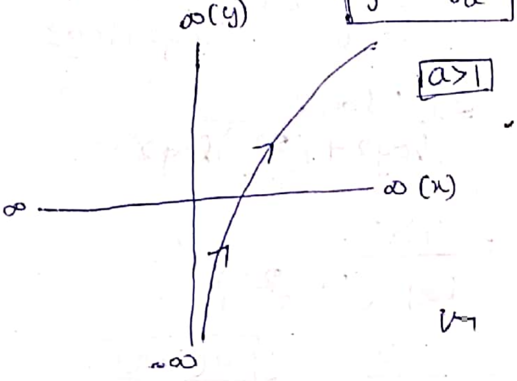
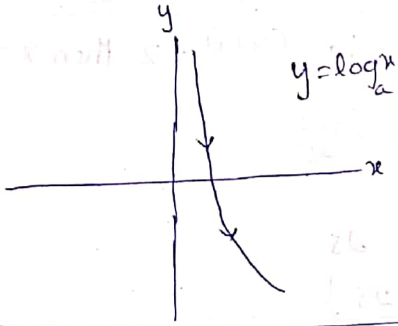
Logarithms

① $\boxed{y=\log_a^x}$



② $y=\log_a^x$ $\boxed{0<a<1}$



Domain of logarithm function : $(0,\infty)$

Range of logarithms : $(-\infty,\infty)$

Inequalities:

① $0<x<y \Rightarrow \log_a^x < \log_a y \Rightarrow a>1$ ↑

② $0<x<y \Rightarrow \log_a^x > \log_a y \Rightarrow 0<a<1$ ↓

③ $0>x>y \Rightarrow \log_a^x > \log_a y \Rightarrow a>1$ ↑

④ $0>x>y \Rightarrow \log_a^x < \log_a y \Rightarrow 0<a<1$ ↓

IMP FORMULAS:

$\log_y^x = \dfrac{\log_a x}{\log_a y}$	$a^{\log_c b} = b^{\log_c a}$
$\log 1 = 0$	$\log_a 0 \to -\infty$ $\boxed{a>1}$; ∞ $\boxed{0<a<1}$
$\log_b^a = \dfrac{1}{\log_a^b}$	$\log_y^x \to$ irrational (∵ x & $y \to$ coprime)

To solve least num for logarithmic function:

$A.M \geq G.M$

$\dfrac{a+b}{2} \geq \sqrt{ab}$

$\boxed{a=\log(f(x)),\ b=\log(f(y))}$ (logarithmic fn)

⑨ $(1+x)^{-1} = 1-x+x^2-x^3+x^4+\dots\infty$

⑩ $(1-x)^{-1} = 1+x+x^2+x^3+x^6\dots\infty$

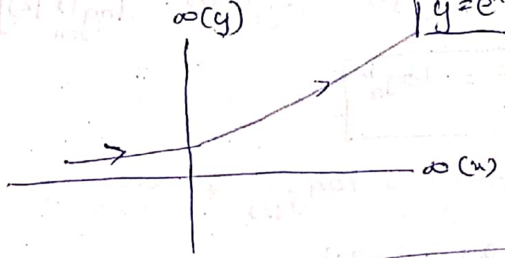
⑪ $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right) = \tanh^{-1}(x) = x+\frac{x^3}{3}+\frac{x^5}{5}+\dots\infty$

⑫ $a+ar+ar^2\dots\infty = \dfrac{a}{1-r}$ $\boxed{|r|<1}$

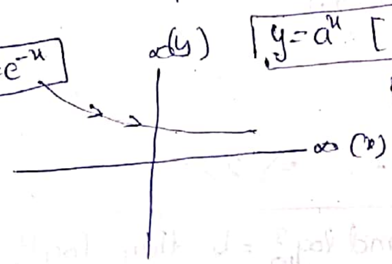
Exponentials:

$\Rightarrow \boxed{y=a^x\ (\because a>1)}$

① $\boxed{y=e^x}$



② $\boxed{y=e^{-x}}$ $\boxed{y=a^x\ [0<a<1]}$



$e=2.718$

Inequalities:

$x<y \Rightarrow a^x>a^y \Rightarrow 0<a<1 \Rightarrow$ ↓

$x<y \Rightarrow a^x<a^y \Rightarrow a>1 \Rightarrow$ ↑

$x>y \Rightarrow a^x>a^y \Rightarrow a>1 \Rightarrow$ ↑

$x>y \Rightarrow a^x<a^y \Rightarrow 0<a<1 \Rightarrow$ ↓

Domain of exponential function $=(-\infty,\infty)$

Range of exponential function $=(0,\infty]$

First principle of derivative:

$$\boxed{f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

$$\boxed{f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}}$$

★IMP: DERIVATIVES OF INFINITE SERIES:

① $e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\frac{x^n}{n!}\ \infty$

② $a^x = 1+x\log_e a+\frac{x^2(\log_e a)^2}{2!}+\dots\infty$

③ $\log_e(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}+\dots\infty$

④ $\log_e(1-x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}\dots\infty$

⑤ $\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots\infty$

⑥ $\cos x = x-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\dots\infty$

⑦ $\sinh x = x+\frac{x^3}{3!}+\frac{x^5}{5!}\dots\infty$

⑧ $\cosh x = x+\frac{x^2}{2!}+\frac{x^4}{4!}\dots\infty$

★ logarithm problems ★

Simplify $\left[\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}\right] =$

$$\boxed{\frac{1}{\log_b a} = \log_a b}$$

$\Rightarrow \log_{xyz} xy + \log_{(xyz)} yz + \log_{xyz} zx$

$\Rightarrow \log_{xyz}(xy \times yz \times zx)$

$= \log_{xyz}(xyz)^2$

$= 2\log_{xyz} \cancel{xyz}\ 1 \Rightarrow 2$

If $\log_{10} 2 = a$ and $\log_{10} 3 = b$ then $\log_5 12 = ?$

$\log_5 12 = \log_5 3 + \log_5 4$

$= \log_5 3 + 2\log_5 2$ $\quad \boxed{\log_b a = \frac{\log_x a}{\log_x b}}$

$= \frac{\log_{10} 3}{\log_{10} 5} + 2\frac{\log_{10} 2}{\log_{10} 5}$

$= \frac{\log_{10} 3}{\log_{10} 10 - \log_{10} 2} + \frac{2\log_{10} 2}{\log_{10} 10 - \log_{10} 2}$

$= \frac{b}{1-a} + \frac{2a}{1-a} \Rightarrow \boxed{\frac{2a+b}{1-a}}$

If $\log_{12} 27 = a$ then $\log_6 16 =$ ——? ★

$\frac{\log 27}{\log 12} = a$

$\frac{3\log 3}{\log 3 + 2\log 2} = a$

$\frac{\log 3 + 2\log 2}{3\log 3} = \frac{1}{a}$

$\frac{2\log 2}{3\log 3} = \frac{1}{a} - \frac{1}{3} = \frac{3-a}{3a}$

$\frac{\log 2}{\log 3} = \frac{3}{2} \times \left(\frac{3-a}{3a}\right) = \frac{9-3a}{6a} = \boxed{\frac{3-a}{2a}}$

$\boxed{\log 2 = \left[\frac{3-a}{2a}\right] \times \log 3} \leftarrow (x)$

or

$\boxed{\log 3 = \left(\frac{2a}{3-a}\right)\log 2 \text{ — (1)}}$

$\log_6 16 = \frac{\log 16}{\log 6} \Rightarrow \frac{4\log 2}{\log 2 + \log 3}$

$\Rightarrow \frac{4\log 2}{\log 2 + \left(\frac{2a}{3-a}\right)\log 2}$

$= \frac{4\cancel{\log 2}}{\cancel{\log 2}\left(1 + \frac{2a}{3-a}\right)}$

$= \frac{4}{\frac{3+a}{3-a}} = \boxed{\frac{4(3-a)}{3+a}}$

$\log_5(x^2+x) - \log_5(x+1) = 2$ then $x =$ ——

$\log_5\left(\frac{x^2+x}{x+1}\right) = \log_5 25$

$\frac{x(\cancel{x+1})}{\cancel{x+1}} = 25$

$\boxed{x = 25}$

$\frac{1}{2}(\log x + \log y) = \log\left[\frac{x+y}{2}\right]$ then

$\frac{1}{2}\log xy = \log\left[\frac{x+y}{2}\right]$

$\cancel{\log}(xy)^{1/2} = \cancel{\log}\left[\frac{x+y}{2}\right]$

$xy = \left(\frac{x+y}{2}\right)^2$

$xy = \frac{x^2+y^2+2xy}{4}$

$4xy = x^2+y^2+2xy$

$x^2+y^2-2xy = 0$

$(x-y)^2 = 0$

$x-y = 0$

$\boxed{x = y}$

The value of $(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9) =$ ——

$\frac{\cancel{\log 4}}{\log 3} \times \frac{\cancel{\log 5}}{\cancel{\log 4}} \times \frac{\cancel{\log 6}}{\cancel{\log 5}} \times \frac{\cancel{\log 7}}{\cancel{\log 6}} \times \frac{\cancel{\log 8}}{\cancel{\log 7}} \times \frac{\log 9}{\cancel{\log 8}}$

$= \frac{\log 9}{\log 3} = \frac{2\cancel{\log 3}}{\cancel{\log 3}} = 2$

If $a = b^x$; $b = c^y$ and $c = a^z$ then value of $\boxed{x \times y \times z}$ = ____

$a = b^x$; $x = \log_b^a$

$b = c^y$; $y = \log_c^b$

$c = a^z$; $z = \log_a^c$

$x \times y \times z = \log_b^a \times \log_c^b \times \log_a^c$

$= \frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a} = 1$

If $a = b^2 = c^3 = d^4$ then value of $\log_a(abcd)$ =

$\boxed{\begin{array}{l} b = a^{1/2} \\ c = a^{1/3} \\ d = a^{1/4} \end{array}}$ $\therefore \log_a(a \cdot a^{1/2} \cdot a^{1/3} \cdot a^{1/4})$

$= \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \log_a^a$ (1)

$= \boxed{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$

$\log 27 = 1.431$ then value of $\log 9$ = ____

$3 \log 3 = 1.431$

$\log 3 = \frac{1.431}{3} = 0.477$

$\log 9 = 2 \log 3 \Rightarrow 2 \times 0.477$

$= 0.954$

$\log_{10}^2 = 0.3010$, the value of $\log_{10}^5$ = ____

$\log_{10}^5 = \log_{10}\left(\frac{10}{2}\right) \Rightarrow \log_{10}^{10} - \log_{10}^2$

$\Rightarrow 1 - \log_{10}^2$

$\Rightarrow 1 - 0.3010$

$\Rightarrow \boxed{0.6990}$

If $\log 3$, $\log(3^x - 2)$, $\log(3^x + 4)$ are in A.P then x = ____

In A.P $\Rightarrow \boxed{C.R = C}$

$\log(3^x - 2) - \log 3 = \log(3^x + 4) - \log(3^x - 2)$

$2 \log(3^x - 2) = \log(3^x + 4) + \log 3$.

$(3^x - 2)^2 = 3^{x+1} + 12$

$3^{2x} + 4 - 4 \times 3^x = 3^{x+1} + 12$ $\boxed{\text{let } (3^x = a)}$

$a^2 + 4 - 4a = 3a + 12$

$a^2 - 8 - 7a = 0$

$\boxed{a^2 - 7a - 8 = 0}$

$a^2 - 8a + a - 8 = 0$

$a(a-8) + 1(a-8) = 0$

$(a+1)(a-8) = 0$

$\boxed{a = -1}$ — X

$\boxed{a = 8}$ ✓

$3^x = 8$

$\boxed{x = \log_3^8}$

If $\log_{10}^a = p$ and $\log_{10}^b = q$, then $\log_{10}(a^p b^q)$ = ____

$\log_{10}(a^p \cdot b^q) = \log_{10} a^p + \log_{10} b^q$

$= p(\log_{10}^a) + q \cdot \log_{10}^b$

$= p(p) + q(q)$

$= \boxed{p^2 + q^2}$

NOTES: $\log_{10} x = 3.996$

characteristic $= 3$

mantissa $= 0.996$

$\log_{10} x = -4.87 \rightarrow -5 + 0.13$

Characteristic $= -5$

Mantissa $= 0.13$

No: of digits in given number = characteristic Num + 1

If $\log_{10}^2 = 0.3010$. The No: of digits before decimal point of the number $(2000)^{2000}$ is

$\log_{10}(2000)^{2000} \Rightarrow 2000 \log_{10} 2000$

$\Rightarrow 2000 \times [3 + \log_{10} 2]$

$\Rightarrow 2000 \times [3 + 0.3010]$

$= 2000 \times [3.3010]$

$= 6602.0$

No: of digits = characteristic Num + 1

$= 6602 + 1$

$= 6603$

* MATRICES *

Matrix $= m \times n$ (m → rows, n → columns)

No. of elements in each row = column (n)

No. of elements in each column = rows (m)

Total elements in matrix $= m \times n$

* properties of Identity matrix * (I)

- I → multiplicative Identity
- $I^n = I$
- $\boxed{I^T = adj(I) = I^{-1} = I}$
- $|I| = 1$ (2×2) ; $= 0$ (3×3)

Additive Inverse of matrix $A = -A$

properties of matrix Addition

→ Follows commutative

→ Follows Associative

→ Null matrix (O) is additive Identity.

→ $A+B = A+C$

$\therefore \boxed{B = C}$

→ $A.B = A.C$

But $\boxed{B \neq C}$

* Matrix multiplication *

$A_{m\times n}$ $\because B_{p\times q}$

$\therefore \boxed{n = p}$ → For $A.B$ ⇒ resultant matrix ($m\times q$)

$\boxed{q = m}$ → For $B.A$ ⇒ " " ($p\times n$)

$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = A \Rightarrow A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$ (for any order)

$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \Rightarrow A \Rightarrow A^n = \boxed{a^{n-1} . A}$ (for any order)

$\begin{bmatrix} a & a & a \\ a & a & a \\ a & a & a \end{bmatrix} \Rightarrow A = A^n = \boxed{(\text{trace of } A)^{n-1} . A}$ (for any order)

Matrix equations

$A^2 - [\text{trace of } A] A + |A| I = 0$ (For 2×2)

$A^3 - \beta_1 A^2 + \beta_2 A - \beta_3 I = 0$ (For 3×3)

β_1 = trace of A

$\beta_2 = M_{11} + M_{22} + M_{33}$ (Minor)

$\beta_3 = |A|$.

$A^2 = I$ ⇒ Involutary matrix

$n \Rightarrow$ odd ⇒ $I.A = A$.

even ⇒ I

$\forall$ $\boxed{A^2 = I}$ $\therefore$ $\boxed{|A| = -1 \\ \text{trace} = 0}$

Eg: $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then $A^{1994} = I$, $A^{1995} = A$.

$A^2 = A$ ⇒ Idempotent matrix

$A^n = 0$ ⇒ Nilpotent matrix

$\boxed{n \rightarrow \text{Index of Nilpotent}}$

* properties of Multiplication *

→ doesn't follow commutative

→ follow Associative.

→ follow distributive.

$AB = BA$ ⇒ commute matrix.

Matrix and its adjoint / or Matrix & its Inverse are always $\boxed{\text{commute}}$

* Transpose *

Transpose of $(A)_{m\times n} = (A)_{n\times m}$

$A^T = A$ ⇒ symmetric matrix ⇒ $\boxed{a_{ij} = a_{ji}}$

$A^T = -A$ ⇒ skew symmetric ⇒ $\boxed{a_{ij} = 0 \; ; \forall \; i = j}$ $\boxed{a_{ij} = -a_{ji} \; \forall \; i \neq j}$

→ sum of all elements in any order of skew symmetric matrix $= 0$

$\boxed{A.A^T = I}$ ⇒ orthogonal matrix

$|A_{ortho}| = \pm 1$

In orthogonal matrix; Sum of squares of all elements in a row / column = 1

→ If A is orthogonal, then A^{-1} and A^T are also orthogonal

properties of transpose :

$(A^T)^T = A$.

$I^T = I$

$(KA)^T = K(A^T)$

$(A \pm B)^T = A^T \pm B^T$

$(AB)^T = B^T . A^T$

$(A.B.C.D)^T = D^T . C^T . B^T . A^T$

$A = \underbrace{\frac{1}{2}(A+A^T)}_{\text{symmetric matrix}} + \underbrace{\frac{1}{2}(A-A^T)}_{\text{skew symmetric matrix.}}$

$A^T = A$ {symmetric} $\Rightarrow A^n =$ symmetric.

$A^T = -A$ {skew symmetric} $= A^n$ — n = even — symmetric; n = odd — skew symmetric

$\boxed{\text{Trace}}$ → sum of diagonal elements

properties:

- $tr(A) = tr(A^T)$
- $tr(O) = 0$.
- $tr(\text{skew symmetric}) = 0$.
- → $\boxed{tr(I_n) = n.}$
- → $tr(kA) = k\,tr(A)$
- $tr(A+A^T) = 2tr(A)$
- $tr(A-A^T) = 0$.
- $tr(ABC) = tr(BAC) = tr(CAB)$

— cofactor of $a_{ij} = (-1)^{i+j}(M_{ij})$

— $(\text{Cofactor of } A)^T$ = Adjoint

$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \Rightarrow Adj(A) = \begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix}$

$\boxed{Adj(A^T) = (adj\,A)^T}$

$A = \begin{bmatrix} 0 & 0 & a \\ 0 & a & 0 \\ a & 0 & 0 \end{bmatrix}$ {Non-diagonal matrix; then

$\boxed{adj(A) = -A}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow Adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

if $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \Rightarrow |A| = abc$

$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \Rightarrow |A| = a^3$

$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} \Rightarrow |A| = adf$

$\begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ f & e & c \end{bmatrix} \Rightarrow |A| = abc$

Note

<u>Note</u>: For skew symmetric;

i) even order $\Rightarrow |A|_n$ = perfect square (a^2)

ii) odd order $\Rightarrow |A|_n = 0$.

<u>For symmetric matrix:</u>

$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \Rightarrow |A| = 3abc - a^3 - b^3 - c^3$

<u>properties of Det:</u>

→ $|A| = |A^T|$

→ $|\text{cofactor matrix}| = |\text{Adjoint}|$ ★

→ $|kA| = k^n|A| \Rightarrow n$ — rows or columns

→ if 3 rows / 3 columns are A.P. or G.P., then it's $\boxed{\det = 0}$ ★

$|A| = 0 \Rightarrow$ Singular matrix

$|A| \neq 0 \Rightarrow$ Non singular matrix.

★ $\begin{bmatrix} x & a & a \\ a & x & a \\ a & a & x \end{bmatrix} = |A| = \boxed{(x+2a)(x-a)^2}$

★ If each row / column have same degree homogeneous function, then det may number / function

→ if it is function, it's degree = degree of product of diagonal elements

$\boxed{\text{Inverse / Multiplicative Inverse}}$

→ Matrix should be Non singular

→ $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

→ $A^{-1} = \frac{Adj(A)}{|A|}$

→ $\boxed{|adj\,A| = |A|^{n-1}}$

$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$

$A = \begin{bmatrix} ab & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & b^2 \end{bmatrix} = |A^{-1}| = \frac{1}{a^3b^3}$

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properties of Inverse :

$\rightarrow (A^{-1})^{-1} = A$

$\rightarrow \boxed{A \cdot A^{-1} = A^{-1} \cdot A = I}$

$\rightarrow I^{-1} = I$

$\rightarrow (A^{-1})^T = (A^T)^{-1}$

$\rightarrow (AB)^{-1} = B^{-1} \cdot A^{-1}$

$\rightarrow \boxed{(ABCD)^{-1} = D^{-1} \cdot C^{-1} \cdot B^{-1} \cdot A^{-1}}$

Properties of Adjoint

$\rightarrow adj(A) \cdot A = A \cdot adj(A) = \boxed{|A| \cdot I}$

$\rightarrow$ A is singular; $|A| = 0$.

$\cdot \boxed{adj(A) \cdot A = A \cdot adj(A)}$

$\rightarrow |adj(A)| = |\cancel{adj}\ \text{cofactor matrix}| = |A|^{n-1}$

$\rightarrow |adj \cdot adj \cdot adj \cdots r\text{ times}| = |A|^{(n-1)^r}$

$\rightarrow adj(A^T) = (adj A)^T$

$\rightarrow adj(KA) = K^{n-1} \cdot adj(A)$

$\rightarrow Adj(AB) = adj(B) \cdot adj(A)$

$\rightarrow Adj(A^n) = (Adj \cdot A)^n$

$\rightarrow \boxed{Adj[adj(A)] = |A|^{n-2} \cdot A}$

Solution of linear Eqn

① parallel lines : $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$

② similar/coincident lines : $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

③ Intersecting lines : $\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$

$\rightarrow$ eqns containing $\boxed{\text{only 1 sol}}$, they are said to be $\boxed{\text{Consistent otherwise Inconsistent}}$

3 Variable Linear Eqn

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$A \cdot X = D$

$\boxed{X = D \cdot A^{-}}$

if $\boxed{A \text{ is singular matrix}}$ $|A| = 0$: then set of eqns has no solutions / infinite solutions.

$\rightarrow Adj(A) \cdot [D] = 0$; $\rightarrow \boxed{\text{System possesses Infinite solutions}}$

~~If A is Non~~

$\rightarrow Adj(A)[D] \neq 0$; $\rightarrow \boxed{\text{System possess no solution.}}$

if A is Non-singular; $|A| \neq 0$; system contains only 1 solution. (Consistent)

If $|A| = 0$; then 'A' has no Inverse

$\boxed{A^{-1} = \frac{adj(A)}{|A|}}$

$\rightarrow$ The No. of all possible matrices with entry as 0 or 1, if order of matrices is 3x3.

Total elements in (3x3) = 9 elements

each matrix is replaced by 0 or 1 = (2).

$\Rightarrow$ No. of possible matrices $= 2^9$

$= 512$

Rank of matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{(3\times3)}$$

Case i) $|A| = 0$

$\Rightarrow \boxed{\text{Rank} \neq \text{order}}$

$\rightarrow$ ∴ consider (2x2) in given matrix.

$\rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}_{2\times2}$

$|A| = -1-1 = \boxed{-2 \neq 0}$

$\boxed{\text{Rank} = 2}$

case ii) $|A| \neq 0$.

$\boxed{\text{Rank} \neq \text{order}}$

Note:

if $|A|_{2\times2} = 0$;

$\boxed{\text{Rank} = 1}$

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: PARTIAL FRACTIONS :

$$\frac{1}{a^2-x^2} = \frac{A}{a+x} + \frac{B}{a-x}$$

$$\boxed{A = \frac{1}{2a} ; B = \frac{1}{2a}}$$

$$\frac{1}{x^2-a^2} = \frac{A}{x-a} + \frac{B}{x+a}$$

$$\boxed{A = \frac{1}{2a} ; B = \frac{-1}{2a}}$$

Det of
$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 5 \quad 4\times4$$

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix}$$

$a=b=c=d=1.$

$$\boxed{(abcd)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)}$$

★ TRIGONOMETRY ★

1 Trigonometric Ratios:

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\mathrm{cosec}^2\theta - \cot^2\theta = 1$

→ $\sin\theta + \sin(\pi+\theta) + \sin(2\pi+\theta) + \dots \sin(n\pi+\theta)$

⇒ $n \Rightarrow$ odd $= 0$

$n \Rightarrow \boxed{\text{even}} \Rightarrow \boxed{\sin\theta}$

→ $\cos\theta + \cos(\pi+\theta) + \cos(2\pi+\theta) + \dots + \cos(n\pi+\theta)$

$n \Rightarrow$ odd $= 0$

$n \Rightarrow$ even $= \boxed{\cos\theta}$

★ COMPOUND ANGLES ★

$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$

$\sin(A-B) = \sin A\cdot\cos B - \cos A\cdot\sin B$

$\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$

$\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B.$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\cdot\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\cdot\tan B}$

$\cot(A+B) = \dfrac{\cot A\cdot\cot B - 1}{\cot B + \cot A}$

$\cot(A-B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$

$\sin(A+B+C) = \sin A\cdot\cos B\cdot\cos C + \sin B\cdot\cos A\cdot\cos C + \sin C\cdot\cos A\cdot\cos B - \sin A\cdot\sin B\cdot\sin C$

$\cos(A+B+C) = \cos A\cos B\cdot\cos C - \cos A\cdot\sin B\cdot\sin C - \cos B\cdot\sin A\cdot\sin C - \cos C\cdot\sin A\cdot\sin B$

$\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A\cdot\tan B\cdot\tan C}{1 - \tan A\cdot\tan B - \tan B\cdot\tan C - \tan C\cdot\tan A}$

$\cot(A+B+C) = \dfrac{\cot A + \cot B + \cot C - \cot A\cdot\cot B\cdot\cot C}{1 - \cot A\cot B - \cot B\cdot\cot C - \cot C\cdot\cot A}$

$\tan(\pi/4 + A) = \dfrac{1+\tan A}{1-\tan A}$

$\tan(\pi/4 - A) = \dfrac{1-\tan A}{1+\tan A}$

$\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A.$

$\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

$\tan(A+B)\cdot\tan(A-B) = \dfrac{\tan^2 A - \tan^2 B}{1 - \tan^2 A\cdot\tan^2 B}$

$\Sigma\cot A\cdot\cot B = 1 + \dfrac{\sin(A+B+C)}{\sin A\cdot\sin B\cdot\sin C}$

$\Sigma\tan A\cdot\tan B = 1 - \dfrac{\cos(A+B+C)}{\cos A\cdot\cos B\cdot\cos C}$

	15° ($\pi/12$)	75° ($5\pi/12$)
sin	$\frac{\sqrt3-1}{2\sqrt2} = \frac{\sqrt6-\sqrt2}{4}$	$\frac{\sqrt3+1}{2\sqrt2} = \frac{\sqrt6+\sqrt2}{4}$
cos	$\frac{\sqrt3+1}{2\sqrt2} = \frac{\sqrt6+\sqrt2}{4}$	$\frac{\sqrt3-1}{2\sqrt2} = \frac{\sqrt6-\sqrt2}{4}$
tan	$\frac{\sqrt3-1}{\sqrt3+1} = 2-\sqrt3$	$\frac{\sqrt3+1}{\sqrt3-1} = 2+\sqrt3$

★ Standard Results ★

I

→ $A+B+C = \pi/2, 3\pi/2, 5\pi/2, \dots (2n+1)\pi/2$; $n \in I$

• $\boxed{\Sigma\tan A\cdot\tan B = 1}$

→ $\tan A\cdot\tan B + \tan B\cdot\tan C + \tan C\tan A = 1$

• $\boxed{\Sigma\cot A = \Pi(\cot A)}$

→ $\cot A + \cot B + \cot C = \cot A\cdot\cot B\cdot\cot C$ $\cancel{\cot A\cdot\cot B + \cot B\cdot\cot C} + \cancel{\cot C\cdot\cot A}.$

→ $A+B+C = 0^\circ\ (n\pi) \Rightarrow \{0, \pi, 2\pi, \dots\}$

$\boxed{\Sigma\tan A = \Pi(\tan A)}$

$\tan A + \tan B + \tan C = \tan A\cdot\tan B\cdot\tan C.$

$\boxed{\Sigma\cot A\cdot\cot B = 1}$

$\cot A\cdot\cot B + \cot B\cdot\cot C + \cot C\cdot\cot A = 1$

$\sin(A+B) + \sin(A-B) = 2\sin A\cdot\cos B.$

$\sin(A+B) - \sin(A-B) = 2\cos A\cdot\sin B$

$\cos(A+B) + \cos(A-B) = 2\cos A\cdot\cos B$

$\cos(A+B) - \cos(A-B) = -2\sin A\sin B.$

II If $\boxed{A+B=90^\circ}$

$[A=B=45^\circ]$

$\sin^2A+\sin^2B=1$ $\quad \boxed{\tan A-\tan B=2\tan(A-B)}$

$\cos^2A+\cos^2B=1$

$\tan A\cdot\tan B=1$

$\cot A\cdot\cot B=1$

$\sin A=\cancel{\sin B}\cos B \Rightarrow \boxed{\sin A-\cos B=0}$

$\cos A=\sin B \Rightarrow \boxed{\cos A-\sin B=0.}$

III $A+B=45^\circ, 225, \ldots\ldots$ $\boxed{(4n+1)\pi/4}$

$\rightarrow (1+\tan A)(1+\tan B)=2$

$\rightarrow (1-\cot A)(1-\cot B)=2$

$\rightarrow \dfrac{(1+\cot A)(1+\cot B)}{\cot A\cdot\cot B}=2$

$A+B=135^\circ, 315^\circ, \ldots$ $\boxed{(4n-1)\pi/4.}$

$\rightarrow (1-\tan A)(1-\tan B)=2$

$\rightarrow (1+\cot A)(1+\cot B)=2$

$\rightarrow \dfrac{(1-\cot A)(1-\cot B)}{\cot A\cdot\cot B}=2$

IV $A+B=180^\circ$

$\rightarrow \cos A+\cos B=0$

$\rightarrow \sin A-\sin B=0.$

$\rightarrow \tan A+\tan B=0.$

$\rightarrow \cot A+\cot B=0.$

V $\tan A+\tan B+k\tan A\cdot\tan B = k$

$\tan A-\tan B-k\tan A\cdot\tan B = k.$

$\tan A+\tan B-k\tan A\cdot\tan B = -k.$

VII $a\cos x+b\sin x=m$

$a\sin x-b\cos x=n.$

$\Rightarrow \boxed{a^2+b^2=m^2+n^2}$

VIII $\boxed{\dfrac{a+b}{a-b}=\dfrac{c+d}{c-d}}$ $\left[\text{if } \dfrac{a}{b}=\dfrac{c}{d}\right]$

componendo dividendo Rule

★ Multiples & Submultiples ★

$\sin 2A = 2\sin A\cdot\cos A = \dfrac{2\tan A}{1+\tan^2A}$

$\sin A = 2\sin A/2\cdot\cos A/2 = \dfrac{2\tan A/2}{1+\tan^2 A/2}$

$\cos 2A = \cos^2A-\sin^2A$
$= 2\cos^2A-1$
$= 1-2\sin^2A$
$= \dfrac{1-\tan^2A}{1+\tan^2A}$

$\cos A = \cos^2 A/2-\sin^2 A/2$
$= 2\cos^2 A/2-1$
$= 1-2\sin^2 A/2$
$= \dfrac{1-\tan^2 A/2}{1+\tan^2 A/2}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2A}$ $\quad \tan A = \dfrac{2\tan A/2}{1-\tan^2 A/2}$

$\sin 3A = 3\sin A-4\sin^3A$

$\cos 3A = 4\cos^3A-3\cos A$

$\tan 3A = \dfrac{3\tan A-\tan^3A}{1-3\tan^2A}$

$1+\cos 2A = 2\cos^2A$

$1+\cos A = 2\cos^2 A/2$

$1-\cos 2A = 2\sin^2A$

$1-\cos A = 2\sin^2 A/2$

$\dfrac{1-\cos 2A}{1+\cos 2A} = \dfrac{2\sin^2A}{2\cos^2A} = \tan^2A$

$1+\cos kA = 2\cos^2(kA/2)$

$1-\cos kA = 2\sin^2(kA/2)$

$\cot A+\tan A = 2\operatorname{cosec}(2A)$

$\cot A-\tan A = 2\cot 2A$

$\sin^4x+\cos^4x = 1-2\sin^2x\cdot\cos^2x = 1-\frac{1}{2}\sin^2(2x)$

$\sin^6x+\cos^6x = 1-3\sin^2x\cdot\cos^2x = 1-\frac{3}{4}\sin^2(2x)$

$\sin^2x+\cos^4x = 1-\frac{1}{4}\sin^2(2x)$

$2B - 3B'$

Ratio	$\pi/10$	$\pi/8$	$\pi/5$	$3\pi/10$	$5\pi/8$	$2\pi/5$
	18°	$22\frac{1}{2}^\circ$	36°	54°	$67\frac{1}{2}^\circ$	72°
Sin	$\frac{\sqrt5-1}{4}$	$\sqrt{\frac{\sqrt2-1}{2\sqrt2}}$	$\frac{\sqrt{10-2\sqrt5}}{4}$	$\frac{\sqrt5+1}{4}$	$\sqrt{\frac{\sqrt2+1}{2\sqrt2}}$	$\frac{\sqrt{10+2\sqrt5}}{4}$
Cos	$\frac{\sqrt{10+2\sqrt5}}{4}$	$\sqrt{\frac{\sqrt2+1}{2\sqrt2}}$	$\frac{\sqrt5+1}{4}$	$\frac{\sqrt{10-2\sqrt5}}{4}$	$\sqrt{\frac{\sqrt2-1}{2\sqrt2}}$	$\frac{\sqrt5-1}{4}$
Tan	×	$\sqrt2-1$	×	×	$\sqrt2+1$	

$$\boxed{\tan 7\tfrac{1}{2}^\circ = \frac{2\sqrt2-(1+\sqrt3)}{\sqrt3-1}}$$

★ $\boxed{\text{Standards}}$ ★

(I) $\boxed{\text{product problems}}$ $\boxed{\text{Angles are in A.P}}$

$\rightarrow \cos\left(\frac{\pi}{n}\right)\cdot\cos\left(\frac{2\pi}{n}\right)\cdot\cos\left(\frac{3\pi}{n}\right)\cdots\cos\left(\frac{(n-1)\pi}{n}\right)$.

Sol: $\Rightarrow$ if n = odd: $\frac{(-1)^{\frac{n-1}{2}}}{2^{n-1}}$; n = even: 0

$\rightarrow \cos\left(\frac{\pi}{2n+1}\right)\cdot\cos\left(\frac{2\pi}{2n+1}\right)\cdots\cos\left(\frac{n\pi}{2n+1}\right)$

Sol: $\frac{1}{2^n}$ $\boxed{n=\text{odd or even}}$

$\rightarrow \cos\left(\frac{\pi}{2n}\right)\cdot\cos\left(\frac{2\pi}{2n}\right)\cdot\cos\left(\frac{3\pi}{2n}\right)\cdots\cos\left(\frac{(n-1)\pi}{2n}\right)$

Sol: $\frac{\sqrt n}{2^{n-1}}$

$\rightarrow \sin(\pi/n)\cdot\sin(2\pi/n)\cdots\sin\left(\frac{(n-1)\pi}{n}\right)$

Sol: $\frac{n}{2^{n-1}}$ $\boxed{n=\text{odd or even}}$

$\rightarrow \sin\left(\frac{\pi}{2n+1}\right)\cdot\sin\left(\frac{2\pi}{2n+1}\right)\cdots\sin\left(\frac{n\pi}{2n+1}\right)$.

Sol: $\frac{\sqrt{2n+1}}{2^n}$

$\rightarrow \sin(\pi/2n)\cdot\sin\left(\frac{2\pi}{2n}\right)\cdot\sin\left(\frac{3\pi}{2n}\right)\cdots\sin\left(\frac{n-1}{2n}\right)\pi$

Sol: $\frac{\sqrt n}{2^{n-1}}$

Case II) $\boxed{\text{when angles are in G.P}}$

$\rightarrow \cos\theta\cdot\cos2\theta\cdot\cos4\theta\cdot\cos8\theta\cdots\cos(2^{n-1}\theta)$

Sol: $\boxed{\frac{\sin 2^n\theta}{2^n\sin\theta}}$

$\rightarrow \cos\left(\frac{\pi}{2^n-1}\right)\cdot\cos\left(\frac{2\pi}{2^n-1}\right)\cdots\cos\left(\frac{2^{n-1}\pi}{2^n-1}\right) = \left(\frac{-1}{2}\right)^n$

$\rightarrow \cos\left(\frac{\pi}{2^n+1}\right)\cdot\cos\left(\frac{2\pi}{2^n+1}\right)\cdots\cos\left(\frac{2^{n-1}\pi}{2^n+1}\right) = \left(\frac{1}{2}\right)^n$

$\boxed{\text{Model-III}}$ Angles nearer to $60^\circ/120^\circ/240^\circ/300^\circ/420^\circ/\ldots$ etc

$\rightarrow \sin\theta\cdot\sin(60^\circ-\theta)\cdot\sin(60^\circ+\theta) = \frac14\sin3\theta$.

$\rightarrow \cos\theta\cdot\cos(60^\circ-\theta)\cdot\cos(60^\circ+\theta) = \frac14\cos3\theta$

$\rightarrow \tan\theta\cdot\tan(60^\circ-\theta)\cdot\tan(60^\circ+\theta) = \tan3\theta$

$\rightarrow \cot\theta\cdot\cot(60^\circ-\theta)\cdot\cot(60^\circ+\theta) = \cot3\theta$.

$\rightarrow \sec\theta\cdot\sec(60^\circ-\theta)\cdot\sec(60^\circ+\theta) = 4\sec3\theta$.

$\rightarrow \operatorname{cosec}\theta\cdot\operatorname{cosec}(60-\theta)\cdot\operatorname{cosec}(60^\circ+\theta) = 4\operatorname{cosec}3\theta$.

$\boxed{\text{IV}}$ – $\boxed{\text{Sum of ratios}}$ $\rightarrow$ $\boxed{\text{angles in A.P}}$

$\rightarrow \cos\theta_1+\cos\theta_2+\cos\theta_3+\ldots+\cos\theta_n$

Sol: $\frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}\cdot\cos\left(\frac{\theta_1+\theta_n}{2}\right)$

$\rightarrow \sin\theta_1+\sin\theta_2+\ldots+\sin\theta_n$

$\Rightarrow \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}\cdot\sin\left(\frac{\theta_1+\theta_n}{2}\right)$.

$\boxed{\beta = \text{common difference}}$

→ $\cos\left(\frac{\pi}{2n+1}\right)+\cos\left(\frac{3\pi}{2n+1}\right)+\dots+\cos\left(\frac{(2n-1)\pi}{2n+1}\right)$

Sol: $\frac{1}{2}$..

→ $\cos\left(\frac{2\pi}{2n+1}\right)+\cos\left(\frac{4\pi}{2n+1}\right)+\dots+\cos\left(\frac{2n\pi}{2n+1}\right)$

Sol: $-\frac{1}{2}$

V

→ if $\frac{\sin\alpha}{\sin\beta}=\frac{m}{n}$; then $\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}=\frac{m+n}{m-n}$

→ $\frac{\cos\alpha}{\cos\beta}=\frac{m}{n}$; then $-\cot\left(\frac{\alpha+\beta}{2}\right).\cot\left(\frac{\alpha-\beta}{2}\right)=\frac{m+n}{m-n}$

→ $\frac{\tan\alpha}{\tan\beta}=\frac{m}{n}$; $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{m+n}{m-n}$

→ $\cot\alpha.\cot\beta=\frac{m}{n}$; $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}=\frac{m+n}{m-n}$

VI $\boxed{S+C \text{ or } S-C}$

$S=\sin A/2$

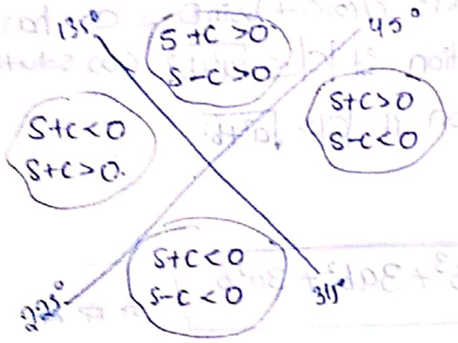
$C=\cos A/2$

$S+C=\sin A/2+\cos A/2=\pm\sqrt{1+\sin A}$

$S-C=\sin A/2-\cos A/2=\pm\sqrt{1-\sin A}$

$2\sin A/2=\pm\sqrt{1+\sin A}\ (+)\ \pm\sqrt{1-\sin A}$

$2\cos A/2=\pm\sqrt{1+\sin A}\ (-)\ \pm\sqrt{1-\sin A}$



VII Based on Formulas / Trig. values of multiple & sub mul

Ref n−

$\sqrt{2+\sqrt{2+\sqrt{2+\dots n}}}=2\cos\frac{\pi}{2^{n+1}}$

$\sqrt{2+\sqrt{2+\sqrt{2+\dots+2\cos\theta}}}=2\cos\frac{\theta}{2^{n}}$

[n = no. of square roots]

[Hyperbolic functions]

$e^x=\left(\frac{e^x+e^{-x}}{2}\right)+\left(\frac{e^x-e^{-x}}{2}\right)$ [ASSUMPTION]

[cosh x] [sinh x]

$\sinh x=\frac{e^x-e^{-x}}{2}$ (odd function)

$\cosh x=\frac{e^x+e^{-x}}{2}$ (even function)

$\tanh x=\frac{e^x-e^{-x}}{e^x+e^{-x}}$ (odd)

$\coth x=\frac{e^x+e^{-x}}{e^x-e^{-x}}$ (odd)

$\operatorname{cosech} x=\frac{2}{e^x-e^{-x}}$ (odd)

$\operatorname{sech} x=\frac{2}{e^x+e^{-x}}$ (even)

$\cosh^2x-\sinh^2x=1$

$\coth^2x-\operatorname{cosech}^2x=1$

$\operatorname{sech}^2x+\tanh^2x=1$

$\sinh(x+y)=\sinh x\cosh y+\cosh x.\sinh y$

$\sinh(x-y)=\sinh x\cosh y-\cosh x.\sinh y$

$\cosh(x+y)=\cosh x.\cosh y+\sinh x.\sinh y$

$\cosh(x-y)=\cosh x.\cosh y-\sinh x.\sinh y$

$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x.\tanh y}$

$\cancel{\coth(x+y)}\ \tanh(x-y)=\frac{\tanh x-\tanh y}{1-\tanh x.\tanh y}$

$\sinh 2x=2\sinh x.\cosh x=\frac{2\tanh x}{1-\tanh^2x}$

$\boxed{\tanh 2x=\frac{2\tanh x}{1+\tanh^2x}}$

$\sinh 3x=3\sinh x-4\sinh^3x$

$\cosh 3x=4\cosh^3x-3\cosh x$

$\tanh 3x=\frac{3\tanh x+\tanh^3x}{1+3\tanh^2x}$

$\cosh 2x=\cosh^2x+\sinh^2x=2\cosh^2-1=1+2\sinh^2x=\frac{1+\tanh^2x}{1-\tanh^2x}$

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$\sinh x + \sinh y = 2\sinh\left(\frac{x+y}{2}\right)\cdot\cosh\left(\frac{x-y}{2}\right)$

$\sinh x - \sinh y = 2\cosh\left(\frac{x+y}{2}\right)\cdot\sinh\left(\frac{x-y}{2}\right)$

$\cosh x + \cosh y = 2\cosh\left(\frac{x+y}{2}\right)\cdot\cosh\left(\frac{x-y}{2}\right)$

$\cosh x - \cosh y = 2\sinh\left(\frac{x+y}{2}\right)\cdot\sinh\left(\frac{x-y}{2}\right)$

$\sinh(x+y)\cdot\sinh(x-y) = \sinh^2 x - \sinh^2 y$
$= \cosh^2 x - \cosh^2 y$

$\cosh(x+y)\cdot\cosh(x-y) = \cosh^2 x + \sinh^2 x$
$\cosh^2 y + \sinh^2 y x$

$\tanh(x+y)\cdot\tanh(x-y) = \frac{\tanh^2 x - \tanh^2 y}{1 - \tanh^2 x\cdot\tanh^2 y}$

Conversions:

Hyp to Trig	Trig to Hyp
$\sinh(ix) = i\sin x$	$\sin(ix) = i\sinh x$
$\cosh(ix) = \cos x$	$\cos(ix) = \cosh x$
$\tanh(ix) = i\tan x$	$\tan(ix) = i\tanh x$
$\mathrm{cosech}(ix) = -i\,\mathrm{cosec}\,x$	$\mathrm{cosech}(ix) = -i\,\mathrm{cosech}\,x$
$\mathrm{sech}(ix) = \sec x$	$\sec(ix) = \mathrm{sech}\,x$
$\coth(ix) = -i\cot x$	$\cot(ix) = i\coth x$

$\sinh^{-1}x = \log_e(x + \sqrt{x^2+1})$

$\cosh^{-1}x = \log_e(x + \sqrt{x^2-1})$

$\tanh^{-1}x = \frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right)$

$\coth^{-1}x = \frac{1}{2}\log_e\left(\frac{x+1}{x-1}\right)$

$\mathrm{sech}^{-1}x = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

$\mathrm{cosech}^{-1}x = \log_e\left(\frac{1+\sqrt{1+x^2}}{x}\right); x>0.$

$= \log_e\left(\frac{1-\sqrt{1+x^2}}{x}\right); x<0.$

$e^{ix} = \cos x + i\sin x$

$e^{-ix} = \cos x - i\sin x$

$\cos x = \frac{e^{ix} + e^{-ix}}{2}$

$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$

Trigonometric Ratios Equations

Ratio	Domain	Range
Sin	$[-\pi/2, \pi/2]$	$[-1, 1]$
cos	$[0, \pi]$	$[-1; 1]$
tan	$(-\pi/2, \pi/2)$	$(-\infty, \infty) - R$

★ General solutions of Ratios ★

(I)

	General solution of θ
$\sin\theta = 0$	$\theta = n\pi; n\in Z$
$\cos\theta = 0$	$\theta = (2n+1)\pi/2; n\in Z$
$\tan\theta = 0$	$\theta = n\pi; n\in Z$

(II)

	G.s. of θ	
$\sin\theta = \sin\alpha$	$\alpha\in[-\pi/2, \pi/2]$ $\theta = n\pi + (-1)^n\alpha$	$n\in Z$
$\cos\theta = \cos\alpha$	$\alpha\in[0,\pi]$: $\theta = 2n\pi \pm \alpha$	$n\in Z$
$\tan\theta = \tan\alpha$	$\alpha\in(-\pi/2, \pi/2)$; $\theta = n\pi + \alpha$	$n\in Z$

(III)

$\sin^2\theta = \sin^2\alpha$	G.s of $\theta =$
$\cos^2\theta = \cos^2\alpha$	
$\tan^2\theta = \tan^2\alpha$	$n\pi \pm \alpha$; $\boxed{n\in Z}$

The equation: $a\cos\theta + b\sin\theta = c$, has

i) a solution if $|c| \le \sqrt{a^2+b^2}$ (∞ solution)

ii) No solution if $|c| > \sqrt{a^2+b^2}$

$\boxed{a,b\in Z}$

$(a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$ ★★★

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cdot\cos\frac{C-D}{2}$

$\cos C - \cos D = -2\sin\frac{C+D}{2}\cdot\sin\frac{C-D}{2}$

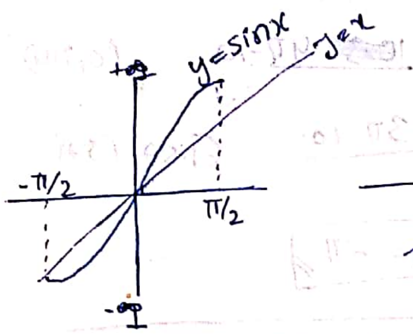
$\sin C + \sin D = 2\sin\frac{C+D}{2}\cdot\cos\frac{C-D}{2}$

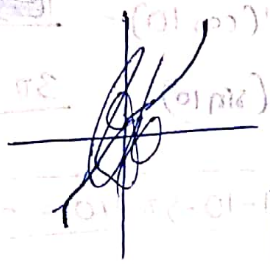
$\sin C - \sin D = 2\cos\frac{C+D}{2}\cdot\sin\frac{C-D}{2}$

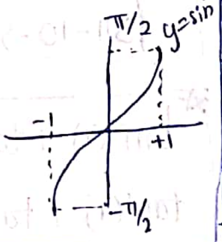
G.S $= \boxed{2n\pi \pm \theta}$

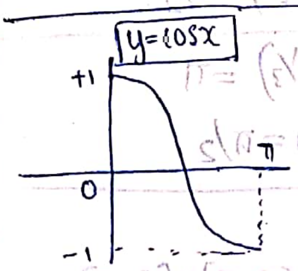
★ INVERSE TRIGONOMETRY ★

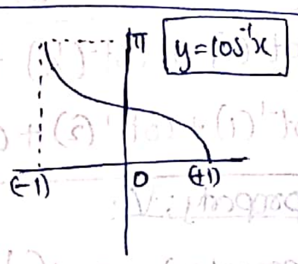
GRAPHS

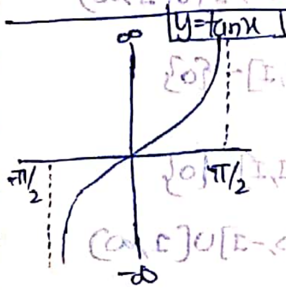


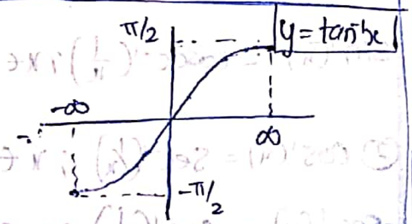


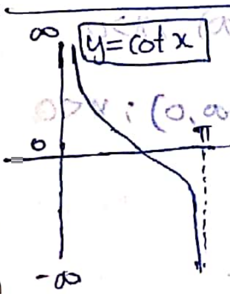


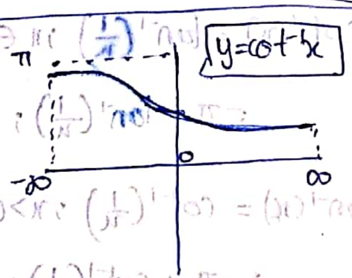












property-1 :

① $\sin[\sin^{-1}x] = x$; $x \in [-1,1]$

② $\cos[\cos^{-1}x] = x$; $x \in [-1,1]$

③ $\tan[\tan^{-1}x] = x$; $x \in [R]$

④ $\cot[\cot^{-1}x] = x$; $x \in [R]$

⑤ $\sec[\sec^{-1}x] = x$; $x \in (-\infty,-1] \cup [1,\infty)$

⑥ $\text{cosec}[\text{cosec}^{-1}x] = x$; $x \in (-\infty,-1] \cup [1,\infty)$.

Domain & Ranges

	Domain	Range
$\sin^{-1}x$	$[-1,1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1}x$	$R = (-\infty,\infty)$	$[-\pi/2, \pi/2]$
$\cot^{-1}x$	$(-\infty,\infty)$	$(0,\pi)$
$\sec^{-1}x$	$R-(-1,1)$	$[0,\pi]-\{\pi/2\}$
$\text{cosec}^{-1}x$	$R-(-1,1)$	$[-\pi/2,\pi/2]-\{0\}$

Notations :

→ $\text{Arc}(\sin x) = \sin^{-1}x$

→ Inverse of $(\sin x) = \sin^{-1}x$

property-II :

→ $\sin^{-1}x$, $\tan^{-1}x$, $\text{cosec}^{-1}x$ → odd functions

→ $\cos^{-1}x$, $\cot^{-1}x$, $\sec^{-1}x$ → neither even nor odd

$\sin^{-1}(-x) = -\sin^{-1}x$; $x \in [-1,1]$

$\tan^{-1}(-x) = -\tan^{-1}(x)$; $x \in R$.

$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$; $x \in (-\infty,-1] \cup [1,\infty)$

$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$; $x \in [-1,1]$

$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$; $x \in R$

$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$; $x \in (-\infty,-1] \cup [1,\infty)$

property : III :

$\sin^{-1}(x) + \cos^{-1}(x) = \pi/2$ $[x \in (-1,1)]$

$\tan^{-1}(x) + \cot^{-1}(x) = \pi/2$ $[x \in R]$

$\sec^{-1}(x) + \text{cosec}^{-1}(x) = \pi/2$ $[x \in (-\infty,-1] \cup [1,\infty)]$

★ $\sin^{-1}(x) + \sin^{-1}(y) = \theta$; then

$\cos^{-1}(x) + \cos^{-1}(y) = \boxed{\pi - \theta}$

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property-IV :

$\sin^{-1}(\sin\theta)=\theta$ $\{\theta\in[-\pi/2,\pi/2]\}$

$\cos^{-1}(\cos\theta)=\theta$ $\{\theta\in[0,\pi]\}$

$\tan^{-1}(\tan\theta)=\theta$ $\{\theta\in(-\pi/2,\pi/2)\}$

$\cot^{-1}(\cot\theta)=\theta$ $\{\theta\in(0,\pi)\}$

$\sec^{-1}(\sec\theta)=\theta$ $\{\theta\in([0,\pi]-\{\pi/2\})\}$

$\mathrm{cosec}^{-1}(\mathrm{cosec}\,\theta)=\theta$ $\{\theta\in[-\pi/2,\pi/2]-\{0\}\}$

Note: If θ is not in specified range, convert into $\boxed{\pi-\theta\,/\,\pi/2-\theta}$ .. etc into specified range

Eg: $\cos^{-1}(\cos(5\pi/3))$ $\boxed{5\pi/3=300}$

for $\cos^{-1}x \Rightarrow$ range $=[0,\pi]$

$\therefore \cos^{-1}\left[\cos\left(2\pi-\frac{5\pi}{3}\right)\right]$

$\cos^{-1}[\cos(\pi/3)]$

$=\pi/3$

$\sin^{-1}(\sin 1)=1$ Range of $\sin^{-1}x=[-\pi/2,\pi/2]$

$\sin^{-1}(\sin 2)=\pi-2$ $(=0.14)$ $=\left[\frac{-3.14}{2},\frac{3.14}{2}\right]$

$\vdots$ $\vdots$ $\vdots$ $[-1.57,1.57]$

$\sin^{-1}(\sin 4)=\pi-4$ $(=0.14)$

$\cos^{-1}(\cos 1)=1$ Range of $\cos^{-1}x=[0,\pi]$ $\frac{3.14}{2}$

$\cos^{-1}(\cos 2)=2$ $=(0\text{ to }3.14)$

$\cos^{-1}(\cos 3)=3$

$\cos^{-1}(\cos 4)=(2\pi-4)$

$\tan^{-1}(\tan 1)=1$ Range of $\tan^{-1}(x)$

$\tan^{-1}(\tan 2)=\pi-2$ $\Rightarrow(-\pi/2,\pi/2)$

$\tan^{-1}(\tan 3)=\pi-3$ $\Rightarrow(-1.57,1.57)$

Note: $\boxed{\text{Take approximate to range in problems.}}$

Eg If $x=\cos^{-1}(\cos 10)$; $y=\sin^{-1}(\sin 10)$ then $y-x\Rightarrow$.

$\cos^{-1}(\cos 10)=\underline{4\pi-10}$ $(0,3.14)$

$\sin^{-1}(\sin 10)=\underline{3\pi-10}$ $(-1.57,1.57)$

$\boxed{3\pi-10-4\pi+10=-\pi}$

★ $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)=\pi$

$\tan^{-1}(1)+\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)=\pi/2$

$\cot^{-1}(1)+\cot^{-1}(1/2)+\cot^{-1}(1/3)=\pi$

$\cot^{-1}(1)+\cot^{-1}(2)+\cot^{-1}(3)=\pi/2$

---

property: V :

(1) $\mathrm{cosec}^{-1}(x)=\sin^{-1}\left(\frac{1}{x}\right)$; $x\in(-\infty,-1]\cup[1,\infty)$

$\sin^{-1}(x)=\mathrm{cosec}^{-1}\left(\frac{1}{x}\right)$; $x\in[-1,1]-\{0\}$

(2) $\cos^{-1}(x)=\sec^{-1}(1/x)$; $x\in[-1,1]-\{0\}$

$\sec^{-1}(x)=\cos^{-1}\left(\frac{1}{x}\right)$; $x\in(-\infty,-1]\cup[1,\infty)$

(3) $\cot^{-1}(x)=\tan^{-1}\left(\frac{1}{x}\right)$; $x\in(0,\infty)$; $x>0$

$=\pi+\tan^{-1}\left(\frac{1}{x}\right)$; $x\in(-\infty,0)$; $x<0$

(4) $\tan^{-1}(x)=\cot^{-1}\left(\frac{1}{x}\right)$; $x>0$

$=-\pi+\cot^{-1}\left(\frac{1}{x}\right)$; $x<0$.

---

property: VI

$\tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left[\frac{x+y}{1-xy}\right]$; $x>0; y>0$ $\boxed{xy<1}$

$=\pi+\tan^{-1}\left[\frac{x+y}{1-xy}\right]$ $x>0, y>0$; $\boxed{xy>1}$

• $\sin^{-1}x + \sin^{-1}(y) = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$.

$x>0, y>0$; $\boxed{x^2+y^2 \le 1}$

• $\sin^{-1}x - \sin^{-1}(y) = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$; $x>0; y>0$

• $\sin^{-1}(x) + \sin^{-1}(y) = \pi - \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$

; $x>0, y>0$; $\boxed{x^2+y^2>1}$

$\cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}\left[xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]$

$\boxed{x>0, y>0}$

$\cos^{-1}(x) - \cos^{-1}(y) = \cos^{-1}\left[xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]$; $x>0, y>0$ $\boxed{x<y}$

$= -\cos^{-1}\left[xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]$; $x>0; y>0$ $\boxed{x>y}$

$\tan^{-1}\left[\frac{x}{y}\right] + \tan^{-1}\left[\frac{y-x}{y+x}\right] = \pi/4$

$\tan^{-1}\left[\frac{x}{y}\right] - \tan^{-1}\left[\frac{x-y}{x+y}\right] = \pi/4$

property-7:

$2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$; $\boxed{\frac{-1}{\sqrt2} < x < \frac{1}{\sqrt2}}$

$2\cos^{-1}(x) = \cos^{-1}(2x^2-1)$; $0 \le x \le 1$

$2\tan^{-1}(x) = \tan^{-1}\left[\frac{2x}{1-x^2}\right]$; $\boxed{-1<x<1}$

$= \pi + \tan^{-1}\left[\frac{2x}{1-x^2}\right]$; $\boxed{1<x<\infty}$

$3\sin^{-1}(x) = \sin^{-1}(3x-4x^3)$; $\boxed{\frac{-1}{2} \le x \le \frac{1}{2}}$

$3\cos^{-1}(x) = \cos^{-1}(4x^3-3x)$; $\frac{1}{2} \le x \le 1$

$3\tan^{-1}(x) = \cancel{\sin}\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$; $\frac{-1}{\sqrt3} < x < \frac{1}{\sqrt3}$

$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$; $-1 \le x \le 1$

$= \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$; $x>0$

$= -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$; $x<0$.

$\tan^{-1}\left[\frac{a}{x}\right] + \tan^{-1}\left[\frac{b}{x}\right] = \pi/2$

then $x = \sqrt{ab}$.

