second order non-linear differential equation
3) a second oder and second degree differential equation
4) a second order homogeneous linear differential equation

22. The differential equation of the family of circles of given radius 'a' and having their centres lying on the x-axis is

1) $1+(y')^2=a^2$
2) $1+(y')^2=a^2(y')^2$
3) $1+(y')^2=\frac{a^2}{y^2}$
4) $1+(y')^2=\frac{a^2}{(y')^2}=0$

23. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=2$ is

1) $y=\frac{x+c}{x}$
2) $y=\frac{-x+c}{x}$
3) $xy=x^2+c$
4) $y=\frac{c+1}{x}$

24. The differential equation obtained by eliminating the arbitrary constants A and B from $y=Ax^2+Bx$ is

1) $y''=0$
2) $y''+y=0$
3) $x^2y''+y=0$
4) $x^2y''-2xy'+2y=0$

The solution of $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$ is

1) $y=Ae^{2x}+Be^{-2x}$
2) $y=Ae^{2x}+Be^{-2x}$
3) $y=(A+Bx^2)e^{2x}$
4) $y=(A+Bx)e^{2x}$

The differential equation obtained from $y=Ae^{3x}+Be^{5x}$ where A and B are parameter is

1) $\frac{d^2y}{dx^2}+8\frac{dy}{dx}+15y=0$
2) $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=0$
3) $\frac{d^2y}{dx^2}-8\frac{dy}{dx}-15y=0$
4) $\frac{d^2y}{dx^2}+8\frac{dy}{dx}-15y=0$

The solution of $x\,dy=(y+x^2+y^2)dx$ is

1) $\log(x^2+y^2)=cx$
2) $y=x-\tan(c+x)$
3) $x=y-\tan(c-x)$
4) $y=x\tan(c+x)$

Particular integral $(D^4-1)y=e^x\cos x$ is

1) $\frac{1}{5}e^x\cos x$
2) $\frac{-1}{7}e^x\cos x$
3) $\frac{1}{7}e^x\cos x$
4) $\frac{-1}{5}e^x\cos x$

The differnctial equation $\frac{d^3y}{dx^3}+\left(\frac{d^2y}{dx^2}\right)^2+\frac{dy}{dx}+y=0$ is

1) a third order linear differential equation
2) a third order non-linear differential equation
3) a third order second degree differential equation
4) a third order homogeneous linear differential equation

The solution of the differential equation $(y+x-1)\frac{dy}{dx}=(x+y+1)$ is

1) $(x-y)-\log(x+y)=c$
2) $(x+y)-\log(x-y)=c$
3) $(x+y)+\log(x-y)=c$
4) $(y-x)-\log(x+y)=c$

31. The solution of $dy=e^{x+y}dx=0$ is

1) $y=x+c$
2) $e^x+e^{-y}=c$
3) $x+y=c$
4) $e^{x+y}=cy$

32. Solve $(y^2e^{xy^2}+4x^3)dx+(2xy\,e^{xy^2}-3y^2)dy=0$

1) $e^{xy^2}+x^3-y^3=c$
2) $e^{x^2y}+x^3-y^3=c$
3) $e^{xy^2}+x^4-y^4=c$
4) $e^{xy^2}+x^4-y^3=c$

33. Solution of $2\frac{dy}{dx}=\frac{y}{x}+\frac{y^2}{x^2}$ is

1) $y-x=c\sqrt{xy}$
2) $y+x=c\sqrt{xy}$
3) $y-x=c\sqrt{x+y}$
4) $y-x=c\sqrt{x-y}$

34. Solve $\frac{dy}{dx}+y\cot x=2\cos x$

1) $2y\cos x+\cos 2x+c=0$
2) $2y\sin x+\cos 2x+c=0$
3) $2y\sin 2x+\cos x+c=0$
4) $2y\sin 2x+\sin x+c=0$

35. Solution of $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=0$ is

1) $e^{3x}(c_1\cos 3x+c_2\sin 3x)$
2) $c_1e^{3x}+c_2e^{2x}$
3) $e^{2x}(c_1\cos 3x+c_2\sin 3x)$
4) None of these

36. Obtain the differential equation of which $y=Ae^x+Be^{3x}+Ce^{5x}$ is a solution

1) $\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0$
2) $\frac{d^3y}{dx^2}-23\frac{dy}{dx}+15y=0$
3) $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0$
4) $\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0$

37. Solve $(xy^2+x)dx+(yx^2+y)dy=0$

1) $(x+1)(y+1)=c$ 2) $(x^2+1)(y^2+1)=c$

3) $(x+1)(y^2+1)=c$ 4) $(x^2+1)(y+1)=c$

38. The order of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=2\frac{d^2y}{dx^2}$$ is

1) 4 2) 2 3) 3 4) 1

39. The general solution of the differential equation

$\frac{dy}{dx}+\frac{y}{x}=1$ is

1) $2xy-x^2=c$ 2) $2y^2-2y=c$

3) $3y^2+xy+c$ 4) $y^2+xy=c$

40. The general solution of the differential equation

$\frac{d^2y}{dx^2}=4\frac{dy}{dx}+13y=0$ is $y(x)=$

1) $e^{3x}(c_1\cos 2x+c_2\sin 2x)$

2) $e^{2x}(c_1\cos 3x+c_2\sin 3x)$

3) $c_1\cos 3x+c_2\sin 3x$ 4) none of these

41. The degree of the differential equation

$ydx-xdy+3x^2y^2e^{x^3}dx=0$ is

1) 6 2) 7 3) 3 4) 4

42. The solution of the differential equation

$ydx-xdy+3x^2y^2e^{x^3}dx=0$ is

1) $\frac{y}{x}+e^{x^3}=c$ 2) $\frac{x}{y}+e^{x^3}=c$

3) $\frac{y}{x}=e^{x^3}=c$ 4) none of these

43. If one solution of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$ is $y_1(x)=e^x$, the other linearly independent solution $y_2(x)$ is

1) e^{2x} 2) $2e^x$ 3) xe^x 4) $-e^x$

44. The solution of $\frac{dy}{dx}+y\cot x=2\cos x$ is

1) $y\cos x=-\frac{1}{2}\cos 2x+c$

2) $y\sin x=-\frac{1}{2}\cos 2x+c$

3) $y\sin x=-\frac{1}{2}\sin 2x+c$

4) $\sin x+\sin 2x=\cos x+c$

45. The solution of $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+\left[x+\log x-x\sin y\right]dy=0$ is

1) $yx+y\log x+x\cos y=c$

2) $y\log x+2y+\cos y-c$

3) $\frac{1}{y}\log x+2cx\cos y=0$

4) $x(x+y)=y\left(1-\frac{\cos y}{\log x}\right)+c$

46. An integrating factor of $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$ is

1) $3x^3y^3$ 2) $\frac{1}{3x^2y^2}$ 3) $\frac{1}{3x^3y^3}$ 4) $\frac{1}{3x^3y^2}$

47. The differential equation that represents all parabolas each of which has a latus-rectum 8a and whose axes are parallel to x-axis is

1) $\frac{d^2y}{dx^2}+a\frac{dy}{dx}=0$ 2) $4a\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0$

3) $4a\frac{dy}{dx}+\frac{d^2y}{dx^2}=1$ 4) $4a\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

48. The solution of $\frac{dy}{dx}+y\cos x=\frac{1}{2}\sin 2x$ is

1) $e^{\cos x}(\sin x+1)+c$

2) $e^{\sin x}(\cos x-\sin x)+c$

3) $e^{\sin x}(\sin x-1)+c$ 4) None

IMPORTANT QUESTIONS KEY

01) 2	02) 4	03) 4	04) 1	05) 1
06) 1	07) 3	08) 3	09) 2	10) 2
11)	12) 1	13) 1	14) 1	15) 1
16) 2	17) 2	18) 4	19) 4	20) 2
21) 2	22) 3	23) 3	24) 4	25) 4
26) 2	27) 4	28) 4	29) 2	30) 4
31) 2	32) 4	33) 2	34) 2	35) 3
36) 3	37) 2	38) 2	39) 1	40) 2
41) 4	42) 2	43) 3	44) 2	45) 1
46) 3	47) 2	48) 4		

PREVIOUS ECET BITS

2007

01. The degree of the differential equation $(dy/dx)^2=3x/4y$ is:

1) 2 2) 1 3) 0 4) 3

02. Solution of $dy/dx=3x^2/y^2$ is:

1) $3x^2=y^2$ 2) $6x=2y$

3) $y^3=3x^3+3e$ 4) $y^3-3x^3+c=0$

03. Integrating factor of $dy/dx+y\tan(x)=x^2e^x\cos(x)$ is:

1) $\sec(x)$ 2) $\cos(x)$ 3) $\tan(x)$ 4) $\sin(x)$

04. The general solution of $y^{11}-3y^1+2y=0$ is:

1) $c_1e^x+c_2e^{3x}$ 2) $c_1e^x+c_2e^{2x}$

3) $c_1x+c_2 2x$ 4) $c_1e^{2x}+c_2e^{3x}$

05. The general solution of $y^{11}-6y^1+9y=0$ is:

1) $(c_1+c_2x)e^{2x}$ 2) $c_1e^{3x}+c_2e^{-3x}$

3) $(c_1+c_2x)e^{3x}$ 4) $c_1e^x+c_2e^{3x}x^2$

06. The general solution of $y^{11}+9y=0$ is:

1) $c_1\sin(3x)+c_2\cos(3x)$

2) $c_1\sin(x)+c_2\cos(x)$

3) $c_1\sin(2x)+c_2\cos(2x)$

4) $c_1\sin(x)-c_2\cos(x)$

2008

07. Degree of the differential equation

$y=x\frac{dy}{dx}+\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is

1) 1 2) 3 3) 2 4) 4

08. The differential equation whose solution $x=A\cos(pt-\alpha)$, where A and are arbitrary constants is given by

1) $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+2x=0$ 2) $\frac{d^2x}{dt^2}+p^2x=0$

3) $\frac{dx}{dt}=x$ 4) $\frac{d^2x}{dt^2}=-p^2$

09. The solution of the differential equation

$x\frac{dy}{dx}+y=0$ is

1) $\frac{x}{y}=c$ 2) $xy=c$ 3) $x-y=c$ 4) $x+y=c$

10. The integrating factor of $x^2y\,dx-(x^3+y^3)dy=0$ is

1) $\frac{1}{y(x^2-y^2)}$ 2) $\frac{1}{y^4}$

3) $\frac{1}{x^2y^2}$ 4) $\frac{1}{xy}$

11. The general solution of the differential equation

$\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}+11\frac{dy}{dx}+6y=0$ is

1) $y=c_1e^{-x}+c_2e^{-2x}+c_3e^{-3x}$

2) $y=(c_1+c_2x+c_3x^2)e^{-x}$

3) $y=c_1e^{x}+(c_2+c_3x)e^{3x}$

4) $y=(c_1+c_2x)e^{x}+c_3e^{4x}$

12. The particular value of $\frac{1}{D^3}\cos x$ is

1) $\sin x$ 2) $-\sin x$ 3) $2\sin x$ 4) $x\sin x$

13. The particular integral of $(D^2-5D+6)y=e^{4x}$ is

1) $\frac{e^{4x}}{2}$ 2) $e^{4x}+e^{6x}$ 3) $\frac{xe^{4x}}{2}$ 4) $\frac{x^2e^{4x}}{2}$

14. The particular integral of the differential equation $(D^2-2D+4)y=x\,e^{2x}$ is

1) $\frac{x^2e^{2x}}{2}$ 2) x^2e^{2x} 3) $\frac{x^3e^{2x}}{6}$ 4) None

2009

15. Solution of $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

1) $\sin^{-1}x+\sin^{-1}y=c$ 2) $\sin^{-1}x-\sin^{-1}y=c$

3) $\sinh^{-1}x+\sinh^{-1}y=c$ 4) $\tan^{-1}x=\tan^{-1}y+c$

16. The integrating factor of $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$ is

1) $\frac{1}{3x^3y^3}$ 2) $\frac{1}{x^3}$ 3) $\frac{1}{y^3}$ 4) $-\frac{1}{3x^3y^3}$

17. Integrating factor of $\frac{dy}{dx}+Py=Q$ to make it an exact equation is

1) $e^{\int P dx}$ 2) $e^{\int Q(y)dy}$ 3) e^{x} 4) e^{y}

18. The particular solution of $(D^2-D-2)y=\sin 2x$ is

1) $\frac{1}{20}(\cos 2x-3\sin 2x)$ 2) $\frac{1}{2}\cos x$

3) $\frac{1}{2}\sin x$ 4) $\frac{1}{8}x\sin 2x$

19. Particular solution of $(D^2-4D+4)y=x^3$ is

1) $\frac{1}{8}(2x^3+6x^2+9x+6)$ 2) $9x^2+6x-25$

3) $12x^2+30x$ 4) x^2+2x+5

20. Particular solution of $(D^2-6D+13)y=8e^{3x}\sin 2x$

1) $-2xe^{3x}\cos 2x$ 2) xe^{3x}

3) $x^2e^{3x}\cos 3x$ 4) $-2xe^{x}\sin 2x$

21. Differential equation of the family of circles having centres on the x axis adn passing through the origin is

1) $y^2-x^2-2xy\frac{dy}{dx}=0$ 2) $x\frac{dy}{dx}=2y$

3) $(x^2-a^2)\frac{dy}{dx}=xy$ 4) $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=y$

