Heat and Thermodynamics

Heat

Fluid ← → Energy

$\boxed{\text{L. Calorie}}$ $\boxed{\text{J. P Joule}}$

units: calorie units: Joule

1 cal = 4.2J

Energy

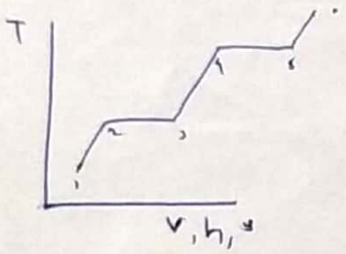
Transit ← → storage

Heat energy, Work energy

$U_p$ ← internal energy → $U_k$

NOTE : Differentional eq$^n$ of First law of T.D is $dQ = dU + dW$

Phase transformation



h = enthalpy
Total energy
s - enthalpy
degree of unstability or
ger. disorderness of gas m

$$ds = \frac{dQ}{T}$$

| Types of heat | Temp. | Phase |
|---|---|---|
| Sensible heat @ $mc(\Delta T)$ | changes | const. |
| lantent heat $mL$ | const. | changes |

Int. K.E → motion
→ molecular collison
→ Temp. ↑ (sensible heat)

~~Int.~~ Phase changes when int. P.E increas

NOTE : $U_P \in$ latent heat

→ Entropy of matter

$S_{gas} > S_{liq} > S_{solid}$

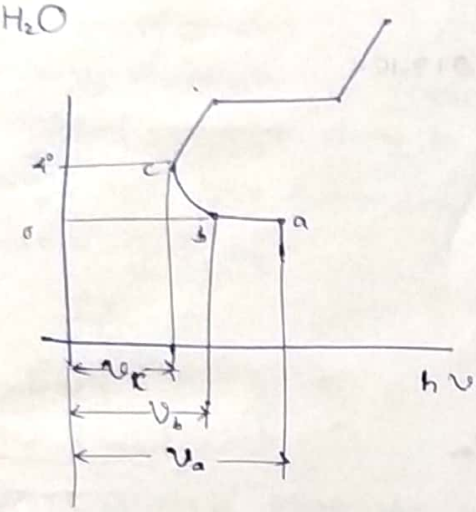
NOTE : $U \propto$ molecular collision

$U \propto T$ | $h \propto T.$

$dU = C_v dT$ | $dh = C_p dT$

$C_v = \frac{dU}{dT}$ | $C_p = \frac{dh}{dT}$

$H_2O$




→ $v_a > v_b > v_c$  $v = \frac{V}{m}$

$d_a < d_b < d_c$  $v = \frac{1}{d}$

$d = \frac{m}{v}$

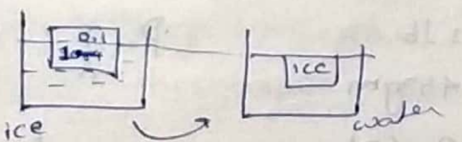
EI : When ice melts in water, then final level water is <u>same</u>

$d_{ice} = 900 kg/m3 = 0.9 gm/cm3 \Rightarrow 0.9 gm = 1cc \Rightarrow 1gm)_{ice} = \frac{1}{0.}$

$d_{water} = 1000 kg/m3 = 1 gm/cm3$ $= 1.$

↓

$1 gm = 1cc$



PHYSICS

Temperature: level of internal energy. K.E

$T \propto U$ * degree of hotness

" " " coldness

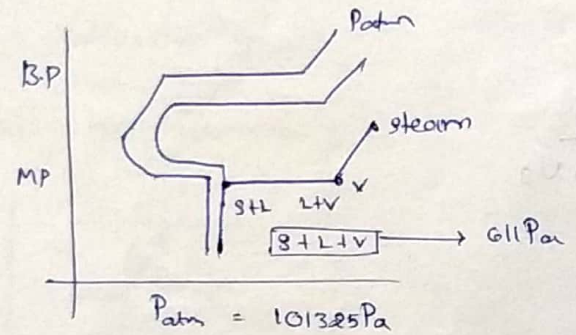
Units: kelvin

kelvin: $\frac{1}{273.16}^{th}$ part of Triple point of water

Where the steam + vapour + liquid are in equilibrium condition under 0.00611 bar and absolute 0°. This point is called triple point of water

$\boxed{S+L+V} \longrightarrow \boxed{\text{Triple point}} = 273.16k$

equlibrium




$P_{atm} = 101325 Pa$

$= 1 \times 10^5 Pa$

$\approx 1 bar$

$P_{atm}$ = 760mm of Hg

= 0.76m of Hg

= 10.34 m of $H_2O$

= 101325 N/m² or Pa

= $1.01325 \times 10^5$ Pa

= 1.01bar

= 14 psi $\left[\frac{\text{pound}}{\text{inch}^2}\right]$

Pressure ⇒ density × gra[vity] × height

= 13,600 × 9.81 × 0.7[6]

= 101325Pa

1 pound = 1 lb

= 453gm

1 pound = 0.453kg

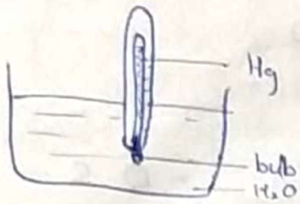
1kg = 2.2 pounds

$P = \frac{F}{A} = \frac{W}{A}$

Zeroth law of thermodynamics

~~When to what is~~

When two bodies are in thermal equilibrium with 3rd body Then they are in equilibrium with each other.




$$T_{Hg} = T_{bulb} = T_{H_2O}$$

Types of thermometers:

1. liquid thermometer
   - Working medium is mercury bcoz it has high surface tension (doesnot stick to glass). → working medium is alcohol

   length is the thermometric property

2. Const. vol. gas thermometer

   Ideal gas eqⁿ.

   $$\frac{P\cancel{V}}{T} = C$$

   $$P \propto T \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

   Charles II law

   Thermometric property is pressure

3. Thermoelectric thermometer [seeback effect]

   When two dissimilar metals at diff. temperatures are joined, a small potential differences generated at the point of contact. this effect is called seeback effect.

   App: • Rapid change in temp.
   • Small bodies, means (insects). ↓ temp.

• Magnetic thermometer.

   Magnetic susceptibility is inversely proportional to temperature

   $\psi \propto \frac{1}{T}$. The temperature at which a metal loses it magnetic property is called curie's temperature

   App: Very low temperature can be measurements.