• $\sin^{-1}\left[\frac{a}{x}\right] + \sin^{-1}\left[\frac{b}{x}\right] = \pi/2$; $\boxed{x = \sqrt{a^2+b^2}}$

PROPERTIES OF △les

$\boxed{\text{Sum of sides in } \triangle le} > \boxed{\text{Third side}}$

$\boxed{\text{diff of two sides in } \triangle le} < \boxed{\text{third side}}$

Perimeter: $(2s) = \boxed{a+b+c}$

$s = \boxed{\text{semiperimeter}}$

$$2(s-a) = b+c-a$$
$$2(s-b) = a+c-b$$
$$2(s-c) = a+b-c$$

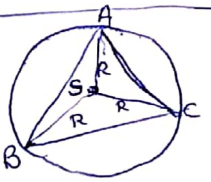
Area: $(\Delta) = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= 2R^2 \sin A \cdot \sin B \cdot \sin C$$
$$= \frac{1}{2} ab \sin C$$
$$= \frac{1}{2} bc \sin A$$
$$= \frac{1}{2} ca \sin B$$
$$= \frac{abc}{4R}$$
$$= r \cdot s$$

Circumradius $(R) = \frac{abc}{4\Delta}$

$\boxed{S \Rightarrow \text{Circumcenter}}$

↳ equidistant from $\boxed{\text{3-vertices}}$



Inradius $(r) = \frac{\Delta}{s}$

$$= 4R \sin A/2 \cdot \sin B/2 \cdot \sin C/2$$
$$= \frac{a}{\cot \frac{B}{2} + \cot \frac{C}{2}}$$
$$= \frac{b}{\cot \frac{A}{2} + \cot \frac{C}{2}}$$
$$= \frac{c}{\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right)}$$
$$= (s-a) \tan A/2$$
$$= (s-b) \tan B/2$$
$$= (s-c) \tan C/2$$

Exradius:

$r_1 \longrightarrow$ opp to $\angle A$
$r_2 \longrightarrow$ opp to $\angle B$
$r_3 \longrightarrow$ opp to $\angle C$

$r_1 = \frac{\Delta}{s-a}$	$r_2 = \frac{\Delta}{s-b}$	$r_3 = \frac{\Delta}{s-c}$
$4R \sin A/2 \cdot \cos B/2 \cdot \cos C/2$	$4R \sin B/2 \cdot \cos A/2 \cdot \cos C/2$	$4R \sin C/2 \cdot \cos A/2 \cdot \cos B/2$
$= \frac{a}{\tan \frac{B}{2} + \tan \frac{C}{2}}$	$\frac{b}{\tan A/2 + \tan C/2}$	$\frac{c}{\tan(A/2) + \tan \frac{B}{2}}$
$= s \tan A/2$	$s \tan B/2$	$s \tan C/2$
$= (s-b) \cot C/2$	$(s-c) \cdot \cot A/2$	$(s-a) \cot B/2$

Standard Results:

$r_1 + r_2 = 4R \cos^2 \frac{C}{2}$	$r_1 - r = 4R \sin^2 A/2$
$r_2 + r_3 = 4R \cos^2 A/2$	$r_2 - r = 4R \sin^2 B/2$
$r_3 + r_1 = 4R \cos^2 B/2$	$r_3 - r = 4R \sin^2 C/2$

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$$

- $\sqrt{r \cdot r_1 \cdot r_2 \cdot r_3} = \Delta$
- $r_1 r_2 + r_2 r_3 + r_3 r_1$ (or) $\sum_{cyc} r_1 r_2 = s^2$
- $r(r_1 + r_2 + r_3) + s^2 = ab + bc + ca$
- $r_1 + r_2 + r_3 - r = 4R$
- $r_2 + r_3 + r - r_1 = 4R \cos A$
- $r_3 + r_1 + r - r_2 = 4R \cos B$
- $r_1 + r_2 + r - r_3 = 4R \cos C$
- $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$
- $\sin A + \sin B + \sin C = \frac{s}{R}$
- $\sum a \cos A = \frac{2\Delta}{R}$

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- $\sum a\cot A = 2(r+R)$
- $\sum \cos A = \dfrac{a^2+b^2+c^2}{4\Delta}$

SINE RULE

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R}$$

$$\boxed{\begin{aligned} a &= 2R\sin A \\ b &= 2R\sin B \\ c &= 2R\sin C \end{aligned}}$$

$a:b:c = \sin A : \sin B : \sin C$

$\sin A : \sin B : \sin C = a:b:c.$

- $\sin A + \sin B + \sin C = s/R$

→ angle opposite to smallest side → $\boxed{\text{smallest angle}}$

→ angle opposite to longest side → $\boxed{\text{Greatest angle}}$

COSINE RULE

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos A : \cos B : \cos C = a(b^2+c^2-a^2) : b(a^2+c^2-b^2) : c(a^2+b^2-c^2).$$

$$\sum bc\cos A = \frac{a^2+b^2+c^2}{2}$$

$$\sum \frac{\cos A}{a} = \frac{a^2+b^2+c^2}{2abc}$$

Tangent Rule:

$$\tan\left[\frac{B-C}{2}\right] = \frac{b-c}{b+c}\cot A/2$$

$$\tan\left[\frac{A-B}{2}\right] = \frac{a-b}{a+b}\cdot\cot\frac{C}{2}$$

$$\tan\left[\frac{C-A}{2}\right] = \frac{c-a}{c+a}\cot B/2$$

projection Rule

$a = b\cos C + \cos B . c$

$b = \cancel{\cos}\ c\cos A + a\cos C$

$c = a\cos B + b\cos A$

Half angle formulae:

$$\sin A/2 = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad \cos A/2 = \sqrt{\frac{s(s-a)}{bc}}$$

$$\sin B/2 = \sqrt{\frac{(s-a)(s-c)}{ac}} \qquad \cos B/2 = \sqrt{\frac{s(s-b)}{ac}}$$

$$\sin C/2 = \sqrt{\frac{(s-b)(s-a)}{ab}} \qquad \cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\tan A/2 = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{(s-b)(s-c)}{\Delta} = \frac{\Delta}{s(s-a)}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} = \frac{(s-a)(s-c)}{\Delta} = \frac{\Delta}{s(s-b)}$$

$$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{(s-a)(s-b)}{\Delta} = \frac{\Delta}{s(s-c)}$$

$$\cot\frac{A}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} = \frac{\Delta}{(s-b)(s-c)} = \frac{s(s-a)}{\Delta}$$

$$\cot\frac{B}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}} = \frac{\Delta}{(s-a)(s-c)} = \frac{s(s-b)}{\Delta}$$

$$\cot\frac{C}{2} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = \frac{\Delta}{\cancel{s}(s-a)(s-b)} = \frac{s(s-c)}{\Delta}$$

$$\cot A = \frac{b^2+c^2-a^2}{4\Delta}$$

$$\cot B = \frac{a^2+c^2-b^2}{4\Delta}$$

$$\cot C = \frac{a^2+b^2-c^2}{4\Delta}$$

Mollweide rule:

$$\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$$

$$\frac{b+c}{a} = \frac{\cos\left(\frac{B-C}{2}\right)}{\sin(A/2)}$$

$$\frac{a+c}{b} = \frac{\cos\left(\frac{A-C}{2}\right)}{\sin\frac{B}{2}}$$

$$\frac{a-b}{c} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}$$

$$\frac{b-c}{a} = \frac{\sin\left(\frac{B-C}{2}\right)}{\cos(A/2)}$$

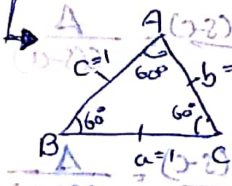
$$\frac{a-c}{b} = \frac{\sin\left(\frac{A-C}{2}\right)}{\cos B/2}$$

$\boxed{\text{Model-1}}$

→ Logic

→ By degree

→ $\begin{bmatrix} a & b & c \\ r_1 & r_2 & r_3 \\ A & B & C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \sqrt{3}/2 \\ 60^\circ & 60^\circ & 60^\circ \end{bmatrix}$



$s = \frac{3}{2}$ $\quad R = \frac{1}{\sqrt{3}}$

$\Delta = \frac{\sqrt{3}}{4}$ $\quad r = \frac{1}{2\sqrt{3}}$

$\boxed{\frac{r}{R} = 1/2}$

$\boxed{r : R : r_1 = 1:2:3}$

$\boxed{\text{Model:2}}$

Tip: • If some ratio given and asked to find sides ratio $(a:b:c)$ → $\boxed{\text{use Trick}}$

• If sides ratio given, asked to find other ratio → $\boxed{\text{Use Formula}}$

→ if $2r_1 : r_2 : 3r_3$ then $a:b:c = \underline{\quad ? \quad}$

$a = \text{Coeff } r_2 + \text{coeff. } r_3 \Rightarrow 4$

$b = \text{Coeff } r_1 + \text{coeff } r_3 \Rightarrow 5$

$c = \text{Coeff. } r_1 + \text{Coeff } r_2 = 3$

$\therefore a:b:c = 4:5:3.$

→ $r_1 : r_2 : r_3 = 1:2:3$ then $a:b:c = \underline{\quad}$

$a = 2+3 = 5$

$b = 3+1 = 4 \quad \therefore a:b:c = 5:4:3$

$c = 2+1 = 3$

$= (5\times1):(4\times2):(3\times3)$

$5:8:9$

→ $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = 1:4:15$

$a:b:c = ?$

$a = 15+4 = 19$

$b = 15+1 = 16 \quad \therefore a:b:c = 19:16:5.$

$c = 4+1 = 5$

$\cot A : \cot B : \cot C = 30:19:6$ then $a:b:c = \underline{\quad}$

$a = \sqrt{19+6} = 5$

$b = \sqrt{30+6} = 6 \quad \therefore (a:b:c = 5:6:7)$

$c = \sqrt{30+19} = 7$

$\tan A : \tan B : \tan C = 1:2:3$, then $a:b:c$

$\boxed{\text{Convert into } \cancel{\text{to}} \text{ cot ratios}}$

$\cot A : \cot B : \cot C = 1 : \frac{1}{2} : \frac{1}{3}$

$a:b:c = \sqrt{\frac{1}{2}+\frac{1}{3}} : \sqrt{1+\frac{1}{3}} : \sqrt{1+\frac{1}{2}}$

$\left(\sqrt{\frac{5}{6}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{3}{2}}\right)\sqrt{6}.$

$: \boxed{\sqrt{5} : 2\sqrt{2} : 3}$

$\sin A : \sin B : \sin C = a:b:c$

$a:b:c = \sin A : \sin B : \sin C$

$a:b:c = 7:8:9;$ $\cos A : \cos B : \cos C = ?$

$\cos A : \cos B : \cos C = a(b^2+c^2-a^2) : b(a^2+c^2-b^2) : c(a^2+b^2-c^2)$

$= \underline{\qquad}$

$a:b:c = 4:5:6$ the $r:R = \underline{\qquad}$

$\frac{r}{R} = \frac{(b+c-a)(a+c-b)(a+b-c)}{2abc}$

$= \frac{15}{28}$

$a:b:c = 5:8:9$, then $r_1 : r_2 : r_3$

$= \frac{1}{b+c-a} : \frac{1}{a+c-b} : \frac{1}{a+b-c}$

$= \left(\frac{1}{12} : \frac{1}{6} : \frac{1}{4}\right) 12$

$= \boxed{1:2:3}$

$a:b:c = 19:16:5$ then $\cot A/2 : \cot B/2 : \cot C/2$

$= \frac{s(s-a)}{\Delta} : \frac{s(s-b)}{\Delta} : \frac{s(s-c)}{\Delta}$

$= s-a : s-b : s-c$

$= 21 : 24 : 36$

$\boxed{7:8:12}$ //

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if $a:b:c = 5:6:7$; the $\cot A : \cot B : \cot C$

$$= \frac{b^2+c^2-a^2}{4\Delta} ; \frac{a^2+c^2-b^2}{4\Delta} ; \frac{a^2+b^2-c^2}{4\Delta}$$

$= b^2+c^2-a^2 ; a^2+c^2-b^2 ; a^2+b^2-c^2$

$= 36+49-25 ; 25+49-36 ; 25+36-49$

$= 60 : 38 : 12$

$= \boxed{30 : 19 : 6}$

$a^2 = 25$, $b^2 = 36$, $c^2 = 49$

Model : 3

$\boxed{r_1 r_2 r_3}$ → are given & asked to find other parameters:

$$r = \frac{1}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}$$

$$\Delta = \sqrt{r \cdot r_1 r_2 r_3}$$

$$S = \sqrt{\Sigma r_1 r_2} = \sqrt{\frac{r_1 r_2 r_3}{r}}$$

$$\frac{r_1+r_2+r_3-r}{4} = R$$

$$a = \sqrt{(r_2+r_3)(r_1-r)}$$

$$b = \sqrt{(r_1+r_3)(r_2-r)}$$

$$c = \sqrt{(r_1+r_2)(r_3-r)}$$

$$\angle A = 2\tan^{-1}\sqrt{\frac{r r_1}{r_2 r_3}} \Rightarrow \sqrt{\frac{r r_1}{r_2 r_3}} = \tan A/2$$

$$\angle B = 2\tan^{-1}\sqrt{\frac{r r_2}{r_1 r_3}} \Rightarrow \sqrt{\frac{r r_2}{r_1 r_3}} = \tan B/2$$

$$\angle C = 2\tan^{-1}\sqrt{\frac{r r_3}{r_1 r_2}} \Rightarrow \sqrt{\frac{r r_3}{r_1 r_2}} = \tan C/2.$$

when a, b, c, given & asked to find other parameters

$$S = \frac{a+b+c}{2} \qquad r_1 = \frac{\Delta}{s-a}$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \qquad r_2 = \frac{\Delta}{s-b}$$

$$r = \Delta/s \qquad r_3 = \frac{\Delta}{s-c}$$

$$R = \frac{abc}{4\Delta}$$

$$\angle A = \cos^{-1}\left[\frac{b^2+c^2-a^2}{2bc}\right]$$

$$\angle B = \cos^{-1}\left[\frac{a^2+c^2-b^2}{2ac}\right]$$

$$\angle C = \cos^{-1}\left[\frac{a^2+b^2-c^2}{2ab}\right]$$

★ **Note:** If sides of Δle are right angle Δles:

i) Area $= \frac{1}{2} \times b \times h$

ii) Circumradius $(R) = \frac{\text{max. side}}{2} = \frac{\text{Hypo.}}{2}$

Sides of the Δle:

(i) $n = \text{odd}$

$n, \frac{n^2+1}{2}, \frac{n^2-1}{2}$

(ii) $n = \text{even}$ $\downarrow 2(n) \to n^2+1, n^2-1$

Eg: $8 = 2 \times (4)$ $\boxed{8, 17, 15}$

(iii) $m, n, \sqrt{m^2+n^2}$

If sides of Δle are given, then

i) $S = \frac{3}{2}(\text{side})$	(iv) $R = \frac{\text{side}}{\sqrt{3}}$
ii) Area $(\Delta) = \frac{\sqrt{3}}{4}(\text{side})^2$	(v) exradius: $r_1 = r_2 = r_3 = \frac{\sqrt{3}}{2}$
iii) Inradius $(r) = \frac{\text{side}}{2\sqrt{3}}$	(vi) Greatest angle $= 60°$

★ $\boxed{\text{lle } \Delta\text{le Trick}}$ ★

$\boxed{\text{Property-1}}$: $\boxed{a=3 \quad b=4 \quad c=5}$

a	b	c
3	4	5

$\boxed{S = \Delta = 6}$; $R = \frac{\text{max. side}}{2} = \frac{5}{2}$.

$\boxed{r=1}$

$\boxed{r_1 = 2 \mid r_2 = 3 \mid r_3 = 6}$

$\boxed{\text{property: 2}}$

$\boxed{\text{All angles known}}$

$A : B : C \Rightarrow 30 : 60 : 90$

$a : b : c \Rightarrow 1 : \sqrt{3} : 2$

$\Delta = \frac{\sqrt{3}}{2}$; $R = 1$.

Note! 1 If sides of Δle are 3, 5, 7,

a	b	c
3	5	7

$\boxed{\text{Greatest angle} = 120°}$

$\boxed{\text{obtuse angled } \Delta\text{le}}$

Note: 2: $a : b : c \Rightarrow 1 : 1 : \sqrt{2}$

$A : B : C \Rightarrow 45 : 45 : 90$

$1 : 1 : 2$

$\boxed{\text{lled Isosceles } \Delta\text{le}}$

[Model-4] [A.P/H.P/G.P]

A.P:

$t_n = a+(n-1)d$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\boxed{\text{each term} \times k \Rightarrow d \times k}$
$\boxed{\text{each term} \div k \Rightarrow d \div k}$

if a, b, c are in A.P, then $\boxed{b = \frac{a+c}{2}}$

H.P: $\frac{1}{A.P} = \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$

$\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \boxed{b = \frac{2ac}{c+a}}$

G.P:

$t_n = a r^{n-1}$

each term $\times k \Rightarrow r$ same
each term $\div k \Rightarrow r$ same

$S_n = \frac{a(r^n-1)}{r-1}$; $r>1$

$\boxed{S_n = n \times a}$; $\boxed{r=1}$

$S_n = \frac{a(1-r^n)}{1-r}$; $r<1$.

a, b, c are in G.P; then $\boxed{b^2 = 4ac}$ → Geometric mean.
$\boxed{b = \sqrt{ac}}$

Arithmetic ~~and~~ Geometric progression

$t_n = [a+(n-1)d]\, b r^{n-1}$

[Model: 5] [Standards]

$\frac{a}{b+c} + \frac{c}{a+b} = 1 \Rightarrow \angle B = 60°$

$\frac{b}{a+c} + \frac{c}{a+b} = 1 \Rightarrow \angle A = 60°$

$\frac{a}{b+c} + \frac{b}{a+c} = 1 \Rightarrow \angle C = 60°$

$\frac{1}{a+b} + \frac{1}{c+a} = \frac{3}{a+b+c} \Rightarrow \angle A = 60°$

$\frac{1}{a+b} + \frac{1}{b+c} = \frac{3}{a+b+c} \Rightarrow \angle B = 60°$

$\frac{1}{a+c} + \frac{1}{c+b} = \frac{3}{a+b+c} \Rightarrow \angle C = 60°$

$b+c = 2a\cos\left[\frac{B-C}{2}\right] \Rightarrow \angle A = 60°$

$a+c = 2b\cos\left[\frac{A-C}{2}\right] \Rightarrow \angle B = 60°$

$a+b = 2c\cos\left[\frac{A-B}{2}\right] \Rightarrow \angle C = 60°$

$a^2+b^2-c^2 = ab \Rightarrow \angle C = 60°$

$a^2+c^2-b^2 = ac \Rightarrow \angle B = 60°$

$b^2+c^2-a^2 = bc \Rightarrow \angle A = 60°$

$(a+b+c)(a+b-c) = 3ab \Rightarrow \angle C = 60°$

$(a+b+c)(a+c-b) = 3ac \Rightarrow \angle B = 60°$

$(a+b+c)(b+c-a) = 3bc \Rightarrow \angle A = 60°$

$\frac{a}{b^2-c^2} + \frac{c}{b^2-a^2} = 0 \Rightarrow \angle B = 60°$

$\frac{a}{c^2-b^2} + \frac{b}{c^2-a^2} = 0 \Rightarrow \angle C = 60°$

$\frac{b}{a^2-c^2} + \frac{c}{a^2-b^2} = 0 \Rightarrow \angle A = 60°$

[Model-6.] [90°]

$r r_1 = r_2 r_3 \Rightarrow \angle A = 90°$

$r r_2 = r_1 r_3 \Rightarrow \angle B = 90°$

$r r_3 = r_2 r_1 \Rightarrow \angle C = 90°$

$r_1 - r = 2R \Rightarrow \angle A = 90°$

$r_2 - r = 2R \Rightarrow \angle B = 90°$

$r_3 - r = 2R \Rightarrow \angle C = 90°$

$r_1 - r = r_2 + r_3 \Rightarrow \angle A = 90°$

$r_2 - r = r_1 + r_3 \Rightarrow \angle B = 90°$

$r_3 - r = r_1 + r_2 \Rightarrow \angle C = 90°$

$\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right) = 2 \Rightarrow \angle A = 90°$

$\left(1-\frac{r_2}{r_1}\right)\left(1-\left(\frac{r_2}{r_3}\right)\right) = 2 \Rightarrow \angle B = 90°$

$\left(1-\frac{r_3}{r_1}\right)\left(1-\frac{r_3}{r_2}\right) = 2 \Rightarrow \angle C = 90°$

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$b+c = 2(r+R) \Rightarrow \angle A = 90^\circ$

$a+c = 2(r+R) \Rightarrow \angle B = 90^\circ$

$a+b = 2(r+R) \Rightarrow \angle C = 90^\circ$

$\boxed{\text{Model-7}} \Rightarrow \boxed{120^\circ}$

$rr_1 = (r_2 r_3)3 \Rightarrow \angle A = 120^\circ$

$rr_2 = (r_1 r_3)3 \Rightarrow \angle B = 120^\circ$

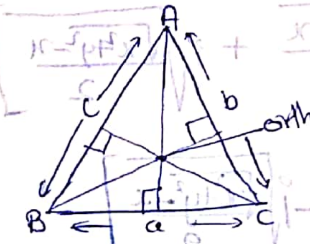
$rr_3 = 3 r_1 r_2 \Rightarrow \angle C = 120^\circ$

$(a+b+c)(a+b-c) = ab \Rightarrow \angle C = 120^\circ$

$(a+b+c)(a+c-b) = ac \Rightarrow \angle B = 120^\circ$

$(a+b+c)(b+c-a) = bc \Rightarrow \angle A = 120^\circ$

Model-8: $\boxed{\text{Altitudes of } \Delta le}$



$\Delta = \frac{1}{2}aP_1 = \frac{1}{2}bP_2 = \frac{1}{2}cP_3$

$P_1 = \frac{2\Delta}{a}$; $P_2 = \frac{2\Delta}{b}$; $P_3 = \frac{2\Delta}{c}$

$\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} = \frac{a+b+c}{2\Delta} = \frac{2s}{2\Delta} = \frac{s}{\Delta} = \frac{1}{r}$

$\frac{1}{P_2} + \frac{1}{P_3} - \frac{1}{P_1} = \frac{b+c-a}{2\Delta} = \frac{1}{r_1}$

$\frac{1}{P_1} + \frac{1}{P_3} - \frac{1}{P_2} = \frac{a+c-b}{2\Delta} = \frac{1}{r_2}$

$\frac{1}{P_1} + \frac{1}{P_2} - \frac{1}{P_3} = \frac{a+b-c}{2\Delta} = \frac{1}{r_3}$

EAMCET:

* $b+c = 3a$; then $\cot\frac{B}{2} \cdot \cot\frac{C}{2} = \frac{3+1}{3-1} = \frac{4}{2} = 2$

$\boxed{\text{convert into tan}}$

if $a+b = kc$; then $\tan A/_2 \cdot \tan B/_2 = \frac{k-1}{k+1}$

if $b+c = ka$, then $\tan B/_2 \cdot \tan C/_2 = \frac{k-1}{k+1}$

if $c+a = bk$ then $\tan\frac{C}{2} \cdot \tan A/_2 = \frac{k-1}{k+1}$

* $b^2+c^2 = 2012a^2$ then $\frac{\cot B + \cot C}{\cot A} = \frac{2}{2011}$

→ $a^2+b^2 = kc^2$, then $\frac{\cot A + \cot B}{\cot C} = \frac{2}{k-1}$

→ $b^2+c^2 = ka^2$ then $\frac{\cot B + \cot C}{\cot A} = \frac{2}{k-1}$

→ $c^2+a^2 = kb^2$ then $\frac{\cot C + \cot A}{\cot B} = \frac{2}{k-1}$

In Δle ABC, $\sum\left(\frac{\sin^2 A + \sin A + 1}{\sin A}\right)$ is always 9

$\sum\left(\sin A + \frac{1}{\sin A} + 1\right)$

$\boxed{AM \geq GM}$

$\frac{\sin A + \frac{1}{\sin A}}{2} \geq \sqrt{\sin A \cdot \frac{1}{\sin A}}$

$\boxed{\sin A + \frac{1}{\sin A} \geq 2}$

$\sum(2+1) \Rightarrow \sum 3$

$\Rightarrow 3+3+3 = 9$

if $A_1 A_2 A_3$ are areas of incircle and excircle then $\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}} = \_\_\_$

$= \frac{1}{\sqrt{\pi r_1^2}} + \frac{1}{\sqrt{\pi r_2^2}} + \frac{1}{\sqrt{\pi r_3^2}}$

$= \frac{1}{\sqrt{\pi}}\left[\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right]$

$= \frac{1}{\sqrt{\pi}}\left[\frac{1}{r}\right]$

$= \frac{1}{\sqrt{\pi r^2}} = \frac{1}{\sqrt{A}}$

: COMPLEX NUMBERS : (C.N)

$\boxed{i \to \text{iota}}$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1 \quad [i^2 \cdot i^2 = (-1)\times(-1) = 1]$

$i^5 = i$

$i^6 = (i^2)^3 = (-1)^3 = -1$

$\frac{1}{i} = -i \;;\; \frac{1}{-i} = i$

$i^{-3} = \frac{1}{i^3} = \frac{-i^2}{i^3} = \frac{-1}{i}$

$i^{4n} = 1$

$i^{4n+1} = i$

$i^{4n+2} = -1$

$i^{4n+3} = -i$

$\frac{1+i}{1-i} = i \qquad \frac{1-i}{1+i} = -i$

$\frac{1}{a-ib} = \frac{a+ib}{a^2+b^2}$

$\frac{1}{a+ib} = \frac{a-ib}{a^2+b^2}$

Tips:

① $i^{77} + i^{204} + i^{-67} + i^{-5} =$ (n ÷ 4)

$i^{n \geq 4} \Rightarrow \frac{n}{4} \Rightarrow (R)$

4)77(19 — 4, 37, 36, (1)

4)67(16 — 4, 27, 24

$= i^1 + i^0 + i^{-3} + i^{-1}$

$\frac{1}{i^3} = \frac{1}{-i}$

$= i + 1 + \cancel{\left(\frac{-1}{i}\right)} + \cancel{\frac{1}{i}}$

$= i + 1$

$i^p \cdot i^q \cdot i^r \cdot i^s = -1$	$i^{0!} = i$	
$i^p + i^q + i^r + i^s = 0$	$(1+i)^2 = 2i$	$(1-i)^2 = -2i$

$1 + i^2 + i^4 + i^6 + i^8 + \dots + (2n+1) \text{ terms} = \boxed{1}$

$1 + i^2 + i^4 + i^6 + \dots + i^{2n}$
- n = even ⇒ $\boxed{1}$
- n = odd ⇒ $\boxed{0}$

Algebraic operations

$z_1 = a+ib$
$z_2 = c+id$

$z_1 \times z_2 = (ac - bd) + i(bc + ad)$

$\frac{z_1}{z_2} = \frac{ac + bd + i(bc - ad)}{c^2 + d^2}$

Additive Inverse of C.N :

$-z$ is additive Inverse of z $\quad (z = a+ib)$

$\boxed{-z = -a - ib}$

• Conjugate of C.N ($\bar{z}$) :

reflection of z about real axis (x-axis)

$\boxed{-x+iy} \updownarrow \boxed{-x-iy}$ $\qquad$ $\boxed{x+iy} \updownarrow \boxed{x-iy}$

$z = a+ib$
$\bar{z} = a-ib$

$|z| = \sqrt{(R.P)^2 + (I.P)^2} \qquad \boxed{z = x+iy}$

$|z| = \sqrt{x^2+y^2}$

modulus of C.N ⟶ distance of a point from (0,0)

Square root of C.N : ($\sqrt{z}$)

$$\sqrt{x+iy} = \pm\left[\sqrt{\frac{\sqrt{x^2+y^2}+x}{2}} + i\sqrt{\frac{\sqrt{x^2+y^2}-x}{2}}\right]$$

$$\sqrt{x-iy} = \pm\left[\sqrt{\frac{\sqrt{x^2+y^2}+x}{2}} - i\sqrt{\frac{\sqrt{x^2+y^2}-x}{2}}\right]$$

$(a+ib)^2 = (a^2-b^2) + i(2ab)$

$(a-ib)^2 = (a^2-b^2) - i(2ab)$

$(a+ib)^3 = (a^3-3ab^2) + i(3a^2b - b^3)$

$(a-ib)^3 = (a^3-3ab^2) - i(3a^2b - b^3)$

Trick

① $\sqrt{8-6i} \Rightarrow \frac{6}{2} = 3 \to 3, 1$

$1^2 - 3^2 = -8$

$\boxed{3^2 - 1^2 = 8}$

$\pm \boxed{3 - i}$

② $\sqrt{-9 + 40i}$

$\frac{40}{2} = 20 \to 4, 5$

$\boxed{4^2 - 5^2 = -9}$

$\pm [4 + 5i]$

Multiplicative Inverse of z $(x+iy)$ =

$\frac{1}{z} = \frac{\bar{z}}{|z|^2} = \frac{x-iy}{x^2+y^2}$

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PART-2

$\boxed{z^2 - (2\times R.P) + |z|^2}$ → form of quadratic equation

Eg: $z = 3+i$; then $z^2 - 6z + 9$.

$z^2 - 2(3) + 10 = 0$

$\boxed{z^2 - 6 + 9 = -1}$

Eg: $z = 3-5i$, then $z^3 - 10z^2 + 58z - 135$

	1	-10	58	-135
6	0	6	-24	0
-34	0	0	-34	136
	1	-4	0	1

1 Ans

Q.E = $z^2 - 2(3)z + 34$
$= \boxed{z^2 - 6z + 34}$

Remainder is Ans.

Eg: $x = i(i+1)$ then $x^4 + 4x^3 + 6x^2 + 4x - 5 = -5$

$x = i^2 + i$

$\boxed{x = i - 1}$

Q.E = $x^2 - 2(-1)x + 2$
$= \boxed{x^2 + 2x + 2}$

	1	4	6	4	-5
-2	0	-2	-4	0	0
-2	0	0	-2	-4	0
	1	2	0	0	-5

Sol: -5

If $z \neq \alpha$ then $\frac{z-\alpha}{z+\alpha}$ is P.I ; then $|z| = \alpha$

properties of conjugate

$\bar{\bar{z}} = z$, $\bar{\bar{\bar{z}}} = \bar{z}$ } → ∴ $\bar{z}^{\,n}$ — n-even = z; n-odd = $\bar{z}$

$z + \bar{z} = 2(R.P \text{ of } z) \Rightarrow \frac{z+\bar{z}}{2}$ | $z - \bar{z} = 2i(I.P \text{ of } z) \Rightarrow \frac{z-\bar{z}}{2i}$

$z = \bar{z} \Rightarrow z$ is real (purely real)

$z = -\bar{z} \Rightarrow z$ is purely imaginary.

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ | $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$

$\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$ | $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

$(\bar{z})^n = \overline{(z^n)}$ | $\bar{i} = -i$; $\overline{-i} = i$

$|z|^2 = z\bar{z}$ | $z_1\bar{z}_2 + z_2\bar{z}_1 = 2 \cdot R.P \text{ of } z_1\bar{z}_2 = 2 \cdot R.P \text{ of } z_2 \cdot \bar{z}_1$

properties of Modulus

$|z| = 0 \Rightarrow z = 0$

$|z| > 0$ (+ve)

$z\bar{z} = |z|^2$

$|\bar{z}| = |-\bar{z}| = |-z| = |z| = \sqrt{x^2+y^2}$

$|z|^n = |z^n|$

$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$|kz| = k|z|$

Cube Root unity

$$x^{1/3} \begin{cases} 1 \\ \frac{-1+i\sqrt{3}}{2}\ (\omega) \\ \frac{-1-i\sqrt{3}}{2}\ (\omega^2) \end{cases}$$

$\boxed{\omega = \frac{-1+i\sqrt{3}}{2}}$

$\boxed{\omega^2 = \frac{-1-i\sqrt{3}}{2}}$

$\therefore 1+\omega+\omega^2 = 0.$ (n ÷ 3)

$1.\omega.\omega^2 = \omega^3 = 1$

$\Rightarrow 1+\omega+\omega^2 = 0$ — $\boxed{1+\omega = -\omega^2}$, $\boxed{1+\omega^2 = -\omega}$, $\boxed{\omega+\omega^2 = -1.}$

$\Rightarrow \omega^3 = 1$

$\Rightarrow \omega^2 = \frac{1}{\omega}$

$\Rightarrow \omega = \frac{1}{\omega^2}$

$\boxed{1, \omega, \omega^2 \Longrightarrow \text{G.P series}}$

$|\omega| = |\omega^2| = |1+\omega| = |1+\omega^2| = |\omega+\omega^2|$

$\omega^{3n} = 1$; $\omega^{3n+1} = \omega$; $\omega^{3n+2} = \omega^2$

$\sqrt{\omega} = \pm\omega^2$; $\sqrt{\omega^2} = \pm\omega$

$\rightarrow \overline{\omega} = \omega^2$; $\overline{\omega^2} = \omega$ $\left(\frac{1}{\omega^2} = \omega\right)$

$\omega^p + \omega^q + \omega^r = 0.$ $[p, q, r \in \text{Consecutives}]$

$\omega^p.\omega^q.\omega^r = 1$ $[p, q, r \in \text{Consecutives}]$

$$\frac{1-i\sqrt{3}}{1+i\sqrt{3}} = \frac{-2\omega}{-2\omega^2} = \frac{1}{\omega} = \omega^2$$

$$\frac{1+i\sqrt{3}}{1-i\sqrt{3}} = \omega$$

$1+i\sqrt{3} = -2\omega^2$; $1-i\sqrt{3} = -2\omega$

$\sqrt{3}+i = -2\omega i$; $\sqrt{3}-i = 2\omega i$

Roots of eqn

$x^2+x+1 = 0$ — $x = \omega$, $x = \omega^2$

$x^2-x+1 = 0$ — $x = -\omega$, $x = -\omega^2$

$x^2+2x+4 = 0$ — $x = 2\omega$, $x = 2\omega^2$

$x^2-2x+4 = 0$ — $x = -2\omega$, $x = -2\omega^2$

product of n^{th} roots of unity $= (-1)^{n-1}$

* Fourth root of unity * $(x^{1/4})$

$$x = (1)^{1/4} \begin{cases} 1 \\ -1 \\ i \\ -i \end{cases}$$

- Sum of four roots $= 0.$
- product of 4 roots $= -1.$
- represents square with $a = \sqrt{2}$
- $\boxed{|1| = |-1| = |i| = |-i| = 1.}$

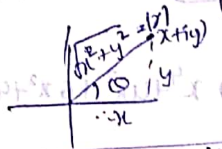
(n ÷ 4)

$\frac{1}{i} = -i \Rightarrow \frac{1}{-i} = i$

$\boxed{\text{locus}} \rightarrow$ name of curve/shape

$\rightarrow$ put $\boxed{z = x+iy}$ in given eqn

Argument / Amplitude (θ) $= \tan^{-1}\left[\frac{y}{x}\right] \therefore [z = x+iy]$



$\cos\theta = \frac{|x|}{r}$ $\therefore |x| = r\cos\theta$

$\sin\theta = \frac{|y|}{r}$ $|y| = r\sin\theta$

$z = x+iy$

$\boxed{z = r(\cos\theta + i\sin\theta)} \longrightarrow$ polar form / modulus Amplitude form

$\boxed{r = e^{i\theta}} \longrightarrow$ euler form

* Properties of Argument *

$x>0; y>0$ $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$x<0; y>0$ $= \pi - \tan^{-1}\left|\frac{y}{x}\right|$

$x<0; y<0$ $= -\pi + \tan^{-1}\left|\frac{y}{x}\right|$

$x>0; y<0$ $= -\tan^{-1}\left|\frac{y}{x}\right|$

$\rightarrow \text{Arg}(a)$; $a>0 = 0°$

$\text{Arg}(a)$; $a<0 = 180° (\pi)$

$\text{Arg}(ai)$; $a>0 = \pi/2$

$\text{Arg}(ai)$; $a<0 = -\pi/2$

$\text{Arg}(0) \longrightarrow$ not defined.

$\text{Arg}(z) = \theta$

$\text{Arg}(1/z) = -\theta$

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$\operatorname{Arg}(z_1 z_2) = \operatorname{Arg}(z_1) + \operatorname{Arg}(z_2)$

$\rightarrow \operatorname{Arg}\left(\frac{z_1}{z_2}\right) = \operatorname{Arg}(z_1) - \operatorname{Arg}(z_2)$

$\operatorname{Arg}(z^n) = n \operatorname{Arg}(z)$

$\operatorname{Arg}(|z|) = 0$.

$\rightarrow$ General Argument $= \theta + 2n\pi$; $(n \in I)$

$\rightarrow$ principle Argument/Amplitude $= (-\pi, \pi]$

$\boxed{-\pi < \theta \leq \pi}$

Demoivre's theorem

$e^{i\theta} = \cos\theta + i\sin\theta = \operatorname{Cis}(\theta)$

$e^{-i\theta} = \cos\theta - i\sin\theta = \operatorname{Cis}(-\theta)$

$\sin\theta + i\cos\theta = i\operatorname{cis}(-\theta)$

$\sin\theta - i\cos\theta = -i\operatorname{Cis}(\theta)$

$\operatorname{Cis}\alpha \cdot \operatorname{Cis}\beta = \operatorname{Cis}(\alpha+\beta)$

$\frac{\operatorname{Cis}\alpha}{\operatorname{Cis}\beta} = \operatorname{Cis}(\alpha-\beta)$

$\operatorname{Cis}^n\alpha = (\operatorname{Cis}\alpha)^n = \operatorname{cis}(\alpha n)$

$\frac{1}{\operatorname{Cis}(\theta)} = \operatorname{Cis}(-\theta)$

$(\cos\theta + i\sin\theta)^n = \operatorname{Cis}(n\theta)$

$(\cos\theta - i\sin\theta)^n = \operatorname{Cis}(-n\theta)$

$(\cos\theta + i\sin\theta)^{-n} = \operatorname{cis}(-n\theta)$

$(\cos\theta - i\sin\theta)^{-n} = \operatorname{Cis}(n\theta)$

TRICK-II

$$\left(\frac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}\right)^n = \operatorname{Cis}(n\theta)$$

$$\left(\frac{1+\cos\theta - i\sin\theta}{1+\cos\theta + i\sin\theta}\right)^n = \operatorname{Cis}(-n\theta)$$

$$\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \operatorname{Cis}\left(\frac{n\pi}{2} - n\theta\right)$$

$$\left(\frac{1+\sin\theta - i\cos\theta}{1+\sin\theta + i\cos\theta}\right)^n = \operatorname{Cis}\left[-\left(\frac{n\pi}{2} - n\theta\right)\right]$$

TRICKS

TRICK-1:

① if $x = \cos\theta + i\sin\theta$ (or)

$x + \frac{1}{x} = 2\cos\theta$ (or)

$x - \frac{1}{x} = 2i\sin\theta$.

then.

$x^{10} + \frac{1}{x^{10}} = 2\cos 10\theta$

$(+) \longrightarrow 2\cos$

$(-) \longrightarrow 2i\sin$

$x^8 - \frac{1}{x^8} = 2i\sin(8\theta)$

$x^n + \frac{1}{x^n} = 2\cos(n\theta)$

$x^m + \frac{1}{x^m} = 2\cos(m\theta)$

② $x = \cos\alpha + i\sin\alpha$ &

$y = \cos\beta + i\sin\beta$.

$\therefore$ i) $x^2y^3 + \frac{1}{x^2y^3} = 2\cos(2\alpha + 3\beta)$

$\frac{x^7}{y^8} - \frac{y^8}{x^7} = 2i\sin(7\alpha - 8\beta)$

$x^m \cdot y^n - \frac{1}{x^m y^n} = 2i\sin(m\alpha + n\beta)$

$\frac{x^m}{y^n} + \frac{y^n}{x^m} = 2\cos(m\alpha - n\beta)$

(x) – Add

(÷) → Subst.

power → Add

power → Arg.

$x^1y^1 + \frac{1}{xy} = 2\cos(\alpha+\beta)$

TRICK-III

$(a+ib)^n + (a-ib)^n = 2r^n\cos n\theta$

$(a+ib)^n - (a-ib)^n = 2ir^n\sin n\theta$

$\boxed{r = \sqrt{x^2+y^2}}$

Eg: $(1+i)^{2n} + (1-i)^{2n} = 2(\sqrt{2})^{2n}\cos n\pi/2$

$(1+i)^n - (1-i)^n = 2i(\sqrt{2})^n\sin n(\pi/4)$

$(1+i\sqrt{3})^n + (1-i\sqrt{3})^n = 2\cdot 2^n\cos n(\pi/6)$

$= 2^{n+1}\cos n\pi/6$.

TRICK - IV] product of ∞ terms :

if $x_r = \text{cis}\frac{\pi}{3^r}$; then $x_1 \cdot x_2 \cdot x_3 \cdots x_\infty$ = ____

$\text{cis}\frac{\pi}{3-1} = \text{cis}\,\pi/_2$

$= (\cos\pi/_2 + i\sin\pi/_2)$

$= 0+i$

$= i$

if $x_k = \text{cis}\left(\frac{\pi}{2^k}\right)$ then $x_1 \cdot x_2 \cdot x_3 \cdots x_\infty$ = ____

$\text{cis}\left(\frac{\pi}{2-1}\right) = \text{cis}(\pi)$

$= (\cos\pi + i\sin\pi)$

$= -1$

$\prod_{r=1}^{\infty}\left[\cos\frac{\pi}{4^r} + i\sin\frac{\pi}{4^r}\right]$ = ____

$= \text{cis}(\pi/3)$

$= (\cos\pi/_3 + i\sin\pi/_3)$

$= \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$= \frac{1+i\sqrt{3}}{2}$

$-\frac{(-1-i\sqrt{3})}{2} = -\omega^2$

NOTE:

if $z = \text{cis}\,\theta$; $|z| = 1$

$z = r\,\text{cis}\,\theta$; $|z| = r$

$z = re^{i\theta}$; $|e^{iz}| = e^{-r\sin\theta}$

$i^i = e^{-\pi/2}$ $\qquad \log_e i = \pi/2$

$i^{-i} = e^{\pi/2}$ $\qquad \log_e(\log_e i) = \log(\pi/2) + i\pi/2$

Solved Questions

① $\cos 1° + \cos 2° + \ldots + \cos 179° =$ ____

$\cos\theta + \cos 2\theta + \cdots \cos n\theta = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \cdot \cos\left(\frac{\theta_1+\theta_n}{2}\right)$

$= \frac{\sin\left(\frac{1(179)}{2}\right)}{\sin(1/2)} \cdot \cos\left(\frac{179+1}{2}\right)_0$

$= 0$

② $\sin\frac{3\pi}{5} + \sin\frac{4\pi}{5} + \sin\frac{6\pi}{5} + \sin\frac{7\pi}{5} =$ ____

$\Rightarrow \sin\frac{3\pi}{5} + \sin\frac{4\pi}{5} + \sin\left(\frac{5\pi}{5}\right) + \sin\frac{6\pi}{5} + \sin\frac{7\pi}{5} - \sin\frac{5\pi}{5}$

$\sin\theta_1 + \sin 2\theta + \ldots \sin n\theta = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cdot \sin\left(\frac{\theta_1+\theta_n}{2}\right)$

$= \frac{\sin\left(\frac{7\pi/5}{2}\right)}{\sin(1/2)} \cdot \sin\left(\frac{\frac{3\pi}{5}+\frac{7\pi}{5}}{2}\right) \Rightarrow \sin\left(\frac{10\pi/5}{2}\right)$

$\sin 2\pi = 0$

③ $\cos A + \sin(270+A) - \sin(270-A) + \cos(180-A)$

$= \cos A - \cos A + \cos A - \cos A$

$= 0$

In ΔABC, $\cos\left[\frac{B+2C+3A}{2}\right] + \cos\left[\frac{A-B}{2}\right] =$

$\Delta le \Rightarrow \boxed{A+B+C = 180}$

$\boxed{C = 180 - A - B}$

$\cos\left[\frac{B+2(180-A-B)+3A}{2}\right] + \cos\left[\frac{A-B}{2}\right]$

$= \cos\left[\frac{B+360-2A-2B+3A}{2}\right] + \cos\left[\frac{A-B}{2}\right]$

$= \cos\left[\frac{360+A-B}{2}\right] + \cos\left[\frac{A-B}{2}\right]$

$= \cos\left[180 + \frac{A-B}{2}\right] + \cos\left[\frac{A-B}{2}\right]$

$= -\cos\left[\frac{A+B}{2}\right] + \cos\left[\frac{A+B}{2}\right]$

$= 0$

⑤ if $(\sin\alpha+\operatorname{cosec}\alpha)^2+(\cos\alpha+\sec\alpha)^2$

$= \_\_\_\_\_\_$

$= \sin^2\alpha+\operatorname{cosec}^2\alpha+2+\cos^2\alpha+\sec^2\alpha+2$

$= 1+\operatorname{cosec}^2\alpha+2+\sec^2\alpha+2$

$= 5+\operatorname{cosec}^2\alpha+\sec^2\alpha$

$= 5+1+\cot^2\alpha+1+\tan^2\alpha$

$= 5+\tan^2\alpha+\cot^2\alpha+2$

$= 7+\tan^2\alpha+\cot^2\alpha$

$\boxed{\therefore (\sin\alpha+\operatorname{cosec}\alpha)^2+(\cos\alpha+\sec\alpha)^2 = 7+\tan^2\alpha+\cot^2\alpha}$

⑥ $\left(\frac{\sqrt3+2\cos A}{1-2\sin A}\right)^{-3}+\left(\frac{1+2\sin A}{\sqrt3-2\cos A}\right)^{-3} = \underline{0.}$

put $A=90°$

$\left(\frac{\sqrt3}{-1}\right)^{-3}+\left(\frac{3}{\sqrt3}\right)^{-3}$

$-\left(\frac{1}{\sqrt3}\right)^3+\left(\frac{1}{\sqrt3}\right)^3 = 0$

⑦ $\operatorname{cosec}\theta+\cot\theta = -3/2$, then $\operatorname{cosec}\theta = \_\_\_\_$

$\operatorname{cosec}^2\theta-\cot^2\theta = 1$

$(\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta) = 1$

$\frac{-3}{2}(\operatorname{cosec}\theta-\cot\theta) = 1$

$\boxed{\operatorname{cosec}\theta-\cot\theta = \frac{-2}{3}}$

$\operatorname{cosec}\theta+\cancel{\cot\theta} = -3/2$ $\quad \frac{-3}{2}-\frac{2}{3}$

$+\operatorname{cosec}\theta-\cancel{\cot\theta} = -2/3$ $\quad \frac{-13}{6}$

$2\operatorname{cosec}\theta = -13/6$

$\operatorname{cosec}\theta = -13/12$

⑧ if $\cos\theta-\sin\theta = \sqrt2\sin\theta$, then $\cos\theta+\sin\theta = \_\_$

$\boxed{(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2 = 2}$

$(\cos\theta+\sin\theta)^2+(2\sin^2\theta) = 2$ $\quad [1-\sin^2\theta]$

$(\cos\theta+\sin\theta)^2 = 2\cos^2\theta$

$\boxed{\cos\theta+\sin\theta = \pm\sqrt2\cos\theta}$

⑨ $\cos\theta-4\sin\theta = 1 \Rightarrow \sin\theta+4\cos\theta = \_\_\_\_$

$\boxed{\sin\theta+4\cos\theta = x}$

$(\cos\theta-4\sin\theta)^2+(\sin\theta+4\cos\theta)^2 = 1+x^2$

$(\cos^2\theta+\sin^2\theta 16 \cancel{-8\sin\theta\cos\theta})+(\sin^2\theta+16\cos^2\theta+\cancel{8\sin\theta\cos\theta}) = 1+x^2$

$\cos^2\theta+\sin^2\theta\cdot16+\sin^2\theta+16\cos^2\theta = 1+x^2$

$\sin^2\theta+\cos^2\theta+16(\sin^2\theta+\cos^2\theta) = 1+x^2$

$\cancel{1}+16 = \cancel{1}+x^2$

$x = \sqrt{16}$

$\boxed{x = \pm4}$

$\therefore \boxed{\sin\theta+4\cos\theta = \pm4}$

⑩ If $\sec\theta+\tan\theta = p\ (p\neq0)$, then $\sin\theta = \_\_\_$

$\sec\theta+\tan\theta = p$

$\sec\theta-\tan\theta = 1/p$

$2\sec\theta = p+\frac{1}{p}$

$2\sec\theta = \frac{p^2+1}{p}$

$\sec\theta = \frac{p^2+1}{2p} = \frac{Hyp}{Adj}$

$\boxed{\sin\theta = \frac{p^2-1}{p^2+1}}$

⑪ If $a\cos\theta+b\sin\theta = c$, then $(a\sin\theta-b\cos\theta)^2 = $ (k)

$(a\cos\theta+b\sin\theta)^2+(a\sin\theta-b\cos\theta)^2 = c^2+k^2$

$a^2\cos^2\theta+b^2\sin^2\theta+a^2\sin^2\theta+b^2\cos^2\theta = c^2+k^2$

$a^2(1)+b^2(1) = c^2+k^2$

$a^2+b^2 = c^2+k^2$

$\boxed{k^2 = a^2+b^2-c^2}$

⑫ if $\sin\alpha+\operatorname{cosec}\alpha = 2$, then $\sin^n\alpha+\operatorname{cosec}^n\alpha = \_\_$

$\sin\alpha+\frac{1}{\sin\alpha} = 2$

$\frac{\sin^2\alpha+1}{\sin\alpha} = 2$

$\sin^2\alpha-2\sin\alpha+1 = 0$

$\boxed{\sin\alpha = 1;\ \operatorname{cosec}\alpha = 1}$

$\boxed{1^n+1^n = 2}$

(13) if $\sin x + \sin^2 x = 1$; therefore;
$\cos^8 x + 2\cos^6 x + \cos^4 x =$ ______

$\sin x = 1 - \sin^2 x = \cos^2 x$

$\boxed{(\cos^4 x)^2 + 2\cos^4 x \cdot \cos^2 x + (\cos^2 x)^2}$ Given
$a^2 + 2\,a\,b + b^2$

$(\cos^4 x + \cos^2 x)^2$
$= (\sin^2 x + \sin x)^2$
$= 1^2$
$= 1$

(14) if $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then
$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$ ______

$\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$
$1 \quad 1 \quad 1$

$\theta = 90° = \theta_1 = \theta_2 = \theta_3$
$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

(15) if $\sec\theta = a + \frac{1}{4a}$ then $\sec\theta + \tan\theta =$

$= \sec\theta + \sqrt{\sec^2\theta - 1}$

$= \sec\theta + \sqrt{\left(a + \frac{1}{4a}\right)^2 - 1}$

$= \left(a + \frac{1}{4a}\right) + \sqrt{\left(\frac{4a^2+1}{4a}\right)^2 - 1}$

$= \frac{4a^2+1}{4a} + \sqrt{\frac{16a^4 + 8a^2 + 1}{16a^2} - 1}$

$= \frac{4a^2+1}{4a} + \sqrt{\frac{16a^4 + 8a^2 + 1 - 16a^2}{16a^2}}$

$= \left(a + \frac{1}{4a}\right) + \left(a - \frac{1}{4a}\right)$

$= 2a$

$\sec\theta + \tan\theta = k$
$\sec\theta - \tan\theta = \frac{1}{k}$

$2\sec\theta = \frac{k^2+1}{k}$

$2\sec\theta = \frac{k^2+1}{2k}$

$= \frac{2k^2+2}{4k}$

$\frac{k}{2} = a$
$k = 2a$

(16) if $x = a\cos^4\theta$, $y = b\sin^4\theta$; then
$\sqrt{x/a} + \sqrt{y/b} =$ ______

$x = a\cos^4\theta$
$\sqrt{x} = \sqrt{a}\,.\cos^2\theta$
$\cos^2\theta = \sqrt{x/a}$

$y = b\sin^4\theta$
$\sqrt{y} = \sqrt{b}\sin^2\theta$
$\sin^2\theta = \sqrt{y/b}$

$\sqrt{x/a} + \sqrt{y/b} = \cos^2\theta + \sin^2\theta = 1$

(17) if $\sin\theta + \cos\theta = p$ and $\tan\theta + \cot\theta = q$, then $q(p^2 - 1)$

$(\tan\theta + \cot\theta)\left[(\sin\theta + \cos\theta)^2 - 1\right]$

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\left[2\sin\theta\cos\theta\right]$

$= \frac{1}{\cancel{\sin\theta\cos\theta}}\left[2\cancel{\sin\theta\cos\theta}\right]$

$= 2$

(18) if $\tan\theta + \sin\theta = m$; $\tan\theta - \sin\theta = n$,
then $(m^2 - n^2)^2 =$ ______

$(m^2 - n^2)^2 = (m+n)^2 (m-n)^2$

$= (2\tan\theta)^2 (2\sin\theta)^2$

$= 4\tan^2\theta \cdot 4\sin^2\theta$

$= 16\tan^2\theta \cdot \sin^2\theta$

$= 16\tan^2\theta\,(1 - \cos^2\theta)$

$= 16\left[\tan^2\theta - \sin^2\theta\right]$

$= 16\left[(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)\right]$

$= 16(mn)$

(19) if $x = \sin 1$; $y = \sin 1^\circ$ then;

$1\,rad = 57^\circ$

$\therefore \boxed{x \approx \sin 57^\circ}$

$\therefore \boxed{x > y}$

Standard Result:

$\Rightarrow 3\left(\cos^4\theta + \sin^4\theta\right) - 2\left[\cos^6\theta + \sin^6\theta\right] = 1$

$a\sin^2\theta + b\cos^2\theta = c \Rightarrow \tan^2\theta = \underline{\ ?\ }$

$a\tan^2\theta + b = c\sec^2\theta$

$a\tan^2\theta + b = c\left[1 + \tan^2\theta\right]$

$a\tan^2\theta + b = c + c\tan^2\theta$

$\tan^2\theta\,[a - c] = c - b$

$\tan^2\theta = \boxed{\dfrac{c-b}{a-c}}$

$\dfrac{\sec\theta + \tan\theta - 1}{\cancel{\sec\theta}\tan\theta - \sec\theta + 1} =$

$\sec\theta + \tan\theta - \left[(\sec\theta + \tan\theta)\cdot(\sec\theta - \tan\theta)\right]$

$\dfrac{\sec\theta + \tan\theta\left[\cancel{1 - \sec\theta + \tan\theta}\right]}{\cancel{\tan\theta - \sec\theta + 1}} = \boxed{\sec\theta + \tan\theta}$

$\dfrac{\cos^3A + \sin^3A}{\cos A + \sin A} + \dfrac{\cos^3A - \sin^3A}{\cos A - \sin A} = \underline{\qquad}$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$

$\dfrac{\cancel{(\cos A + \sin A)}(\cos^2A - 2\sin A\cdot\cos A + \sin^2A)}{\cancel{\cos A + \sin A}} = (1)$

$\dfrac{\cancel{(\cos A - \sin A)}(\cos^2A + 2\cos A\sin A + \sin^2A)}{\cancel{\cos A - \sin A}} = (2)$

$\therefore (\cos^2A - 2\sin A\cancel{\cos A} + \sin^2A) + (\cos^2A + \cancel{2\cos A\cdot\sin A} + \sin^2A)$

$= 1 + 1$

$= 2$

$\Rightarrow$ if $\csc\theta = p + \dfrac{1}{4p}$ then $\csc\theta + \cot\theta = \underline{\qquad}$

★ $\csc\theta + \cot\theta = k$

$\csc\theta - \cot\theta = 1/k$

$2\csc\theta = k^2 + 1/k$

$\csc\theta = \dfrac{k^2+1}{2k} = \dfrac{2k^2 + 2}{4k}$

$\boxed{\csc\theta = \dfrac{k}{2} + \dfrac{1}{2k}}$

$\csc\theta = p + \dfrac{1}{4p}$

$\dfrac{k}{2} = p$

$\boxed{k = 2p} \Rightarrow \boxed{\csc\theta + \cot\theta = 2p}$

$\Rightarrow \sqrt{1 - \sin^2 100^\circ}\,;\ \sec 100^\circ = \underline{\qquad}$

$(-\cos 100^\circ)\cdot\sec 100^\circ =$

$\downarrow$

$\cos^2(90 + 10) =$

$\boxed{-\sin 10}$ (Q2)

INVERSEE TRIGONOMETRY.

* EXERCISE QUESTIONS *

(1) $\sec^2[\tan^{-1}(2)] + \csc^2[\cot^{-1}(3)] =$ ____

$= 1 + \tan^2(\tan^{-1}(2)) + 1 + \cot^2(\cot^{-1}(3))$

$= 1 + (2)^2 + 1 + (3)^2$

$= 1+4+1+9$

$= 15$

*** Standard Result:**

if $\cos^{-1}\left[\frac{x}{a}\right] + \cos^{-1}\left[\frac{y}{b}\right] = \theta$, then

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\theta = \sin^2\theta}$$

(2) If $\sin^{-1}(x/5) + \csc^{-1}(5/4) = \pi/2$; then

$x =$ ____

$\sin^{-1}(x/5) = \alpha \Rightarrow \sin\alpha = x/5 \Rightarrow \tan\alpha = \frac{x}{\sqrt{25-x^2}}$

$\csc^{-1}(5/4) = \beta \Rightarrow \csc\beta = 5/4;\ \tan\beta = \frac{4}{3}.$

$\boxed{\alpha+\beta = \pi/2}$ $\boxed{\tan\alpha \cdot \tan\beta = 1}$

$\frac{x}{\sqrt{25-x^2}} \cdot \frac{4}{3} = 1$

$\frac{x^2}{25-x^2} \cdot \frac{16}{9} = 1$

$16x^2 = 9(25-x^2)$

$\cancel{16x^2 = 9x^2 \frac{9\times25}{16}}$

$\cancel{x^2} =$

$25x^2 = 9\times25$

$x^2 = \frac{9\times25}{25}$

$\boxed{x = \pm 3}$

$\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}(x)$

$\boxed{\therefore x = 1/2, 0.}$

(3) The value of x where $x>0$, and

$\tan[\sec^{-1}(1/x)] = \sin(\tan^{-1}2)$

$\sec^{-1}(1/x) = \alpha \Rightarrow \sec\alpha = \frac{1}{x};\ \tan\alpha = \frac{\sqrt{1-x^2}}{x}$

$\tan^{-1}(2) = \beta \Rightarrow \tan\beta = \frac{2}{1};\ \sin\beta = \frac{2}{\sqrt{5}}$

$\boxed{\tan(\alpha) = \sin\beta}$

$\frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$

$\frac{1-x^2}{x^2} = \frac{4}{5}$

$\frac{1}{x^2} - 1 = 4/5 \Rightarrow \frac{1}{x^2} = 4/5 + 1 = 9/5$

$x^2 = 5/9 \Rightarrow \boxed{x = \pm\sqrt{5/3}}$

$\tan^{-1}\left[\frac{x-1}{x-2}\right] + \cot^{-1}\left[\frac{x+2}{x+1}\right] = \pi/4.$

$\therefore x = \pm 1/\sqrt{2}$

Range of $\sin^{-1}(5x) = \boxed{[-\pi/2, \pi/2]}$

* properties of Δles *

In ΔABC, if $a = \sqrt{3}+1$, $B = 30°$, $C = 45°$, then $c =$ ____

$A = 180 - B - C$

$= 180 - 30 - 45$

$A = 105°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$ (1)

$\frac{\sqrt{3}+1}{\sin(90+15°)} = \sqrt{2}c$

$\frac{\sqrt{3}+1\,(2\sqrt{2})}{\sqrt{3}+1} = \frac{\cancel{\sqrt{2}}}{1} \times c$

$\boxed{c = 2}$

② In ΔABC, if $b=20, c=21$ and $\sin A = 3/5$

then $a =$ ______

$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$ $\quad \sin A = 3/5$

$\cos(A) = 4/5$

$\therefore a^2 = b^2+c^2-2bc\cos A$

$a^2 = 400+441 - 2(20)(21)\cdot\frac{4}{5}$

$a^2 = 169$

$a = \sqrt{169}$

$\boxed{a = 13}$

In ΔABC, if $a=2$; $B=120°$, $C=30°$ then

$\Delta =$

$\boxed{A = 30°}$

$\Delta = \frac{1}{2}ab\sin C$

$\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$

$\dfrac{2b}{\sqrt{3}} = 4$

$\boxed{b = 2\sqrt{3}}$

$\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{\cancel{2}} \times \cancel{2} \times \cancel{2}\sqrt{3} \times \frac{1}{\cancel{2}}$ $= \boxed{\sqrt{3} \text{ Sq.units}}$

In ΔABC, if $c^2=a^2+b^2$; $2s = a+b+c$, then

$4s(s-a)(s-b)(s-c) =$ ______

given Δle is right angled at C $(\because c^2=a^2+b^2)$

$\Delta = \frac{ab}{2}$

$4\Delta^2$

$= 4\left(\frac{ab}{2}\right)^2$

$= \cancel{4}\dfrac{a^2b^2}{\cancel{4}}$

$= a^2b^2$

In ΔABC, $2\cos B = a/c$, then Δle ______

$\cancel{2}\left(\dfrac{a^2+c^2-b^2}{\cancel{2}a\cancel{c}}\right) = \dfrac{a}{\cancel{c}}$

$a^2+c^2-b^2-a^2=0$

$c^2 - b^2 = 0$

$\boxed{c = b}$

$\boxed{\text{Isosceles } \Delta\text{le}}$

In ΔABC, $\tan\frac{A}{2} = 5/6$; $\tan\frac{C}{2} = \frac{2}{5}$, then

$\tan A/2 \cdot \tan C/2 = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}} \cdot \sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$

$\dfrac{5}{6} \times \dfrac{2}{5} = \sqrt{\dfrac{(s-b)(s-b)}{s^2}}$

$\dfrac{1}{3} = \boxed{\dfrac{s-b}{s}}$

$s = 3s-3b$

$3b = 2s$ $\quad \boxed{a,b,c \text{ are in A.P}}$

$3b = a+b+c$

$2b = a+c$

$\boxed{b = \frac{a+c}{2}}$

☆ if a,b,c are in A.P, then $\tan A/2 \cdot \tan C/2 = 1/3$

In ΔABC, $r_1r_2r_3 =$ ______

$\dfrac{\Delta}{s-a} \cdot \dfrac{\Delta}{s-b} \cdot \dfrac{\Delta}{s-c} \times \dfrac{s}{s}$

$\boxed{\Delta = rs}$

$= \left(\dfrac{s\Delta^3}{\Delta^2}\right) = s(\Delta)$

$= s(rs)$

$= rs^2$

$r_1r_2r_3 = rs^2$

$\boxed{\sqrt{\dfrac{r_1r_2r_3}{r}} = s}$

$(r_1-r)(r_2-r)(r_3-r) \Rightarrow 4Rr^2$

TRANSFORMATIONS :

$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) =$ ______

$\therefore 4\cos\left[\frac{\alpha+\beta}{2}\right].\cos\left[\frac{\beta+\gamma}{2}\right].\cos\left[\frac{\gamma+\alpha}{2}\right]$

$\cos(\alpha+\beta+\gamma) + \cos(\alpha-\beta-\gamma) + \cos(\beta-\gamma-\alpha)$

$+ \cos(\gamma-\alpha-\beta) = \boxed{4\cos\alpha.\cos\beta.\cos\gamma}$

$\cos 40^\circ + \cos 80^\circ + \cos 160^\circ =$ ______

$\left(2\cos\frac{40+80}{2}.\cos\frac{80-40}{2}\right) + \cos(180-20)$

$2\cos 60^\circ.\cos 20^\circ + \cos(180-20^\circ)$

$= \cos 20^\circ - \cos 20^\circ$

$= 0^\circ$

$\cos 12^\circ + \cos 84^\circ + \cos 132^\circ + \cos 156^\circ =$ ______

$= 2\cos 48.\cos 36 + 2\cos 144.\cos 12^\circ$

$= 2\cos 48.\cos 36 - 2\cos 36.\cos 12^\circ$

$= 2\cos 36^\circ[\cos 48 - \cos 12]$

$= 2\cos 36^\circ[-2\sin 30^\circ.\sin 18^\circ]$

$= \cancel{2}.\frac{\sqrt5+1}{\cancel{4}_2}\left[-\cancel{2}\frac{1}{\cancel{2}}.\frac{\sqrt5-1}{4}\right]$

$= 2.\frac{\sqrt5+1}{4}\left[\frac{-\sqrt5+1}{4}\right]$

$= \cancel{2}\frac{-4}{16_8} = -1/2$

$\cos 6^\circ.\sin 24^\circ.\cos 72^\circ =$ ______

$\left[\frac{1}{2}.2\sin 24^\circ.\cos 6^\circ\right].\cos 72^\circ$

$\frac{1}{2}[\sin 30 + \sin 18^\circ].\cos 72^\circ$

$\frac{1}{2}\left[\frac{1}{2} + \frac{\sqrt5-1}{4}\right].\frac{\sqrt5-1}{4}$

$\frac{\sqrt5-1}{16}\ \frac{\sqrt5-1}{4}\left(\frac{1}{4} + \frac{\sqrt5-1}{8}\right).\frac{\sqrt5-1}{4}$

$\frac{\sqrt5-1}{16} + \frac{(\sqrt5-1)^2}{32}$

$= \frac{1}{8}$

$2\sqrt5-2+$

$\sin 12^\circ.\sin 24^\circ.\sin 48^\circ.\sin 84^\circ =$ ______

$\frac{1}{2}\left[\cancel{2\sin 24.\sin 12}\right].\frac{1}{2}\left(\cancel{2\sin 84.\sin 48^\circ}\right) =$ ______

$\frac{1}{2}\left[\cancel{2\cos 6^\circ}\ \cos\right.$

$\frac{1}{2}\left[\cancel{\cos 12^\circ - \cos 36^\circ}\right].\frac{1}{2}\left[\cancel{\cos 36^\circ - \cos 132^\circ}\right]$

$(\sin 48^\circ.\sin 12^\circ).(\sin 84^\circ.\sin 24^\circ)$

$\frac{1}{2}(2\sin 48.\sin 12).\frac{1}{2}(2\sin 84.\sin 24^\circ)$

$= \frac{1}{2}(\cos 36^\circ - \cos 60^\circ).\frac{1}{2}(\cos 60^\circ - \cos 108^\circ)$

$= \frac{1}{4}\left[\frac{\sqrt5+1}{4} - \frac{1}{2}\right]\left[\frac{1}{2} + \frac{\sqrt5-1}{4}\right]$

$= \frac{1}{4}\left[\frac{\sqrt5-1}{4}\right]\left[\frac{\sqrt5+1}{4}\right]$

$= \frac{1}{64}\times 4 = \frac{1}{16}$

$\cos 6^\circ.\cos 42^\circ.\cos 60^\circ.\cos 66^\circ.\cos 78^\circ.$

$= \cos 6^\circ.\cos 66^\circ\ .\ \cos 42^\circ.\cos 78^\circ.\ \cos 60^\circ$

$= \frac{\cos 6^\circ.\cos 54^\circ.\cos 66^\circ\ .\ \cos 18^\circ.\cos 42^\circ.\cos 78^\circ}{\cos 54^\circ.\cos 18^\circ}.\cos 60^\circ$

$= \frac{\frac{1}{4}\cos 18^\circ.\ \frac{1}{4}\cos 54^\circ}{\cos 54.\cos 18^\circ}\times\frac{1}{2}$

$= \frac{1}{16}\times\frac{1}{2} = \frac{1}{32}$

$\Rightarrow \cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cdot \cos 16^\circ =$

$= \cos^2 76 + 1 - \sin^2 16^\circ - \frac{1}{2}\left[2\cos 76^\circ \cdot \cos 16^\circ\right]$

$= 1 + \cos^2 76 - \sin^2 16 - \frac{1}{2}\left[\cos 92 + \cos 60^\circ\right]$

$= 1 + \cos 92^\circ \cdot \cos 60^\circ - \frac{1}{2}\cos 92^\circ - \frac{1}{2}\cdot\frac{1}{2}$.