2010

22. The degree and order of the differential equation $y=x\frac{dy}{dx}+\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

1) 2 & 1 2) 1 & 2 3) 2 & 2 4) 1 & 1

23. The general solution of differential equation $(x+y)\frac{dy}{dx}+y=0$ is

1) $(x+y)-x^2=c$ 2) $(x+y)^2+x^2=c$

3) $(x+y)^2-x^2=c$ 4) $x^2+y^2=c$

24. If $y=Ae^{x}+Be^{2x}$, where A & B are orbitrary constant, then the differential equation is.

1) $y_2+3y_1+2y=0$ 2) $y_2+3y_1-2y=0$

3) $y_2-3y_1-2y=0$ 4) $y_2-3y_1+2y=0$

25. The complementary solution of $(D^4+2D^3+D^2)y=0$ is

1) $y=c_1+c_2x+c_3e^{x}+c_4x\,e^{x}$

2) $y=c_1+c_2x+c_3e^{-x}+c_4xe^{-x}$

3) $y=c_1+c_2x+c_3e^{-x}+c_4x\,e^{x}$

4) $y=(c_1+c_2x)e^{x}+(c_3+c_4x)e^{-x}$

26. The particular solution of $(D^2+4)y=\sin 2x$ is

1) $\frac{-x\cos 2x}{4}$ 2) $\frac{-x\sin 2x}{4}$

3) $\frac{x\cos 2x}{4}$ 4) $\frac{x\sin 2x}{4}$

27. The solution of the differential equation $(x^2-ay)dx=(ax-y^2)dy$ is

1) $x^3-3axy+y^3+c$

2) $x^3-6axy+y^3+c$

3) $x^3+y^3=c$ 4) $x^3-y^3=c$

28. The solution of $\frac{1}{D^2+D+1}x^2$

1) x^2+2x+4 2) x^2+2x+2

3) x^2-2x 4) x^2-2x-4

29. The solution of $\frac{1}{D-a}e^{ax}$ is

1) $(x-1)e^{ax}$ 2) $(x+1)e^{ax}$

3) xe^{ax} 4) xe^{-ax}

2011

30. The differential equation of the family of curves $y=Ae^{x}+Be^{-x}$ for different values of A & B is

1) $\frac{d^2y}{dx^2}=y$ 2) $\frac{d^2y}{dx^2}+y=0$

3) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$ 4) $\frac{dy}{dx}+P(x)y=q(x)$

31. Solution of the differential equation $\frac{dy}{dx}=\frac{y^2}{xy-x^2}$ is

1) $y=ce^{xy}$ 2) $x=ce^{y/x}$

3) $\log y=c$ 4) $e^{x-y}=y$

32. The solution of the linear third order equation $\frac{d^3y}{dx^3}-7\frac{d^2y}{dx^2}+16\frac{dy}{dx}-12y=0$

1) $y=c_1e^{3x}+c_2e^{x}+c_3e^{4x}$

2) $y=c_1e^{3x}+c_2xe^{x}+c_3e^{4x}$

3) $y=c_1e^{2x}+c_2xe^{3x}+c_3e^{4x}$

4) $y=c_1e^{3x}+(c_2+c_3x)e^{2x}$

2012

33. The order of $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}-3y=x$ is

1) 1 2) 4 3) 3 4) 2

34. The degree of $\left[\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=a\frac{d^2y}{dx^2}$ is

1) 4 2) 2 3) 1 4) 3

35. The family of straight lines passing through the origin is represented by the differential equation

1) $ydx+xdy=0$ 2) $xdy-ydx=0$

3) $xdx+ydy=0$ 4) $xdx-ydy=0$

36. The differential equation $\frac{dy}{dx}+\frac{ax+hy+g}{hx+by+f}=0$ is called

1) Homogeneous 2) Exact

3) Linear 4) Legender

37. The solution of differential equation $\frac{dy}{dx}=e^{-x^2}-2xy$ is

1) $y.e^{-x^2}=x+c$ 2) $ye^{x}=x+c$

3) $ye^{x^2}=x+c$ 4) $y=x+c$

38. The complementary function of $\left(D^3+D^2+D+1\right)y=10$ is

1) $C_1\cos x+C_2\sin x+C_3e^{-x}$
2) $C_1\cos x+C_2\sin x+C_3e^{x}$
3) $C_1+C_2\cos x+C_3\sin x$
4) $\left(C_1+C_2x+C_3x^2\right)e^{x}$

39. Particular Integral of $(D-1)^4 y=e^x$ is

1) $x^4 e^x$ 2) $\frac{x^4}{24}e^{-x}$ 3) $\frac{x^4}{12}e^x$ 4) $\frac{x^4}{24}e^x$

2013

40. The degree of the differential equation $\left(x^3y''\right)^4-4x^2\left(y''\right)^8+6xy'-10y=\cos 4x$ is

1) 8 2) 3 3) 4 4) 1

41. The differential equation of the family of circles with center at the origin is

1) $yy'+x=0$ 2) $xy+y'=0$
3) $xy-y'=0$ 4) $y''+x=0$

42. The solution of $\frac{dy}{dx}=\frac{x+y+1}{2x+2y+2}$ is

1) $2y=c$ 2) $x-2y=c$
3) $y=2x^2+c$ 4) $x^2+y^2=c^2$

43. The solution of $\frac{dy}{dx}+\frac{2y}{x}=\frac{2\cos 2x}{x^2}$ is

1) $xy^2=\cos 2x+c$
2) $xy=4\sin 2x+c$
3) $xy=4\cos 2x+c$
4) $x^2y=\sin 2x+c$

44. The solution of the equation $\left(D^2-1\right)^2 y=0$ is

1) $y=c_1\cos x+c_2\sin x+c_3e^x+c_4e^{-x}$
2) $y=\left(c_1x+c_2\right)\sin x+\left(c_3e^x+c_4\right)\cos x$
3) $y=c_1+c_2x+c_3e^x+c_4e^{-x}$
4) $y=\left(c_1x+c_2\right)e^x+\left(c_3x+c_4\right)e^{-x}$

45. The particular integral of the differential equation $\left(D^4-1\right)y=4\sin x$ is

1) $\cos x$ 2) $x\cos x$ 3) $x\sin x$ 4) $\sin x$

46. The particular integral of the differential equation $\left(D^2-4D+4\right)y=2e^x$ is

1) $2e^x$ 2) x^2e^{2x} 3) xe^{2x} 4) $2xe^{2x}$

2014

47. The differential equation formed by eliminating the arbitrary constants a and b in the relation $y=a\cos(nx+b)$ is

1) $\frac{d^2y}{dx^2}+n^2y=0$ 2) $\frac{d^3y}{dx^3}-x^3y=0$
3) $\frac{dy}{dx}+ny=0$ 4) $\frac{d^2y}{dx^2}-y=0$

48. The solution of $\frac{dy}{dx}=e^{x-y}$

1) $e^x-e^{-y}+c=0$ 2) $e^{x-y}+c$
3) $e^x+e^{-y}+c=0$ 4) $e^x-e^y+e^c=0$

49. The solution of the differential equation $\tan x\frac{dy}{dx}+y=\sec x$ is

1) $y\sin x-x=c$ 2) $y\cot x+x=c$
3) $y=\tan x+c$ 4) $y\,\mathrm{cosec}\,x=x+c$

50. If $y_1=e^x$ and $y_2=e^{-x}$ are two solutions of the homogeneous differential equation; then

1) $y_3=e^{2x}$ and $y_4=e^{-2x}$ are also solutions of the equation
2) $y_3=xe^x$ and $y_4=xe^{-x}$ are also solutions of the equation
3) $y_3=\cosh x$ and $y_4=\sinh x$ are also solutions of the equation
4) $y_3=\cos x$ and $y_4=\sin x$ are also solutions of the equation

51. The particular integral (P.I) of the equation $\left(D^2+D-6\right)y=5e^{2x}+6$ is

1) $xe^{2x}-1$ 2) $xe^{2x}+1$
3) $5xe^{2x}+1$ 4) $xe^{2x}-1$

52. The particular integral of $\left(D^2+16\right)y=8\cos 4x$ is

1) $\cos 4x$ 2) $x\sin 4x$
3) $-\frac{1}{4}\sin 4x$ 4) $-\frac{1}{4}\cos 4x$

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53. The degree of the differential equation $\left(\frac{dy}{dx}\right)^3$ - $\left(\frac{d^2y}{dx^2}\right)^{3/2}$ + 5x=0 is

1) 3 2) 6 3) 1 4) 2

54. The differntial equation formed by eliminating the arbitrary constant a from $a^2y+ax+8=0$ is

1) $8y_1^2-xy_1+y=0$ 2) $8y_1^2+xy_1+y=0$
3) $8y_1^2-xy_1-y=0$ 4) $8y_1^2+2xy_1-y=0$

55. Solution of the equatio $(1+y^2)dx+(1+x^2)dy=0$ is

1) $ran^{-1}x+\tan^{-1}y=c$
2) $\tan^{-1}x+\tan^{-1}y=0$
3) $y+\frac{y^3}{3}+x+\frac{x^3}{3}=c$
4) $\sin^{-1}x+\sin^{-1}y=0$

56. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$ under the condition that y=1 when x=1 is

1) $4xy=x^3+3$ 2) $4xy=x^4+3$
3) $4xy=y^3+3$ 4) $4xy=y^4+3$

57. The paricular Integral of $\left(D^2+a^2\right)$ y= cos ax is

1) $xe^{\frac{-x}{3}}$ 2) $e^{\frac{-x}{3}}$ 3) $-xe^{\frac{-x}{3}}$ 4) $xe^{\frac{x}{3}}$

58. The complimentary Function of $(D^2+4D+5)y=13e^x$is

1) $e^{-2x}(c_1\cos x+c_2\sin x)$
2) $e^{2x}(c_1\cos x+c_2\sin x)$
3) $e^x(c_1\cos 2x+c_2\sin 2x)$
4) $e^{-x}(c_1\cos 2x+c_2\sin 2x)$

59. The particular Integral of $(D^2-2D+1)y=\cosh x$is

1) $\frac{x^2e^x}{4}+\frac{e^{-x}}{8}$ 2) $\frac{x^2e^{-x}}{4}+\frac{e^x}{8}$
3) $\frac{x^2e^x}{4}-\frac{e^{-x}}{8}$ 4) $\frac{x^2e^{-x}}{4}-\frac{e^x}{8}$

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60. Differential equation corresponding to $y=\sqrt{5x+c}$ is

1) $y^2=5x+c$ 2) $y'=\frac{2.5}{\sqrt{5x+c}}$
3) $yy'=5$ 4) $yy'=2.5$

61. The differntial equation: $(y')^2+5y^{1/3}=x$ is

1) linear of order 1 and degree 2
2) non-linear of order 1 and degree 2
3) linear of order 1 and degree 6
4) non-linear of order 1 and degree 6

62. The differntial equation: $(x+x^8+ay^2)dx+(y^8-y+bxy)dy=0$ is exact if

1) b = a 2) b= 2a
3) a = 1, b = 3 4) $b\neq 2a$

63. Complementary function of $y''+4y=0$ is

1) $\cos 2x+\sin 2x$
2) $C_1\cos 2x+C_2\sin 2x$
3) $C_1\cos x+C_2\sin x$
4) $C_1\cos 4x+C_2\sin 4x$

64. Integrating factor of differential equation; $x^2y'=3x^2-2xy+1$ is

1) x 2) $\frac{1}{x}$ 3) $\frac{1}{x^2}$ 4) x^2

65. Particular integral of $(D^2+4)y=\cos 2x$ *is*

1) $\frac{\sin 2x}{4}$ 2) $\frac{\cos 2x}{4}$
3) $\frac{x\sin 2x}{4}$ 4) $\frac{x\cos 2x}{4}$

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66. The differential equations of the family of circles touching y-axis at the origin is

1) $y^2-x^2-2xyy'=0$
2) $(x^2-y^2)y'-2xy=0$
3) $yy'+y^2=x^2$ 4) $2yy'-y^2=x^2$

67. The solution of the differential equaton ydx-2xdy=0 represents a family of

1) straight lines 2) parabolas
3) circles 4) catenaries

68. If y=x is solution of $x^2y''+xy'-y=0$ then the second linearly independent solution of the equation is

1) x^2 2) $\frac{1}{x}$ 3) $\frac{1}{x^2}$ 4) x^n

69. Which of the following is an integrating factor of $\frac{dy}{dx}(x+y+1)=1$?

1) e^x 2) e^y 3) e^{-x} 4) e^{-y}

70. The differential equation whose solution is Ax^2+By^2, where A,B are arbitrary constants is of

1) 1^{st} order and 1^{st} degree
2) 2^{nd} order and 1^{st} degree
3) 2^{nd} order and 2^{nd} degree
4) 1^{st} order and 2^{nd} degree