   NOTES: High temperatures can be measured pyrometer, thermopile, polometer etc.

| | | | LFP | UFP | UFP − LFP |
|---|---|---|---|---|---|
| 1. | Centigrade | °C | 0 | 100 | 100 |
| 2. | Kelvin | K | 273 | 373 | 100 |
| 3. | Fah-renheit | °F | 32 | 212 | 180 |
| 4. | Rankine | °Ra | 492 | 672 | 180 |
| 5. | Reamer | °R | 0 | 80 | 80 |

Thermometric principle $\dfrac{T - LFP}{F.I\,[UFP - LFP]} = C$

$$\frac{°C - 0}{100 - 0} = \frac{K - 273}{373 - 273} = \frac{°F - 32}{212 - 32} = \frac{°Ra - 492}{672 - 492} = \frac{°R - 0}{80 - 0}$$

$\Rightarrow \quad °C = \frac{5}{9}(F - 32)$

$\Rightarrow \quad C_2 - C_1 = \frac{5}{9}(\Delta F)$

$\Delta C = \frac{5}{9}\Delta F$

$\Rightarrow \quad \Delta C = \Delta K$

$\frac{C}{100} = \frac{F - 32}{212 - 32}$

$\frac{C}{\cancel{100}_{5}} = \frac{F - 32}{\cancel{180}_{9}}$

$C = \frac{5}{9}(F - 32)$

$C_2 = \frac{5}{9}F_2 - \cancel{\frac{5}{9} \times 32}$

$C_1 = \frac{5}{9}F_1 - \cancel{\frac{5}{9} \times 32}$

$\Delta C = \frac{5}{9}\Delta F$

°C & °F are same at −40

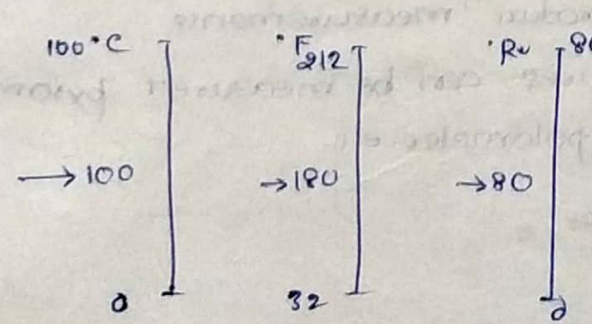
" " °R " " 0

" " K " " " 574.26

" " °Ra " " " −25.6

1° temp is max. in Reamer

min. in Fah-renheit



$1°F < 1°C < 1$

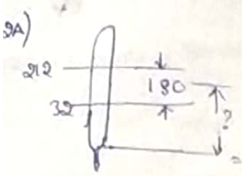
$1°R > 1°C > °$

~~1°F > 1°C > 1°R >~~

1A) $C = \frac{5}{9}(F-32)$

Ans:- $F = 320$

2A)




$\frac{9}{10} \times 180$

$= 162$
$+32$
$= 164$

A)

$20cm = 180$ dist. b/w fixed point

$5cm = ?$

$= 45$

from 0 point $45+32$

$= 77$

)

$$\boxed{R_t = R_0[1+\alpha T]}$$

$$4 = 2[1+0.0125\,t]$$

$t = 80$

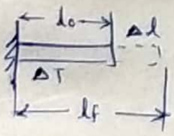
## Expansion of Solids

linear expansion ($\alpha$)

Areal (or) Superficial expansion ($\beta$)

Volume (or) cubical " ($\gamma$)

$$\text{Coefficient of expansion} = \frac{\text{change in dimension}}{\text{original dimension} \times \Delta T}$$




$$\alpha = \frac{l_f - l_o}{l_f \times \Delta T}$$

$$l_f = l_o[1+\alpha\Delta t]$$

$$A_f = A_o(1+\alpha\Delta t)$$

$$V_f = V_o(1+\alpha\Delta t)$$

→ Relation

$\alpha : \beta : \gamma = 1:2:3$

$\beta = 2\alpha$

$\gamma = 3\alpha$

Expansion of liquids

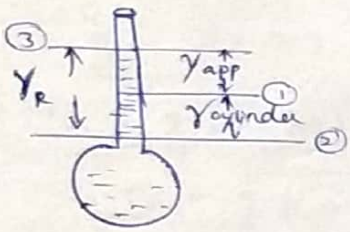
1. Real vol. expansion ⟶ depends on material of liq.
2. Apparent vol. " ⟶ " " " " " &
" " container

NOTE :- Metals — castiron, rubber etc

$\alpha$ [+ve] — $\alpha$ is [-ve]

$\gamma_{app} > \gamma_R$

$\Rightarrow \gamma_R = \gamma_{app} - \gamma_{cylinder}$




$\gamma_R = \gamma_{app} + \gamma_{cylinder}$

$$\boxed{\gamma_R = \gamma_{app} + 3\alpha}$$

NOTE : $\gamma_{cylinder} = \gamma_R$

$\gamma_{app}$ remains same

# Expansion of Gases

Volume of coefficent of expansion ($\alpha$) → Pressure const. → Regnau apparat

Pressure of coefficient of expansion ($\beta$) - Vol. const. → Jolly's b apparat

⇒ $V_f = V_0(1 + \alpha t)$

$V_1 = \cancel{V_0}(1 + \alpha t_1)$

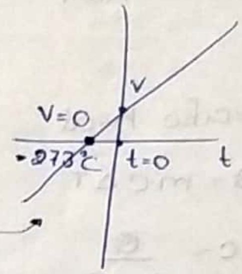
$V_2 = \cancel{V_0}(1 + \alpha t_2)$

$$\frac{V_1}{V_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$$

$V_1 + V_1 \alpha t_2 = V_2 + \alpha t_1$

$$\alpha = \frac{V_1 - V_2}{V_2 t_1 - V_1 t_2} \quad (\text{or}) \quad \alpha = \frac{V_2 - V_1}{V_1 t_2 - V_2 t_1}$$

$$\beta = \frac{P_1 - P_2}{P_2 t_1 - P_1 t_2}$$




$P = 760\text{mm of Hg}$
$V = 22.4\,l$
$T = 0°C = 273k$

$V_f = V_0(1 + \alpha t)$

$0 = \cancel{V_0}(1 + \alpha t)$

$1 + \alpha t = 0$

$$\alpha = \frac{-1}{t}$$

$$\alpha = \frac{-1}{-273.16} = \frac{1}{273.16} = \beta$$

Vanderwaall's eqn:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

a = molecular coef. attraction
b = vol. occupied by mol.