$= 1 + \cancel{\frac{1}{2}\cos 92^\circ} - \cancel{\frac{1}{2}\cos 92^\circ} - \frac{1}{4}$

$= 1 - \frac{1}{4}$

$= 3/4$

$\Rightarrow \frac{\sin A + \sin 5A + \sin 9A}{\cos A + \cos 5A + \cos 9A} = ?$

$= \frac{\sin 5A + \sin 9A + \sin A}{\cos 5A + \cos 9A + \cos A}$

$= \frac{\sin 5A + 2\sin 5A \cdot \cos 4A}{\cos 5A + 2\cos 5A \cdot \cos 4A}$

$= \frac{\sin 5A\left(1 + \cancel{2\cos 4A}\right)}{\cos 5A\left[\cancel{1 + 2\cos 4A}\right]} = \frac{\sin 5A}{\cos 5A} = \tan 5A$

$\Rightarrow \frac{\cos A + \cos 3A + \cos 5A + \cos 7A}{\sin A + \sin 3A + \sin 5A + \sin 7A}$

$= \frac{(\cos 5A + \cos A) + (\cos 7A + \cos 3A)}{(\sin 5A + \sin A) + (\sin 7A + \sin 3A)}$

$= \frac{2\cos 3A \cdot \cos 2A + 2\cos 5A \cdot \cos 2A}{2\sin 3A \cdot \cos 2A + 2\sin 5A \cdot \cos 2A}$

$= \frac{\cancel{2\cos 2A}\,[\cos 3A + \cos 5A]}{\cancel{2\cos 2A}\,[\sin 3A + \sin 5A]}$

$= \frac{\cos 5A + \cos 3A}{\sin 5A + \sin 3A} = \frac{2\cos 4A \cdot \cancel{\cos A}}{2\sin 4A \cdot \cos 2A}$

$= \boxed{\cot 4A}$

$\Rightarrow \frac{\sin(n+1)\alpha - \sin(n-1)\alpha}{\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha} = A$

$= \frac{2\cos n\alpha \cdot \sin\alpha}{2\cos n\alpha + 2\cos n\alpha \cdot \cos\alpha}$

$= \frac{\sin\alpha \cdot \cancel{2\cos n\alpha}}{\cancel{2\cos n\alpha}\,[1 + \cos\alpha]}$

$= \frac{\sin\alpha}{1+\cos\alpha} = \frac{\cancel{2}\sin\alpha/2 \cdot \cos\alpha/2}{\cancel{2}\cos^2\alpha/2}$

$= \tan\alpha/2$

$\Rightarrow \frac{\sin 70^\circ + \cos 40^\circ}{\cos 70 + \sin 40^\circ}$

$= \frac{\cos 20 + \cos 40^\circ}{\cos 70 + \cos 50^\circ} = \frac{\cancel{2}\cos 30^\circ \cdot \cancel{\cos 10^\circ}}{\cancel{2}\cos 60^\circ \cdot \cancel{\cos 10^\circ}}$

$= \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$

$A + C = 2B$; $\frac{\cos C - \cos A}{\sin A - \sin C} =$

$\frac{2\sin\frac{C+A}{2} \cdot \cancel{\sin\frac{A-C}{2}}}{2\cos\frac{C+A}{2} \cdot \cancel{\sin\frac{A-C}{2}}} = \tan\frac{C+A}{2}$

$= \tan B$

$\sin\alpha = \sin\beta$ and $\cos\alpha = \cos\beta$, then =

$\sin\alpha - \sin\beta = 0$. — ①

$\cos\alpha - \cos\beta = 0$ — ②

$2\cos\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2} = 0$ — ①

$-2\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2} = 0$ — ②

$\therefore \boxed{\sin\frac{\alpha-\beta}{2} = 0}$

$$\Rightarrow \cos A = \frac{3}{4} \Rightarrow 8\,2\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{5A}{2}\right) =$$

$$= 16\left[2\sin\frac{A}{2}\cdot\sin\frac{5A}{2}\right]$$

$$= 16\left[\cos 2A - \cancel{3}\cos 3A\right]$$

$$= 16\left[2\cos^2 A - 1 - 4\cos^3 A + 3\cos A\right]$$

$$= 16\left[\frac{18}{16} - 1 - \cancel{4}\left[\frac{27}{\cancel{64}_{16}}\right] + \frac{9}{4}\right]$$

$$= 16\left[\frac{18}{16} - \frac{27}{16} - 1 + \frac{9}{4}\right]$$

$$= 16\left[\frac{9}{4} - \frac{1}{16} - 1\right]$$

$$= \cancel{16}\left[\frac{36 - 9 - 16}{\cancel{16}}\right] = 36 - 25 = 11$$

$\Rightarrow \sin\alpha + \sin\beta = a;\ \cos\alpha + \cos\beta = b;$ then $\sin(a+b) =$

$$\frac{a}{b} = \frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta}$$

$$= \frac{2\sin\frac{\alpha+\beta}{2}\cdot\cancel{\cos\left(\frac{\alpha-\beta}{2}\right)}}{2\cos\frac{\alpha+\beta}{2}\cdot\cancel{\cos\left(\frac{\alpha-\beta}{2}\right)}}$$

$$\boxed{\frac{a}{b} = \tan\frac{\alpha+\beta}{2}}$$

$$\sin(\alpha+\beta) = \frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)} = \frac{2\left(a/b\right)}{1+\frac{a^2}{b^2}}$$

$$= \frac{2a}{\cancel{b}} \times \frac{b^{\cancel{2}}}{a^2+b^2} = \boxed{\frac{2ab}{a^2+b^2}}$$

$$\cos x + \cos y + \cos\alpha = 0$$

$$\boxed{\cos x + \cos y = -\cos\alpha} \quad \text{— (1)}$$

$$\sin x + \sin y + \sin\alpha = 0$$

$$\boxed{\sin x + \sin y = -\sin\alpha} \longrightarrow (2)$$

$$\frac{(1)}{(2)} = \frac{\cos x + \cos y}{\sin x + \sin y} = \frac{\cancel{-}\cos\alpha}{\cancel{-}\sin\alpha} = \cot\alpha.$$

$$\frac{\cancel{2}\cos\left(\frac{x+y}{2}\right)\cdot\cancel{\cos\left(\frac{x-y}{2}\right)}}{2\sin\left(\frac{x+y}{2}\right)\cdot\cancel{\cos\left(\frac{x-y}{2}\right)}} = \cot\alpha$$

$$= \boxed{\cot\left[\frac{x+y}{2}\right] = \cot\alpha}$$

$$\Rightarrow \cos x + \cos y = 1/3 \quad \text{— (1)}$$

$$\sin x + \sin y = 1/4 \quad \text{— (2)}$$

$\therefore \cos(x+y) \Rightarrow ?$

$(1) \div (2)$

$$\frac{\cos x + \cos y}{\sin x + \sin y} = \frac{4}{3}$$

$$\frac{2\cos\left(\frac{x+y}{2}\right)\cdot\cancel{\cos\left(\frac{x-y}{2}\right)}}{2\sin\left(\frac{x+y}{2}\right)\cdot\cancel{\cos\left(\frac{x-y}{2}\right)}} = \frac{4}{3}$$

$$\cot\left(\frac{x+y}{2}\right) = \frac{4}{3}$$

$$\boxed{\tan\left(\frac{x+y}{2}\right) = \frac{3}{4}.}$$

$$\boxed{\cos A = \frac{1-\tan^2 A/2}{1+\tan^2 A/2}} \Rightarrow \cos(x+y) = \frac{1 - 9/16}{1 + 9/16}$$

$$= \frac{7}{16} \times \frac{16}{25} = \boxed{7/25}$$

If n is odd integer, then;

$$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n + \left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$$

$$= \left[\frac{2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)}\right]^n + \left[\frac{2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)}\right]^n$$

$$\left[\cot\left(\frac{A-B}{2}\right)\right]^n + \left[-\cot\left(\frac{A-B}{2}\right)\right]^n = 0.$$

If $\boxed{n = \text{even}}$

$$= \boxed{2\cot^n\left[\frac{A-B}{2}\right]}$$

* COMPLEX NUMBERS *

If θ is Real, then modulus of $\frac{1}{1+\cos\theta+i\sin\theta}$ is

$$\left|\frac{1}{1+\cos\theta+i\sin\theta}\right| = \left|\frac{1}{2\cos^2\theta/2+i\,2\sin\theta/2.\cos\theta/2}\right|$$

$$= \left|\frac{1}{2\cos\theta/2\left[\cos\theta/2+i\sin\theta/2\right]}\right|$$

$$= \frac{1}{2\cos\theta/2}(1)$$

$$= \boxed{\frac{1}{2}\sec\theta/2}$$

$$\Rightarrow \left|\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}\right| =$$

$$\left|\frac{(2-i)^2-(2+i)^2}{(4-i^2)^2}\right| = \left|\frac{-8i}{25}\right| = \frac{8}{25}$$

if α and β are real then $\left|\frac{\alpha+i\beta}{\beta+i\alpha}\right| =$

$$\frac{\sqrt{\alpha^2+\beta^2}}{\sqrt{\alpha^2+\beta^2}} = 1$$

$i^2+i^4+i^6+\dots+(2n+1)$ terms.

$\Rightarrow \boxed{G.P = \text{series}}$

$$S_\infty = \frac{a(1-r^n)}{1-r} = \frac{i^2\left[1-(i^2)^{2n+1}\right]}{1-i^2}$$

$$= \frac{-1[1+1]}{[1+1]} = -1$$

If $u+iv = \frac{3i}{x+iy+2}$, then y in terms of u, v = ——

$$x+2+iy = \frac{3i}{u+iv}$$

$$= \frac{3i}{u+iv}\cdot\frac{(u-iv)}{u-iv}$$

$$= \frac{3iu+3v}{u^2+v^2}$$

$$= \underbrace{\frac{3u}{u^2+v^2}}_{y}i + \underbrace{\frac{3v}{u^2+v^2}}_{x}$$

$$\boxed{y = \frac{3u}{u^2+v^2}}$$

if $(1+\cos\theta+i\sin\theta)(1+\cos2\theta+i\sin2\theta) = x+iy$ then $y =$ ——.

$$=(2\cos^2\theta/2+i\,2\sin\theta/2\cos\theta/2)(2\cos^2\theta+i\,2\sin\theta\cos\theta)$$

$$= 2\cos\theta/2\left[\cos\theta/2+i\sin\theta/2\right].2\cos\theta\left[\cos\theta+i\sin\theta\right]$$

$$= 4\cos\theta/2.\cos\theta\left[\cos(\theta/2+\theta)+i\sin(\theta+\theta/2)\right]$$

$$= 4\cos\theta/2.\cos\theta\left[\cos3\theta/2+i\sin3\theta/2\right]$$

$$x = 4\cos\theta/2.\cos\theta.\cos3\theta/2$$

$$y = 4\cos\theta/2.\cos\theta.\sin3\theta/2$$

$$\frac{y}{x} = \tan\frac{3\theta}{2}$$

$$\boxed{y = x\tan\frac{3\theta}{2}}$$

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TRIGONOMETRIC Equations

The values of satisfying $\text{cosec}\,\theta + 2 = 0$. in $(0, 2\pi) =$ ____

$\text{Cosec}\,\theta = -2$

$\sin\theta = \frac{-1}{2}$

$\boxed{\theta = -\pi/6}$

$\boxed{\theta = n\pi - (-1)^n \pi/6}$

$n = 1 \Rightarrow \pi + \pi/6 \Rightarrow 180 + 30 = 210°$

$n = 2 = 2\pi + \pi/6 \Rightarrow 360 - 30 = 330°$

The most general value of θ; which satisfies both equations $\tan\theta = -1$; and $\cos\theta = \frac{1}{\sqrt{2}}$ is

$\tan\theta = -1 \Rightarrow (-\pi/4)$

$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow (\pi/4)$

common solution $= 2n\pi \pm \theta$

$= 2n\pi + 2\pi - \pi/4$

$= 2n\pi + \frac{7\pi}{4}$

If $\sin^{10}x - \cos^{10}x = 1$; then $x =$ ____

$\sin^{10}x = 1 + \cos^{10}x$

$\sin^{10}x \le 1 = 1 + \cos^{10}x \le 1$

$= \cos^{10}x \le 0$

$= \cos^{10}x = 0$

$\Rightarrow \cos x = 0 \therefore x = \pi/2$

$\boxed{\theta = (2n+1)\pi/2}$

If $4\cos\theta - \sec\theta = 2\tan\theta$; then $\theta =$ ____

$4\cos\theta - \frac{1}{\cos\theta} = \frac{2\sin\theta}{\cos\theta}$

$4\cos^2\theta - 1 = 2\sin\theta$

$4(1 - \sin^2\theta) - 1 = 2\sin\theta$

$4 - 4\sin^2\theta - 1 = 4\sin\theta$

$4\sin^2\theta + 4\sin\theta - 3 = 0$

$4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$

$4\sin^2\theta - 2\sin\theta + 6\sin\theta - 3 = 0$

$2\sin\theta(2\sin\theta - 1) + 3(2\sin\theta - 1) = 0$

$(2\sin\theta + 3)(2\sin\theta - 1) = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \pi/6$

$\boxed{\theta = n\pi + (-1)^n \pi/6;\ n \in Z}$

$\Rightarrow \cos^2\theta + \cos\theta - 1 = 0$; $\theta = ?$

$2\cos^2\theta + 2\cos\theta - \cos\theta - 1 = 0$.

$2\cos\theta(\cos\theta + 1) - 1(\cos\theta + 1) = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0$	$\cos\theta + 1 = 0$
$\cos\theta = \frac{1}{2}$	$\cos\theta = -1$
$\boxed{\theta = 2n\pi \pm \frac{\pi}{3}}$	$\boxed{\theta = 2n\pi \pm \pi}$

If $2\sin\theta + \tan\theta = 3\sin\theta \cdot \cos\theta$; then $\theta =$ ____

$2\sin\theta + \frac{\sin\theta}{\cos\theta} = 3\sin\theta \cdot \cos\theta$

$2\sin\theta \cdot \cos\theta + \sin\theta = 3\sin\theta \cdot \cos^2\theta$

$3\sin\theta \cdot \cos^2\theta - 2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta\,[3\cos^2\theta - 2\cos\theta - 1] = 0$

$\sin\theta\,[3\cos^2\theta - 3\cos\theta + \cos\theta - 1] = 0$.

$\sin\theta\,[3\cos\theta(\cos\theta - 1) + 1(\cos\theta - 1)] = 0$

$\sin\theta\,[(3\cos\theta + 1)(\cos\theta - 1)] = 0$.

$\sin\theta = 0 \mid \cos\theta = 1$ $\therefore \theta = 0° \Rightarrow 2n\pi \pm \pi$ or $n\pi$.

Solution of $\theta \Rightarrow \sqrt{3}(\cot\theta + \tan\theta) = 4$

$$\cot\theta + \tan\theta = \frac{4}{\sqrt{3}}$$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{4}{\sqrt{3}}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta.\sin\theta} = \frac{4}{\sqrt{3}}$$

$$\frac{1\times 2}{2\sin\theta.\cos\theta} = \frac{4}{\sqrt{3}}$$

$$\frac{2}{\sin 2\theta} = \frac{4}{\sqrt{3}}$$

$$\sin 2\theta = \sqrt{3}/2$$

$$2\theta = \pi/3$$

$$\boxed{\theta = \pi/6}$$

$\Rightarrow$ If $8\sin^2\theta + 10\sin\theta.\cos\theta + 3\cos^2\theta = 0$; $\theta = ?$ $(8\times3) = 24$ $(12)(2)$

$$8\sin^2\theta + 12\sin\theta.\cos\theta - 2\sin\theta\cos\theta - 3\cos^2\theta = 0.$$

$$4\sin\theta(2\sin\theta + 3\cos\theta) - \cos\theta(2\sin\theta + 3\cos\theta) = 0$$

$$(4\sin\theta - \cos\theta)(2\sin\theta + 3\cos\theta) = 0$$

$4\sin\theta - \cos\theta = 0$	$2\sin\theta + 3\cos\theta = 0$
$4\tan\theta - 1 = 0$	$2\tan\theta + 3 = 0$
$\tan\theta = \frac{1}{4}$	$\tan\theta = \frac{-3}{2}$
$\theta = \tan^{-1}(\frac{1}{4})$	$\theta = \tan^{-1}(-3/2)$
$n\pi + \tan^{-1}(\frac{1}{4})$	$n\pi - \tan^{-1}(3/2)$

$\Rightarrow 3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$; then $\theta =$ ___

$3\times3 = 9$ $(3\sqrt{3})(\sqrt{3})$

$$3\cos^2\theta - 3\sqrt{3}\sin\theta\cos\theta + \sqrt{3}\sin\theta.\cos\theta - 3\sin^2\theta = 0$$

$$3\cos\theta(\cos\theta - \sqrt{3}\sin\theta) + \sqrt{3}\sin\theta(\cos\theta - \sqrt{3}\sin\theta) = 0$$

$$(3\cos\theta + \sqrt{3}\sin\theta)(\cos\theta - \sqrt{3}\sin\theta) = 0$$

$3\cos\theta + \sqrt{3}\sin\theta = 0$	$\cos\theta - \sqrt{3}\sin\theta = 0$
$3 + \sqrt{3}\tan\theta = 0$	$1 - \sqrt{3}\tan\theta = 0$
$\tan\theta = \frac{-3}{\sqrt{3}} = -\sqrt{3}$	$\tan\theta = \frac{1}{\sqrt{3}}$
$\theta = -\pi/3$	$\theta = \pi/6$
$\boxed{n\pi + \pi/3}$	$\boxed{n\pi + \pi/6}$

$\Rightarrow 1 + \sin^2\theta = 3\sin\theta\cdot\cos\theta$; then $\theta =$ ___

$$\sin^2\theta + \cos^2\theta + \sin^2\theta = 3\sin\theta.\cos\theta$$

$$2\sin^2\theta - 3\sin\theta\cos\theta + \cos^2\theta = 0$$

$$2\sin^2\theta - 2\sin\theta.\cos\theta - (1\sin\theta\cos\theta) + \cos^2\theta = 0$$

$$2\sin\theta(\sin\theta - \cos\theta) - \cos\theta(\sin\theta - \cos\theta) = 0$$

$$(2\sin\theta - \cos\theta)(\sin\theta - \cos\theta) = 0.$$

$2\sin\theta - \cos\theta = 0$	$\sin\theta - \cos\theta = 0$
$2\tan\theta - 1 = 0$	$\theta = \pi/4$
$\tan\theta = \frac{1}{2}$	$\boxed{n\pi + \pi/4}$
$\theta = \tan^{-1}(1/2)$	
$\boxed{n\pi + \tan^{-1}(1/2)}$	

All values of x satisfying $\sin 2x + \sin 4x = 2\sin 3x$ are

$\sin 2x + \sin 4x = 2\sin 3x$

$2\sin 3x \cdot \cos x = 2\sin 3x$

$2\sin 3x \cdot \cos x - 2\sin 3x = 0$

$2\sin 3x(\cos x - 1) = 0$

$2\sin 3x = 0$ | $\cos x - 1 = 0$

$\sin 3x = 0$ | $\cos x = 1$

$3x = n\pi$ | $x = 0°$

$\boxed{x = n\pi/3}$ | $x = \boxed{0 = 2n\pi.}$

If $2\sin 2x - \sin x = 0$, then $x =$

$2 \cdot 2\sin x \cdot \cos x - \sin x = 0$

$4\sin x \cdot \cos x - \sin x = 0.$

$\sin x(4\cos x - 1) = 0$

$\sin x = 0$ | $\cos x = 1/4$

$\boxed{n\pi}$

$\cos\theta + \cos 2\theta + \cos 3\theta = 0$; then $\theta =$ ______

$\cos\theta + \cos 3\theta + \cos 2\theta = 0$

$2\cos 2\theta \cdot \cos\theta + \cos 2\theta = 0$

$\cos 2\theta[2\cos\theta + 1] = 0$

$\cos 2\theta = 0$ | $2\cos\theta + 1 = 0$

$2\theta = (2n+1)\pi/2$ | $\cos\theta = -1/2$

$\theta = (2n+1)\pi/4$

$\boxed{\theta = -\pi/6}$ $\quad 2\pi - \frac{\pi}{6}$

$\boxed{\theta = 2n\pi \pm \left(\frac{2\pi}{3}\right)}$

$\frac{\pi}{2} - \frac{\pi}{6}$

$\frac{4\pi}{12}$

$\Rightarrow 4\sin x \cdot \sin 2x \cdot \sin 4x = \sin 3x$, then $\theta =$ ______

$2\sin 4x \cdot 2\sin x \cdot \sin 2x = \sin 3x$

$2\sin 4x[\cos x - \cos 3x] = \sin 3x$

$2\sin 4x \cdot \cos x - 2\sin 4x \cos 3x = \sin 3x$

$\sin 5x + \cancel{\sin 3x} - \sin 7x - \sin x = \cancel{\sin 3x}$

$\sin 5x - \sin 7x - \sin x = 0$

$-2\cos 6x \cdot \sin x - \sin x = 0.$

$-\sin x(2\cos 6x + 1) = 0.$

$\sin x = 0$ | $2\cos 6x + 1 = 0$

$\boxed{x = n\pi}$ | $6x = -\frac{1}{2}$ $\quad \frac{2\pi}{3}$ $\quad 6x = 2n\pi \pm \frac{\pi}{3}$

$\frac{\pi}{6}$ $\quad \boxed{\frac{n\pi}{3} \pm \pi/9}$

The sum of solutions in $(0, 2\pi)$ of equation $\cos x \cdot \cos(\pi/3 - x) \cdot \cos(\pi/3 + x) = \frac{1}{4}$

$\cos x[\cos^2 x - \sin^2 \pi/3] = \frac{1}{4}$

$\frac{\pi}{2}$

$\cos x\left[\cos^2 x - \frac{3}{4}\right]$

$4\cos^3 x - 3\cos x = 1$

$\cos 3x = 1$

$3x = 2n\pi$

$x = \frac{2n\pi}{3}$

Solution $(0, 2\pi) = \frac{2\pi}{3}, \frac{4\pi}{3} =$

$\therefore$ sum $= \frac{2\pi}{3} + \frac{4\pi}{3} = \frac{6\pi}{3} = 2\pi$

If α, β are diff values of x satisfying $a\cos x + b\sin x = c$, then $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a}$

MULTIPLES & SUBMULTIPLES.

$\Rightarrow \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} =$

$\frac{3\sin\theta - 4\sin^3\theta}{\sin\theta} - \frac{(4\cos^3\theta - 3\cos\theta)}{\cos\theta}$

$= \frac{\cancel{\sin\theta}(3-4\sin^2\theta)}{\cancel{\sin\theta}} - \frac{\cancel{\cos\theta}(4\cos^2\theta - 3)}{\cancel{\cos\theta}}$

$= 3 - 4\sin^2\theta - 4\cos^2\theta + 3$

$= 6 - 4(1)$

$= 2$

$\Rightarrow \frac{\cos 3\theta - \sin 3\theta}{\cos\theta + \sin\theta}$

$= \frac{4\cos^3\theta - 3\cos\theta - 3\sin\theta + 4\sin^3\theta}{\cos\theta + \sin\theta}$

$= \frac{4(\cos^3\theta + \sin^3\theta) - 3(\cos\theta + \sin\theta)}{\cos\theta + \sin\theta}$

$= \frac{4[\cos\theta + \sin\theta(\cos^2\theta - \cos\theta.\sin\theta + \sin^2\theta)] - 3(\cos\theta + \sin\theta)}{\cos\theta + \sin\theta}$

$= \frac{4[\cos\theta + \sin\theta(1 - \cos\theta.\sin\theta] - 3(\cos\theta + \sin\theta)}{\cos\theta + \sin\theta}$

$= \frac{4(\cos\theta + \sin\theta)}{\cos\theta + \sin\theta} \cdot \frac{1 - \cos\theta.\sin\theta}{\cancel{\cos\theta + \sin\theta}} - \frac{3(\cos\theta + \sin\theta)}{\cos\theta + \sin\theta}$

$= \frac{4(1 - \cos\theta.\sin\theta)}{\cancel{\cos\theta + \sin\theta}} - 3$

$= 4 - 4\cos\theta.\sin\theta - 3$

$= 1 - 4\cos\theta.\sin\theta$

$= \boxed{1 - 2\sin 2\theta}$

$\Rightarrow \frac{3\cos\theta + \cos 3\theta}{3\sin\theta - \sin 3\theta}$

$\frac{\cancel{3\cos\theta} + 4\cos^3\theta - \cancel{3\cos\theta}}{\cancel{3\sin\theta} - \cancel{3\sin\theta} + 4\sin^3\theta} = \cot^3\theta$

~~$3\cos\theta$~~ [

$\frac{\sin^3 A + \sin 3A}{\sin A} + \frac{\cos^3 A - \cos 3A}{\cos A} =$

Sol:

$\frac{\sin^3 A + 3\sin A - 4\sin^3 A}{\sin A} + \frac{\cos^3 A - 4\cos^3 A + 3\cos A}{\cos A}$

$\sin^2 A + 3 - 4\sin^2 A + \cos^2 A - 4\cos^2 A + 3$

$= 3 - 3\sin^2 A + 3 - 3\cos^2 A$

$= 6 - 3(1)$

$= 3$

$\Rightarrow \frac{\sin 3A}{1 + 2\cos 2A}$

$= \frac{3\sin A - 4\sin^3 A}{1 + 2(1 - 2\sin^2 A)}$

$= \frac{\sin A(3 - 4\sin^2 A)}{1 + 2 - 4\sin^2 A}$

$= \frac{\sin A(3 - 4\sin^2 A)}{(3 - 4\sin^2 A)} = \sin A$

$\Rightarrow$ if $180° < \theta < 270°$, $\sin\theta = -4/5$, then $\tan\theta/2 =$ ______

$\boxed{\cos\theta = 3/5}$

$\frac{180}{2} < \frac{\theta}{2} < \frac{270}{2} \Rightarrow \boxed{90° < \frac{\theta}{2} < 135°}$

$\boxed{90+\theta}$

$\tan\theta/_2 = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-3/5}{1+3/5}} = \sqrt{\frac{2}{8}} = \boxed{\frac{-1}{2}}$

$\Rightarrow$ if $\tan x = \frac{b}{a}$; then $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} =$ ______

$= \sqrt{\frac{1+b/a}{1-b/a}} + \sqrt{\frac{1-b/a}{1+b/a}}$

$= \sqrt{\frac{1+\tan x}{1-\tan x}} + \sqrt{\frac{1-\tan x}{1+\tan x}}$

$= \frac{1+\cancel{\tan x}+1-\cancel{\tan x}}{\sqrt{1-\tan^2 x}}$

$= \frac{2}{\sqrt{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}}$

$= \frac{2\cos x}{\sqrt{\cos 2x}}$

If $\frac{\sin\alpha}{a} = \frac{\cos\alpha}{b}$, then $a\sin 2\alpha + b\cos 2\alpha =$

$\boxed{\frac{\sin\alpha}{\cos\alpha} = \frac{a}{b} = \tan\alpha}$

$a\left[\frac{2\tan\alpha}{1+\tan^2\alpha}\right] + b\left[\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right]$

$a\left[\frac{2a/b}{1+\frac{a^2}{b^2}}\right] + b\left[\frac{1-a^2/b^2}{1+a^2/b^2}\right]$

$a\left[\frac{2a\times \cancel{b^2}}{\cancel{b}(b^2+a^2)}\right] + b\left[\frac{b^2-a^2}{b^2+a^2}\right]$

$a\left[\frac{2a\cdot b}{a^2+b^2}\right] + b\left[\frac{b^2-a^2}{b^2+a^2}\right]$

$\frac{2a^2b + b^3 - a^2b}{a^2+b^2}$

$= b\left[\frac{2a^2+b^2-a^2}{a^2+b^2}\right]$

$= b\left[\frac{\cancel{a^2+b^2}}{\cancel{a^2+b^2}}\right] = b$

If $\tan\theta = 3/4$; then value of $\tan 2\theta + \sec 2\theta =$ —

$= \tan 2\theta + \sec 2\theta$

$= \frac{2\tan\theta}{1-\tan^2\theta} + \frac{1+\tan^2\theta}{1-\tan^2\theta}$

$= \frac{2\tan\theta + 1 + \tan^2\theta}{1-\tan^2\theta}$

$= \frac{(\tan\theta+1)^{\cancel{2}}}{(1-\tan\theta)(\cancel{1+\tan\theta})} = \frac{\tan\theta+1}{1-\tan\theta} = \frac{1+3/4}{1-3/4}$

$= 7$

Std.:

$\tan x + \tan\left(x+\frac{\pi}{3}\right) + \tan\left(x+\frac{2\pi}{3}\right) = 3$

$\boxed{\tan 3x = 1}$

$\Rightarrow$ if $\tan 35° = k$, then $\frac{\tan 145° - \tan 125°}{1+\tan 145°\cdot\tan 125°}$

$\therefore \tan(145-125) = \boxed{\tan 20°}$

$\tan 20 = \cot 70 = \frac{1}{\tan 70}$

$= \frac{1}{\tan(2\times 35)} = \frac{1-\tan^2 35}{2\tan 35} = \frac{1-k^2}{2k}$

$\Rightarrow \operatorname{cosec}\theta = \frac{p+q}{p-q}$ then $\cot(\pi/4 + \theta/2)$

$\frac{1}{\sin\theta} = \frac{p+q}{p-q}$

$\frac{1+\sin\theta}{1-\sin\theta} = \frac{\cancel{2}p}{\cancel{2}q}$

$\frac{1+\sin\theta}{1-\sin\theta} = p/q$

$\frac{\sin^2\theta/2 + \cos^2\theta/2 + 2\cos\theta/2 \cdot \sin\theta/2}{\sin^2\theta/2 + \cos^2\theta/2 - 2\cos\theta/2 \cdot \sin\theta/2} = p/q$

$\frac{(\sin\theta/2 + \cos\theta/2)^2}{(\sin\theta/2 - \cos\theta/2)^2} = p/q$

$\frac{\sin\theta/2 + \cos\theta/2}{\sin\theta/2 - \cos\theta/2} = \sqrt{p/q}$

$\tan(\pi/4 + \theta/2) = \sqrt{p/q}$

$\boxed{\cot(\pi/4 + \theta/2) = \sqrt{q/p}}$

$\Rightarrow \sin 12^\circ . \sin 24^\circ , \sin 48^\circ . \sin 84^\circ$

$\frac{\sin 12^\circ . \sin 48^\circ . \sin 72^\circ . \sin 24^\circ . \sin 84^\circ , \sin 36^\circ}{\sin 72 . \sin 36}$

$= \frac{\frac{1}{4}\cancel{\sin 36} \cdot \frac{1}{4}\cancel{\sin 72}}{\cancel{\sin 72} . \cancel{\sin 36}} = \frac{1}{16}$

$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ =$ ______

$\tan 9 + \tan 81^\circ - (\tan 27^\circ + \tan 63)$

$\tan 9 + \cot 9 - (\tan 27 + \cot 27)$

$\frac{\sin 9}{\cos 9} + \frac{\cos 9}{\sin 9} - \left(\frac{\sin 27}{\cos 27} + \frac{\cos 27}{\sin 27}\right)$

$\frac{1}{\cos 9 . \sin 9} - \left(\frac{1}{\cos 27 . \sin 27}\right)$

$\frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$

$\frac{2 \times 4}{\sqrt{5}-1} - \frac{2 \times 4}{\sqrt{5}+1}$

$8\left[\frac{\sqrt{5}+1 - \sqrt{5}+1}{5-1}\right] = 8 \times \frac{2}{4} = 4$

A quadratic eqn. whose roots are $\sin^2 18^\circ$, $\cos^2 36^\circ$ are

$\boxed{x^2 - (\alpha+\beta)x + \alpha.\beta}$

$\sin^2 18^\circ + \cos^2 18 = \left(\frac{\sqrt{5}-1}{4}\right)^2 + \left(\frac{\sqrt{5}+1}{4}\right)^2$

$= \frac{1}{16}\left[2(5+1)\right]$

$= \frac{1}{16} \times 12 = 3/4.$

$\sin^2 18^\circ . \cos^2 36^\circ = \left(\frac{\sqrt{5}-1}{4}\right)^2 . \left(\frac{\sqrt{5}+1}{4}\right)^2$

$= \left(\frac{5-1}{16}\right)^2 = \left(\frac{4}{16}\right)^2 = \frac{1}{16}$

$\therefore x^2 - \left(\frac{3}{4}\right)x + \frac{1}{16} = 0$

$\boxed{16x^2 - 12x + 1 = 0}$

$(1+\cos\pi/10)(1+\cos 3\pi/10)(1+\cos 7\pi/10)(1+\cos 9\pi/10)$

$\Rightarrow$ $(1+\cos\pi/10)(1+\cos 3\pi/10)(1+\cos(\pi - 3\pi/10))(1+\cos(\pi - \pi/10))$

$(1+\cos\pi/10)(1+\cos 3\pi/10)(1-\cos 3\pi/10)(1-\cos\pi/10)$

$(1-\cos^2\pi/10)(1-\cos^2 3\pi/10)$

$(\sin^2 \pi/10)(\sin^2 3\pi/10)$

$\sin^2 18^\circ . \sin^2 54^\circ$

$= \left(\frac{\sqrt{5}-1}{4}\right)^2 . \left(\frac{\sqrt{5}+1}{4}\right)^2$

$= \left(\frac{4}{16}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

COMPOUND ANGLES:

$\sec 255° = —$

$= \sec(270-15)$

$= -\text{cosec}\,15° \Rightarrow \frac{-2\sqrt{2}}{\sqrt{3}-1}$

$= \frac{-\cancel{2}\sqrt{2}(\sqrt{3}+1)}{\cancel{2}} = \boxed{-\sqrt{6}-\sqrt{2}}$

$\Rightarrow \sin^2 52\frac{1}{2}° - \sin^2 22\frac{1}{2}°$

$\sin^2 A - \sin^2 B = \sin(A+B).\sin(A-B)$

$= \sin(52\frac{1}{2}+22\frac{1}{2}) \cdot \sin(52\frac{1}{2}-22\frac{1}{2})$

$= \sin 75 . \sin 30$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{3}+4}{4\sqrt{2}}}$

$\Rightarrow \cos\alpha . \sin(\beta-\gamma) + \cos\beta . \sin(\gamma-\alpha) + \cos\gamma . \sin(\alpha-\beta) =$

$\cos\alpha\ \cancel{\sin\beta . \cos\gamma} - \cancel{\cos\alpha\ \sin\gamma}.\cos\beta + \cancel{\cos\beta . \sin\gamma}.$
$\cancel{\cos\alpha} - \cos\beta\ \cancel{\sin\alpha . \cos\gamma} + \cancel{\cos\gamma\ \sin\alpha\ \cos\beta} - \cos\gamma$
$\cancel{\sin\beta . \cos\alpha}. = 0$

If A, B are acute angles, $\tan A = \frac{n}{n+1}$; $\tan B = \frac{1}{2n+1}$ then $(A+B) =$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{n}{n+1} + \frac{1}{2n+1}}{1 - \frac{n}{n+1} \cdot \frac{1}{2n+1}} = \frac{2n^2+n+n+1}{(n+1)(2n+1) - n}$

$= \frac{\cancel{n(2n+1)+16}\quad 2n^2+n+n+1}{2n^2 \cancel{+n} + 2n + 1 - \cancel{n}}$

$\tan(A+B) = \frac{\cancel{2n^2+n+n+1}}{2n^2 \cancel{+2n} + 1} = 1$

$\boxed{A+B = \pi/4}$

If $\cos(A-B) = 3/5$; $\tan A . \tan B = 2$, then $\cos(A+B) = ?$

$\tan A . \tan B = 2$

$\frac{\sin A . \cancel{\cos}\sin B}{\cos A . \cos B} = 2$

$\frac{a}{b} = \frac{c}{d}$

$\boxed{\frac{a+b}{a-b} = \frac{c+d}{c-d}}$

$\frac{\cos A \cos B}{\sin A . \sin B} = \frac{1}{2}$

$\frac{\cos A . \cos B + \sin A . \sin B}{\cos A \cos B - \sin A . \sin B} = \frac{1+2}{1-2}$

$\frac{\cos(A-B)}{\cos(A+B)} = -3$

$\frac{\frac{3}{5}}{\cos(A+B)} = -3$

$\boxed{\cos(A+B) = -1/5}$

$\Rightarrow \cos\theta + \cos(120+\theta) + \cos(120-\theta) =$

$\cos\theta + 2\cos 120\theta . \cos\theta$

$\cos\theta + 2\cos(180-60) . \cos\theta$

$= \cos\theta - 2\cos 60° \cos\theta$

$= \cos\theta - \cos\theta$

$= 0.$

$A+B+C = 180°$, the $\sum \frac{\cot A + \cot B}{\tan A + \tan B}$

$\sum \frac{\frac{1}{\tan A} + \frac{1}{\tan B}}{\tan A + \tan B} = \sum \frac{\cancel{\tan A + \tan B}}{\tan A . \tan B . (\cancel{\tan A + \tan B})}$

$= \sum \frac{1}{\tan A . \tan B} = \sum \cot A . \cot B$

$\therefore \sum \cot A \cot B = 1 \quad (\because A+B+C = n\pi)$

☆ Partial Fractions ☆

→ If the remainders of polynomial $f(x)$ when divided by $x-1$, $x-2$ are 2, 5, then remainder of $f(x)$ when divided by $(x-1)(x-2)$ is

$f(1) = 2$

$f(2) = 5$

$f(x) = \underset{(x-1)(x-2)}{\text{Divisor}} \times \text{quotient} + R(x).$

$f(1) = 0 \times \text{quotient} + R(x)$

$2 = R(x) = \boxed{a(1)+b} = C \because ax+b)$

$\boxed{a+b=2}$ — ①

$f(2) = (x-1)\cancel{(x-2)}^{0} \text{ quotient} + r(x)$

$5 = 2a+b \boxed{(ax+b)}$

$\boxed{2a+b=5}$ — ②

$2a+b = 5$
$-a+b = 2$
$a = 3$

$b = -1$ form of $r(x) = ax+b$
$= \boxed{3x-1}$

$\Rightarrow \frac{x^3}{(2x-1)(x+2)(x-3)} = A + \frac{B}{2x-1} + \frac{C}{x+2} + \frac{D}{x-3}$

$A = \frac{\text{Highest degree cofficient in Numerator}}{\text{Highest degree coefficient in Denominator}}$

$\boxed{A = \frac{1}{2}}$

$\frac{x+1}{(2x-1)(3x+1)} = \frac{A}{2x+1} + \frac{B}{3x+1} \Rightarrow 16A+9B$

$\boxed{x=5}$

$\frac{6}{(9)(6)} = \frac{A}{9} + \frac{B}{16}$

$\boxed{6 = \frac{16A+9B}{\cancel{9\times 6}}}$ $= 6$//
$\cancel{9\times 6}$

$\Rightarrow \frac{3x^2+x+1}{(x-1)^4} = \frac{a}{x-1} + \frac{b}{(x-1)^2} + \frac{c}{(x-1)^3} + \frac{d}{(x-1)^4}$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = $ ______

$\boxed{\text{put } x = 0}$

$1 = -a+b-c+d$

$\boxed{1 = b+d-(a+c)} \Rightarrow \begin{bmatrix} \cancel{3}5 \\ 7\cancel{6} \end{bmatrix} \begin{bmatrix} 0 & 3 \\ 7 & 5 \end{bmatrix}$

$\Rightarrow \frac{x^2+5}{(x^2+2)^2} = \frac{1}{x^2+2} + \frac{k}{(x^2+2)^2}$: then $k =$ ____

$\boxed{\text{put } x = 0}$

$\frac{5}{4} = \frac{1}{2} + \frac{k}{4}$

$\frac{5}{4} - \frac{1}{2}$

$\frac{5-2}{\cancel{4}} = \frac{k}{\cancel{4}}$

$\boxed{k=3}$

$\Rightarrow \frac{x+1}{x^4(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+2}$;

then $B+D+E =$ ______

$\boxed{\text{put } x=-1}$

$0 = -A+B-C+D+E$

$\boxed{B+D+E = A+C}$

$\Rightarrow \frac{1}{(x-1)^2(x+1)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-1)^2} + \frac{B}{x+1}$

$A_1 = \frac{1}{(n-1)!}\left[\frac{d^{n-1}}{dx}\left(\frac{1}{x+1}\right)\right]_{x=1}$

$= \frac{1}{(2-1)!}\left[\frac{d^{2-1}}{dx}\left(\frac{1}{x+1}\right)\right]_{x=1}$

$= \frac{1}{1}\left(\frac{-1}{(x+1)^2}\right)_{x=1}$

$\boxed{A_1 = \frac{-1}{4}}$

$A_2 = \frac{1}{(2-2)!}\left[\frac{d^{2-2}}{dx}\left(\frac{1}{1+x}\right)\right]_{x=1}$

$= \frac{1}{x+1}\left(\frac{1}{2}\right)$

$\boxed{A_2 = \frac{1}{2}}$

$B = \frac{1}{4}$//

☆ PERIODICITY AND EXTREME VALUES ☆

T.F	Domain	Range	period
$\sin x$	R	$[-1, 1]$	2π
$\cos x$	R	$[-1, 1]$	2π
$\tan x$	$R - \{(2n+1)\cdot\pi/2\}$	R	π
$\cot x$	$R - \{n\pi\}$	R	π
$\sec x$	$R - \{(2n+1)\pi/2\}$	$R - [-1, +1]$	2π
$\operatorname{cosec} x$	$R - \{n\pi\}$	$R - [-1, +1]$	2π

properties of periodic function :

(I) period of $f(x) = T$; then period of

$\frac{1}{f(x)}$

$\sqrt{f(x)}$

$c^{f(x)}$

$a^{f(x)} (a>0)$ — T only.

$-f(x)$

$Af(x)$

$Af(x+B)$

$Af(x+B)+C$

period of $Af(Kx+B)+C$ is $\frac{T}{|K|}$

(II). Algebraic functions are non periodic.

$x^2, \frac{1}{|x|}, \frac{1}{1+x}, (1+x)^2$

. functions of $\sin\sqrt{x}$, $\cos(x^2)$, $\tan(1/x)$, $x\sin x$, $x^2\sec x$, $x+\sec x$, $\longrightarrow$ $\boxed{\text{Non-periodic}}$

• period of $x - [x] = \{x\} \Rightarrow 1$.

• let period of $g(x) = T_1$, period of $h(x) = T_2$] period of $f(x) =$ l.c.m. of (T_1, T_2)

$\Rightarrow f(x) \Rightarrow g(x) + h(x)$

$= g(x) - h(x)$

$= \frac{g(x)}{h(x)}$

$= g(x) \times h(x)$

Note: l.c.m. will fail, if argument coefficient $= 1$

Eg: $f(x) = |\sin x| + |\cos x|$

$\boxed{T = \pi/2}$

$\sin^4 x + \cos^4 x \Rightarrow \pi/2$

(III) $\sin^n x, \cos^n x, \sec^n x, \operatorname{cosec}^n x$ — $n =$ even $p = \pi$; $n =$ odd $P = 2\pi$

$\tan^n x, \cot^n x \longrightarrow \boxed{P = \pi} \Rightarrow$ (n may be even/odd)

period of $|\sin x|, |\cos x|, |\tan x|, |\cot x|, |\sec x|, |\operatorname{cosec} x| \Rightarrow P = \pi$

$f(x) = a\sin^n x + b\operatorname{cosec}^n x$

n	a=b	a≠b
even	$\pi/2$	π
odd	2π	2π

$f(x) = a\tan^n x + b\cot^n x$

n	a=b	a≠b
even	$\pi/2$	π
odd	π	π

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$f(x) = a|\sin x| + b|\cos x|$

$= a|\tan x| + b|\cot x|$

$= a|\sec x| + b|\csc x|$

$\Rightarrow$ $\boxed{p = \pi/2 \text{ if } a = b}$

$\boxed{p = \pi \text{ if } a \neq b}$

$\rightarrow$ if $f(x+10) + f(x+4) = 7$; then

period of $f(x) = 2(10-4) = \underline{12}$

Note: period of $f(x) = 2$ (diff of arguments)

Extreme values. (Range)

$x \leq k \Rightarrow x_{max} = k$.

$x \geq k \Rightarrow x_{min} = k$.

* $\boxed{-1 \leq \cos nx \leq 1}$

$\boxed{-1 \leq \sin nx \leq 1}$

$\boxed{0 \leq \sin^2 x \leq 1}$

$\boxed{0 \leq |\sin x| \leq 1}$

Range of standard Trig. function:

$\boxed{a\cos x + b\sin x + c}$ $\rightarrow$ valid for same argument

(min value, max values) $= \left[c - \sqrt{a^2+b^2},\ c + \sqrt{a^2+b^2}\right]$

Eg: Range of $27^{\cos 2x} \cdot 81^{\sin 2x}$

$= (3)^{3\cos 2x} \cdot 3^{4\sin 2x}$

$= 3^{3\cos 2x + 4\sin 2x}$

$= 3^{\left(0 - \sqrt{9+16},\ 0 + \sqrt{9+16}\right)}$

$= 3^{(-5, 5)}$

$= (3^{-5}, 3^{5})$

Range based on AM/GM inequality.

AM $\geq$ GM

$$\boxed{\frac{a+b}{2} \geq \sqrt{ab}}$$

$\rightarrow$ if functions are reciprocal, we should use this case

① $9\tan\theta + 16\cot\theta$; Find range:

$$\frac{9\tan\theta + 16\cot\theta}{2} \geq \sqrt{9\tan\theta \cdot 16\cot\theta}$$

$$\frac{9\tan\theta + 16\cot\theta}{2} \geq 12$$

$$9\tan\theta + 16\cot\theta \geq 24$$

Range of $9\tan\theta + 16\cot\theta = [24, \infty)$

② $A+B+C = \pi$, then least value of $\tan A \cdot \tan B \cdot \tan C$ ($= E$)

$$\frac{\tan A + \tan B + \tan C}{3} \geq \sqrt[3]{\tan A \cdot \tan B \cdot \tan C}$$

$$\frac{\Sigma \tan A}{3} \geq \sqrt[3]{E}$$

$$\frac{E}{E^{1/3}} \geq 3$$

$$E^{2/3} \geq 3$$

$$E \geq 3^{3/2} \Rightarrow \sqrt{27} = \sqrt{3 \times 3 \times 3} = 3\sqrt{3}$$

★ ANALYTICAL GEOMETRY ★

★ Straight Lines ★

point slope form: $(m) = \frac{y_2 - y_1}{x_2 - x_1}$

straight lines

parallel
$m_1 = m_2$

$\perp$
$\boxed{m_1 m_2 = -1}$

Slope of line $ax + by + c = 0$ is $\frac{-a}{b}$

$\boxed{|a|+|b| \neq 0}$ " x intercept $= \frac{-c}{a}$

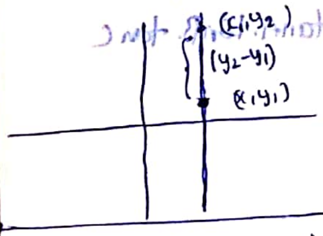
" y intercept $= \frac{-c}{b}$

distance b/w 2 points (x_1, y_1); (x_2, y_2)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

distance b/w 2 point (x_1, y_1) and (x_2, y_2) on a line parallel to y-axis is $|y_2 - y_1|$

$\boxed{x_1 = x_2}$



distance b/w 2 points (x_1, y_1) and (x_2, y_2) on a line parallel to x-axis is $\boxed{|x_2 - x_1|}$

$\boxed{y_1 = y_2}$

coordinates of the point dividing line segment joining (x_1, y_1) & (x_2, y_2) externally in ratio $m:n$.

is $\left[\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right]$

whereas internally:

$\left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$

centroid of $\Delta le \Rightarrow \left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$

centroid $\rightarrow$ intersection all medians
It divides each median in 2:1 ratio.

$A(x_1, y_1)$; $B(x_2, y_2)$; $C(x_3, y_3)$

Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$\rightarrow$ if $\Delta = 0$; then points, A, B, C are collinear.

$\rightarrow$ Area of Δle having $(0,0)$, (x_1, y_1) (x_2, y_2) is
$\frac{1}{2}|x_1 y_2 - x_2 y_1|$

$\rightarrow$ Area of Δle formed by $ax + by + c = 0$ with coordinate axes $\Rightarrow \frac{1}{2}\left|\frac{c^2}{ab}\right|$

$\rightarrow$ Area of Δle formed by $(0,0)$; $(x, 0)$, $(0, y)$
$= \frac{1}{2}|xy|$

equation of line passing through $(0,0)$, having slope $m \Rightarrow \boxed{y = mx}$

; having intercept 'c' on y axis $\Rightarrow \boxed{y = mx + c}$

equation of line $A(x_1, y_1)$; $B(x_2, y_2)$

$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$

(or)

$\boxed{y - y_1 = m(x - x_1)}$

Eqn of line having x-intercept $\Rightarrow a$
" y-intercept $\Rightarrow b$

$\therefore \boxed{\frac{x}{a} + \frac{y}{b} = 1}$

$\boxed{\text{Slope} = \frac{-b}{a}}$

Mathematician who introduced coordinate geometry $=$ rene descartes

Eqn. of straight line passing through (x_1, y_1) and parallel to $ax+by+c=0$:

$$\boxed{ax+by = ax_1+by_1}$$

Eqn. of straight line passing through (x_1,y_1) and $\perp$ to $ax+by+c=0$.

$$\boxed{bx-ay = bx_1-ay_1}$$

The Ratio that line $\boxed{ax+by+c}=0$ divides line segment joining $A(x_1,y_1)$ & $B(x_2,y_2)$

$$= -\left[\frac{ax_1+by_1+c}{ax_2+by_2+c}\right]$$

x_1, y_1; $ax+by+c$; x_2, y_2.

$\perp$le distance: $\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$\parallel$-distance: $\dfrac{|c_1-c_2|}{\sqrt{a^2+b^2}}$

distance b/w $(0,0)$ and line $ax+by+c=0$ is

$$=\frac{|c|}{\sqrt{a^2+b^2}}$$

$A(x_1,y_1)$; $B(x_2,y_2)$; $C(x_3,y_3)$ $\quad a,b,c \rightarrow$ length of Δle

incenter of Δle $ABC = \left[\dfrac{ax_1+bx_2+cx_3}{a+b+c}, \dfrac{ay_1+by_2+cy_3}{a+b+c}\right]$

$$m = \tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

Eqn. of straight line whose distance from origin is 'p', and makes an angle α with +ve x-axis:

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$

equation of straight line passing through (x_1,y_1) and making an angle θ with x-axis

$$\boxed{\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}}$$

$\boxed{ax^2+2hxy+by^2=0} \Rightarrow$ represents two real & different straight lines; if $\boxed{h^2>ab}$

$h^2=ab \Rightarrow$ coincident lines

$h^2<ab \Rightarrow$ Imaginary lines

Angle b/w pair of lines

$$\tan\alpha = \frac{2\sqrt{h^2-ab}}{a+b}$$

$\rightarrow$ equation of line whose segment b/w coordinate axes is divided by point (x_1,y_1) in the ratio $= m:n$ is

$$\boxed{\frac{mx}{x_1} + \frac{ny}{y_1} = m+n}$$

$\rightarrow$ If $Q(h,k)$ is the foot of perpendicular from $P(x_1,y_1)$ on a straight line $ax+by+c=0$ then;

$$\boxed{\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}}$$

$P(x_1,y_1)$; $Q(h,k)$; $ax+by+c$

$\rightarrow$ If $Q(h,k)$ is the image of point $P(x_1,y_1)$ with respect to straight line $ax+by+c=0$ then;

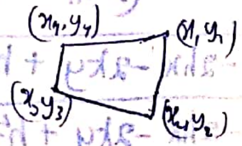
$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

$ax+by+c$; $P(x_1,y_1)$; $Q(h,k)$

To find area of any polygon of n sides with given coordinates

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & \cdots & x_n & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_5 & \cdots & y_n & y_1 \end{vmatrix}$$

Eg: Area of quadrilateral



Sol: $\Delta = \dfrac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$

$$= \frac{1}{2}\left|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)\right|$$

★Circles★

General form of equation:

$$\boxed{ax^2+by^2+2hxy+2gx+2fy+c=0}$$

$\boxed{a=b\neq 0}$ $\quad \boxed{2hxy=0} \Downarrow \boxed{h=0}$

$\boxed{a=b=1}$

radius $=\sqrt{g^2+f^2-c}$

centre $=[-g,-f]$

eqn of circle with centre $(0,0)$ =

$$\boxed{x^2+y^2=r^2}$$

Circle with centre (h,k) =

$$x^2+y^2-2hx-2ky+h^2+k^2=r^2$$

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$

→ eqn. of circle passing through $(0,0)$

$$\Rightarrow \boxed{x^2+y^2+2gx+2fy=0}$$

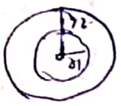
→ eqn of circle, whose centre lies on X-axis

$$\Rightarrow \boxed{x^2+y^2+2gx+c=0}$$

→ eqn of circle whose centre lies on Y-axis

$$\Rightarrow \boxed{x^2+y^2+2fy+c=0}$$

Concentric circles: differ in $\boxed{\text{'C' term}}$



(h,k)

$$(x-h)^2+(y-k)^2=r_1^2$$
$$(x-h)^2+(y-k)^2=r_2^2$$

$$\boxed{\begin{aligned}&x^2+y^2-2hx-2ky+h^2+k^2-r_1^2=0\\&x^2+y^2-2hx-2ky+h^2+k^2-r_2^2=0\end{aligned}}$$

$g^2-c=0 \rightarrow$ X-axis touches the circle

$f^2-c=0 \rightarrow$ Y-axis touches the circle

$g^2-c<0 \rightarrow$ circle doesn't meet X-axis

$f^2-c<0 \rightarrow$ circle doesn't meet y-axis

Length of chord:

 $= 2\sqrt{r^2-l^2}$ → l distance from centre to chord

↓ radius

eqn. of circle $=\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$

→ if $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3), D(x_4,y_4)$ said to be $\boxed{\text{concyclic}}$ if they lied on same circle

equation of circle passing through 3-different points

Eg: $(0,0), (a,0), (0,b)$

→ should take 2-pair of points that in which coordinates are ~~ea~~ unequal.

$(0,0)$ $(a,0)$ ✗

$(a,0)$ $(0,b)$ ✓

$(0,b)$ $(0,0)$ ✗

$(a,0)(0,b)$

eqn. $=(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$(x-a)(x)+y(y-b)=0$

$x^2-ax+y^2-yb=0$

$$\boxed{x^2+y^2-ax-by=0}$$

Length of Intercepts:

$$\boxed{x^2+y^2+2gx+2fy+c=0}$$

x-intercept $\Rightarrow \boxed{2\sqrt{g^2-c}}$

y-intercept $\Rightarrow \boxed{2\sqrt{f^2-c}}$

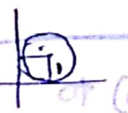
Circle with x-axis as tangent:

$$\boxed{x^2+y^2+2gx+2fy+g^2=0}$$

Circle with y axis as tangent

$$\boxed{x^2+y^2+2gx+2fy+f^2=0}$$

eqn. of circle touching both the axes:



$(\pm r, \pm r)$

eqn: $x^2+y^2 \pm 2rx \pm 2ry + r^2 = 0$.

$\boxed{x^2+y^2+2gx+2fy+c}$

$\therefore \boxed{C = r^2 = g^2 = f^2}$

Notations:

$S \equiv x^2+y^2+2gx+2fy+c=0$

$S_1 \equiv xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

$S_2 \equiv xx_2+yy_2+g(x+x_2)+f(y+y_2)+c=0$

$S_{12} \equiv x_1x_2+y_1y_2+g(x_1+x_2)+f(y_1+y_2)+c=0.$

$S_{11} = x_1^2+y_1^2+2gx_1+2fy_1+c=0$

eqn. of tangent:

$$\boxed{S_1 \equiv xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0}$$

Let point $A(x_1y_1)$

if $S_{11}=0$; A lies on circle

if $S_{11}>0$; A lies outside circle

if $S_{11}<0$; A lies inside circle

eqn. of tangent for $x^2+y^2=r^2$ having slope (m)

$$\boxed{y = mx \pm r\sqrt{1+m^2}}$$

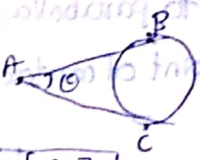
Condition of tangency:

$c = \pm r\sqrt{1+m^2}$

$$\boxed{c^2 = r^2(1+m^2)}$$

→ eqn. of tangent for $(x-h)^2+(y-k)^2=r^2$, having slope m' = $\boxed{(y-k) = m(x-h) \pm r\sqrt{1+m^2}}$

length of tangent = $\sqrt{S_{11}}$



$$\boxed{\theta = 2\tan^{-1}\left[\frac{r}{\sqrt{S_{11}}}\right]}$$

★ Relative positions of 2 circles ★

condition	Description
$C_1C_2 > r_1+r_2$	circles donot intersect
$C_1C_2 = r_1+r_2$	circles touch each other externally
$\lvert r_1-r_2\rvert < C_1C_2 < r_1+r_2$	2-circles intersect at two different points
$C_1C_2 = \lvert r_1-r_2\rvert$	2-circles touch each other internally
$C_1C_2 < \lvert r_1-r_2\rvert$	do not intersect, but one circle will be completely inside the other
$C_1C_2 = 0$	Concentric circles

Common Tangents :-

CONDITION	NO. OF TANGENTS
$C_1C_2 > r_1+r_2$	4
$C_1C_2 = r_1+r_2$	3
$\lvert r_1-r_2\rvert < C_1C_2 < r_1+r_2$	2
$C_1C_2 = \lvert r_1-r_2\rvert$	1
$C_1C_2 < \lvert r_1-r_2\rvert$	0

→ Combined equation of pair of tangents drawn from external point (P) (x_1,y_1) to the circle $S=0$ is

$$\boxed{S.S_{11} = S_1^2}$$

Types of curves:

parabola: ⇒ $\boxed{\text{eccentricity } (e) = 1}$

~~eqn of~~ A straight line $\boxed{y=mx+c}$ is the tangent to a $\boxed{\text{parabola}}$ $\boxed{y^2=4ax}$; then

$$\boxed{c = \frac{a}{m}}$$

→ eqn. of tangent to parabola $\boxed{y^2=4ax}$ is $\boxed{y = mx + \frac{a}{m}}$; it's point of contact $= \left[\frac{a}{m^2}, \frac{2a}{m}\right]$

Ellipse: $\boxed{\text{eccentricity } (e) < 1}$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \quad [S]$$

Equations of ellipse:

$$S_1 \equiv \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0.$$

$$S_{12} \equiv \frac{x_1x_2}{a^2} + \frac{y_1y_2}{b^2} - 1 = 0$$

$$S_{11} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 = 0$$

let point $P(x_1,y_1)$:

→ P lies outside ellipse ⇒ $\boxed{S_{11} > 0}$

→ P lies on ellipse ⇒ $\boxed{S_{11} = 0}$

→ P lies inside ellipse ⇒ $\boxed{S_{11} < 0}$

→ straight line $\boxed{y=mx+c}$ represents a tangent to ellipse $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$; then

$$\boxed{c^2 = a^2m^2 + b^2}$$

→ Eqn. of tangent at $P(x_1,y_1)$ to ellipse $\boxed{S=0}$ is $\boxed{S_1=0}$

$$\boxed{\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1}$$

→ Eqn. of Normal at $P(x_1,y_1)$ to ellipse $\boxed{S=0}$ is

$$\boxed{\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2}$$

→ Eqn. of tangent at $P(\theta)$ to ellipse $(S)=0$

$$\boxed{\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1}$$

→ Eqn. of Normal at $P(\theta)$ to ellipse $(S)=0$;

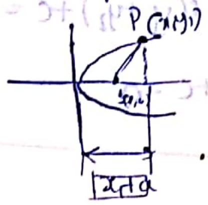
$$\boxed{\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2}$$

Eqn. of tangent at point $P(x_1,y_1)$ to parabola $(S)=0$ given by $\boxed{S_1=0}$

$$\boxed{yy_1 - 2a(x+x_1) = 0}$$

Eqn. of Normal at $P(x_1,y_1)$ on parabola $(S)=0$ given by, $\boxed{y - y_1 = \frac{-y_1}{2a}(x - x_1)}$

Focal distance of parabola $y^2=4ax$ whose focus $(S) = (a,0)$ and point $P(x_1,y_1)$ is $\boxed{x_1+a}$



products of lengths of perpendiculars from foci on any tangent to the ellipse $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$ is $\boxed{b^2}$

* Hyperbola *

$\boxed{\text{eccentricity } (e) > 1}$

→ $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1} \Rightarrow (S)$

→ parametric equation of hyperbola

$$\boxed{\begin{aligned} x &= a\sec\theta \\ y &= \pm b\tan\theta \end{aligned}}$$

Equations of hyperbola :

$S_1 \Rightarrow \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1 = 0$

$S_{12} \Rightarrow \frac{x_1x_2}{a^2} - \frac{y_1y_2}{b^2} - 1 = 0$

$S_{11} \Rightarrow \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 = 0.$

position of point (P) $[x_1, y_1]$ on Hyperbola :

$S_{11} > 0 \Rightarrow$ point lies on outside

$S_{11} < 0 \Rightarrow$ point lies inside

$S_{11} = 0 \Rightarrow$ point lies on Hyperbola.