71. The general solution of the differential equation $\frac{d^2x}{dt^2}-4\frac{dx}{dt}+5x=0$ is

1) $x=(c_1\cos t+c_2\sin t)e^{2t}$
2) $t=(c_1\cos t+c_2\sin x)e^{2t}$
3) $t=(c_1\cos 2t+c_2\sin 2t)e^{t}$
4) $t=(c_1\cos 2t+c_2\sin 2x)e^{x}$

72. The particular integral of $\frac{d^2y}{dx^2}-y=\cosh x$ is

1) $\frac{x\sinh x}{4}$ 2) $\frac{x\sinh x}{2}$
3) $\frac{x(xe^x-e^{-x})}{4}$ 4) $\frac{x\cosh x}{4}$

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73. Solution of yxdy=$(y^2-1)dx$ is

1) $y^2=\frac{x^2}{2}+1$ 2) $y^2=cx^2+1$
3) $y=\sqrt{x+\frac{1}{2}}$ 4) $y=cx^2=x$

74. Solution of $e^x\cot y\,dx+(1-e^x)\operatorname{cosec}^2 y\,dy=0$ is

1) $e^{-x}\cot y=C$ 2) $(e^x-1)\cot y=C$
3) $e^x+\cot y x=C$
4) $(e^x-\cot y)+1=C$

75. Solution of $(D^2-2D+1)y=e^{-x}$*is*

1) $y=(c_1+c_2x)e^x+\frac{1}{4}e^{-x}$
2) $y=(c_1+c_2x)e^{-x}+\frac{1}{4}e^x$
3) $y=(c_1+c_2x)e^x+\frac{1}{2}e^x$
4) $y=c_1\cos x+c_2\sin x+\frac{1}{4}e^{-x}$

76. Solution of $xe^{x^2+y}=y\frac{dy}{dx}$ *is*

1) $(y+1)e^{-y}+\frac{1}{2}e^{x^2}=C$ 2) $ye^{-y}+\frac{1}{2}e^{x^2}=C$
3) $\left(y+\frac{1}{2}e^{x^2}\right)e^{-y}=C$ 4) $y=e^{-y}+1\frac{1}{2}e^{x^2}$

77. The solution of ydx+$(x+x^2y)dy=0$ is

1) logy=cx 2) $\log y=\frac{1}{xy}+c$
3) $-\frac{1}{xy}+\log y=c$ 4) $\log\left(\frac{y}{x}\right)=c$

78. If $y-\cos x\frac{dy}{dx}=y^2(1-\sin x)\cos x;\ y(0)=1,$ *then* $y\left(\frac{\pi}{3}\right)=$

1) 0 2) 1 3) 2 4) $\sqrt{3}$

A.P ECET-2017

79. $y=cx-c^2$ is the general solution of the differential equation

1) $\left(\frac{dy}{dx}\right)^2-x\left(\frac{dy}{dx}\right)+y=0$
2) $\frac{d^2y}{dx^2}=0$ 3) $\frac{dy}{dx}=c$
4) $\left(\frac{dy}{dx}\right)^2+x\left(\frac{dy}{dx}\right)+y=0$

80. The genral solution of the differential equation $\frac{dy}{dx}+\frac{y}{3}=1$ is

1) $y=3+ce^{\frac{x}{3}}$ 2) $y=3+ce^{-\frac{x}{3}}$
3) $3y=c+e^{\frac{x}{3}}$ 4) $3y=c+e^{-\frac{x}{3}}$

81. The differential equation corresponding to the family of curves $y=ae^{bx}$, where a and b are arbitrary constants, is ______

1) $\frac{d^2y}{dx^2}=y\frac{dy}{dx}$ 2) $y\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$
3) $y\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$ 4) $\frac{dy}{dx}-y^2=0$

82. An integrating factor of the differential equation $(x^2y+y+1)dx+(x+x^3)dy=0$ is

1) e^x 2) x^2 3) $\frac{1}{x}$ 4) x

83. The differential equation whose solution is Ax^2+By^2, where A,B are arbitary constants are of----

1) 1^{st} order and 1^{st} degree
2) 2^{nd} order and 1^{st} degree
3) 2^{nd} order 2^{nd} degree 4) 1^{st} order and 2^{nd} degree

84. The general solution of the differential equation $\frac{d^2x}{dt^2}-4\frac{dx}{dt}+5x=0$ is

1) $x=(c_1\cos t+c_2\sin t)e^{2t}$
2) $t=(c_1\cos x+c_2\sin x)e^{2x}$
3) $x=(c_1\cos 2t+c_2\sin 2t)e^{t}$
4) $t=(c_1\cos 2x+c_2\sin 2x)e^{x}$

85. The particular integral of $(D-2)^2y=\sin 2x$ is

1) $\frac{\cos 2x}{8}$ 2) $\frac{\sin 2x}{8}$
3) $\frac{-\cos 2x}{2}$ 4) $\frac{-\sin 2x}{2}$

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86. The general solution of $x\frac{dy}{dx}=y[\log y-\log x+1]$ is

1) $y=Ce^x$ 2) $y=Ce^y$
3) $y=xe^{cx}$ 4) $x=Ce^{y/x}$

87. A and B are arbitrary constants, the differential equation having $xy = Ae^{x} + Be^{-x} + x^2$ as its general solution is

1) $y'' + 2xy' - xy + x^2 = 0$
2) $xy'' + y' - xy - 2 = 0$
3) $xy'' + 2y' - 2xy + 3x^2 - 2 = 0$
4) $xy'' + 2y' - xy + x^2 - 2 = 0$

88. The solution of $\left(e^{-2\sqrt{x}} - y\right)\dfrac{dx}{dy} = \sqrt{x}$

1) $y = e^{-2\sqrt{x}}\left(2\sqrt{x} + C\right)$
2) $y = e^{-2\sqrt{x}} + \sqrt{x} + C$
3) $y = e^{-2\sqrt{x}} + e^{\sqrt{x}}\sqrt{x} + C$
4) $y = e^{2\sqrt{x}} + \log x + C$

89. The solution of cosx dy = (sinx-y) ydx

1) y = sec x tan x +C
2) $y^{-1} Co\sec x = \cot x + C$
3) $y^{-1}\sec x = \tan x + C$
4) y= log sin x+C

90. The solution of $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 0$ satisfying $y(0)=1$ and $y'(0)=0$ is

1) $y = e^{-2x}\left[\cos x + 2\sin x\right]$
2) $y = e^{-x}\left[2\cos x + \sin x\right]$
3) $y = e^{2x}\left[2\cos x + 3\sin x\right]$
4) $y = e^{x}\left[\cos x + 2\sin x\right]$

91. $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 2e^{x}$; with y(0)=1; $y'(0) = 1$ satisfies

1) $y = c_1e^{2x} + c_2e^{3x} + e^{x}$
2) $y = 2e^{2x} + 3e^{3x} + e^{x}$
3) $y = e^{2x} + 2e^{3x} + e^{-x}$
4) $y = e^{x}$

92. The solution of (y log x -2) ydx = xdy

1) $y = x\left(\log x + C\right)$
2) $y = \dfrac{1}{x}\log x + x + C$
3) $\dfrac{1}{y} = x\log x + x + Cx$
4) $\dfrac{1}{y} = x^2\log x + x + C$

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93. The solution of $\sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = 0$ is

1) $\cos^{-1}x + \cos^{-1}y = c$
2) $\sinh^{-1}x + \cosh^{-1}y = c$
3) $\cos^{-1}x + \sec^{-1}x = c$
4) $\sin^{-1}x + \sin^{-1}y = c$

94. The solution of exact differential equation $2xy\,dx + x^2dy = 0$ is

1) $x^2y^2 = c$ 2) $x^2y = c$ 3) $x^3y = c$ 4) $x^2y^3 = c$

95. The complementary function of $\left(D^2 + 3D + 2\right)y = 8\sin 5x$ is

1) $c_1e^{-x} + c_2e^{-2x}$
2) $c_1e^{x} + c_2e^{2x}$
3) $c_1e^{-x} + c_2e^{2x}$
4) $c_1e^{2x} + c_2e^{3x}$

96. The solution of $\dfrac{dy}{dx} + y = e^{-x}$ is

1) $(x+c)e^{-x}$
2) $(x-c)e^{x}$
3) $(x+c)e^{x}$
4) $(x+c)e^{-2x}$

97. The particualr integral of $\left(D^2 + 5D + 6\right)y = e^{x}$ is

1) $\dfrac{-e^{-x}}{12}$ 2) $\dfrac{6^{2x}}{12}$ 3) $\dfrac{e^{x}}{12}$ 4) $\dfrac{e^{x}}{6}$

98. The solution of $\dfrac{dy}{dx} = (4x + y + 1)^2$ is

1) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{4x+y+1}{2}\right) = x + c$
2) $\dfrac{1}{2}\cot^{-1}\left(\dfrac{4x+y+1}{2}\right) = x + c$
3) $-\dfrac{1}{2}\tan^{-1}\left(\dfrac{4x+y+1}{2}\right) = x + c$
4) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{4x-y-1}{2}\right) = x + c$

99. Form the differential equation by eliminating the arbitary constant a from $ay^2 = x^3$

1) $\dfrac{dy}{dx} = \dfrac{3y}{2x}$ 2) $\dfrac{dy}{dx} = \dfrac{2x}{3y}$
3) $\dfrac{dy}{dx} = \dfrac{x}{y}$ 4) $\dfrac{dy}{dx} = \dfrac{2y}{x}$

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100. The solution of $\left(x + 2y^3\right)\dfrac{dy}{dx} = y$

1) $y = x^3 + cx$ 2) $x = y^3 + cy$
3) $x = y^2 + cy$ 4) $y = x^3 + cy^2$

101. The general solution of $\dfrac{dy}{dx} = \dfrac{x^2+4x-9}{x+2}$ is____

1) $y = (x+2)^2 - 13\log|x+2| + c$
2) $y = (x+2)^2 - 5\log|x+2| + c$
3) $y = \dfrac{x^2}{2} + 2x + 13\log|x+2| + c$
4) $y = \dfrac{x^2}{2} + 2x - 13\log|x+2| + c$

102. The differnetial equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where c being a positive parameter is of____

1) order 3 2) order 2
3) degree 3 4) degree 1

103. The differential equation formed by eliminating the arbitrary constants a and b from the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is

1) $xy' = 1$ 2) $xy'' = 0$ 3) $y'' = 0$ 4) $y'' = 1$

104. The solution of the differential equation $\dfrac{dy}{dx} = \left(1+x^2\right)\left(1+y^2\right)$ is____

1) $Tan^{-1}y = x + \dfrac{x^3}{3} + c$
2) $Tan^{-1}y = x - \dfrac{x^3}{3} + c$
3) $Cot^{-1}y = x + \dfrac{x^3}{3} + c$
4) $Sin^{-1}y = x + \dfrac{x^3}{3} + c$

105. The solution of the differential equation y dx - x dy + log x dx is____

1) c x + y + (1-log x) = 0
2) c x-y - (1+log x) =0
3) c y + x+ log x-1 =0
4) c x-y + (1+log x) =0

106. The general solution of the equation $\left(D^2 - D - 2\right)y = \sin 2x$, $\left(D = \dfrac{d}{dx}\right)$ is____

1) $y = c_1e^{-x} + c_2e^{2x} + \dfrac{1}{20}(\cos 2x - 3\sin 2x)$
2) $y = c_1e^{-x} + c_2e^{-2x} + \dfrac{1}{20}(\cos 2x + 3\sin 2x)$
3) $y = c_1e^{-x} + c_2e^{2x} + \dfrac{1}{20}(\cos 2x - 3\sin 3x)$
4) $y = c_1e^{x} + c_2e^{-2x} + \dfrac{1}{20}(\cos 2x + 3\sin 2x)$

107. The particular integral of $\left(D^2 - 5D + 6\right)y = e^{4x}$ is____

1) e^{4x} 2) $-e^{4x}$ 3) $\dfrac{1}{2}e^{4x}$ 4) $\dfrac{1}{4}e^{4x}$

PUT YOUR FULL EFFORTS DON'T WORRY ABOUT RESULTS THEY ARE BOUND TO COME TO YOU

PREVIOUS ECET BITS KEY

01) 2	02) 4	03) 1	04) 2	05) 3
06) 1	07) 3	08) 2	09) 2	10) 2
11) 1	12) 2	13) 1	14) 4	15) 1
16) 1	17) 1	18) 1	19) 1	20) 1
21) 1	22) 1	23) 3	24) 4	25) 2
26) 1	27) 1	28) 3	29) 3	30) 1
31) 2	32) 4	33) 3	34) 4	35) 2
36) 2	37) 3	38) 1	39) 4	40) 3
41) 1	42) 2	43) 4	44) 4	45) 2
46) 1	47) 1	48) 4	49) 1	50) 3
51) 1	52) 2	53) 1	54) 1	55) 1
56) 2	57) 1	58) 1	59) 1	60) 4
61) 4	62) 2	63) 2	64) 4	65) 3
66) 1	67) 2	68) 2	69) 4	70) 2
71) 1	72) 2	73) 2	74) 2	75) 1
76) 1	77) 2	78) 3	79) 1	80) 2
81) 3	82) 4	83) 2	84) 1	85) 1
86) 3	87) 4	88) 1	89) 3	90) 1
91) 4	92) 4	93) 4	94) 2	95) 1
96) 1	97) 3	98) 1	99) 1	100) 2
101) 4	102) 3	103) 3	104) 1	105) 2
106) 1	107) 3			

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SPACE FOR IMPORTANT NOTES

LAPLACE TRANSFORMS

SYNOPSIS

1. **The Laplace transform** of a function f(t) denoted by $L\{f(t)\}$ is defined by
$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s)$$

2. If $L\{f(t)\} = \bar{f}(S)$ and $L\{g(t)\} = \bar{g}(S)$ then
$$L[c_1.f(t) + c_2.g(t)] = c_1.\bar{f}(s) + c_2\bar{g}(s)c_1.L[f(t)] + c_2.L[g(t)]$$
$$= c_1.\bar{f}(s) + c_2\bar{g}(s)$$
where c_1 and c_2 are constants.