a = 0
b = 0

→ $PV = RT$ [Ideal gas eqn]

K.E theory assumptions

molecules are perfectly elastic rigid molecules

Ideal gas
- i) a = 0 [molecular attraction is zero]
- ii) b = 0 [vol. occupied is negligeble w.r.t container]

$PV = nR_uT$

$PV = \cancel{n} R_u T$

$PV = \frac{m}{M} R_u T$

* no. of mol = $\frac{\text{mass}}{\text{mol. wt}}$

$PV = mRT$

$n = \frac{\cancel{\text{mol}}\text{ no. of mol.}}{\text{Avagodro's no.}} \rightarrow \cancel{8.314 \text{ mol}}$

$\hookrightarrow 6.0232 \times 10^{23}$

$P = dRT$

$Pv = RT$

$R_u$ = universal gas const.

= 8.314 J/mole k

$R_u = MR$

R = characteristic gas eqn

$R_{air} = \frac{R_u}{M_{air}} = \frac{8.314}{28} \simeq 0.287 \text{ kJ/kgk}$

Specific heat [$C_p$ or $C_v$]

$Q = mC\Delta T$

$C = \frac{Q}{C. \Delta T}$

It is the amount of heat required to raise temperature through 1°c of unit mass.

$C_p > C_v$

$C_p - C_v = R$

$MC_p - MC_v = MR$ [~~Molecular~~ sp. heat]

Joule's law

$U = F(\text{Temp}$

$\uparrow\downarrow U = \uparrow\downarrow T$

$dU \propto dT$

$dU = C_v dT$

$C_v = \frac{dU}{dT}$

$\uparrow\downarrow H \propto \uparrow\downarrow T$

$d\phi \propto dH$

$C = \frac{dH}{dT}$

$\frac{C_p}{C_v} = \gamma$ — (adiabatic index)

$C_p = \frac{\gamma R}{\gamma - 1}$, $C_v = \frac{R}{\gamma - 1}$

Translation of K.E of molecule.

$V_{rms} = \sqrt{\frac{3R_uT}{M}}$

$K.E = \frac{1}{2} m \times V_{rms}^2$

$= \frac{1}{2} \times m \times \frac{3R_uT}{M}$

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$$\boxed{K.E = \frac{3}{2} n R_u T}$$

$E = \frac{3}{2} n R_u T$

$= \frac{3}{2} \times \frac{\text{no. of molecules}}{N} \times R_u \times T$

$= \frac{3}{2} \times \frac{R_u T}{N}$ — R

$$\boxed{E = \frac{3}{2} KT}$$

Boltzman const.

$K = \frac{\text{Universal Gas const.}}{\text{Avogador's no}}$

Interms of k

density of gas: ________ when mass of mol. = m

$d = \frac{\text{mass}}{\text{vol}} = \frac{m \times \text{no. of molecule}}{V}$

$= \frac{P}{RT} \times \text{no. of mol.}$ $\Rightarrow \frac{m}{V} = \frac{P}{RT}$

$= \frac{P}{RT} \times \frac{mN}{M}$ $\Rightarrow \frac{m}{M} = \frac{\text{no. of mol.}}{N}$

$= \frac{PmN}{R_u T}$

$= \frac{mP}{\frac{R_u}{N} T}$

$d = \frac{mP}{KT}$

Boyle's law: $\frac{PV}{\cancel{T}} = C \rightarrow P \propto \frac{1}{V} \rightarrow PV = C \rightarrow P^1V^1 = P^2V^2 \rightarrow [xy = c]$ $[T = c]$

Isothermal

Charle's I law: $\frac{\cancel{P}V}{T} = C \rightarrow V \propto T \rightarrow \frac{V}{T} = C \rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2} \rightarrow [P = C]$

Isobaric

Charles II law: $\frac{P\cancel{V}}{T} = C \rightarrow P \propto T \rightarrow \frac{P}{T} = C \rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow [V = C]$

Isochoric → Gay laussac's law

Very slow process

PHYSICS

Reversible Adiabatic process

Reversible ↓ Frictionless

Adiabatic ↓ no heat transfer

very fast process

$dQ = 0$

$$\boxed{\begin{matrix} PV^{\gamma} = C \\ TV^{\gamma-1} = C \end{matrix}} \longleftarrow \left(\frac{V_1}{V_2}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

---

10) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ $\boxed{P=C}$

11) $T_1 = 0°C$

$T_2 = 100°C$

↓10cm

$$\frac{l \times \cancel{\pi r^2}}{l \times \cancel{\pi r^2}} = \frac{T_1}{T_2}$$

760mm of Hg

~~101325 Pa~~

$T_1 = 300k$

$T_2 \longleftarrow P_2 = 2P_1$

$$\frac{T_1}{T_2} = \frac{\cancel{P_1}}{2\cancel{P_1}}$$

$$\frac{T_1}{T_2} = \frac{300k}{}$$

$T_2 = 2T_1$

$= 600k$

$-273$

$327$ k

Specific heat CP > CV

$Q = mC\Delta t$

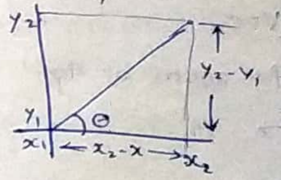
$C = \frac{Q}{m\Delta t}$

$C = Q$ if $m = \Delta t = 1$

$CP - CV = R$ [specific heat]

$MCP - MCV = RU$ [

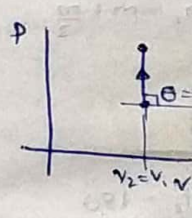
Slopes



$\tan\theta = \frac{Y_2 - Y_1}{X_2 - X_1} =$

Const. Vol.

