✱ TRIGONOMETRY upto transformation ✱

Type - 1   method of solving :-

① If $\tan\theta = b/a$ then $a\cos 2\theta + b\sin 2\theta =$ ______

ⓐ a ⓑ b ⓒ $2b/a$ ⓓ $2a/b$.

Sol ✱ $\boxed{\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta}}$ = $\dfrac{b}{a} = \dfrac{1-\cos 2\theta}{\sin 2\theta}$ ⇒ $b\sin 2\theta = a - a\cos 2\theta$

∴ $b\sin 2\theta + a\cos 2\theta =$ ⓐ

logic :- whenever the problem in that form directly the answer is $\boxed{\cos \cdot \text{coeff}}$

→ If $\dfrac{\cos\theta}{3} = \dfrac{\sin\theta}{4}$ then $3\cos 2\theta + 4\sin 2\theta =$ ______

ⓐ 6 ⓑ 8 ⓒ 3 ⓓ 4.

Sol $\cancel{\cos\theta}$ $\cancel{\frac{\sin\theta}{\cos\theta}}$ $4\cos\theta = 3\sin\theta$ ⇒ $\dfrac{\sin\theta}{\cos\theta} = \dfrac{4}{3} = \tan\theta = \dfrac{4}{3}$

∴ logic cos coeff $= 3$

→ If $\tan\left(\frac{\theta}{2}\right) = \dfrac{m}{n}$ then $m\sin\theta + n\cos\theta =$ ______

ⓐ m ⓑ n ⓒ $m+n$ ⓓ $m-n$.

Sol logic :- n (or), $\tan\theta/2 = \boxed{\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{m}{n}}$ ⇒ $n - n\cos\theta = m\sin\theta$

⇒ $\boxed{m\sin\theta + n\cos\theta = n}$

→ If $\dfrac{\cos 3\theta}{\cos\theta} = 2$ then $\dfrac{\sin 3\theta}{\sin\theta} =$

ⓐ 4 ⓑ 0 ⓒ 2 ⓓ 1

$\boxed{\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} = 2}$

$(\sin A\cos B - \cos A\sin B) = \sin(A-B)$

⇒ $\dfrac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta} = 2$

$\dfrac{\sin 3\theta}{\sin\theta} - 2 = 2$ $= \dfrac{\sin(2\theta)}{\sin\theta\cos\theta}$ ⇒ $\dfrac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}} = 2$

∴ $\boxed{\dfrac{\sin 3\theta}{\sin\theta} = 4}$

previous concept

If $\frac{\tan 3\theta}{\tan\theta} = k$ then $\frac{\sin 3\theta}{\sin\theta} =$ ______

(a) k (b) $\frac{2k}{k-1}$ (c) $\frac{2k}{k+1}$ (d) $\frac{k}{k-1}$

Sol

$$\boxed{\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2}$$

$$\frac{\tan 3\theta}{\tan\theta} = k$$

$$= \frac{\sin 3\theta}{\cos 3\theta} \times \frac{\cos\theta}{\sin\theta} = k$$

$$= \cancel{\frac{\sin 3\theta \cos\theta}{\cos\theta \sin\theta}} = \frac{k}{}$$

$$= \cancel{\frac{\sin 3\theta}{k \sin\theta} - \frac{\cos 3\theta}{\cos\theta}}$$

$$= \frac{\sin 3\theta}{\cos 3\theta} = \frac{k \sin\theta}{\cos\theta}$$

$$= \boxed{\frac{\sin 3\theta}{k \sin\theta} = \frac{\cos 3\theta}{\cos\theta}}$$

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\sin 3\theta}{k \sin\theta} = 2$$

$$= \frac{\sin 3\theta}{\sin\theta}\left(1 - \frac{1}{k}\right) = 2$$

$$= \frac{\sin 3\theta}{\sin\theta} \cdot \frac{k-1}{k} = 2$$

$$\therefore \frac{\sin 3\theta}{\sin\theta} = \boxed{\frac{2k}{k-1}}$$

→ If $3\cos\theta + 4\sin\theta = 5$; $0 < x < \frac{\pi}{2}$ then $\tan\theta =$ ______

Sol $a\cos\theta + b\sin\theta = c$ then

Step-① Divide B-S with $\cos\theta$

② put $\tan\theta = t$

③ Solve the Quad eqn in t → α, → β

$$= \frac{a\cancel{\cos\theta}}{\cancel{\cos\theta}} + \frac{b\sin\theta}{\cos\theta} = \frac{c}{\cos\theta}$$

$$= a + b\tan\theta = c/\cos\theta$$

$$= 3 + 4\tan\theta = \frac{5}{\cos\theta} \Rightarrow \cancel{4\tan\theta = 2} \quad 3 + 4\tan\theta = \frac{5}{\sec\theta}$$

= (lengthy process) ✗ dont follow.

S.C :-

$$a\cos\theta + b\sin\theta = c$$

If $a^2 + b^2 = c^2$ then $\tan\theta = \frac{\text{coeff of } \sin\theta}{\text{coeff of } \cos\theta}$

$$3^2 + 4^2 = 5^2$$

$$\boxed{\tan\theta = \frac{4}{3}}$$

If $8\cos\theta + 15\sin\theta = 17$ then $\cot\theta =$ ______

$\cot\theta = \frac{8}{15}$ $\boxed{\text{By using S.C}}$ $a^2 + b^2 = c^2$

when $a\cos\theta + b\sin\theta = c$

If $0 < x < \pi$; $\cos x + \sin x = \frac{1}{2}$ then $\tan x =$ ______ (2)

ⓐ $-\left(\frac{4+\sqrt{7}}{2}\right)$ ⓑ $-\left(\frac{4-\sqrt{7}}{3}\right)$ ⓒ $-\left(\frac{4+\sqrt{7}}{4}\right)$ ⓓ none.

Divide $\cos\theta$ on both sides

$1 + \tan x = \frac{1}{2}\sec x$ (S.O.B.S)

$= (1+\tan x)^2 = \frac{1}{4}\sec^2 x \Rightarrow (1+\tan x)^2 = \frac{1}{4}(1+\tan^2 x)$

put $\tan x = t$

$\Rightarrow (1+t)^2 = \frac{1}{4}(1+t^2) \Rightarrow 4 + 4t^2 + 8t - 1 - t^2 = 0$

$= 3t^2 + 8t + 3 = 0$ $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$t = \cancel{-8 \pm \sqrt{4(3)(3)}}$

$t = \frac{-8 \pm \sqrt{64-36}}{6} = \frac{-8 \pm \sqrt{28}}{6}$

$t = \frac{-\cancel{8}4 \pm 2\sqrt{7}}{\cancel{6}3} = \frac{-4-2\sqrt{7}}{3}$ or $\frac{-4+2\sqrt{7}}{3} = -\left(\frac{4-2\sqrt{7}}{3}\right)$

If $0 < x < \frac{\pi}{2}$; $\cos x + \sin x = \frac{1}{5}$ then $\tan x =$ ______

$1 + \tan x = \frac{1}{5}\sec x$

$= (1+t)^2 = \frac{1}{25}(1+t^2) \Rightarrow 25 + 25t^2 + 50t - 1 - t^2 = 0$

$\Rightarrow 24t^2 + 50t + 24 = 0$

$= 12t^2 + 25t + 12 = 0$

$144t^2$: $16t$, $9t$

$\cancel{= \frac{-12 \pm \sqrt{625 + 576}}{24} = \frac{-12 \pm \sqrt{1201}}{24}}$

$\cancel{= \frac{-1-\sqrt{1201}}{2} \text{ or } -\left(\frac{1-\sqrt{1201}}{7}\right)}$

144 × 4 = 576; 576 + 625 = 1201

$= 12t^2 + 16t + 9t + 12 = 0$

$= 3t(4t+3) + 4(4t+3) = 0$

$(3t+4)(4t+3) = 0$

$t = -\frac{4}{3}, -\frac{3}{4}$ ∴ IV Q $\theta < \frac{\pi}{2}$ ∴

∴ no solution

Important Questions

⇒ If $\tan A = \frac{x\sin B}{1 - x\cos B}$, $\tan B = \frac{y\sin A}{1-y\cos A}$ then $\frac{\sin A}{\cancel{\cos B}}$ $\frac{\sin A}{\sin B}$

(a) x/y (b) y/x (c) $\frac{2y}{x}$ (d) none.

Sol $\tan A = \frac{x\sin B}{1-x\cos B}$

$\frac{\sin A}{\cos A} = \frac{x\sin B}{1-x\cos B}$ ⇒ $\sin A - x\sin A\cos B = x\cos A\sin B$

⇒ $\sin A = x(\underbrace{\sin A\cos B + \cos A\sin B}_{\sin(A+B)})$

$= \sin A = x(\cancel{\sin(A+B)})$

$\sin B = y(\cancel{\sin(A+B)})$

∴ $\boxed{\frac{\sin A}{\sin B} = \frac{x}{y}}$.

⇒ If $\sin(x+3\alpha) = 3\sin(\alpha - x)$ then $\tan x =$ ____

Sol $\sin x\cos 3\alpha + \cos x\sin 3\alpha = 3(\sin\alpha\cos x - \cos\alpha\sin x)$

$\sin x\cos 3\alpha + 3\cos\alpha\cdot\sin x = 3\sin\alpha\cos x - \cos x\sin 3\alpha$

⇒ $\sin x(\cos 3\alpha + 3\cos\alpha) = \cos x(3\sin\alpha - \sin 3\alpha)$

$\sin x(4\cos^3\alpha - \cancel{3\cos\alpha} + \cancel{3\cos\alpha}) = \cos x(\cancel{3\sin\alpha} + 4\sin^3\alpha - \cancel{3\sin\alpha})$

$\frac{\sin x}{\cos x} = \frac{4\sin^3\alpha}{4\cos^3\alpha} = \tan^3\alpha$.