3. **Laplace Transform of Elementary Functions:**

(i) $L(K) = \frac{K}{s}$, where K is a constant

(ii) $L(e^{at}) = \frac{1}{s-a}$

(iii) $L(e^{-at}) = \frac{1}{s+a}$

(iv) $L(\sin at) = \frac{a}{s^2+a^2}$

(v) $L(\cos at) = \frac{s}{s^2+a^2}$

(vi) $L(\sinh at) = \frac{a}{s^2-a^2}$

(vii) $L(\cosh at) = \frac{s}{s^2-a^2}$

(viii) $L(t^n) = \frac{n!}{s^{n+1}}$, if n is a positive integer.

(ix) $L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}$ where $n > -1$

(a) $L(\sqrt{t}) = \frac{\sqrt{\pi}}{2s^{3/2}}$

(b) $L\left(\frac{1}{\sqrt{t}}\right) = \sqrt{\frac{\pi}{s}}$

(x) $L(\sin^2 at) = \frac{2a^2}{s(s^2+4a^2)}$

(xi) $L(\cos^2 at) = \frac{1}{2s} + \frac{s}{2(s^2+4a^2)}$

4. **First Shifting Theorem :**

(i) If $L[f(t)] = \bar{f}(s)$ then $L\left[e^{at}f(t)\right] = \bar{f}(s-a)$

i.e., $L\left[e^{at}f(t)\right] = \left[\bar{f}(s)\right]_{s \to s-a}$

(ii) If $L[f(t)] = \bar{f}(s)$ then $L\left[e^{-at}f(t)\right] = \bar{f}(s+a)$

i.e., $L\left[e^{-at}f(t)\right] = \left[\bar{f}(s)\right]_{s \to s+a}$

5. **Second Shifting Theorem :**

If $L[f(t)] = \bar{f}(s)$ and $g(t) = \begin{cases} 0, & \text{if } t < a \\ f(t-a) & \text{if } t > a \end{cases}$

then $L[g(t)] = e^{-as}\bar{f}(s)$

6. **Change of Scale Property :**

If $L\{f(t)\} = \bar{f}(S)$ then

(i) $L[f(at)] = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$

(ii) $L\left[f\left(\frac{t}{a}\right)\right] = a\bar{f}(as)$

7. Laplace Transform of Derivatives :

If $L[f(t)]=\bar{f}(s)$, then

(i) $L[f'(t)]=s.\bar{f}(s)-f(0)$

(ii) $L[f''(t)]=s^2\bar{f}(s)-s.f(0)-f'(0)$

(iii) $L[f'''(t)]=s^3\bar{f}(s)-s^2f(0)-s.f'(0)-f''(0)$

8. Laplace Transform of the Integral of a function :

If $L\{f(t)\}=\bar{f}(s)$ then

(i) $L\left[\int_0^t f(u)du\right]=\frac{1}{s}\bar{f}(s)$

(ii) $L\left[\int_0^t\int_0^t f(u)du\,du\right]=\frac{1}{s^2}\bar{f}(s)$

9. Multiplicatin by powers of t:

(i) If $L[f(t)]=\bar{f}(s)$ then

(a) $L[t f(t)]=(-1)\frac{d}{ds}\left[\bar{f}(s)\right]$

(b) $L[t^n f(t)]=(-1)^n\frac{d^n}{ds^n}\left[\bar{f}(s)\right]$

(ii) $L[t\sin at]=\frac{2as}{(s^2+a^2)^2}$

(iii) $L[t\cos at]=\frac{s^2-a^2}{(s^2+a^2)^2}$

10. Division by t :

If $L[f(t)]=\bar{f}(s)$, then $L\left[\frac{f(t)}{t}\right]=\int_s^\infty \bar{f}(s)ds$, provide the integral exists.

(i) $L\left[\frac{e^{-at}-e^{-bt}}{t}\right]=\log\left(\frac{s+b}{s+a}\right)$

(ii) $L\left[\frac{\cos at-\cos bt}{t}\right]=\frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)$

11. Inverse Laplace Transform :

If $L[f(t)]=\bar{f}s$, then $f(t)$ is known as the inverse Laplace Transform of $\bar{f}(s)$ and is denoted by $L^{-1}\left[\bar{f}(s)\right]$

$\therefore f(t)=L^{-1}\left[\bar{f}(s)\right]$

L^{-1} is known as the Inverse Laplace Transform operator.

12. Inverse Laplace Transform of Some functions:

(i) $L^{-1}\left\{\frac{1}{s}\right\}=1$

(ii) (a) $L^{-1}\left\{\frac{1}{s-a}\right\}=e^{at}$ (b) $L^{-1}\left\{\frac{1}{s+a}\right\}=e^{-at}$

(iii) (a) $L^{-1}\left\{\frac{1}{s^n}\right\}=\frac{t^{n-1}}{(n-1)!}$

(b) $L^{-1}\left\{\frac{1}{s^n}\right\}=\frac{t^{n-1}}{\Gamma(n)}$

(iv) (a) $L^{-1}\left\{\frac{1}{s^2+a^2}\right\}=\frac{1}{a}\sin at$

(b) $L^{-1}\left\{\frac{1}{s^2-a^2}\right\}=\frac{1}{a^2}\sinh at$

(v) (a) $L^{-1}\left\{\frac{s}{s^2+a^2}\right\}=\cos at$

(b) $L^{-1}\left\{\frac{s}{s^2-a^2}\right\}=\cosh at$

13. If $L[f(t)]=\bar{f}(s)$ and $L[g(t)]=\bar{g}(s)$ then

$L^{-1}\left[c_1\bar{f}(s)+c_2\bar{g}(s)\right]=c_1L^{-1}\left[\bar{f}(s)\right]+c_2L^{-1}\left[\bar{g}(s)\right]=c_1f(t)+c_2g(t)$

where c_1 and c_2 are constants.

14. First Shifting Theorem of Inverse Laplace Transform (ILT) :

If $L^{-1}\left[\bar{f}(s)\right]=f(t)$ then

(a) $L^{-1}\left[\bar{f}(s-a)\right]=e^{at}f(t)=e^{at}.L^{-1}\left[\bar{f}(s)\right]$

(b) $L^{-1}\left[\bar{f}(s+a)\right]=e^{-at}f(t)=e^{-at}.L^{-1}\left[\bar{f}(s)\right]$

(i) $L^{-1}\left[\frac{1}{(s-a)^n}\right]=e^{at}.\frac{t^{n-1}}{(n-1)!}$

(ii) $L^{-1}\left[\frac{1}{(s+a)^n}\right]=e^{-at}.\frac{t^{n-1}}{(n-1)!}$

(iii) $L^{-1}\left[\frac{1}{(s-a)^2+b^2}\right]=\frac{1}{b}e^{at}\sin bt$

(iv) $L^{-1}\left[\frac{1}{(s+a)^2+b^2}\right]=\frac{1}{b}e^{-at}\sin bt$

(v) $L^{-1}\left[\frac{s-a}{(s-a)^2+b^2}\right]=e^{at}\cos bt$

(vi) $L^{-1}\left[\frac{s+a}{(s+a)^2+b^2}\right]=e^{-at}\cos bt$

(vii) $L^{-1}\left[\frac{b}{(s+a)^2+b^2}\right]=\frac{1}{b}e^{-at}\sin bt$

15. Second Shifting Theorem of I.L.T:

If $L^{-1}\left[\bar{f}(s)\right]=f(t)$, then

$L^{-1}\left[e^{-as}\bar{f}(s)\right]=f(t-a)u(t-a)$ where $u(t-a)$ is the unit step function.

16. Change of Scale Property of I.L.T :

If $L^{-1}\left[\bar{f}(s)\right]=f(t)$, then

$L^{-1}\left[\bar{f}(as)\right]=\frac{1}{a}f\left(\frac{t}{a}\right),(a>0)$

17. Inverse Laplace Transform of Derivatives :

If $L^{-1}\left[\bar{f}(s)\right]=f(t)$, then

$L^{-1}\left[\bar{f}^{(n)}(s)\right]=(-1)^n t^n f(t)$ where

$\bar{f}^{(n)}(s)=\frac{d^n}{ds^n}\left[\bar{f}(s)\right]$, n=1, 2, 3, ...

(i) $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]=\frac{e^{-bt}-e^{-at}}{t}$

(ii) $L^{-1}\left[\tan^{-1}\frac{1}{s}\right]=L^{-1}\left[\cot^{-1}s\right]\frac{\sin t}{t}$

18. Inverse Laplace Transform of Integrals :

If $L^{-1}\left[\bar{f}(s)\right]=f(t)$ then $L^{-1}\left[\int_s^\infty \bar{f}(s)ds\right]=\frac{f(t)}{t}$

19. Multiplication by powers of 's' :

(i) If $L^{-1}\left[\bar{f}(s)\right]=f(t)$ and $f(0)=0$ then $L^{-1}\left[s\bar{f}(s)\right]=f'(t)$

(ii) If $L^{-1}\left[\bar{f}(s)\right]=f(t)$ and $f(0)=f'(0)=0$ then $L^{-1}\left[s^2\bar{f}(s)\right]=f''(t)$

20. Division by s:

If $L^{-1}\left[\bar{f}(s)\right]=f(t)$ then $L^{-1}\left[\frac{\bar{f}(s)}{s}\right]=\int_0^t f(u)du$

21. (i) $L^{-1}\left[\frac{1}{\left(s^2+a^2\right)^2}\right]=\frac{1}{2a^3}(\sin at-at\cos at)$

(ii) $L^{-1}\left[\frac{s}{\left(s^2+a^2\right)^2}\right]=\frac{t}{2a}\sin at$

(iii) $L^{-1}\left[\frac{s}{(s^2-a^2)^2}\right]=\frac{t}{2a}\sinh at$

(iv) $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]=\frac{1}{2a}(\sin at+at\cos at)$

22. Convolution Theorem:

If $L[f(t)]=\bar{f}(s)$ and $L[g(t)]=\bar{g}(s)$ then $L[f(t)*g(t)]=\bar{f}(s).\bar{g}(s)$

(or) $L^{-1}\left[\bar{f}(s).\bar{g}(s)\right]=f(t)*g(t)$ where $f(t)*g(t)$ is called convolution of f(t) and g(t) and is defined as $f(t)*g(t)=\int_0^t f(u)g(t-u)\,du$.