$\left.\frac{dP}{dV}\right]_{V=C} = \infty$



Adiabatic [

$PV^{\gamma} = C$

$\frac{dP}{dV} = -\gamma$

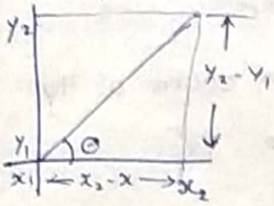
(ii) $\frac{dV}{V} \times 100$

$\frac{dP}{P} \times 100$

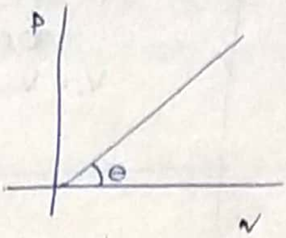
$\frac{dP}{dV} =$

$100 \times \left[\frac{dP}{P}\right] = -$

Slopes



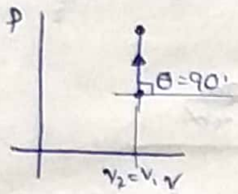

$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$




$\tan\theta = \frac{dy}{dx} = \frac{dP}{dV}$

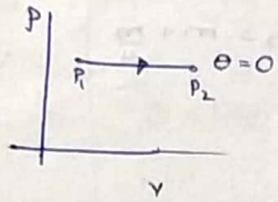
Const. Vol.

$\left.\frac{dP}{dV}\right]_{V=C} = \infty$




Const. Pressure

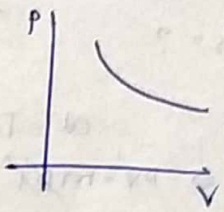
$\left.\frac{dP}{dV}\right]_{P=C} = \infty$




Const. temp.

$PV = C$, $xy = c$

Rect. hyperbola




$d(PV) = d(c)$

$P\,dV + V\,dP = 0$

$\frac{dP}{dV} = \frac{-P}{V}$

Adiabatic [Reversible]

$PV^\gamma = C$

$\frac{dP}{dV} = \frac{-\gamma P}{V}$

ECET

$\frac{\text{Slope of adiabatic}}{\text{" " Isothermal}} =$

$\frac{-\gamma\, \cancel{P/V}}{\cancel{-P/V}} = \gamma$

1) $\frac{dV}{V} \times 100 = -2.5$

$\frac{dP}{P} \times 100 = ?$

Diatomic $= \gamma = \frac{7}{5}$

$\frac{dP}{dV} = \frac{-\gamma P}{V}$

$100 \times \left[\frac{dP}{P}\right] = -\gamma \left[\frac{dV}{dV}\right] \times 100$

$= -\frac{7}{5}\left(-\frac{5}{2}\right)$

$= 3.5$

(3)

$P \propto T^c$

$P^{\frac{1}{\gamma}} \propto T^{\frac{1}{\gamma-1}}$

$P \propto T^{\frac{\gamma}{\gamma-1}}$

$c = \frac{\gamma}{\gamma-1} \quad \left[\gamma = \frac{5}{3}\right]$

$c = \frac{5/3}{5/3 - 1}$

$c = 2.5$

(42)

$T = 27^\circ C$
$= 300 K$
$V_1 = V$

$V = C$

$P = 30cm$ of Hg

$\rightarrow 1/4$

$101325 Pa = 30cm$ of Hg

$P = 760mm$ of Hg

$P = \frac{76\cancel{0}}{30\cancel{0}} \times 101325$

(36) $V = C$

$P_1 = 30cm$ of Hg $\quad m_1 = m$

$P_2 = ?$

$m_2 = m + \frac{m}{4}$

at $T = C$

$\Rightarrow P\cancel{V} = m\cancel{RT}$

$\frac{30}{P_2} = \frac{4\cancel{m}}{5\cancel{m}}$

$P_2 = \frac{150}{4}$

$= 37.5$

(37) $P_1 = 90 \quad m_1 = m$

$P_2 = ? \quad m_2 = m + \frac{m}{3}$

$\frac{90}{P_2} = \frac{\cancel{3m}}{4\cancel{m}}$

$\frac{\cancel{90}^{45} \times \cancel{3}}{\cancel{2}} = P_2 = 180$

$\frac{90}{P_2} = \frac{m}{m - \frac{m}{3}}$

$\frac{90}{P_2} = \frac{3}{2} \Rightarrow P_2 = 60$

(38) $V = C$

$P_1 = 12 atm \quad P_2 = ?$

$T_1 = 27C \quad T_2 = 127^\circ C$

$m_1 = m \quad m_2 = m/2$

$P\cancel{V} = m\cancel{R}T$

$\frac{P}{mT} = c$

$\frac{12}{m \times 27} = \frac{2P_2}{\frac{m}{2} \times 127}$

$\frac{12}{\cancel{m} \times 27} = \frac{P_2}{\cancel{m}/2 \times 127}$

$P_2 = \frac{\cancel{12}^{26} \times 127 \times \cancel{m}}{\cancel{2} \times \cancel{m} \times \cancel{27}_{9}}$

$\frac{{}^4\cancel{12} \times 127 \times 2}{\cancel{27}_{\cancel{9}\,3}} = P_2$

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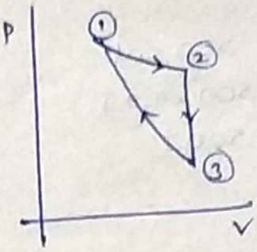
# Thermodynamics

Thermodynamics is a science which deals with conversion of heat energy to work energy or viceversa in a system

$\oint dQ \propto \oint dW$

Heat supply = Work done




$Q_{1-2} + Q_{2-3} + Q_{3-1} = W_{1-2} + W_{2-3} + W_{3-1}$

**Work**

| +ve | -ve |
| --- | --- |
| Expansion | compression |
| generation | consumption |
| by the sys. | on the sys. |

**Heat**

| +ve | -ve |
| --- | --- |
| 1. supplied | rejected |
| 2. gained | loss |

→ D.E of I law of T.D is $\boxed{dQ = dU + dW}$

$Q \propto W$

$W \propto Q$

$W \propto$ Heat [H]

$W = JH$

$W = \frac{\text{Joule}}{\text{cal}} \times \text{cal}$

$W = J$

$1\ J = 4.2\ \text{cal}$

**Application:** A bullet of mass 'm' moving with velocity 'v' is made to stop by metal plate, then change in temp. of bullet is ______