∴ $\boxed{\frac{\sin x}{\cos x} = \tan^3\alpha}$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\sin 3\theta = \cancel{3\sin\theta - 4\sin^3\theta}$

$3\sin\theta - 4\sin^3\theta$

⇒ If $\frac{\sin^4 x}{a} + \frac{\cos^4 x}{b} = \frac{1}{a+b}$ then $\frac{\sin^8 x}{a^3} + \frac{\cos^8 x}{b^3} =$ ____

Sol $\left(\frac{a+b}{a}\right)\sin^4 x + \left(\frac{a+b}{b}\right)\cos^4 x = 1$

$= \left(\frac{a+b}{a}\right)\sin^4 x + \left(\frac{a+b}{b}\right)\cos^4 x = \cos^2 x + \sin^2 x$

$= \left(\frac{a+b}{a}\sin^2 x\right)\sin^2 x + \left(\frac{a+b}{b}\cos^2 x\right)\cos^2 x = \cos^2 x + \sin^2 x$

$\cancel{\frac{a+b}{a}\sin^2 x = 1\sin^2 x}$ $\cancel{\frac{a+b}{b}\cos^2 x = 1\cos^2 x}$

$\frac{a+b}{a}\sin^2 x = 1 \Rightarrow \sin^2 x = \frac{a}{a+b}$; $\cos^2 x = \frac{b}{a+b}$

$\sin^8 x = \frac{a^4}{(a+b)^4}$ $\quad \cos^8 x = \frac{b^4}{(a+b)^4}$

$\therefore \frac{\cancel{a^4}}{(a+b^4)^4 \cancel{a^3}} + \frac{\cancel{b^4}}{(a+b)^4 \cancel{b^3}} = \frac{a}{(a+b)^4} + \frac{b}{(a+b)^4}$

$= \frac{\cancel{a+b}}{(a+b)^4} = \boxed{\frac{1}{(a+b)^3}}$

---

3 If $\frac{\sin^2 t}{2} + \frac{\cos^2 t}{3} = \frac{1}{5}$ then $\tan^2 t =$ ______

Sol $\frac{\sin^2 x}{a} + \frac{\cos^2 x}{b} = \frac{1}{a+b}$ $\quad a = 2,\ b = 3$

$\sin^2 x = \frac{a}{a+b}$ $\quad \cos^2 x = \frac{b}{a+b}$

$= \frac{2}{5}$ $\quad = \frac{3}{5}$

$\therefore \tan^2 t = \frac{2}{\cancel{5}} \times \frac{\cancel{5}}{3} = \frac{2}{3}$

Type - 2 :- Super '9' Tricks (AP/GP/nearto 60 angles) π problems.

Case - 1 Series of product of Sine/Cosine terms when angles are in A.P

① $\sin(\pi/7)\sin(2\pi/7)\sin(3\pi/7)\sin(4\pi/7)\sin(5\pi/7)\sin(\frac{6\pi}{7})$ ______

Sol when $\sin(\pi/n)\sin(2\pi/n)\sin(3\pi/n) \ldots\ldots \sin\left(\frac{(n-1)\pi}{n}\right) = \frac{n}{2^{n-1}}$

$\therefore n = 7 = \frac{7}{2^6} = \frac{7}{64}$ $\quad$ (n-1) should be same

$\boxed{\text{Super trick - 1}}$ — when (n-1) is ~~equal~~. Same for N x value of last

② $\sin(\pi/5)\sin(2\pi/5)\sin(3\frac{\pi}{5})\sin(4\frac{\pi}{5}) =$ ______

Sol $n = 5 = \frac{5}{2^4} = \frac{5}{16}$ $\quad 5 - 1 = 4$

4 is equal to nr 4.

Super trick - 2

3) $\sin\left(\frac{\pi}{7}\right) \sin\left(\frac{2\pi}{7}\right) \sin\left(\frac{3\pi}{7}\right) =$ ______

Sol $7-3 = 4 \neq 3$ then use. S.T-2

$\sin\left(\frac{\pi}{2n+1}\right) \sin\left(\frac{2\pi}{2n+1}\right) \sin\left(\frac{3\pi}{2n+1}\right) \ldots\ldots \sin\left(\frac{n\pi}{2n+1}\right) = \frac{\sqrt{2n+1}}{2^n}$

Note (n value should be equal to last value of Nr. π coeff.)

$= \frac{\sqrt{2(3)+1}}{2^3} = \frac{\sqrt{7}}{8}$

→ $\sin\left(\frac{\pi}{5}\right) \sin\left(\frac{2\pi}{5}\right)$

Sol $\sin\left(\frac{\pi}{2(2)+1}\right) \sin\left(\frac{2\pi}{2(2)+1}\right) = \frac{\sqrt{5}}{4}$

Super trick - 3

→ $\sin\left(\frac{\pi}{14}\right) \sin\left(\frac{2\pi}{14}\right) \sin\left(\frac{3\pi}{14}\right) \sin\left(\frac{4\pi}{14}\right) \sin\left(\frac{5\pi}{14}\right) \sin\left(\frac{6\pi}{14}\right) =$ ______

Sol by using (S.t-3)

~~$\sin\pi$~~ n is even then $= m \times (n)$ - is equal to $n-1$ of last value then ~~then~~

S.T-3 → $\boxed{\sin\left(\frac{\pi}{2n}\right) \sin\left(\frac{2\pi}{2n}\right) \sin\left(\frac{3\pi}{2n}\right) \ldots\ldots \sin\left(\frac{(n-1)\pi}{2n}\right) = \frac{\sqrt{n}}{2^{n-1}}}$

$= \sin\frac{\pi}{2\times 7} = 7-1 = 6 =$ Nr coeff of last $= 6$

$= (n=7)$ then $\frac{\sqrt{7}}{2^6} = \frac{\sqrt{7}}{64}$

→ $\sin\left(\frac{\pi}{8}\right) \sin\left(\frac{2\pi}{8}\right) \sin\left(\frac{3\pi}{8}\right)$

Sol $2\times(4) = 8$ $4-1 = 3$ then $\frac{\sqrt{4}}{2^3} = \frac{2}{8} = \frac{1}{4}$

Super trick -4

$\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{3\pi}{11}\right)\ldots\ldots\cos\left(\frac{10\pi}{11}\right).$

$\cos\left(\frac{\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right)\cos\left(\frac{3\pi}{n}\right)\ldots\ldots\cos\frac{(n-1)\pi}{n} = \frac{(-1)^{\frac{n-1}{2}}}{2^{n-1}}$; n is odd

$= 0$; n is even.

$11-1 = 10$ ✓

$= \quad n=11 \Rightarrow \frac{(-1)^{\frac{11-1}{2}}}{2^{11-1}} = \frac{-1^5}{2^{10}} = \frac{-1}{2^{10}}$

(2) $\cos\left(\frac{\pi}{15}\right)\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{3\pi}{15}\right)\ldots\ldots\cos\left(\frac{14\pi}{15}\right).$

Sol $n=15 \Rightarrow \frac{(-1)^{\frac{15-1}{2}}}{2^{15-1}} = \frac{(-1)^7}{2^{14}} = \frac{-1}{2^{14}}$

(3) $\cos\left(\frac{\pi}{22}\right)\cos\left(\frac{2\pi}{22}\right)\cos\left(\frac{3\pi}{22}\right)\ldots\cos\left(\frac{21\pi}{22}\right) = \_\_\_\_$

Sol $n=22$ even $\therefore 0$

Super trick - 5

(1) $\cos\left(\pi/7\right)\cos\left(2\pi/7\right)\cos\left(3\pi/7\right) = \_\_\_\_$

Sol $n-1 \Rightarrow 7-3 = 4 \neq 3$ $\quad n=7$ (odd)

$\boxed{\text{by using S-T-5}} \rightarrow \boxed{\cos\left(\frac{\pi}{2n+1}\right)\cos\left(\frac{2\pi}{2n+1}\right)\cos\left(\frac{3\pi}{2n+1}\right)\ldots\cos\left(\frac{n\pi}{2n+1}\right) = \frac{1}{2^n}}$

$= \cos\left(\frac{\pi}{2(3)+1}\right)$ $\quad 3 = 3$ (n=) coff of last term.

$\therefore n=3$ then $\frac{1}{2^3} = \frac{1}{8}$

$\rightarrow \cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\ldots\cos\left(\frac{5\pi}{11}\right)$

Sol $n = 2\times 5+1$; $n=5$ then $\frac{1}{2^5} = \frac{1}{32}$

Super trick - 6

(1) $\cos\left(\frac{\pi}{10}\right)\cos\left(\frac{2\pi}{10}\right)\cos\left(\frac{3\pi}{10}\right)\cos\left(\frac{4\pi}{10}\right) = \_\_\_\_$

Sol even $2\times\underset{n}{5}$ $\quad n-1 \Rightarrow 5-1 = 4$ then $\cos\left(\frac{\pi}{2\pi}\right)\cos\left(\frac{2\pi}{2n}\right)\ldots\cos\left(\frac{(n-1)\pi}{2n}\right)$

$= \frac{\sqrt{5}}{2^4} = \frac{\sqrt{5}}{16} = \frac{\sqrt{n}}{2^{n-1}}$

$\Rightarrow \cos\left(\frac{\pi}{8}\right) \cos\left(\frac{2\pi}{8}\right) \cos\left(\frac{3\pi}{8}\right)$

Sol: $2\times 4 \Rightarrow 4-1=3$ ✓ $n=4$

then $= \frac{\sqrt{4}}{2^3} = \frac{\cancel{2}}{\cancel{8}}\frac{1}{4} = \frac{1}{4}$ //

$\Rightarrow$ Questions based on Super '6' tricks using some manupulations (missing angles)

(IIT)

$\Rightarrow \sin\left(\frac{\pi}{14}\right) \sin\left(\frac{3\pi}{14}\right) \sin\left(\frac{5\pi}{14}\right) \sin\left(\frac{7\pi}{14}\right) \sin\left(\frac{9\pi}{14}\right) \sin\left(\frac{11\pi}{14}\right) \sin\left(\frac{13\pi}{14}\right)$