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PRACTICE SET - I

01. $L(2e^{3t}-e^{-3t})=$

1) $\frac{s+9}{s-9}$ 2) $\frac{s-9}{s+9}$ 3) $\frac{s+9}{s^2-9}$ 4) $\frac{s-9}{s^2+9}$

02. $L(3\sin 4t-2\cos 5t)=$

1) $\frac{3}{s^2+4}-\frac{2s}{s^2+5}$ 2) $\frac{12}{s^2+16}-\frac{2s}{s^2+25}$ 3) $\frac{12}{s^2-16}-\frac{2s}{s^2-25}$ 4) $\frac{8}{s^2+16}+\frac{4s}{s^2+25}$

03. $L(3\cosh 5t-4\sinh 5t)=$

1) $\frac{3s-20}{s^2-25}$ 2) $\frac{3s-20}{s^2+25}$ 3) $\frac{3s-22}{s^2+25}$ 4) $\frac{3s+20}{s^2-25}$

04. $L\left[(t+1)^2\right]=$

1) $1+\frac{2}{s}+\frac{2}{s^2}$ 2) $\frac{1}{s}+\frac{2}{s^2}+\frac{2}{s^3}$ 3) $\frac{1}{s}+\frac{2}{s^2}+\frac{1}{s^3}$ 4) $\frac{1}{s}-\frac{2}{s^2}+\frac{2}{s^3}$

05. $L\left[\frac{e^{-at}-1}{a}\right]=$

1) $\frac{1}{s(s+a)}$ 2) $\frac{1}{s(s-a)}$ 3) $\frac{-1}{s(s-a)}$ 4) $\frac{-1}{s(s+a)}$

06. $L(\sin^2 3t)=$

1) $\frac{3}{s(s^2+4)}$ 2) $\frac{6}{s(s^2+9)}$

3) $\frac{9}{s(s^2+25)}$ 4) $\frac{19}{s(s^2+36)}$

07. $L(\cos^2 2t)=$

1) $\frac{s^2+8}{s^2+16}$ 2) $\frac{s^2-8}{s^2+16}$ 3) $\frac{s^2+8}{s(s^2+16)}$ 4) $\frac{s(s^2+8)}{(s^2+16}$

08. $L\left[(\sin 2t-\cos 2t)^2\right]=$

1) $\frac{1}{s}-\frac{4}{s^2+16}$ 2) $\frac{1}{s}+\frac{4}{s^2+16}$ 3) $\frac{1}{s}+\frac{4}{s^2-16}$ 4) $\frac{1}{s}+\frac{2}{s^2-16}$

09. $L(\sinh^2 2t)=$

1) $\frac{8}{s(s^2-16)}$ 2) $\frac{8}{s(s^2+16)}$ 3) $\frac{4}{s(s^2+16)}$ 4) none

10. $L(\cosh^2 3t)=$

1) $\frac{1}{s}+\frac{s}{s^2-36}$ 2) $\frac{1}{s}-\frac{s}{s^2+36}$ 3) $\frac{1}{2}\left[\frac{1}{s}+\frac{s}{s^2-36}\right]$ 4) $\frac{1}{2}\left[\frac{1}{s}-\frac{s}{s^2+36}\right]$

11. $L(\sin t\cos t)=$

1) $\frac{1}{s^2+4}$ 2) $\frac{2}{s^2+4}$ 3) $\frac{4}{s^2+4}$ 4) $\frac{2}{s^2-4}$

12. $L(\sin 2t\cos 3t)=$

1) $\frac{s^2-2}{(s^2+1)(s^2+25)}$

2) $\frac{2(s^2-5)}{(s^2+1)(s^2+25)}$

3) $\frac{2(s^2+5)}{(s^2+1)(s^2+25)}$

4) $\frac{2(s^2+3)}{(s^2+1)(s^2+25)}$

13. $L(\sin 2t\sin 3t)=$

1) $\frac{2s}{(s^2+1)(s^2+25)}$

2) $\frac{4s}{(s^2+1)(s^2+25)}$

3) $\frac{8s}{(s^2+1)(s^2+25)}$

4) $\frac{12s}{(s^2+1)(s^2+25)}$

14. $L(\sin^3 t)$

1) $\frac{1}{s^2+1}-\frac{1}{s^2+9}$

2) $\frac{1}{s^2+1}+\frac{1}{s^2+9}$

3) $\frac{2}{3}\left(\frac{1}{s^2+1}-\frac{1}{s^2+9}\right)$

4) $\frac{3}{4}\left(\frac{1}{s^2+1}-\frac{1}{s^2+9}\right)$

15. If $f(t)=\begin{cases} e^{t}, & 0<t<1 \\ 0, & t>1 \end{cases}$ then L[f(t)] =

1) $\frac{e^{1-s}-1}{1-s}$ 2) $\frac{1-e^{-1-s}}{1-s}$

3) $\frac{e^{1-s}-1}{s-1}$ 4) $\frac{1+e^{-1-s}}{1-s}$

16. $L(t^2 e^{2t})$ =

1) $\frac{1}{(s-2)^2}$ 2) $\frac{1}{(s-2)^3}$

3) $\frac{2}{(s-2)^3}$ 4) $\frac{3}{(s-2)^3}$

17. $L(t^3 e^{-3t})$ =

1) $\frac{1}{(s+3)^3}$ 2) $\frac{3}{(s+3)^3}$

3) $\frac{6}{(s+3)^3}$ 4) $\frac{6}{(s+3)^4}$

18. $L[t^7 e^{15t}]$ =

1) $\frac{6!}{(s-15)^7}$ 2) $\frac{7!}{(s-15)^8}$

3) $\frac{7!}{(s+15)^7}$ 4) $\frac{7!}{(s+15)^8}$

19. $L[e^{-at}(1-at)]$ =

1) $\frac{1}{s+a}$ 2) $\frac{s}{(s+a)^2}$

3) $\frac{a}{(s+a)^2}$ 4) $\frac{1}{a(s+a)^2}$

20. $L(e^{-t}\sin 2t)$ =

1) $\frac{2}{s^2+2s+5}$ 2) $\frac{1}{s^2+2s+5}$

3) $\frac{1}{s^2-2s+5}$ 4) $\frac{2}{s^2+4s+5}$

21. $L(e^{-2t}\cos t)$ =

1) $\frac{s-2}{s^2+4s+5}$ 2) $\frac{s-1}{s^2+4s+5}$

3) $\frac{s+2}{s^2+4s+5}$ 4) $\frac{s+2}{s^2-4s+5}$

22. $L(e^{2t}\cos 3t)$ =

1) $\frac{s-2}{s^2-4s+13}$ 2) $\frac{s+2}{s^2+4s+13}$

3) $\frac{s+2}{s^2+4s-13}$ 4) $\frac{s-2}{s^2+4s-13}$

23. $L(e^{-t}\cosh t)$ =

1) $\frac{s-1}{s(s+2)}$ 2) $\frac{s+1}{s(s+2)}$

3) $\frac{s+1}{s(s-2)}$ 4) $\frac{s-1}{s(s-2)}$

24. $L[(e^{2t}(\cos 4t+3\sin 4t)]$ =

1) $\frac{1}{s^2-4s+20}$ 2) $\frac{1}{s^2+4s+20}$

3) $\frac{s}{s^2+4s+20}$ 4) $\frac{s}{s^2-4s+20}$

25. $L[e^{-t}\sin t\cos t]$=

1) $\frac{1}{s^2-2s+5}$ 2) $\frac{1}{s^2+2s+5}$

3) $\frac{1}{s^2+2s-5}$ 4) $\frac{1}{s^2-2s+5}$

26. $L[e^{-t}\sin^2 t]$ =

1) $\frac{1}{s+1}+\frac{s+1}{s^2+2s+5}$

2) $\frac{1}{s-1}+\frac{s-1}{s^2-2s+5}$

3) $\frac{1}{2}\left[\frac{1}{s+1}-\frac{s+1}{s^2+2s+5}\right]$

4) $\frac{1}{2}\left[\frac{1}{s+1}+\frac{s+1}{s^2+2s+5}\right]$

27. $L[e^{t}\cos^2 t]$ =

1) $\frac{1}{s+1}+\frac{s+1}{s^2+2s+5}$

2) $\frac{1}{s-1}+\frac{s-1}{s^2-2s+5}$

3) $\frac{1}{2}\left[\frac{1}{s+1}+\frac{s+1}{s^2+2s+5}\right]$

4) $\frac{1}{2}\left[\frac{1}{s-1}+\frac{s-1}{s^2-2s+5}\right]$

28. If $f(t)=\begin{cases} 0, & \text{if } t<a \\ e^{t-a}, & \text{if } t>a \end{cases}$ then L[f(t)] =

1) $\frac{e^{as}}{s-1}$ 2) $\frac{e^{-as}}{s-1}$

3) $\frac{e^{as}}{s+1}$ 4) $\frac{se^{-as}}{s-1}$

29. $f(t)=\begin{cases} 0, & \text{if } t<\frac{\pi}{3} \\ \cos\left(t-\frac{\pi}{3}\right), & \text{if } t>\frac{\pi}{3} \end{cases}$ then L[f(t)]

1) $\frac{1}{s^2+1}e^{-\pi s/3}$ 2) $\frac{1}{s-1}e^{-\pi s/3}$

3) $\frac{s}{s^2+1}e^{-\pi s/3}$ 4) $\frac{s}{s^2-1}e^{\pi s/3}$

30. If $f(t)=\begin{cases} 0, & \text{if } t<\frac{2\pi}{3} \\ \sin\left(t-\frac{2\pi}{3}\right), & \text{if } t>\frac{2\pi}{3} \end{cases}$ then L[f(t)]=

1) $\frac{1}{s^2+1}e^{-2\pi s/3}$ 2) $\frac{1}{s^2+1}e^{2\pi s/3}$

3) $\frac{s}{s^2+1}e^{2\pi s/3}$ 4) $\frac{s}{s^2-1}e^{-2\pi s/3}$

31. If $L[f(t)]=\frac{1}{s}e^{-1/s}$ then L[f(5t)] =

1) $\frac{5}{s}e^{-1/s}$ 2) $\frac{1}{5s}e^{-1/5s}$

3) $\frac{1}{s}e^{-5/s}$ 4) $\frac{1}{s}e^{-1/s}$

32. If $L[f(t)]=\frac{20-4S}{S^2-4S+20}$ then L[f(3t)] =

1) $\frac{20-4S}{S^2-4S+20}$ 2) $\frac{40-4S}{S^2-12S+80}$

3) $\frac{40-6S}{S^2+12S-80}$ 4) $\frac{60-4S}{S^2-12S+80}$

33. If $L\left[\frac{\sin t}{t}\right]=\tan^{-1}\left(\frac{1}{s}\right)$ then $L\left[e^{-t}\frac{\sin 3t}{t}\right]$ =

1) $\tan^{-1}\left(\frac{3}{s}\right)$ 2) $\tan^{-1}\left(\frac{3}{s-1}\right)$

3) $\tan^{-1}\left(\frac{3}{s+1}\right)$ 4) $\tan^{-1}\left(\frac{s}{3}\right)$

34. L[t sint] =

1) $\frac{1}{s^2+1}$ 2) $\frac{s}{s^2+1}$

3) $\frac{2s}{s^2+1}$ 4) $\frac{2s}{(s^2+1)^2}$

35. L [t cos 2 t]=

1) $\frac{s}{s^2+4}$ 2) $\frac{s}{(s^2+4)^2}$

3) $\frac{s^2-4}{(s^2+4)^2}$ 4) $\frac{s^2+4}{(s^2+4)^2}$

36. L t cosh t] =

1) $\frac{s^2+1}{(s^2-1)^2}$ 2) $\frac{s^2-1}{(s^2+1)^2}$

3) $\frac{s-1}{(s^2+1)^2}$ 4) none

37. $L[t^2 e^{-2t}] =$

1) $\frac{1}{(s+2)^2}$ 2) $\frac{2}{(s+2)^2}$

3) $\frac{2}{(s+2)^3}$ 4) $\frac{2}{(s-2)^3}$

38. $L[t^2 \cos t] =$

1) $\frac{2(s^2-3)}{(s^2+1)^2}$ 2) $\frac{2(s^2-3)}{(s^2+1)^3}$

3) $\frac{2s(s^2+3)}{(s^2+1)^3}$ 4) $\frac{2s(s^2-3)}{(s^2+1)^3}$

39. $L[t^3 e^{3t}] =$

1) $\frac{2}{(s-3)^3}$ 2) $\frac{2}{(s+3)^3}$

3) $\frac{6}{(s-3)^4}$ 4) $\frac{6}{(s+3)^5}$

40. $L\left(\frac{\sin t}{t}\right) =$

1) $\tan^{-1} s$ 2) $\cot^{-1} s$

3) $-\cot^{-1} s$ 4) $-\tan^{-1} s$

41. $L\left[\frac{\sin 4t}{t}\right] =$

1) $\tan^{-1}\left(\frac{s}{4}\right)$ 2) $\cot^{-1}\left(\frac{s}{4}\right)$

3) $-\cot^{-1}\left(\frac{s}{4}\right)$ 4) $\cot^{-1}(4s)$

42. $L\left[\frac{\sinh t}{t}\right] =$

1) $\log\left(\frac{s-1}{s+1}\right)$ 2) $\log\left(\frac{s+1}{s-1}\right)$

3) $\frac{1}{2}\log\left(\frac{s+1}{s-1}\right)$ 4) $\frac{1}{2}\log\left(\frac{s-1}{s+1}\right)$

43. $L\left[\frac{1-e^{-t}}{t}\right] =$

1) $\log\left(\frac{s+1}{s}\right)$ 2) $\log\left(\frac{s-1}{s}\right)$

3) $\log\left(\frac{s}{s+1}\right)$ 4) $\log\left(\frac{s}{s-1}\right)$

44. $L\left[\frac{1-e^{2t}}{t}\right] =$

1) $\log\left(\frac{s-2}{s}\right)$ 2) $\log\left(\frac{s+2}{s}\right)$

3) $2\log\left(\frac{s-2}{s}\right)$ 4) $\frac{1}{2}\log\left(\frac{s}{s-2}\right)$