$W = JH$

$\frac{1}{2} \cancel{m} v^2 = J \cancel{m} s \theta$

$\theta = \frac{v^2}{2Js}$

Sensible heat = temp.cha → $ms\theta$

latent " = phase cha = $mL$

2. Rise in temp. of water, mass 'm' if P.E of water fall is converted to heat energy

$W = JH$

$P.E = JH$

$\cancel{m} gh = J \times \cancel{m} s \theta$

$\Rightarrow \theta = \frac{gh}{Js}$

~~m = 4.2~~

$W = JH$

$\frac{1}{2}mv^2 = JH$

$\frac{1}{2} \times \cancel{4.2} \times 10^3 \times 10^3 = 4.2 \times H$

$H = 500$ cal

$H = 500 \times 4.2$ J

$= 500$ J

(2)

$\frac{1}{2} \times m \times v^2 = J m s \theta$

$\frac{1}{2} \times \cancel{m} \times 210 \times 210 = J \times \cancel{m} \times \theta$ $\quad S = 0.03$

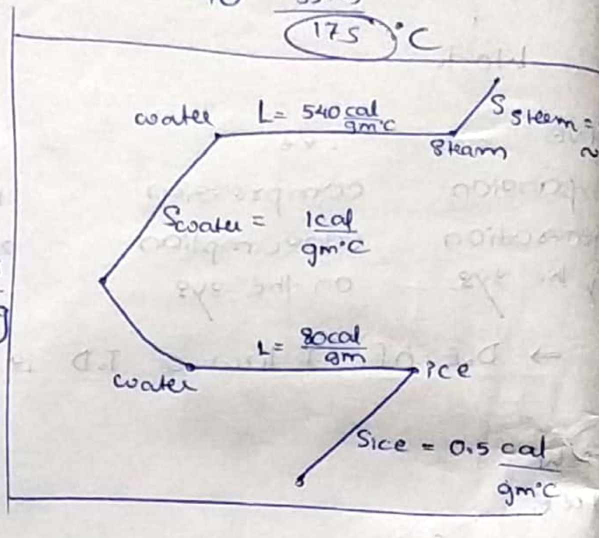
$0.03 \times 10^3 \frac{cal}{kg^\circ C}$

$\theta = \frac{210 \times 210 \times}{0.03 \times 10^3 \times 2 \times 4.2}$

$= \frac{210 \times 210 \times 100}{0.03 \times 1000 \times 2 \times 4.2}$

$= \cancel{70} \quad 35 \times 5$

175 °C




(3)

$P = \frac{W}{t} = \frac{J \times H}{t}$

$= \frac{J \times m \times L}{t}$

$= \frac{1 \times 100 \text{ gm}}{60} \times \frac{336 \times 10^3 \text{ J}}{10^3 \times g}$

$P = J/s = 560$ watts

(4) $m = 42 \times 10^3$ g

$H = 980$ cal.

$W = mgh = 4.2 \times 980$

$42 \times 10^3 \times 9.8 \times h = 4.2 \times 980$

$h = 10$

(5)

$\cancel{mgh} = JH$

$mgh = 1 \times ms\theta$

$9.8 \times 840 = \frac{4200 \text{ J}}{10^3 \times k}$

$9.8 \times 840 = \frac{4200}{10^3}\theta$

$9.8 \times 20$

98

196 . 19.6

1.96

(7)

~~$9.8 \times 8.4 = 4.2 \times 0.1 \times 10^3 ; 4.2$~~

$mg(h_1 - h_2) = JH$

$\cancel{m}g(h_1 - h_2) = 4.2 \times \cancel{m} \times s \times \theta$

$\theta = \frac{9.8(8.4 - 4.2)}{0.1 \times 10^3 \times 4.2}$

$\theta = 0.098$ °C

# PHYSICS

VECTORS:- According to magnitude & direction, the physical quantities are divided into three types.

1. Vectors
2. Scalars
3. Tensors

→ Vectors: The physical quantity which have both magnitude and direction.

Ex:- displacement, velocity ~~and~~, acceleration, weight, torque, momentum impulse, intensity of gravitation field, electric field, magnetic field, magnetic moment, magnetic induction, current density and amplitude.

→ Characters:

1. Vectors follow the ~~main~~ laws of vector addition
2. Vectors can be resolved into components
3. The magnitude of vector changes according to the direction

→ Scalar: The physical quantity which have only magnitude without direction (uni-directional) called as scalars

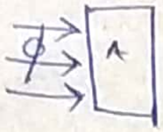
Ex: mass, length, time, distance, speed, volume, density, work, power, energy, heat temperature, velocity of sound, velocity of light **Pressure**, surface tension, viscosity, magnetic pole strength, magnetic flux, electric current, potential resistance, induction, capacitance,

→ Tensors:

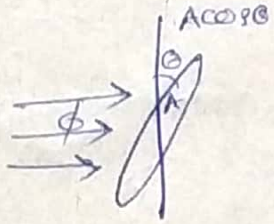
The physical quantity which are neither vector nor scalar called tension.

Ex: Moment of inertia, stress & strain.

Surface area is both scalar and vector




$\phi = BA$



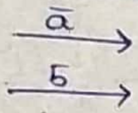

$\phi = BA\cos\theta$

→ Unit area is vector
→ large area is scalar

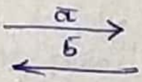
→ vectors: The line segments which represents the vector phy. quantity both in mag & direction is known as vectors.