• Rectangular hyperbola = $\boxed{x^2 - y^2 = a^2}$

$e = \sqrt{2}$

→ Eqn. of tangent to hyperbola $(S) = 0$, at $P(x_1, y_1)$

is $\boxed{S_1 = 0}$ $\boxed{\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1}$

→ Eqn. of tangent at $P(\theta)$ to $S = 0$, ⇒ is

$$\boxed{\frac{x}{a}(\sec\theta) - \frac{y}{b}(\tan\theta) = 1}$$

→ Eqn. of normal to $\boxed{S=0}$ at $P(x_1, y_1)$ is

$$\boxed{\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = (a^2+b^2).}$$

→ Eqn of Normal at $P(\theta)$ to $S = 0$ is

$$\boxed{\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2}$$

straight line $\boxed{y = mx + c}$ represents tangent to hyperbola $\boxed{S=0}$ if when

$\boxed{c^2 = a^2m^2 - b^2}$

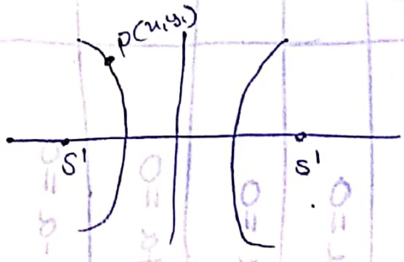
→ eqn. of tangent to $\boxed{S=0}$; having slope(m) is $\boxed{y = mx \pm \sqrt{a^2m^2 - b^2}}$

→ If e and e_1 are the eccentricities of hyperbola and it's conjugate hyperbola respectively then $\boxed{\frac{1}{e^2} + \frac{1}{e_1^2} = 1}$

If $S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is hyperbola and

$S' \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is conjugate hyperbola,

then eqn. of asymptote is $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0}$



If P is point on hyperbola $(S) = 0$ with foci S and S' then $\boxed{S'P - SP = 2a}$

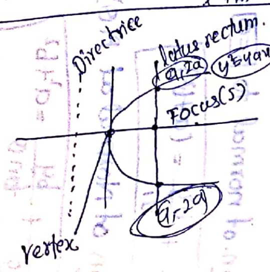
Forms of parabolas :

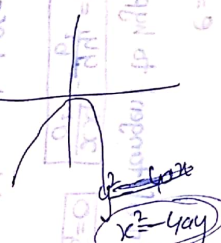
Forms of parabolas	Focus(S)	Equation of directrices	Axis	Vertex
① $y^2=4ax$; $[a>0]$	$(a,0)$	$x+a=0.$	$y=0$	$(0,0)$
② $y^2=-4ax$; $[a>0]$	$(-a,0)$	$x-a=0$	$y=0$	$(0,0)$
③ $x^2=4ay$; $[a>0]$	$(0,a)$	$y+a=0$	$x=0.$	$(0,0)$
④ $x^2=-4ay$; $[a>0]$	$(0,-a)$	$y-a=0$	$x=0.$	$(0,0)$
⑤ $(y-k)^2=4a(x-h)$; $[a>0]$	$(a+h,k)$	$x-h+a=0$	$y-k=0.$	(h,k)
⑥ $(y-k)^2=-4a(x-h)$; $[a>0]$	$(h-a,k)$	$x-h-a=0$	$y-k=0$	(h,k)
⑦ $(x-h)^2=4a(y-k)$; $[a>0]$	$(h,a+k)$	$y-k+a=0$	$x-h=0$	(h,k)
⑧ $(x-h)^2=-4a(y-k)$; $[a>0]$	$(h,k-a)$	$y-k-a=0$	$x-h=0$	(h,k)
$(x-\alpha)^2+(y-\beta)^2=\frac{(lx+my+n)^2}{l^2+m^2}$	(α,β)	$lx+my+n=0$	$m(x-\alpha)+l(y-\beta)=0$	Any point.

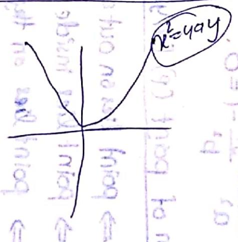
★ points :

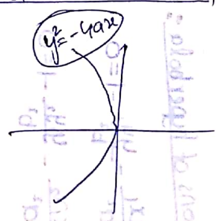
$\boxed{y^2=4ax}$

- parametric equations of parabola :- $\boxed{\begin{array}{l} x=at^2 \\ y=2at \end{array}}$
- length of latus rectum of $\boxed{y^2=4ax \Rightarrow 4a}$



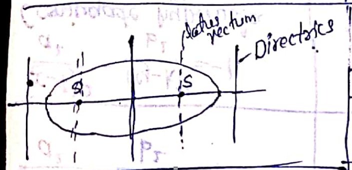






Forms of Ellipses :

Ellipses	Major axis is along	length of major axis	length of minor axis	centre (C)	Foci (Focus) S	S'	Equation of directrics	Eccentricity	length of latus rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $[a>b>0]$	x-axis $(y=0)$	$2a$	$2b$	$(0,0)$	$(ae,0)$	$(-ae,0)$	$x=\frac{a}{e}, -\frac{a}{e}$	$\sqrt{\frac{a^2-b^2}{a^2}}$	$\frac{2b^2}{a}$
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $[0<a<b]$	y-axis $(x=0)$	$2b$	$2a$	$(0,0)$	$(0, be)$	$(0,-be)$	$y=\frac{b}{e}, -\frac{b}{e}$	$\sqrt{\frac{b^2-a^2}{b^2}}$	$\frac{2a^2}{b}$
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}$; $[a>b>0]$	$y=k$	$2a$	$2b$	(h,k)	$(h+ae, k)$	$(h-ae, k)$	$x=h+\frac{a}{e}$, $x=h-\frac{a}{e}$	$\sqrt{\frac{a^2-b^2}{a^2}}$	$\frac{2b^2}{a}$
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}$; $[0<a<b]$	$x=h$	$2b$	$2a$	(h,k)	$(h, be+k)$	$(h, k-be)$	$y=k+\frac{b}{e}$, $y=k-\frac{b}{e}$	$\sqrt{\frac{b^2-a^2}{b^2}}$	$\frac{2a^2}{b}$



Imp points :

→ equation of auxillary circle for $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $[a>b]$ ⇒ $\boxed{x^2+y^2=a^2}$; director circle $\boxed{x^2+y^2=a^2+b^2}$

→ " " " " $[b>a]$ ⇒ $\boxed{x^2+y^2=b^2}$

→ if length of latus rectum $=\frac{1}{2}$ (major axis) ⇒ $e=\frac{1}{\sqrt{2}}$; length of latus rectum $=\frac{1}{2}$ (minor axis) ; $e=\frac{\sqrt{3}}{2}$

→ If length of major axis $=3$ (length of minor axis) ⇒ $e=\frac{2\sqrt{2}}{3}$

→ parametric equations of ellipse $=\boxed{\begin{matrix}x=a\cos\theta \\ y=b\sin\theta\end{matrix}}$

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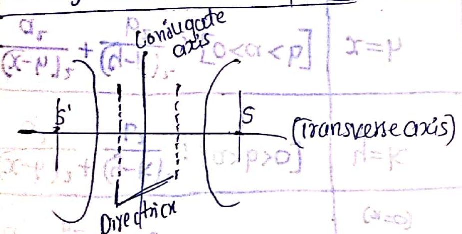
Hyperbola:

Forms of hyperbola	Transverse axis is along	Length of major axis	Conjugate axis is along	Length of minor axis	Centre (C)	Focii (S)	Eqn. of directrix	Eccentricity
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	x-axis	2a	y-axis	2b	(0,0)	$(\pm ae, 0)$	$x=\pm\frac{a}{e}$	$\sqrt{\frac{a^2+b^2}{a^2}}$
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ (Conjugate hyperbola)	y-axis	2b	x-axis	2a	(0,0)	$\cancel{y}(0, \pm be)$	$y=\pm\frac{b}{e}$	$\sqrt{\frac{a^2+b^2}{b^2}}$
$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\cancel{x}$ $y=k$	2a	$x=h$	2b	(h,k)	$(h\pm ae, k)$	$x=h\pm\frac{a}{e}$	$\sqrt{\frac{a^2+b^2}{a^2}}$
$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=-1$ Conjugate hyperbola	$x=h$	2b	$y=k$	2a	(h,k)	$(h, k\pm be)$	$y=k\pm\frac{b}{e}$	$\sqrt{\frac{a^2+b^2}{b^2}}$

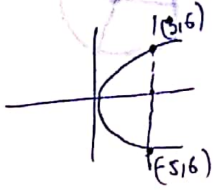


ends of latus rectum are $\left(\pm ae, \pm\frac{b^2}{a}\right)$

length of latus rectum $= \frac{2b^2}{a}$

PROBLEMS

→ Two ends of latus rectum of parabola are (3,6), & (-5,6), The focus of parabola is



$\boxed{\text{Focus} = \text{mid point of latus rectum}}$

$\left(\frac{3-5}{2}, \frac{6+6}{2}\right) \Rightarrow \left(\frac{-2}{2}, \frac{12}{2}\right) \Rightarrow (-1, 6)$

The curve represented by $x = 2(\cos\theta + \sin\theta)$, $y = 5(\cos\theta - \sin\theta)$.

$x = 2(\cos\theta + \sin\theta) \Rightarrow \frac{x}{2} = \cos\theta + \sin\theta$ —(1)

$y = 5(\cos\theta - \sin\theta) \Rightarrow \frac{y}{5} = \cos\theta - \sin\theta$ —(2)

$(1)^2 + (2)^2$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{25} = 2(1)$

$\boxed{\frac{x^2}{4} + \frac{y^2}{25} = 2} \Rightarrow \boxed{\text{ellipse}}$

For circle $ax^2 + y^2 + bx + dy + 2 = 0$, center $(1,2)$ then $2b + 3d =$ ______

$\boxed{a = 1}$

$\therefore x^2 + y^2 + bx + dy + 2 = 0.$ $\quad 2g = b$

Centre $= (-g, -f)$

$(1,2) = \left(\frac{-b}{2}, \frac{-d}{2}\right)$

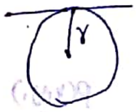
$\therefore \boxed{\begin{matrix} b = -2 \\ d = -4 \end{matrix}} \Rightarrow 2b + 3d$

$= 2(-2) + 3(-4)$

$= -4 - 12$

$= -16$

The equation of circle with centre at (2,3) and touching line $3x - 4y + 1 = 0$



g. $r = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$\sqrt{g^2 + f^2 - c} = \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}$

$\sqrt{4 + 9 - c} = \frac{|3(2) - 4(3) + 1|}{5} = \frac{6 - 12 + 1}{5} = \left|\frac{-5}{5}\right| = 1$

$4 + 9 - c = 1$

$c = 13 - 1$

$\boxed{C = 12}$ $\quad \therefore \boxed{x^2 + y^2 - 4x - 6y + 12 = 0}$

The distance b/w foci of an ellipse is 8 and the distance b/w directrices is 18. The eccentricity is ______

sol distance foci $= (h + ae, 0)(h - ae, 0) = 2ae$

$\Rightarrow 2ae = 8$ —(1)

distance b/w directrices $= \left(h + \frac{a}{e}, h - \frac{a}{e}\right)$

$\frac{2a}{e} = 18$ —(2)

$(1) \times (2) = 2a\cancel{e} \times \frac{2a}{\cancel{e}} = 8 \times 18$

$\Rightarrow 4a^2 = 8 \times 18$

$a^2 = 36$

$\boxed{a = 6}$

$\Rightarrow 2ae = 8$

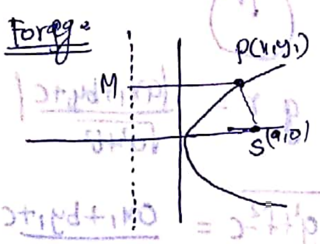
$2(6) \times e = 8$

$e = \frac{8}{2 \times 6} = \frac{2}{3}$

The equation of conic with focus at $C(1,-1)$, directrix along $\boxed{x-y+1=0}$ with eccentricity $\sqrt{2}$ is

$\cancel{(x-1)^2+(y+1)^2 = 2}$

For eg:



$$\boxed{\frac{PS}{PM}=e}$$

Sol: $\frac{PS}{PM}=e \Rightarrow (PS)^2 = e^2(PM)^2$

$$(x-1)^2+(y+1)^2 = 2\left(\frac{x-y+1}{\sqrt{1+1}}\right)^2$$

$$x^2-2x+1+y^2+2y+1 = \cancel{2}\frac{(x-y+1)^2}{\cancel{2}}$$

$$\cancel{x^2}-2x+1+\cancel{y^2}+2y+1 = \cancel{x^2}+\cancel{y^2}+1-2xy-2y+2x$$

$$-2x+2y+2 = -2xy-2y+2x$$

$$-2x+2y+2+2xy+2y-2x=0$$

$$-4x+4y+2xy+2-1=0$$

$\cancel{-2x+2y+xy+1=0}$

$$\boxed{-4x+4y+2xy+1=0}$$

eqn. of tangent to parabola $y^2=16x$ at $(1,4)$ is

$$y^2=16x$$

$$2y\cdot\frac{dy}{dx}=16$$

$$\frac{dy}{dx}=\frac{16}{2y}=\frac{8}{y}=2$$

$$(y-y_1)=m(x-x_1)$$

$$(y-4)=2(x-1)$$

$$y-4=2x-2$$

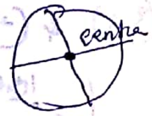
$$2x-2-y+4$$

$$\boxed{2x-y+2=0}$$

Eqn. of circle with radius 10 and whose two diameters $x+y-6=0$ and $x+2y-4=0$

$\cancel{x+y-6=0}$

$x+2y-4=0$



$$x+2y=4$$

$$-x+y=6$$

$$y=-2 \quad \boxed{x=8}$$

center (=) $\Rightarrow (8,-2)$

$$(x-8)^2+(y+2)^2=10^2$$

$$x^2-16x+64+y^2+4y+4=100$$

$$x^2+y^2-16x+4y+68=100$$

$$\boxed{x^2+y^2-16x+4y-32=0}$$

The point on curve $y^2=x$, the tangent at which makes an angle $45°$, with x-axis will be given by ____

$$\frac{dy}{dx} \Rightarrow 2y\,dy/dx=1$$

$$dy/dx=\frac{1}{2y}=\tan 45°=1$$

$$\boxed{y=\frac{1}{2}}$$

$$\boxed{x=\left(\frac{1}{2}\right)^2=\frac{1}{4}}$$

$$(x,y)=\left(\frac{1}{4},\frac{1}{2}\right)$$

The eccentricity of ellipse $9x^2+5y^2-18x-20y-16$

$$9x^2-18x+5y^2-20y-16=0$$

$$9(x^2-2x)+5(y^2-4y)-16=0$$

$$9(x^2-2x+1-1)+5(y^2-4y+4-4)-16=0$$

$$9((x-1)^2-1)+5((y-2)^2-4)-16=0$$

$$9(x-1)^2-9+5(y-2)^2-(5\times4)-16=0$$

$$9(x-1)^2+5(y-2)^2-29-16=0$$

$$\frac{9(x-1)^2}{45}+\frac{5(y-2)^2}{45}=1$$

$$\frac{(x-1)^2}{5}+\frac{(y-2)^2}{9}=1 \qquad \boxed{b>a}$$

$$\Downarrow a^2 \qquad \Downarrow b^2$$

$$e=\sqrt{\frac{b^2-a^2}{b^2}}=\sqrt{\frac{9-5}{9}}=\frac{\sqrt4}{\sqrt9}=\frac{2}{3}$$

If the eccentricity of hyperbola is $\sqrt3$, then the eccentricity of its conjugate hyperbola =

$$\boxed{\frac{1}{e^2}+\frac{1}{e_1^2}=1}$$

$$\frac{1}{3}+\frac{1}{e_1^2}=1$$

$$\Rightarrow \frac{1}{e_1^2}=\frac{2}{3}$$

$$\boxed{e_1^2=\sqrt{3/2}}$$

Eqn. to the Normal line to the curve $y=x\log x$ parallel to $\boxed{x-y=0}$

$$\frac{dy}{dx}=x\cdot\frac{1}{x}+\log x \Rightarrow \boxed{1+\log x}$$

Slope normal $=\frac{-1}{1+\log x}$

$$\Rightarrow \frac{-1}{1+\log x}=1$$

$$\log x=-2$$

$$\boxed{x=e^{-2}}$$

$$y=e^{-2}\log_e e^{-2} \Rightarrow \boxed{-2e^{-2}}$$

$$(x,y)=(e^{-2},-2e^{-2})$$

let;

$$x-y+k=0$$

$$e^{-2}+2e^{-2}+k=0$$

$$\boxed{k=-3e^{-2}}$$

$$\therefore \boxed{x-y-3e^{-2}=0}$$

eqn. of line, whose segment b/w coordinate axes is divided by point $\left(\frac12,\frac13\right)$, in 2:3 ratio is

$$\frac{mx}{x_1}+\frac{ny}{y_1}=m+n$$

$$\frac{2x}{\frac12}+\frac{3y}{\frac13}=5$$

TIP: Substitute point in eqn.

$$\boxed{4x+9y=5}$$

If the eqns to the locus of points equidistant from points $(-2,3)$, $(6,-5)$ is $ax+by+c=0$ where $a>0$, then ascending order of a,b,c =

$$(x+2)^2+(y-3)^2=(x-6)^2+(y+5)^2$$

$$\cancel{x^2}+4x+4+\cancel{y^2}-6y+9=\cancel{x^2}-12x+36+\cancel{y^2}+10y+25$$

$$4x-6y+13=10y-12x+61$$

$$10y-12x+61-4x+6y-13=0$$

$$16y-16x+48=0$$

$$y-x+3=0$$

$$\boxed{x-y-3=0}$$

$$a=1 \quad b=-1 \quad c=-3$$

$$\boxed{-3>-1>1 \quad c>b>a}$$

If P and q are ⊥ distances from the origin to the straight line $x\sec\theta-y\operatorname{cosec}\theta=a$ and $x\cos\theta+y\sin\theta=a\cos2\theta$, then ______

$$p=\frac{|a|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}=\frac{a}{\frac{1}{\sin\theta\cos\theta}}$$

$$\boxed{p=a\sin\theta\cos\theta \Rightarrow \frac{a}{2}\sin2\theta}$$

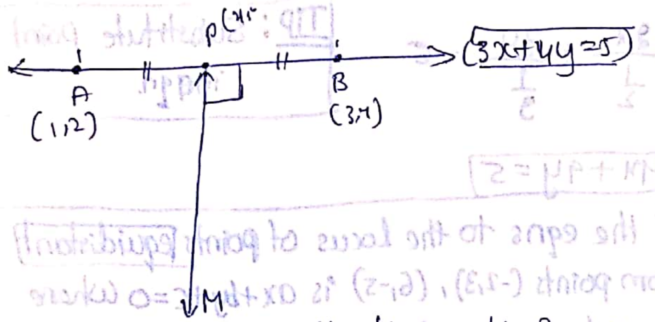
$$\boxed{4p^2=a^2\sin^2 2\theta} \quad —①$$

$$q=\frac{a\cos2\theta}{\sqrt{\cos^2\theta+\sin^2\theta}}=a\cos2\theta$$

$$q^2=a^2\cos^2 2\theta \quad —②$$

$$\boxed{4p^2+q^2=a^2}$$

The point on line $3x+4y=5$ which is equidistant from $(1,2)$ and $(3,4)$ is



slope of $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{4-2}{3-1} = \frac{2}{2} = 1$

Slope of $MP = -1$

~~$y-y_1 = m(x-x_1)$~~

$(x,y_1) =$ mid of AB

$x_1 = \frac{3+1}{2} \Rightarrow 2$

$y_1 = \frac{4+2}{2} \Rightarrow 3$

$y-3 = -1(x-2)$

$y-3 = -x+2$

$x-2+y-3=0$

$\boxed{x+y-5=0}$ —— ① $\Rightarrow \boxed{3x+3y-15=0}$

$\boxed{3x+4y=5}$ —— ②

$3x+3y = 15$

$-3x+4y = 5$

$-y = 10$

$\boxed{y=-10}$

$\boxed{x=15}$

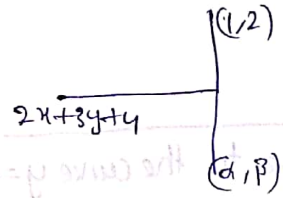
→ The coordinate of image of origin 'O' with respect to straight line $x+y+1=0$, is

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$

$\frac{h-0}{1} = \frac{k-0}{1} = \frac{-2(c)}{2}$

$\boxed{h=k=-1}$ $\boxed{(h,k)=(-1,-1)}$

→ $2x+3y+4=0$, is the perpendicular bisector of segment Joining points $(A) \Rightarrow (1,2)$ and $B(\alpha,\beta)$ then value of $(\alpha+\beta) =$ ______



$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$

$\frac{\alpha-1}{2} = \frac{\beta-2}{3} = \frac{-2(2+6+4)}{13}$

$\frac{\alpha-1}{2} = \frac{\beta-2}{3} = \frac{-2\times12}{13}$

$\frac{\alpha-1}{2} = \frac{-2\times12}{13}$ | $\frac{\beta-2}{3} = \frac{-2\times12}{13}$

$\alpha-1 = \frac{-48}{13}$ | $\beta-2 = -\frac{72}{13}$

$\alpha = \frac{-48}{13}+1$ | $\beta = -\frac{72}{13}+2$

$\alpha = \frac{-35}{13}$ | $\beta = \frac{-46}{13}$

72
$+26$
46

$\boxed{\alpha+\beta = \frac{-81}{13}}$

⇒ The angle b/w lines $x\cos\alpha + y\sin\alpha = P_1$ and $x\cos\beta + y\sin\beta = P_2$ where $\alpha > \beta$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\boxed{m_1 = \frac{-\cos\alpha}{\sin\alpha} \qquad m_2 = \frac{-\cos\beta}{\sin\beta}}$$

$$= \left|\frac{\frac{-\cos\alpha}{\sin\alpha} + \frac{\cos\beta}{\sin\beta}}{1 + \frac{\cos\alpha}{\sin\alpha}\cdot\frac{\cos\beta}{\sin\beta}}\right|$$

$$= \frac{\sin\alpha.\cos\beta - \sin\beta\cos\alpha}{\sin\alpha.\sin\beta + \cos\alpha.\cos\beta}$$

$$\tan\theta = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$

$$\tan\theta = \tan(\alpha-\beta)$$

$$\boxed{\theta = \alpha - \beta}$$

If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$, $x + 4cy + c = 0$ are concurrent, then a, b, c are in ______

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$$(3bc - 4bc) - 2a(c-b) + a(4c - 3b) = 0$$

$$-bc - 2ac + 2ab + 4ac - 3ab = 0$$

$$-bc + 2ac - ab = 0$$

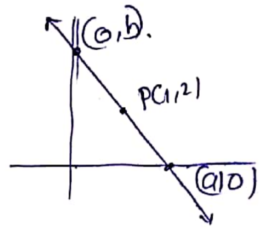
$$2ac = ab + bc$$

$$2ac = b(a+c)$$

$$\frac{2}{b} = \frac{a+c}{ac}$$

$$\boxed{\frac{2}{b} = \frac{1}{a} + \frac{1}{c}} \Rightarrow HP$$

⇒ A straight line through P(1,2) is such that it's intercept b/w axes is bisected at P. The eqⁿ is



$$\left.\begin{array}{l} \frac{a+0}{2} = 1 \\ \frac{b+0}{2} = 2 \end{array}\right] \rightarrow \boxed{\begin{array}{l} a = 2 \\ b = 4 \end{array}}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2} + \frac{y}{4} = 1$$

$$4x + 2y = 8$$

$$\boxed{2x + y = 4}$$

The value of "k" such that the lines are concurrent $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ & $8x - 11y - 33 = 0$

$$\begin{vmatrix} 2 & -3 & k \\ 3 & -4 & -13 \\ 8 & -11 & -33 \end{vmatrix} = 0$$

$$2(132 - 143) + 3(-99 + 104) + k(-33 + 32) = 0$$

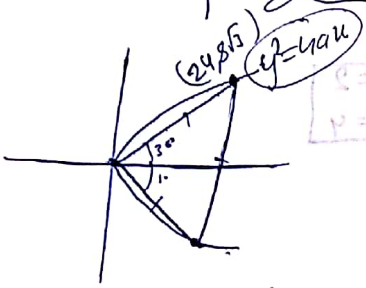
$$2(-11) + 3(5) + k(-1) = 0$$

$$-22 + 15 - k = 0$$

$$k = -22 + 15$$

$$\boxed{k = -7}$$

An equilateral Δle is inscribed in parabola $y^2 = 8x$, with one of it's vertices is the vertex of parabola. Then length of side of Δle = ______



$y^2 = 8x$

$4a = 8$

$\boxed{a = 2}$

parametric equations:

$(at^2, 2at)$

$\Rightarrow (2t^2, 4t)$

$\tan 30° = \frac{4t}{2t^2} = \frac{2}{t} = \frac{1}{\sqrt{3}}$

$\boxed{t = 2\sqrt{3}}$

$\Rightarrow 2(2\sqrt{3})^2, 4(2\sqrt{3})$

$\Rightarrow (24, 8\sqrt{3})$

$= \sqrt{(24)^2 + (8\sqrt{3})^2}$

$= \sqrt{576 + 192}$

$= \sqrt{768}$

$\boxed{= 16\sqrt{3}}$

$\Rightarrow$ Min. distance from point $(4,2)$ to the parabola $y^2 = 8x$ is ______

let A be the point on curve; $\boxed{y = P}$

$\Downarrow$

$(x, y) \to \left(\frac{P^2}{8}, P\right)$

distance b/w $\left(\frac{P^2}{8}, P\right)$ and $(4,2)$.

$$d^2 = \left(\frac{P^2}{8} - 4\right)^2 + (P-2)^2$$

diff. w.r.t. P.

$$2d \cdot \frac{dd}{dP} = \cancel{2}\left[\frac{P^2}{8} - 4\right]\frac{2P}{8_4} + 2(P-2)$$

$\Downarrow$

$$0 = \frac{P}{2}\left[\frac{P^2}{8} - 4\right] + 2(P-2)$$

$$0 = \frac{P^3 - 32P}{16} + 2P - 4$$

$$0 = P^3 - \cancel{32P} + \cancel{32P} - 64$$

$P^3 = 64$

$\boxed{P = 4}$

$\therefore$ Min. distance:

$$d^2 = \left(\frac{P^2}{8} - 4\right)^2 + (P-2)^2$$

$$d^2 = \cancel{(8-4)^2 + (6)^2} \quad (2-4)^2 + (2)^2$$

$$= (-2)^2 + (2)^2$$

$$= 8$$

$$d = \sqrt{8} = 2\sqrt{2}$$

The pole of line $2x+3y-4=0$ with the respect to parabola $y^2=4x$.

Note: ~~pto~~ polar $\Rightarrow$ $\boxed{yy_1-2a(x+x_1)=0}$

$\boxed{\text{equal to tangent eqn}}$

Pole $\Rightarrow$ $\boxed{\text{point on polar}}$

$\boxed{y^2=4x} \Rightarrow \boxed{a=1}$

$yy_1-2a(x+x_1)=0$

$yy_1-2(x+x_1)=0$

$yy_1-2x-2x_1=0$

$\boxed{2x+2x_1-yy_1=0}$

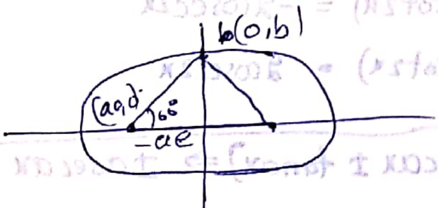
$\Downarrow$ compare with given line

$2x+3y-4=0$

$2x-yy_1+2x_1=0$

$\therefore$ $\boxed{\begin{matrix} y_1=-3 \\ x_1=-2 \end{matrix}}$ pole $=(-2,-3)$

$\Rightarrow$ S and T are foci of an ellipse and B is an end of the minor axis. If STB is an equilateral Δle, then $e=$ ____



$(\tan 60°)=\sqrt{3}=\frac{b}{ae}$

Slope

$\sqrt{3}=\frac{b}{ae}$

$3=\frac{b^2}{a^2e^2}$

$e^2=\frac{b^2}{3a^2}$

$e=\sqrt{\frac{a^2-b^2}{a^2}}$

$\frac{b^2}{3a^2}=\frac{a^2-b^2}{a^2}$

$b^2=3a^2-3b^2$

$4b^2=3a^2$

$\boxed{b^2=\frac{3}{4}a^2}$

$\Downarrow$

$e=\sqrt{\frac{a^2-b^2}{a^2}}$

$e=\sqrt{\frac{a^2-\frac{3}{4}a^2}{a^2}}$

$e=\sqrt{\frac{1}{4}\cancel{a^2}/\cancel{a^2}}$

$\boxed{e=\frac{1}{2}}$

If the foci of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and the hyperbola $\frac{x^2}{4}-\frac{y^2}{b^2}=1$, coincide, then $b^2=$ ____

$\boxed{e_{ellipse}=e_{hyperbola}}$

$\frac{a^2-b^2}{\cancel{a^2}\, a^2}=\frac{a^2+b^2}{\cancel{a^2}\, a^2}$

$\frac{25-16}{25}=\frac{4+b^2}{4}$

$\frac{9}{25}=\frac{4+b^2}{4}$

$\frac{9\times 4}{25}-4=b^2$

$\frac{36}{25}-4$

focus of hyperbola focus = focus of ellipse ae

$\sqrt{\frac{25-16}{25}}\times 5=\sqrt{\frac{4+b^2}{4}}\times 2$

$25-16=4+b^2$

$9=4+b^2$

$\boxed{5=b^2}$

★ DIFFERENTIATION ★

Formulas:

$x^n = nx^{n-1}$

$\sqrt{x} = \frac{1}{2\sqrt{x}}$

$\frac{1}{x} = \frac{-1}{x^2}$

$\frac{1}{\sqrt{x}} = \frac{-1}{2x\sqrt{x}}$

$k = 0$

$e^x = e^x$

$a^x = a^x \log a$

$\log x = \frac{1}{x}$

$\frac{1}{x} = \frac{-1}{x^2}$

$\log_a^x = \frac{1}{x \log a}$

$\sin x = \cos x$

$\cos x = -\sin x$

$\tan x = \sec^2 x$

$\cot x = -\mathrm{cosec}^2 x$

$\sec x = \sec x . \tan x$

$\mathrm{cosec}\, x = -\cot x . \mathrm{cosec}\, x$

$\sinh x = \cosh x$

$\cosh x = \sinh x$

$\tanh x = \sec^2 h x$

$\coth x = -\mathrm{cosech}^2 x$

$\mathrm{sech}\, x = -\mathrm{sech}\, x . \tanh x$

$\mathrm{cosech}\, x = -\coth x . \mathrm{cosech}\, x$

$\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

$\tan^{-1} x = \frac{1}{1+x^2}$

$\cot^{-1} x = \frac{-1}{1+x^2}$

$\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

$\mathrm{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$

$\sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}$

$\cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$

$\tanh^{-1} x = \frac{1}{1-x^2}$

$\frac{d}{dx} e^x . f(x) = e^x [f(x) + f'(x)]$

$\frac{u}{v} = \frac{vu' - uv'}{v^2}$

$uv = uv' + vu'$

$\frac{d}{dx}\left[\frac{ax+b}{cx+d}\right] = \frac{ad-bc}{(cx+d)^2}$

$\frac{d}{dx}\left[\frac{af(x)+b}{cf(x)+d}\right] = \frac{ad-bc}{(cf(x)+d)^2} , f'(x)$

Eg: $y = \frac{a+b\sin x}{a-b\sin x}$

$y = \frac{b\sin x + a}{-b\sin x + a} = \delta = \frac{2ab}{(a-b\sin x)^2} (\cos x).$

$\frac{d}{dx}(U^U) = \boxed{U^U . U'(1+\log U)}$

$\frac{d}{dx} U^V = \boxed{U^V . \left[V . \frac{U'}{U} + V' \log U\right]}$

$\frac{d}{dx}\left[\frac{1+x}{1-x}\right] = \frac{2}{(1-x)^2}$

$\frac{d}{dx}\left[\frac{1-x}{1+x}\right] = \frac{-2}{(1+x)^2}$

$\frac{d}{dx}\left[\frac{1+\sin x}{1-\sin x}\right] = \frac{2\cos x}{(1-\sin x)^2}$

$\frac{d}{dx}\left[\frac{1-\sin x}{1+\sin x}\right] = \frac{-2\cos x}{(1+\sin x)^2}$

$\frac{d}{dx}\left[\frac{1+e^x}{1-e^x}\right] = \frac{2e^x}{(1-e^x)^2}$

$\frac{d}{dx}\left[\frac{1-e^x}{1+e^x}\right] = \frac{-2e^x}{(1+e^x)^2}$

$\therefore \frac{1+f(x)}{1-f(x)} \Rightarrow \delta = \frac{2[f'(x)]}{[1-f(x)]^2}$

$\frac{1-f(x)}{1+f(x)} \Rightarrow \delta = \frac{-2f'(x)}{[1+f(x)]^2}$

$\frac{d}{dx}\left[\frac{e^x+e^{-x}}{e^x-e^{-x}}\right] = \frac{-4}{(e^x-e^{-x})^2}$

$\frac{d}{dx}\left[\frac{e^x-e^{-x}}{e^x+e^{-x}}\right] = \frac{4}{(e^x+e^{-x})^2}$

$\frac{d}{dx}\left[\frac{e^{ax}+e^{-ax}}{e^{ax}-e^{-ax}}\right] = \frac{-4a}{(e^{ax}-e^{-ax})^2}$

$\frac{d}{dx}\left[\frac{e^{ax}-e^{-ax}}{e^{ax}+e^{-ax}}\right] = \frac{4a}{(e^{ax}+e^{-ax})^2}$

$\frac{d}{dx}[\log(\sec x + \tan x)] = \sec x$

$\frac{d}{dx}[\log(\sec x - \tan x)] = -\sec x$

" $[\log(\mathrm{cosec}\, x + \cot x)] = -\mathrm{cosec}\, x$

" $[\log(\mathrm{cosec}\, x - \cot x)] = \mathrm{cosec}\, x$

$\log(\sec 2x + \tan 2x) = 2\sec 2x$

$\log(\sec 2x - \tan 2x) = -2\sec 2x$

$\log(\mathrm{cosec}\, 2x + \cot 2x) = -2\mathrm{cosec}\, 2x$

$\log(\mathrm{cosec}\, 2x - \cot 2x) = 2\mathrm{cosec}\, 2x$

★ $y = \log(\sec ax \pm \tan ax) \Rightarrow \pm a\sec ax$

★ $y = \log(\mathrm{cosec}\, ax + \cot ax) \Rightarrow -a\,\mathrm{cosec}\, ax$

★ $y = \log(\mathrm{cosec}\, ax - \cot ax) \Rightarrow a\,\mathrm{cosec}\, ax$

$y = \log\sqrt{\frac{1+x}{1-x}} \Rightarrow \frac{1}{1-x^2}$

$y = \log\sqrt{\frac{1-x}{1+x}} \Rightarrow \frac{-1}{1-x^2}$

$y = \log\sqrt{\frac{1+\sin ax}{1-\sin ax}} = a\sec ax$

$y = \log\sqrt{\frac{1-\sin ax}{1+\sin ax}} = -a\sec ax$

$y = \log\sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{1}{1-\sin x} . \cos x$
$= \sec x$

$y = \log\sqrt{\frac{1-\sin x}{1+\sin x}} = -\sec x$

$y = \log\sqrt{\frac{1+\cos ax}{1-\cos ax}} = -a\,\mathrm{cosec}\, ax$

$y = \log\sqrt{\frac{1-\cos ax}{1+\cos ax}} = a\,\mathrm{cosec}\, ax$

$y = \log\sqrt{\frac{1+\cos x}{1-\cos x}} = -\mathrm{cosec}\, x$

$y = \log\sqrt{\frac{1-\cos x}{1+\cos x}} = \mathrm{cosec}\, x$

$$\frac{d}{dx}\left[\log(x+\sqrt{x^2+a^2})\right] = \frac{1}{\sqrt{x^2+a^2}}$$

$$\text{"} \log(x-\sqrt{x^2+a^2}) = \frac{-1}{\sqrt{x^2+a^2}}$$

$$\log(x+\sqrt{x^2-a^2}) = \frac{1}{\sqrt{x^2-a^2}}$$

$$\log(x-\sqrt{x^2-a^2}) = \frac{-1}{\sqrt{x^2-a^2}}$$

$$\frac{d}{dx}\left[\underbrace{x+\sqrt{x^2+a^2}}_{\boxed{y}}\right]^n \Rightarrow \frac{ny}{\sqrt{x^2+a^2}}$$

$$y=(x+\sqrt{x^2-a^2})^n \Rightarrow \frac{ny}{\sqrt{x^2-a^2}}$$

$$y=(x-\sqrt{x^2+a^2})^n \Rightarrow \frac{-ny}{\sqrt{x^2+a^2}}$$

$$y=(x-\sqrt{x^2-a^2})^n \Rightarrow \frac{-ny}{\sqrt{x^2-a^2}}$$

$$y=\log(\sqrt{\operatorname{cosec}x+1}+\sqrt{\operatorname{cosec}x-1}) \Rightarrow \delta = \frac{-1}{2}\operatorname{cosec}x$$

$$y=\log(\sqrt{\operatorname{cosec}x+1}-\sqrt{\operatorname{cosec}x-1}) \Rightarrow \delta = \frac{1}{2}\operatorname{cosec}x$$

Observation problems

① $y=\log\left[\frac{1+x}{1-x}\right]^{\frac{1}{4}} - \frac{1}{2}\tan^{-1}(x)$

$$\frac{dy}{dx} = \frac{1}{4}\times\frac{1}{\frac{1+x}{1-x}}\times\frac{2}{(1-x)^2} - \frac{1}{2}\cdot\left(\frac{1}{1+x^2}\right)$$

$$=\frac{1}{2}\left[\frac{1}{1-x^2}-\frac{1}{1+x^2}\right]$$

$$=\frac{1}{2}\left[\frac{2x^2}{1-x^4}\right] \Rightarrow \boxed{\frac{x^2}{1-x^4}}$$

② $y=\log\left(\frac{\sin^m x}{\cos^n x}\right)$

$y = m\log\sin x - n\log\cos x$

$$\frac{dy}{dx} = m\frac{1}{\sin x}\cdot\cos x - n\frac{1}{\cos x}(-\sin x)$$

$$\Rightarrow \boxed{m\cot x + n\tan x}$$

③ $y=(x+a)(x^2+a^2)(x^4+a^4)(x^8+a^8)$

$$\therefore y = \frac{(x-a)(x+a)(x^2+a^2)(x^4+a^4)(x^8+a^8)}{x-a}$$

$$y=\frac{x^{16}-a^{16}}{x-a}$$

$$\frac{dy}{dx} = \frac{(x-a)16x^{15}-(x^{16}-a^{16})(1)}{(x-a)^2}$$

$$=\frac{16x^{16}-a16x^{15}-x^{16}+a^{16}}{(x-a)^2}$$

$$\boxed{\delta = \frac{15x^{16}-a16x^{15}+a^{16}}{(x-a)^2}}$$

④ $f(x)=e^x$

$g(x) = \tan^{-1}(x)$

$h(x) = f[g(x)]$

Then $\frac{h'(x)}{h(x)}$

$h(x) = e^{\tan^{-1}(x)}$

$h'(x) = e^{\tan^{-1}(x)}\cdot\frac{1}{1+x^2}$

$$\therefore \frac{h'(x)}{h(x)} = \frac{\cancel{e^{\tan^{-1}(x)}}}{1+x^2}\times\frac{1}{\cancel{e^{\tan^{-1}(x)}}} = \frac{1}{1+x^2}$$

⑤ $f(x)=\left(\frac{x^a}{x^b}\right)^{a+b}\cdot\left(\frac{x^b}{x^c}\right)^{b+c}\cdot\left(\frac{x^c}{x^a}\right)^{c+a}$ then $f'(x)=$

Sol: $\Rightarrow x^{(a-b)(a+b)}\cdot x^{(b-c)(b+c)}\cdot x^{(c-a)(c+a)}$

$\Rightarrow x^{a^2-b^2}\cdot x^{b^2-c^2}\cdot x^{c^2-a^2}$

$\Rightarrow x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$

$f'(x)=0$

⑥ $y=\sin x^\circ$

$= \sin\frac{\pi}{180}x$

$$\frac{dy}{dx} = \cos\frac{\pi}{180}x\cdot\left(\frac{\pi}{180}\right)$$

$$=\frac{\pi}{180}\cos x^\circ$$

$y = \sin^2 x^\circ$

$\frac{dy}{dx} = 2\sin x^\circ \cdot \cos x^\circ \left(\pi/180\right)$

$y = \sec(x^\circ + 30^\circ)$

$\frac{dy}{dx} = \frac{\pi}{180}\sec(x^\circ+30^\circ) \cdot \tan(x+30^\circ)$

$\Rightarrow f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$ then $f'(a) =$ ———

Sol:

$f'(x) = \frac{1 \cdot a}{2\sqrt{ax}} - \frac{a^2 \cdot a}{2ax\sqrt{ax}}$

$f'(a) = \frac{a}{2\sqrt{aa}} - \frac{a^2 \cdot a}{2a \cdot a \cdot \sqrt{aa}}$

$= \frac{1}{2} - \frac{1}{2}$

$\boxed{f'(a) = 0}$

$\Rightarrow f(x) = \frac{1}{1+x^{\beta-\alpha}+x^{\gamma-\alpha}} + \frac{1}{1+x^{\gamma-\beta}+x^{\alpha-\beta}} + \frac{1}{1+x^{\alpha-\gamma}+x^{\beta-\gamma}}$

then $f'(x) =$ ———

$f(x) = \frac{1}{1+\frac{x^\beta}{x^\alpha}+\frac{x^\gamma}{x^\alpha}} + \frac{1}{1+\frac{x^\alpha}{x^\beta}+\frac{x^\gamma}{x^\beta}} + \frac{1}{1+\frac{x^\alpha}{x^\gamma}+\frac{x^\beta}{x^\gamma}}$

$f(x) = \frac{x^\alpha}{x^\alpha+x^\beta+x^\gamma} + \frac{x^\beta}{x^\alpha+x^\beta+x^\gamma} + \frac{x^\gamma}{x^\alpha+x^\beta+x^\gamma}$

$f(x) = \frac{x^\alpha+x^\beta+x^\gamma}{x^\alpha+x^\beta+x^\gamma} = 1.$

$\boxed{f'(x) = 0}$

$y = \sin^n x \Rightarrow f = n\sin^{n-1}(x) \cdot (\cos x)$

$y = \sin(nx) \Rightarrow f = n(\cos nx)$

$y = \cos^n x \Rightarrow f = n\cos^{n-1}(x)\,[-\sin x]$

$y = \cos(nx) \Rightarrow f = -n\sin(nx)$

$\Rightarrow y = \sin^n x \cdot \sin(nx)$

$f = \sin^n x \cdot n\cos(nx) + \sin(nx) \cdot n\sin^{n-1} x \cdot \cos x$

$f = n\sin^{n-1}(x)\,[\sin x \cdot \cos nx + \sin nx \cdot \cos x]$

$f = n\sin^{n-1}(x)\,[\sin(nx+x)]$

$\boxed{f = n\sin^{n-1}(x)\,[\sin(n+1)x]}$

$\Rightarrow y = \sin^n x \cdot \cos nx$

$f = \sin^n x\,(-n\sin(nx)) + \cos nx \cdot n\sin^{n-1}(x) \cdot \cos x$

$f = -\sin^n x \cdot n\sin(nx) + \cos nx \cdot n\sin^{n-1}(x) \cdot \cos x$

$f = n\sin^{n-1}(x)\,[-\sin x \cdot \sin nx + \cos nx \cdot \cos x]$

$f = n\sin^{n-1}(x)\,[\cos nx \cdot \cos x - \sin(nx)\sin x]$

$f = n\sin^{n-1}(x)\,[\cos(nx+x)]$

$\boxed{f = n\sin^{n-1}(x)\,[\cos(n+1)x]}$

$y = \cos^n x \cdot \cos nx$

$f = \cancel{n\cos^{n-1}(x)\sin x}$

$f = -\cos^n x \cdot n\sin(nx) + \cos nx \cdot -n\cos^{n-1}(x)(\sin x)$

$f = -\cos^n x \cdot n\sin(nx) - \cos nx \cdot n\cos^{n-1}(x) \cdot (\sin x)$

$f = -n\cos^{n-1}(x)\,[\cos x\sin nx + \sin x \cdot \cos nx]$

$f = -n\cos^{n-1}(x)\,[\sin(nx+x)]$

$\boxed{f = -n\cos^{n-1}(x)\,[\sin(n+1)x]}$

$y = \cos^n x \cdot \sin(nx)$

$f = \cos^n x \cdot n\cos(nx) + \sin(nx) \cdot -n \cdot \cos^{n-1}(x) \cdot \sin x$

$f = \cos^n x \cdot n\cos(nx) - \sin(nx) \cdot n\cos^{n-1}(x) \cdot \sin(x)$

$f = n\cos^{n-1}(x)\,[\cos x \cdot \cos nx - \sin(nx) \cdot \sin x]$

$f = n \cdot \cos^{n-1}(x)\,[\cos(nx+x)]$

$\boxed{f = n \cdot \cos^{n-1}(x)\,[\cos(n+1)x]}$

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T.S.R

(1) $y = \sin^n x \cdot \sin(nx)$

$\boxed{\delta = n\sin^{n-1}(x)\,[\sin(n+1)x]}$

(2) $y = \sin^n x \cdot \cos(nx)$

$\boxed{\delta = n\sin^{n-1}(x)\,[\cos(n+1)x]}$

(3) $y = \cos^n x \cdot \cos nx$

$\boxed{\delta = -n\cos^{n-1}x\,[\sin(n+1)x]}$

(4) $y = \cos^n x \cdot \sin(nx)$

$\boxed{\delta = n\cos^{n-1}(x)\cdot[\cos(n+1)x]}$

T.S.R:

$y = \sqrt{\frac{1+\sin x}{1-\sin x}} = \delta = \frac{1}{1-\sin x}$

$y = \sqrt{\frac{1-\sin x}{1+\sin x}} \Rightarrow \delta = \frac{-1}{1+\sin x}$

$y = \sqrt{\frac{1+\cos x}{1-\cos x}} \Rightarrow \delta = \frac{-1}{1-\cos x}$

$y = \sqrt{\frac{1-\cos x}{1+\cos x}} \Rightarrow \delta = \frac{1}{1+\cos x}$

$y = \sqrt{\frac{1+x}{1-x}} \Rightarrow \delta = \frac{1}{\sqrt{1-x^2}\,(1-x)}$

$y = \sqrt{\frac{1-x}{1+x}} \Rightarrow \delta = \frac{-1}{\sqrt{1-x^2}\,(1+x)}$

$y = \log_7 x$

$y = \frac{\log x}{\log 7}$

$\delta = \frac{1}{\log 7}\cdot\frac{1}{x}$

$\boxed{\delta = \frac{1}{x\log 7}}$

$y = \log_x 7$

$y = \frac{\log 7}{\log x}$

$y = \log 7 \times \frac{1}{\log x}$

$\delta = \log 7 \cdot \frac{-1}{(\log x)^2}\cdot\frac{1}{x}$

$\boxed{\delta = \frac{-\log 7}{x(\log x)^2}}$

$y = \log_7(x^2+1)$

$y = \frac{\log(x^2+1)}{\log 7}$

$\delta = \frac{1}{\log 7}\cdot\frac{1}{x^2+1}\cdot 2x$

$\boxed{\delta = \frac{2x}{(x^2+1)(\log 7)}}$

$y = \log_{(x^2+1)} 7$

$y = \frac{\log 7}{\log(x^2+1)}$

$\delta = \log 7 \cdot \frac{-1}{(\log(x^2+1))^2}$

$\frac{1}{(x^2+1)}\cdot 2x$

$\boxed{\delta = \frac{-2x\cdot\log 7}{(x^2+1)[\log(x^2+1)]^2}}$

$y = \sin^{-1}\left[\frac{a+b\sin x}{b+a\sin x}\right] \Rightarrow \delta = \frac{\sqrt{b^2-a^2}}{b+a\sin x}$

$y = \sin^{-1}\left[\frac{b+a\sin x}{a+b\sin x}\right] \Rightarrow \delta = \frac{\sqrt{a^2-b^2}}{a+b\sin x}$

$y = \cos^{-1}\left[\frac{a+b\cos x}{b+a\cos x}\right] \Rightarrow \delta = \frac{\sqrt{b^2-a^2}}{b+a\cos x}$

$y = \cos^{-1}\left[\frac{b+a\cos x}{a+b\cos x}\right] \Rightarrow \delta = \frac{\sqrt{a^2-b^2}}{a+b\cos x}$

$y = \tanh^{-1}\left[\frac{x^2-1}{x^2+1}\right]$

$\delta = \frac{1}{1-\left(\frac{x^2-1}{x^2+1}\right)^2}\cdot\frac{4x}{(x^2+1)^2}$

$\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}$

$= \frac{2x(x^2+1-x^2+1)}{(x^2+1)^2}$

$\delta = \frac{4x}{(x^2-1)^2}$

$\delta = \frac{1}{\frac{(x^2+1)^2-(x^2-1)^2}{(x^2+1)^2}}\cdot\frac{4x}{\cancel{(x^2+1)^2}}$

$\delta = \frac{4x}{(x^2+1)^2-(x^2-1)^2} \Rightarrow \frac{4x}{4x^2} = \frac{1}{x}$

$\Rightarrow y = \log_7 x + \log_x 7 + \log_x 7 + \log_7 7$

$y = \frac{\log x}{\log 7} + \frac{\log 7}{\log x} + 2$

$\boxed{\delta = \frac{1}{x \log 7} - \frac{\log 7}{x(\log x)^2}}$

If $y = 2^{ax}$ and $\frac{dy}{dx} = \log 256$ at $x=1$

Then $\boxed{a = 2}$

$\frac{dy}{dx} = 2^{ax} \log(2) \times a.$

$\left.\frac{dy}{dx}\right)_{at\ x=1} = 2^a \log 2 (a)$

$\log 256 = a 2^a \log 2$

$\log 2^8 = a 2^a \log 2$

$8 \cancel{\log 2} = a 2^a \cancel{\log 2}$

$\therefore$ put $a=2$

$8 = 2 \cdot 2^2$ $\quad \therefore \boxed{a=2}$

$\boxed{8 = 8}$

$y = \log\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)$

$\delta = \frac{1}{\frac{1+\sqrt{x}}{\cancel{1-\sqrt{x}}}} \cdot \frac{\cancel{2}}{(1-\sqrt{x})x} \cdot \frac{1}{\cancel{2}\sqrt{x}}$

$\delta = \frac{1}{\sqrt{x}(1-x)}$ //

$y = 5^{-x}$

$\delta = -5^{-x} \log 5$

$\delta = 5^{-x} \log(1/5)$

$y = 8^{-x}$

$\delta = 8^{-x} \log\left(\frac{1}{8}\right)$

$\delta = -8^{-x} \log 8$

$y = 2^{2^x}$

$\delta = 2^{2^x} \log 2 . \ 2^x . \log 2$

$\delta = 2^{2^x} . 2^x . (\log 2)^2$

$\boxed{\delta = 2^x . y \ (\log 2)^2}$

If $h(x) = e^{e^x}$; then $\frac{h'(x)}{h(x)} =$ ____

$\frac{h'(x) = \cancel{e^{e^x}} \cdot e^x}{h(x) = \cancel{e^{e^x}}} = e^x \Rightarrow \boxed{\log(h(x))}$

Substitution method

I

① $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1} x$

② $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}(x)$

③ $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}(\tan 2\theta) = 2\theta = 2\tan^{-1}(x)$

II

① $y = \sin^{-1}(3x - 4x^3) = \sin^{-1}(\sin 3\theta) = 3\theta = 3\sin^{-1}(x)$

② $y = \cos^{-1}(4x^3 - 3x) = \cos^{-1}(\cos 3\theta) = 3\theta = 3\cos^{-1}(x)$

③ $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = \tan^{-1}(\tan 3\theta) = 3\theta = 3\tan^{-1}(x)$

Tip: whenever we find $\sqrt{1+x}$, $\sqrt{1-x}$, $\left(\sqrt{\frac{1+x}{1-x}}\right)$, $\left(\sqrt{\frac{1-x}{1+x}}\right)$

put $\boxed{x = \cos\theta}$

$\cos 2\theta = 2\cos^2\theta - 1$	$\cos 2\theta = 1 - 2\sin^2\theta$
$1 + \cos 2\theta = 2\cos^2\theta$	$1 - \cos 2\theta = 2\sin^2\theta$
$1 + \cos\theta = 2\cos^2\frac{\theta}{2}$	$1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

① Q. $y = \sin^{-1}\sqrt{\frac{1-x}{2}}$

$\boxed{x = \cos\theta}$

$y = \sin^{-1}\sqrt{\frac{2\sin^2\theta/2}{2}}$

$y = \cancel{\sin^{-1}}(\cancel{\sin}\ \theta/2)$

$y = \theta/2$

$y = \frac{\cos^{-1} x}{2} = \delta = \frac{-1}{2\sqrt{1-x^2}}$ //

$\sin^{-1}\sqrt{\frac{1-x}{2}}$

$\cos^{-1}\sqrt{\frac{1+x}{2}}$

$\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

$\operatorname{cosec}^{-1}\sqrt{\frac{2}{1-x}}$

$\sec^{-1}\sqrt{\frac{2}{1+x}}$

$\cot^{-1}\left(\sqrt{\frac{1+x}{1-x}}\right)$

$\Big] - \frac{1}{2}\cos^{-1}(x).$

$\sin^{-1}(x) = \cos^{-1}\sqrt{1-x^2}$

$= \tan^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right]$

$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$

$= \tan^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right] \Rightarrow$ put $\boxed{x = \sin\theta}$

$y = \tan^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right] \Rightarrow$ put $\boxed{x = \cos\theta}$

$y = \cot^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right] \Rightarrow$ put $\boxed{x = \sin\theta}$

$y = \cot^{-1}\left[\frac{x}{\sqrt{1-x^2}}\right] \Rightarrow$ put $x = \cos\theta$

$\Rightarrow y = \sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ $\quad \boxed{x = \cos\theta}$ $\quad \sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}}$ $\tan\frac{\theta}{2}$

~~$y = \sin[2\tan^{-1}(\tan\theta/2)$~~

$y = \sin\left(\cancel{2}\,\frac{1}{\cancel{2}}\cos^{-1}x\right)$

$y = \sin(\sin^{-1}\sqrt{1-x^2})$

$y = \sqrt{1-x^2}$

$\delta = \frac{1}{\cancel{2}\sqrt{1-x^2}}\cdot(-\cancel{2}x) = \frac{-x}{\sqrt{1-x^2}}$

$y = \sin^2\left[\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$

$= \sin^2\left(\frac{1}{2}\cos^{-1}x\right)$

$= \frac{1-\cos\cancel{2}\left(\frac{1}{\cancel{2}}\cos^{-1}x\right)}{2}$

$= \frac{1-x}{2}$

$\delta = -\frac{1}{2}$ //

$y = \sin(3\sin^{-1}x)$

$= \sin[\sin^{-1}(3x-4x^3)]$

$= 3x-4x^3$

$\delta = \boxed{3-12x^2}$

$y = \cos(3\cos^{-1}x)$

$= \cos(\cos^{-1}(4x^3-3x))$

$\boxed{\delta = 12x^2-3}$

$y = \sin^{-1}\left(\frac{3x}{2} - \frac{x^3}{2}\right)$

$= \sin^{-1}\left(3\left(\frac{x}{2}\right) - 4\left(\frac{x^3}{8}\right)\right)$

$= \sin^{-1}\left[3\left(\frac{x}{2}\right) - 4\left(\frac{x}{2}\right)^3\right]$ $\quad \frac{x}{2} = \sin\theta$

$= \sin^{-1}[\sin 3\theta]$ $\quad \boxed{\theta = \sin^{-1}x/2}$

$= 3\theta$

$= 3\sin^{-1}x/2$

$\delta = 3\frac{1}{\sqrt{1-x^2/4}}\cdot\frac{1}{2}$

$\delta = \frac{3}{\cancel{2}}\,\frac{1\times\cancel{2}}{\sqrt{4-x^2}} = \frac{3}{\sqrt{4-x^2}}$

$y = \cos^{-1}\left[\frac{4x^3}{27} - 3\right]$

$= \cos^{-1}\left[4\left(\frac{x}{3}\right)^3 - 3\left(\frac{x}{3}\right)\right]$ $\quad \frac{x}{3} = \cos\theta$

$\boxed{\theta = \cos^{-1}\left(\frac{x}{3}\right)}$

$y = 3\theta$

$y = 3\cos^{-1}\frac{x}{3}$

$\delta = \cancel{3}\frac{(-1)}{\sqrt{1-x^2/9}}\cdot\frac{1}{\cancel{3}} = \frac{-3}{\sqrt{9-x^2}}$ //

$y = \sin^{-1}(x/a)$

$\delta = \frac{1}{\sqrt{1-x^2/a^2}}\cdot(1/a)$

$\delta = \frac{1}{\sqrt{a^2-x^2}}\times\cancel{a}\cdot\frac{1}{\cancel{a}}$

$\boxed{\delta = \frac{1}{\sqrt{a^2-x^2}}}$

$y = \cos^{-1}(x/a)$

$\delta = \frac{-1}{\sqrt{1-x^2/a^2}}\cdot 1/a$

$\boxed{\delta = \frac{-1}{\sqrt{a^2-x^2}}}$

$y = \sin^{-1}(x/a) \Rightarrow \delta = \frac{1}{\sqrt{a^2-x^2}}$

$y = \cos^{-1}(x/a) \Rightarrow \delta = \frac{-1}{\sqrt{a^2-x^2}}$

$\sin^{-1}(-x) = -\sin^{-1}x$ | $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

$\tan^{-1}(-x) = -\tan^{-1}x$ | $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$

$\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1}x$ | $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

☆ $\boxed{\text{conditions are not necessary}}$ ☆

① $y = \cos^{-1}\left[\dfrac{x - x^{-1}}{x + x^{-1}}\right]$

$= \cos^{-1}\left[\dfrac{x^2-1}{x^2+1}\right]$

$= \cos^{-1}\left[\dfrac{-(1-x^2)}{1+x^2}\right]$

$= \pi - \cos^{-1}\left[\dfrac{1-x^2}{1+x^2}\right]$ $\quad x = \tan\theta$

$= \pi - \cos^{-1}(\cos 2\theta)$

$= \pi - 2\theta$

$= \pi - 2\tan^{-1}(x)$

$\delta = 0 - \dfrac{2(1)}{1+x^2}$

$\boxed{\delta = \dfrac{-2}{1+x^2}}$

② $y = \cos^{-1}\left[\dfrac{x^{2n}-1}{x^{2n}+1}\right]$ $\quad \boxed{x^n = \tan\theta}$