45. $L\left[\frac{e^{-t}-e^{-2t}}{t}\right] =$

1) $\log\left(\frac{s-2}{s+1}\right)$ 2) $\log\left(\frac{s+2}{s+1}\right)$

3) $\log\left(\frac{s+2}{s-1}\right)$ 4) $\log\left(\frac{s-1}{s-2}\right)$

46. $L\left[\frac{1-\cos t}{t}\right] =$

1) $\log\left(\frac{s^2+1}{s}\right)$ 2) $\log\left(\frac{s^2+1}{s^2}\right)$

3) $\frac{1}{2}\log\left(\frac{s^2+1}{s^2}\right)$ 4) $\frac{1}{2}\log\left(\frac{s^2-1}{s^2}\right)$

47. $L\left[\int_0^t \sin 2t\,dt\right] =$

1) $\frac{2}{s^2+4}$ 2) $\frac{2s}{s^2+4}$

3) $\frac{2s}{s^2-4}$ 4) $\frac{2}{s(s^2+4)}$

48. $L\left[\int_0^t \cos t\,dt\right] =$

1) $\frac{s}{s^2+1}$ 2) $\frac{1}{s^2+1}$

3) $\frac{1}{s(s^2+1)}$ 4) none

49. $L\left[\int_0^t e^{-t}\cos t\,dt\right] =$

1) $\frac{s}{s^2+2s+2}$ 2) $\frac{s+1}{s(s^2+2s+2)}$

3) $\frac{s-1}{s(s^2-2s+2)}$ 4) none

50. $L\left[\int_0^t \frac{\sin t}{t}\,dt\right] =$

1) $\tan^{-1} s$ 2) $s\tan^{-1} s$

3) $\frac{1}{s}\tan^{-1} s$ 4) $\frac{1}{s}\cot^{-1} s$

PRACTICE SET - I KEY

01) 3	02) 2	03) 1	04) 2	05) 4
06) 4	07) 3	08) 1	09) 1	10) 3
11) 1	12) 2	13) 4	14) 4	15) 1
16) 3	17) 4	18) 2	19) 2	20) 1
21) 3	22) 1	23) 2	24) 4	25) 2
26) 3	27) 4	28) 2	29) 3	30) 1
31) 3	32) 4	33) 2	34) 4	35) 3
36) 1	37) 3	38) 4	39) 3	40) 2
41) 2	42) 3	43) 1	44) 1	45) 2
46) 3	47) 4	48) 2	49) 2	50) 4

PRACTICE SET - II

01. The value of $\int_0^\infty t e^{-3t}\,dt =$

1) $\frac{1}{3}$ 2) $\frac{1}{9}$ 3) $\frac{2}{9}$ 4) $\frac{2}{3}$

02. The value of $\int_0^\infty e^{-4t}\sin 3t\,dt =$

1) $\frac{1}{10}$ 2) $\frac{2}{15}$ 3) $\frac{3}{25}$ 4) $\frac{4}{25}$

03. The value of $\int_0^\infty t^3 e^{-2t}\,dt =$

1) $\frac{1}{8}$ 2) $\frac{2}{8}$ 3) $\frac{3}{8}$ 4) $\frac{5}{8}$

04. The value of $\int_0^\infty e^{-t}\sin 2t\,dt =$

1) $\frac{1}{5}$ 2) $\frac{2}{5}$ 3) $\frac{3}{5}$ 4) $\frac{4}{5}$

05. $L^{-1}\left[\frac{1}{3s-5}\right] =$

1) e^{5t} 2) $\frac{1}{3}e^{-5t}$ 3) $\frac{1}{3}e^{-\frac{5}{3}t}$ 4) $\frac{1}{3}e^{\frac{5}{3}t}$

06. $L^{-1}\left[\frac{2s+1}{s^2-9}\right] =$

1) 2 cos 3t + sin 3t

2) 2 cos 3t - sin 3t

3) 2 cosh 3t + $\frac{1}{3}$ sinh 3t

4) 2 sinh 3t - $\frac{1}{3}$ cosh 3t

07. $L^{-1}\left[\frac{3s-12}{s^2+8}\right] =$

1) $3[\cos 2\sqrt{2}t - \sqrt{2}\sin 2\sqrt{2}t]$

2) $3[\cos\sqrt{2}t - 2\sin\sqrt{2}t]$

3) $3[\sin 2\sqrt{2}t - \sqrt{3}\cos 2\sqrt{2}t]$

4) none

08. $L^{-1}\left[\frac{2s-5}{s^2-4}\right] =$

1) 2 sinh 2t - 5 cosh 2t

2) 2 cosh 2t -5 sinh 2t

3) 2 cosh 2t - $\frac{5}{2}$ sinh 2t

4) 2 cosh 2t + $\frac{5}{2}$ sinh 2t

09. $L^{-1}\left[\frac{s^2-3s+8}{s^3}\right] =$

1) $1-3t+2t^2$ 2) $1-3t+4t^2$

3) $t-4t^2$ 4) $1-3t+2t^3$

10. $L^{-1}\left[\frac{s^2+9s-9}{s^3-9s}\right] =$

1) 1+3sinh 3t 2) 1+9sinh 9t

3) 1-3sinh 2t 4) 1-3sin 3t

11. $L^{-1}\left[\frac{3s-8}{4s^2+25}\right] =$

1) $\frac{3}{5}\cos\frac{5t}{2}+\frac{4}{5}\sin\frac{5t}{2}$

2) $\frac{3}{4}\cos\frac{5t}{2}-\frac{4}{5}\sin\frac{5t}{2}$

3) $\frac{3}{4}\cosh\frac{5t}{2}-\frac{4}{5}\sinh\frac{5t}{2}$

4) $\frac{3}{4}\sin\frac{5t}{2}-\frac{4}{5}\cos\frac{5t}{2}$

12. $L^{-1}\left[\left(\frac{\sqrt{s}-1}{s}\right)^2\right] =$

1) $1+t-4\sqrt{t}$ 2) $1-t+4\sqrt{t}$

3) $1-t+4\sqrt{\frac{t}{\pi}}$ 4) $1+t-4\sqrt{\frac{t}{\pi}}$

13. $L^{-1}\left[\frac{1}{(s-2)^3}\right] =$

1) $e^{2t}\frac{t^3}{3!}$ 2) $e^{2t}\frac{t^2}{2!}$

3) $e^{-2t}\frac{t^3}{3!}$ 4) $e^{-2t}\frac{t^4}{4!}$

14. $L^{-1}\left[\frac{s}{(s+1)^5}\right] =$

1) $\frac{e^{-t}}{24}(4t^3-t^4)$ 2) $\frac{e^{-t}}{24}(t^4-4t^3)$

3) $\frac{e^{-t}}{12}(t^4-4t^3)$ 4) $\frac{e^{-t}}{24}(t^4+4t^3)$

15. $L^{-1}\left[\frac{s}{(s-3)^5}\right] =$

1) $\frac{e^{3t}}{24}(4t^3+3t^4)$ 2) $\frac{e^{3t}}{24}(4t^3-3t^4)$

3) $\frac{e^{-3t}}{24}(4t^3-3t^4)$ 4) $\frac{e^{-3t}}{24}(4t^3+3t^4)$

16. $L^{-1}\left[\frac{1}{s^2+4s+20}\right] =$

1) $e^{t}\sin 2t$ 2) $\frac{1}{2}e^{-2t}\sin 2t$

3) $\frac{1}{2}e^{-2t}\sin 4t$ 4) $\frac{1}{4}e^{-2t}\sin 4t$

17. $L^{-1}\left[\frac{2s+3}{s^2+2s+2}\right] =$

1) $e^{-t}(2\cos t-\sin t)$ 2) $e^{-t}(2\cos t+\sin t)$

3) $e^{t}(2\cos t-\sin t)$ 4) $e^{-t}(\cos t+2\sin t)$

18. $L^{-1}\left[\frac{3s-2}{s^2-4s+20}\right] =$

1) $e^{2t}(3\cos 4t+\sin 4t)$ 2) $e^{2t}(3\cos 4t-\sin 4t)$

3) $e^{-2t}(3\cos 4t-\sin 4t)$ 4) $e^{2t}(3\cos 2t+\sin 2t)$

19. $L^{-1}\left[\frac{7}{(2s+1)^3}\right] =$

1) $\frac{7}{16}t^2e^{-t/2}$ 2) $\frac{7}{8}te^{-t/2}$

3) $\frac{7}{16}te^{t/2}$ 4) $\frac{7}{16}t^2e^{t/2}$

20. $L^{-1}\left[\frac{1}{9s^2+6s+1}\right] =$

1) $\frac{1}{3}e^{-t/3}$ 2) $\frac{1}{9}e^{-t/3}$ 3) $\frac{t}{9}e^{-t/3}$ 4) $\frac{t}{12}e^{t/3}$

21. $L^{-1}\left[\frac{s^2}{(s-2)^3}\right] =$

1) $e^{2t}(1+4t+2t^2)$ 2) $e^{2t}(1-4t+2t^2)$

3) $e^{2t}(1-2t+4t^2)$ 4) $e^{-2t}(1-2t+4t^2)$

22. $L^{-1}\left[\frac{3s+7}{s^2-2s-3}\right] =$

1) $e^{-t}-e^{3t}$ 2) $4e^{-3t}-e^{t}$

3) $4e^{3t}-e^{-t}$ 4) $4e^{-3t}-e^{t}$

23. $L^{-1}\left[\frac{s}{(s-3)^2+1}\right] =$

1) $e^{3t}(\cos t+3\sin t)$ 2) $e^{3t}(\sin t+3\cos t)$

3) $e^{-3t}(\cos t-3\sin t)$ 4) $e^{-3t}(\sin t-3\cos t)$

24. $L^{-1}\left[\frac{1}{(s+1)^2}\right] =$

1) $\frac{e^t}{t}$ 2) te^t 3) te^{-t} 4) $\frac{t}{2}e^{-t}$

25. $L^{-1}\left[\frac{2}{(s-a)^3}\right] =$

1) te^{at} 2) t^2e^{at}

3) $\frac{t^2}{2}e^{at}$ 4) $\frac{t^2}{2}e^{-at}$

26. $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right] =$

1) $t\sinh t$ 2) $2t\sinh t$

3) $\frac{2}{t}\sinh t$ 4) $\frac{2}{t}\cosh t$

27. $L^{-1}\left[\frac{s}{(s+2)^2}\right] =$

1) $e^{2t}(1-t)$ 2) $e^{-2t}(1-2t)$

3) $e^{-2t}(2t-1)$ 4) $e^{2t}(2t-1)$

28. $L^{-1}\left[\frac{s}{(s+2)^2+4}\right] =$

1) $e^{-2t}(\cos 2t-\sin 2t)$

2) $e^{-2t}(\cos 2t+\sin 2t)$

3) $e^{-2t}(\sin 2t-\cos 2t)$

4) $e^{2t}(\sin 2t-\cos 2t)$

29. $L^{-1}\left[\frac{1}{s(s+1)}\right] =$

1) $1-e^{-t}$ 2) $1-e^{t}$

3) $1+e^{-t}$ 4) $1-\frac{1}{2}e^{t}$

30. $L^{-1}\left[\frac{1}{s(s-3)}\right] =$

1) $3(e^{3t}-1)$ 2) $3(1-e^{3t})$

3) $\frac{1}{3}(1-e^{-3t})$ 4) $\frac{1}{3}(e^{3t}-1)$

31. $L^{-1}\left[\frac{1}{s(s^2+a^2)}\right] =$

1) 1-cosat 2) $\frac{1}{a^2}$(1-cosat)

3) a^2(1+cosat) 4) a^2(1-sinat)