Sol: (a) $\frac{1}{128}$ (b) $\frac{1}{64}$ (c) $\frac{-1}{64}$ (d) $\frac{-1}{128}$

Sol: $\dfrac{\sin\left(\frac{\pi}{14}\right) \sin\left(\frac{2\pi}{14}\right) \sin\left(\frac{3\pi}{14}\right) \cdots\cdots \sin\left(\frac{13\pi}{14}\right)}{\sin\frac{\cancel{2\pi}}{\cancel{14}} \sin\left(\frac{\cancel{4\pi}}{\cancel{14}}\right) \sin\left(\frac{\cancel{6\pi}}{\cancel{14}}\right) \sin\left(\frac{8\pi}{\cancel{14}}\right) \sin\left(\frac{\cancel{10}\pi}{\cancel{14}}\right) \sin\left(\frac{\cancel{12\pi}}{\cancel{14}}\right)}$

above series in case - 1 $14-1=13$ ✓

$n=14$ then $\frac{n}{2^{n-1}} = \dfrac{\frac{14}{2^{13}}}{\frac{7}{2^6}} = \frac{\cancel{14}}{\cancel{2^{13}}} \times \frac{2^6}{\cancel{7}}$

$n=7$

$= \frac{1}{2^6} = \frac{1}{64}$ //

$\Rightarrow \sin\left(\frac{\pi}{7}\right) \sin\left(\frac{2\pi}{7}\right) \sin\left(\frac{4\pi}{7}\right)$

Sol: $\cancel{\dfrac{\sin\left(\frac{\pi}{7}\right) \sin\left(\frac{2\pi}{7}\right) \sin\left(\frac{3\pi}{7}\right) \sin\left(\frac{4\pi}{7}\right)}{\sin\left(\frac{3\pi}{7}\right)}} = \sin\left(\frac{\pi}{7}\right) \sin\left(\frac{2\pi}{7}\right) \sin\left(\pi - \frac{3\pi}{7}\right)$

$n = 2(3)+1$ $= \sin\left(\frac{\pi}{7}\right) \sin\left(\frac{2\pi}{7}\right) \sin\left(\frac{3\pi}{7}\right) = \sqrt{\frac{2n+1}{2^n}}$

$= \sqrt{\frac{7}{2^3}} = \frac{\sqrt{7}}{8}$

$\Rightarrow \cos\left(\frac{2\pi}{7}\right) \cos\left(\frac{4\pi}{7}\right) \cos\left(\frac{8\pi}{7}\right)$.

Sol: $\cos\left(\frac{2\pi}{7}\right) \cos\left(\frac{\pi - 3\pi}{7}\right) \cos\left(\pi + \frac{\pi}{7}\right)$

$= \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{\pi}{7} \Rightarrow \cos\frac{\pi}{7} \cos\frac{2\pi}{7} \cos\frac{3\pi}{7}$

$2(3)+1 = \frac{1}{2^n} = \frac{1}{2^3} = \frac{1}{8}$

$\sin\left(\frac{\pi}{16}\right)\sin\left(\frac{2\pi}{16}\right)\sin\left(\frac{5\pi}{16}\right)\sin\left(\frac{7\pi}{16}\right)$

(a) $\frac{1}{8\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{4\sqrt{2}}$ (d) none.

$\sin\left(\frac{\pi}{16}\right)\sin\left(\frac{2\pi}{16}\right)\sin\left(\frac{3\pi}{16}\right)\sin\left(\frac{4\pi}{16}\right)\sin\frac{5\pi}{16}\sin\left(\frac{6\pi}{16}\right)\sin\left(\frac{7\pi}{16}\right) = \frac{\sqrt{n}}{2^{n-1}}$

$\sin\frac{2\pi}{16}$ $\quad \sin\left(\frac{4\pi}{16}\right)$ $\quad \sin\left(\frac{6\pi}{16}\right)$

$n = 8 - 1 = 7$ apply case-3 on both Nr & dr.

$$= \frac{\frac{\sqrt{8}}{2^7}}{\frac{\sqrt{4}}{2^3}} = \frac{\sqrt{8}}{2^4} \times \frac{\cancel{2}}{\sqrt{4}} = \frac{\sqrt{2}\times\sqrt{4}}{2^4\times\sqrt{4}} = \frac{\sqrt{2}}{16}$$

$$= \cancel{\frac{\sqrt{2}}{2\times 8}} \quad \cancel{\frac{\sqrt{2}}{2\times}} = \frac{\sqrt{2}}{16} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{16\sqrt{2}}$$

$$= \frac{1}{8\sqrt{2}}$$

Case-11 Super "3" tricks (for product of cosine terms when angle are in G.P.

$\cos\theta\cos 2\theta\cos 4\theta\cos 8\theta \cdots \cos(2^{n-1}\theta) = \frac{\sin(2^n\theta)}{2^n\cdot\sin\theta}$ (n → no of cosine terms)

G.P

1. $\cos\theta\cos 2\theta\cos 4\theta\cos 8\theta$
2. $\cos\left(\frac{\pi}{2^n-1}\right)\cos\left(\frac{2\pi}{2^n-1}\right)\cos\left(\frac{4\pi}{2^n-1}\right)\cdots\cos\left(\frac{2^{n-1}\pi}{2^n-1}\right) = -\frac{1}{2^n}$ (angle higher)
3. $\cos\left(\frac{\pi}{2^n+1}\right)\cos\left(\frac{2\pi}{2^n+1}\right)\cos\left(\frac{4\pi}{2^n+1}\right)\cdots\cos\left(\frac{2^{n-1}\pi}{2^n+1}\right) = \frac{1}{2^n}$

1. $\cos\left(\frac{\pi}{15}\right)\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)$

Sol: $\cos\left(\frac{\pi}{2^4-1}\right)$ $\quad \cos\left(\frac{2^3\pi}{2^4-1}\right) = -\frac{1}{2^n} = -\frac{1}{2^4} = -\frac{1}{16}$

2. $\cos\left(\frac{\pi}{65}\right)\cos\left(\frac{2\pi}{65}\right)\cos\left(\frac{4\pi}{65}\right)\cos\left(\frac{8\pi}{65}\right)\cos\left(\frac{16\pi}{65}\right)\cos\left(\frac{32\pi}{65}\right)$.

Sol: $\cos\frac{2^5\pi}{2^6+1}$

$\boxed{n=6}$

$$= \frac{1}{2^6} = \frac{1}{64}$$

$\Rightarrow$ $16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right)$

Sol $\cancel{16 \cos}$

$\frac{2^3 = 8}{2^4-1}$ (not satisfying) then $\cos\left(\frac{\pi+\pi}{15}\right)$

$= -\cos\frac{\pi}{5}$

$= 16\left[\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)-\cos\left(\frac{\pi}{15}\right)\right]$

$= 16 --\left(\frac{1}{2^4}\right) = \cancel{16}\frac{1}{\ } \quad \cancel{16}\times\frac{1}{\cancel{16}} = 1$

IIT

$\Rightarrow$ $\cos\left(\frac{\pi}{2^2}\right)\cos\left(\frac{\pi}{2^3}\right)\cos\left(\frac{\pi}{2^4}\right)\ldots\ldots\cos\left(\frac{\pi}{2^{10}}\right)\sin\left(\frac{\pi}{2^{10}}\right)$

Sol GP but ~~case~~ it is not in 2^n-1 or 2^n+1

then use Short trick - 1 $\frac{\sin(2^n\theta)}{2^n\sin\theta}$ $n = 9$

$\theta = \frac{\pi}{2^{10}}$

$= \frac{\sin \cancel{2^9}\frac{\pi}{\cancel{2^{10}}\,2}}{2^9 \cancel{\sin\frac{\pi}{2^{10}}}} \times \cancel{\sin\frac{\pi}{2^{10}}} = \frac{1}{2^9} = \frac{1}{512}$

**

$\Rightarrow$ $\cos\left(\frac{\pi}{10}\right)\cos\left(\frac{2\pi}{10}\right)\cos\left(\frac{4\pi}{10}\right)\cos\left(\frac{8\pi}{10}\right)\cos\left(\frac{16\pi}{10}\right)$

Sol $10 \neq 2^n-1;\ 2^n+1$

$\frac{\sin(2^n\theta)}{2^n\sin\theta} = \frac{\sin\left(2^5\frac{\pi}{10}\right)}{2^5\sin\left(\frac{\pi}{10}\right)} = \frac{\sin\left(3\pi+\frac{2\pi}{10}\right)}{32\sin\left(\frac{\pi}{10}\right)}$

$= \frac{-\sin\frac{2\pi}{10}}{32\sin\frac{\pi}{10}}$

$= \frac{-\sin 36^\circ}{32\sin(18^\circ)} = \frac{\frac{-\sqrt{10-2\sqrt5}}{\cancel{4}}}{32\times\frac{\sqrt5-1}{\cancel{4}}}$

$= \frac{\sqrt{10-2\sqrt5}}{32(1-\sqrt5)}$

-3 Angles nearest to $60^\circ / 120^\circ / 240^\circ / 300^\circ / 420^\circ$ /

$\sin\theta \; \sin(60^\circ-\theta) \; \sin(60^\circ+\theta) = \frac{1}{4}\sin 3\theta$

$\cos\theta \; \cos(60^\circ-\theta) \; \cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$

$\tan\theta \; \tan(60^\circ-\theta) \; \tan(60^\circ+\theta) = \tan 3\theta$

→ $\cos 5^\circ \; \cos 55^\circ \; \cos 65^\circ$

Sol $\frac{1}{4}\cdot\cos(3\times5^\circ) = \frac{1}{4}\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}} \quad \frac{1}{4}\times\frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{8\sqrt{2}}$

→ $\sin 10^\circ \; \sin 30^\circ \; \sin 50^\circ \; \sin 70^\circ$

Sol $\frac{1}{2}\cdot\frac{1}{4}\sin(3\times10) = \frac{1}{8}\times\frac{1}{2} = \frac{1}{16}$

IIT

→ $\tan 6^\circ \; \tan 42^\circ \; \tan 66^\circ \; \tan 78^\circ$

Sol

$$\frac{\tan 6^\circ \; \tan 54^\circ \; \tan 66^\circ \; \tan 78^\circ \times \tan 42^\circ}{\tan 54^\circ}$$

$$= \tan 18^\circ \times \frac{\tan 18^\circ \times \tan 78^\circ \times \tan 42^\circ}{\tan 54^\circ \; \tan 18^\circ} = \frac{\cancel{\tan 18^\circ \; \tan 54^\circ}}{\cancel{\tan 54^\circ \; \tan 18^\circ}} = 1$$

$= 1$

→ $\left(1+\cos\frac{\pi}{9}\right)\left(1+\cos\frac{3\pi}{9}\right)\left(1+\cos\frac{5\pi}{9}\right)\left(1+\cos\frac{7\pi}{9}\right)$.