$= \cos^{-1}\left[\dfrac{(x^n)^2-1}{(x^n)^2+1}\right]$

$= \cos^{-1}\left[-\dfrac{(1-(x^n)^2)}{1+(x^n)^2}\right]$

$= \pi - \cos^{-1}\left[\dfrac{1-(x^n)^2}{1+(x^n)^2}\right]$

$= \pi - 2\tan^{-1}(x^n)$

$\delta = 0 - 2\dfrac{1}{1+x^{2n}} \times nx^{n-1}$

$\Rightarrow \boxed{\dfrac{-2}{x^{2n}+1}}$

$\Rightarrow \boxed{\dfrac{-2nx^{n-1}}{x^{2n}+1}}$

$y = \tan^{-1}\left[\dfrac{6x - 8x^3}{1-12x^2}\right]$

$= \tan^{-1}\left[\dfrac{3(2x) - (2x)^3}{1-3(2x)^2}\right]$ $\quad 2x = \tan\theta$

$= \tan^{-1}[\tan 3\theta]$

$= 3\theta$

$= 3\tan^{-1}(2x)$

$\delta = 3 \cdot \dfrac{1}{1+4x^2} \cdot 2$

$\boxed{\delta = \dfrac{6}{4x^2+1}}$

$y = \tan^{-1}\left[\dfrac{3a^2x - x^3}{a^3 - 3ax^2}\right]$

$y = \tan^{-1}\left[\dfrac{3a^2x/a^3 - x^3/a^3}{a^3/a^3 - 3ax^2/a^3}\right]$

$= \tan^{-1}\left[\dfrac{3(x/a) - (x/a)^3}{1 - 3(x/a)^2}\right]$ $\quad$ put $x/a = \tan\theta$

$= \tan^{-1}(\tan 3\theta)$

$= 3\theta$

$= 3\tan^{-1}(x/a)$

$\delta = 3\dfrac{1}{1 + x^2/a^2} \cdot \dfrac{1}{a}$

$\delta = \dfrac{3a}{\frac{a^2+x^2}{a^2}} \Rightarrow \dfrac{3a^3}{a^2+x^2}$ //

Important substitutions :

① $\sqrt{1-x^2} \Rightarrow x = \sin\theta$ or $\cos\theta$

② $\sqrt{1+x^2} \Rightarrow x = \tan\theta$ or $\cot\theta$

③ $\sqrt{x^2-1} \Rightarrow x = \sec\theta$ or $\mathrm{cosec}\,\theta$

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① $\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$ / or $a\cos\theta$

② $\sqrt{a^2+x^2} \Rightarrow x = a\tan\theta$ or $a\cot\theta$

③ $\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$

① $y = \tan^{-1}\left[\dfrac{1-\sqrt{1-x^2}}{x}\right]$; put $\boxed{x = \sin\theta}$

$y = \tan^{-1}\left[\dfrac{1-\cos\theta}{\sin\theta}\right]$

$y = \tan^{-1}\left[\dfrac{\cancel{2}\sin^2\theta/2}{\cancel{2}\sin\theta/2 . \cos\theta/2}\right]$

$y = \tan^{-1}\left[\tan\theta/2\right]$

$y = \theta/2 \Rightarrow \dfrac{\sin^{-1}x}{2}$

$\delta = \left(\dfrac{1}{\sqrt{1-x^2}}\right)\dfrac{1}{2}$

$= \dfrac{1}{2}\sqrt{\dfrac{1}{1-x^2}}$

$\boxed{\delta = \dfrac{1}{2\sqrt{1-x^2}}}$ //

$y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}-1}{x}\right]$ $\boxed{x = \tan\theta}$

$= \tan^{-1}\left[\dfrac{\sec\theta - 1}{\tan\theta}\right]$

$= \tan^{-1}\left[\dfrac{1-\cos\theta}{\sin\theta}\right]$

$= \tan^{-1}\left[\dfrac{\cancel{2}\sin^2\theta/2}{\cancel{2}\sin\theta/2 . \cos\theta/2}\right]$

$= \cancel{\tan^{-1}}\left[\cancel{\tan}\ \theta/2\right]$

$y = \theta/2 \Rightarrow \dfrac{\tan^{-1}(x)}{2}$

$\delta = \boxed{\dfrac{1}{2(1+x^2)}}$ //

$y = \tan^{-1}\left[\dfrac{\sqrt{1-x^2}-1}{x}\right]$ put $x = \sin\theta$

$= \tan^{-1}\left[\dfrac{\cos\theta - 1}{\sin\theta}\right]$

$= -\tan^{-1}\left[\dfrac{1-\cos\theta}{\sin\theta}\right]$

$= -\cancel{\tan^{-1}}\left(\cancel{\tan}\ \theta/2\right)$

$y = -\theta/2 \Rightarrow \dfrac{-1}{2} . \sin^{-1}(x)$

$\delta = \dfrac{-1}{2}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

$\delta = \dfrac{-1}{2\sqrt{1-x^2}}$ //

$y = \tan^{-1}\left[\dfrac{x}{1+\sqrt{1+x^2}}\right]$; put $\boxed{x = \tan\theta}$

$y = \tan^{-1}\left[\dfrac{\tan\theta}{1+\sec\theta}\right]$

$y = \tan^{-1}\left[\dfrac{\sin\theta}{1+\cos\theta}\right]$

$y = \tan^{-1}\left[\dfrac{\cancel{2}\sin\theta/2 . \cos\theta/2}{\cancel{2\sin^2\theta}/\cancel{2}\cos^2\theta/2}\right]$

$y = \cancel{\tan^{-1}}\left[\cancel{\tan}\ \theta/2\right]$

$y = \dfrac{1}{2}\tan^{-1}(x)$

$\delta = \dfrac{1}{2}\left(\dfrac{1}{1+x^2}\right)$

$\boxed{\delta = \dfrac{1}{2(1+x^2)}}$ //

$y = \tan^{-1}\left[\dfrac{x}{1-\sqrt{1+x^2}}\right]$ $\boxed{x = \tan\theta}$

$= \tan^{-1}\left[\dfrac{\tan\theta}{1-\sec\theta}\right]$

$= \tan^{-1}\left[\dfrac{\sin\theta}{\cos\theta - 1}\right]$

$= -\tan^{-1}\left[\dfrac{\sin\theta}{1-\cos\theta}\right]$

$= -\tan^{-1}\left[\dfrac{\cancel{2}\sin\theta/2 . \cos\theta/2}{\cancel{2}\sin^{2}\theta/2}\right]$

$= -\tan^{-1}\left[\cot\theta/2\right]$

$= -\cancel{\cot}\ \phi$

$= -\cancel{\tan^{-1}}\left[\cancel{\tan}\left(\pi/2 - \theta/2\right)\right]$

$y = -\pi/2 + \theta/2$

$\delta y = -\pi/2 + \dfrac{\tan^{-1}x}{2}$

$\delta = 0 + \dfrac{1}{2(1+x^2)}$

$\boxed{\delta = \dfrac{1}{2(1+x^2)}}$

$$y = \tan^{-1}\left[\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right]$$

put $\boxed{x = \cos\theta}$

$$y = \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta/2}+\sqrt{2\sin^2\theta/2}}{\sqrt{2\cos^2\theta/2}-\sqrt{2\sin^2\theta/2}}\right]$$

$$y = \tan^{-1}\left[\frac{\sqrt{\cos^2\theta/2}+\sqrt{\sin^2\theta/2}}{\sqrt{\cos^2\theta/2}-\sqrt{\sin^2\theta/2}}\right]$$

$$y = \tan^{-1}\left[\frac{\cos\theta/2+\sin\theta/2}{\cos\theta/2-\sin\theta/2}\right]$$

$$y = \cancel{\tan^{-1}}\left[\cancel{\tan}(\pi/4+\theta/2)\right]$$

$$y = \pi/4 + \frac{\cos^{-1}x}{2}$$

$$\delta = 0 + \frac{-1}{2\sqrt{1-x^2}}$$

$$\boxed{\delta = \frac{-1}{2\sqrt{1-x^2}}} \,//$$

$$y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] \quad ☆☆$$

put $\boxed{x^2 = \cos\theta}$

$\therefore$ ~~$y = \pi/4 + \frac{\cos^{-1}(x)}{2}$~~

~~$\delta = -2$~~

$$y = \pi/4 + \theta/2 \qquad \theta = \cos^{-1}(x^2)$$

$$y = \pi/4 + \frac{1}{2}\cos^{-1}(x^2)$$

$$\delta = 0 + \frac{1}{\cancel{2}}\frac{-1}{\sqrt{1-x^4}}\cdot \cancel{2}x$$

$$\boxed{\delta = \frac{-x}{\sqrt{1-x^4}}} \,//$$

$$y = \tan^{-1}\left[\frac{x+\sqrt{a^2-x^2}}{x-\sqrt{a^2-x^2}}\right]$$

$$\tan(\pi/4+\theta) = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$

$$\tan(\pi/4-\theta) = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$\boxed{x = a\cos\theta}$

$\boxed{\frac{x}{a} = \cos\theta}$

$$y = \tan^{-1}\left[\frac{a\cos\theta+\sqrt{a^2-a^2\cos^2\theta}}{a\cos\theta-\sqrt{a^2-a^2\cos^2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right]$$

$$= \cancel{\tan^{-1}}\left[\cancel{\tan}(\pi/4+\theta)\right]$$

$$= \pi/4 + \theta$$

$$= \pi/4 + \cos^{-1}(x/a)$$

$$\delta = 0 + \frac{-1}{\sqrt{a^2-x^2}}$$

$$\boxed{\delta = \frac{-1}{\sqrt{a^2-x^2}}} \,//$$

$$y = \tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right] \qquad \boxed{x = a\sin\theta}$$

$$= \tan^{-1}\left[\frac{a\sin\theta}{\sqrt{a^2\sin^2\theta - a^2\sin^2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\sin\theta}{\cos\theta}\right]$$

$$= \tan^{-1}[\tan\theta]$$

$$= \theta$$

$$= \sin^{-1}(x/a)$$

$$\boxed{\delta = \frac{1}{\sqrt{a^2-x^2}}} \,//$$

$$y = \cancel{\tan^{-1}}\ \sin^{-1}\left[\sqrt{\frac{x^2}{a^2+x^2}}\right]$$

$$\boxed{x = a\tan\theta}$$

$$y = \cancel{\tan^{-1}}\ \sin^{-1}\left[\frac{\cancel{a}\tan\theta}{\cancel{a}\sec\theta}\right]$$

$$y = \cancel{\tan^{-1}}\ \sin^{-1}[\sin\theta]$$

$$y = \theta \Rightarrow \tan^{-1}(x/a)$$

$$\delta = \frac{1}{1+x^2/a^2} \Rightarrow \frac{a^2}{a^2+x^2}\cdot\frac{1}{a}$$

$$\boxed{\delta = \frac{a}{a^2+x^2}}$$

$$y = \sin^{-1}(2x\sqrt{1-x^2})$$

$$\boxed{x = \sin\theta}$$

$$y = \sin^{-1}(2\sin\theta\cos\theta)$$

$$y = 2\theta \Rightarrow 2\sin^{-1}x$$

$$\boxed{\delta = \frac{2}{\sqrt{1-x^2}}}$$

$$y = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$$ Then dy/dx at $x=e$, $\Rightarrow \boxed{1/e}$

$$\boxed{\log x = \tan\theta}$$

$$y = \cos^{-1}[\cos 2\theta]$$

$$y = 2\theta \Rightarrow 2\tan^{-1}(\log x)$$

$$\delta = 2\frac{1}{1+(\log x)^2}\cdot\frac{1}{x}$$

$$\delta = \frac{2}{x[1+(\log x)^2]}$$

$$\delta_e = \frac{2}{e[2]} \frac{1}{2e}$$

$$y = \cos^{-1}\left[\frac{9-x^2}{9+x^2}\right]$$

$$= \cos^{-1}\left[\frac{1-(x/3)^2}{1+(x/3)^2}\right]$$

$$\boxed{x/3 = \tan\theta}$$

$$y = 2\theta \Rightarrow 2\tan^{-1}(x/3)$$

$$\delta = 2\frac{1}{1+x^2/9}\cdot\frac{1}{3}$$

$$\delta = \frac{2\times 9^3}{9+x^2}\cdot\frac{1}{\cancel{3}} = \frac{6}{9+x^2}$$

$$y = \tanh^{-1}\left[\frac{x}{\sqrt{1+x^2}}\right]$$ $\cosh^2\theta - \sinh^2\theta = 1$

put $\boxed{x = \sinh\theta}$

$$= \tanh^{-1}\left[\frac{\sinh\theta}{\cosh\theta}\right]$$

$$= \tanh^{-1}(\tanh\theta)$$

$$= \theta \Rightarrow \sinh^{-1}(x)$$

$$\delta = \frac{1}{\sqrt{x^2+1}}$$

$$\delta = \frac{1}{\sqrt{1+x^2}}$$

★ $\tan^{-1}\left[\frac{x+y}{1-xy}\right] = \tan^{-1}(x) + \tan^{-1}(y)$

★ $\tan^{-1}\left[\frac{x-y}{1+xy}\right] = \tan^{-1}(x) - \tan^{-1}(y)$

① $y = \tan^{-1}\left[\frac{1+x}{1-x}\right]$

$$= \tan^{-1}(1) + \tan^{-1}(x)$$

$$= \pi/4 + \tan^{-1}(x)$$

$$\boxed{\delta = \frac{1}{1+x^2}}$$

$y = \tan^{-1}\left[\frac{a^3+x^3}{1-a^3x^3}\right]$

$y = \tan^{-1}(a^3) + \tan^{-1}(x^3)$

$\delta = 0 + \frac{1}{1+x^6}\cdot(3x^2)$

$\boxed{\delta = \frac{3x^2}{1+x^6}}$

$y = \tan^{-1}\left[\frac{2+3x}{3-2x}\right]$

$= \tan^{-1}\left[\frac{2/3 + 3/3(x)}{3/3 - 2x/3}\right]$

$= \tan^{-1}\left[\frac{\frac{2}{3}+x}{1-\frac{2}{3}x}\right]$

$y = \tan^{-1}\left(\frac{2}{3}\right) + \tan^{-1}(x)$

$\delta = \frac{1}{1+x^2}$ //

$y = \tan^{-1}\left[\frac{5-4x}{4+5x}\right]$

$= \tan^{-1}\left[\frac{5/4 - x}{1+\frac{5}{4}x}\right]$

$= \tan^{-1}(5/4) - \tan^{-1}(x)$

$\delta = 0 - \frac{1}{1+x^2}$

$\delta = \frac{-1}{1+x^2}$

$y = \tan^{-1}\left[\frac{4x}{1+5x^2}\right]$ $\quad \left[\frac{x+y}{1-xy}\right]$ or $\frac{x-y}{1+xy}$

$= \tan^{-1}\left(\frac{5x-x}{1+5x^2}\right)$

$y = \tan^{-1}(5x) - \tan^{-1}(x)$

$\delta = \frac{5}{1+25x^2} - \frac{1}{1+x^2}$

$y = \tan^{-1}\left[\frac{2+3x}{3-2x}\right] + \tan^{-1}\left[\frac{4x}{1+5x^2}\right]$

$y = \tan^{-1}\left(\frac{2/3+x}{1-2/3x}\right) + \tan^{-1}\left(\frac{5x-x}{1+5x^2}\right)$

$= \tan^{-1}\left(\frac{2}{3}\right) + \cancel{\tan^{-1}(x)} + \tan^{-1}(5x) - \cancel{\tan^{-1}(x)}$

$\delta = 0 + \frac{5}{1+25x^2}$

$\boxed{\delta = \frac{5}{25x^2+1}}$ //

$y = \tan^{-1}\left[\frac{5ax}{a^2-6x^2}\right]$

$= \tan^{-1}\left[\frac{5x/a}{1-6x^2/a^2}\right]$

$= \tan^{-1}\left[\frac{2x/a + 3x/a}{1-6x^2/a^2}\right]$

$\therefore y = \tan^{-1}\left(\frac{2x}{a}\right) + \tan^{-1}\left(\frac{3x}{a}\right)$

$\delta = \frac{1}{1+\left(\frac{2x}{a}\right)^2}\cdot\left(\frac{2}{a}\right) + \frac{1}{1+\left(\frac{3x}{a}\right)^2}\left(\frac{3}{a}\right)$

$\delta = \frac{a^{\cancel{2}}}{4x^2+a^2}\frac{2}{\cancel{a}} + \frac{a^{\cancel{2}}}{9x^2+a^2}\frac{3}{\cancel{a}}$

$\delta = \frac{2a}{4x^2+a^2} + \frac{3a}{9x^2+a^2}$ //

$y = \cot^{-1}\left(\frac{6x^2+1}{x}\right)$

$y = \tan^{-1}\left(\frac{x}{1+6x^2}\right)$

$= \tan^{-1}\left(\frac{3x-2x}{1+6x^2}\right)$

$y = \tan^{-1}(3x) - \tan^{-1}(2x)$

$\delta = \frac{3}{9x^2+1} - \frac{2}{4x^2+1}$

$$y = \tan^{-1}\left[\frac{a\cos x + b\sin x}{b\cos x - a\sin x}\right]$$

$$= \tan^{-1}\left[\frac{\frac{a}{b} + \frac{b\sin x}{a\cos x}}{1 - \frac{b\sin x}{a\cos x}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{a}{b} + \frac{b}{a}\tan x}{1 - \frac{b}{a}\tan x \frac{a}{b}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{a}{b} + \tan x}{1 - \frac{a}{b}\tan x}\right]$$

$$= \tan^{-1}(a/b) + \tan^{-1}(\tan x)$$

$$\delta = 0 + \frac{1}{1+x^2}\ 1$$

$$\boxed{\delta = \frac{1}{1+x^2}}\ 1$$

$$y = \tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right] = -1$$

$$\therefore \cancel{y = \tan^{-1}\left[\frac{a f(x) + b(f(x))}{b f(x) - a(f(x))}\right] = +1\ (f'(x))}$$

$$\cancel{= \tan^{-1}\left[\frac{a f(x) + b f(x)}{b f(x) + a f(x)}\right] = -f'(x)}$$

$$y = \tan^{-1}\left[\frac{a\cos(f(x)) + b\sin f(x)}{b\cos f(x) - a\sin f(x)}\right] = +1(f'(x))$$

$$y = \tan^{-1}\left[\frac{a\cos f(x) - b\sin f(x)}{b\cos f(x) + a\sin f(x)}\right] = -1[f'(x)]$$

Basic standards of differentiation:

$$\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = n \Rightarrow dy/dx = \frac{\left[\frac{n^2-2}{2}\right](y-x)}{y - \left[\frac{n^2-2}{2}\right]x}$$

$$\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = n \Rightarrow dy/dx = \frac{\frac{n^2+2}{2}(y-x)}{y - \left(\frac{n^2+2}{2}\right)x}$$

: Methods of differentiation:

(1) Logarithmic differentiation:

used when $y = f(x)^{g(x)},\ f(x).g(x),\ \frac{f(x)}{g(x)} \dots$

⇓

Function

(1)Q. $y = x^x$

$\log y = \log x^x$

$\log y = x\log x$

$\frac{1}{y}.dy/dx = x.\frac{1}{x} + \log x\,(1)$

$\frac{1}{y}.dy/dx = (1+\log x)$

$dy/dx = y(1+\log x)$

$\boxed{= x^x(1+\log x)}$

(2)Q. $y = x^{-x}$

$\log y = -x\log x$

$\frac{1}{y}.dy/dx = -x.\frac{1}{x} + \log x(-1)$

$\frac{1}{y}.dy/dx = -(1+\log x)$

$dy/dx = -y(1+\log x)$

$\boxed{dy/dx = -x^{-x}(1+\log x)}$

(3)Q. $y = x^{\sin x}$

$\frac{1}{y}.dy/dx = \sin x\frac{1}{x} + \log x(\cos x)$

$dy/dx = y\left(\frac{1}{x}.\sin x + \cos x\log x\right)$

$\boxed{= x^{\sin x}\left(\frac{1}{x}.\sin x + \cos x\log x\right)}$

$y = x^{\cos x}$

$\log y = \log x^{\cos x}$

$\log y = \cos x.\log x$

$\frac{1}{y}.dy/dx = \cos\frac{1}{x} + \log x.(-\sin x)$

$dy/dx = y\left(\frac{1}{x}.\cos x - \sin x.\log x\right)$

$\boxed{= x^{\cos x}\left(\frac{1}{x}\cos x - \sin x.\log x\right)}$

$\cancel{y = x^{\sin x} + x^{\cos x}}$

Q. ① $y = x^{1/x}$
② $y = x^{\sqrt{x}}$
③ $y = x^{\log x}$
④ $y = x^{2x}$
⑤ $y = x^{\sin x}$
⑥ $y = x^{\cos x}$
⑦ $y = x^{\tan^{-1} x}$
⑧ $y = x^{\sin^{-1} x}$
⑨ $y = x^{\cos^{-1} x}$
⑩ $y = x^{\sin x} + x^{\cos x}$
⑪ $y = x^{\sin x} + \sin x^{x}$
⑫ $y = (ax+b)^{cx+d}$

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① $y = x^{1/x}$

$\log y = \log x^{1/x}$

$\log y = \frac{1}{x}.\log x$

$\frac{1}{y}.dy/dx = \frac{1}{x}.\frac{1}{x} + \log x\left(\frac{-1}{x^2}\right)$

$\frac{1}{y}.dy/dx = \frac{1}{x^2} - \log x\left(\frac{1}{x^2}\right)$

$dy/dx = y\frac{1}{x^2}(1-\log x)$ $\quad \frac{1}{x}-2 = \frac{1-2x}{x}$

$= x^{1/x}.\frac{1}{x^2}(1-\log x)$

$= x^{\frac{1-2x}{2}}(1-\log x)$

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② $y = \sqrt{x}$

$\log y = \log\sqrt{x}$

$\log y = \frac{1}{2}\log x$

$\frac{1}{y}.\frac{dy}{dx} = \frac{1}{2}\frac{1}{x}$

$dy/dx = y.\frac{1}{2x}$

$\boxed{dy/dx = \frac{\sqrt{x}}{2x}}$

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③ $y = x^{\log x}$

$\log y = \log x^{\log x}$

$\log y = \log x.\log x$

$\frac{1}{y}.dy/dx = 2\log x\left(\frac{1}{x}\right)$

$dy/dx = y.\frac{2\log x}{x}$

$= x^{\log x}.\frac{2\log x}{x}$

$\boxed{dy/dx = x^{\log x-1}[2\log x]}$

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④ $y = x^{2x}$

$\log y = 2x.\log x$

$\frac{1}{y}.dy/dx = 2x.\frac{1}{x} + \log x(2)$

$\frac{1}{y}dy/dx = 2 + 2\log x.$

$dy/dx = x^{2x}(2)(1+\log x)$

$\boxed{dy/dx = 2x^{2x}[1+\log x]}$

$y = x^{\tan^{-1}(x)}$

$\log y = \tan^{-1}(x).\log x$

$\frac{1}{y}.dy/dx = \tan^{-1}(x).\frac{1}{x} + \log x\frac{1}{1+x^2}$

$dy/dx = x^{\tan^{-1}(x)}\left[\frac{\tan^{-1}x}{x} + \frac{\log x}{1+x^2}\right]$

$y = x^{\sin x} + \sin x^{x}.$

$y = x^{\sin x}$

$\frac{dy}{dx} = x^{\sin x}\left[\frac{1}{x}.\sin x + \cos x.\log x\right]$

$y = \sin x^{x}$

$\log y = x\log\sin x$

$\frac{1}{y}.dy/dx = x.\frac{1}{\sin x}.\cos x. + \log\sin x.$

$dy/dx = (\sin x)^{x}[x\cot x + \log\sin x]$

D of y = D of $x^{\sin x}$ + D of $(\sin x)^{x}$

$$y=(ax+b)^{cx+d}$$

$$\log y = (cx+d)\log(ax+b)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = (cx+d)\frac{1}{(ax+b)}\cdot(a) + \log(ax+b)\cdot c$$

$$\frac{1}{y}\frac{dy}{dx} = \left(\frac{cx+d}{ax+b}\right)x + \log(ax+b)\cdot c$$

$$\frac{dy}{dx} = (ax+b)^{cx+d}\boxed{\left[\frac{cx+d}{ax+b}\right]x + c\cdot\log(ax+b)}$$

$$y = \sin x\cdot\sin 2x\cdot\sin 3x\cdot\sin 4x,$$

$$\log y = \log(\sin x\cdot\sin 2x\cdot\sin 3x\cdot\sin 4x)$$

$$\log y = \log\sin x + \log\sin 2x + \log\sin 3x + \log\sin 4x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\sin x}\cdot\cos x + \frac{1}{\sin 2x}2\cos 2x + \frac{1\times 3\cos 3x}{\sin 3x} + \frac{4\cos 4x}{\sin 4x}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{\cos x}{\sin x} + \frac{2\cos 2x}{\sin 2x} + \frac{3\cos 3x}{\sin 3x} + \frac{4\cos 4x}{\sin 4x}$$

$$\boxed{\frac{dy}{dx} = y(\cot x + 2\cot 2x + 3\cot 3x + 4\cot 4x)}$$

$$y = \cos x\cdot\cos 2x\cdot\cos 3x\cdot\cos 4x.$$

$$\log y = \log(\cos x\cdot\cos 2x\cdot\cos 3x\cdot\cos 4x)$$

$$\log y = \log\cos x + \log\cos 2x + \log\cos 3x + \log\cos 4x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1(-\sin x)}{\cos x} + \frac{1(-2\sin 2x)}{\cos 2x} + \frac{1(-3\sin 3x)}{\cos 3x} + \frac{1(-4\sin 4x)}{\cos 4x}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = -(\tan x + 2\tan 2x + 3\tan 3x + 4\tan 4x)$$

$$\boxed{\frac{dy}{dx} = -y(\tan x + 2\tan 2x + 3\tan 3x + 4\tan 4x)}$$

★

$$y = x^{x^x}$$

$$\log y = \log x^{x^x}$$

$$\log y = x^x\log x$$

$$\frac{1}{y}\frac{dy}{dx} = x^x\cdot\frac{1}{x} + \log x[x^x(1+\log x)].$$

$$= x^x\cdot\frac{1}{x} + x^x\log x + x^x\cdot(\log x)^2$$

$$= x^x\left[\frac{1}{x} + \log x + (\log x)^2\right]$$

$$\frac{dy}{dx} = x^{x^x}\cdot x^x\left[\frac{1}{x} + \log x + (\log x)^2\right]$$

$$y = (x^x)^x$$

$$\log y = x\log x^x$$

$$\frac{1}{y}\cdot dy/dx = x\cdot\frac{1}{x^x}[x^x(1+\log x)] + \log x^x(1)$$

$$\frac{1}{y}\cdot dy/dx = x + x\log x + \log x^x$$

$$dy/dx = (x^x)^x[x + x\log x + \log x^x]$$

$$dy/dx = (x^x)^x[x + x\log x + x\log x]$$

$$= (x^x)^x[x + 2x\log x]$$

$$\boxed{dy/dx = (x^x)^{x+1}[1+2\log x]}$$

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2. Implicit Differentiation *

A function $y = f(x,y)$ defined by $F(x,y)=0$.

$$\frac{dy}{dx} = \left[\frac{-\partial f/\partial x}{\partial f/\partial y}\right]$$

if $x\sqrt{1+y} + y\sqrt{1+x} = 0$ then $dy/dx = $ ___

$x\sqrt{1+y} = -y\sqrt{1+x}$

S.O.B

$x^2(1+y) = y^2(1+x)$

$x^2+x^2y = y^2+xy^2$

$x^2+x^2y-y^2-xy^2=0$.

$x^2-y^2+xy(x-y)=0$

$(x+y)(x-y)+xy(x-y)=0$

$(x-y)(x+y+xy)=0$.

$x-y=0$

$x=y$

$dy/dx=1$

$x+y+xy=0$

$dy/dx = -\frac{\partial f/\partial x}{\partial f/\partial y}$

$x+y+xy=0$

$x+y(1+x)=0$

$y(1+x)=-x$.

$y=\frac{-x+0}{1+x}$

$dy/dx = \frac{-1}{(1+x)^2}$

$= \frac{-(1+0+y)}{0+1+x} = \frac{-(1+y)}{(1+x)}$

$\boxed{(x+y)^{m+n} = x^m.y^n \Rightarrow dy/dx = y/x}$

$(x+y)^{m+n} = x^m.y^n$

$\log(x+y)^{(m+n)} = \log(x^m.y^n)$

$(m+n)\log(x+y) = \log x^m + \log y^n$

$(m+n)\log(x+y) = m\log x + n\log y$

$(m+n)\log(x+y) - m\log x - n\log y = 0$.

$$\frac{dy}{dx} = -\frac{\left((m+n)\frac{1}{x+y}\cdot(1) - \frac{m}{x} - 0\right)}{\left((m+n)\frac{1}{x+y}(1) - \frac{n}{y}\right)}$$

$$= -\left[\frac{\frac{m+n}{x+y}-\frac{m}{x}}{\frac{m+n}{x+y}-\frac{n}{y}}\right]$$

$$-\left[\frac{\frac{\cancel{mx}+nx-\cancel{mx}-my}{x(x+y)}}{\frac{my+\cancel{ny}-nx-\cancel{ny}}{y(x+y)}}\right]$$

$$= \frac{-y}{x}\left[\frac{nx-my}{my-nx}\right]$$

$$= \frac{y}{x}\left[\frac{\cancel{my-nx}}{\cancel{my-nx}}\right]$$

$= y/x$

$\boxed{y = x^m.y^n = a^{m+n} \Rightarrow \delta = \frac{-m}{n}(y/x)}$

$\Rightarrow (x+y)^5 = x^3.y^2 \Rightarrow dy/dx = \cancel{\frac{x}{y}}\ y/x$

$\Rightarrow$ If $(x+y)^k = x.y$ and $dy/dx = y/x \Rightarrow$ the $k = 1+1$
$= 2$

~~$\sin y = x$~~

$\sin y = x\sin(a+y)$

$\frac{\sin y}{\sin(a+y)} = x$

$x - \frac{\sin y}{\sin(a+y)} = 0$

$$\frac{dy}{dx} = -\left[\frac{1-0}{0-\frac{\sin a}{\sin^2(a+y)}}\right]$$

$$\boxed{\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}.}$$

$$\frac{d}{dy}\left[\frac{\sin y}{\sin(a+y)}\right] = \frac{\sin(a+y).\cos y - \sin y.\cos(a+y)}{\sin^2(a+y)}$$

$$\frac{\sin(a)}{\sin^2(a+y)}$$

$y \Rightarrow e^x + e^y = e^{x+y}$. Then $dy/dx = \boxed{-e^{y-x}}$

$e^x + e^y = e^x . e^y$

$\cancel{e^x + e^y - e^x e^y} = 0$

$\frac{e^x + e^y}{e^x + e^y} = 1$

$\frac{1}{e^y} + \frac{1}{e^x} = 1$

$e^{-y} + e^{-x} = 1$

$\frac{dy}{dx} = \frac{-[0 - e^{-x}]}{-e^{-y} + 0} = \frac{-e^{-x}}{e^{-y}}$

$= -e^{-x} . e^{y}$

$\Rightarrow -e^{-x+y}$

$\boxed{dy/dx = -e^{y-x}.}$

$2^x + 2^y = 2^{x+y} \Rightarrow dy/dx = -2^{y-x}.$

$2^x + 2^y = 2^x . 2^y$

$\frac{2^x + 2^y}{2^x . 2^y} = 1$

$2^{-y} + 2^{-x} = 1$

$\frac{dy}{dx} = -\left(\frac{0 + 2^{-x}\log 2(-1)}{2^{-y}\log 2(-1) + 0} \right)$

$= -\left[\frac{-2^{-x}\log 2}{-2^{-y}\log 2} \right] = -2^{-x} . 2^y$

$= -2^{y-x}$

$3^x + 3^y = 3^{x+y} \Rightarrow$ then $dy/dx = (-3)^{y-x}$

$5^x + 5^y = 5^{x+y} \Rightarrow$ then $dy/dx = (-5)^{y-x}$

$x^y = e^{(x-y)} \Rightarrow \frac{\log x}{(1+\log x)^2}$

$\log x^y = \log e^{x-y}$

$y \log x = (x-y)$

$y \log x = x - y$

$y \log x - x + y = 0.$

$y(1+\log x) = x$

$y = \frac{x}{1+\log x}$

$dy/dx = \frac{1+\log x(1) - x\left(\frac{1}{x}\right)}{(1+\log x)^2}$

$dy/dx = \frac{\log x}{(1+\log x)^2}$

$x = y\log(xy)$ Then dy/dx:

$x = y(\log x + \log y)$

$x = (y\log x + y\log y)$

$x - y\log x - y\log y = 0.$

$dy/dx = -\left[\frac{1 - y/x - 0}{0 - \log x - [y \cdot \frac{1}{y} + \log y(1)]} \right]$

$= -\left[\frac{\frac{x-y}{x}}{0 - \log x - 1 - \log y} \right]$

$= -\left[\frac{\frac{x-y}{x}}{0 - 1 - \log x - \log y} \right]$

$= +\left[\frac{x-y}{x(1+\log x+\log y)} \right]$

$\boxed{dy/dx = \frac{x-y}{x(1+\log xy)}}$

$y^2 - 2x^2 = y$, Then dy/dx $(1,-1)$ is ______

$y^2 - 2x^2 - y = 0$

$dy/dx = -\left[\frac{0 - 4x - 0}{2y - 0 - 1}\right]$

$dy/dx = \frac{4x}{2y-1}$

$\frac{dy}{dx}\Big]_{(1,-1)} = \frac{4(1)}{-2-1} = \frac{-4}{3}$ //

$x^y = y^x$ Then dy/dx = ______

$y\log x = x\log y$

$y\log x - x\log y = 0.$

$\frac{dy}{dx} = -\left[\frac{\frac{y}{x} - \log y}{\log x - \frac{x}{y}}\right]$

$= -\left[\frac{\frac{y - x\log y}{x}}{\frac{y\log x - x}{y}}\right]$

$= -\frac{y}{x}\left[\frac{y - x\log y}{y\log x - x}\right]$

$\boxed{dy/dx = \frac{y}{x}\left[\frac{x\log y - y}{y\log x - x}\right]}$

If $x^y \cdot y^x = 1$

$\log(x^y \cdot y^x) = 0.$

$\log x^y + \log y^x$

$y\log x + x\log y = 0.$

$\frac{dy}{dx} = -\left[\frac{(y/x + \log y)}{(\log x + \frac{x}{y})}\right]$

$\frac{dy}{dx} = -\left[\frac{\frac{y + x\log y}{x}}{\frac{y\log x + x}{y}}\right]$

$\frac{dy}{dx} = -\frac{y}{x}\left[\frac{y + x\log y}{y\log x + x}\right]$

$\boxed{\frac{dy}{dx} = \frac{y}{x}\left[\frac{x\log y - y}{y\log x + x}\right]}$ //

① $\sinh^{-1}(x) + \sinh^{-1}(y) = 1 \Rightarrow dy/dx = \sqrt{\frac{1+y^2}{1+x^2}}$

② $\sin(x+y) = \log(x+y) \Rightarrow dy/dx = -1$

③ $x^2 + y^2 + 3xy - 7 = 0 \Rightarrow dy/dx = \frac{-(2x+3y)}{2y+3x}$

④ $\sin(xy) + 4\cos xy = 5 \Rightarrow dy/dx = -y/x$

⑤ $\sqrt{x} + \sqrt{y} = \sqrt{xy} \Rightarrow dy/dx$

⑥ $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$; Then $dy/dx\ (a,b)$ = ______

⑦ $\frac{x}{y} - \frac{y}{x} = c \Rightarrow$ Then dy/dx.

⑧ $y = e^{xy} \Rightarrow$ Then dy/dx.

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(1) $\sinh^{-1}(x) + \sinh^{-1}(y) = 1$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+y^2}} \cdot dy/dx = 0.$$

$$\frac{1}{\sqrt{1+y^2}} \cdot dy/dx = \frac{-1}{\sqrt{1+x^2}}$$

$$\boxed{dy/dx = -\sqrt{\frac{1+y^2}{1+x^2}}}$$

(2) $\sin(x+y) = \log(x+y) \Rightarrow dy/dx = ?$

$$\sin(x+y) - \log(x+y) = 0$$

$$\frac{dy}{dx} = -\left[\frac{\cos(x+y)(1) - \frac{1}{(x+y)}(1)}{\cos(x+y)(1) - \frac{1}{(x+y)}(1)}\right]$$

$$\frac{dy}{dx} = -1$$

(3) $x^2+y^2+3xy-7=0 \Rightarrow dy/dx =$

$$\frac{dy}{dx} = -\left[\frac{2x+0+3y-0}{0+2y+3x-0}\right]$$

$$dy/dx = -\left[\frac{2x+3y}{2y+3x}\right]$$

(4) $\sin(xy) + x\cos xy = 5$; $dy/dx =$ ———

$$\frac{dy}{dx} = -\left[\frac{y\cos xy + x(-\sin xy)y}{x\cos xy + x(-\sin xy)x}\right]$$

$$dy/dx = -\left[\frac{y\cos xy - xy\sin xy}{x\cos xy - xx\sin xy}\right]$$

(5) $\sqrt{x} + \sqrt{y} = \sqrt{xy}$

$$\sqrt{x} + \sqrt{y} - \sqrt{xy} = 0.$$

$$\frac{dy}{dx} = -\left[\frac{\frac{1}{2\sqrt{x}} + 0 - \sqrt{y}\cdot\frac{1}{2\sqrt{x}}}{0 + \frac{1}{2\sqrt{y}} - \sqrt{x}\frac{1}{2\sqrt{y}}}\right]$$

$$= -\left[\frac{\frac{1}{2\sqrt{x}}(1-\sqrt{y})}{\frac{1}{2\sqrt{y}}(1-\sqrt{x})}\right]$$

$$dy/dx = -\sqrt{\frac{y}{x}}\left[\frac{1-\sqrt{y}}{1+\sqrt{x}}\right]$$

(6) $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \Rightarrow$ Then $dy/dx =$ ———

$$\frac{x^n}{a^n} + \frac{y^n}{b^n} = 2$$

$$\frac{dy}{dx} = -\left[\frac{\frac{1}{a^n} nx^{n-1} + 0}{0 + \frac{1}{b^n} ny^{n-1}}\right]$$

$$\frac{dy}{dx} = -\left(\frac{b}{a}\right)^n \left[\frac{\cancel{n}x^{n-1}}{\cancel{n}y^{n-1}}\right]$$

$$\frac{dy}{dx} = -\left[\frac{b}{a}\right]^n \left[\frac{x^{n-1}}{y^{n-1}}\right]$$

(7) $\frac{x}{y} - \frac{y}{x} = c \Rightarrow$ Then $dy/dx =$ ———

$$\frac{dy}{dx} = -\left[\frac{\frac{1}{y} - y\left(\frac{-1}{x^2}\right)}{x\cdot\frac{-1}{y^2} - \frac{1}{x}}\right]$$

*

$$= \cancel{-}\left[\frac{\frac{1}{y} + \frac{y}{x^2}}{\cancel{-}\frac{x}{y^2} + \frac{1}{x}}\right]$$

$$= \left[\frac{x^2+y^2}{x^2+y^2}\right]$$

$$\boxed{dy/dx = 1}$$

$y = e^{xy} \Rightarrow dy/dx = $ ______

$\cancel{y = e^x + e}^y$.

$\log y = \log e^{xy}$

$\log y = xy(1)$

$\frac{1}{y} \cdot \frac{dy}{dx} = y(1) + x \cdot dy/dx$

$\left(\frac{1}{y} - x\right) \cdot dy/dx = y$

$dy/dx = \frac{y}{\left(\frac{1}{y} - x\right)}$

$\boxed{dy/dx = \frac{y^2}{1 - xy}}$

3. parametric Functions

let $x = f(t)$
$y = g(t)$

Then $dy/dx = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$.

① $x = \theta - \frac{1}{\theta}$; $y = \theta + \frac{1}{\theta}$; Then dy/dx

$dy/dx = \frac{1 - \frac{1}{\theta^2}}{1 + \frac{1}{\theta^2}} = \frac{\theta^2 - 1}{\theta^2 + 1} = \boxed{\frac{x}{y}}$

② $x = ct$
$y = \frac{c}{t}$

$\frac{dy}{dx} = \frac{\frac{-c}{t^2}}{c} \Rightarrow \frac{-\cancel{c}}{t^2} \times \frac{1}{\cancel{c}} = \frac{-1}{t^2}$

$x = a(t + \sin t)$
$y = a(1 - \cos t)$

$\frac{dy}{dx} = \frac{a(0 + \sin t)}{a(1 + \cos t)} = \frac{\sin t}{1 + \cos t}$

$= \frac{\cancel{2}\sin t/2 \cdot \cos t/2}{\cancel{2}\cos^2 t/2} = \tan t/2$

$x = a\cos^3\theta$
$y = a\sin^3\theta$

$\frac{dy}{dx} = \frac{\cancel{a}\, \cancel{3}\sin^2\theta \cdot \cos\theta}{\cancel{a}\, \cancel{3}\cos^2\theta \cdot (-\sin\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$

$x = 3\cos\theta - 2\cos^3\theta$
$y = 3\sin\theta - 2\sin^3\theta$

$\frac{dy}{dx} = \frac{3\cos\theta - 6\sin^2\theta \cdot \cos\theta}{3(-\sin\theta) - 6\cos^2\theta(-\sin\theta)}$.

$= \frac{3\cos\theta - 6\sin^2\theta \cdot \cos\theta}{-3\sin\theta + 6\cos^2\theta \cdot \sin\theta}$

$= \frac{3\cos\theta[1 - 2\sin^2\theta]}{3\sin\theta[-1 + 2\cos^2\theta]}$

$= \frac{3\cos\theta}{3\sin\theta}\left[\frac{1 - 2\sin^2\theta}{2\cos^2\theta - 1}\right]\}\cos 2\theta$

$\boxed{\frac{dy}{dx} = \cancel{\tan\theta}\ \cot\theta}$

$x = e^t(\cos t + \sin t)$
$y = e^t(\cos t - \sin t)$

$\frac{d}{dx} e^x f(x) \Rightarrow e^x[f(x) + f'(x)]$

$\cancel{dy}$ $x = e^t\cos t + e^t\sin t$
$y = e^t\cos t - e^t\sin t$

$\frac{dy}{dx} = \frac{e^t[\cos t - \sin t] - e^t[\sin t + \cos t]}{e^t[\cos t - \sin t] + e^t[\sin t + \cos t]}$

$= \frac{\cancel{\cos t} - \sin t - \sin t - \cancel{\cos t}}{\cos t - \cancel{\sin t} + \cancel{\sin t} + \cos t}$

$= \frac{-\cancel{2}\sin t}{\cancel{2}\cos t} = -\tan t$

$x = a(\cos\theta + \cancel{\tan}\log\tan\frac{\theta}{2})$

$y = a\sin\theta$

$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\theta/2}\cdot\sec^2\theta/2\cdot\frac{1}{2}\right)$

$= a\left(-\sin\theta + \frac{\cos\theta/2}{\sin\theta/2}\cdot\frac{1}{\cos^2\theta/2}\cdot\frac{1}{2}\right)$

$= a\left(-\sin\theta + \frac{1}{2\sin\theta/2.\cos\theta/2}\right)$

$= a\left(-a\sin\theta + \frac{1}{\sin\theta}\right)$

$= a\left(\frac{1-\sin^2\theta}{\sin\theta}\right) = a\frac{\cos^2\theta}{\sin\theta}$ //

$\frac{dy}{d\theta} = a.\cos\theta.$

$\therefore \frac{dy}{d\theta} = \cancel{a}\cos\theta \times \frac{\sin\theta}{\cancel{a}\cos^2\theta} = \tan\theta$ //

$x = a\cos t + t\sin t$
$y = a\sin t - t\cos t$ $\Big\} \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\frac{dx}{dt} = a(-\sin t) + (t\cos t + \sin t)$

$\frac{dy}{dt} = a\cos t - [t(-\sin t) + \cos t]$

$= a\cos t + t\sin t - \cos t.$

$= a\cos t - \cos t + t\sin t$

$\cancel{\frac{dx}{dt} = \sin t}$ [1

$\frac{dy}{dt} = \boxed{\cos t(a-1) + t\sin t}$

$\frac{dx}{dt} = -a\sin t + t\cos t + \sin t$

$= \sin t - a\sin t + t\cos t$

$= \sin t(1-a) + t\cos t$

$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\left(\frac{dx}{dt}\right)^2 = \left(\sin t(1-a) + t\cos t\right)^2$

$= \sin^2 t(1-a)^2 + t^2\cos^2 t + 2\sin t(1-a).t\cos t$

$\left(\frac{dy}{dt}\right)^2 = \left(\cos t(a-1) + t\sin t\right)^2$

$= \cos^2 t(a-1)^2 + t^2\sin^2 t + 2\cos t(a-1).t\sin t$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (a-1)^2 + t^2[\sin^2 t + \cos^2 t]$

$= (a-1)^2 + t^2$

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(a-1)^2 + t^2}$ //

$x = a(\cos t + t\sin t)$

$y = a(\sin t - t\cos t)$

$\frac{dx}{dt} = a(-\cancel{\sin t} + t\cos t + \cancel{\sin t})$

$= at\cos t$

$\frac{dy}{dt} = a\cancel{(-\sin t}\; a(\cos t - [t(-\sin t) + \cos t])$

$= a(\cancel{\cos t} + t\sin t - \cancel{\cos t})$

$= at\sin t$

$\sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} = \sqrt{a^2t^2\cos^2 t + a^2t^2\sin^2 t}$

$= \sqrt{a^2t^2(1)}$

$= at$ //

$$x = \sec\theta - \cos\theta$$
$$y = \sec^n\theta - \cos^n\theta \quad \left(\frac{dy}{dx}\right)^2$$

$$\frac{dy}{d\theta} = n\sec^{n-1}\theta.\sec\theta.\tan\theta. - n\cos^{n-1}\theta.(-\sin\theta)$$
$$= n\sec^{n-1}\theta.\sec\theta.\tan\theta + n\cos^{n-1}\theta.\sin\theta$$
$$= n\sec^{n-1}\theta\ \sec\theta.\tan\theta + n\cos^{n-1}\theta.\tan\theta.\cos\theta$$
$$= n\tan\theta\left[\sec^n\theta + \cos^n\theta\right]$$

$$\frac{dx}{d\theta} = \sec\theta.\tan\theta + \sin\theta$$
$$= \sec\theta.\tan\theta + \tan\theta.\cos\theta$$
$$= \tan\theta\left[\sec\theta + \cos\theta\right]$$
$$= \cancel{\tan\theta\left[\frac{1+\cos^2\theta}{\cos\theta}\right]}$$

$$\frac{dy}{dx} = \frac{n\tan\theta\left[\sec^n\theta + \cos^n\theta\right]}{\tan\theta\left[\sec\theta + \cancel{\tan}\,\theta\right]}$$
$$= n\left[\frac{\sec^n\theta + \cos^n\theta}{\sec\theta + \frac{\cancel{\tan\theta}}{\cos\theta}}\right]$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{n^2\left[\sec^n\theta + \cos^n\theta\right]^2}{\left(\sec\theta + \frac{\cancel{\tan\theta}}{\cos\theta}\right)^2}$$
$$= n^2\left[\frac{(\sec^n\theta - \cancel{\tan}\cos^n\theta)^2 + 4\sec^n\theta.\cos^n\theta}{(\sec\theta - \frac{\cancel{\tan\theta}}{\cos\theta})^2 + 4\sec\theta.\cos\theta}\right]$$

$$\boxed{= n^2\left[\frac{y^2+4}{x^2+4}\right]}$$

$$x = \sin t.\cos 2t \qquad dy/dx \text{ at } t = \pi/4$$
$$y = \cos t.\sin 2t$$

$$\frac{dy}{dt} = \cos t.\ \cos 2t.2 + \sin 2t(-\sin t)$$
$$= 2\cos t.\cos 2t - \sin t\sin 2t$$

$$\frac{dx}{dt} = \sin t\ (-\sin 2t)2 + \cos 2t.\cancel{\sin t}\cos t$$
$$= -2\sin t.\sin 2t + \cos 2t.\cancel{\sin t}\cos t$$

$$\left.\frac{dy}{dx}\right|_{t=\pi/4} = \left[\frac{2\cos t.\cos 2t - \sin t.\sin 2t}{\cos 2t.\cancel{\sin t}\cos t - 2\sin t.\sin 2t}\right]$$
$$= \frac{2\cos\pi/4 + \cancel{\cos 2\pi/4} - \sin\pi/4.\sin 2\pi/4}{\cancel{\cos(2\pi/4)}.\cos\pi/4 - 2\sin\pi/4.\sin 2\pi/4}$$
$$= \left[\frac{0 - \frac{1}{\sqrt2}}{0 - 2\frac{1}{\sqrt2}}\right]$$
$$= \frac{\frac{-1}{\sqrt2}}{\frac{-2}{\sqrt2}} = \frac{1}{2}$$

$$\cancel{x = \cos\theta(1+\cos\theta)}$$
$$x = \cos\theta\,(1+\cos\theta)$$
$$y = \sin\theta\,(1+\sin\theta) \qquad dy/dx \text{ at } \theta = \pi/2$$

$$\frac{dy}{dx} = \frac{\sin\theta(0+\cos\theta) + (1+\sin\theta).\cos\theta}{\cos\theta(0-\sin\theta) + (1+\cos\theta)-\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \frac{1(0) + (1+1)\,0}{10+0} = 0$$

4. Derivative of function with another function

$$\frac{dy}{dx} = \frac{f'(x)}{g'(x)}$$

The derivative of $\sin x$ w.r.t $\cos x$.
$\Downarrow f(x)$, $g(x)$

$$\frac{dy}{dx} = \frac{f'(x)}{g'(x)} = \frac{\cos x}{-\sin x} = -\cot x.$$

The derivative of $e^{f(x)}$ w.r.t $f(x)$ is ____

$$\frac{dy}{dx} = \frac{e^{f(x)} \cdot \cancel{f'(x)}}{\cancel{f'(x)}} = e^{f(x)}$$

② $e^{\sin x}$ w.r.t $\sin x \Rightarrow \frac{dy}{dx} = e^{\sin x}$

$e^{\cos x}$ .w.r.t. $\cos x$. $dy/dx = e^{\cos x}$

$e^{\tan^{-1}x}$ w.r.t $\tan^{-1}x \Rightarrow dy/dx = e^{\tan^{-1}x}$

$e^{\tanh^{-1}x}$ .w.r.t. $\tanh^{-1}x$ $dy/dx = e^{\tanh^{-1}x}$

$\sin^{-1}\left[\frac{2x}{1+x^2}\right]$ w.r.t $\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$

$\boxed{x = \tan\theta}$

$$\frac{dy}{dx} = \frac{\sin^{-1}[\sin 2\theta]}{\cos^{-1}[\cos 2\theta]} = \frac{2\theta}{2\theta} = 1$$

$\tan^{-1}\left[\frac{2x}{1-x^2}\right]$ w.r.t $\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right] \Rightarrow dy/dx = 1$

$$\left.\begin{array}{l} \sin^{-1}\left[\frac{2x}{1+x^2}\right] \\ \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right] \\ \tan^{-1}\left[\frac{2x}{1-x^2}\right] \end{array}\right\} = 2\tan^{-1}x$$

$\tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right] \Leftarrow 3\tan^{-1}x$

$\sin^{-1}(3x-4x^3) \xrightarrow{w.r.t.} \cos^{-1}(4x^3-3x)$

$\boxed{x = \sin\theta}$ $\Downarrow 3\sin^{-1}x$.

$\boxed{x = \cos\theta}$ $= 3\cos^{-1}x$.

$$\frac{dy}{dx} = \frac{\cancel{3\sin^{-1}}\ 3 \cdot \frac{1}{\sqrt{1-x^2}}}{3 \cdot \frac{-1}{\sqrt{1-x^2}}} = -1$$

- $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t $\sqrt{1+3x}$ at $x = -\frac{1}{3}$; $dy/dx = 0$
- $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\tan^{-1}\left[\frac{2x\sqrt{1-x^2}}{1-2x^2}\right]$ at $x=0$ $dy/dx = \frac{1}{4}$

5. Differentiation of Inifinite series:

$y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x)}}}} \ldots \infty$

$y = \sqrt{f(x) + y}$

$y^2 = f(x) + y$

$y^2 - y = f(x)$

$2y \cdot dy/dx - dy/dx = f'(x)$

$dy/dx\,[2y-1] = f'(x)$

$$\boxed{dy/dx = \frac{f'(x)}{2y-1}}$$

$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x}}} \ldots \infty$

$$dy/dx = \frac{\cos x}{2y-1}$$

$y = \sqrt{f(x) \cdot \sqrt{f(x) \cdot \sqrt{f(x)}}} \cdots \infty$

$dy/dx =$

$y^2 = f(x) \cdot y$

$y = f(x)$

$\boxed{dy/dx = f'(x)}$

$y = \sqrt{x \cdot \sqrt{x \cdot \sqrt{x \cdot \sqrt{\cdots}}}} \infty$

$dy/dx = 1$

$y = \sqrt{\sin x \cdot \sqrt{\sin x \sqrt{\sin x \cdots \infty}}}$

$dy/dx = \cos x$ //

$y = x^{x^{x^{\cdot^{\infty}}}}$

$y = x^y$

$\log y = y \log x$

$\frac{1}{y} \cdot dy/dx = y \cdot \frac{1}{x} + \log x \cdot dy/dx$

$dy/dx \left(\frac{1}{y} - \log x\right) = \frac{y}{x}$

$dy/dx \left(\frac{1 - y\log x}{y}\right) = y/x$

$dy/dx = y/x \left[\frac{y}{1 - y\log x}\right]$

$$\boxed{\frac{dy}{dx} = \frac{y^2}{x[1 - y\log x]}}$$

$y = \sqrt{x}^{\sqrt{x}^{\cdots\infty}}$

$y = \sqrt{x}^y$

$\log y = \log(\sqrt{x})^y$

$\log y = \cancel{y} \log(x)^{y/2}$

$\log y = \frac{y}{2} \log x$

$\frac{1}{y} \cdot dy/dx = \frac{y}{2} \frac{1}{x} + \log x \frac{1}{2} dy/dx$

$\frac{dy}{dx}\left(\frac{1}{y} - \frac{\log x}{2}\right) = \frac{y}{2x}$

$\frac{dy}{dx}\left[\frac{2 - y\log x}{2y}\right] = \frac{y}{2x}$

$\frac{dy}{dx} = \frac{y}{\cancel{2}x}\left[\frac{\cancel{2}y}{2 - y\log x}\right] \Rightarrow \frac{y^2}{x[2 - y\log x]}$

$$\boxed{dy/dx = \frac{2y^2}{4x - 2xy\log x}}$$

$y = \sin x^{\sin x^{\cdots\infty}}$

$y = \sin x^y$

$\log y = \log(\sin x)^y$

$\log y = y \log \sin x$

$\frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x \cdot dy/dx$

$\frac{dy}{dx}\left(\frac{1}{y} - \log \sin x\right) = y \cot x$

$\frac{dy}{dx}\left[\frac{1 - y\log \sin x}{y}\right] = y \cot x$

$\frac{dy}{dx} = y\cot x \left[\frac{y}{1 - y\log \sin x}\right]$

$$\boxed{dy/dx = \frac{y^2 \cot x}{1 - y\log \sin x}}$$

$$\Rightarrow y = a^{x^{a^{x \cdots \infty}}}$$

$$y = a^{x^y}$$

$$\log y = \log a^{x^y}$$

$$\log y = x^y \log a$$

$$\log(\log y) = \log(x^y \cdot \log a)$$

$$\log(\log y) = y \log x + \log(\log a)$$

$$\frac{1}{\log y} \cdot \frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} + \cancel{\frac{1}{\log}} \; 0.$$

$$\frac{dy}{dx}\left[\frac{1}{y \log y} - \log x\right] = \frac{y}{x}$$

$$\frac{dy}{dx}\left[\frac{1 - y\log x \cdot \log y}{y \log y}\right] = \frac{y}{x}$$

$$\frac{dy}{dx} = \left[\frac{y}{x}\right] \frac{y \log y}{1 - y \log x \cdot \log y}$$

$$\boxed{\frac{dy}{dx} = \frac{y^2 \log y}{x(1 - y\log x \cdot \log y)}}$$

$$\Rightarrow y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{\cdots \infty}}}$$

$$y = x + \frac{1}{y}$$

$$y = \frac{xy + 1}{y}$$

$$y^2 = xy + 1$$

$$y^2 - xy - 1 = 0$$

$$2y \frac{dy}{dx} \cancel{- (1) \cdot dy/dx - 0} = 0$$

$$dy/\cancel{dx} \cancel{[2y - 1]} = 0.$$

$$y^2 - xy - 1 = 0$$

$$2y \cdot \frac{dy}{dx} - [x \cdot dy/dx + y(1)] - 0 = 0$$

$$2y \cdot \frac{dy}{dx} - x\, dy/dx - y = 0$$

$$2y \frac{dy}{dx} - x\, dy/dx = y$$

$$dy/dx\, [2y - x] = y$$

$$\boxed{\frac{dy}{dx} = \frac{y}{2y - x}}$$

$$y = x^2 + \cfrac{1}{x^2 + \cfrac{1}{x^2 \cdots \infty}}$$

$$y = x^2 + \frac{1}{y}$$

$$y - \frac{1}{y} = x^2$$

$$\frac{dy}{dx} + \frac{1}{y^2} \cdot dy/dx = 2x.$$

$$dy/dx \left[\frac{y^2 + 1}{y^2}\right] = 2x$$

$$\boxed{dy/dx = \frac{2xy^2}{y^2 + 1}}$$

$y = e^x + e^{x+e^{x \cdots \infty}}$

$y = e^{x+y}$

$\log y = \log(e)^{x+y}$

$\log y = x+y$

$\frac{1}{y}\cdot\frac{dy}{dx} = 1 + dy/dx$

$\frac{dy}{dx}\left(\frac{1}{y}-1\right) = 1$

$\frac{dy}{dx}\left(\frac{1-y}{y}\right) = 1$

$\boxed{\frac{dy}{dx} = \frac{y}{1-y}}$

$x = e^{y+e^{y+\cdot\infty}}$

$x = e^{y+x}$

$\log x = \log(e)^{y+x}$

$\log x = y + x$

$\frac{1}{x} = \frac{dy}{dx} + 1$

$\frac{dy}{dx} = \frac{1}{x} - 1$

$\boxed{\frac{dy}{dx} = \frac{1-x}{x}.}$

★SUPER TRICKS★

①

$\tan(\pi/4 + x) = \frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$

$\tan(\pi/4 - x) = \frac{1-\tan x}{1+\tan x} = \frac{\cos x - \sin x}{\cos x + \sin x}$

① $y = \tan^{-1}\left[\frac{1+\tan x}{1-\tan x}\right] \Rightarrow \tan^{-1}\left[\tan(\pi/4 + x)\right]$

$= \pi/4 + x \Rightarrow dy/dx = 1$

② $y = \tan^{-1}\left[\frac{1-\tan x}{1+\tan x}\right] \Rightarrow \tan^{-1}\left[\tan(\pi/4 - x)\right]$

$= \pi/4 - x = dy/dx = -1$

II

$y = \tan^{-1}\left[\frac{1+\sin 2x}{\cos 2x}\right]$

$y = \tan^{-1}\left[\frac{\cos x + \sin x}{\cos x - \sin x}\right]$

$y = \tan^{-1}(\tan(\pi/4 + x))$

$dy/dx = 1$

$1+\sin 2x = (\sin x + \cos x)^2$

$\cos 2x = \cos^2 x - \sin^2 x$

$\Downarrow$

$(\cos x + \sin x)(\cos x - \sin x)$

$y = \tan^{-1}\left[\frac{1-\sin 2x}{\cos 2x}\right]$

$= \tan^{-1}\left[\frac{(\sin x - \cos x)^2}{(\cos x + \sin x)(\cos x - \sin x)}\right]$

$= \tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right]$

$= dy/dx = -1$

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$\tan^{-1}\left[\frac{1+\sin x}{\cos x}\right]$

$\tan^{-1}\left[\frac{(\cos x/2+\sin x/2)^2}{(\cos x/2+\sin x/2)(\cos x/2-\sin x/2)}\right]$

$\tan^{-1}\left[\tan(\pi/4+x/2)\right]$

$dy/dx = \frac{1}{2}$

$1+\sin x = (\cos x/2+\sin x/2)^2$

$\cos x = \cos^2 x/2 - \sin^2 x/2$

$= (\cos x/2+\sin x/2)\times(\cos x/2-\sin x/2)$

$y = \tan^{-1}\left[\frac{1+\cos 2x}{\sin 2x}\right]$

$y = \tan^{-1}\left(\frac{2\cos^2 x}{2\sin x.\cos x}\right)$

$y = \tan^{-1}\left[\frac{\cos x}{\sin x}\right]$

$y = \tan^{-1}\left[\tan(90-x)\right]$

$y = 90-x$

$\boxed{dy/dx = -1}$

★ partial Derivatives ★

If $u = \log(x^3+y^3)$; then $\frac{\partial u}{\partial x}$ = ——

$\frac{\partial u}{\partial x} = \frac{1}{x^3+y^3}(3x^2) = \frac{3x^2}{x^3+y^3}$

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$

$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)$

$\frac{\partial^2 u}{\partial y \cdot \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$

$\frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)$

if $u = \sqrt{x^2+y^2} \Rightarrow \frac{\partial^2 u}{\partial x \cdot \partial y} = ?$

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) \Rightarrow$

$\frac{\partial u}{\partial y} = \frac{1}{\cancel{2}\sqrt{x^2+y^2}}\cdot(\cancel{2}y) = \frac{y}{\sqrt{x^2+y^2}}$

$\frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2}}\right) \Rightarrow y\cdot\frac{-1}{2(x^2+y^2)\cdot\sqrt{x^2+y^2}}(2x)$

$= \frac{-xy}{(x^2+y^2)^{3/2}}$

$dy/dx = \frac{-\partial f/\partial x}{\partial f/\partial y}$

$a^x + a^y = a^b$ then $dy/dx =$

$\frac{dy}{dx} = \frac{-(a^x\log a)}{(a^y\log a)} = -a^{x-y}$

Eulers theorem:

If a function is homogeneous, let $u = f(x,y)$ is homogeneous in which all terms must have same degree.