32. $L^{-1}\left[\frac{1}{s(s^2-a^2)}\right] =$

1) $1-\cosh at$ 2) $\frac{1}{a^2}(1-\cosh at)$

3) $\frac{1}{a^2}(\cosh at-1)$

4) $\frac{1}{a^2}(1+\cosh at)$

33. $L^{-1}\left[\frac{1}{s^2(s+1)}\right] =$

1) $1-t+e^{-t}$ 2) $1-t-e^{-t}$

3) $t+1-e^{-t}$ 4) $t-1+e^{-t}$

34. $L^{-1}\left[\frac{1}{s^2(s+1)^2}\right] =$

1) $t(1+e^{-t})$ 2) $\frac{1}{t}(1+e^{-t})$

3) $t(1-e^{-t})$ 4) $\frac{1}{t}(1-e^{t})$

35. $L^{-1}\left[\frac{1}{(s-1)(s+2)}\right] =$

1) $\frac{1}{3}(e^{t}-e^{-2t})$ 2) $\frac{1}{2}(e^{t}+e^{-2t})$

3) $\frac{1}{2}(e^{-2t}-e^{t})$ 4) $\frac{1}{3}(e^{-t}+e^{2t})$

36. $L^{-1}\left[\frac{1}{(s+1)(s+3)}\right] =$

1) $\frac{1}{2}(e^{t}-e^{3t})$ 2) $\frac{1}{2}(e^{-t}-e^{-3t})$

3) $\frac{1}{3}(e^{t}-e^{3t})$ 4) $\frac{1}{3}(e^{-t}-e^{-3t})$

37. $L^{-1}\left[\frac{1}{s^2-5s+6}\right] =$

1) $e^{2t}-e^{3t}$ 2) $e^{2t}+e^{3t}$

3) $e^{-2t}+e^{-3t}$ 4) $e^{3t}-e^{2t}$

38. $L^{-1}\left[\frac{1}{s^2-3s+2}\right] =$

1) $e^{2t}-e^{t}$ 2) $e^{-2t}+e^{-t}$

3) $e^{t}-e^{2t}$ 4) $\frac{1}{2}(e^{t}-e^{2t})$

39. $L^{-1}\left[\frac{1}{s^2+6s+5}\right] =$

1) $\frac{1}{4}(e^{-t}-e^{-5t})$ 2) $\frac{1}{2}(e^{-t}-e^{-5t})$

3) $\frac{1}{2}(e^{-t}+e^{-5t})$ 4) $\frac{1}{3}(e^{-t}+e^{-5t})$

40. $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right] =$

1) $\frac{1}{b-a}(e^{-at}-e^{-bt})$ 2) $\frac{1}{a-b}(e^{-at}-e^{-bt})$

3) $\frac{1}{a-b}(e^{at}-e^{bt})$ 4) $\frac{1}{b-a}(e^{at}-e^{bt})$

41. $L^{-1}\left[\frac{1}{(s-a)(s-b)}\right] =$

1) $\frac{1}{b-a}(e^{-at}-e^{-bt})$ 2) $\frac{1}{a-b}(e^{-at}-e^{-bt})$

3) $\frac{1}{a-b}(e^{at}-e^{bt})$ 4) $\frac{1}{b-a}(e^{at}-e^{bt})$

42. For the differential equation y''' +2y''-y'-2y = 0 with y(0) =1, y'(0) = y''(0) = 2, the Laplace transform of y is

1) $\frac{s^2+4s+5}{s^3+2s^2-s-2}$ 2) $\frac{s^2-4s+5}{s^3+2s^2-s-2}$

3) $\frac{s^2+4s+5}{s^3-2s^2-s+2}$ 4) $\frac{s^2+4s-5}{s^3+2s^2+s-2}$

43. For the differential equation y''+2y'-3y = sin t with y(0) = y'(0) = 0, the Laplace transform of y is

1) $\frac{1}{(s^2-1)(s^2+2s-3)}$ 2) $\frac{1}{(s^2+1)(s^2+2s-3)}$

3) $\frac{1}{(s^2+1)(s^2-2s-3)}$ 4) $\frac{s}{(s^2+1)(s^2-2s+3)}$

44. The solution of the differential equation y'' + y = 0 with y(0) = 0 and y'(0) = 2 is

1) y = sin t 2) y = 2 sin t

3) y = $\frac{1}{2}$ sin t 4) y = -$\frac{1}{2}$ sin t

45. The solution of the differential equation y'' - 4y' + 5y = 0 with y(0) = 1, y'(0) = 2 is

1) y = e^{2t} cost 2) y = e^{t} cost

3) y = e^{-2t} cost 4) y = e^{-t} cost

46. For the differential equation y''+3y'+2y = e^{-t} with y(0) = y'(0) = 0, the Laplace transform of y is

1) $\frac{1}{(s^2-1)(s^2+2)}$ 2) $\frac{1}{(s^2+1)(s+2)}$

3) $\frac{1}{(s+1)^2(s+2)}$ 4) $\frac{1}{(s-1)^2(s+2)}$

47. For the differential equation y''-y=a cosh t with y(0) = y'(0) = 0, with Laplace transform of y is

1) $\frac{a}{s^2-1}$ 2) $\frac{a}{(s^2-1)^2}$

3) $\frac{a}{(s^2+1)^2}$ 4) $\frac{as}{(s^2+1)^2}$

PRACTICE SET - II KEY

01) 2	02) 3	03) 3	04) 2	05) 3
06) 3	07) 1	08) 3	09) 1	10) 1
11) 2	12) 4	13) 2	14) 1	15) 1
16) 3	17) 2	18) 1	19) 1	20) 3
21) 1	22) 3	23) 1	24) 3	25) 2
26) 3	27) 2	28) 1	29) 1	30) 4
31) 2	32) 3	33) 4	34) 1	35) 1
36) 2	37) 4	38) 1	39) 1	40) 1
41) 3	42) 1	43) 2	44) 2	45) 1
46) 3	47) 2			

PUT YOUR FULL
EFFORTS
DON'T WORRY ABOUT
RESULTS
THEY ARE BOUND TO
COME TO YOU

PREVIOUS ECET BITS

01. L [sin at - at cos at] =
where (L : Lapalce transform]

1) $\frac{2a^3}{(S^2+a^2)^2}$ 2) $\frac{a}{(S^2+a^2)^2}$

3) $\frac{a^3}{(S^2+a^2)^2}$ 4) $\frac{2a^3}{(a^2+S^2)^2}$

02. $L^{-1}\left[\frac{S-1}{S^2+2S}\right]=$

1) $\frac{-5}{2}+\frac{3}{2}e^{-2t}$ 2) $1+\frac{3}{2}e^{-2t}$

3) $\frac{1}{2}(3e^{-2t}-1)$ 4) $\frac{e^{-2t}-3}{2}$

03. $L(e^{2t}\sin at)=$

1) $\frac{S}{S^2+a^2}$ 2) $\frac{a}{S^2+a^2}$

3) $\frac{a}{(S-2)^2+a^2}$ 4) does not exist

T.S ECET- 2017

04. The Laplace transform of $\left\{\frac{e^{-at}t^{n-1}}{(n-1)!}\right\}=$

1) $\frac{e^{-at}}{(s+a)^n}$ 2) $\frac{1}{(s+a)^n}$

3) $\frac{1}{(s-a)^n}$ 4) $\frac{e^{at}}{(s-a)^n}$

05. The inverse Laplace transforms of $\left\{\frac{1}{(8s-27)^{1/3}}\right\}=$

1) $\frac{e^{(1/2)t}t^{-2/3}}{2\Gamma\left(\frac{1}{3}\right)}$ 2) $\frac{e^{(8/27)t}t^{-3/2}}{2\Gamma\left(\frac{1}{3}\right)}$

3) $\frac{e^{(2/3)t}t^{-3/2}}{2\Gamma\left(\frac{1}{3}\right)}$ 4) $\frac{e^{(27/8)t}t^{-2/3}}{2\Gamma\left(\frac{1}{3}\right)}$

06. The inverse Lapalce tranform of $\left\{\frac{s+3}{s^2+6s+25}\right\}=$

1) $e^{-3t}\cos 4t$ 2) $e^{3t}\sin 4t$ t

3) $e^{3t}\cos 4t$ 4) $e^{-3t}\cos 3t$

T.S ECET- 2018

07. If L[f(t)] denotes the Laplace Transform of f(t), then $L\left[t^2e^{-2t}\right]=$

1) $\frac{1}{(s+2)^3}$ 2) $\frac{2}{(s+2)^3}$

3) $\frac{1}{(s+2)^4}$ 4) $\frac{2}{(s+2)^2}$

08. If the Laplace tranform of a function $f(t)$ is F(S), then $\int_0^\infty f(t)\,dt=$

1) F(1) 2) $F(\infty)$ 3) F(0) 4) F(S-1)

PREVIOUS ECET BITS - KEY

01) 4	02) 3	03) 3
04) 2	05) 4	06) 1
07) 2	08) 3	

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FOURIER SERIES

SYNOPSIS

1. Suppose a given function f(x) defined in an interval of length 2π , say $[-\pi,\pi]$, or $[0,2\pi]$, or $[c,c+2\pi]$ and which satisfies certain conditions can be expressed in the trigonometric series as

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

where a's and b's are constants. Such a series is known as Fourier series for f(x) and the constants a_0, a_n, b_n (n = 1,2,3, ...) are called Fourier coefficients of f(x).

2. **Euler's Formulae:**
The Fourier series for the function f(x) in the interval $(c, c+2\pi)$ is given by

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

where $a_0=\frac{1}{\pi}\int_c^{c+2\pi}f(x)dx$

$$a_n=\frac{1}{\pi}\int_c^{c+2\pi}f(x)\cos nx\,dx$$

$$b_n=\frac{1}{\pi}\int_c^{c+2\pi}f(x)\sin nx\,dx$$

These values of a_0, a_n and b_n are known as Euler's formulae.

3. The Fourier series for f(x) in $(0,2\pi)$ is given by

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

where $a_0=\frac{1}{\pi}\int_c^{2\pi}f(x)dx$

$$a_n=\frac{1}{\pi}\int_0^{2\pi}f(x)\cos nx\,dx$$

$$b_n=\frac{1}{\pi}\int_0^{2\pi}f(x)\sin nx\,dx$$

4. The Fourier series for f(x) in the interval $(-\pi,\pi)$ is given by

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

where $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx$$

5. **Fourier Series of Even and Odd Functions:**
i) A function f(x) is said be even if f(-x) = f(x) and odd if f(-x) = -f(x).
ii) The Fourier series of an even function contains "only cosine terms".
iii) The Fourier series of an odd function contains "only sine terms".
iv) **Fourier Cosine series:**
If f(x) is an even function, then Fourier series of f(x) is given by

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx$$

where $a_0=\frac{2}{\pi}\int_0^{\pi}f(x)dx$

and $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$

v) Fourier sine series:

If f(x) is an odd function, then Fourier series of f(x) is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

where $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

6. Change of Interval

i) Fourier series of f(x) in the interval $(0,2l)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where $a_0 = \frac{1}{l}\int_0^{2l} f(x)dx$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

ii) Fourier series of f(x) in the interval $(-l,l)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where $a_0 = \frac{1}{l}\int_{-l}^{l} f(x)dx$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

iii) Fourier series of Even and Odd Functions in $(-l,l)$:

a) If f(x) is even in $(-l,l)$ then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

where $a_0 = \frac{2}{l}\int_0^{l} f(x)dx$

$$a_n = \frac{2}{l}\int_0^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

b) If f(x) is odd in $(-l,l)$ then

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where $b_n = \frac{2}{l}\int_0^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$

7. Half-Range Expansions:

(i) The half-range fourier sine series expansion of f(x) in $(0,l)$ is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)dx$$

where $b_n = \frac{2}{l}\int_0^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$

(ii) The half-range Fourier Cosine series expansion of f(x) in $(0,l)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

where $a_0 = \frac{2}{l}\int_0^{l} f(x)dx$

and $a_n = \frac{2}{l}\int_0^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

(iii) The half-range Fourier Sine series in $(0,\pi)$ is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

where $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

(iv) The half-range Fourier Cosine series in $(0,\pi)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

where $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx$

and $a_n = \frac{2}{\pi}\int_0^{l} f(x)\cos nx\,dx$

8. Some Important Results:

(i) $\sin n\pi = \sin 2n\pi = 0$

(ii) $\cos n\pi = (-1)^n$, $\cos 2n\pi = 1$

(iii) $\sin(2n+1)\frac{\pi}{2} = (-1)^n$

(iv) $\cos(2n+1)\frac{\pi}{2} = 0$

(v) If $f(x) = e^{ax}$ is expanded as a Fourier series in $(-\pi,\pi)$ then $a_0 = \frac{2\sinh a\pi}{a\pi}$

(vi) If $f(x) = e^{ax}$ is expanded as Fourier series in $(0,2\pi)$, then $a_0 = \frac{e^{2a\pi}-1}{a\pi}$

PUT YOUR FULL EFFORTS
DON'T WORRY ABOUT RESULTS
THEY ARE BOUND TO COME
TO YOU

PRACTICE SET - I

01. If $f(x) = x^2$ is expanded as a Fourier series in $(0,2\pi)$, the value of $a_0 =$

1) $\frac{2\pi^2}{3}$ 2) $2\pi^2$ 3) $\frac{4\pi^2}{3}$ 4) $\frac{8\pi^2}{3}$

02. If $f(x) = x-\pi$ is expanded as a Fourier series in $(-\pi,\pi)$, the value of $a_0 =$

1) π 2) $-\pi$ 3) -2π 4) 2π

03. If $f(x) = \pi^2 - x^2$ is expanded as a Fourier series in $(-\pi,\pi)$, the value of $\frac{a_0}{2} =$

1) $\frac{\pi^2}{3}$ 2) $\frac{2\pi^2}{3}$ 3) $\frac{4\pi^2}{3}$ 4) $\frac{8\pi^2}{3}$