Sol $(1+\cos 20^\circ)\;(1+\cos 60^\circ)\;(1+\cos 100^\circ)\;(1+\cos 140^\circ)$

$= \frac{3}{2}\left(2\cos^2 10^\circ\right)\left(2\cos^2 50^\circ\right)\left(2\cos^2 70^\circ\right) \quad \left[\because 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right]$

$= \cancel{\frac{3}{2}\left(2\times\frac{1}{4}\cos 30^\circ\right)^2 = \left(\frac{3}{2}\times\frac{1}{2}\frac{\sqrt{3}}{2}\right)^2 = \left(\frac{3\sqrt{3}}{8}\right)^2 = \frac{9\times3}{64}} = \frac{27}{64}$

$= \frac{3}{2}\cdot 2\left(\cos^2 10^\circ \cos^2 50 \cos^2 70^\circ\right)$

$= 3\times\left(\frac{1}{4}\cos^2 30^\circ\right) = 3\times\frac{1}{4}\times\frac{3}{4} = \frac{9}{16}$

Case - 11 :- Summation of series of sine / cosine terms of AP angles
(1/2 problem)

Super 2 results

S.R - I $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \dots + \sin(\alpha+(n-1)\beta)$

$= \dfrac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \cdot \sin\left(\dfrac{F+L}{2}\right)$

n → no of sine terms

β = common diff of angle

S.R - II $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) \dots + \cos(\alpha+(n-1)\beta)$

$= \dfrac{\sin(n\beta/2)}{\sin(\beta/2)} \times \cos\left(\dfrac{F+L}{2}\right)$

n = no of cosine terms

β = common diff of angle

Corollary :-

① $\dfrac{\sin\theta_1 + \sin\theta_2 + \dots \sin\theta_n}{\cos\theta_1 + \cos\theta_2 + \dots + \cos\theta_n} = \tan\left(\dfrac{F+L}{2}\right)$ $\theta_1, \theta_2, \theta_3 \dots \theta_n$ are in AP.

② n → odd
$\cos\left(\frac{\pi}{2n+1}\right) + \cos\left(\frac{3\pi}{2n+1}\right) + \cos\left(\frac{5\pi}{2n+1}\right) + \dots + \cos\left(\frac{(2n-1)\pi}{2n+1}\right) = +\frac{1}{2}$ (odd)
d → odd

③ n → even
$\cos\left(\frac{2\pi}{2n+1}\right) + \cos\left(\frac{4\pi}{2n+1}\right) + \cos\left(\frac{6\pi}{2n+1}\right) + \dots + \cos\left(\frac{2n\pi}{2n+1}\right) = -\frac{1}{2}$ (even)
d → odd

→ $\sin\left(\frac{\pi}{18}\right) + \sin\left(\frac{2\pi}{18}\right) + \sin\left(\frac{3\pi}{18}\right) + \dots + \sin\left(\frac{35\pi}{18}\right)$.

Sol: $\frac{2\pi}{18} - \frac{\pi}{18} = \frac{\pi}{18}$; $\frac{3\pi}{18} - \frac{2\pi}{18} = \frac{\pi}{18}$

S.R - I $n = 35$, $\beta = \frac{\pi}{18}$ $L = \frac{35\pi}{18}$ $F = \frac{\pi}{18}$

$= \dfrac{\sin(n\beta/2)}{\sin(\beta/2)} \sin\left(\dfrac{F+L}{2}\right)$; $\sin\left(\frac{35\pi}{18} + \frac{\pi}{18}\right) = \frac{36\pi}{18}$

$= \sin 2\pi = 0$

$= \dfrac{(\ )}{(\ )} \times 0 = 0$

$\cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{3\pi}{7}\right) + \cos\left(\frac{5\pi}{7}\right) = \_\_\_\_$

Sol: $\frac{2\pi}{7}$ ✓ $\frac{2\pi}{7}$ — $\frac{odd}{even} = +\frac{1}{2}$.

→ $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$.

Sol: $\frac{2\pi}{7}$ ✓ $\frac{2\pi}{7}$ — $\frac{even}{odd} = -\frac{1}{2}$.

→ $\dfrac{\sin A + \sin 2A + \sin 3A + \cdots + \sin(2001A)}{\cos A + \cos 2A + \cos 3A + \cdots + \cos(2001A)}$

Sol: $\tan\left(\frac{F+L}{2}\right) = \tan\left(\frac{A+2021}{2}\right) = \tan \frac{\cancel{2022}^{1011} A}{\cancel{2}}$

$= \tan(1011A)$.

→ $\dfrac{\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 89^\circ}{\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin 85^\circ}$

Sol: $\cot \frac{\cancel{90}^{45}}{\cancel{2}} = \frac{1}{\tan 45^\circ} = \frac{1}{1} = 1$.

→ $\cos\left(\frac{2\pi}{n}\right) + \cos\left(\frac{4\pi}{n}\right) + \cdots + \cos\left(\frac{2(n-1)\pi}{n}\right) = -1$

Sol: $\frac{2\pi}{n}$.

S-R-2:- $\sin\frac{2\pi}{n}$ $\dfrac{\sin\left(\frac{(n-1)\cancel{2}\pi/n}{\cancel{2}}\right)}{\sin\left(\frac{2\pi/n}{2}\right)}$ $\cos\left[\frac{\frac{2\pi}{n} + \frac{2n\pi}{n} - \frac{2\pi}{n}}{2}\right]$

$\cos(\pi)$

$= \dfrac{\sin\left(\pi - \frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)} \cdot \dfrac{\cancel{\sin\frac{\pi}{n}}}{\cancel{\sin\frac{\pi}{n}}} (-1)$

$= -1$.

→ $\sin\left(\frac{2\pi}{n}\right) + \sin\left(\frac{4\pi}{n}\right) + \cdots + \sin\left(\frac{2(n-1)\pi}{n}\right)$.

Sol: $\cancel{\frac{2\pi}{n}} + \frac{2n\pi}{n} - \cancel{\frac{2\pi}{n}}$ $\sin 2\pi = 0$

$(\;) \times 0 = 0$.

$\rightarrow \sum_{r=1}^{n-1} \cos^2\left(\frac{r\pi}{n}\right)$.

5 :-

Sol $\cos^2\theta = \frac{1+\cos 2\theta}{2}$.

$= \frac{1}{2}\sum_{r=1}^{n-1}\left(1+\cos\frac{2r\pi}{n}\right)$

$= \frac{1}{2}\left(\sum_{r=1}^{n-1} 1 + \sum_{r=0}^{n-1}\cos\frac{2r\pi}{n}\right)$.

$= \frac{1}{2}\left[(n-1)+\left(\cos\frac{2r\pi}{n}\right]\right. = \frac{1}{2}\left[(n-1)+\underbrace{\cancel{\cos\frac{2(n-1)\pi}{n}}}_{-1}\right]$

$= \cancel{\frac{1}{2}[(n-1)+1]} = \frac{1}{2}[n-1-1] = \frac{1}{2}[n-2]$

$= \left[\frac{n}{2}-1\right]$.

$\rightarrow \sum_{r=1}^{n-1} \sin^2\left(\frac{r\pi}{n}\right)$

So. $\frac{n}{2}$.

~~Ktc~~

$\boxed{\frac{1-\cos 2\theta}{2} = \sin^2\theta}$

$\rightarrow \cos\left(\frac{\pi}{7}\right)+\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{3\pi}{7}\right)+\cdots\cdots+\cos\frac{7\pi}{7}$.

Sol $\cancel{-\frac{1}{2}} + \cancel{\frac{1}{2}} + \underbrace{\cancel{\cos\pi}}_{-1} = -1$.

$\rightarrow \cos\left(\frac{2\pi}{17}\right)+\cos\left(\frac{4\pi}{17}\right)+\cos\left(\frac{6\pi}{17}\right)+\cdots\cdots+\cos\left(\frac{16\pi}{17}\right)$.

Sol $-\frac{1}{2}$.

$\rightarrow \cos\left(\frac{\pi}{19}\right)+\cos\left(\frac{3\pi}{19}\right)+\cos\left(\frac{5\pi}{19}\right)+\cdots+\cos\left(\frac{17\pi}{19}\right)$

Sol $\frac{1}{2}$.