Eg: $u = x^2 + xy$

$u = x^2 + 2xy + y^2$

$u = \sqrt{x} + \sqrt{y} + \sqrt{x}$

Then $\boxed{x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = nu}$

n = degree of function (u) $\Downarrow$ $f(x,y)$.

If $u = x^2 + xy + y^2$, then $x \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} =$

$= nu$

$= 2(x^2 + xy + y^2)$ //

If $u = \frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}$ then $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} =$ ____

$n = \cancel{2} \quad 2 - \frac{1}{2} = \frac{3}{2}$

$\therefore \Rightarrow \frac{3}{2} u$

(1) $\Rightarrow$ If $u = \sin^{-1}[f(x,y)] \Rightarrow$ then $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} =$

$\underbrace{\sin u}_{\Downarrow} = f(x,y)$

$z = \sin u = f(x,y)$.

$\frac{\partial z}{\partial x} = \cos u \cdot \frac{\partial u}{\partial x}$

$\frac{\partial z}{\partial y} = \cos u \cdot \frac{\partial u}{\partial y}$.

$z = f(x,y)$

$x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = nz$.

$x \cos u \frac{\partial u}{\partial x} + y \cdot \cos u \cdot \frac{\partial u}{\partial y}$

$= n \sin u$

$\boxed{x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = n \cdot \tan u}$

$\boxed{n = \text{degg. of } f(x,y)}$

(2) $\Rightarrow$ If $u = \operatorname{cosec}^{-1}[f(x,y)]$

then $x \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = \boxed{-n \operatorname{Tan} u}$

(3) If $u = \cos^{-1}[f(x,y)]$; then $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = -n \cot u$

(4) $u = \sec^{-1}[f(x,y)]$; then $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = n \cot u$

(5) $u = \tan^{-1}[f(x,y)]$; then $x \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = \frac{n}{2} \cdot \sin 2u$.

(6) $u = \cot^{-1}[f(x,y)]$; then $\frac{x \cdot \partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = \frac{-n}{2} \sin 2u$

If $u = \sin^{-1}\left[\frac{x^3 + y^3}{x + y}\right]$ then $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} =$?

$\Rightarrow n \tan u$

$= (3-1) \tan\left(\sin^{-1}\left[\frac{x^3 + y^3}{x + y}\right]\right)$

$\Rightarrow 2 \tan u$

If $u = \log_a(f(x,y))$; then $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} =$ ____

$a^u = f(x,y) = z$

$\frac{\partial z}{\partial x} = a^u \cdot \log a \cdot \frac{\partial u}{\partial x}$

$\frac{\partial z}{\partial y} = a^u \cdot \log a \frac{\partial u}{\partial y}$

$z = f(x,y)$

$x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = nz$

$x\, a^u \cdot \log a \cdot \frac{\partial u}{\partial x} + y \cdot a^u \cdot \log a \frac{\partial u}{\partial y}$

$= n \cdot a^u$.

$x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{n \cdot \cancel{a^u}}{\cancel{a^u} \cdot \log a}$

$\boxed{= \frac{n}{\log_e a}}$

$\Rightarrow \boxed{n \cdot \log_a e}$

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If $u = \log_5 (x^3+y^3+x^2y)$.

then $x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$ ____

$\boxed{n=3}$

$\Rightarrow \frac{n}{\log_e a} \Rightarrow \frac{3}{\log_e 5} \Rightarrow 3 \log_5 e$

If $u = a^{f(x,y)} \Rightarrow$ then $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} =$

$\log_a u = f(x,y) = z$

$\frac{\partial z}{\partial x} = \frac{1}{u \log a} \cdot \frac{\partial u}{\partial x}$

$\frac{\partial z}{\partial y} = \frac{1}{u \log a} \cdot \frac{\partial u}{\partial y}$

$z = f(x,y)$

$x \cdot \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = n \cdot z$.

$x \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = n \cdot \log_a u \cdot u \cdot \log a$

$\Rightarrow \boxed{n . \log_a u . u \log a}$

$\Rightarrow n . \frac{\log u}{\log a} u . \log a$

$\Rightarrow \boxed{n u \log_e u} \Rightarrow \boxed{n u \log_e u}$

If $u = e^{\sqrt{x}+\sqrt{y}} \Rightarrow x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$ ____

$\Rightarrow n u \log_e u$

$= \boxed{\frac{1}{2} u \log_e u}$

★ Functions ★

- Transcedental functions
  - All functions except $\boxed{\text{Algebraic}}$
- Algebraic function [polynomial function]

(1) Identity function:

$f: A \to A$

$\boxed{\text{Domain} = \text{Codomain}}$

Eg: $\boxed{f(x) = x}$

(2) polynomial Function:

$P_n(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots a_n$

→ $a_0, a_1, a_2, \dots \in R$

→ power index of variable $(n) \geq 0$.

Eg: x^2+x+3.

(3) Greatest Integer function / stepfunction:

$f(2.5) = [2.5] \Rightarrow 2$

$f(-2.5) = [-2.5] = -3$

$f(-2012.3) = [-2012.3] = -2013$.

(4) Modulus Function

$f(x) = |x|$

$|x| = x ; x > 0$

$|x| = -x ; x < 0$

$|0| = 0 ; x = 0$.

Signum function

$f(x) = \frac{|x|}{x} = 1 ; x > 0$

$= -1 ; x < 0$

$\boxed{\text{odd function}}$

Inverse Function:

$f: A \to B$

$f^{-1}: B \to A$

Eg: $y = x+3$; then $f^{-1}(x) =$ ____

$y = x+3$

$x = y-3$

$\boxed{f^{-1}(x) = x-3}$

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$f(x)$	$f^{-1}(x)$
$ax+b$	$\frac{x-b}{a}$
$\sqrt{a^2-x^2}$	$\sqrt{a^2-x^2}$
a^x	$\log_a^x$
x^n	$\sqrt[n]{x}$
$\frac{ax+b}{cx-a}$	$\frac{ax+b}{cx-a}$

$f(x) = \log_a^x$

$y = \log_a^x$

$x = a^y$

$\boxed{f'(x) = a^x}$

odd / even functions:

$f(-x) = -f(x) \Rightarrow f(x)$ is odd

$f(-x) = f(x) \Rightarrow f(x)$ is even

odd	even
$\sin x$ x $\tan x$	$\cos x$ x^2 $\sqrt{a^2-x^2}$ $\lvert x\rvert$

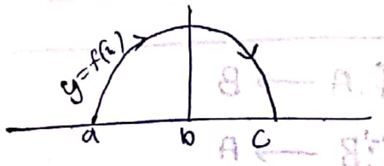
odd × odd = even

odd × even = odd

odd ÷ even = odd

even ÷ odd = odd

Monotonic function: function only increasing or only decreasing in interval (a,b).



$(a,b) \rightarrow$ mono tonic ↑

$(b,c) \rightarrow$ monotonic ↓

★ Limits ★

Basics:

A limit will be exists if left hand approach = Right hand approach.

i.e, $\boxed{\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)}$

~~$x \to 2$~~ For eg: $\lim_{x\to 2} \sqrt{4-x^2}$

$x \to 2^- \Rightarrow x = 1.99 \Rightarrow \sqrt{4-(1.99)^2} = \sqrt{+ve} = 0.$

$x \to 2^+ \Rightarrow x = 2.001 \Rightarrow \sqrt{4-(2.001)^2} = \sqrt{-ve} =$ Imagn

$\therefore$ left hand aproach ≠ Right hand approach

$\therefore$ $\boxed{\text{Limit doesn't exist}}$

standard forms of existence of limits.

(1) $\lim_{x\to a} \sqrt{f(x)}$ — $f(a)=0 \Rightarrow$ doesn't Exist; $f(a) \neq 0 \Rightarrow \boxed{\text{sol: } \sqrt{f(a)}}$

(2) $\lim_{x\to a} \frac{1}{f(x)}$ — $f(a)=0 \Rightarrow$ D.E; $f(a)\neq 0 \Rightarrow \boxed{\text{sol: } \frac{1}{f(a)}}$

(3) $\lim_{x\to a} \frac{|f(x)|}{f(x)}$ — $f(a)=0 \Rightarrow$ D.E; $f(a)\neq 0 \Rightarrow$ sol: 1.

(4) $\lim_{x\to a} [f(x)]$ — $f(a) \Rightarrow$ Non integer; sol: $\boxed{[f(a)]}$; $f(a) \Rightarrow$ Integer $\Rightarrow \boxed{\text{D.E}}$

$\lim \tan x$, $\lim \sec x$ — D.E for odd multiples of $\pi/2$

~~lt tan x~~ $\lim \cot x$, $\lim \operatorname{cosec} x$ $\rightarrow$ D.E for all multiples of π

$\lim \sin x$, $\lim \cos x$ — exist for all R.

$x \to \infty$ (or) $x \to -\infty$; where $f(x), g(x) \in$ $\boxed{\text{Algebraic functions}}$

$\boxed{\text{common highest degree coeff.}}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}.$$

Sol: $\dfrac{\text{Highest deg. coeff. in Numerator}}{\text{Highest deg. coeff. in Denominator}}$

Eg: $\lim_{x\to\infty} \dfrac{\boxed{x^{16}} + x^3 + x^5 + 2x + 2020}{2\boxed{x^{16}} + x^2 + 5x + 2020}$

Sol: $= \dfrac{1}{2}$

$\lim_{x\to\infty} \dfrac{x^{16} + x + 5}{x+1}$

$\Rightarrow \dfrac{1}{0} \Rightarrow \boxed{\text{undefined or } \infty}$

$x \to \infty$ (or) $x \to -\infty$; where $f(x), g(x) \in$ $\boxed{\text{Exponential functions}}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}$$

$\boxed{\text{Common highest base}}$

Sol: $\dfrac{\text{coeff. of highest base in Numerator}}{\text{Coeff. of highest base in Denominator}}$

Eg: $\lim_{x\to\infty} \dfrac{2^x + e^x + 5^x + 7^{x+1}}{1(7^x) + 8^x + 5^x + e^x + 2^x}$ $\Rightarrow 7^x(7)$

Sol: $\dfrac{7}{1} \Rightarrow 7$ //

$\lim_{x\to\infty} \dfrac{e^x + 1}{2^x + 1}$ $\quad e = 2.735$

Sol: $\dfrac{1}{0} = \infty$

$\lim_{x\to\infty} \dfrac{2^x + 1}{3^x + 1} = \dfrac{0}{\cancel{3}} = 0.$

$\boxed{\text{Format - 1}}$

$$\lim_{x\to 0} \frac{(1) f_1(ax) \pm (1) f_2(bx)}{(1) g_1(cx) \pm (1) g_2(dx)}$$

where $f_1, f_2, g_1, g_2 \in \phi$ $\boxed{\text{set of functions}}$

$\{ x, \sin x, \tan x, \sin^{-1}x, \tan^{-1}x, \sinh x, \tanh x, \sinh^{-1}x, \tanh^{-1}x \}$

" , " $\in$ $\boxed{\text{1st degree functions}}$

Sol: $\boxed{\dfrac{(1) a \pm (1) b}{(1) c \pm (1) d}}$

Eg: $\lim_{x\to 0} \dfrac{5\sinh^{-1}(20)x + 2\tan^{-1}x}{6x + 2\sin x}$

Sol: $\dfrac{5(20) + 2(1)}{6(1) + 2(1)} \Rightarrow \dfrac{102}{8} = \dfrac{51}{4}$

Eg: $\lim_{x\to 0} \left\{ \dfrac{2\tan x + x}{2\tan^{-1}x + 2x} \right\}$

Sol: $\dfrac{2(1) + 1}{2(1) + 2(1)} \Rightarrow \dfrac{3}{4}$ //

Eg: $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$ $\qquad \lim_{x\to 0} \dfrac{\tan x}{x} = 1$

$\lim_{x\to 0} \dfrac{\sin ax}{x} = a$ $\qquad \lim_{x\to 0} \dfrac{\tan ax}{x} = a$

$\lim_{x\to 0} \dfrac{2\sin^2 \frac{mx}{2}}{x^2}$

$\Rightarrow 2\left[\lim_{x\to 0} \left(\dfrac{\sin \frac{mx}{2}}{x}\right)^2\right]$

$\Rightarrow 2\dfrac{m^2}{4} \Rightarrow \boxed{m^2/2}$

$\lim_{x\to 0} \dfrac{\tan^2 \frac{mx}{2}}{16x^2}$

$\Rightarrow \lim_{x\to 0} \left(\dfrac{\tan \frac{mx}{2}}{4x}\right)^2$

$\Rightarrow \left(\dfrac{m}{8}\right)^2 = \boxed{\dfrac{m^2}{64}}$

Note: $f(0) = 0$ or $g(0) = 0$; $\rightarrow$ we get $\frac{0}{0}$.

$\boxed{\frac{0}{0}, \frac{\infty}{\infty}, 0^0, 1^\infty, 0\times\infty, \infty-\infty, \infty+\infty}$

↳ Indeterminant forms

Format - 2

$$\lim_{x\to 0} \frac{[f_1(ax)]^m \cdot [f_2(bx)]^n}{[g_1(cx)]^p \cdot [g_2(dx)]^q}$$

where $f_1, f_2, g_1, g_2 \in \phi$. Then :

1) if $\boxed{m+n = p+q}$; sol: $= \dfrac{a^m \cdot b^n}{c^p \cdot d^q}$

2) if $\boxed{m+n > p+q}$; sol: 0.

3) if $\boxed{m+n < p+q}$; sol: ∞, D.E

Eg: $\lim_{x\to 0} \dfrac{2x^{(2)} \cdot (\tan^{-1}(100x))^1}{\sin x \cdot \sinh^{-1}(10x)}$

$\boxed{\begin{matrix} m+n = 3 \\ p+q = 2 \end{matrix}}$ $\therefore \boxed{m+n > p+q}$

$\therefore$ sol: 0.

$$\lim_{x\to 0} \frac{\sin^3 x * [\tan^{-1}x]^7}{x^4 * (\sin^{-1}x)^6}$$

$\boxed{\begin{matrix} m+n = 10 \\ p+q = 10 \end{matrix}}$ $\therefore$ sol: $\dfrac{(1)^3 \cdot (1)^7}{(1)^4 \cdot (1)^6} = 1.$

$$\lim_{x\to 0} \frac{x^5 \cdot \tan^{-1}x}{\sin^6 x \cdot (\sin^{-1}x)^2}$$

$\boxed{\begin{matrix} m+n = 6 \\ p+q = 8 \end{matrix}}$ $\boxed{m+n < p+q}$

sol: $\boxed{\infty, \text{ or D.E}}$

$\lim_{x\to 0} f(x)^{g(x)} =$ sol: 1

where $f(x), g(x) \in \phi$.

Eg: $x^{\sin x}\Big\}_{\lim_{x\to 0}} = 1$

$\lim_{x\to 0} (\sin^{-1}x)^{\tan^{-1}x} = 1$

Standard Forms

① $\lim_{x\to a} \dfrac{x^n - a^n}{x-a} \Rightarrow$ sol: $\boxed{n.a^{n-1}}$

$\lim_{f(x)\to a} \dfrac{[f(x)]^n - a^n}{f(x) - a} \Rightarrow$ sol: $\boxed{na^{n-1}}$

② $\lim_{x\to 0} \dfrac{a^x - 1}{x} \Rightarrow \log_e a$

$\lim_{f(x)\to 0} \dfrac{a^{f(x)} - 1}{f(x)} \Rightarrow \log_e a$

③ $\lim_{x\to 0} (1+ax)^{b/x} = \boxed{e^{ab}}$

$\lim_{f(x)\to 0} [1+af(x)]^{\frac{b}{f(x)}} = \boxed{e^{ab}}$

④ $\lim_{x\to\infty} \left(1+\dfrac{a}{x}\right)^{bx} = \boxed{e^{ab}}$

$\lim_{f(x)\to\infty} \left[1+\dfrac{a}{f(x)}\right]^{b.f(x)} = \boxed{e^{ab}}$

⑤ $\lim_{x\to\infty} \left(\dfrac{x+a}{x+b}\right)^{x+c} \Rightarrow e^{a-b}.$

$\lim_{f(x)\to\infty} \left(\dfrac{f(x)+a}{f(x)+b}\right)^{f(x)+c} \Rightarrow e^{a-b}$

$\lim_{x\to\infty} \left(\dfrac{x+a}{x-a}\right)^x \Rightarrow e^{a-(-a)} = e^{2a}.$

⑥ $\lim_{x\to\infty} \left(1 + \dfrac{1}{ax+b}\right)^{cx+d} \Rightarrow \boxed{e^{c/a}}$

$\lim_{f(x)\to\infty} \left(1+\dfrac{1}{af(x)+b}\right)^{cf(x)+d} = \boxed{e^{c/a}}$

L-Hospital Rule:

$$\lim_{x\to a} f(x)/g(x)$$

$\Rightarrow$ if $\frac{f(a)}{g(a)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$

then $\lim_{x\to a} \frac{f'(x)}{g'(x)}$

$\Rightarrow$ if $\frac{f'(a)}{g'(a)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$ then

$\lim_{x\to a} \frac{f''(x)}{g''(x)}$.

:Theorems:

(1) $\lim_{x\to a} (k) = k$.

- $\lim_{x\to a} k[f(x)] = k\left[\lim_{x\to a} f(x)\right]$
- $\lim_{x\to a} f(x) \pm g(x) \Rightarrow \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
- $\lim_{x\to a} f(x).g(x). \Rightarrow \lim_{x\to a} f(x) . \lim_{x\to a} g(x)$
- $\lim_{x\to a} f(x)/g(x) \Rightarrow \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
- $\lim_{x\to a} f(x)^{g(x)} = e^{\lim_{x\to a} g(x). \log f(x)}$ ★
- $\lim_{x\to a} f(x) = \lim_{x\to 0} f(a-x)$ ★

$$\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{1/x} \Rightarrow \sqrt{ab}$$

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x} \Rightarrow \sqrt[3]{abc}$$

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{1/x} = \sqrt[4]{a.b.c.d}$$

$$\lim_{x\to 1} x^{\frac{1}{x-1}}.$$

$$e^{\lim_{x\to 1} \frac{1}{x-1}.\log x}$$

$$e^{\lim_{x\to 1} \frac{\log x}{x-1}} \Rightarrow \lim_{x\to 1} \frac{1}{x} = 1$$

$\Rightarrow e^1 = e$//.

$$\lim_{x\to 0} \left(\frac{2^x+3^x}{4}\right)^{1/x}$$

$$e^{\lim_{x\to 0} \frac{1}{x}.\log\left(\frac{2^x+3^x}{4}\right)} \Rightarrow \lim_{x\to 0} \frac{\log\left(\frac{2^x+3^x}{4}\right)}{x} = \infty$$

$= e^\infty \Rightarrow \infty$//

:More standard forms:

$$\lim_{n\to\infty} \frac{1^k+2^k+3^k+4^k+\dots+n^k}{n^{k+1}} = \frac{1}{k+1}$$

$\boxed{k>0}$

$$\lim_{x\to 0} \frac{\cos ax - \cos bx}{x^2} \Rightarrow b^2-a^2$$

$$\lim_{x\to 0} \frac{\cos ax - \cos bx}{\cos cx - \cos dx} \Rightarrow \frac{b^2-a^2}{d^2-c^2}$$

$$\lim_{x\to 0} [1+af(bx)]^{\frac{c}{g(dx)}} \Rightarrow e^{\frac{abc}{d}}$$

$\boxed{f(x), g(x) \in \phi}$

eg: $\lim_{x\to 0} (1+x)^{1/x} = e^{\frac{1\times1\times1}{1}} = e$

$$\lim_{x\to 0} (1+3\sin^{-1}2x)^{\frac{2}{\sin^{-1}3x}} \Rightarrow e^{\frac{3\times2\times2}{3}} \Rightarrow e^4//$$

$$\lim_{x\to 0} \frac{\sqrt[m]{1+f_1(ax)} - \sqrt[n]{1+f_2(bx)}}{x} = \frac{a}{m} - \frac{b}{n}$$

$\boxed{f_1, f_2 \in \phi.}$

Eg: $\lim_{x\to 0} \frac{\sqrt[5]{1+\sin 3x} - \sqrt[4]{1+\sin^{-1}(3x)}}{x} \Rightarrow \frac{3}{5} - \frac{3}{4}$

$\Rightarrow \boxed{\frac{-3}{20}}$

$\lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \Rightarrow \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$

Applications of L.Hospital Rule:

$\lim_{x\to a} \frac{x^m - a^m}{x^n - a^n} \Rightarrow \lim_{x\to a} \frac{m x^{m-1}}{n x^{n-1}} \rightarrow \boxed{\frac{m}{n} a^{m-n}}$

$\lim_{x\to 0} \frac{a^x - 1}{b^x - 1} \Rightarrow \lim_{x\to 0} \frac{a^x \log a}{b^x \log b} \Rightarrow \boxed{\log_b a}$

$\lim_{x\to 0} \frac{a^x + b^x - 2}{x} \Rightarrow \lim_{x\to 0} \frac{a^x \log a + b^x \log b}{1}$

$\Rightarrow \boxed{\log(ab)}$

$\lim_{x\to a} \frac{1}{[f(x)]^n}$
- if $f(a)=0$ & n is odd — sol: DE
- n is even $\Rightarrow$ sol: ∞

If an error of 3% in side of cube, the % error in volume: ____

let side $= x$

$\boxed{\frac{\Delta x}{x} = 0.03}$

$V = x^3$

$\Delta V = 3x^2 . \Delta x \Rightarrow 3x^2 \times 0.03x$

$\frac{\Delta V}{V} = \frac{3x^2 \times 0.03x}{x^3} \times 100$

$= 3 \times 0.03 \times 100$

$= 9\%$

Standard Limits:

- $\lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{1-\cos ax}{x} = 0$
- $\lim_{x\to 0} \frac{1-\cos ax}{x^2} = \frac{a^2}{2}$
- $\lim_{x\to 0} \frac{1-\cos ax}{1-\cos bx} = a^2/b^2$
- $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$
- $\lim_{n\to 0} (1+n)^{1/n} = e.$
- $\lim_{x\to 0} \frac{a^x - b^x}{x} = \log(a/b)$
- $\lim_{x\to 0} \frac{\log(x+1)}{x} = 1$
- $\lim_{x\to 0} \frac{\log(ax+1)}{x} = a$ $\quad \frac{\frac{1}{ax+1}\cdot a}{1}$
- $\lim_{x\to 0} \frac{a^x - 1}{b^x - 1} = \log_b a.$
- $\lim_{x\to\infty} \log x = \infty$ $\quad \lim_{x\to 0^+} \log x = -\infty$

If the increase in side of a square is 1%, find the % of change in area of square ____

$A = x^2$ $\quad \frac{\Delta x}{x} = 0.01$

$\boxed{\Delta A = 2x . \Delta x}$ $\quad \boxed{\Delta x = 0.01 \times x}$

$= 2x \times 0.01 \times x$

% of error $= \frac{\Delta A}{A} = \frac{2x \times 0.01 \times x}{x^2} \times 100$

$= 2\%$

Continuity & Discontinuity:

If f(x) is continuous at $x=a$, then

$\lim_{x\to a} f(x) = f(a)$

$\boxed{\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)} \Rightarrow f(a).$

→ If $\lim_{x\to a} f(x)$ doesn't exists, then f(x) is discontinuous at $x=a$.

The function $f(x) = \tan x$ is discontinuous at ____

a) $\pi/4$ b) π ~~c) $\pi/2$~~ d) π

$\lim \tan x$ doesn't exist for odd multiples of $\pi/2$.

The function $f(x) = [x]$ is discontinuous at

a) 3.4 b) 2.5 c) 1.5 ~~d) 7~~

$\lim_{x\to a} [x]$ D.E (f(a)⇒ I)

The function $= \frac{1}{x}$ is discontinuous at

a) 1 b) -1 c) 2 ~~d) 0~~

$\boxed{\lim_{x\to 0} \frac{1}{x} = \frac{1}{0} = \infty} \Rightarrow$ D.E

If $f(x) = \frac{\sin 3x}{x}$ is continuous at $x=0$.

then $f(0) =$ ____

$\lim_{x\to 0} \frac{\sin 3x}{x} \Rightarrow \frac{3}{1} \Rightarrow \boxed{f(0)=3}$

If $f(x) = (1+3x)^{1/x}$ is continuous at $x=0$

then $f(0) =$ ____

$\lim_{x\to 0} (1+3x)^{1/x} \Rightarrow e^{3\times 1} = \boxed{e^3 \Rightarrow f(0)}$

If $f(x) = \frac{\log x}{x-1}$ is continuous at $x=1$

then $f(1) =$ ____

$\lim_{x\to 1} \frac{\log x}{x-1} \Rightarrow \lim_{x\to 1} \frac{1}{x} = 1.$

$\therefore \boxed{f(1)=1}$

★ Maxima & Minima ★

let $y=f(x)$, then

$f'(x)>0$ $\boxed{\text{slope} > 0} \Rightarrow$ f(x) is increasing

$f'(x)<0$ $\boxed{\text{slope} < 0} \Rightarrow$ f(x) is decreasing.

$f'(x)=0$ $\boxed{\text{stationary}}$ $\boxed{\text{slope} = 0} \Rightarrow$ f(x) is stationary.

Maximum & Minimum Values of f(x)

Step 1: find $f'(x)$.

→ equate to $\boxed{f'(x)=0}$

→ we get stationary points $\boxed{x=a,b}$

Step 2:

→ find $f''(x)$

→ if $f''(a)<0 \rightarrow f(x)$ has max. value at $x=a$.

→ if $f''(a)>0 \rightarrow f(x)$ has min. value at $x=0$.

$\therefore$ max. value of $f(x) = f(a)$

min. value of $f(x) = f(b)$.

→ stationary point & stationary values $\in R$

$f(x) = a\sin x + b\cos x + c$

max. value $\Rightarrow c + \sqrt{a^2+b^2}$

min. value $\Rightarrow c - \sqrt{a^2-b^2}$

The max. value of $a\cos^2 x + b\sin^2 x$, $a>b$; = a

The min. value of $a\cos^2 x + b\sin^2 x$; $a \ngtr b$; = b

$\boxed{f(x) = a\tan x + b\cot x}$

$\therefore$ Min. value $= 2\sqrt{ab}$

$\therefore$ Min value at $x = \tan^{-1}\left(\sqrt{b/a}\right)$

$\boxed{f(x) = a\sec x + b\operatorname{cosec} x}$

$\therefore$ min value $= \left(a^{2/3} + b^{2/3}\right)^{3/2}$

$\therefore$ Min value at $x = \tan^{-1}\left(\sqrt[3]{a/b}\right)$.

$\boxed{f(x) = a^2\sec^2 x + b^2\operatorname{cosec}^2 x}$

min. value $= (a+b)^2$

min value at $x = \tan^{-1}\left(\sqrt{b/a}\right)$

If sum of two numbers $= k$, and sum of their squares is minimum, then numbers are $\{k/2, k/2\}$

$a>0, b>0, x>0$;
least value of $f(x) = ax + \frac{b}{x}$ is $2\sqrt{ab}$.

if $ab = 6$, min. value of $a+b$ is ______

$\frac{a+b}{2} \geq \sqrt{ab}$ $\Rightarrow 2\sqrt{6}$

$\boxed{a+b \geq 2\sqrt{6}}$

if $a+b = 8$, then max. value of ab =

$\frac{a+b}{2} \geq \sqrt{ab}$

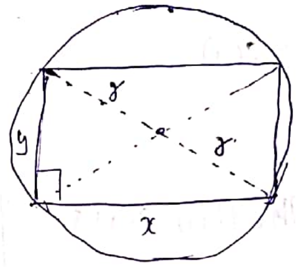
$4 \geq \sqrt{ab}$

$ab \leq 16$

$\therefore$ ab max value of $ab = 16$.

Concept-1:

Max. area of Rectangle in a circle of Radius (r) units.



$x^2 + y^2 = 4r^2$

$y^2 = 4r^2 - x^2$

$\boxed{y = \sqrt{4r^2 - x^2}}$

Area $= y \times x$

$\boxed{A = x\sqrt{4r^2 - x^2}}$

Area should be max.

$$\frac{dA}{dx} = x \cdot \frac{1}{2\sqrt{4r^2 - x^2}} \cdot (-2x) + \sqrt{4r^2 - x^2}\,(1)$$

$$= \frac{-x^2}{\sqrt{4r^2 - x^2}} + \sqrt{4r^2 - x^2}$$

$$\Rightarrow \frac{-x^2 + 4r^2 - x^2}{\sqrt{4r^2 - x^2}} = 0$$

$$\Rightarrow 4r^2 - 2x^2 = 0$$

$$2r^2 - x^2 = 0$$

$$x^2 = 2r^2$$

$$\boxed{x = \sqrt{2}r}$$

$$y = \sqrt{4r^2 - 2r^2}$$

$$y = \sqrt{2r^2}$$

$$\boxed{y = \sqrt{2}r}$$

$\therefore \left.\begin{matrix} x = \sqrt{2}r \\ y = \sqrt{2}r \end{matrix}\right\}$ represents square.

$$\boxed{\begin{aligned} x \times y &= \sqrt{2}r \cdot \sqrt{2}r \\ &= 2r^2 \end{aligned}}$$

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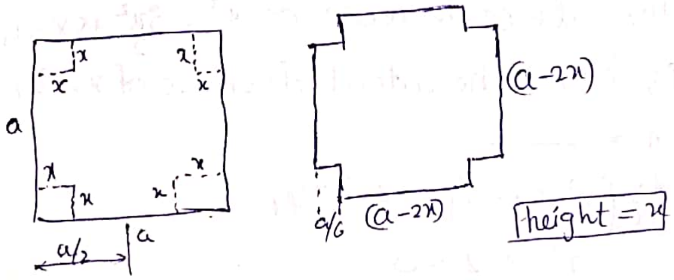
Concept-2:

The max. value of open top box made from a square metal sheet of $a \times a$



$$\text{Volume} = (a-2x)(a-2x)\,x$$

$$\boxed{V = x(a-2x)^2}$$

$$\frac{dv}{dx} = x(2)(a-2x)(-2) + (a-2x)^2$$

$$= -4x(a-2x) + (a-2x)^2$$

$$= (a-2x)\left[a-2x-4x\right] = 0$$

$$a-2x=0 \qquad\qquad a-6x=0$$

$$x = a/2 \qquad\qquad \boxed{x = a/6}$$

Not possible

max. volume: $\frac{a}{6}\left[a-2\left(\frac{a}{6}\right)\right]^2$

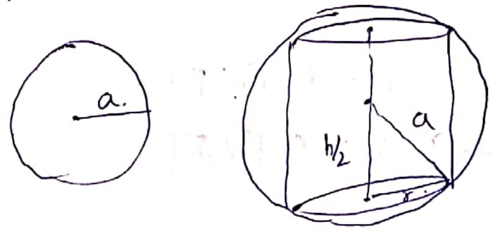
$$= \frac{a}{6}\left[a-\frac{a}{3}\right]^2$$

$$= \frac{a}{6_3} \cdot \frac{4a^2}{9} \Rightarrow \frac{2a^3}{27}$$

$$\boxed{\therefore \text{max. volume} = \frac{2a^3}{27}}$$

Concept-3

Max. volume of cylinder is made from spherical solid of radius (a).



$$\frac{h^2}{4} + r^2 = a^2$$

$$r^2 = a^2 - \frac{h^2}{4}$$

$$\boxed{r = \sqrt{a^2 - \frac{h^2}{4}}}$$

$$V = \pi r^2 h$$

$$= \pi\left[a^2 - \frac{h^2}{4}\right]h$$

$$\frac{dv}{dh} = \left[h\left(-\frac{h}{2}\right) + \left(a^2 - \frac{h^2}{4}\right)(1)\right]$$

$$= -\frac{h^2}{2} + a^2 - \frac{h^2}{4}$$

$$= \frac{-2h^2 + 4a^2 - h^2}{4} = 0$$

$$\Rightarrow -3h^2 + 4a^2 = 0.$$

$$4a^2 = 3h^2$$

$$h^2 = \frac{4a^2}{3}$$

$$\boxed{h = \sqrt{\frac{4a^2}{3}} = \frac{2a}{\sqrt{3}}}$$

$$r = \sqrt{a^2 - \frac{4a^2/3}{4}}$$

$$r = \sqrt{a^2 - a^2/3}$$

$$r = \sqrt{\frac{2a^2}{3}}$$

$$\boxed{r = \sqrt{\frac{2}{3}}\,a}$$

$$\text{Max volume} = \pi\left(\frac{2}{3}\right)a^2 \cdot \frac{2a}{\sqrt{3}}$$

$$\boxed{\text{Max volume} = \frac{4\pi}{3\sqrt{3}}a^3}$$

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Inequalities:

$(x-\alpha) < 0 \Rightarrow x \in (-\infty, \alpha)$

$(x-\alpha) \le 0 \Rightarrow x \in (-\infty, \alpha]$

$(x-\alpha)(x-\beta) < 0 \; ; \; x \in (\alpha, \beta)$

$(x-\alpha)(x-\beta) \le 0 \; ; \; x \in [\alpha, \beta]$

$(x-\alpha)(x-\beta) > 0 \; ; \; x \in (-\infty, \alpha) \cup (\beta, \infty)$

$(x-\alpha)(x-\beta) \ge 0 \; ; \; x \in (-\infty, \alpha] \cup [\beta, \infty)$

strictly increasing $\rightarrow f'(x) > 0$

increasing $\rightarrow f(x) \ge 0$

strictly decreasing $\rightarrow f'(x) < 0$

decreasing $\rightarrow f(x) \le 0$

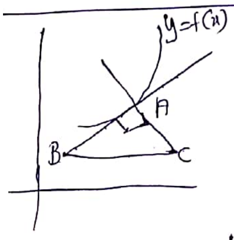
★ Tangents and Normals ★

length of Tangent $= \frac{y_1}{|m|}\sqrt{1+m^2}$

length of Normal $= y_1\sqrt{1+m^2}$

length of Subtangent $= \cancel{y_1\sqrt{1+m^2}} \; \frac{y_1}{|m|}$

length of Subnormal $= y_1 |m|$



$\Delta ABC = \frac{1}{2} \times$ length of Tangent $\times$ Length of Normal

$= \frac{1}{2} \times \frac{y_1}{|m|}\sqrt{1+m^2} \times y_1\sqrt{1+m^2}$

$= \frac{1}{2}\frac{y_1^2}{|m|}(1+m^2)$

Solved Examples:

Standards in Differentiation:

$$\frac{d}{dx}\left(\sin^m nx \cdot \cos^n mx\right)$$

$$\delta = mn \sin^{m-1} nx \cdot \cos^{n-1} mx \cdot \cos(m+n)x$$

$$\frac{d}{dx} = \left(\sin^m nx \cdot \sin^n mx\right)$$

$$\delta = mn \sin^{m-1} nx \cdot \sin^{n-1} mx \cdot \sin(m+n)x$$

$$\frac{d}{dx}\left(\cos^m nx \cdot \sin^m mx\right)$$

$$\delta = mn \cos^{m-1} nx \cdot \sin^{n-1} mx \cdot \cos(m+n)x$$

$$\frac{d}{dx}\left(\cos^m nx \cdot \cos^n mx\right)$$

$$\delta = -nm \cos^{m-1} nx \cdot \cos^{n-1} mx \cdot \sin(m+n)x$$

$$y = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) \Rightarrow \delta = \sqrt{a^2+x^2}$$

$$y = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) \Rightarrow \delta = \sqrt{a^2-x^2}$$

$$y = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) \Rightarrow \delta = \sqrt{x^2-a^2}$$

If $u = f(x,y)$ is homogeneous function of deg. n then: $\boxed{n = \text{degree}}$ ★

i) $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = nu$

ii) $x \cdot \frac{\partial^2 u}{\partial x^2} + y \cdot \frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial x}$

iii) $x \cdot \frac{\partial^2 u}{\partial x \cdot \partial y} + y\frac{\partial^2 u}{\partial y^2} = (n-1) \cdot \frac{\partial u}{\partial y}$

iv) $x^2 \cdot \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \cdot \frac{\partial^2 u}{\partial y^2} = n(n-1)u$

'IMP. Problems':

If $u = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} =$

0

$$\frac{d}{dx}\left[x^n \log x\right]$$

$$\Rightarrow n x^{n-1}\left[1 + n \log x\right]$$

→ The radius of a circular plate is increasing at the rate of 0.01 cm/sec, when the radius is 12cm. The rate at which the area increases is ______

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$= 2\pi(12)(0.01) \Rightarrow \boxed{0.24\pi \ cm^2/s.}$$

$z = \log(\tan x + \tan y)$, then

$\sin 2x \frac{\partial z}{\partial x} + \sin 2y \frac{\partial z}{\partial y} =$ <u>2</u>

$$\frac{\partial z}{\partial x} = \frac{1}{(\tan x + \tan y)} \cdot (\sec^2 x)$$

$$\frac{\partial z}{\partial y} = \frac{1}{(\tan x + \tan y)}(\sec^2 y)$$

$$\therefore \frac{2\sin x \cdot \cos x \cdot \sec^2 x}{\tan x + \tan y} + \frac{2\sin y \cdot \cos y \cdot \sec^2 y}{\tan x + \tan y}$$

$$= 2\left[\frac{\tan x}{\tan x + \tan y} + \frac{\tan y}{\tan x + \tan y}\right]$$

$$= 2\left[\cancel{\frac{\tan x + \tan y}{\tan x + \tan y}}\right]$$

$$= 2$$

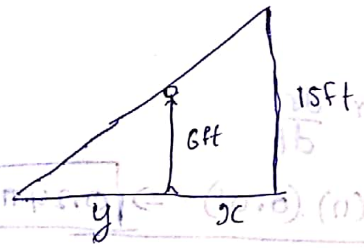
If $u = x(y-z) + y(z-x) + z(x-y)$,

then $\boxed{\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0}$

$$\Rightarrow \lim_{x\to 0}\left[\frac{2^x + 2^{2x} + 2^{3x}}{3}\right]^{1/x}$$

$$\sqrt[3]{2\cdot 2^2\cdot 2^3} \Rightarrow \sqrt[3]{64} = 4$$

⇒ A man of height 6ft is running away from base of a street light that is at 15ft high. if he moves at the rate 18 ft per sec. then the rate at which his shadow changing is ____



$$\frac{15}{6} = \frac{x+y}{y}$$

$$5y = 2x + 2y$$

$$2x = 3y$$

$$2\frac{dx}{dt} = 3\cdot\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{2}{3}\cdot dx/dt \Rightarrow \frac{2}{3}\times 18 = 12 \text{ ft/s}$$

if $u = \cos^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ then $\sum x u_x =$

$\Downarrow$

$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}}$

$= -n\cot u$

$= -\frac{1}{2}\cot u$.

A spherical balloon is expanding, if the radius is increasing at the rate of 2 inches per min., then the rate at which the volume is increasing when the radius is 5 inches.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 4\pi r^2 \cdot dr/dt$$

$$= 4\pi(5)^2\cdot 2$$

$$\boxed{\frac{dv}{dt} = 200\pi}$$

If the side of a cube increases at the rate of 0.02 cm/s, then rate of increases in its volume when side is 10cm.

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2\cdot\frac{dx}{dt}$$

$$= 3\times(10)^2\times 0.02$$

$$= \boxed{6 \text{ cc/sec}}$$

If the rate of change in the area of a circle is equal to rate of change in its radius, then radius = ____

$$\boxed{\frac{dA}{dt} = \frac{dr}{dt}}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r\cdot\frac{dr}{dt}$$

$$1 = 2\pi r$$

$$\boxed{r = \frac{1}{2\pi}}$$

* Standards of Limits * & Formulas :

Formula-1 :

$$\lim_{x\to a} \frac{x^n - a^n}{x-a} = n a^{n-1}.$$

$$\lim_{x\to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$$

$$\lim_{x\to 0} \frac{(1+x)^m - 1}{x} = m$$

$$\lim_{x\to 0} \frac{(1+x)^m - 1}{(1+x)^n - 1} = \frac{m}{n}$$

Formula ; 2

$$\Sigma n = 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$\Sigma n^2 = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\Sigma n^3 = 1^3+2^3+3^3+\dots+n^3 = \frac{n^2(n+1)^2}{4}$$

$$\Sigma n^4 = \left(\frac{n(n+1)(2n+1)}{6}\right)\cdot\left(\frac{3n^2+3n-1}{5}\right)$$

Formula ; 3

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x\to 0} \frac{e^{ax} - 1}{x} = a$$

$$\lim_{x\to 0} \frac{e^{ax} - e^{bx}}{x} = a-b$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \log a$$

$$\lim_{x\to 0} \frac{a^x - b^x}{x} = \log a/b$$

$$\lim_{x\to 0} x(a^{1/x} - 1) = \log a$$

$$\lim_{x\to 0} \left(\frac{a^{1/x} - 1}{b^{1/x} - 1}\right) = \log_b a$$

$$\lim_{x\to 0} x\left(a^{1/x} - b^{1/x}\right) = \log \frac{a}{b}.$$

Formula-4

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta\to 0} \frac{\sin a\theta}{\theta} = a$$

$$\lim_{\theta\to 0} \frac{\tan a\theta}{\theta} = a$$

$$\lim_{x\to 0} \frac{\sin^{-1}x}{x} = 1$$

$$\lim_{x\to 0} \frac{\tan^{-1}x}{x} = 1$$

$$\lim_{x\to 0} \frac{\sinh x}{x} = 1$$

$$\lim_{x\to 0} \frac{\tanh x}{x} = \lim_{x\to 0} \frac{\tanh^{-1}x}{x} = \lim_{x\to 0} \frac{\sinh^{-1}x}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin ax \pm \sin bx}{\sin cx \pm \sin dx} = \frac{a\pm b}{c\pm d}$$

$$\lim_{x\to 0} \frac{\tan ax \pm \tan bx}{\tan cx \pm \tan dx} = \frac{a\pm b}{c\pm d}$$

$$\lim_{x\to 0} \frac{1-\cos mx}{1-\cos nx} = \left(\frac{m}{n}\right)^2$$

Formula ; 5

$$\lim_{x\to 0} \frac{\sin x^\circ}{x} = 1^\circ = \pi/180$$

$$\lim_{x\to 0} \frac{\sin x^\circ}{x^\circ} = 1$$

$$\lim_{x\to 0} \frac{\tan x^\circ}{x} = \pi/180$$

~~*INTEGRATION*~~

Formula : 6

$$\lim_{x\to 0}(1+x)^{1/x} = e$$

$$\lim_{x\to 0}(1+ax)^{1/x} = e^{a}$$

$$\lim_{x\to 0}(1+x)^{a/x} = e^{a}$$

$$\lim_{x\to \infty}\left(1+\frac{1}{x}\right)^{x} = e$$

$$\lim_{x\to \infty}\left(1+\frac{a}{x}\right)^{x} = e^{a}$$

$$\lim_{x\to \infty}\left(1+\frac{1}{kx}\right)^{x} = e^{1/k}$$

$$\lim_{x\to \infty}\left(1+\frac{1}{ax+b}\right)^{cx+d} = e^{c/a}$$

$$\lim_{x\to 0}(\cos x+a\sin bx)^{1/x} = e^{ab}$$

$$\lim_{x\to 0}(\sec x+a\tan bx)^{1/x} = e^{ab}$$

$$\lim_{x\to \infty}\left(\frac{x+a}{x+b}\right)^{x} = e^{a-b}$$

$$\lim_{x\to \infty}\left(\frac{x+a}{x-b}\right)^{x} = e^{a+b}$$

$$\lim_{x\to \infty}\left[\frac{x^2+bx+c}{x^2+kx+d}\right]^{x} = e^{b-k}.$$

$$\lim_{x\to \infty}\left[\frac{ax^2+bx+c}{ax^2+dx+e}\right]^{px} = e^{\frac{p(b-d)}{a}}$$

$$\lim_{x\to \infty}\left[1+\frac{1}{a+bx}\right]^{cx+d} = e^{c/b}$$

$$\lim_{x\to \infty}\left[\frac{a_1x^2+b_1x+c_1}{a_2x^2+b_2x+c_2}\right]^{\frac{d_1x+c_1}{d_2x+c_2}} = \left(\frac{a_1}{a_2}\right)^{d_1/d_2}$$

Formula : 7

$$\lim_{x\to \infty}\left[\sqrt{x^2+ax+b}-x\right] = \frac{a}{2}$$

$$\lim_{x\to 0}\frac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x} = \frac{1}{n}a^{\frac{1}{n-1}}$$

$$\lim_{x\to 0}\frac{\sqrt[n]{a+bx}-\sqrt[n]{a}}{x} = \frac{b}{n}a^{\frac{1}{n-1}}$$

$$\lim_{x\to 0}\frac{\sqrt[n]{a+x+x^2+\dots x^m}-\sqrt[n]{a}}{x} = \frac{1}{n}a^{\frac{1}{n-1}}$$

Formula : 8

$$\lim_{x\to 0}\frac{\sqrt[n]{1+\sin x}-1}{x} = \frac{1}{n}.$$

$$\lim_{x\to 0}\frac{\sqrt[n]{1+\sin x}-\sqrt[n]{1-\sin x}}{x} = \frac{2}{n}$$

$$\lim_{x\to 0}\frac{\sqrt[n]{a\sin x+1}-1}{x} = \frac{a}{n}$$

$$\lim_{x\to a}\frac{|x-a|}{x-a} = \text{doesn't exist}$$

$$\lim_{x\to a^+}\frac{|x-a|}{x-a} = 1$$

$$\lim_{x\to a^-}\frac{|x-a|}{x-a} = -1$$

$\lim_{x\to a}[x] \Rightarrow$ D.E if a = any integer

$\lim_{x\to}\{x\} \Rightarrow x-[x]$ = D.E, if a = any integer

$$\lim_{x\to\infty} \frac{[ax+b]}{x} = a \text{ where } \boxed{a>0}$$

$$\lim_{x\to\infty} \frac{[x]}{x} = 1$$

$$\lim_{n\to\infty} \frac{[1x]+[2x]+\dots+[nx]}{n^2} = \frac{x}{2}$$

$$\lim_{n\to\infty} \frac{[1^2x]+[2^2x]+[3^2x]\dots+[n^2x]}{n^3} = \frac{x}{3}$$

$$\lim_{x\to 0} \left(\frac{a^x+b^x}{2}\right)^{1/x} = \sqrt{ab}$$

$$\lim_{x\to 0} \left(\frac{a^{1/x}+b^{1/x}}{2}\right)^{x} = \sqrt{ab}$$

$$\lim_{x\to 0} \left(\frac{a^x+b^x}{2}\right)^{n/x} = (\sqrt{ab})^n$$

$$\lim_{x\to 0} \left(\frac{a^x+b^x+c^x}{3}\right)^{1/x} = \sqrt[3]{abc}$$

$$\lim_{x\to 0} \left(\frac{a^{1/x}+b^{1/x}+c^{1/x}}{3}\right)^{x} = \sqrt[3]{abc}$$

$$\lim_{x\to 0} \left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{n}{x}} = [\sqrt[3]{abc}]^n$$

$$\lim_{x\to 0} \left[\frac{a_1^x+a_2^x+\dots a_n^x}{n}\right]^{1/x} = \sqrt[n]{a_1a_2\dots a_n}$$

$$\lim_{x\to 0} \left(\frac{a_1^{1/x}+a_2^{1/x}+\dots a_n^{1/x}}{n}\right)^{x} = \sqrt[n]{a_1a_2a_3\dots a_n}$$

$$\lim_{x\to 0} \left(\frac{a_1^x+a_2^x\dots+a_n^x}{n}\right)^{k/x} = \left(\sqrt[n]{a_1a_2\dots a_n}\right)^k$$

$$\lim_{x\to a} \frac{f(x)}{x} = \frac{0}{0}$$

where $f(x)$ & exponential functions.

Sol: $\cdot \log\left[\frac{\text{product of +ve term bases}}{\text{product of -ve term bases}}\right]$

$$\lim_{x\to 0} \frac{a^x-1}{x} \Rightarrow \log\left(\frac{a}{1}\right) = \log a$$

$$\lim_{x\to 0} \frac{3^x+2^x-5^x-1}{x} \Rightarrow \log\left(\frac{3\times 2}{5}\right)$$

$$\Rightarrow \log\frac{6}{5}$$

$$\lim_{x\to 0} \frac{(ab)^x-a^x-b^x+1}{x^2} = \log a \cdot \log b.$$

$$\lim_{x\to 0} \frac{6^x-2^x-3^x+1}{x^2} = \log 2 \cdot \log 3.$$

Errors and Approximation ☆

$$\%\text{error} = \boxed{\frac{\Delta x}{x}\times 100}$$

If an error of 0.02cm is made while measuring the radius 1cm of circle, then % error made while calculating its area appr. = ______

$$\%\text{error} = \frac{\Delta A}{A}\times 100.$$

$$A = \pi(1)^2 = \pi$$

$$\Delta A = 2\pi r.\Delta r \Rightarrow 2\pi(1)\,0.02$$

$$\%\text{ error} = \frac{2\pi\times 0.02}{\pi}\times 100$$

$$= 4\%$$

If there is an error of 0.05 sq cm in the surface area of sphere, then error in volume when radi = 40cm

$$\boxed{\Delta V = 4\pi r^2\Delta r.}$$

$$\Delta V = \frac{4\pi\times(40)^2\times 0.05}{8\times\pi\times 40}$$

$$\Delta V = \frac{40\times 0.05}{2}$$

$$= 20\times\frac{5}{100} = \boxed{1\,c.c.}$$

$$\Delta S = 8\pi r.\Delta r.$$

$$\frac{\Delta S}{8\pi r} = \Delta r$$

$$\frac{0.05}{8\times\pi\times 40} = \Delta r.$$

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★ INTEGRATION ★

$$\cancel{\frac{1}{1+\frac{x^2}{y^2}} \cdot \frac{1}{y} \cdot \frac{y}{x^2}} \qquad 1+\frac{y^2}{x^2}$$

$$\cancel{\frac{y^2}{y^2+x^2}} \cdot \frac{x}{y}$$

Formula-1 :

$\int \sin x = -\cos x + C$

$\int \cos x = \sin x + C$

$\int \sec^2 x = \tan x + C$

$\int \operatorname{cosec}^2 x = -\cot x + C$

$\int \sec x . \tan x = \sec x + C$

$\int \operatorname{cosec} x . \cot x = -\operatorname{cosec} x + C$

Formula : 2 :

$\int k . dx = kx + C$

$\int x^n . dx = \frac{x^{n+1}}{n+1}$

$\int \frac{1}{x} . dx = \log|x| + C$

$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$

$\int e^x = e^x$

$\int a^x = \frac{a^x}{\log a}$

$\int \log x = x[\log x - 1]$

$\int \sqrt{x} = \frac{2}{3} x\sqrt{x} = \frac{2}{3} x^{3/2}$

$\int f(x).g(x).dx$

$= f(x).\int g(x).dx - \int f'(x).\left(\int g(x).dx\right).dx$

$\int x \sin x .$

$x . \int \sin x . dx - \int 1 \left(\int \sin x . dx\right) . dx$

$(x)(-\cos x) + \int \cos x . dx$

$-x\cos x + \sin x + C$

$\boxed{\sin x - x\cos x}$

Formula : 3 :

$\int \tan x = \log|\sec x| + C = -\log|\cos x| + C$

$\int \cot x = \log|\sin x| + C = -\log|\operatorname{cosec} x| + C$

$\int \sec x . dx = \log|\sec x + \tan x| + C$

$= \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|$

$\int \operatorname{cosec} x . dx = \log|\operatorname{cosec} x - \cot x| + C$

$= \log\left|\tan\frac{x}{2}\right| + C$

Formula-4 :

$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1}x = -\cos^{-1}x$

$\int \frac{1}{1+x^2} = \tan^{-1}x = -\cot^{-1}x$

$\int \frac{1}{|x|\sqrt{x^2-1}} = \sec^{-1}x = -\operatorname{cosec}^{-1}x$

Formula : 5 :

$\int \sinh x \rightarrow \cosh x + C$

$\int \cosh x \rightarrow \sinh x + C$

$\int \tanh x \rightarrow \log|\cosh x| + C$

$\int \coth x \rightarrow \log|\sinh x| + C$

$\int \operatorname{sech} x \rightarrow 2\tan^{-1}(e^x) + C$

$\int \operatorname{cosech} x \rightarrow \log\left|\tanh\frac{x}{2}\right| + C$

$$\int \operatorname{sech}^2 x = \tanh x + c$$

$$\int \operatorname{cosech}^2 x = -\coth x + c$$

$$\int \operatorname{sech} x \cdot \tanh x = -\operatorname{sech} x + c$$

$$\int \operatorname{cosech} x \cdot \coth x = -\operatorname{cosech} x + c$$

$$\int \frac{1}{\sqrt{1+x^2}} = \cancel{\operatorname{sech}^{-1} x}\ \sinh^{-1} x + c$$

$$\int \frac{1}{\sqrt{x^2-1}} = \cosh^{-1} x + c$$

Formula : 6 :

- $$\int \frac{1}{(x+a)(x+b)} = \frac{1}{b-a} \log\left|\frac{x+a}{x+b}\right| + c$$

- $$\int \frac{1}{(x-a)(x^2+b)} = \frac{1}{a^2+b}\left[\frac{1}{x-a} - \frac{x+a}{x^2+b}\right]$$

- $$\int \frac{1}{(ax+b)(cx+d)} = \frac{1}{ad-bc} \log\left|\frac{ax+b}{cx+d}\right|$$

- $$\int \frac{1}{(x^2+a^2)(x^2+b^2)} = \frac{1}{b^2-a^2}\left[\frac{1}{a}\tan^{-1}\frac{x}{a} - \frac{1}{b}\tan^{-1}\frac{x}{b}\right]$$

- $$\int \frac{x}{(x^2+a^2)(x^2+b^2)} = \frac{1}{2(b^2-a^2)} \log\left|\frac{x^2+b^2}{x^2+a^2}\right|$$

- $$\int \frac{1}{x(x^n+1)} = \frac{1}{n} \log\left|\frac{x^n}{1+x^n}\right| + c$$

- $$\int \frac{1}{x(1-x^n)} = \frac{1}{n} \log\left|\frac{x^n}{1-x^n}\right| + c$$

Formula : 7 :

$$\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$$

$$\int \frac{1}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}} = \sin^{-1}(x/a)$$

$$\int \frac{1}{\sqrt{a^2+x^2}} = \sinh^{-1}(x/a) = \log\left|x+\sqrt{a^2+x^2}\right|$$

$$\int \frac{1}{\sqrt{x^2-a^2}} = \cosh^{-1}(x/a) = \log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}(x/a) + c$$

$$\int \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}(x/a) + c$$

$$\int \sqrt{a^2+x^2} = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}(x/a) + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-(f(x))^2}} = \sin^{-1}\left[\frac{f(x)}{a}\right]$$

$$\int \frac{f'(x)}{\sqrt{a^2+[f(x)]^2}} = \sinh^{-1}\left[\frac{f(x)}{a}\right]$$

$$\int \frac{f'(x)}{\sqrt{[f(x)]^2-a^2}} = \cosh^{-1}\left[\frac{f(x)}{a}\right]$$

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Formula : 8 :-

$$\int \frac{1}{a^2\sin^2x + b^2\cos^2x} = \frac{1}{ab}\tan^{-1}\left(\frac{a\sin x}{b\cos x}\right) + c$$

$$\int \frac{1}{a\sin x + b\cos x}\cdot dx = \frac{1}{\sqrt{a^2+b^2}}\log\left|\tan\left(\frac{x}{2}+\frac{1}{2}\tan^{-1}\left(\frac{b}{a}\right)\right)\right| + c$$

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx =$$

$$\frac{ac+bd}{c^2+d^2}x + \frac{ad-bc}{c^2+d^2}\log|c\cos x + d\sin x| + c$$

$$\int x^n\cdot\log x\,dx = \frac{x^{n+1}}{n+1}\left[\log x - \frac{1}{n+1}\right] + c$$

$\boxed{n \neq -1}$

- $\int e^x[f'(x) + f(x)]\,dx = e^x[f(x)] + c$
- $\int e^{ax}\left[f(x) + \frac{f'(x)}{a}\right]\cdot dx = \frac{e^{ax}f(x)}{a} + c$
- $\int (xf'(x) + f(x))\cdot dx = x f(x) + c$
- $\int e^{ax}\sin(bx+c) =$

$$\Rightarrow \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c) - b\cos(bx+c)\right]$$

- $\int e^{ax}\cos(bx+c)$

$$\frac{e^{ax}}{a^2+b^2}\left[a\cos(bx+c) + b\sin(bx+c)\right]$$

Formula - 9 :

$$\int \sin^{-1}x = x\sin^{-1}x + \sqrt{1-x^2}$$

$$\int \cos^{-1}x = x\cos^{-1}x - \sqrt{1-x^2}$$

$$\int \tan^{-1}x = x\tan^{-1}x - \frac{1}{2}\log(1+x^2)$$

$$\int \cot^{-1}x = x\cot^{-1}x + \frac{1}{2}\log(1+x^2)$$

$$\int \sec^{-1}x = x\sec^{-1}x - \cosh^{-1}x + c$$

$$\int \operatorname{cosec}^{-1}x = x\operatorname{cosec}^{-1}x + \cosh^{-1}x$$

$$\int x^{-n}\log x\,dx = \frac{x^{1-n}}{1-n}\left[\log x - \frac{1}{1+n}\right] + c$$

$\int x^n e^{ax}$ then $I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a}I_{n-1}$ $\boxed{n \in +ve}$

$\int \sin^n x$; then $\Rightarrow I_n = \frac{-\sin^{n-1}x\cdot\cos x}{n} + \left(\frac{n-1}{n}\right)I_{n-2}$ $\boxed{n \geq 2}$

$\int \cos^n x$; then $\Rightarrow I_n = \frac{\cos^{n-1}x\cdot\sin x}{n} + \left(\frac{n-1}{n}\right)I_{n-2}$ $\boxed{n \geq 2}$

$\int \tan^n x$; then $I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$; $\boxed{n \geq 2}$

$$\boxed{\int x\cdot e^{ax} = \frac{e^{ax}[ax-1]}{a^2}}$$

PART-1 Integration of special standards

Case i) $\int \frac{dx}{ax^2+bx+c}$

Tip: $\boxed{\text{Take } \boxed{x^2} \text{ coefficient common}}$

Eg: $\int \frac{1}{2x^2+x-1} \cdot dx$

$\Rightarrow \frac{1}{2(x^2+\frac{x}{2}-\frac{1}{2})} \cdot dx$

$\Rightarrow \frac{1}{2(x^2+2\cdot(\frac{1}{2})\cdot\frac{x}{2}-\frac{1}{2})}$

$\Rightarrow \frac{1}{2(x^2+2\cdot\frac{x}{4}+\frac{1}{16}-\frac{1}{16}-\frac{1}{2})}$

$\Rightarrow \frac{1}{2[(x+\frac{1}{4})^2-\frac{9}{16}]}$

$\Rightarrow \frac{1}{2}\int \frac{1}{(x+\frac{1}{4})^2-9/16} \cdot dx.$

$\Rightarrow \frac{1}{2}\int \frac{1}{(x+\frac{1}{4})^2-(\frac{3}{4})^2} \cdot dx.$

$= \frac{1}{2}\left(\frac{1}{2\times\frac{3}{4}}\log\left|\frac{x+\frac{1}{4}-\frac{3}{4}}{x+\frac{1}{4}+\frac{3}{4}}\right|\right.$

$= \frac{1}{3}\log\left|\frac{\frac{2x-1}{2}}{x+1}\right| + c$

$\int \frac{1}{\sqrt{5-4x-x^2}} \cdot dx$

$\frac{1}{\sqrt{-(x^2+4x-5)}} \cdot dx$

$\frac{1}{\sqrt{-(x^2+4x+4-4-5)}} \cdot dx$

$= \frac{1}{\sqrt{-[(x+2)^2-9]}} \cdot dx$

$= \int \frac{1}{\sqrt{9-(x+2)^2}} \cdot dx$

$= \sin^{-1}\left[\frac{x+2}{3}\right] + C$ //

$\int \sqrt{x^2-4x+9} \cdot dx$

$\int \sqrt{x^2-4x+4-4+9} \cdot dx$

$\int \sqrt{(x+2)^2+5} \cdot dx$

$\boxed{\int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}(\frac{x}{a})}$

$\Rightarrow \frac{x+2}{2}\sqrt{(x+2)^2+5} + \frac{5}{2}\sinh^{-1}\left[\frac{x+2}{\sqrt{5}}\right] + C$

$$\Rightarrow \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} \cdot dx$$

$$\int \frac{\cos x}{\sqrt{(\sin x + 1)^2 - 4}} \cdot dx$$

put $\boxed{\sin x = t}$
$\boxed{\cos x \cdot dx = dt}$

$$= \int \frac{1}{\sqrt{(t-1)^2 - 4}} \cdot dt$$

$$= \cosh^{-1}\left[\frac{\sin x - 1}{2}\right] \,//$$

Case ii) :

$$\int \frac{f'(x)}{f(x)} = \log f(x) + C$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)} + C$$

$$\int f'(x) \cdot f(x)^n = \frac{f(x)^{n+1}}{n+1}$$

$$\int f'(x)\sqrt{f(x)} = \frac{f(x)^{3/2}}{3/2} = \frac{2}{3} f(x)^{3/2}$$

Eg: $\int \frac{px+q}{ax^2+bx+c} \cdot dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}}$

$\int p(x) + q \; (\sqrt{ax^2+bx+c}) \cdot dx.$

Write Numerator term $= \lambda \frac{dy}{dx}(D_r) + \mu$.