04. If $f(x) = \frac{\pi - x}{2}$ is expanded as a Fourier series in $(0,2\pi)$, then $\frac{a_0}{2} =$

1) 0 2) π 3) $\frac{\pi}{2}$ 4) 2π

05. If $f(x) = x + x^2$ is expanded as a Fourier series in $(-\pi,\pi)$, then $a_0 =$

1) $\frac{\pi^2}{3}$ 2) $\frac{2\pi^2}{3}$ 3) $\frac{-\pi^2}{3}$ 4) $\frac{-4\pi^2}{3}$

06. If $f(x) = \frac{(\pi - x)^2}{4}$ is expanded as a Fourier series in $(0,2\pi)$, then value of $a_0 =$

1) $\frac{\pi^2}{2}$ 2) $\frac{\pi^2}{3}$ 3) $\frac{\pi^2}{4}$ 4) $\frac{\pi^2}{6}$

07. If $f(x) = x\sin x$ is expanded as a Fourier series in $(0,2\pi)$, then $a_0 =$

1) 1 2) 2 3) -2 4) -4

08. If $f(x) = e^x$ is expanded as a Fourier series in $(-\pi,\pi)$, the value of $a_0 =$

1) $\frac{\sinh\pi}{\pi}$ 2) $\frac{2\sinh\pi}{\pi}$

3) $\frac{\sinh\pi}{2\pi}$ 4) $\frac{e^{2\pi}-1}{\pi}$

09. If $f(x) = e^x$ is expanded as a Fourier series in $(0, 2\pi)$, then $a_0 =$

1) $\frac{\sinh \pi}{2\pi}$ 2) $\frac{2\cosh \pi}{\pi}$ 3) $\frac{e^{2\pi}+1}{2\pi}$ 4) $\frac{e^{2\pi}-1}{2\pi}$

10. If $f(x) = e^{-x}$ is expanded as a Fourier series in $(0, 2\pi)$, then $a_0 =$

1) $\frac{e^{-2\pi}-1}{2}$ 2) $\frac{1-e^{-2\pi}}{2}$ 3) $\frac{1+e^{-2\pi}}{2}$ 4) $\frac{1+e^{-2\pi}}{\pi}$

11. If $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$ is expressed as a Fourier series then $a_0 =$

1) 0 2) 2k 3) $\frac{k}{2}$ 4) $\frac{k}{4}$

12. If $f(x) = \begin{cases} -\pi, & if -\pi < x < 0 \\ 0, & if\ 0 < x < \pi \end{cases}$ is expressed as a Fourier series then $a_0 =$

1) $\frac{\pi}{2}$ 2) $-\frac{\pi}{2}$ 3) $-\frac{\pi}{4}$ 4) 2π

13. If $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$ is expressed as a Fourier series then $a_0 =$

1) $\frac{1}{\pi}$ 2) $\frac{2}{\pi}$ 3) 2π 4) $\frac{\pi}{3}$

14. If $f(x) = \begin{cases} 1, & if\ 0 < x < \pi \\ 0, & if\ \pi < x < 2\pi \end{cases}$ is expressed as a Fourier series then $a_0 =$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) $\frac{1}{6}$

15. If $f(x) = \begin{cases} 1, & if\ 0 < x < \pi \\ 2, & if\ \pi < x < 2\pi \end{cases}$ is expressed as a Fourier series then $a_0 =$

1) 1 2) 2 3) 3 4) 4

16. If $f(x) = x$ is expressed as a Fourier series in $(-\pi, \pi)$, then $a_n =$

1) 0 2) $\frac{1}{n}(-1)^{n+1}$ 3) $\frac{2}{n}(-1)^{n+1}$ 4) $\frac{4}{n}(-1)^n$

17. If $f(x) = x^2$ is expressed as a Fourier series in $(-\pi, \pi)$ then $a_0 =$

1) $\frac{\pi^2}{3}$ 2) $\frac{2\pi^2}{3}$ 3) $\frac{4\pi^2}{3}$ 4) 0

18. If $f(x) = x^2$ is expressed as a Fourier series in $(-\pi, \pi)$ then $b_n =$

1) $\frac{\pi^2}{3}$ 2) $\frac{2(-1)^n}{n^2}$ 3) $\frac{4(-1)^n}{n^2}$ 4) 0

19. If $f(x) = x \sin x$ is expressedas a Fourier series in $(-\pi, \pi)$ then $b_n =$

1) 0 2) 2 3) $\frac{2(-1)^n}{n^2+1}$ 4) $\frac{4(-1)^{n+1}}{n^2+1}$

20. If $f(x) = \sin ax$ is expressedas a Fourier series in $(-\pi, \pi)$ then $a_n =$

1) 0 2) $\frac{2n\sin a\pi}{\pi(n^2-a^2)}$ 3) $\frac{n\sin a\pi}{2\pi(n^2-a^2)}$ 4) none of these

PRACTICE SET - I KEY

01) 4	02) 3	03) 2	04) 1	05) 2
06) 4	07) 3	08) 2	09) 4	10) 2
11) 1	12) 2	13) 2	14) 1	15) 3
16) 1	17) 2	18) 4	19) 1	20) 1

PRACTICE SET - II

01. If $f(x) = \sqrt{1-\cos x}$ is expanded as a Fourier series in $(-\pi, \pi)$, the value of $a_0 =$

1) $\frac{\sqrt{2}}{\pi}$ 2) $\frac{2\sqrt{2}}{\pi}$ 3) $\frac{3\sqrt{2}}{\pi}$ 4) $\frac{4\sqrt{2}}{\pi}$

02. If $f(x) = x^3$ is expressed as a Fourier series in $(-\pi, \pi)$, the value of $a_n =$

1) 0 2) $2\left(\frac{\pi^2}{n} - \frac{3}{n^3}\right)$ 3) $2\left(\frac{\pi^2}{n} - \frac{6}{n^3}\right)$ 3) $4\left(\frac{\pi^2}{n} + \frac{6}{n^3}\right)$

03. If $f(x) = |\sin x|$ is expressed as a Fourier series in $(-\pi, \pi)$, the value of $a_0 =$

1) $\frac{1}{\pi}$ 2) $\frac{2}{\pi}$ 3) $\frac{3}{\pi}$ 4) $\frac{4}{\pi}$

04. If $f(x) = x - x^2$ is expressed as a Fourier series in $(-1, 1)$, then $a_0 =$

1) $\frac{1}{3}$ 2) $\frac{-1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{-2}{3}$

05. If $f(x) = x^2$ is expressed as a Fourier series in $(-1, 1)$, then $a_0 =$

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{-2}{3}$ 4) $\frac{4}{3}$

06. If $f(x) = x^2 - 2$ is expressed as a Fourier series in $(-2, 2)$, then $a_0 =$

1) $\frac{-1}{3}$ 2) $\frac{-2}{3}$ 3) $\frac{-4}{3}$ 4) $\frac{-8}{3}$

07. If $f(x) = 1 - x^2$ is expressed as a Fourier series in $(-1, 1)$, then $b_n =$

1) $\frac{2}{3}$ 2) $\frac{4}{\pi n^2}\cos n\pi$ 3) $(-1)^n \frac{4}{\pi n^2}\cos n\pi$ 4) 0

08. If $f(x) = 4 - x^2$ is expressed as a Fourier series in $(-2, 2)$, then $a_0 =$

1) $\frac{4}{3}$ 2) $\frac{8}{3}$ 3) $\frac{16}{3}$ 4) $\frac{32}{3}$

09. If $f(x) = e^{-x}$ is expressed as a Fourier series in $(-1, 1)$, then $\frac{a_0}{2} =$

1) sinh 1 2) 2 sinh 1 3) 2 cosh 1 4) $\frac{1}{2}$cosh 1

10. If $f(x) = \pi x$ is expressed as a Fourier series in the interval $(0, 2)$, then $a_0 =$

1) π 2) $\frac{\pi}{2}$ 3) 2π 4) 4π

11. If $f(x) = \frac{1}{2}(\pi - x)$ is expressed as a Fourier series in the interval $(0, 2)$, then $a_0 =$

1) $\pi - 1$ 2) $\frac{\pi-1}{2}$ 3) $\frac{\pi-2}{2}$ 4) $\frac{1-\pi}{2}$

12. If $f(x) = 2x - x^2$ is expressed as a Fourier series in the interval $(0, 2)$ then $a_0 =$

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{5}{3}$ 4) $\frac{4}{3}$

13. If $f(x) = |x|$ is expressed as a Fourier series in $(-2, 2)$ then $b_n =$

1) 0

2) $\frac{2}{n^2\pi^2}\cos(2n+1)\frac{\pi x}{2}$

3) $\frac{4}{n^2\pi^2}\cos(2n+1)\frac{\pi x}{2}$

4) $\frac{4(-1)^n}{n^2\pi^2}\cos(2n+1)\frac{\pi x}{2}$

14. If the half-range cosine series expansion of $f(x) = x^2$ in $(0, \pi)$, the value of $a_0 =$

1) $\frac{\pi^2}{3}$ 2) $\frac{2\pi^2}{3}$ 3) $\frac{4\pi^2}{3}$ 4) $\frac{8\pi^2}{3}$

15. In the half-range sine series expansion of $f(x) = k$ in $(0, \pi)$ then value of $a_n =$

1) $\frac{k}{\pi(2n-1)}\sin nx$ 2) $\frac{2k}{\pi(2n-1)}\sin nx$ 3) $\frac{4k}{\pi(2n-1)}\sin nx$ 4) 0

16. In the half-range cosine series expansion of $f(x) = \pi - x$ in $(0, \pi)$, the value of a_0 =

1) π 2) 2π 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

17. In the half-range cosine series expansion of $f(x) = x(\pi - x)$ in $(0, \pi)$, the value of a_0 =

1) $\frac{\pi^2}{2}$ 2) $\frac{\pi^2}{3}$ 3) $\frac{\pi^2}{4}$ 4) $\frac{\pi^2}{6}$

18. In the half-range Fourier cosine series expansion of $f(x) = \sin x$ in $(0, \pi)$, the value of a_0 =

1) $\frac{1}{\pi}$ 2) $\frac{2}{\pi}$ 3) $\frac{3}{\pi}$ 4) $\frac{4}{\pi}$

19. In the half-range Fourier cosine series expansion of $f(x) = x^3$ in the interval $(0, l)$, is

1) $\frac{l}{2}$ 2) $\frac{l^2}{2}$ 3) $\frac{l^3}{2}$ 4) $\frac{l^4}{4}$

20. In the half-range Fourier cosine series expansion of $f(x) = x(2-x)$ in the interval $(0,2)$, the value of a_0 =

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) $\frac{5}{3}$

21. In the half-range Fourier cosine series expansion of $f(x) = x - x^2$ in $0<x<1$, the value of a_0 =

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{1}{6}$ 4) $\frac{5}{3}$

PRACTICE SET - II KEY

01) 4	02) 1	03) 4	04) 4	05) 2
06) 3	07) 4	08) 3	09) 1	10) 3
11) 1	12) 4	13) 1	14) 2	15) 4
16) 1	17) 2	18) 4	19) 3	20) 3
21) 1				

PREVIOUS ECET BITS

01. The Fourier series for $f(x) = x$ in $(-\pi, \pi)$ is

1) $2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx$ 2) $\sum_{n=1}^{\infty}(-1)^n \sin nx$

3) $2\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)}\sin nx$ 4) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{(n+1)}\sin(n+1)x$

02. In the half range series for $f(x) = \sin x$ in $0<x<\pi$, the coefficient a_0 is

1) $\frac{2}{\pi}$ 2) $\frac{4}{\pi}$ 3) $\frac{1}{\pi}$ 4) 0

T.S ECET-2017

03. If

$$f(x)=\begin{cases}0; -\pi \le x \le 0\\ \sin x; 0 \le x \le \pi\end{cases}, \quad f(x+2\pi)=f(x)$$

and $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

then a_0 =

1) $\frac{1}{\pi}$ 2) 1 3) 0 4) $\frac{2}{\pi}$

T.S ECET-2018

04. $f: R \to R$, $f(x) = x^2, -\pi \le x \le \pi$ and $f(x+2\pi) = f(x), \forall x \in R$. If the fourier series of $f(x)$ is represented as $f(x)=\sum_{n=0}^{\infty}a_n \cos nx$, then a_0 = ____

1) $\frac{2\pi^2}{3}$ 2) $\frac{\pi^2}{3}$ 3) $\frac{4\pi^2}{3}$ 4) $\frac{5\pi^2}{3}$

05. $f(t) = 2t^2 - 5, -2 \le t \le 2$ and $f(t+4) = f(t)$. If $2t^2 - 5 = \sum_{n=0}^{\infty}A_n Cos\left(\frac{n\pi t}{2}\right)$, then A_1 =

1) 0 2) $\frac{-32}{\pi^2}$ 3) $\frac{1-(-1)^n}{n}\frac{2}{\pi^2}$ 4) $\frac{16}{\pi^2}$

PREVIOUS ECET BITS - KEY

01) 1	02) 2	03) 4
04) 2	05) 2	

SPACE FOR IMPORTANT NOTES

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