- 5 :- * Conditional Identities * (Sum of Trig func of 3 angles)

Master "5" Formulas

Sum to product

M-1 : $\sin\alpha + \sin\beta + \sin\gamma = 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\gamma+\alpha}{2}\right) + \sin(\alpha+\beta+\gamma)$

M-2 : $\cos\alpha + \cos\beta + \cos\gamma = 4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right) \pm \cos(\alpha+\beta+\gamma)$

product to Sum

M-3 : $\sin A \sin B \sin C = \frac{1}{4}\left[-\sin(A+B+C) + \Sigma \sin(B+C-A)\right]$

M-4 : $\cos A \cos B \cos C = \frac{1}{4}\left[\cos(A+B+C) + \Sigma \cos(B+C-A)\right]$

M-5 : $\tan A \tan B \tan C = \tan C - \tan A - \tan B$ (Here $A+B=C$)

→ If $A+B+C=180°$ then $\underset{\alpha}{\sin 2A} + \underset{\beta}{\sin 2B} + \underset{\gamma}{\sin 2C} =$ ______

Sol use M-1 ~~$4\sin\left(\frac{2A+2B}{2}\right)\sin\left(\frac{2B+2C}{2}\right)\sin\left(\frac{2C+2A}{2}\right) + \sin 2A + 2B +$~~ $+ \underbrace{\sin(2A+2B+2C)}_{360° = 0}$

$4\sin(A+B)\sin(B+C)\sin(C+A) + \underbrace{\sin(2A+2B+2C)}_{360° = 0}$

$\boxed{4\sin C \sin A \sin B}$

→ If $A+B+C=180°$ then $\cos 2A + \cos 2B + \cos 2C$

Sol use M-2 . $4\cos(A+B)\cos(B+C)\cos(C+A) \pm \underbrace{\cos(2A+2B+2C)}_{360° = 1}$

$= 4 - \cos C - \cos A - \cos B \pm 1$

$= 4 - (\cos A \cos B \cos C) \pm 1$

$= -4\cos A \cos B \cos C \pm 1$

→ If $A+B+C=0$ then $\sin 2A + \sin 2B + \sin 2C$.

Sol $4\sin(A+B)\sin(B+C)\sin(C+A) + \underbrace{\sin(0)}_{0}$

$= 4\sin(-C)\sin(-A)\sin(-B)$

$= -4\sin A \sin B \sin C$

→ If $A+B+C=270^\circ$ then $\cos 2A+\cos 2B+\cos 2C+4\sin A\sin B\sin C$

Sol $4\cos(A+B)\cos(B+C)\cos(C+A) - \cos(2A+2B+2C) + 4\sin A\sin B\sin C$

540°

$4\cos(270^\circ-C)$ .. .. ..

$-\cancel{4\sin C\sin A\sin B} + 1 + \cancel{4\sin A\sin B\sin C}$

$= 1$

→ If $A+B+C=180^\circ$ then $\sin 2A+\sin 2B-\sin 2C$

Sol ~~4 sin A~~ $\sin 2A+\sin 2B-\sin(180+2C)$

$= \cancel{\sin 2A+\sin 2B+\sin 2C}$

$\cancel{4\sin(A+B)\sin(B+C)\sin(C+A)+\sin 2A+2B+2}$

$= 4\sin(A+B)\sin\left(\frac{2B+2C+180}{2}\right)\sin\left(\frac{180+2C+2A}{2}\right)$

$= 4\sin(A+B)\sin(B+C+90)\sin(A+C+90^\circ)$

$= 4\sin C\sin(180-A+90)\sin(180-B+90^\circ)$ $+\sin(2A+2B+180+2C)$

$= \boxed{4\sin C\cos A\cos B}$

$\sin(2(A+B+C+90^\circ)$

$180+90$

$\sin(540^\circ)=0$ $\frac{270}{\frac{2}{540}}$

→ If $A+B+C=180^\circ$ then $\cos^2 A+\cos^2 B+\cos^2 C$

Sol $\cancel{\frac{1+\cos 2A}{2}+\frac{1+\cos 2B}{2}+\frac{1+\cos 2C}{2}}$

$= \cancel{\frac{1}{2}[3+(\cos 2A+\cos 2B+\cos 2C)]}$

$= \cancel{\frac{1}{2}[3+(-1-4\cos A\cos B\cos C)]}$

$= \frac{1}{2}[1 ...$ use verification method,

$\frac{\cancel{180}\ 60^\circ}{3}$ $=$ $A=60^\circ$ $B=60^\circ$ $C=60^\circ$

$=$ $\cancel{\frac{3}{4}+\frac{3}{4}}$ $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ $=\frac{3}{4}$

$\frac{3}{4}$ is present in any option it is right answer.

(iii) If $A+B+C=180°$ (or) In a Δ^{le} ABC & $\cos 3A+\cos 3B+\cos 3C=1$

ⓐ $\perp^{red}$ ⓑ $\perp^{red}$ (or) Iso ⓒ Δ^{le} ⓓ obtuse angle Δ^{le}.

$4\cos\left(\frac{3A+3B}{2}\right)$ $\cos(\quad)$ $\cos(\quad)$ $-\cos(3A+3B+3C)$ $\sin 0 = -1$

$4\cos\left(\frac{3\pi-3C}{2}\right)$

$= -4\sin\left(\frac{3A}{2}\right)\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right)$ $\cancel{1+1} = 1$

$= -4\sin\left(\frac{3A}{2}\right)\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right) = 0$

product of 3 numbers $= 0$ mean any of them leads to 0.

assume $\sin\frac{3A}{2} = 0$ but in Δ^{le} 0° is impossible then

$\sin 180° = 0$ ✓ possible

$\frac{3A}{2} = \cancel{180}$ $\boxed{A = 120°}$ (or), $B = 120°$ (or) $C = 120°$

∴ one angle should be more than 90° means "obtuse angle"

Conclusion Type - 1/2 problems + (mixed type)

Type - 1 method of solving

Type - 2 super tricks (Σ/Π problems).

→ If $3\sin\theta + 4\cos\theta = 5$ then $3\cos\theta - 4\sin\theta =$ ______

Sol S.C $a\cos\theta + b\sin\theta = c$ $a\cos\theta + b\sin\theta = c$

$b\sin\theta - a\cos\theta =$ $b\cos\theta - a\sin\theta = d$ } $\perp^r$ lines

then $a^2+b^2 = c^2+d^2$

$= \underbrace{3^2+4^2}_{25} = \cancel{5^2} + d^2 \Rightarrow d^2 = 0 \Rightarrow \boxed{d=0}$

→ If $\cos\theta + \sin\theta = \sqrt{2}\sin\theta$ then $\cos\theta - \sin\theta = ?$

Sol $~~1+1 = 2+d^2 \Rightarrow~~$ $1+1 = 2\sin^2\theta + d^2$

$= 2 - 2\sin^2\theta = d^2 \Rightarrow 2\cos^2\theta = d^2$

$\Rightarrow d = \sqrt{2}\cos\theta$

⇒ ~~If 3cos 3sinθ + 4cosθ = 5~~ then $3\cos\theta - 4\sin\theta = ?$

⇒ If $7\cos\theta - 24\sin\theta = 25$, then $24\cos\theta + 7\sin\theta = d$?

Sol $9 + 16 =$

Sol $49 + 576 = 625 + d^2$ ⇒ $\boxed{d = 0}$

⇒ $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$ then $\cos^2 x =$ ______

$\therefore \cos 2x = 2\cos^2 x - 1$

Sol $5(\sec^2 x - 1 - \cos^2 x) = 2(2\cos^2 x - 1) + 9$

$= 5\left(\frac{1}{\cos^2 x} - 1 - \cos^2 x\right) = 2(2\cos^2 x - 1) + 9.$ put $\cos^2 x = t$

$= 5\left(\frac{1}{t} - \frac{1}{1} - \frac{t}{1}\right) = 2(2t - 1) + 9$

$= 5\left(\frac{1 - t - t^2}{t}\right) = 2(2t - 1) + 9$

$5 - 5t - 5t^2 = 2t(2t - 1) + 9t$

$= 5 - 5t - 5t^2 = 4t^2 - 2t + 9t$

$= 5 - 5t - 5t^2 - 4t^2 + 2t - 9t = 0$

$= -9t^2 - 12t + 5 = 0$ ⇒ $9t^2 + 12t - 5 = 0$

$-45t^2$; $+15$, -3

$9t^2 + 15t - 3t - 5 = 0$

$3t(3t + 5) - 1(3t + 5)$

$(3t - 1)(3t + 5) = 0$

$t = -5/3$ or $t = 1/3$

$\therefore \boxed{\cos^2 x = \frac{1}{3}}$

⇒ $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$

(a) 1 (b) 2 (c) 4 (d) none

Sol $\frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\frac{2\sin 10^\circ \cos 10^\circ}{2}} = \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\frac{\sin 20^\circ}{2}}$ ($\therefore 2\sin A\cos A = \sin 2A$)

$\boxed{\begin{array}{l} \text{If } a\cos x \pm b\sin x = \sqrt{a^2+b^2}\,\sin(\alpha \pm x) \\ \alpha = \tan^{-1}\left(\frac{a}{b}\right) \end{array}}$

$= \cancel{\frac{\sqrt{1+3}\,\sin(30^\circ - 10^\circ)}{\frac{\sin 20^\circ}{2}}} = \frac{1}{2}\ \sqrt{4}\ \sin(30^\circ - 10^\circ) \times 2$

$= \left(\frac{\pi}{18}\right) \sin\left(\frac{5\pi}{18}\right) \sin\left(\frac{7\pi}{18}\right)$

$\sin 10 \quad \sin 50 \quad \sin 70$

$\theta \quad 60-\theta \quad 60+\theta$

$= \frac{1}{4}\sin(3\times 10) = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$

→ $10\sin^4 x + 15\cos^4 x = 6$ then $27\,\mathrm{cosec}^6\alpha + 8\sec^6\alpha =$ ______

Sol: divide by "6" on both sides to get $\sin^2 x + \cos^2 x = 1$

$\left(\frac{10}{6}\sin^2\alpha\right)\sin^2 x + \left(\frac{15}{6}\cos^2\alpha\right)\cos^2 x = 1\,\sin^2 x + 1\cos^2 x$

$1 = \frac{10}{6}\times\left(\frac{6}{10}\right) \qquad 1 = \frac{15}{6}\times\left(\frac{6}{15}\right)$

$\sin^2\alpha = \frac{6}{10} = \frac{3}{5} \qquad \cos^2\alpha = \frac{6}{15} = \frac{2}{5} \qquad = \frac{5}{5} = 1$ (Identity satisfied)

then $27\,\mathrm{cosec}^6\alpha + 8\sec^6\alpha$

$= 27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3$

$= 125 + 125 = 250$

→ If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$ then $\cos 2\alpha + \cos 2\beta$

(a) $2\cos(\alpha+\beta)$ (b) $-2\cos(\alpha+\beta)$ (c) $2\cos\left(\frac{\alpha+\beta}{2}\right)$ (d) $-2\sin(\alpha+\beta)$