Types of vectors

1. like vectors

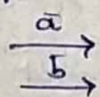



2. Unlike vectors

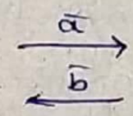



3. Collinear vectors


4. ~~Unlike~~ Eq. vectors

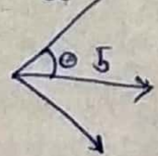



5. Opposite vectors

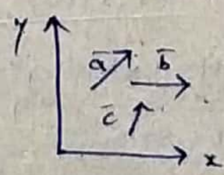



$\bar{a} = -\bar{b}$

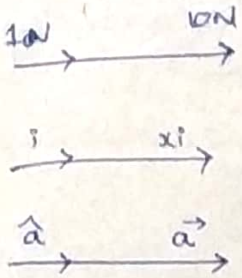
6. Coinitial vectors




7. Co-planar vectors



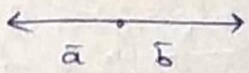
8) Unit vector




unit vector = $\frac{\text{vector}}{\text{magnitude}}$

$$\hat{a} = \frac{\vec{a}}{a} = \frac{\vec{a}}{|\vec{a}|}$$

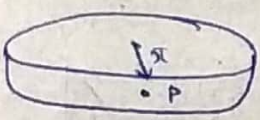
9) Null vector



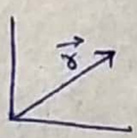

$|\bar{a}| = |\bar{b}|$

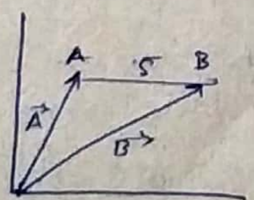
$|\bar{R}| = 0$

10) Position vector




$$\vec{r} = xi + yj + zk$$

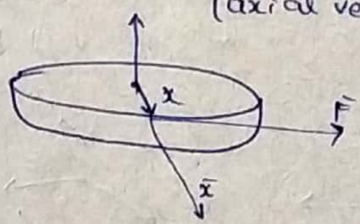



$$\vec{s} = \vec{B} - \vec{A}$$

11) Pseudo vector

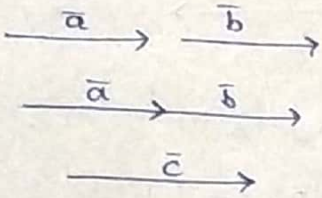
(axial vectors)




Torque, $\bar{\tau} = \bar{r} \times \bar{F}$

Addition of vectors:

1. For collinear vectors, the vectors & algebraic addition is same

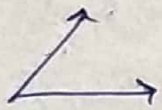


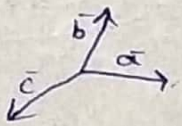

$\bar{c} = \bar{a} + \bar{b}$

Triangle law:- When a two sides of $\Delta^{le}$ represents the two vectors then the third side " " result-ant vector in magnitude & ~~dire~~ but opposite in direction

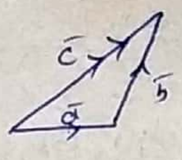
(or)

When three sides of a $\Delta^{le}$ represents three vectors in regular order then the sys. will be in eq$^m$ states.

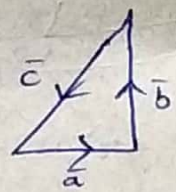





eq$^m$ state




$\bar{c} = \bar{a} + \bar{b}$

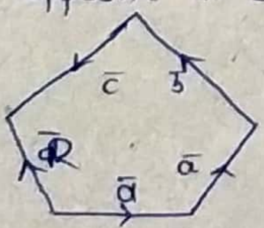



eq$^m$ state

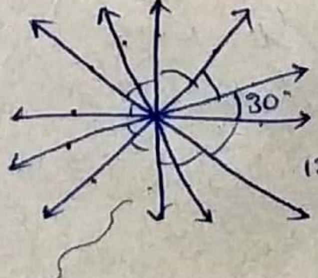
$\frac{\bar{a}}{a} = \frac{\bar{b}}{b} = \frac{\bar{c}}{c}$

Polygon law:

When all sides of a polygon represents the vectors, then the closing end represents the resultant vector in magnitude but opposite in direction.




$\bar{R} = \bar{d} + \bar{a} + \bar{b} + \bar{c}$



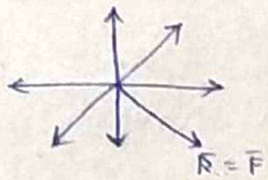

$12 \times 30° = 360°$

$\bar{R} = 0$

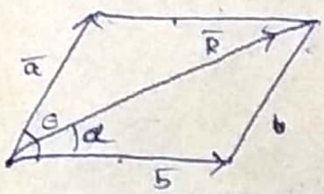
→ When twelve forces on F each acts on a particle at an angle of 30° b/w two successive forces, then the resultant is zero

→ ~~When 11 force~~

→ When ∥ forces ~~acting~~ on a particle, then the Resultant force is $R = F$




∥gm law :- When two sides of a ∥gm represent two vectors then the diagonal represents the Resultant vector in mag. & direct.




$\bar{R} = \sqrt{\bar{a}^2 + \bar{b}^2 + \bar{a}\bar{b}\cos\theta}$

$\theta = \tan^{-1}\left(\dfrac{\bar{a}\sin\theta}{\bar{a} + \bar{b}\cos\theta}\right)$

If $\bar{a} = \bar{b} = \bar{P}$

$\bar{R} = 2\bar{a}\cos\theta/2$

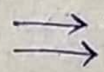
~~$\theta = \tan^{-1}\left(\dfrac{[illegible]}{2\cos\theta/2}\right)$~~

SPECIAL CASES

Magnitude | Direction

1. $\theta = 0°$

$\bar{R} = \bar{a} + \bar{b}$



$\alpha = 0$

2. $\theta = 90°$

$\bar{R} = \sqrt{a^2 + b^2}$

$\alpha = 45°$

3. $\theta = 180°$

$\bar{R} = \bar{a} - \bar{b}$

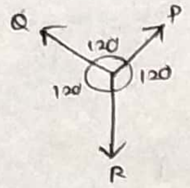
$\alpha = 0$

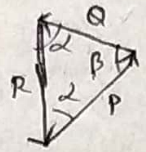
→ The min. no. of eq mag vectors required for eq$^m$ is
→ " " " " ueq. & coplanar vectors " " " " 3

$\boxed{|\bar{c}| \leq |\bar{a}| + |\bar{b}|}$

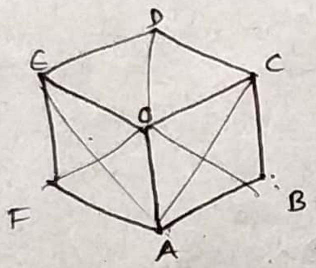
→ " " " " non coplanar vectors " " " " 4

When 3 eq. magnitude forces are in eq$^m$ then the angle b/w other two forces is 120°