Find λ and μ and solve eqn.

Eg: $\int \frac{4x-1}{x^2 - 4x + 5}$

$$4x - 1 = \lambda \frac{d}{dx}(x^2 - 4x + 5) + \mu$$

$$4x - 1 = \lambda(2x - 4) + \mu$$

comparing coefficients :

$2\lambda = 4$
$\boxed{\lambda = 2}$

$-1 = -4\lambda + \mu$
$\boxed{\mu = 7}$

$$\therefore \int \frac{2(2x-4) + 7}{x^2 - 4x + 5}$$

$$2\int \frac{2x-4}{x^2-4x+5} + 7\int \frac{1}{x^2-4x+5} \cdot dx$$

$$2\log(x^2-4x+5) + 7\int \frac{1}{(x-2)^2+1} \cdot dx$$

$$\boxed{2\log(x^2-4x+5) + 7\tan^{-1}(x-2) + C}$$

Eg:

$\int x\sqrt{1+x-x^2}\cdot dx$

$\boxed{x = \lambda\,(d/dx(D_r)) + \mu}$ ⟸ common in all 3 forms !!

$x = \lambda(1-2x) + \mu$

$1 = -2\lambda$ | $0 = \lambda + \mu$

$\boxed{\lambda = -1/2}$ | $0 = -1/2 + \mu$

$\mu = \frac{1}{2}$.

$\therefore \int \left(-\frac{1}{2}(1-2x) + \frac{1}{2}\right)\sqrt{1+x-x^2}\cdot dx$

$\int -\frac{1}{2}(1-2x)(\sqrt{1+x-x^2})\,dx + \frac{1}{2}\int\sqrt{1+x-x^2}\cdot dx$

$-\frac{1}{2}\left[\frac{(1+x-x^2)^{3/2}}{\frac{3}{2}}\right] + \frac{1}{2}\int\sqrt{-(x^2-x-1)}\cdot dx$

$-\frac{1}{3}(1+x-x^2)^{3/2} + \frac{1}{2}\int\sqrt{-\left(x-\frac{1}{2}\right)^2 - (5/4)}\cdot dx$

$-\frac{1}{3}(1+x-x^2)^{3/2} + \frac{1}{2}\int\sqrt{5/4 - \left(x-\frac{1}{2}\right)^2}\,dx$

$\boxed{-\frac{1}{3}(1+x-x^2)^{3/2} + \frac{1}{2}\left[\frac{x-1/2}{2}\sqrt{\frac{5}{4}-(x-1/2)^2} + \frac{5}{8}\sin^{-1}\frac{(x-\frac{1}{2})}{\sqrt{5}/2}\right] + c}$

Case iii)

$\int\sqrt{\frac{px+q}{rx-s}}\cdot dx = \int\left(\sqrt{\frac{px+q}{rx-s}} \times \frac{px+q}{px+q}\right)\cdot dx$

$= \int\frac{px+q}{\sqrt{ax^2+bx+c}}\cdot dx$

Case iv)

$\int\frac{\phi(x)}{(px+q)(\sqrt{ax+b})}\cdot dx$

$\int\frac{\phi(x)}{(px^2+qx+r)(\sqrt{ax+b})}\cdot dx$

$\int\phi(x)\cdot\sqrt{ax+b}\cdot dx$

put $\sqrt{ax+b} = t$. (or) $\boxed{ax+b = t^2}$

where $\phi(x)$ is a polynomial in (x)

Eg: $\int\left(\frac{x+2}{(x^2+3x+3)\sqrt{x+1}}\right)\cdot dx$

$\boxed{x+1 = t^2} \Rightarrow x = t^2-1$

$\boxed{dx = 2t\,dt}$.

$\Rightarrow \int\frac{(t^2-1)+2}{((t^2-1)^2 + 3(t^2-1)+3)\cancel{t}}\cdot 2\cancel{t}\,dt$

$2\int\frac{t^2+1}{t^4 - 2t^2 + 1 + 3t^2 - \cancel{3} + \cancel{3}}\,dt$

$2\int\frac{\cancel{t^2+1}}{\cancel{u^2-2t}+1}\cdot dt$

$2\int\frac{t^2+1}{t^4-2t^2+1+3t^2}\cdot dt$

$2\int\frac{t^2+1}{t^4+t^2+1}\cdot dt$

$2\int\frac{1+1/t^2}{t^2+1+1/t^2}\cdot dt$

$2\int\frac{1+1/t^2}{(t-1/t)^2+(\sqrt{3})^2}\cdot dt$ $\qquad t - 1/t = u$; $\left(1+\frac{1}{t^2}\right)\cdot dt = du$

$2\int\frac{du}{u^2+(\sqrt{3})^2}\cdot dt$

$\frac{2}{\sqrt{3}}\tan^{-1}\frac{u}{\sqrt{3}} \Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{3}t}\right) \Rightarrow$

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Eg: $\int \frac{1}{(x+1)\sqrt{x-2}}\,dx$

$x-2 = t^2$

$dx = 2t\,dt$

$\Rightarrow \int \frac{1}{(x+1)\,\cancel{t}}\,2\cancel{t}\,dt$

$2\int \frac{1}{x-2+3}\,dt$

$2\int \frac{1}{t^2+(\sqrt{3})^2}\,dt$

$2\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{\sqrt{x-2}}{\sqrt{3}}\right]$

$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\sqrt{\frac{x-2}{3}} + C$ //

Case: V $\int \frac{\phi(x)}{(px+q)\left(\sqrt{ax^2+bx+c}\right)}\,dx$

put $\boxed{px+q = 1/t}$

Eg: $\int \frac{1}{(x-1)\sqrt{x^2+x+1}}\,dx$

put $\boxed{x-1 = \frac{1}{t}}$

$dx = \frac{-1}{t^2}\,dt$

$\int \frac{\frac{-1}{t^{\cancel{2}}}\,dt}{\frac{1}{\cancel{t}}\left(\sqrt{\left(\frac{1}{t}+1\right)^2+\left(\frac{1}{t}+1\right)+1}\right)}$

$\int \frac{-1/t\;dt}{\sqrt{\frac{1}{t^2}+\frac{2}{t}+1+\frac{1}{t}+1+1}}$

$\int \frac{-1/t}{\sqrt{\frac{1}{t^2}+\frac{3}{t}+3}}$

$\int \frac{-1/\cancel{t}\;dt}{\frac{1}{\cancel{t}}\sqrt{3t^2+3t+1}}$

$-\int \frac{1}{\sqrt{3t^2+3t+1}}\,dt$

$-\int \frac{1}{\sqrt{3\left(t^2+\cancel{t}+\frac{1}{3}\right)}}\,dt$

~~$t^2+\frac{t}{2}\cdot 2$~~

$t^2+2\cdot\frac{1}{2}t+\frac{1}{4}$

$-\int \frac{1}{\sqrt{3\left(\left(t+\frac{1}{2}\right)^2+\frac{1}{12}\right)}}\,dt$

$-\frac{1}{4}+\frac{1}{3}$

$\frac{-1}{\sqrt{3}}\int \frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{1}{12}}}\,dt$

$\frac{-1}{\sqrt{3}}\sinh^{-1}\left[\frac{\frac{1}{x-1}+\frac{1}{2}}{\frac{1}{\sqrt{12}}}\right]$

$\boxed{= \frac{-1}{\sqrt{3}}\sinh^{-1}\sqrt{12}\left(x+1\right)}$ //

Case 6:

$$\int \frac{ax+b}{\sqrt{cx+d}} \; ; \; \int \frac{\sqrt{cx+d}}{ax+b} ; \int (ax+b)\sqrt{cx+d} \; ;$$

$$\int \frac{1}{(ax+b)\sqrt{cx+d}}$$

put $\boxed{cx+d = t^2}$

Case 7:

$$\int \frac{1}{(ax^2+bx+c)\sqrt{px^2+qx+c}}$$

Case 8:

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}} ; \int \sqrt{(x-a)(x-b)} \cdot dx ;$$

$$\int \sqrt{\frac{x-a}{b-x}} \cdot dx$$

put $\boxed{x = a\cos^2\theta + b\sin^2\theta}$

$\boxed{dx = d\theta . \sin\theta . \cos\theta . 2(b-a)}$

Format-1:

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}}$$

$$\int \frac{d\theta . \sin\theta . \cos\theta . 2(b-a)}{\sin\theta . \cos\theta . \sqrt{(b-a)(a-b)}}$$

$$2\sqrt{\frac{b-a}{a-b}} \int d\theta$$

$$\left[\theta = \sin^{-1}\sqrt{\frac{x-a}{b-a}}\right]$$

$$2\sqrt{\frac{b-a}{a-b}} \cdot \theta$$

$$= \boxed{2\sqrt{\frac{b-a}{a-b}} \cdot \sin^{-1}\sqrt{\frac{x-a}{b-a}}}$$

Format: 2:

$$\int \sqrt{(x-a)(x-b)} \cdot dx$$

$$\int (a\cos^2\theta + b\sin^2\theta - a)(a\cos^2\theta + b\sin^2\theta - b) \cdot dx$$

$$\int \sin\theta . \cos\theta . \sqrt{(b-a)(a-b)} \cdot d\theta . \sin\theta . \cos\theta . 2(b-a)$$

$$\sqrt{(b-a)(a-b)} . 2(b-a) \int \sin^2\theta \cos^2\theta . d\theta$$

$$\sqrt{(b-a)(a-b)} \cdot 2(b-a) \cdot \frac{1}{32}(4\theta - \sin 4\theta)$$

$$\sqrt{(b-a)(a-b)} \cdot 2(b-a) \cdot \frac{1}{32}\left(4 . \sin^{-1}\sqrt{\frac{x-a}{b-a}} - \sin 4\left(\sin^{-1}\sqrt{\frac{x-a}{b-a}}\right)\right)$$

Format: 3:

$$\int \sqrt{\frac{x-a}{b-x}} \cdot dx.$$

$$\int \sqrt{\frac{(b-a)\sin^2\theta}{(b-a)\cos^2\theta}} \cdot d\theta . \sin\theta\cos\theta . 2(b-a)$$

$$(b-a)\int \tan\theta \cdot \sin 2\theta \, d\theta$$

$$(b-a)\int \tan\theta \; \frac{2\tan\theta}{1+\tan^2\theta} \cdot d\theta$$

$$(b-a)\int \frac{2\tan^2\theta}{\sec^2\theta} \cdot d\theta$$

$$2(b-a)\int \sin^2\theta \cdot d\theta$$

$$2(b-a)\int \frac{1-\cos 2\theta}{2} \cdot d\theta$$

$$(b-a)\int 1-\cos 2\theta \, d\theta$$

$$(b-a)\left[\theta - \frac{\sin 2\theta}{2}\right].$$

$$\frac{b-a}{2}\left[2\theta - \sin 2\theta\right]$$

$$\frac{b-a}{2}\left[2\sin^{-1}\sqrt{\frac{x-a}{b-a}} - \sin 2\sin^{-1}\sqrt{\frac{x-a}{b-a}}\right]$$

Case: 9:

$$\int \frac{1}{a+b\cos x}, \quad \int \frac{1}{a+b\sin x}$$

$$\Rightarrow \int \frac{1}{a+b.\cancel{\tan x}\left[\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right]}$$

$$= \int \frac{1}{a+b\left(\frac{1-t^2}{1+t^2}\right)} \qquad \tan\frac{x}{2} = t$$

$$\frac{1}{2}\sec^2\frac{x}{2}.dx = dt$$

$$= \int \frac{1+t^2}{a(1+t^2)+b(1-t^2)} \cdot \frac{2}{1+t^2}$$

Case: 10:

$$\int \frac{1}{a+b\cos^2 x}, \quad \int \frac{1}{a+b\sin^2 x},$$

$$\int \frac{1}{a+b\cos^2 x+c\sin^2 x}, \quad \int \frac{1}{(a\sin x+b\cos x)^2}$$

$\Rightarrow$ Multiply with $\sec^2 x$ (Num & deno.)

Special formations

$$\boxed{\int x^m (a+bx^n)^p.dx}$$

Standards.

$$\int a^x.e^x.dx = \frac{a^x e^x}{\log_e(ae)}$$

$$= \boxed{\frac{(ae)^x}{1+\log a}}$$

$$\int \frac{\cos 2x}{\cos^2 x.\sin^2 x}.dx$$

$$= \frac{2\cos^2 x-1}{\cos^2 x.\sin^2 x}$$

$$= \frac{2\cos^2 x}{\cos^2 x.\sin^2 x} - \frac{\sin^2 x+\cos^2 x}{\cos^2 x.\sin^2 x}$$

$$\int (2\mathrm{cosec}^2 x - \sec^2 x - \mathrm{cosec}^2 x)$$

$$\int (\mathrm{cosec}^2 x - \sec^2 x).dx$$

$$\int \boxed{-\cot x - \tan x + c}$$

$$\int \frac{1}{1+\sin x}$$

$$\int \frac{1}{1+\sin x} \cdot \frac{(1-\sin x)}{(1-\sin x)}.dx$$

$$\int \frac{1-\sin x}{1-\sin^2 x}$$

$$\int \frac{1-\sin x}{\cos^2 x}$$

$$\int \frac{1}{\cos x}\left(\frac{1-\sin x}{\cos x}\right)$$

$$\int \sec x(\sec x - \tan x)$$

$$\int \sec^2 x - \sec x.\tan x$$

$$\int \boxed{\tan x - \sec x + c}$$

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$$\int \left(\frac{1-\sin x}{1+\sin x}\right).dx$$

$$\int \left(\frac{1-\sin x}{1+\sin x}\right)\frac{(1+\sin x)}{(1-\sin x)}.dx$$

$$\int \frac{(1-\sin x)^2}{1-\sin^2 x}$$

$$\int \frac{(1-\sin x)^2}{\cos^2 x}$$

$$\int \left(\frac{1-\sin x}{\cos x}\right)^2 . dx$$

$$\int (\sec x - \tan x)^2 . dx$$

$$\int \sec^2 x + \tan^2 x - 2\sec x \tan x. + c$$

$$\tan x + \tan x - x - 2\sec x + c$$

$$\boxed{2\tan x - 2\sec x - x + c}$$

$$\int \sec^2 x . \operatorname{cosec}^2 x \, dx$$

$$\int \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x}.dx$$

$$\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} dx$$

$$\int (\sec^2 x + \operatorname{cosec}^2 x) \, dx$$

$$\boxed{\tan x - \cot x + c}$$

$$\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$$

$$\Rightarrow \frac{2}{3(a-b)}\left[(x+a)\sqrt{(x+a)} + (x+b)\sqrt{(x+b)}\right]$$

$$\int \frac{x^3 \tan^{-1} x^4}{1+x^8}$$

put $x^4 = t$

$$\boxed{dx . 4x^3 = dt}$$

$$\frac{1}{4}\int \frac{\cancel{4x^3} \tan^{-1}(t)}{1+t^2} . \frac{dt}{\cancel{4x^3}}$$

$$\frac{1}{4} \frac{(\tan^{-1}(t))^2}{2}$$

$$= \frac{[\tan^{-1}(x^4)]^2}{8}$$

$$\int \frac{dx}{\sin(x-a) \cdot \sin(x-b)}$$

$$\frac{1}{\sin(a-b)} \log\left[\frac{\sin(x-a)}{\sin(x-b)}\right]$$

$$\int \frac{\sin x . \cos x}{a^2\cos^2 x + b^2 \sin^2 x} dx$$

$$= \boxed{\frac{1}{2(a^2-b^2)} \log(a^2\cos^2 x + b^2 \sin^2 x)}$$

$$\int \frac{(a-b)\sin x + (a+b)\cos x}{a\sin x + b\cos x} dx$$

$$x + \log(a\sin x + b\cos x)..$$

$$\int \frac{1}{1+\cot x} . dx$$

$$\int \frac{\sin x}{\cos x + \sin x} dx \qquad \left(\frac{a\cos x + b\sin x}{c\cos x + d\sin x}\right)$$

$$\int \frac{0x\cos x + \sin x}{\cos x + \sin x} . dx$$

$$\boxed{\frac{1}{2}x - \frac{1}{2}\log(\cos x + \sin x)} + c$$

$$\int \frac{1}{\cos(x-a)\cdot\cos(x-b)}\cdot dx$$

$$\boxed{\frac{1}{\sin(a-b)}\cdot\log\left[\frac{\cos(x-a)}{\cos(x-b)}\right]}$$

$$\int \frac{f'(x)}{f(x)\log[f(x)]}\,dx = \log[\log[f(x)]]$$

: Definite Integrals:

$$\int_{-a}^{a} f(x).dx = \int_0^a [f(x)+f(-x)].dx.$$

$\Rightarrow 2\int_0^a f(x) \Rightarrow \boxed{f(x) \Rightarrow \text{even}}$

$\Rightarrow 0 \Rightarrow \boxed{f(x) = \text{odd}}$

$$\int_0^{2a} f(x) = \int_0^a [f(x)+f(2a-x)].dx$$

$= 2\int_0^a f(x)$, if $f(2a-x) = f(x)$

$= 0 \Rightarrow$ if $f(2a-x) = -f(x)$

$$\int_0^a f(x).dx = 2\int_0^{a/2} f(x).dx \Rightarrow \text{if } f(a-x) = f(x).$$

$= 0 \Rightarrow$ if $f(a-x) = -f(x)$

$$\int_0^{\pi/2} \frac{f(\alpha)}{f(\alpha)+f(\beta)}.dx = \pi/4$$

$\alpha = \cos x,\ \beta = \sin x$

$\alpha = \tan x \quad \beta = \cot x$

$\alpha = \sec x,\quad \beta = \operatorname{cosec} x.$

$$\int_a^b x.f(x) = \frac{a+b}{2}\int_a^b f(x).dx$$

$\Rightarrow$ if $(f(a+b-x)) = f(x).$

$$\int_0^\pi x.\sin x = \frac{\pi}{2}\int_0^\pi \sin x$$

$= -\frac{\pi}{2}(\cos x)_0^\pi$

$= -\frac{\pi}{2}(-1-1)$

$= \pi$

- $\int_0^{\pi/2} \log\tan x.dx = 0$

- $\int_{-a}^{a} |x|.dx = a^2$

- $\int_{-a}^{a} \frac{|x|}{x}.dx = 0.$

- $\int_1^n [x] = 1+2+3+\ldots+(n-1).$

Eg $\int_1^{100} [x] = 1+2+3+\ldots+99$

$= \frac{99(99+1)}{2} = 50\times 99$

$= 4950$

$$\int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx$$

$$= \frac{n-1}{n} \times \frac{n-3}{n-2} \times \frac{n-5}{n-4} \times \ldots \times \pi/2 \quad \{n = \text{even}\}$$

$$= \frac{n-1}{n} \times \frac{n-3}{n-2} \times \frac{n-5}{n-4} \times \ldots \times \frac{2}{3} \quad \{n = \text{odd}\}$$

$$\int_0^{\pi/2} \sin^4 x \, dx = \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} = 3\pi/16.$$

$$\int_0^{\pi/2} \sin^5 x \, dx = \frac{2}{3} \times \frac{4}{5} \times \frac{2}{3} = \frac{16}{45}$$

$$\int_0^{\pi/2} \cos^6 x \, dx = \frac{5}{\cancel{6}_2} \times \frac{\cancel{3}^1}{4} \times \frac{1}{2} \times \frac{\pi}{2} = \frac{5\pi}{32}$$

standards

	$n = \text{odd}$	$n = \text{even}$
$\int_0^{\pi} \sin^n x$	$2\int_0^{\pi/2} \sin^n x.$	$2\int_0^{\pi/2} \sin^n x$
$\int_0^{\pi} \cos^n x$	0	$2\int_0^{\pi/2} \cos^n x$
$\int_{-\pi/2}^{\pi/2} \sin^n x$	0	$2\int_0^{\pi/2} \sin^n x$
$\int_{-\pi/2}^{\pi/2} \cos^n x$	$2\int_0^{\pi/2} \cos^n x$	$2\int_0^{\pi/2} \cos^n x$
$\int_0^{2\pi} \sin^n x$, $\int_0^{2\pi} \cos^n x$	0	$4\int_0^{\pi/2} \sin^n x$, $4\int_0^{\pi/2} \cos^n x$
$\int_{-\pi}^{\pi} \sin^n x$, $\int_{-\pi}^{\pi} \cos^n x$	0	$4\int_0^{\pi/2} \sin^n x$, $4\int_0^{\pi/2} \cos^n x$

$$\int_0^{\pi/2} \sin^m x . \cos^n x . dx$$

$$= \frac{[(m-1)(m-3)(m-5)(m-7)] \ldots (1) \text{ or } (2)] \times [(n-1)(n-3)(n-5)(n-7) \ldots (1) \text{ or } (2)]}{(m+n)(m+n-2)(m+n-4) \ldots (2) \text{ or } (1).} \times K$$

$K = \pi/2$; when both m & n = even

$K = 1$; when any one is odd or both are odd.

(1) $\int_0^{\pi/2} \sin^3 x . \cos^2 x . dx$

$$\frac{(2)(1)}{(5)(3)} \times 1 = \frac{2}{15}$$

(2) $\int_0^{\pi/2} \sin^5 x . \cos^6 x . dx.$

$$\frac{(4)(2) \times (\cancel{5})(\cancel{3})(1)}{(11)(9)(7)(\cancel{5})(\cancel{3})} = \frac{8}{693}$$

$$I_n = \int_0^{\pi/4} \tan^n x . dx$$

$$= \int_0^{\pi/4} (\tan^n x + \tan^{n-2} x) . dx = \frac{1}{n-1}$$

$$= \int_{\pi/4}^{\pi/2} (\cot^n x + \cot^{n-2} x) . dx = \frac{1}{n-1}.$$

	m,n are odd	m is odd, n is even	m is even, n is odd	m,n are even
$\int_0^{\pi} \sin^m x \cdot \cos^n x$	0	$2\int_0^{\pi/2} \sin^m x \cos^n x$	0	$2\int_0^{\pi/2} \sin^m x \cdot \cos^n x$
$\int_{-\pi/2}^{\pi/2} \sin^m x \cdot \cos^n x$	0	0	$2\int_0^{\pi/2} \sin^m x \cdot \cos^n x$	$2\int_0^{\pi/2} \sin^m x \cdot \cos^n x$
$\int_0^{2\pi} \sin^m x \cos^n x$	0	0	0	$4\int_0^{\pi/2} \sin^m x \cdot \cos^n x$
$\int_{-\pi}^{\pi} \sin^m x \cdot \cos^n x$	0	0	0	$4\int_0^{\pi/2} \sin^m x \cdot \cos^n x$

Limit Summation:

$$\boxed{\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f(r/n) = \int_0^1 f(x)\cdot dx}$$

Eg. $\cancel{\lim_{n\to\infty} \sum_{r=1}^{n} f(r/n)} =$

$$\lim_{n\to\infty} \frac{1^k + 2^k + 3^k + \cdots + n^k}{n^{k+1}}.$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{r^k}{n^k}\right) \Rightarrow \int_0^1 x^k \cdot dx$$

$$\left(\frac{x^{k+1}}{k+1}\right)_0^1$$

$$= \frac{1}{k+1}.$$

$$\lim_{n\to\infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}\right]$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n+r}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{1}{1+\left(\frac{r}{n}\right)}\right) \Rightarrow \int_0^1 \frac{1}{1+x}$$

$$= \log(1+x)_0^1$$

$$= \log_e 2$$

$$\lim_{x\to 0} (\sin x)^x$$

$$y = (\sin x)^x$$

$$\log y = \lim_{x\to 0} x \log \sin x$$

$$= \lim_{x\to 0} \frac{\log \sin x}{\frac{1}{x}}$$

$$= \lim_{x\to 0} \frac{\cot x}{-1/x^2} \Rightarrow \frac{-x^2}{\frac{\tan x}{x}} = \frac{0}{1} = 0$$

$$\log y = 0$$
$$= e^0 = 1$$

$$y = (\sin x)^x$$

$$\log y = x \log \sin x$$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{\sin x}\cdot\cos x + \log \sin x$$

$$\frac{dy}{dx} = (\sin x)^x (x \cot x + \log \sin x)$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r/n}{\sqrt{\frac{n^2+r^2}{n^2}}}$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r/n}{\sqrt{1+r^2/n^2}}$$

$$\Rightarrow \int_0^2 \frac{x}{\sqrt{1+x^2}}\,dx =$$

$$= \frac{1}{2}\int_0^2 \frac{2x}{\sqrt{1+x^2}}\,dx$$

$$= \left(\frac{1}{2}\, 2\sqrt{1+x^2}\right)_0^2$$

$$= \left(\sqrt{1+x^2}\right)_0^2 = \sqrt{5}-1$$

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+n^2}\right)$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{n^2}{n^2+r^2}\right)$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{1}{1+r^2/n^2}$$

$$\int_0^1 \frac{1}{1+x^2}\,dx$$

$$= \left(\tan^{-1}(x)\right)_0^1 = \pi/4 - 0$$

$$= \pi/4$$

$$\lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{1/n}$$

$$\text{let } A = \left(\frac{n!}{n^n}\right)^{1/n}$$

$$\log A = \lim_{n\to\infty} \log\left(\frac{n!}{n^n}\right)^{1/n}$$

$$= \lim_{n\to\infty} \frac{1}{n}\log\left(\frac{n!}{n^n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\log\left[\frac{n\cdot(n-1)(n-2)\cdot(n-3)\cdots}{n\cdot n\cdot n\cdot n\cdots}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n}\log\left[\frac{n-r}{n}\right]$$

$$\lim_{n\to\infty} \sum_{r=0}^{n-1} \frac{1}{n}\log\left(1-\frac{r}{n}\right)$$

$$\int_0^1 \log(1-x)\,dx$$

$$= \left[\frac{(1-x)\log(1-x)-(1-x)}{-1}\right]_0^1$$

$$= -1.$$

$$\log A = -1$$

$$A = e^{-1}$$

$$\boxed{A = 1/e}$$

$$\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{\frac{2n^4+1}{5n^5+1}}$$

$$\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{\left(\frac{1}{n}\right)\left(\frac{2n^5+n}{5n^5+1}\right)}$$

$$\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{\frac{1}{n}} \quad \lim_{n\to\infty} \left(\frac{2n^5+n}{5n^5+1}\right)$$

$$= \left(\frac{1}{e}\right)^{\frac{2}{5}}$$

: Standard Results :

$$\int_0^{\pi/4} \log(1+\tan\theta) = \frac{\pi}{8}\log 2$$

$$\int_0^{\pi/2} \log\sin x = \int_0^{\pi/2} \log\cos x = -\frac{\pi}{2}\log 2$$

$$\int_0^{\pi/2} \frac{1}{a^2\sin^2 x + b^2\cos^2 x} = \frac{\pi}{2ab}$$

$$\int_0^{\pi} \frac{x}{a^2\sin^2 x + b^2\cos^2 x} = \frac{\pi^2}{2ab}$$

$$a>0; \int_0^{\infty} e^{-ax}\cdot\cos bx \cdot dx = \frac{a}{a^2+b^2}$$

$$a>0, \int_0^{\infty} e^{-ax}\cdot\sin bx \cdot dx = \frac{b}{a^2+b^2}$$

$$\int_0^{\infty} \frac{1}{(x+\sqrt{x^2-1})^n}\cdot dx = \int_0^{\pi/2} \frac{\sec^2 x}{(\sec x+\tan x)^n} = \frac{n}{n^2-1}$$

$$\int_a^b \sqrt{(x-a)(b-x)}\, dx = \frac{\pi}{8}(b-a)^2$$

$$\int_a^b \frac{1}{\sqrt{(x-a)(b-x)}}\cdot dx = \pi$$

$$\int_a^b \frac{1}{x\sqrt{(x-a)(b-x)}}\cdot dx = \frac{\pi}{\sqrt{ab}}$$

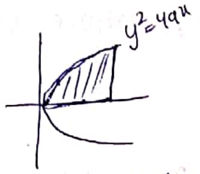
$$\int_a^b \sqrt{\frac{x-a}{b-x}}\, dx = \int_a^b \sqrt{\frac{b-x}{x-a}}\, dx = \frac{\pi}{2}(b-a)$$

★ Application of Integration ★

Important standards:

① $y^2 = 4ax$ (parabola)

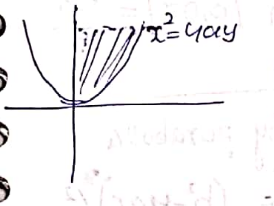
at $x=0$ & $x=a$ on x-axis



∴ Area bounded by curve $y^2=4ax$ at $x=0$ and $x=a$; $\Rightarrow \frac{4a^2}{3}$

→ at lattus rectum $\Rightarrow 2 \times \frac{4a^2}{3} = \frac{8a^2}{3}$

② $x^2 = 4ay$ at $y=0$ and $y=a$

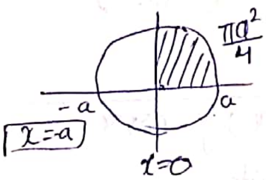


Area bounded by curve $x^2=4ay$ at $y=0$ and $y=a$

$\Rightarrow \frac{4a^2}{3}$

→ at it's lattus rectum $= \frac{8a^2}{3}$

③ Area of circle $x^2+y^2=a^2$ {at origin}

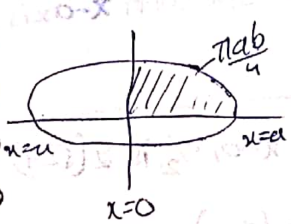


① at $x=0, a \Rightarrow \frac{\pi a^2}{4}$

② at $x=-a, a \Rightarrow \frac{\pi a^2}{2}$

③ area enclosed within the curve $\Rightarrow \pi a^2$

③ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

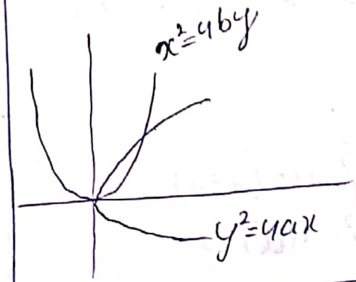


① Area under curve b/w $x=0$ and $x=a$ on x-axis $= \pi ab/4$

② b/w $x=-a, a \Rightarrow \frac{\pi ab}{2}$

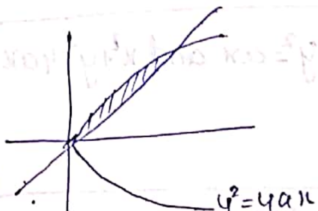
③ Area enclosed $= \pi ab$

Area enclosed by 2-parabolas:

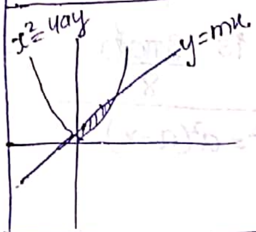


Area $= \boxed{\frac{16ab}{3}}$

Area b/w parabola and straight lines:



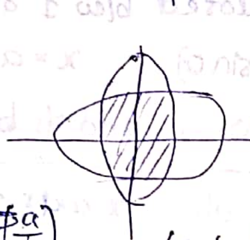
Area $= \frac{8}{3}\left(\frac{a^2}{m^3}\right)$



Area $= \frac{8}{3}(a^2m^3)$

Area enclosed b/w 2-ellipses

$a^2x^2+b^2y^2=1$
$b^2x^2+a^2y^2=1$



Area $=$ ~~$\frac{4}{ab}\tan^{-1}\left(\frac{a}{b}\right)$~~ $\quad 4ab\tan^{-1}(b/a)$

Area under $\sqrt{x}+\sqrt{y}=\sqrt{a}$ b/w $x=0$ and $x=a$ or $y=0$ and $y=a$ is $\frac{a^2}{6}$

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★ More standards on Areas ★

area bounded by 1 arc of curve $y = \sin ax$ or $y = \cos ax$ and x-axis is $\frac{2}{a}$ sq. units.

---

area enclosed b/w $y^2 = 4a(x+a)$ & $y^2 = 4a(b-x)$

$\Rightarrow \frac{8}{3}\sqrt{ab}\,(a+b)$

---

area common to curves $y^2 = ax$ and $x^2+y^2 = 4ax$ is $a^2\left[3\sqrt{3} + \frac{4\pi}{3}\right]$.

---

area of curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ (or) $[x = a\cos^3\theta$ or $y = b\sin^3\theta]$ is $\frac{3\pi ab}{8}$.

---

area enclosed by curve $xy^2 = a^2(a-x)$ and y-axis is πa^2

---

area enclosed b/w 1 arch of cycloid $x = a(\theta - \sin\theta)$ or $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ and its base is $\boxed{3\pi a^2}$

---

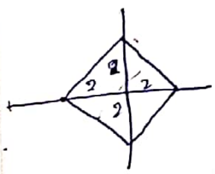
area lying above x-axis included b/w circle $x^2+y^2 = 2ax$ and the parabola $y^2 = ax$ is $a^2\left(\frac{\pi}{4} - \frac{2}{3}\right)$

---

area bounded by $|x|+|y| = 1 \Rightarrow 2$ sq.un

" " $|x|+|y| = 2 \Rightarrow 8$ sq.

$\boxed{\text{rhombus}}$



Area of rhombus $\Rightarrow \frac{1}{2} \times (d_1 \times d_2)$

$= \frac{1}{2}(4 \times 4)$

$= 8$

$\boxed{P_1 = P_2}$

$\text{Area} = \frac{P_1 \cdot P_2}{\sin\theta} = \frac{P_1^2}{\sin\theta}$

Area of curve $\frac{|x|}{a} + \frac{|y|}{b} = 1$ where $a, b > 0$ is $2ab$ (straight line)

$\frac{|x|}{a} + \frac{|y|}{b} = 2 \Rightarrow \frac{1}{2}(4a \times 4b)$

$= 8ab$

---

area of region bounded by $y = \sin ax$ & x-axis in $[0, n\pi]$ is $\frac{2n}{a}$

$\therefore y = \cos ax$ and x-axis in $[0, n\pi] = \frac{2n}{a}$

---

area of region bounded by parabola $y = ax^2+bx+c$ & x-axis $= \frac{(b^2-4ac)^{3/2}}{6a^2}$

" $x = ay^2+by+c$ & y-axis $= \frac{(b^2-4ac)^{3/2}}{6a^2}$

---

area bounded b/w $y = \sin x$; $y = \cos x$ and y-axis is $\sqrt{2}-1$ sq.units and x-axis is $2\sqrt{2}$ sq.units.

Area of one of the curve linear Δle formed b/w $y = \sin x$; $y = \cos x$ with x-axis is $2-\sqrt{2}$.

$y = \sin x$; $y = \cos x$ from $x = 0, \pi/2$ is $2(\sqrt{2}-1)$

" " $x = (0, \pi)$ is 1

~~area of I arch~~

area enclosed b/w parabolas $y^2=4a(x+a)$ and $y^2=4a(a-x)$ is $\frac{16a^2}{3}$

---

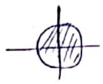
area common to two curves $y^2=ax$ and $x^2+y^2=4ax$ is $a^2\left(3\sqrt{3}+\frac{4\pi}{3}\right)$

---

whole area of asteroid $x^{2/3}+y^{2/3}=a^{2/3}$ (or) $x=a\cos^3\theta,\ y=a\sin^3\theta \Rightarrow \frac{3\pi a^2}{8}$

:Volumes:

Volume of curve $x^2+y^2=a^2$



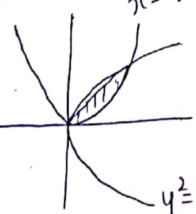
$\Rightarrow V=\frac{4}{3}\pi a^3$

Volume of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

i) $V=\frac{4}{3}\pi b^2 a$ (on x-axis ie, major axis)

ii) $V=\frac{4}{3}\pi a^2 b$ (on y-axis ie, minor axis)

$\rightarrow$



$V=\frac{96}{5}\pi a^{5/3}.b^{4/3}$. $\boxed{\text{On-x-axis}}$

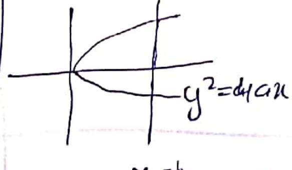
$V=\frac{96}{5}\pi a^{4/3}.b^{5/3}$ $\boxed{\text{On y-axis}}$

Volume of $y^2=4ax$ and $y=mx$

$V=\frac{32}{3}\pi\left[\frac{a^3}{m^4}\right] \rightarrow$ x-axis

$V=\frac{8}{3}\pi a^2 m^3$ (x-axis)

Volume generated by revolving the area of parabola $y^2=4ax$ bounded by ordinate $x=h$ about it's axis



$\pi\int_0^h 4ax.dx \Rightarrow \frac{4a\pi}{2}(x^2)_0^h$

$=\boxed{2\pi a h^2}$

$y=A\sin x$; $x=[0,\pi]$

$V=\frac{A^2\pi^2}{2} \Rightarrow$ (x-axis)

$y=A\cos x$; $x=[0,\pi]$

$V=\frac{A^2\pi^2}{2} \Rightarrow$ (y-axis)

---

volume of right circular cone of height 'h'

$=\frac{1}{3}\pi r^2 h$

= with semivertical angle (α) $=\frac{\pi}{3}h^3.\tan^2\alpha$.

Volume of spherical cap of height cut off from a sphere of radius 'a' is $\pi h^2\left(a-\frac{h}{3}\right)$

---

volume generated by revolution of equilateral Δle of side 'a' about one of its side is $\frac{\pi a^3}{16}$

Volume of solid formed by cycloid $x=a(\theta-\sin\theta)$; $y=a(1-\cos\theta)$ about base.

$=5\pi^2a^3$

$\Rightarrow$ about y-axis $\Rightarrow 6\pi^3a^3$

area included b/w parabola $y^2=4ax$ and the y axis and lines $y=\pm h$ is rotated about Y-axis ; Volume thus generated $= \frac{\pi h^5}{40a^2}$

Mean & R.M.S :

Mean of function $f(x)$ in $[a,b]$ is

$$\boxed{\frac{1}{b-a}\int_a^b f(x).dx.}$$

Mean square value of function $f(x)$ over $x=a$ to $x=b$

$$\frac{1}{b-a}\int_a^b [f(x)]^2.dx$$

R.M.S. of function $f(x)$ =

$$\sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2.dx.}$$

Mean value of $\sin(ax+\alpha).\sin(ax+\beta)$ as x varies from 0 to π/a $= \frac{1}{2}\cos(\alpha-\beta)$

$\Rightarrow \cos(ax+\alpha).\cos(ax+\beta)$ from 0 to $\frac{\pi}{a} = \frac{1}{2}\cos(\alpha-\beta)$

R.M.S. of $f(x) = a\sin px + b\cos px$ as x varies from 0 to 2π . is $\sqrt{\frac{a^2+b^2}{2}}$.

R.M.S. of value of $a+b\sin\theta$ is $\sqrt{\frac{2a^2+b^2}{2}}$

R.M.S. of current $i(t) = \frac{E}{R}+a\sin pt$ where $E, R, a, p \in$ constant is $\sqrt{\frac{a^2}{2}+\frac{E^2}{R^2}}$

R.M.S. of $a\cos\theta$ over a half wave $= \frac{a}{\sqrt{2}}$

R.M.S of $a\cos\omega t$ over a period $= \frac{a}{\sqrt{2}}$

R.M.S of $a\sin\omega t$ over a complete wave $= \frac{a}{\sqrt{2}}$.

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Exercise :

(1) area bounded by curve $y = 4x - x^2 - 3$ with x-axis is

$$\text{Area} = \frac{(b^2-4ac)^{3/2}}{6a^2}$$

$$= \frac{(16 - 4(-1)(-3))^{3/2}}{6(1)}$$

$$= \frac{(2^2)^{3/2}}{6} = \frac{8}{6} = \frac{4}{3} \text{ sq.units}$$

(2) $\int_0^{\pi/2} \frac{1}{1+\tan x} \cdot dx$

$$\int_0^{\pi/2} \frac{1}{1+\frac{\sin x}{\cos x}} \cdot dx = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} \cdot dx - I$$

$$2I = \int_0^{\pi/2} 1 \cdot dx$$

$$2I = \frac{\pi}{2}$$

$$I = \pi/4$$

(3) $\int_0^{\pi/2} \frac{\sin^2 x}{(1+\cos x)^2} \cdot dx$

$$\int_0^{\pi/2} \frac{1-\cos^2 x}{(1+\cos x)^2} \cdot dx$$

$$\int_0^{\pi/2} \frac{1-\cos x}{1+\cos x} \cdot dx$$

$$\int_0^{\pi/2} \tan^2 \frac{x}{2} \cdot dx$$

$$\int_0^{\pi/2} \sec^2 \frac{x}{2} - \int_0^{\pi/2} 1 \cdot dx$$

$$2\left[\tan \frac{x}{2}\right]_0^{\pi/2} - (x)_0^{\pi/2}$$

$$= 2 - \pi/2$$

(4) The area bounded by the curve $y = x^2 - 6x + 8$ and x-axis is

$$= \frac{(b^2-4ac)^{3/2}}{6a^2}$$

$$= \frac{(36-32)^{3/2}}{6} = \frac{8}{6} = \frac{4}{3}$$

(5) $\int \frac{x^5}{x^2+1} \cdot dx$

$$x^2+1 \,\overline{)\, x^5 \,}\; \quad x^3 - x$$
$$\underline{-x^5 + x^3}$$
$$-x^3$$
$$\underline{+x^3 - x}$$
$$(x)$$

$$\Rightarrow (x^3 - x) + \frac{x}{x^2+1}$$

$$\boxed{Q + \frac{R}{\text{Divisor}}} = \frac{\text{Divident}}{\text{Divisor}}$$

$$\int x^3 dx - \int x \cdot dx + \frac{1}{2}\int \frac{2x}{x^2+1} \cdot dx$$

$$\frac{x^4}{4} + \frac{x^2}{2} + \frac{1}{2}\log(x^2+1)$$

(6) $\int_0^{\pi/4} \frac{\sin^9 x}{\cos^{11} x} dx =$

$$\Rightarrow \frac{\sin^9 x}{\cos^9 x} \cdot \frac{1}{\cos^2 x}$$

$$= \int_0^{\pi/4} \tan^9 x \cdot \sec^2 x$$

$$\int f(x)^n \cdot f'(x) = \frac{f(x)^{n+1}}{n+1}$$

$$= \left[\frac{\tan^{10} x}{10}\right]_0^{\pi/4}$$

$$= \frac{1}{10}$$

(7) $\int \frac{1}{x \log x [\log(\log x)]} \cdot dx = \log[\log(\log x)]$

(8) $\int_0^{\infty} \frac{1}{\left(x + \sqrt{x^2+1}\right)^3} \Rightarrow \frac{n}{n^2-1} = \frac{3}{9-1} = \frac{3}{8}$

$\int_0^k \frac{\cos x}{1+\sin^2 x} \cdot dx = \pi/4$, then $k =$ —

put ~~cos x =~~ $t = \sin x$

$dt = \cos x \cdot dx$

$\int^k \frac{dt}{1+t^2} \cdot dt$

$\left(\tan^{-1}(t)\right]_0^k$

$\left[\tan^{-1}(\sin x)\right]_0^k = \pi/4$

$\tan^{-1}(\sin k) = \pi/4$

$\sin(k) = \tan \pi/4$

$\sin(k) = 1$

$k = \sin^{-1}(1)$

$\boxed{k = \pi/2}$ //

$\int \frac{1-x^2}{1+x^2} \cdot dx.$

$\frac{1+x^2-2x^2}{1+x^2}$

$1 - \frac{2x^2}{1+x^2}$

$1 - 2\frac{[x^2+1-1]}{x^2+1}$

$\int\left(1-2 - \frac{2}{x^2+1}\right)$

$x - 2x + 2\tan^{-1}(x)$

$\boxed{-x + 2\tan^{-1}(x)}$

$\int \frac{1}{(1-x^2)^{3/2}} \cdot dx$

$\boxed{x = \sin\theta}$ $dx = \cos\theta \cdot d\theta$

$\frac{1}{(\cos^2\theta)^{3/2}}$

$\int \frac{1}{\cos^3\theta} \cdot \cos\theta \cdot d\theta$

$\int \sec^2\theta \cdot d\theta$

$= \tan\theta \Rightarrow \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{x}{\sqrt{1-x^2}}$ //

$\int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1} \cdot dx$

$\sqrt{x} = t$

$\sqrt{9} = 3$

$\sqrt{4} = 2$

$\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} \cdot dx = dt$

$\boxed{dx = 2t \cdot dt}$

$\int_4^9 \frac{t(2t)}{t-1} dt$

$2\int_4^9 \frac{t^2}{t-1} \cdot dt$

$t-1 \,\big)\, t^2 \,(\, t+1$

$\underline{-t^2 - t}$

(t)

$\underline{t - 1}$

(1)

$2\left[\int_4^9 (t+1) + \frac{1}{(t-1)} \cdot dt\right]$

$2\left[t^2/2 + t + \log(t-1)\right]_4^9$

$2\left[t^2/2 + t + \log(t-1)\right]_2^3$

$2\left[\frac{9}{2} + 3 + \log 2 - \frac{4}{2} - 2 - \cancel{\log 1}^{0}\right]$

$2\left[\frac{15}{2} + \log 2 - 4\right]$

$2\left[\frac{7}{2} + \log 2\right]$

$= 7 + 2\log 2$ //

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The area b/w parabola $y^2 = x$ and $x^2 = y$ revolves about x-axis. The volume of solid so generated is

$\frac{96}{5}\pi a^3 \Rightarrow a = \frac{1}{4}$

$\frac{96}{5}\pi \times \left(\frac{1}{4}\right)^3 = \frac{\cancel{96}^{19}}{5} \times \pi \times \frac{1}{\cancel{64}_{16}} = 0.3\pi$

$\int \frac{f(x)\, g'(x) - f'(x)\, g(x)}{f(x) \cdot g(x)} [\log g(x) - \log f(x)]\, dx$

$\Rightarrow \frac{1}{2}\left[\log\left(\frac{g(x)}{f(x)}\right)\right]^2$

$f_n(x) = \log \cdot \log \cdot \log \cdots \log x$ (repeated n times)

then $\int (x f_1(x) \cdot f_2(x) \cdots f_n(x))^{-1} \cdot dx$

$= f_{(n+1)}(x) + C$ //

★ Differential Equations ★

Linear differential Eqn.

- → Variable separable
- → reducible Variable Separable
- → Homogeneous D.E
- → Non Homogeneous D.E
- → Exact diff. Eqn.
- → Lebintz linear differential Eqn
- → Bernouli's D.E.

Higher order D.E

- → $X = e^{ax+b}$
- → $X = k.xe^{0}$
- → $X = \cos ax$ or $\sin ax$ or 0
- → $X = x^{n}$ (polynomial or monomial)

PART-I LINEAR DE

~~i) Variable~~

$\boxed{\text{No. of arbitrary constants} = \text{order of D.E}}$

i) Variable separable:

→ $\frac{dy}{dx} = x^2+x$

$\int dy = \int x^2+x \cdot dx$

$\boxed{y = \frac{x^3}{3}+\frac{x^2}{2}+C}$

→ $y = cx$

$\boxed{\frac{dy}{dx} = c}$

$y = \frac{dy}{dx}x$

$\boxed{y = y_1 x}$

→ $y = Ae^{x}+Be^{-x}$

$\frac{dy}{dx} = Ae^{x}-Be^{-x}$

$\frac{d^2y}{dx^2} = Ae^{x}+Be^{-x}$

$\frac{d^2y}{dx^2} = y \Rightarrow \boxed{y_2 - y = 0}$

Standard forms:

i) $d(x.y) = x.dy + y.dx$

ii) $d\left(\frac{y}{x}\right) = \frac{x.dy - y.dx}{x^2}$

iii) $d\left(\tan^{-1}(y/x)\right) = \frac{x.dy - y.dx}{x^2+y^2}$

Eg: • $x.dy + y.dx = x^2+1.dx$

$\int d(x.y) = \int x^2+1.dx$

$xy = \frac{x^3}{3}+x$

$\boxed{x^3+3x-3xy+C=0}$

• $x.dy - y.dx = (x^2+x^3).dx$

$\frac{x\,dy - y.dx}{x^2} = (x+1).dx$

$\int d(y/x) = \int (x+1).dx$

$\boxed{\frac{y}{x} = \frac{x^2}{2}+x+C}$

• $x \cdot dy - y\,dx = (x^3 + y^2 x) \cdot dx$

$x\,dy - y \cdot dx = (x^2+y^2)x \cdot dx$

$$\int \frac{x \cdot dy - y\,dx}{x^2+y^2} = \int x \cdot dx$$

$$\int d\left(\tan^{-1} y/x\right) = \frac{x^2}{2} + c$$

$$\boxed{\tan^{-1}(y/x) = \frac{x^2}{2} + c}$$

$\rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$

$$\int \frac{dy}{dy} = \int \frac{dx}{x}$$

$\log y = \log x + \log c$

$\log y = \log cx$

$\boxed{y = cx}$

$\rightarrow \dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

$$\int dy = \int \frac{f'(x)}{f(x)} \cdot dx$$

$y = \log f(x) + \log c$

$\boxed{y = \log[f(x) \times c]}$

$\rightarrow \dfrac{dy}{dx} = \dfrac{x}{1+x^2}$

$dy = \dfrac{1}{2} \dfrac{2x}{1+x^2} \cdot dx$

$$\int dy = \frac{1}{2} \int \frac{2x}{1+x^2} \cdot dx$$

$$\boxed{y = \frac{1}{2} \log(1+x^2) + \log c}$$

★ $\dfrac{dy}{dx} = \left[e^{f(x)-g(y)} \times \dfrac{f'(x)}{g'(y)}\right]$ or

$\left[a^{f(x)-g(y)} \times \dfrac{f'(x)}{g'(y)}\right]$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{a^{f(x)}}{a^{g(y)}} \cdot \dfrac{f'(x)}{g'(y)}$

$$\int a^{g(y)} \cdot g'(y) \cdot dy = \int a^{f(x)} \cdot f'(x) \cdot dx$$

$$\frac{a^{g(y)}}{\log a} = \frac{a^{f(x)}}{\log a} + \frac{1}{\log c}$$

$$\boxed{a^{g(y)} = a^{f(x)} + \log_c a}$$

Eg: $\cos y \cdot \dfrac{dy}{dx} = a^{\sin x - \sin y} \cdot \cos x$

$$\int a^{\sin y} \cdot \cos y \cdot dy = \int a^{\sin x} \cdot \cos x\, dx$$

$$\boxed{a^{\sin y} = a^{\sin x} + c'}$$

$\rightarrow \sec^2 y \dfrac{dy}{dx} = 2^{\tan x - \tan y} \cdot \sec^2 x$

$$\int 2^{\tan y} \cdot \sec^2 y \cdot dy = \int 2^{\tan x} \cdot \sec^2 x \cdot dx$$

$$\boxed{2^{\tan y} = 2^{\tan x} + c'}$$

$\rightarrow \dfrac{1}{y} \cdot \dfrac{dy}{dx} = \dfrac{e^{\log x - \log y}}{x}$

$$\int e^{\log y} \cdot \frac{1}{y} \cdot dy = \int e^{\log x} \cdot \frac{1}{x} \cdot dx$$

$e^{\log y} = e^{\log x} + c$

$\boxed{y = x + c'}$

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$$\frac{dy}{dx} = \frac{y\log y}{x\log x}$$

$$\int \frac{dy}{y\log y} = \int \frac{dx}{x\log x}$$

$$\log(\log y) = \log(\log x) + \log c$$

$$\log(\log y) = \log(c\log x)$$

$$\log y = \log x^c$$

$$\boxed{y = x^c}$$

$$\Rightarrow \boxed{\log_x y = c}$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1}y = \sin^{-1}x + \sin^{-1}c$$

$$\cancel{\sin^{-1}}y = \cancel{\sin^{-1}}\left(x\sqrt{1-c^2} + c\sqrt{1-x^2}\right)$$

$$\boxed{y = x\sqrt{1-c^2} + c\sqrt{1-x^2}}$$

---

ii) Reducible variable Separable:

$$\frac{dy}{dx} = f(ax+by+c)$$

$$\Rightarrow \boxed{ax+by+c = z}$$

$$a + b\frac{dy}{dx} = \frac{dz}{dx}$$

$$\boxed{\int \frac{dz}{a+bf(z)} = \int dx.}$$

$$\Rightarrow \frac{dy}{dx} = (x+y)^2$$

$$\int \frac{dz}{1+z^2} = \int dx$$

$$\tan^{-1}(z) = x$$

$$\tan^{-1}(x+y) = x + c$$

$$x+y = \tan(x+c)$$

$$\boxed{y = \tan(x+c) - x}$$

$$\frac{dy}{dx} = \sin(x+y)$$

$$\int \frac{dz}{1+\sin z} = \int dx$$

$$\frac{1}{1+\sin z} \Rightarrow \frac{1-\sin z}{1-\sin^2 z} \Rightarrow \frac{1-\sin z}{\cos^2 z}$$

$$\sec^2 z - \tan z\sec z$$

$$\int \sec^2 z - \tan z\sec z\, dz = x$$

$$\tan z - \sec z = x + c$$

$$\boxed{\tan(x+y) - \sec(x+y) = x + c}$$

$$\frac{dy}{dx} = x + 2y$$

$$\int \frac{dz}{1+2z} = \int dx$$

$$\frac{\log(2z+1)}{2} = x + c$$

$$\boxed{\frac{\log(2(x+2y)+1)}{2} = x + c}$$

iii) Homogeneous D.E

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)} \; ; \; \left.\begin{matrix} f(x,y) \\ g(x,y) \end{matrix}\right\} \text{Homogeneous functions.}$$

$$\boxed{y = vx}$$

$$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$\boxed{\int \frac{dv}{F(v) - v} = \int \frac{dx}{x}}$$

Eg: $\frac{dy}{dx} = \frac{y}{x} + \cos^2 \frac{y}{x}$

$F(v) = v + \cos^2 v.$

$$\int \frac{dv}{v + \cos^2 v - v} = \int \frac{dx}{x}$$

$$\int \sec^2 v \cdot dv = \log x$$

$$\tan v = \log x + \log c$$

$$\boxed{\tan(y/x) = \log cx.}$$

$$\frac{dy}{dx} = \frac{y}{x} + \frac{y^3}{x^3}$$

$F(v) = v + v^3$

$$\int \frac{dv}{v + v^3 - v} = \int \frac{dx}{x}$$

$$\frac{-v^{-2}}{2} = \log cx$$

$$\frac{-1}{2v^2} = \log cx$$

$$\frac{-1}{2(y/x)^2} = \log cx$$

$$\frac{-x^2}{2y^2} = \log cx$$

$$\boxed{2y^2 \log cx + x^2 = 0}$$

iv) Non-Homogeneous D.E

$$\frac{dy}{dx} = \frac{ax + by + c}{a'x + b'y + c'}$$

Case i) $\boxed{b + a' = 0.}$

Sol: $a'xy + b'\frac{y^2}{2} - a\frac{x^2}{2} + c'y - cx = k$

$\boxed{k = \text{arbitrary constant}}$

Case ii) $\boxed{\frac{a}{a'} = \frac{b}{b'} = m.}$

Sol: $\int \frac{v + c'm}{(a + bm)v + (bmc + amc')} \cdot dv = \int . dx$

$\boxed{v = ax + by}$

Eg: $\frac{dy}{dx} = \frac{x - y + 1}{x + y + 2}$

$a = 1 \quad b = -1 \quad c = 1$

$a' = 1 \quad b' = 1 \quad c' = 2$

$\boxed{b + a' = 0}$

Sol: $a'xy + b'\frac{y^2}{2} + a\frac{x^2}{2} + c'y - cx = k$

$$\boxed{xy + \frac{y^2}{2} - \frac{x^2}{2} + 2y - x = k}$$

$$\frac{dy}{dx} = \frac{2x - 3y + 1}{3x + y + 2}$$

$a = 2 \quad b = -3 \quad c = 1$

$a' = 3 \quad b' = 1 \quad c' = 2$

$$a'xy + b'\frac{y^2}{2} - a\frac{x^2}{2} + c'y - cx = k$$

$$\boxed{3xy + \frac{y^2}{2} - x^2 + 2y - x = k}$$

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$$\frac{dy}{dx} = \frac{x+y+1}{x+y+2} \qquad \boxed{v = x+y}$$

$a = 1 \quad b = 1 \quad c = 1$

$a' = 1 \quad b' = 1 \quad c' = 2 \qquad m = \frac{a}{a'} = \frac{b}{b'} = 1$

Sol:
$$\int \frac{(v + c'm)}{(a+bm)v + (bmc + amc')} \cdot dv = \int \cdot dx$$

$$\int \frac{v+2}{(1+1)v + (1+2)} \cdot dv = x + c$$

$$\int \frac{v+2}{2v+3} \cdot dv = x+c$$

$$\frac{1}{2}\int \frac{2v+3}{2v+3} + \frac{1}{2v+3} \cdot dv = x+c$$

$$\frac{1}{2}\int \left(1 + \frac{1}{2v+3}\right) \cdot dv = x+c$$

$$\frac{1}{2}\left(v + \frac{\log(2v+3)}{2}\right) = x+c$$

$$\frac{v}{2} + \frac{\log(2v+3)}{4} = x+c$$

$$\frac{x+y}{2} + \frac{\log(2(x+y)+3)}{4} = x+c$$

(V) Exact D.E:

$M(x,y).dx + N(x,y)\,dy = 0.$

$(M.dx + N\,dy) = 0.$

To be exact D.E = $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$

Sol: $\underbrace{\int M.dx}_{y=\text{constant}} + \underbrace{\int N.dy}_{\text{integrate terms which doesn't include 'x' terms.}} = C$

Eg: ① $e^y dx + (xe^y + 2y).dy = 0.$

$\frac{\partial M}{\partial y} = e^y$

$\frac{\partial N}{\partial x} = e^y$

Sol. $\int M.dx + \int N.dy$

$\int e^y.dx + \int 2y.dy = c$

$\boxed{xe^y + y^2 = c}$

② $(3x^2y + y/x)\,dx + (x^3 + \log x).dy = 0.$

$\frac{\partial M}{\partial y} = 3x^2 + \frac{1}{x}.$

$\frac{\partial N}{\partial x} = 3x^2 + \frac{1}{x}$

Sol: $\int M.dx + \int N.dy = c$

$\int 3x^2y + y/x.dx + \int (x^3 + \log x).dy = c$

$\boxed{x^3y + y\log x = c}$

③ $(\cos x\cos y - \cot x).dx - (\sin x.\sin y).dy = 0$

$\frac{\partial M}{\partial y} = (\cos x)$

$\frac{\partial N}{\partial x} = (\cos x)$

Sol: $\int M.dx + \int N.dy = c$

$\int \cos x\cos y - \cot x.dx - \int 0.dy = c$

$\boxed{\cos y.\sin x - \log\sin x = c}$

④ $(y - x^3).dx + (x + y^3).dy = 0.$

$\frac{\partial M}{\partial y} = (1)$

$\frac{\partial N}{\partial x} = (1)$

Sol: $\int M.dx + \int N.dy = c$

$\int y - x^3.dx + \int y^3.dy = c$

$\boxed{xy - \frac{x^4}{4} + \frac{y^4}{4} = c}$

Integrating Factors for Non-exact:

Form: 1:

$Mdx + Ndy = 0.$

$I.F = \frac{1}{(M\times x) + (N\times y)} \Rightarrow \frac{1}{Mx + Ny}$

Form: 2

$f(x,y)\,y.dx + f(x,y)\,x.dy = 0$

$I.F = \frac{1}{(M\times x) - (N\times y)}$

$M \in = f(x,y)xy$

$N \in = f(x,y)x x$

Form: 3: If I.F is not obtaining then check

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x) \Rightarrow \therefore e$

$I.F = e^{\int f(x).dx}$

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(vi) Lebinitz linear D.E :

$$\frac{dy}{dx} + y(P(x)) = Q(x).$$

Integration factor $= e^{\int P(x).dx}$

Sol: $\boxed{y \times I.F = \int Q.(x).I.F.\, dx + C}$

Eg ① $\frac{dy}{dx} + \frac{y}{x} = 5$

I.F: $e^{\int 1/x.dx} = e^{\log_e x} = x$

Sol: $yx = \int x.5.dx$

$$\boxed{xy = \frac{5x^2}{2} + C}$$

② $\frac{dy}{dx} + \frac{y}{x} = e^x$

I.F $= x$

Sol: $yx = \int x.e^x$

$$\boxed{yx = e^x[x-1] + C}$$

$$\boxed{\int e^x.x^n = e^x\left[x^n - nx^{n-1} + n(n-1)x^{n-2} - \dots\right]}$$

③ $\frac{dy}{dx} + y\cot x = \operatorname{cosec} x.$

I.F. $= e^{\int \cot x\, dx} = e^{\log_e \sin x} = \sin x$

$y\sin x = \int \operatorname{cosec} x.\sin x.dx$

$$\boxed{y\sin x = x + c}$$

Eg:

$\frac{di}{dt} + i = 5$

I.F. $= e^{\int 1 dt} = e^t$

$ie^t = \int e^t.5.dt$

$ie^t = 5e^t + C$

$$\boxed{i = 5 + ce^{-t}}$$

vii) Bernouli's Eqn:

$$\boxed{\frac{dy}{dx} + y\,P(x) = Q(x)\,y^n}$$

Assume $\boxed{y^{1-n} = z}$

$$\boxed{\frac{dz}{dx} + (1-n)z\,P(x) = (1-n)\,Q(x)}$$

Eg: ① $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$.