Sol: verification method

$\alpha = 0°,\ \beta = 180°$

$\alpha = 90°,\ \beta = -90°$

$LHS = \cos 2(0°) + \cos 2(180°)$

$= 1 + 1 = 2$

RHS (a) $2\cos(180°) = -2$ ✗

(b) $-2\cos(180°) = 2$

(c) $2\cos 90° = 0$ ✗

(d) $-2\sin(180°) = 0$ ✗

→ If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\Sigma \cos(\alpha - \beta)$

Sol verification method

→ $\boxed{\alpha = 0^\circ,\ \beta = 120^\circ\ \ \gamma = 240^\circ}$ Ideal values to get = 0

$\cos(0 - 120^\circ) + \cos(-120^\circ) + \cos(240^\circ)$

$= -\frac{1}{2} - \frac{1}{2} - \frac{1}{2} = -3/2$

→ If $t_n = \sin^n\theta + \cos^n\theta$ then $\dfrac{t_7 - t_5}{t_5 - t_3} =$ ______

(a) $\frac{t_1}{t_2}$ (b) $\frac{t_3}{t_1}$ (c) $\frac{t_5}{t_3}$ (d) $\frac{t_3}{t_5}$

Sol

$$\frac{\sin^7\theta + \cos^7\theta - \sin^5\theta - \cos^5\theta}{\sin^5\theta + \cos^5\theta - \sin^3\theta - \cos^3\theta}$$

→ $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

→ $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

→ $\sin^2\theta + \cos^4\theta = 1 - \sin^2\theta\cos^2\theta$

→ $\cos^2\theta + \sin^4\theta = 1 - \sin^2\theta\cos^2\theta$

$$= \frac{\sin^5\theta(\sin^2\theta - 1) + \cos^5\theta(\cos^2\theta - 1)}{\sin^3\theta(\sin^2\theta - 1) + \cos^3\theta(\cos^2\theta - 1)}$$

$$= \frac{+\sin^5\theta\cos^2\theta + \sin^2\theta\cos^5\theta}{+\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}$$

$$= \frac{\cancel{\cos^2\theta\sin^2\theta}(\sin^3\theta + \cos^3\theta)}{\cancel{\cos^2\theta\sin^2\theta}(\sin\theta + \cos\theta)} \quad \therefore = \frac{t_3}{t_1}$$

→ $\dfrac{\sin 2A + \sin 5A + \sin 8A}{\cos 2A + \cos 5A + \cos 8A} = \tan\left(\dfrac{2A + 8A}{2}\right) = \tan(5A).$

→ $\dfrac{\cot 2011A + \cot 2012A + \cot 2013A}{\sin 2011A + \sin 2012A + \sin 2013A} = \cot(2012A)$

→ $\dfrac{\cos 9x + \cos 7x + \cos 5x + \cos 3x}{\sin 9x + \sin 7x + \sin 5x + \sin 3x} = \cot(6x).$

If $5\sin x + 4\cos x = 3$ then $|4\sin x - 5\cos x|$

(a) $4\sqrt{2}$ (b) $2\sqrt{4}$ (c) ± 3 (d) $\pm 4\sqrt{2}$.

$5^2 + 4^2 = 3^2 + d^2 \Rightarrow$ ~~$25 - 9 = d^2 = 16$~~

~~$d^2 = 16$~~ $25 + 16 = 9 + d^2 \Rightarrow$ $41 - 9 = d^2$

$\therefore d = \sqrt{32} = \sqrt{16 \times 2} = |\pm 4\sqrt{2}| = 4\sqrt{2}$

Type - 8 :- verification method :-

→ The value of the expression.

$$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

(a) $\sin A \cos A + 1$ (b) $\sec A \operatorname{cosec} A + 1$ (c) $\tan A + \cot A$ (d) ~~$\sec A + \operatorname{cosec} A$~~.

Sol: way - 1

take $A = -45°$

$LHS = \frac{-1}{1+1} + \frac{-1}{1+1}$

$= \frac{-1}{2} - \frac{1}{2} = \frac{-2}{2} = -1.$

(a) ~~$+ \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + 1 = \frac{1}{2} + 1 = \frac{3}{2}$~~ $\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + 1$

(a) ~~$\frac{1}{2} + 1 = \frac{2+1}{2} = \frac{3}{2}$~~ ✗

(b) ~~$(-\sqrt{2})(\sqrt{2}) + 1 = 2 + 1 =$~~

(a) $\frac{-1}{\sqrt{2}} \cdot \frac{-1}{\sqrt{2}} + 1$

$= \frac{1}{2} + 1$

way - 2

$\tan A = 2$ $\sin A = \frac{2}{\sqrt{5}}$

$\cot A = \frac{1}{2}$ $\cos A = \frac{1}{\sqrt{5}}$

$LHS = \frac{2}{1 - \frac{1}{2}} + \frac{1/2}{1 - 2}$

$= \frac{2}{\frac{1}{2}} + \frac{\frac{1}{2}}{-1} = 4 - \frac{1}{2} = 7/2$

RHS = (a) $\frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} + 1 = \frac{2}{5} + 1 = \frac{7}{5}$ ✗

(b) $\sqrt{5} \cdot \frac{\sqrt{5}}{2} + 1 = 6$ ✗

(c) $2 + \frac{1}{2} = 5/2$ ✗

$\frac{5}{2} + 1 = \frac{5+2}{2}$

(d) $\sqrt{5} + \sqrt{5}$

$= 2\sqrt{5}$

(b) $= \frac{7}{2}$

→ $\frac{1}{\tan 3x - \tan x} - \frac{1}{\cot 3x - \cot x}$

(a) $\sin 2x$ (b) $\cos 2x$ (c) $\cot 2x$ (d) $\sec 2x$

Sol any const is even
any function in trigonometric ratios except cos & sec is odd
cos even sec even

$= \frac{\text{even}}{\text{odd} - \text{odd}} - \frac{\text{even}}{\text{odd} - \text{odd}}$

$=$ odd − odd = odd function.

(a) ✓ (b) × (c) ~~×~~ ✓ (d) ×

$x = 45^\circ$

$\Rightarrow \frac{1}{-1-1} - \frac{1}{-1-1} = \cancel{\frac{1}{-2}} + \cancel{\frac{1}{2}} = 0$.

(a) 1 (c) 0 ✓

→ $\frac{\sin 2\theta}{1 + \cos 2\theta}$

(a) $\sin\theta$ (b) $\tan\theta$ (c) $\cos\theta$ (d) $\sec\theta$

Sol $\frac{\text{odd}}{\text{even} + \text{~~odd~~ even}} = \frac{\text{odd}}{\text{~~odd~~ even}} = \text{odd}$

(a) odd (b) odd (c) × (d) ×

$\theta = 45^\circ \Rightarrow \frac{1}{1+0} = 1$

(a) × (b) 1 ✓

→ $\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$ then $\sin(2021x) + \sin(2021y)$

(a) 0 (b) 1 (c) −1 (d) 2021

Sol If $x = -y$ ~~sin x + si~~ $\sin(-y) + \sin y = \sqrt{3}\,(\cancel{\cos y} - \cancel{\cos y})$

$0 = 0$

then $\sin(2021(-y)) + \sin(2021y)$

$= 0$

$\cos^2(\theta-45^\circ) + \cos^2(\theta+15^\circ) - \cos^2(\theta-15^\circ)$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

$\theta = 0^\circ$

$\cos^2(-45) + \cancel{\cos^2 15^\circ} - \cancel{\cos^2 15}$

$\frac{1}{2}$

⇒ If $x+y=z$ then $\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x \cos y \cos z$

(a) 1 (b) 0 (c) $\cos^2 x$ (d) $\sin^2 x$

Sol $x = y = z = 0$

$1+1+1-2 = 1$

⇒ $\cos\alpha \sin(\beta-\gamma) + \cos\beta \sin(\gamma-\alpha) + \cos\gamma \sin(\alpha-\beta)$

(a) 0 (b) 1 (c) 2 (d) $4\cos\alpha\cos\beta\cos\gamma$

Sol $\alpha = \beta = \gamma = 0$

$0+0+0 = 0$

⇒ $\sin^2 A\cos^2 B + \cos^2 A \sin^2 B + \sin^2 A \sin^2 B + \cos^2 A \cos^2 B$

(a) 0 (b) 1 (c) 2 (d) 4

Sol $\theta = 0^\circ$ ⇒ $0+0+0+1 = 1$

⇒ $\tan x \tan(x-60^\circ) + \tan(x-60^\circ)\tan(x+60^\circ) + \tan(x+60^\circ)\tan x$.

$-\sqrt{3} \quad \sqrt{3} = -3$

(a) 1 (b) −3 (c) 3 (d) 0

Sol $\theta = 0^\circ$ = $0 - 3 + 0 = -3$.