When 3 coplanar forces is eq$^m$ they follows lami's theorem




$\overline{AB} + \overline{AC} + AD + AE + AF$

$= 9 \times \overline{AO}$

$\vec{AD} = 2\vec{AO}$

$\vec{AB} + \vec{AF} = \vec{AO}$

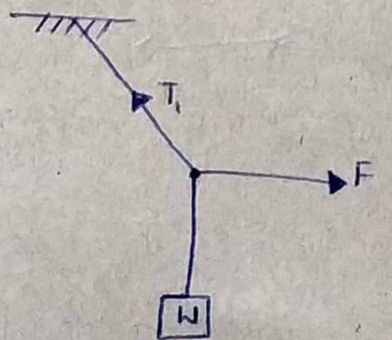
$\vec{AF} + \vec{AO} = \vec{AE}$

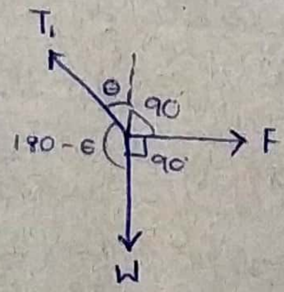
$\vec{AB} + \vec{AO} = \vec{AC}$

$\Rightarrow \vec{AO} + \vec{AO} + \vec{AO} + 2\vec{AO} + \vec{AO}$

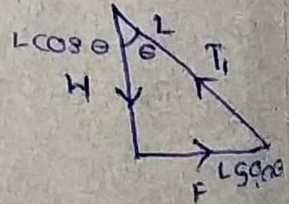
$= 6\vec{AO}$

→ When a pendulum is pulled a side, then it follows $\Delta^{le}$ law as well as lami's theorem




Lamis theorem




$\Delta^{le}$ law

According to $\Delta^{le}$ law

$$\frac{W}{L\cos\theta} = \frac{T_1}{L} = \frac{F}{\sin\theta}$$

The unit forces are eq. according to $\Delta^{le}$ law

According to lami's theorem

$$\frac{T_1}{\sin 90^\circ} = \frac{F}{\sin(180^\circ-\theta)} = \frac{W}{\sin(90+\theta)}$$

$$\frac{T_1}{1} = \frac{F}{\sin\theta} = \frac{W}{\cos\theta}$$

$$\frac{T_1}{\sin\alpha} = \frac{F}{\sin\beta} = \frac{W}{\sin\gamma}$$

→ Laws of vector addition

1 Commutative law.

$$\bar{a}+\bar{b} = \bar{b}+\bar{a}$$

2 Associative law

[Vectors can't be divided]

$$(\bar{a}+\bar{b})+\bar{c} = \bar{a}+(\bar{b}+\bar{c})$$

3. Distributive law

$$m(\bar{a}+\bar{b}) = m\bar{a}+m\bar{b}$$

| law | add | sub | dot. Pr | cross Pr |
|---|---|---|---|---|
| ~~force~~ Commutative | ✓ | × | ✓ | × |
| Associative | ✓ | × | × | × |
| Distributive | ✓ | ✓ | ✓ | ✓ |

The max & min resultant forces ~~and are~~ ratio is 7:4

The Force ratio are

$$F_{max} : F_{min} = 7:4$$

$$F_{max} = F_1 + F_2 = 7$$

$$F_{min} = F_1 - F_2 = 4$$

$$= 2F_1 = 11$$

$$F_1 = 11/2 \;;\; F_2 = 3/2$$

$F_1 : F_2 = 11 : 3$

$F_1 = \frac{F_{max} + F_{min}}{2}$

$F_2 = \frac{F_{max} - F_{min}}{2}$

Q) Two forces of

$F_1 = F_2 = 10\,N$

$R = 2\cos \alpha/2 \times F$

$= \cancel{10}\sqrt{3}$

$=$

Q) The R of two eq forces is $\sqrt{2} \times F$, the angle b/w forces is ______

$\cancel{\alpha = \tan^{-1}\left(\frac{F\sin\theta}{\ }\right)}$

$R = 2P\cos\alpha/2$

$\sqrt{2} \times \cancel{P} = 2 \times \cancel{P} \cos(\alpha/2)$

$\frac{\sqrt{2}}{2} = \cos(\alpha/2)$

$\alpha/2 = \cos^{-1}\frac{\sqrt{2}}{2}$

$\alpha/2 = \cos^{-1}(1/\sqrt{2})$

$= 45$

$\alpha = 90°$

Q) The Ratio of two forces 3:5, The R is 35N then they are at 60°, those F ______

$3:5 \Rightarrow 3x : 5x$

$35 = \sqrt{9x^2 + 25x^2 + 2 \cdot 15x^2 \cos 60°}$

$35^2 = 9x^2 + 25x^2 + \frac{15x^2}{2}$

$35 = \sqrt{49x^2}$ 35

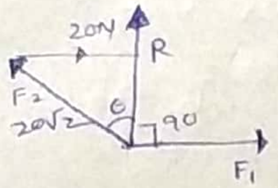
$x = 35/49 \Rightarrow$

$35 = 7x$

$x = 5$

$F_1 = 15N$ ; $F_2 = 25N$

Q) Two forces of 20N & $20\sqrt{2}$ N have resultant $\perp^{le}$ to smaller force, the angle b/w force is ______



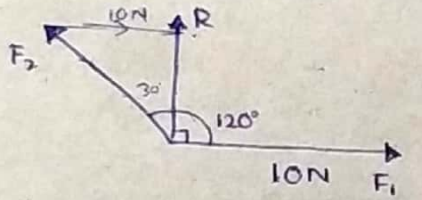

$[\alpha = 90° + \theta]$

$\sin\theta = \frac{\cancel{20}}{\cancel{20}\sqrt{2}}$

$\theta = 45$

$\alpha = 135°$

Q) Two forces of 10N & greater force are at 120°, the resultant is $\perp^{le}$ to smaller "




· gH
· A/H
O/A

$\sin 30° = \frac{10}{F_2}$

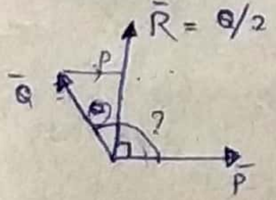
~~$F_2 = \frac{5N}{1/2}$~~ ×

$F_2 = \frac{10}{1/2} \times 2$

$=$

$F_2 = 20N$

Q) Two vectors $\bar{P}$ & $\bar{Q}$




~~$\cos\theta = Q/2$~~

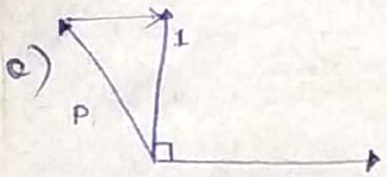
$\cos\theta = \frac{\cancel{Q}/2}{\cancel{Q}}$

$\theta = 60°$

$\alpha = 150°$

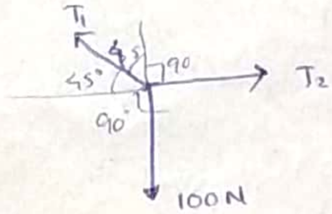
Two forces P .