$\boxed{n=-3}$

$$\frac{dz}{dx} + \frac{\cancel{4}^{2} z}{\cancel{8}x} = 4x$$

$$\frac{dz}{dx} + z\frac{2}{x} = 4x$$

$$I.F = e^{2\int\frac{1}{x}\cdot dx} \Rightarrow x^2$$

$$z.x^2 = \int x^2.4x.\,dx$$

$$x^2 z = 4\left[\frac{x^4}{4}\right] + c$$

$$x^2 z = x^4 + c$$

$$\boxed{y^4 x^2 = x^4 + c}$$

② $3y' + xy = xy^{-2}$

$$3.\frac{dy}{dx} + y(x) = xy^{-2}$$

$$\frac{dy}{dx} + \frac{yx}{3} = \frac{xy^{-2}}{3} \quad \boxed{n=-2}$$

$$\frac{dz}{dx} + \frac{\cancel{3}xz}{\cancel{3}} = \cancel{3}\frac{x}{\cancel{3}}$$

$$\frac{dz}{dx} + zx = x$$

$$I.F = e^{\int x.dx} = e^{x^2/2}$$

$$z.e^{x^2/2} = \int e^{x^2/2}.x.dx \qquad \left[\because \int e^{f(x)}.f'(x) = e^{f(x)}\right]$$

$$\boxed{y^3 e^{x^2/2} = e^{x^2/2} + c}$$

③ $xy' + y = x^3y^6$

$$z = y^{1-6} = y^{-5}$$

$$y' + \frac{y}{x} = x^2y^6.$$

$$\frac{dy}{dx} + \frac{y}{x} = x^2y^6 \quad \boxed{n=6}$$

$$\frac{dz}{dx} + z(-5)\left(\frac{1}{x}\right) = -5x^2$$

$$I.F = e^{-5\int 1/x.dx} = x^{-5}$$

$$z.x^{-5} = -5\int x^{-5}.x^2.\,dx$$

$$z.x^{-5} = -5\,\frac{x^{-2}}{(-2)} + c$$

$$\boxed{\frac{1}{x^5y^5} = \frac{5}{2x^2} + c}$$

PART-2

2^{nd} Order and Higher order D.E

Solution = C.F + P.I.

General form:

$$a_0\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_ny = f(x)$$

$$\underbrace{(a_0D^n + a_1D^{n-1} + \dots + a_n)}_{\text{Auxillary Equation} = f(D)}y = f(x) \text{ (or) } X$$

If roots of Auxillary equation are known, then based on nature of roots, C.F =

Nature of roots	C.F.
① real & distinct	$c_1e^{m_1x} + c_2e^{m_2x} + \dots\dots$
② real and equal	$(c_1 + c_2x + c_3x^2 + c_4x^3)e^{mx}$
③ Complex & Conjugate $\alpha \pm i\beta$	$e^{\alpha x}(c_1\sin\beta x + c_2\cos\beta x)$

Particular Integral (P.I)

case i)

Nature of X	P.I
$X = 0$	$P.I = 0$
$X = e^{ax+b}$.	$P.I = \frac{1}{f(a)}e^{ax+b}$; $f(a) \neq 0$. $= \frac{x}{f'(a)}.e^{ax+b}$; $f(a) = 0$ $= \frac{x^2}{f''(a)}.e^{ax+b}$; $f'(a) = 0$.

case ii) $X = \cos(ax+b)$ or $\sin(ax+b)$

$$P.I = \frac{1}{f(-a^2)}.\sin(ax+b) \qquad \boxed{D^2 = -a^2} \quad \boxed{f(-a^2) \neq 0}$$

$$= \frac{x}{f'(-a^2)}.\sin(ax+b) \qquad \boxed{D^2 = -a^2} \quad \boxed{f(-a^2) = 0}$$

Eg: $(D^2 + 2D + 1)y = \sin x$

$a = 1 \Rightarrow D^2 = -a^2 = -1$

$P.I = \frac{1}{f(-a^2)}.\sin x$

$= \frac{1}{-1+2D+1}.\sin x$

$= \frac{1}{2D}\sin x$

$= \frac{1}{2}\int \sin x$

$P.I = \frac{1}{2} - \cos x$

$f(-a^2) = 0$

if $f(D) = \frac{\sin ax / \cos ax}{D^2 + a^2}$

for $\sin ax = \frac{x}{2}\int \sin ax$

for $\cos ax = \frac{x}{2}\int \cos ax$

② $(D^4 + 2D^2 + 3D + 1)y = \cos x$

$a = 1$

$\boxed{D^2 = -1}$

$= \frac{1}{(-1)^2 - 2 + 3D + 1}.\cos x$

$= \frac{1}{3D}.\cos x$

$= \frac{1}{3}\int \cos x\,dx$

$= \frac{1}{3}\sin x$ //

$\Rightarrow$ $X = e^{ax}\sin bx$ or $e^{ax}\cos bx$

$$P.I = \frac{1}{f(D+a)}\sin bx$$

$\Rightarrow$ $X = e^{ax}x^n$.

$$P.I = e^{ax}\frac{1}{f(D+a)}x^n$$

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$(D^2+3D+1)y = \sin 3x$

$\boxed{D^2 = -9}$

$P.I. = \frac{1}{-9+3D+1} \cdot \sin 3x$

$= \frac{1}{3D-8} \cdot \sin 3x$

$= \frac{3D+8}{9D^2-64} \cdot \sin 3x$

$= \frac{3D+8}{-81-64} \cdot \sin 3x$

$= \frac{3D+8}{-145} \cdot \sin 3x$

$= \frac{-1}{145}(9\cos 3x + 8\sin 3x)$

Case iii)

$X = K$ (constan)

$= K \times e^0.$

$P.I = \frac{1}{f(D)} \cdot k$

$= \frac{1}{f(0)} \cdot k \quad \boxed{f(0) \neq 0}$

$= \frac{x}{f'(0)} \cdot k \quad \boxed{f(0) = 0}$

$= \frac{x^2}{f''(0)} \cdot k ; \quad \boxed{f'(0) = 0}$

Eg: $\frac{d^2y}{dx^2} + y = 5$

$f(D) = D^2+1$

$= \frac{1}{f(0)} \cdot 5$

$= 5$//

$(D^2+D+1)y = 10$

$= \frac{1}{f(0)} \cdot 10$

$= 10$//

Case iv):

$X = x^m \quad (m>0)$ {polynomial or monomial}

$f(0) = [1+\phi(D)]^{-1}$

$(1+x)^n = 1+nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \dots$

$(1+x)^{-1} = 1-x+x^2-x^3+x^4-x^5+x^6+\dots$

$(1-x)^{-1} = 1+x+x^2+x^3+x^4+\dots$

Eg: $(D^2+1)y = x^2$

$f(D) = D^2+1$

$X = x^2$

$P.I = [1+\phi(D)]^{-1} \cdot x^m$

$= (1+D^2)^{-1} \cdot x^2$

$= (1-D^2+D^4+\dots)\, x^2$

$\boxed{P.I = x^2-2+0.}$

$(D^2+2)y = x^3+x+1$

$P.I = [1+\phi(D)]^{-1}\, x^m$

$= \frac{1}{(2+D^2)} \cdot x^m$

$= \frac{1}{2}\left[\cancel{2}\left(1+\frac{D^2}{2}\right)^{-1} \cdot x^3+x+1\right]$

$= \frac{1}{2}\left[1-\frac{D^2}{2}+\frac{D^4}{4}\dots\right] x^3+x+1$

$= \frac{1}{2}\left[x^3+x+1-3x+0\right]$

$= \frac{1}{2}\left[x^3-2x+1\right]$

$(1-x)^{-2} = 1+2x+3x^2+4x^3+\dots$

$(1+x)^{-2} = 1-2x+3x^2-4x^3+5x^4-\dots$

$(1-x)^{-3} = 1+3x+6x^2+10x^3+15x^4+\dots$

$(1+x)^{-3} = 1-3x+6x^2-10x^3+15x^4+\dots$

Solved examples :

① order and degree of differential Eqn.

$$\left[\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3\right]^{6/5} = 6y$$

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = (6y)^{5/6}$$

$\boxed{\begin{array}{l}\text{order} = 2\\ \text{degree} = 1\end{array}}$

② The degree and order of

$$\left(\frac{dy}{dx} + \frac{d^2y}{dx^2}\right)^{5/2} = \left(a\frac{d^2y}{dx^2}\right)^{2/3}$$

$$\left(\frac{dy}{dx} + \frac{d^2y}{dx^2}\right)^{\frac{5}{\cancel{2}}\times \cancel{6}^{3}} = \left(a\frac{d^2y}{dx^2}\right)^{\frac{2}{\cancel{3}}\times \cancel{6}^{2}}$$

$$\left(\frac{dy}{dx} + \frac{d^2y}{dx^2}\right)^{15} = \left(a\frac{d^2y}{dx^2}\right)^{4}$$

$\boxed{\begin{array}{l}\text{degree} = 15\\ \text{order} = 2\end{array}}$

③ The order of D.E whose solution is given by $y = (C_1+C_2)\cos(x+C_3) - C_4 e^{x+C_5}$

$\Rightarrow 5-2 = 3$ //

④ The order and degree of

$$\left(\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2}\right)^{5/3} = a\frac{d^2y}{dx^2} \text{ are } (p, q)$$

$\Rightarrow p+q =$ ______

$$\left(\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2}\right)^{5} = \left(a\frac{d^2y}{dx^2}\right)^{3}$$

order = 4
degree = 5 $\Rightarrow p+q = 9$ //

⑤ The D.E obtained by eliminating the arbitrary constants A and B from $y = A + Bx^2$.

$$\frac{dy}{dx} = 0 + 2Bx$$

$$\frac{d^2y}{dx^2} = 2B$$

$$\boxed{B = \frac{1}{2}y''}$$

$$y_1 = \cancel{2}\,\frac{y_2}{\cancel{2}}\,x$$

$\boxed{y_2 x - y_1 = 0} \Rightarrow$ //

⑥ The D.E $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \sin y = 0$ is

$\boxed{\text{Non linear} - 2^{nd} \text{ order equation}}$

⑦ $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\cancel{x}} + e^y = 0$ is

$\Rightarrow 2^{nd}$ order non linear diff. Eqn.

⑧ $\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} + y = 0$

$\boxed{3^{rd} \text{ order Non linear D.E.}}$

⑨ Solution of $(y+x+1)\cdot\frac{dy}{dx} = x+y+1$

$$\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$$

$a=1$ $b=1$ $c=1$
$a'=1$ $b'=1$ $c'=-1$ $\Bigg\rangle$ $\frac{a}{a'} = \frac{b}{b'} = 1 = m$

$$\int \frac{v + c'm}{(a+bm)v + (bmc + amc')}\cdot dv = \int x\cdot dx$$

$$\int \frac{v + (-1)}{2v + (1-1)}\cdot dv = x + c$$

$$\int \frac{v-1}{2v}\cdot dv = x + c$$

$\frac{1}{2}v - \frac{1}{2}\log v = x + c$

$\boxed{v = x+y}$

$\frac{x+y}{2} - \frac{\log(x+y)}{2} = x + c$

$x+y - \log(x+y) = 2x + c$

$\boxed{(y-x) - \log(x+y) = c}$

(10) The Integrating factor of

$x^2y \cdot dx - (x^3+y^3) \cdot dy = 0.$

Sol: given eqn is not exact

$I.F = \frac{1}{Mx + Ny}$

$= \frac{1}{x^3y + (-yx^3 - y^4)}$

$= \frac{-1}{y^4}$ //

(11) The Integrating factor of

$y(xy + 2x^2y^2) \cdot dx + x(xy - x^2y^2) \cdot dy = 0.$

$\boxed{f(x,y)\, y \cdot dx + f(x,y)\, x \cdot dy = 0}$

$\Rightarrow I.F = \frac{1}{Mx - Ny}$

$= \frac{1}{xy(xy + 2x^2y^2) - xy(xy - x^2y^2)}$

$= \frac{1}{\cancel{x^2y^2} + 2x^3y^3 - \cancel{x^2y^2} + x^3y^3}$

$= \frac{1}{3x^3y^3}$ //

(12) The P.I. of $\frac{1}{D^2+D+1}x^2$

$= \frac{1}{(1-D)^{-1}}x^2$

$= (1-D)x^2$

$= \boxed{x^2 - 2x}$

(13) The order of $x^3 \cdot \frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} - 3y = x$

$= 3$

(14) The degree of $\left[\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = a\frac{d^2y}{dx^2}$

$\left[\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = a\frac{d^2y}{dx^2}$

$\left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right)^3 = a\left(\frac{d^2y}{dx^2}\right)^2$

$\boxed{\begin{array}{l} \text{order} = 2 \\ \text{degree} = 3 \end{array}}$

(15) family of straight lines passing through origin is represented by.

$y = mx$

$\frac{dy}{dx} = m.$

$\frac{dy}{dx} = \frac{y}{x}$

$\boxed{y\,dx - x\,dy = 0}$

(16) The D.E of family of circles with center at origin.

$x^2 + y^2 = a^2$

$2x + 2y \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{y}.$

$yy' = -x + c$

$\boxed{yy' + x = 0}$

(17) $\frac{dy}{dx} = \frac{x+y+1}{2x+2y+2}$ $\left\{\frac{ax+by+c}{a'x+b'y+c'}\right\}$

$\frac{a}{a'} = \frac{b}{b'} = m = \frac{1}{2}$

$a=1 \quad b=1 \quad c=1$

$a'=2 \quad b'=2 \quad c'=2$

$\boxed{v = x+y}$

$$\int \frac{v+c'm}{(a+bm)v + (bmc+amc')} \cdot dv = x+c$$

$$\int \frac{v+2(\frac{1}{2})}{(1+\frac{1}{2})v + (\frac{1}{2}+1)} \cdot dv = x+c$$

$$\frac{2}{3}\int \frac{v+1}{v+1} \cdot dv = x+c$$

$$\frac{2}{3} v = x+c$$

$$\frac{2}{3}(x+y) = x+c$$

$$2x+2y = 3x+c$$

$$2y-x = c$$

$$\boxed{x-2y = c}$$

★ TEST QUESTIONS ★

① If 'c' is a parameter, then the differential Eqn. of family of curves $x^2 = c(y+c)^2$

$x^2 = c(y+c^2)^2$

$\boxed{x = \sqrt{c}\,(y+c)}$

$1 = \sqrt{c}\,\frac{dy}{dx}$

$\boxed{\sqrt{c} = \frac{dx}{dy}}$

$$x = \frac{dx}{dy}\left[y + \left(\frac{dx}{dy}\right)^2\right]$$

$$x = y\cdot\frac{dx}{dy} + \left(\frac{dx}{dy}\right)^3$$

$$x \times \left(\frac{dy}{dx}\right)^3 = \left[y\cdot\frac{dx}{dy} + \left(\frac{dx}{dy}\right)^3\right]\left(\frac{dy}{dx}\right)^3$$

$$x\cdot\left(\frac{dy}{dx}\right)^3 = y\cdot\left(\frac{dy}{dx}\right)^2 + 1$$

$$\Rightarrow \boxed{x\cdot\left(\frac{dy}{dx}\right)^3 - y\cdot\left(\frac{dy}{dx}\right)^2 - 1}$$

② The solution of equation $\frac{dy}{dx} + 2y\tan x = \sin x$ (Satisfying $y=0$) when $x = \pi/3$.

$$I.F. = e^{2\int \tan x \cdot dx} = e^{2\log \sec x} = \cancel{\sin x}\ \sec^2 x$$

$$y.\sec^2 x = \int \sec^2 x \cdot \sin x \cdot dx$$

$$y\sec^2 x = \int \sec x \cdot \tan x \cdot dx$$

$$y\sec^2 x = \sec x + C$$

$$0 = 2+C$$

$$C = -2$$

$$\therefore y\sec^2 x = \sec x - 2$$

$$y = \frac{\sec x}{\sec^2 x} - \frac{2}{\sec^2 x}$$

$$y = \cos x - 2\cos^2 x$$

$$y = \cos x - 2 + 2\sin^2 x$$

$$\boxed{y = 2\sin^2 x + \cos x - 2}$$

③ The differential equation formed by eliminating a and b from the equation

$$y = e^x (a\cos x + b\sin x).$$

$$\frac{dy}{dx} = e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$$

$$\boxed{\frac{dy}{dx} = y + e^x(-a\sin x + b\cos x)}$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-a\sin x + b\cos x) + e^x(-a\cos x - b\sin x).$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-a\sin x + b\cos x) - e^x(a\cos x + b\sin x)$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} - e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x)$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} - e^x(a\cos x + b\sin x) + \frac{dy}{dx} - y$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} - y + \frac{dy}{dx} - y$$

$$\boxed{\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y}$$

$$\boxed{\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0}$$

④ The degree and order of D.E $y = \sqrt{c(x+\sqrt{c})}$ are ______

$$y = \sqrt{c(x+\sqrt{c})} \quad \text{--- ①}$$

$$y' = \sqrt{c} \cdot \frac{1}{2\sqrt{x+\sqrt{c}}} \quad \text{--- ②}$$

① × ②

$$yy' = \sqrt{c} \cdot \frac{1}{2\sqrt{x+\sqrt{c}}} \cdot \sqrt{c} \cdot \sqrt{x+\sqrt{c}}$$

$$yy' = \frac{c}{2}$$

$$\boxed{c = \frac{2yy'}{2}}$$

$$y^2 = c(x+\sqrt{c})$$

$$y^2 = \frac{2yy'}{2}\left(x + \sqrt{2yy'}\right)$$

$$y = 2y'\left(x + \sqrt{2yy'}\right)$$

$$\left(\frac{y}{2y'} - x\right)^2 = 2yy'$$

$$\frac{y - 2xy'}{4y_1^2} = 2yy'$$

$$y - 2xy' = 8y(y')^3$$

$\therefore$ $\boxed{\text{order} = 1, \ \text{degree} = 3}$

⑤ The solution of D.E

$(2x-3y+5)\,dx + (9y-6x-7)\,dy = 0$

$(9y-6x-7)\,dy = -(2x-3y+5)\cdot dx$

$\frac{dy}{dx} = \frac{\cancel{3y}-(2x-3y+5)}{-(6x-9y+7)}$

$\frac{dy}{dx} = \frac{2x-3y+5}{6x-9y+7}$

$\boxed{m = \frac{1}{3}}$

$\int \frac{(v+c'm)}{(a+bm)v+(bmc+amc')}\,dv = \int dx$

$\int \frac{v+7(\frac{1}{3})}{[2+(-3)(\frac{1}{3})]v+[(-3)(\frac{1}{3})(5)+(2)(\frac{1}{3})(7)]}\,dv = x+c$

$\int \frac{\frac{3v+7}{3}}{v+[-5+\frac{14}{3}]}\,dv = x+c$

$\int \frac{3v+7}{3v-1}\,dv = x+c$

$\int 1 + 8\int \frac{1}{3v-1} = x+c$

$v + \frac{8\log(3v-1)}{3} = x+c$

$3v + 8\log(3v-1) = 3x+c$

$3(2x-3y) + 8\log[3(2x-3y)-1] = 3x+c$

$6x-9y+8\log[6x-9y-1] = 3x+c$

$\boxed{3x-9y+8\log[6x-9y-1] = c}$

⑥ Solution of D.E

$\sqrt{1-y^2}\cdot dx + x\cdot dy - \sin^{-1}y\cdot dy = 0.$

$\sqrt{1-y^2}\cdot dx = (\sin^{-1}y - x)\cdot dy.$

$\frac{dx}{dy} = \frac{\sin^{-1}y}{\sqrt{1-y^2}} - \frac{x}{\sqrt{1-y^2}}$

$\frac{dx}{dy} + \frac{x}{\sqrt{1-y^2}} = \frac{\sin^{-1}y}{\sqrt{1-y^2}}$

$I.F = e^{\int \frac{1}{\sqrt{1-y^2}}\cdot dy} = e^{\sin^{-1}y}.$

$x\cdot e^{\sin^{-1}y} = \int e^{\sin^{-1}y}\cdot\frac{\sin^{-1}y}{\sqrt{1-y^2}}\cdot dy$

$x\cdot e^{\sin^{-1}y} = e^{\sin^{-1}y}(\sin^{-1}y-1)+c$

$\int te^t\cdot dt \Rightarrow e^t[t-1]$

$\boxed{x = \cancel{e^{\sin^{-1}y}}\sin^{-1}y - 1 + ce^{-\sin^{-1}y}.}$

⑦ Solution of D.E

$(1+y^2) + (x-e^{\tan^{-1}y})\frac{dy}{dx} = 0.$

$(1+y^2)\cdot dx + x\,dy - e^{\tan^{-1}y}\cdot dy = 0.$

$(1+y^2)\,dx = (e^{\tan^{-1}y} - x)\,dy$

$\frac{dx}{dy} = \frac{e^{\tan^{-1}y}}{1+y^2} - \frac{x}{1+y^2}$

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$

$I.F = e^{\int \frac{1}{1+y^2}\cdot dy} = e^{\tan^{-1}y}$

$x\,e^{\tan^{-1}y} = \int e^{\tan^{-1}y}\cdot\frac{e^{\tan^{-1}y}}{1+y^2}$

$\cancel{x\,e^{\tan^{-1}y} = e^{\tan^{-1}y}[\tan^{-1}y - 1] + c}$

$\boxed{x = \cancel{e^{\tan^{-1}y}\tan^{-1}y - 1 + ce^{-\tan^{-1}y}}}$

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$\Rightarrow$

$$xe^{\tan^{-1}y} = \int e^{\tan^{-1}y} \cdot \frac{e^{\tan^{-1}y}}{1+y^2} \cdot dy.$$

$$xe^{\tan^{-1}y} = \int \frac{e^{2\tan^{-1}y}}{1+y^2} \cdot dy$$

$$xe^{\tan^{-1}y} = \frac{1}{2} e^{2\tan^{-1}y} + c$$

$$2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c'$$

$$\boxed{2xe^{\tan^{-1}y} - e^{2\tan^{-1}y} = c'}$$

(8) Solution of D.E $\frac{dy}{dx} = (4x+y+1)^2$ when $y(0) = 1$

$$\frac{dy}{dx} = (4x+y+1)^2$$

$$\boxed{4x+y+1 = 2}$$

$$\int \frac{dz}{a+b\,f(z)} = \int dx.$$

$$\int \frac{dz}{4+z^2} = x + c$$

$$\frac{1}{2} \tan^{-1} \frac{z}{2} = x + c$$

$$\frac{1}{2} \tan^{-1} \left(\frac{4x+y+1}{2}\right) = x + c$$

$$\frac{1}{2} \tan^{-1} \left(\frac{0+1+1}{2}\right) = c$$

$$\frac{1}{2} \tan^{-1}(1) = c$$

$$2c = \frac{\pi}{4}$$

$$\boxed{c = \frac{\pi}{8}}$$

$$\therefore \frac{1}{2} \tan^{-1} \left[\frac{4x+y+1}{2}\right] = x + \frac{\pi}{8}$$

$$\tan^{-1} \left[\frac{4x+y+1}{2}\right] = 2x + \frac{\pi}{4}$$

$$4x+y+1 = 2\tan\left(2x + \frac{\pi}{4}\right)$$

$$\boxed{y = 2\tan\left(2x + \frac{\pi}{4}\right) - 4x - 1}$$

(9) The general solution of

$$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$$

$$e^{\int \tan x \cdot dx} = e^{\log \sec x} = \sec x$$

$$y \times \sec x = \int (2x + x^2 \tan x) \sec x \cdot dx$$

$$y\sec x = \int 2x\sec x \cdot dx + \int x^2 \tan x \cdot \sec x \cdot dx$$

$$y\sec x = \int 2x\sec x \cdot dx + \left[x^2 \sec x - \int 2x \sec x\right]$$

$$y\sec x = \cancel{\int 2x\sec x\, dx} + x^2 \sec x - \cancel{\int 2x \sec x\, dx}$$

$$y\sec x = x^2 \sec x + c$$

$$y = x^2 \cancel{\sec x} + c\cos x$$

$$\boxed{y = x^2 + c\cos x}$$

(10) The general solution of D.E $\frac{dy}{dx} = \frac{1}{x+y+1}$ is : ________

$$\frac{dy}{dx} = (x+y+1)^{-1}$$

$$\boxed{v = x+y+1}$$

$$\int \frac{dz}{1+\frac{1}{v}} = \int dx$$

$$\int \frac{v}{v+1} \cdot dv = x + c$$

$$\int \frac{v+1}{v+1} - \frac{1}{v+1} \cdot dv = x + c$$

$$v - \log v = x + c$$

$$\cancel{x} + y + 1 - \log(x+y+1) = \cancel{x} + c$$

$$y + 1 - \log(x+y+1) = c$$

$1 - c = \log k$

$$y = \log(x+y+1) + c - 1$$

$$y = \log(x+y+1) - \log k$$

$$\boxed{y = \log\left[\frac{x+y+1}{k}\right]}$$

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(11) D.E = $\frac{dy}{dx} = 1+x+y+xy$; Soln ?

$\frac{dy}{dx} = 1+x+y(x+1)$

$\frac{dy}{dx} = (1+x)(1+y)$.

$\int \frac{dy}{1+y} = \int dx \cdot 1+x$

$\log(1+y) = x + \frac{x^2}{2}$

$1+y = e^{(x+\frac{x^2}{2}+c)}$

$1+y = e^{(x+\frac{x^2}{2})} \cdot e^c$

$y = e^{(x+\frac{x^2}{2})} \cdot e^c - 1$ $\boxed{e^c = k}$

$\boxed{y = k e^{(x+\frac{x^2}{2})} - 1}$

(12) Integrating factor of D.E

$(1-x^2)\frac{dy}{dx} + xy = \frac{x^4}{(1+x^5)}(\sqrt{1-x^2})^3$

$\Rightarrow \frac{dy}{dx} + \frac{xy}{1-x^2} = \frac{x^4(\sqrt{1-x^2})^3}{(1+x^5)(1-x^2)}$.

I.F. = $e^{\int \frac{x}{1-x^2} dx}$

$= e^{\frac{-1}{2}\int \frac{-2x}{1-x^2} dx}$

$= e^{\frac{-1}{2}\log(1-x^2)}$

I.F = $\frac{1}{\sqrt{1-x^2}}$ //

(13) The D.E. of

$y = ae^x + bxe^x + cx^2e^x$.

$y' = ae^x + b[xe^x + e^x] + c[x^2e^x + e^x 2x]$

$y' = ae^x + bxe^x + be^x + cx^2e^x + c2xe^x$

$y' = \underline{ae^x + bxe^x + cx^2e^x} + be^x + c2xe^x$

$\boxed{y' = y + be^x + c.2x.e^x}$

$y'' = y' + be^x + c[2xe^x + 2e^x]$

$y'' = y' + \underline{be^x + c2xe^x} + 2ce^x$

$\boxed{y'' = y' + y' - y + 2ce^x}$

$y'' = 2y' - y + 2ce^x$

$y''' = 2y'' - y' + 2ce^x$

$y''' = 2y'' - y' + (y'' - 2y' + y)$

$y''' = 2y'' - y' + y'' - 2y' + y$

$y''' = 3y'' - 3y' + y$

$\boxed{y''' - 3y'' + 3y' - y = 0}$

(14) If $dx+dy = (x+y)(dx-dy)$, then $\log(x+y) =$ ______

$$dx+dy = xdx - xdy + ydx - ydy$$

$$dx - x\,dx - y\,dx = -xdy - ydy - dy$$

$$dx\,[1-x-y] = -dy\,[x+y+1]$$

$$-dx\,[x+y-1] = -dy\,[x+y+1]$$

$$\frac{dy}{dx} = \frac{x+y-1}{x+y+1}$$

$$\int \frac{v+(1)}{(1+(1))v+(-1+1)}\,dv = x+c$$

$$\int \frac{v+1}{2v}\,dv = x+c$$

$$\frac{1}{2}v + \frac{1}{2}\log v = x+c$$

$$\frac{x+y}{2} + \frac{\log(x+y)}{2} = x+c$$

$$x+y+\log(x+y) = 2x+c'$$

$$\log(x+y) = 2x-x-y+c'$$

$$\boxed{\log(x+y) = x-y+c'}$$

(15) If $y = \sin(7\sin^{-1}x)$, then $(1-x^2)y_2 - xy_1 =$ ______

$$y = \sin(7\sin^{-1}x)$$

$$y' = \cos(7\sin^{-1}x)\cdot\frac{7}{\sqrt{1-x^2}}$$

$$(\sqrt{1-x^2})\,y' = 7\cos(7\sin^{-1}x)$$

$$\boxed{(1-x^2)(y_1')^2 = 49\cos^2(7\sin^{-1}x)}$$

$$\cancel{(1-x^2)\,2y_1y_2 - 2xy_1^2} =$$

$$(1-x^2)(y_1^2) = 49\,[1-\sin^2(7\sin^{-1}x)]$$

$$\boxed{(1-x^2)(y_1^2) = 49\,[1-y^2]}$$

$$\left((1-x^2)\,2y_1y_2 - 2xy_1^2\right) = 49\,[-2yy_1]$$

$$\boxed{(1-x^2)\,y_2 - xy_1 = -49y}$$

(16) $\log y = \tan^{-1}x$, then $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} =$ ______

$$\log y = \tan^{-1}x$$

$$\frac{1}{y}\cdot y_1 = \frac{1}{1+x^2}$$

$$(1+x^2)y_1 = y$$

$$(1+x^2)y_2 + 2xy_1 = y_1$$

$$(1+x^2)y_2 + 2xy_1 - y_1 = 0.$$

$$(1+x^2)y_2 + (2x-1)y_1 = 0$$

$$\boxed{(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0}$$

(17) $y=\sqrt{\cos 2x}$, then $y \cdot \frac{d^2y}{dx^2} + 2y^2 =$ ______

$y^2 = \cos 2x$

$2yy_1 = -2\sin 2x$

$yy_1 = -\sin 2x$

$yy_2 + (y_1)^2 = -2\cos 2x$

$yy_2 + (y_1)^2 = -2y^2$

$yy_2 + 2y^2 = -(y_1)^2$

$$\boxed{y \cdot \frac{d^2y}{dx^2} + 2y^2 = -\left(\frac{dy}{dx}\right)^2}$$

(18) If $y = e^{m\sin^{-1}x}$, then $(1-x^2)y_3 - 3xy_2 =$ ______

$y = e^{m\sin^{-1}x}$

$y_1 = e^{m\sin^{-1}x} \cdot m\frac{1}{\sqrt{1-x^2}}$

$y_1(\sqrt{1-x^2}) = m e^{m\sin^{-1}x}$

$y_1(\sqrt{1-x^2}) = m e^{m\sin^{-1}x}$

$y_1(\sqrt{1-x^2}) = my$

$y_1^2(1-x^2) = m^2y^2$

$y_1^2(-2x) + (1-x^2)\,2y_1y_2 = m^2\,2yy_1$

$(1-x^2)\,2y_1y_2 - y_1^2\,2x = m^2\,2yy_1$

$(1-x^2)y_2 - xy_1 = m^2y$

$(1-x^2)y_3 + y_2(2x) - xy_2 - y_1 = m^2y_1$

$(1-x^2)y_3 - 2y_2x - xy_2 - y_1 = m^2y_1$

$(1-x^2)y_3 - 3xy_2 - y_1 = m^2y_1$

$(1-x^2)y_3 - 3xy_2 = m^2y_1 + y_1$

$= \boxed{y_1(m^2+1)}$ //

The P.I. of $(D^2+4D-3)y = x^2$

$$P.I = \frac{1}{(D^2-4D+3)} \cdot x^2$$

$$= \frac{1}{3\left(1+\frac{D^2-4D}{3}\right)} x^2$$

$$\frac{1}{3}\left(1+\frac{D^2-4D}{3}\right)^{-1} x^2$$

$$\frac{1}{3}\left[1 - \frac{(D^2-4D)}{3} + \frac{(D^2-4D)^2}{9}\right] x^2$$

$$\frac{1}{3}\left[x^2 - \frac{(2-8x)}{3} + \frac{16}{9}(2)\right]$$

$$\frac{1}{3}\left[x^2 + \frac{8x}{3} + \frac{32}{9} - \frac{2}{3}\right]$$

$$\frac{1}{3}\left[x^2 + \frac{8x}{3} + \frac{26}{9}\right]$$

$$= \boxed{\frac{x^2}{3} + \frac{8x}{9} + \frac{26}{27}}$$

The P.I of $(D^2-4D+3)y = e^{2x}\sin 3x$

$$P.I. = \frac{1}{f(D+2)} \cdot e^{2x}\sin 3x$$

$$= \frac{1 \cdot e^{2x}\sin 3x}{(D+2)^2 - 4(D+2) + 3}$$

$$= \frac{e^{2x}\sin 3x}{D^2 + \cancel{4D} + 4 - \cancel{4D} - 8 + 3}$$

$$= \frac{e^{2x}\sin 3x}{D^2 - 8 + 7}$$

$$= \frac{e^{2x}\sin 3x}{D^2 - 1}$$

$\boxed{D^2 = -a^2}$

$$= \frac{e^{2x}\sin 3x}{-10}$$ //

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The P.I. of $(D^2-5D+6)y = xe^{4x}$

$$P.I = \frac{xe^{4x}}{f(D+4)} \qquad \frac{e^{ax}V}{f(D)} = \frac{e^{ax}V}{f(D+a)}$$

$$= \frac{xe^{4x}}{(D+4)^2-5(D+4)+6}$$

$$= \frac{xe^{4x}}{D^2+8D+16-5D-20+6}$$

$$= \frac{xe^{4x}}{D^2+3D+2}$$

$$= \frac{xe^{4x}}{2\left[1+\frac{D^2+3D}{2}\right]}$$

$$= \frac{1}{2}e^{4x}\left[1+\left(\frac{D^2+3D}{2}\right)\right]^{-1} \times x.$$

$$= \frac{1}{2}e^{4x}\left[1-\frac{D^2+3D}{2}+\frac{(D^2-3D)^2}{4}\right]x$$

$$= \frac{1}{2}e^{4x}\left[x-\frac{[(0)+3]}{2}\right]$$

$$= \frac{1}{2}e^{4x}\left[\frac{2x-3}{2}\right]$$

$$= \frac{e^{4x}[2x-3]}{4} \quad //$$

The P.I. of $(D^2+4)y = x\sin x$

$$\boxed{\frac{xV}{f(D)} = \left[x-\frac{f'(D)}{f(D)}\right]\frac{V}{f(D)}}$$

$$P.I = \left[x-\frac{2D}{(D^2+4)}\right]\frac{\sin x}{D^2+4}$$

$$= \frac{x\sin x}{3} - \frac{2\cos x}{(D^2+4)(D^2+4)}$$

$$\boxed{= \frac{x\sin x}{3} - \frac{2\cos x}{9}}$$

The solution of $\sec^2 y \cdot \frac{dy}{dx} + 2x\tan y = x^3$

$$\frac{dy}{dx} + y\,(P(x)) = Q(x)$$

derivative $\Rightarrow \boxed{y.I.f = \int I.f.\,Q(x).dx}$

$$\sec^2 y\,\frac{dy}{dx} + 2x\tan y = x^3$$

$$\tan y.e^{x^2} = \int x^3.e^{x^2}.dx$$

$$e^{x^2}\tan y = \int \frac{t}{2}e^t.dx \qquad x^2 = t$$

or $2x\,dx = dt$

$$e^{x^2}\tan y = \frac{1}{2}\left[e^t(t-1)\right] \qquad dx = \frac{dt}{2x}$$

$$2e^{x^2}\tan y = e^{x^2}(x^2-1)+c$$

$$\boxed{2\tan y = (x^2-1)+c}$$

Test-02

(1) If $y = A(x) \cdot e^{-\int p \cdot dx}$

$$\boxed{y \cdot e^{\int p \cdot dx} = A(x)}$$

$\frac{dy}{dx} + y(p(x)) = Q(x) \Rightarrow$ then $A'(x) = \text{---}$

$y \cdot e^{\int p\,dx} = \int e^{\int p\,dx} Q(x)$.

$A(x) = \int e^{\int p\,dx} \cdot Q(x)$

$$\boxed{A'(x) = Q(x)\, e^{\int p \cdot dx}}$$

(2) general solution of $\sin y \cdot \frac{dy}{dx} = \cos y(1 - x\cos y)$

$\sin y \cdot \frac{dy}{dx} = \cos y - x\cos^2 y$

$\frac{\sin y}{\cos^2 y} \cdot \frac{dy}{dx} = \frac{\cos^2 y}{\cos^2 y} - x$

$\tan y \sec y \cdot \frac{dy}{dx} = \sec y - x$

$\tan y \sec y \cdot \frac{dy}{dx} - \sec y = -x$

Sol: $\sec y \cdot I.F = \int I.F. (-x) \cdot dx$

$I.F = e^{-\int 1\,dx} = e^{-x}$.

$\sec y \cdot e^{-x} = \int e^{-x} x \cdot dx$

$\sec y\, e^{-x} = -\int e^{-x} \cdot x \cdot dx$

~~$\sec y \cdot e^{-x} = -[e^{-x}(x-1)]$~~

~~$\sec y\, e^{-x} = e^{-x} - xe^{-x}$~~

$\sec y\, e^{-x} = xe^{-x} + e^{-x} + c$

$$\boxed{\sec y = x + 1 + ce^{x}}$$

(3) Solution of

$\frac{dy}{dx} = 1 - \cos(y-x) \cdot \cot(y-x)$.

$\boxed{y - x = z}$

$\int \frac{dz}{a + b\,t(z)} = \int dx$

$\int \frac{dz}{-1 + (1 - \cos z \cdot \cot z)} = x + c$

$\int \frac{dz}{-1 + 1 - \cos z \cot z} = x + c$

$-\int \frac{dz}{\cos z \cdot \cot z} = x + c$

$-\int \frac{\sin z}{\cos^2 z} = x + c$

$-\int \tan z \cdot \sec z = x + c$

$-\sec z = x + c$

$x + \sec z = c$

$\boxed{x + \sec(y-x) = c}$

(4) The solution of $(y\sin x + y)\frac{dy}{dx} - \cos^2 x = 0$

$(y\sin x + y) \cdot \frac{dy}{dx} = \cos^2 x$

$y(1 + \sin x) \cdot dy = \cos^2 x \cdot dx$

$\int y \cdot dy = \int \frac{\cos^2 x}{1 + \sin x} \cdot dx$

$\frac{y^2}{2} = \int (1 - \sin x)\,dx$

$\frac{y^2}{2} = x - (-\cos x) + c$

$\frac{y^2}{2} = x + \cos x + c$

$$\boxed{y^2 = 2x + 2\cos x + c'}$$

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⑤ If $x(\log x)\cdot\frac{dy}{dx}+y=\log x^2$ and $y(e)=0$ then $y(e^2)=$ ____

$x\log x\cdot\frac{dy}{dx}+y=\log x^2$

$\frac{dy}{dx}+\frac{y}{x\log x}=\frac{\log x^2}{x\log x}$

$If = e^{\int\frac{1}{x\log x}dx} = e^{\log_e(\log x)}$

$=\log x$

$y\log x = \int\frac{\log x^2}{x\log x}\cdot\log x\cdot dx$

$y\log x = \int\frac{\log x^2}{x}\cdot dx$

$y\log x = 2\int\frac{\log x}{x}\cdot dx$

$y\log x = \cancel{2}\frac{(\log x)^2}{\cancel{2}}+c$

$y\log x = (\log x)^2+c$

$0 = 1+c$

$\boxed{c=-1}$

$y\log x = (\log x)^2-1$

$y\log e^2 = (\log e^2)^2-1$

$2y = 4-1$

$\boxed{y=\frac{3}{2}}$ at $\boxed{x=e^2}$

⑥ The solution of $\frac{dy}{dx}+2yx=2y$ through $(2,0)$

$\frac{dx}{dy}+2yx=2y$

$\frac{dx}{dy}=2y-2yx$

$\frac{dx}{dy}=2y(1-x)$

$\cancel{\int\frac{dy}{2y}=(1-x)\cdot dx}$

$\int\frac{dx}{1-x}=\int dy\cdot 2y$

$-\log(x-1)=y^2+c$ (at 2,0)

$\therefore\boxed{c=0}$

$-\log(x-1)=y^2$

$\boxed{(x-1)=e^{-y^2}}$

⑦ The solution of D.E $3xy'-3y+(x^2-y^2)^{1/2}=0$ at $y(1)=1$ is

$3x\cdot\frac{dy}{dx}=3y-\sqrt{x^2-y^2}$

$3x\cdot\frac{dy}{dx}=3y-x\sqrt{1-\frac{y^2}{x^2}}$

$\frac{dy}{dx}=\frac{y}{x}-\frac{\sqrt{1-\frac{y^2}{x^2}}}{3}$ $\boxed{V=\frac{y}{x}}$

$\int\frac{dv}{F(v)-v}=\int\frac{dx}{x}$

$\int\frac{dv\,3}{3V-\sqrt{1-V^2}-\cancel{3V}}=\log x+\cancel{\log}\,c$

$-\int\frac{dv}{\sqrt{1-v^2}}=\frac{1}{3}\log x+c$

$\boxed{3\cos^{-1}\left(\frac{y}{x}\right)=\log x}$ at $\boxed{y(1)=1;\ c=0}$

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(8) Solution of D.E $y' = \frac{1}{e^{-y} - x}$

$$y' = \frac{1}{e^{-y} - x}$$

$$\frac{dy}{dx} = \frac{1}{e^{-y} - x}$$

$$\frac{dx}{dy} = e^{-y} - x$$

$$\frac{dx}{dy} + x = e^{-y}$$

$$I.F = e^{\int 1 . dy} \Rightarrow e^{y}$$

$$x . e^{y} = \int e^{y} . e^{-y} . dy$$

$$x e^{y} = y + c.$$

$$\boxed{x = e^{-y}(y+c)}$$

(9) $\frac{dy}{dx} = \frac{x \log x^2 + x}{\sin y + y \cos y} = \underline{\quad ? \quad}$

$$\int \sin y + y \cos y . dy = \int (x \log x^2 + x) . dx$$

$$\cancel{-\cos y} + [y \sin y + \cancel{\cos y}] = \int x \log x^2 + \frac{x^2}{2}$$

$$y \sin y = \frac{1}{2}[x^2 \log x^2 - x^2] + \frac{x^2}{2}$$

$$y \sin y = \frac{x^2}{2} \log x^2 - \cancel{\frac{x^2}{2}} + \cancel{\frac{x^2}{2}}$$

$$\boxed{y \sin y = x^2 \log x + C}$$

$x^2 = t$

$2x . dx = dt$

$dx = \frac{dt}{2x}$

$\cancel{x} \log t \frac{dt}{2\cancel{x}}$

$\frac{1}{2} \int \log t \, dt$

$\frac{1}{2}[t \log t - t]$

(10) D.E. having $xy = ax^2 + bx + c$ having solution as ______

$$xy = ax^2 + bx + C$$

$$y_1 = 2ax + b$$

$$y_2 = 2a$$

$$\boxed{y_3 = 0} \quad \therefore \boxed{\frac{d^3y}{dx^3} = 0}$$

(11) A D.E satisfying the relation $x = A\cos(mt - \alpha)$.

$$\frac{dx}{dt} = -A\sin(mt - \alpha)m$$

$$\frac{d^2x}{dt^2} = -Am\cos(mt - \alpha)m$$

$$\frac{d^2x}{dt^2} = -Am^2\cos(mt - \alpha).$$

$$\frac{d^2x}{dt^2} = -m^2 x$$

$$\therefore \boxed{\frac{d^2x}{dt^2} + m^2 x = 0}$$

(12) D.E. of $y = a\cos(\log x) + b\sin(\log x)$.

$$y = a\cos(\log x) + b\sin(\log x)$$

$$y_1 = -a\sin(\log x)\frac{1}{x} + b\cos(\log x).\frac{1}{x}$$

$$\boxed{xy_1 = -a\sin\log x + b\cos\log x}$$

$$xy_2 + y_1 = -a\cos\log x . \frac{1}{x} - b\sin\log x \frac{1}{x}$$

$$x^2y_2 + xy_1 = -(a\cos\log x + b\sin\log x)$$

$$x^2y_2 + xy_1 = -y$$

$$\boxed{x^2y_2 + xy_1 + y = 0}$$

(13) General solution of

$$\frac{dy}{dx} = \frac{y^2-2y+2}{x^2-2x+5}$$

$$\int \frac{dy}{y^2-2y+2} = \int \frac{dx}{x^2-2x+5}$$

$$\int \frac{dy}{(y-1)^2+1} = \int \frac{dx}{(x-1)^2+4}$$

$$\boxed{\tan^{-1}(y-1) = \frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right) + c}$$

(14) The D.E of family of straight lines $y = mx + \frac{a}{m}$ where m is a parameter is

$$\boxed{y_1 = m + 0}$$

$$y = y_1 x + \frac{a}{y_1}$$

$$yy_1 = (y_1)^2 x + a$$

$$\boxed{x\frac{d^2y}{dx^2} - y\cdot\frac{dy}{dx} = -a}$$

(15) The D.E of family of curves given by $y = e^{3x}(Ax+B)$.

$$y = e^{3x}(Ax+B)$$

$$y_1 = 3e^{3x}(Ax+B) + Ae^{3x}$$

$$y_1 = 3y + Ae^{3x}$$

$$y_2 = 3y_1 + 3Ae^{3x}$$

$$y_2 = 3y_1 + 3(y_1 - 3y)$$

$$y_2 = 3y_1 + 3y_1 - 9y$$

$$y_2 = 6y_1 - 9y$$

$$\boxed{\frac{d^2x}{dy^2} - 6\frac{dy}{dx} + 9y = 0}$$

(16) The D.E of system of curves given by

$$\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$$

$$\frac{2x}{a^2} + \frac{2yy_1}{a^2+\lambda} = 0$$

$$\frac{\cancel{2}x}{a^2} = \frac{-\cancel{2}yy_1}{a^2+\lambda}.$$

$$\boxed{\frac{-x}{a^2y_1} = \frac{y}{a^2+\lambda}} \text{ — (1).}$$

$$\frac{x^2}{a^2} - \frac{xy}{a^2y_1} = 1$$

$$x^2 - \frac{xy}{y_1} = a^2$$

$$\boxed{x^2 - \frac{xy}{dy/dx} = a^2}$$

Solution of $y'' - y' - 2y = 0$; $\boxed{y(0)=0}$ $\boxed{y'(0)=4}$

$$(D^2 - D - 2)y = 0.$$

$$D^2 - 2D + D - 2$$

$$D(D-2) + 1(D-2)$$

$$(D+1)(D-2) = 0$$

$$D = -1, 2$$

$$y = c_1e^{2x} + c_2e^{-x} \Rightarrow \boxed{y(0)=0}$$

$$\boxed{0 = c_1 + c_2}$$

$$y' = 2c_1e^{2x} - c_2e^{-x} \quad \boxed{y'(0)=4}$$

$$4 = 2c_1 - c_2$$

$$4 = 3c_1 \Rightarrow \boxed{c_1 = \frac{4}{3}} \quad \boxed{c_2 = \frac{-4}{3}}$$

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Test-03

① The solution of D.E $x.\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ is ______

$$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}$$

$$\boxed{\frac{y}{x} = v}$$

$$\int \frac{dv}{v - \tan v - v} = \int \frac{dx}{x}$$

$$\int \frac{dv}{\tan v} = -\log x + \log k$$

$$\int \cot v .\, dv = -\log x + \log k$$

$$\log \sin v = \log \frac{k}{x}$$

$$\sin v = \frac{k}{x}$$

$$v = \sin^{-1}\left[\frac{k}{x}\right]$$

$$\boxed{y = x\sin^{-1}[k/x]}$$

② The solution of D.E $y.dx - xdy + 3x^2y^2e^{x^3}.dx = 0$ satisfying $y=1$ when $x=1$, is ______

$$y\,dx - x\,dy + 3x^2y^2e^{x^3} = 0$$

$$\frac{y\,dx - x\,dy}{y^2} + 3x^2e^{x^3} = 0.$$

$$d\left(\frac{x}{y}\right) + d\left(e^{x^3}\right) = 0$$

$$\frac{x}{y} + e^{x^3} = c$$

put $x = y = 1$: $\boxed{1 + e = c}$

$$\frac{x}{y} + e^{x^3} = 1 + e$$

$$\frac{x}{y} + e^{x^3} - (1+e) = 0$$

$$\boxed{x + y\left[e^{x^3} - (1+e)\right] = 0}$$

③ The solution of D.E $(x+2y^3)\frac{dy}{dx} = y$

$$y\frac{dx}{dy} = x + 2y^3$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$

$$I.F. = e^{-\int \frac{1}{y}.dy} = \frac{1}{y}$$

$$x.\frac{1}{y} = \int 2y^2.\frac{1}{y}.dy$$

$$\frac{x}{y} = y^2 + c$$

$$\boxed{x = y^3 + cy}$$

④ D.E of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{2x}{a^2} = -\frac{2y}{b^2}.\frac{dy}{dx}$$

$$\boxed{\frac{-b^2}{a^2} = \frac{y}{x}.dy/dx}$$

$$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left[\frac{x\,dy/dx - y}{x^2}\right] = 0$$

$$\boxed{xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\,dy/dx = 0}$$

⑤ $\frac{dx}{dy} + \frac{x}{y} = x^2 \Rightarrow$ ~~DE~~ . Solution = ?

$$\frac{dx}{dy}\frac{1}{x^2} + \frac{1}{y}.\frac{1}{x} = 1$$

let $\frac{1}{x} = t$

$$\frac{-1}{x^2}.\frac{dx}{dy} = \frac{dt}{dy} \Rightarrow \frac{1}{x^2}\frac{dx}{dy} = -\frac{dt}{dy}$$

$$\frac{-dt}{dy} + \frac{t}{y} = 1$$

$$\frac{dt}{dy} - \frac{t}{y} = -1$$

$$I.F = e^{-\int \frac{1}{y} \cdot dy} = e^{-\log y} = \frac{1}{y}$$

$$t \cdot \frac{1}{y} = \int (-1) \frac{1}{y} dy$$

$$\frac{t}{y} = -\log y + c$$

$$t = -y\log y + cy$$

$$\boxed{\frac{1}{x} = cy - y\log y}$$

⇒ Solution of $\cos(x+y) \cdot dy = dx$ =

$$\frac{dx}{dy} = \cos(x+y)$$

$$\boxed{x+y=z}$$

$$\int \frac{dz}{1+\cos z} = \int dy$$

$$\frac{1-\cos z}{1-\cos^2 z}$$

$$\frac{1-\cos z}{\sin^2 z}$$

$$\text{cosec}^2 z - \cot z \cdot \text{cosec}\, z$$

$$\int \text{cosec}^2 z - \cot z \cdot \text{cosec}\, z \cdot dz = y + c$$

(or)

$$\int \frac{dz}{2\cos^2 \frac{z}{2}} = y + c$$

$$\frac{1}{2} \int \sec^2 \frac{z}{2} \cdot dz = y + c$$

$$\frac{1}{2} 2 \tan \frac{z}{2} = y + c$$

$$\tan \left(\frac{x+y}{2}\right) = y + c$$

$$\boxed{y = \tan \left(\frac{x+y}{2}\right) - c}$$

⇒ Solution of $x \frac{dy}{dx} = y + xe^{y/x}$, with $\boxed{y(1)=0}$

$$x \frac{dy}{dx} = y + xe^{y/x}$$

$$\frac{dy}{dx} = \frac{y}{x} + e^{y/x}$$

$$\boxed{y/x = v}$$

$$\int \frac{dv}{F(v) - v} = \int \frac{dx}{x}$$

$$\int \frac{dv}{v + e^v - v} = \log x + c$$

$$\int e^{-v} \cdot dv = \log x + c$$

$$-e^{-v} = \log x + c$$

$$-e^{-y/x} = \log x + c \qquad \boxed{\cancel{C=1}} \quad \boxed{C=-1}$$

$$\boxed{\log x + e^{-y/x} = 1}$$

⇒ solution of $\cos y \cdot \frac{dx}{dy} + (x\sin y - 1) = 0$

$$\cos y \frac{dx}{dy} + x\sin y - 1 = 0.$$

$$\frac{dx}{dy} + x\tan y - \sec y = 0$$

$$\frac{dx}{dy} + x \cdot \tan y = \sec y$$

$$I.F\ e^{\int \tan y \cdot dy} = e^{\log \sec y} = \sec y.$$

$$x \sec y = \int \sec y \cdot \sec y \cdot dy$$

$$x \sec y = \int \sec^2 y \cdot dy$$

$$\boxed{x \sec y = \tan y + c}$$

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⇒ Integreating factor of

$(1+y+x^2y)dx + (x+x^3)dy = 0$

$\underline{\cancel{Ds}}$ $\boxed{Mdx + Ndy = 0}$

$\frac{\partial M}{\partial y} = 1+x^2$; $\frac{\partial N}{\partial x} = 1+3x^2$

$\therefore \boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$

$I.F = \frac{1}{Mx - Ny}$

$= \frac{1}{x+\cancel{xy}+\cancel{x^3y}-\cancel{xy}-\cancel{x^3y}}$

$\boxed{I.F = \frac{1}{x}}$ //

⇒ solution of $\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$.

$\frac{\sin y}{\cos y} \cdot \frac{dy}{dx} = 2\sin x \cdot \cos y$

$\int \frac{\sin y}{\cos^2 y} \cdot dy = 2\int \sin x \, dx$

$t = \cancel{\sin y}$ $\boxed{t = \cos y}$

$-\int \frac{1}{t^2} \cdot dt = -2\cos x + c$

$\cos y = t$

$-\sin y \cdot dy = dt$

$dy = \frac{-dt}{\sin y}$

$\frac{1}{t} = -2\cos x + c$

$\sec y = -2\cos x + C$

~~secy~~

$\boxed{\sec y + 2\cos x = c}$

$\frac{t^{-2+1}}{t^{-2+1}}$

t^{-1}

⇒ $(x^2+y^2)dx = 2xy \cdot dy$

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

$\boxed{y = vx}$

$\frac{dy}{dx} = \frac{x^2+x^2v^2}{2xvx^2}$

$\frac{dy}{dx} = \frac{1+v^2}{2v} = F(v)$

$\therefore \int \frac{dv}{F(v)-v} = \int \frac{dx}{x}$

$\int \frac{dv}{\frac{1+v^2}{2v} - v} = \log x + \log c$

$\int \frac{dv}{\frac{1-v^2}{2v}} = \log cx$

$-\int \frac{2v}{1-v^2} dv = \log cx$

~~$\log v = \log cx$~~

~~$\frac{1}{v} = cx$~~

~~$\frac{x}{y} = cx$~~

$-\log(1-v^2) = \log cx$

$-\log\left(\frac{x^2-y^2}{x^2}\right) = \log x + \log c$

$-\log\left[x^2-y^2\right] + 2\log x = \log x + \log c$

$\log x = \log c + \log(x^2-y^2)$

$\boxed{x = c(x^2-y^2)}$

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⇒ solution of

$y^2 dx + (x^2 - xy + y^2)\,dy = 0.$

$(x^2 - xy + y^2)\,dy = -y^2\,dx$

$\frac{dy}{dx} = \frac{-y^2}{x^2 - xy + y^2}$

put $\boxed{y = vx}$

$\frac{dy}{dx} = \frac{-v^2x^2}{x^2 - x^2v + x^2v^2}$

$\boxed{\frac{dy}{dx} = \frac{-v^2}{1 - v + v^2} = F(v)}$

$\int \frac{dv}{F(v) - v} = \int \frac{dx}{x}.$

$\int \frac{dv}{\frac{-v^2}{1-v+v^2} - v} = \log x + C.$

$\int \frac{dv}{\frac{-v^2 - v + v^2 + v^3}{1-v+v^2}} = \log x + c$

$\int \frac{1 - v + v^2}{-v^3 - v}\,dv = \log x + c$

$\int \frac{-(v^2+1) + v}{v(v^2+1)}\,dv = \log x + c$

$\int \left(\frac{-1}{v} + \frac{1}{v^2+1}\right) dv = \log x + c$

$-\log v + \tan^{-1} v = \log x + c$

$\tan^{-1} v = \log xv + c$

$\boxed{\tan^{-1}\left(\frac{y}{x}\right) = \log y + c}$

⇒ solution of

$x\,dx + y\,dy = x^2y\,dy - xy^2\,dx$

$x\,dx + xy^2\,dx = x^2y\,dy - y\,dy$

$x(1+y^2)\,dx \times 2 = y(x^2-1)\,dy \times 2$

$\int \frac{2x}{x^2-1}\,dx = \int \frac{2y}{(y^2+1)}\,dy$

$\log(x^2-1) = \log(y^2+1) + \log c$

$\boxed{(x^2-1) = c(y^2+1)}$

⇒ solution of

$\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x \cdot dy = 0.$

$\frac{\sec^2 x \tan y}{\tan x \cdot \tan y}\,dx + \frac{\sec^2 y \tan x}{\tan x \cdot \tan y}\,dy = 0$

$\int \frac{\sec^2 x}{\tan x}\,dx + \int \frac{\sec^2 y}{\tan y}\,dy = 0$

$\log \tan x + \log \tan y = 0$

$\log(\tan x \cdot \tan y) = \log 1$

$\boxed{\tan x \cdot \tan y = 1}$

⇒ solution of $(1+\cos x)\,dy = (1-\cos x)\,dx =$

$(1+\cos x)\,dy = (1-\cos x)\,dx$

$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}.$

$\frac{dy}{dx} = 2\tan^2\frac{x}{2}$

$\frac{dy}{dx} = \sec^2\frac{x}{2} - 1$

$\int dy = \int \sec^2\frac{x}{2} - 1 \cdot dx$

$\boxed{y = 2\tan\frac{x}{2} - x + c}$

Test-04 :

(1) G.s. of DE

$$yy' = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right], \text{ where}$$

ϕ is an arbitrary constant.

$$yy' = x\left[\frac{y^2}{x^2} + \frac{\phi\left[\frac{y^2}{x^2}\right]}{\phi'\left[\frac{y^2}{x^2}\right]}\right]$$

$$\left(\frac{y}{x}\right)y' = \left[\frac{y^2}{x^2} + \frac{\phi\left[y^2/x^2\right]}{\phi'\left[y^2/x^2\right]}\right]$$

$$vy' = \left[v^2 + \frac{\phi(v^2)}{\phi'[v^2]}\right]$$

$$\boxed{\int\frac{dv}{F(v)-v} = \int\frac{dx}{x}}$$

$$\int\frac{dv}{\frac{v^2+\frac{\phi(v^2)}{\phi'(v^2)}}{v} + v} = \log x + \log\sqrt{c}$$

$$\int\frac{dv\,(v)}{\cancel{v^2} + \frac{\phi(v^2)}{\phi'(v^2)} - \cancel{v^2}} = \log(x\sqrt{c}).$$

$$\frac{1}{2}\int 2v\,\frac{\phi'(v^2)}{\phi(v^2)}\cdot dv = \log(x\sqrt{c})$$

$$\frac{1}{2}\log\phi(v^2) = \log(x\sqrt{c}).$$

$$\phi(v^2) = x^2c$$

$$\boxed{\phi\left(\frac{y^2}{x^2}\right) = x^2c}$$

degree of DE

$$\left(\frac{dy}{dx}\right)^2 = \frac{3x}{4y}$$

$$\left(\frac{dy}{dx}\right) = \sqrt{\frac{3x}{4y}}$$

$$\boxed{\text{degree} = 1}$$

Solution of $x\frac{dy}{dx}+\cot y=0$.

at $y=\pi/4$; $x=\sqrt{2}$

$x.\frac{dy}{dx}+\cot y=0$.

$\cot y = -x.\frac{dy}{dx}$

$-\int\frac{dx}{x} = \int \tan y.dy$

$-\int\frac{dx}{x} = \int \tan y.dy$

$-\log x+\log c = \log \sec y$

$\log c = \log \sec y + \log x$

$\cancel{\log} c = x\sec y$

$\cancel{\log}\ c = \frac{x}{\cos y}$

$x = c\cos y$

$\sqrt{2} = c\left(\frac{1}{\sqrt{2}}\right)$

$\boxed{c=2}$

$\therefore \boxed{x=2\cos y}$

$\Rightarrow$ solution of $\frac{dy}{dx}-\frac{2y}{x+1}=(x+1)^3$.

$\Rightarrow e^{-2\int\frac{1}{x+1}dx} \Rightarrow e^{\log\frac{1}{(x+1)^2}} \Rightarrow \frac{1}{(x+1)^2}$

$\frac{y}{(x+1)^2} = \int\frac{(x+1)^3}{(x+1)^2}.dx$

$\frac{y}{(x+1)^2} = \cancel{\frac{x^2}{2}+x}\ \frac{(x+1)^2}{2}+c$

$\boxed{y = \frac{(x+1)^4}{2}+c(x+1)^2}$

$\Rightarrow (3e^{3x}y-2x)dx + e^{3x}dy=0$.

$e^{3x}dy = (2x-3e^{3x}y).dx$

$e^{3x}.\frac{dy}{dx} = 2x-3e^{3x}y$

$e^{3x}.\frac{dy}{dx}+3e^{3x}y = 2x$

$\frac{dy}{dx}+3y = 2xe^{-3x}$

$I.F = e^{3\int 1dx} = e^{3x}$

$ye^{3x} = 2\int x.e^{-3x}.dx.e^{3x}$

$ye^{3x} = 2\left(\frac{x^2}{2}\right)+c$

$\boxed{ye^{3x}=x^2+c}$