⇒ $x+y+z = xyz$ then $\Sigma \frac{2x}{1-x^2}$

(a) $\frac{8xyz}{(1-x^2)(1-y^2)(1-z^2)}$ (b) xyz (c) $3xyz$ (d) $\frac{8}{(1-x^2)(1-y^2)(1-z^2)}$

Sol Super tip - 1 $\boxed{x+y+z = xyz \text{ then } x=y=z=\sqrt{3}}$

$\frac{2x}{1-x^2} + \frac{2x}{1-x^2} + \frac{2x}{1-x^2}$ ⇒ $\frac{2\sqrt{3}}{1-3} = -\sqrt{3}\times 3$

LHS $= -3\sqrt{3}$

RHS :- (c) $\dfrac{3\times 3\sqrt{3}}{(1-3)(1-3)(1-3)}$

$= \dfrac{9\sqrt{3}}{(-2)(-2)(-2)} = \dfrac{9\sqrt{3}}{-8} = \dfrac{3\sqrt{3}}{-2}$, $= \dfrac{\overset{3}{\cancel{24}}\sqrt{3}}{-\cancel{8}} = -3\sqrt{3}$

(a) $\dfrac{8xyz}{(1-x^2)(1-y^2)(1-z^2)} = \dfrac{8\times 3\sqrt{3}}{(-2)(-2)(-2)}$

(b) $\times$ $(3\sqrt{3})$

(c) $\times$ $(24\sqrt{3})$

(d) $\times$

→ If $xy+yz+zx=1$ then $\sum \dfrac{x}{1+x^2}$

(a) $\dfrac{2}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}}$ (b) $\dfrac{2}{\sqrt{(1-x^2)(1-y^2)(1-z^2)}}$ (c) $\dfrac{2xyz}{(1+x^2)(1+y^2)(1+z^2)}$ (d) $2xyz$

Sol:

$$\boxed{\begin{array}{l}\text{Super trick - 2 :- } xy+yz+zx=1 \\ x=y=z=\dfrac{1}{\sqrt{3}}\end{array}}$$

$\dfrac{\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}\times 3 = \dfrac{\frac{1}{\sqrt{3}}}{1+\frac{1}{3}} = \dfrac{\frac{1}{\sqrt{3}}}{\frac{4}{3}} = \left(\dfrac{1}{\sqrt{3}}\times\dfrac{3}{4}\right)\times 3$

$= \dfrac{3}{4\sqrt{3}}\times 3 = \dfrac{9}{4\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}} \quad \dfrac{\overset{3}{\cancel{9}}\sqrt{3}}{4\times\cancel{3}} = \dfrac{3\sqrt{3}}{4}$

(a) $\cancel{\dfrac{2}{\left(1+\frac{1}{\sqrt{3}}\right)}}$

$\dfrac{2}{\sqrt{\frac{4}{3}+\frac{4}{3}+\frac{4}{3}}} = \dfrac{2}{\sqrt{\frac{12}{3}}} \quad \cancel{\dfrac{2}{2}} = 1$

(a) $\dfrac{2}{\sqrt{\frac{4}{3}\times\frac{4}{3}\times\frac{4}{3}}} = \dfrac{2}{\sqrt{\frac{4^2}{3^2}\times\frac{4}{3}}} = \dfrac{2}{\frac{4}{3}\times\frac{2}{\sqrt{3}}}$

$= \dfrac{2}{\frac{8}{3\sqrt{3}}} = \dfrac{\cancel{2}\times 3\sqrt{3}}{\cancel{8}_4}$

$= \dfrac{3\sqrt{3}}{4}$ ✓

(b) $\times$ (c) $\times$ (d) $\times$

$xy + yz + zx = 1$ then $\sum \frac{x+y}{1-xy}$ (13)

a) $\frac{1}{xyz}$ (b) $\frac{-1}{xyz}$ (c) $\frac{2}{xyz}$ (d) $\frac{4}{xyz}$.

Sol $x = y = z = \frac{1}{\sqrt{3}}$ $= \cancel{\frac{\sqrt{3}+\sqrt{3}}{1-3}} = \cancel{\frac{2\sqrt{3}}{-2}}$

$= \frac{\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{\cancel{6}\,3}{\cancel{2}\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{3\sqrt{3}}{\cancel{(\sqrt{3})^2}} = 3\times\sqrt{3} = 3\sqrt{3}.$

(a) $\frac{1}{\frac{1}{3}\cdot\frac{1}{\sqrt{3}}}$ $\frac{1}{\frac{1}{3\sqrt{3}}} = 1\times3\sqrt{3} = 3\sqrt{3}$

(b) ✗

(c) ✗

(d) ✗

→ If $\sin\alpha + \sin\beta + \sin\gamma = 3$ then $\Sigma \tan\left(\frac{\alpha}{2}\right) =$ ______

(a) 3 (b) 2 (c) 1 (d) 0

Sol $\alpha = \beta = \gamma = 90°$ $\Sigma \tan 45°$

$= 1 + 1 + 1 = 3$ $= 1+1+1 = 3.$

→ $\cos\alpha + \cos\beta + \cos\gamma + 3 = 0$ then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$ ______

Sol $\cos\alpha + \cos\beta + \cos\gamma = -3$

$\cos\pi = -1$: $-1-1-1 = -3$

$\sin^2\pi = 0 \Rightarrow 0+0+0 = 0$

→ If $\sin x + \sin y = 2$ then $\cos x + \cos y =$ ______

Sol $x = y = 90°$ $\cos 90° = 0$ $= 0+0 = 0$

$1+1 = 2$

→ If $\cos x + \cos y = 2$ then $\sin x + \sin y =$ ______

Sol $x = y = 0°$: $1+1 = 2$ $\sin 0° = 0$

$= 0+0 = 0$

→ If $3\sin x + 4\cos y + 5\sin z = 12$ then $\tan\left(\frac{x}{2}\right) + 5\cot\left(\frac{z}{2}\right) - 6\sin(y)$

(a) 3 (b) −6 (c) 2 (d) 6

Sol: coeff $= 3+4+5 = 12 = 12$

"to get 12" ∴ $x = 90°$ $y = 0°$ $z = 90°$.

$=$ then $\tan\left(\frac{90}{2}\right) + 5\cot\left(\frac{90}{2}\right) - \cancel{6\sin(0)}$ → 0

$= 1 + 5 = 6$

→ If $\sin\alpha + \operatorname{cosec}\alpha = 2$ then $\sin^{2022}\alpha + \operatorname{cosec}^{2022}\alpha =$ ______

Sol: $\alpha = 90°$

$1 + 1 = 2$ ⇒ $1^{2022} + 1^{2022} = 1 + 1 = 2$

→ If $\cos\alpha + \sec\alpha = 2$ then $\cos^{11}\alpha + \sec^{11}\alpha =$ ______

Sol: $\alpha = \beta = 0°$ $= 1 + 1 = 2$ ✓ $1^{11} + 1^{11} = 2$

→ If $\tan\alpha + \cot\alpha = 2$ then $\sqrt{\tan\alpha} + \sqrt{\cot\alpha} =$ ______

Sol: $\alpha = \beta = 45°$ $= 1 + 1 = 2$

$\sqrt{1} + \sqrt{1} = 2$

→ If $\sin\alpha + \operatorname{cosec}\alpha = 5$ then $\sin^2\alpha + \operatorname{cosec}^2\alpha =$ ______

Sol: $(\sin\alpha + \operatorname{cosec}\alpha) = 5$

S.O.B.S

$= \sin^2\alpha + \operatorname{cosec}^2\alpha + 2\cancel{\sin\alpha\operatorname{cosec}\alpha} = 5^2$

$= \sin^2\alpha + \operatorname{cosec}^2\alpha = 25 - 2 = 23$

→ If $\tan\alpha + \cot\alpha = 3$ then $\tan^2\alpha + \cot^2\alpha =$ ______

Sol: ~~$\tan^2\alpha + \cot^2\alpha = 0$~~

$\tan\alpha + \cot\alpha = 3$

(S.O.B.S)

$\tan^2\alpha + \cot^2\alpha + 2\cancel{\tan\alpha\cot\alpha} = 9$

$= 7$

$\tan\theta + \cot\theta = 5$ then (i) $\tan^2\theta + \cot^2\theta = 25$

(ii) $\tan^3\theta + \cot^3\theta =$ ______

(iii) $\tan^4\theta + \cot^4\theta =$ ______

$\tan\theta + \cot\theta = 5$

(C.O.B.S)

(ii) $\tan^3\theta + \cot^3\theta + 3\cancel{\tan\theta\cot\theta}(\underbrace{\tan\theta + \cot\theta}_{5}) = 125$

$\Rightarrow \tan^3\theta + \cot^3\theta = 125 - 15$

$= 110$ ..

(iii) $\cancel{\tan^2\theta + \cot^2\theta = 25}$ $\quad \tan\theta + \cot\theta = 5$

(S.O.B.S)

$= \tan^2\theta + \cot^2\theta + \cancel{2\tan\theta\cot\theta} = 25$

$= \tan^2\theta + \cot^2\theta = 23$

S.O.BS

$= \tan^4\theta + \cot^4\theta = (23)^2 - 2$

$= 529 - 2 = 527$ ..

If $f_k(x) = \frac{1}{k}(\sin^k\alpha + \cos^k\alpha)$; $x \in R$ then $f_4(x) - f_6(x) =$ ______

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) 0 (d) $\frac{1}{24}$

Sol $\alpha = 0°$ $\quad k = 4$ $\quad k = 6$

$= \cancel{\frac{1}{4}(0^4}$ $\quad \frac{1}{4}(0^4 + 1^4)$ | $\frac{1}{6}(0^6 + 1^6)$

$= \frac{1}{4} \times 1 = \frac{1}{4}$ | $= \frac{1}{6}$

$= \frac{1}{4} - \frac{1}{6} = \frac{6-4}{24} = \cancel{\frac{2}{24}} = \frac{1}{12}$ ..

$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$ ______

(a) 0 (b) 13 (c) 6 (d) 1

$\Rightarrow x = 0°$ $= 3(1) + 6(1) + 4(1) = 13$ ..

previous eamcet

$\Rightarrow$ $2(\sin^6 x+\cos^6 x)-3(\sin^4 x+\cos^4 x)+1=$ ______ (a) 0 (b) -1 (c)

Sol $x=0°$ $\Rightarrow$ $2(1)-3(1)+1$

$=$ ~~2-3+1 = -1~~ $-1+1=0$

$\rightarrow$ In a $\triangle^{le}$ ABC $\cos\left(\frac{B+2C+3A}{2}\right)+\cos\left(\frac{A-B}{2}\right)=$ ______

(a) 0 (b) 1 (c) 2 (d) -1

Sol $A+B+C=180$ $A=B=C=60°$

~~$A+B=180-C$~~ $=\cos\left(\frac{60+120+180}{2}\right)$

~~$\cos(A+B)=\cos(180-C)$~~

$=$ ~~$\cos A\cos B-\sin A$~~ $=\cos\left(\frac{\overset{180}{\cancel{360}}}{2}\right)$

$=-1+\cos\left(\frac{60-60}{2}\right)$ $0°$

$=-1+1=0$

$\Rightarrow$ $\tan^2 x+\tan^2 y+\tan^2 y\tan^2 z+\tan^2 z\tan^2 x+2\tan^2 x\tan^2 y\tan^2 z=1$ then

$\Sigma\sin^2 x=$ ______ (a) 1 (b) 0 (c) 2 (d) 3

Sol $x=z=45°$ $=0+0+1+0=1=1$
$y=0°$

$\therefore$ $\Sigma\sin^2 x=\sin^2 x+\sin^2 y+\sin^2 z$

$=\frac{1}{2}+0+\frac{1}{2}=\frac{2}{2}=1$

$\Rightarrow$ If $\sin\alpha+\sin\beta+\sin\gamma=0$ then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=$ ______

~~(a) $\alpha=\beta$~~ (a) 3 (b) $\sqrt{3}$ (c) 1 (d) none

Sol $\alpha=\beta=\gamma=0$ $\Rightarrow$ $1+1+1=3$. ✗

~~$\alpha=45$ $\beta=45°$ $\gamma=0°$~~ because $\sin\theta$ lies -1 to 1

$=$ ~~$\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{2}{2\sqrt{2}}$~~ 0 is in between it is not max value.

α,β,γ is dependendent question

$\therefore$ answer is none

$\sin^2\theta + \sin^2(60-\theta) + \sin^2(60+\theta) = $ ______

(a) 3 (b) $\frac{1}{2}$ (c) $3/2$ (d) $3/4$.

$\theta = 0°$ = $0 + \frac{3}{4} + \frac{3}{4} = \frac{\cancel{6}}{\cancel{4}}\ \frac{3}{2}$ //

above problem based Question :-

$\sin^2 10 + \sin^2 50 + \sin^2 70° = $ ______

$\underset{\theta}{\sin^2 10°} + \sin^2(60 - \underset{\theta}{10°}) + \sin^2(60 + \underset{\theta}{10°})$

replace $\theta = 0°$.

$= 3/2$ //

$\cos^2\theta + \cos^2(60° - \theta) + \cos^2(60° + \theta)$

(a). 1 (b) $\frac{1}{2}$ (c) $3/2$ (d) None.

$\theta = 0° = 1 + \frac{1}{4} + \frac{1}{4} = 1 + \frac{\cancel{2}}{\cancel{4}}\frac{1}{2} = 3/2$ //

If $\cos^3\theta + \cos^3(120° + \theta) + \cos^3(240° - \theta) = k\cos 3\theta$ then $k = $ ______

Sol: $\theta = 0°$

$= 1 + \cos^3(90 + 30°) + \cos^3(270 - 30°) = k\ \cancel{\cos 3 \times 0}\ 1$

$= 1 - \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3 = k$

$= 1 - \frac{1}{8} - \frac{1}{8} = k \Rightarrow \frac{8-1-1}{8} = k \Rightarrow 6 = 8k$

$\Rightarrow k = \frac{\cancel{6}\ 3}{\cancel{8}\ 4} \Rightarrow \boxed{k = \frac{3}{4}}$ //

→ If $A + B = \frac{\pi}{4}$ then $(1 + \tan A)(1 + \tan B) = $ ______

Sol: $A = 0°$ $B = 45°$. $= (1 + 0)(1 + 1)$

$1(2) = 2$ //

$=$

→ If $\sec\theta = 5x + \frac{1}{20x}$ then $\sec\theta + \tan\theta = $ ______ logic :- put × 2

$= 10x$ or $\frac{1}{10x}$.

→ If $\operatorname{cosec}\theta = 2a + \frac{1}{8a}$ then $\operatorname{cosec}\theta + \cot\theta = $ ______

$= 4x$ (or) $\frac{1}{4x}$.

$\Rightarrow \left(\frac{+\sqrt{3}+2\cos A}{1-2\sin A}\right)^{-3} + \left(\frac{1+2\sin A}{\sqrt{3}-2\cos A}\right)^{-3} =$

(a) 1 (b) $\sqrt{3}$ (c) 0 (d) -1

$\frac{3}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{3\sqrt{2}}{(\sqrt{2})^2}$

Sol put $A = 90^\circ$

$= \cancel{\left(\frac{-\sqrt{3}+2}{1}\right.} \left(\frac{-\sqrt{3}}{1-2}\right)^{-3} + \left(\frac{1+2}{\sqrt{3}}\right)^{-3}$

$= \cancel{-(+\sqrt{3})^{-3} + (\sqrt{3})^{-3}}$

$= 0$

$\rightarrow \frac{\sec 8\theta - 1}{\sec 4\theta - 1}$ (a) $\frac{\tan 8\theta}{\tan 2\theta}$ (b) $\frac{\tan 2\theta}{\tan 8\theta}$ (c) $\frac{\cot 8\theta}{\cot 4\theta}$ (d) $\frac{\cot 4\theta}{\cot 8\theta}$

Sol put $\theta = 15^\circ$

$= \cancel{\frac{\sec 120^\circ - 1}{\sec 60^\circ - 1} \Rightarrow \frac{-\operatorname{cosec} 30^\circ - 1}{\frac{\sqrt{3}}{2} - 1} = \frac{-2-1}{\frac{\sqrt{3}-2}{2}} = \frac{-3}{\frac{\sqrt{3}-2}{2}}}$

$= \frac{\sec 120^\circ - 1}{\sec 60^\circ - 1} \Rightarrow \frac{-\operatorname{cosec} 30^\circ - 1}{2-1} = \frac{-2-1}{2-1} = \frac{-3}{1} = -3$

(a) $\frac{\tan(90^\circ+30)}{\tan 30^\circ} = \frac{-\sqrt{3}}{\frac{1}{\sqrt{3}}} = -\sqrt{3} \times \sqrt{3} = -3$ ✓

(b) ×

(c) $\frac{\cot(90+30^\circ)}{\cot 60^\circ} = \frac{-\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = -1$ ×

(d) ×

$\operatorname{cosec}\theta = \frac{p+q}{p-q}$ than $\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$ is ______

(a) p/q (b) q/p (c) $\sqrt{p/q}$ (d) $\sqrt{q/p}$.

$\theta = 30^\circ$, $2 = \frac{p+q}{p-q}$ apply C.r & D.r.

~~than cot $\frac{\pi}{4}$+~~ $= \frac{2+1}{2-1} = \frac{p+q+p-q}{p+q-p+q} = 3 = \frac{2p}{2q}$

$\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

$= \cot\left(\frac{\pi}{4} + \frac{\cancel{30}\,15^\circ}{\cancel{2}}\right) = \cot 60^\circ = \frac{1}{\sqrt{3}}$

(a) $3 \times$ (b) $\frac{1}{3} \times$ (c) $\sqrt{3} \times$ (d) $\sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$ ✓ "

→ If $\theta \in Q_3$ then $\sqrt{4\sin^4\theta + \sin^2 2\theta} + 4\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

(a) 2 (b) -2 (c) 0 (d) 1

Sol

θ is lies btwn $180 < \theta < 270^\circ$

$\theta = 225^\circ$

$= \sqrt{4\sin^4 225^\circ + \sin^2 450^\circ} + 4\cos^2\left(\frac{\pi}{4} - \frac{225}{2}\right)$

$= \sqrt{4 \times \left(\frac{1}{\sqrt{2}}\right)^4 + 1} + 4\cos^2 \frac{2\pi - 900}{8}$

$= \cancel{4} \times \frac{1}{\cancel{4}}\sqrt{1+1} + 4$

$= \sqrt{2} + 4\left(\cos^2 67\text{-}5^\circ\right)$

$= \sqrt{2} + 4^2\left(\frac{\sqrt{2}-1}{2\sqrt{2}}\right)$

$= \sqrt{2}+2 - \sqrt{2} = 2$ "

$\frac{450}{2}$

$\sin 270 - 225$

45

$4\sqrt{2}$

$\frac{4}{900}$

$\frac{2(\pi - 450)}{4}$

→ $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha \Rightarrow k = ?$

Sol

$\alpha = 45^\circ$

$= \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 = k + 1 + 1$

$= \cancel{\left(\frac{1}{2} + 2\right)^2 = \frac{1}{4} + 4 \Rightarrow \frac{1+16}{4} = \frac{17}{4} + \frac{17}{4} = \frac{34}{4}}$

$\cancel{\frac{17 - 15}{2}}$

$= \left(\frac{1+2}{\sqrt{2}}\right)^2 = \left(\frac{3}{\sqrt{2}}\right)^2 = \frac{9}{2}$

$= \frac{9}{2} + \frac{9}{2} = \frac{18}{2} = 9 = k+2$

$\therefore \boxed{k = 7}$ "

→ If $\tan A = \frac{n}{n+1}$, $\tan B = \frac{1}{2n+1}$ $(0° < A, B < 90°)$ then $A+B = 1$

Sol

put $n = 1$

$\tan A = \frac{1}{2}$

$\tan B = \frac{1}{3}$ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \frac{\cancel{5/6}}{\cancel{5/6}} = 1$

$\tan(A+B) = 1$

$A+B = \tan^{-1}(1)$

$A+B = 45°$ "

→ If $\sin A$, $\cos A$, $\tan A$ are in G.P then $\cot^6 A - \cot^2 A =$ ______

(a) 1 (b) 2 (c) 4 (d) 0

Sol Suppose a, b, c → G.P = $b^2 = ac$

→ A.P = $2b = a+c$

→ H.P = $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\cos^2 A = \sin A \tan A$

$\cos^2 A = \frac{\sin^2 A}{\cos A}$ divide $\sin^2 A$ on both sides

$= \boxed{\cot^2 A = \sec A}$

$\cancel{\cot^2 A}$

$\cot^6 A - \cot^2 A$

$\sec^3 A - \sec^2 A$

$\sec A(\sec^2 A - 1)$ $\sec^2 - \tan^2 = 1$

$\sec A(\tan^2 \theta)$

$= \cancel{\cot^2 A}\ \cancel{\tan^2 A}$

$= 1$ "