a)




Sol: $P = \sqrt{1^2 + 1^2}$

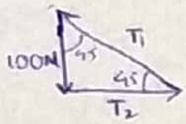
$= \sqrt{2}$




$\frac{T_1 =}{\sin 90°} = \frac{100N}{\sin 135°} = \frac{T_2}{\sin(135°)}$

$T_1 = \frac{100}{1/\sqrt{2}}$ $\quad T_2 = 100N$

$T_1 = 100\sqrt{2}$




$\sin(90+45)$

$+\cos 45$

$1/\sqrt{2}$

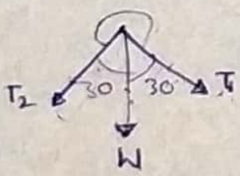
$\frac{T_1}{\sin 45} = \frac{100}{\sin 90°}$

$T_1 = 100\sqrt{2}$ $\quad \cancel{T_2 = 100 \times \sin 45}$

$T_2 = 100$

$\frac{T_1}{\sin 45} = \frac{T_2}{\sin 45}$

a) A photo frame of m-3kg is hanged with a wire passing over a nail, the ∠ b/w two segment is 60°. Tension in wire




$\frac{T_1}{\sin 30} = \frac{T_2}{\sin 30} \quad \frac{W}{\sin 120}$

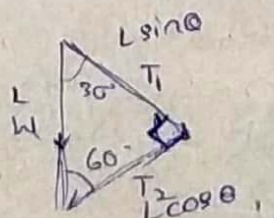
~~180 - 60~~

$\frac{T_1}{\sin 30°} = \frac{W}{\sin 120°}$

$T_1 = 3 \times \frac{1/\cancel{2}}{\sqrt{3}/\cancel{2}}$

$T_1 = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$T_1 = \sqrt{3} = T_2$




$\cancel{\frac{T_1}{30} = \frac{W}{}}$

$\cancel{\sin 30} = \frac{T_1}{W}$

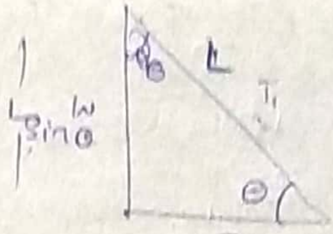
$\frac{T_1}{L \sin 30} = \frac{W}{L} =$

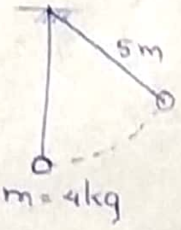
$\Sigma H =$
$\Sigma V$

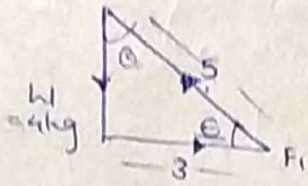
$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$\frac{T_1}{L} = \frac{W}{L\sin\theta} = \frac{T_2}{L\cos\theta}$



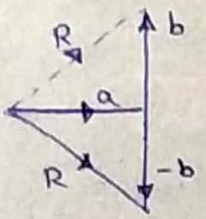




$\cos\theta = \frac{3}{5}$

F

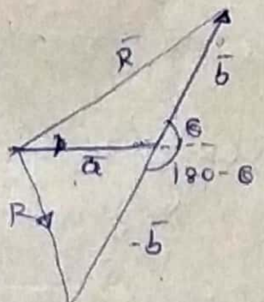
→ Subtraction of Vectors

If $\bar{a} + \bar{b}$ magnitude = $\bar{a} - \bar{b}$ magnitude, then $\alpha = 90^\circ$




$\bar{R} = \bar{a} + (-\bar{b})$
$= \bar{a} + \bar{b}$

Subtraction of vector is the addition of -ve vector




$R = \sqrt{a^2 + b^2 + 2ab\cos(180 - \theta)}$

$\bar{R} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$\bar{R} = |\bar{a} - \bar{b}|$
$= \sqrt{\phantom{x}}$

$a = 3i + 3j$
$b = i + j$

$|\bar{a} - \bar{b}| =$
$\bar{a} - \bar{b} = 2i + 2j$
$|\bar{a} - \bar{b}| = \sqrt{2^2 + 2^2}$
$= 2\sqrt{2}$

If $a = b = P$

$R = \sqrt{2P^2 - 2P^2\cos\theta}$

$\sqrt{2}P\sqrt{(1-\cos\theta)}$

$\sqrt{2}P\sqrt{2\sin^2\theta/2}$

$\Rightarrow R = 2P\sin\theta/2$

$\alpha = \tan^{-1}\left(\frac{b\sin(180-\theta)}{a+b\cos(180-\theta)}\right)$

$= \tan^{-1}\left[\frac{b\sin\theta}{(a-b\cos\theta)}\right]$

$= \tan^{-1}\left[\frac{b\sin\theta}{a-b\cos\theta}\right]$

$\Rightarrow |\bar{a}+\bar{b}| = |\bar{a}-\bar{b}|$

$+2ab\cos\theta = -2ab\cos\theta$

$4ab\cos\theta = 0$

$\theta = 90°$

---

$\bar{P} = \bar{Q} = \bar{R} = \bar{P}$, then $\theta$

$\downarrow$

$\bar{a}+\bar{b}$

$R = 2P\cos\theta/2$

$P = 2P\cos\theta/2$

$\theta = 120°$

$\bar{P} = \bar{Q} = \bar{R} = \bar{P}$ then $\theta$

$\downarrow$

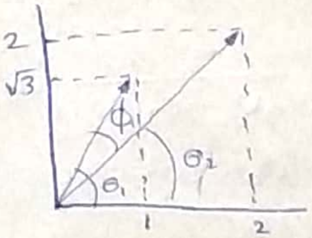
$|\bar{a}-\bar{b}|$

$R = 2P\sin\theta/2$

$P = 2P\sin\theta/2$

$\theta = 60°$

NOTE : change in velocity eq to subtraction of vectors.




$\phi = \theta_1 - \theta_2$

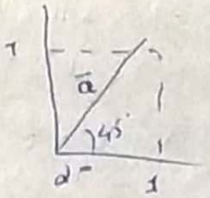
$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) - \tan^{-1}\left(\frac{2}{1}\right)$

$= 60 - 45$

$= 15°$

→ $\bar{a} = i + j$

$x = 1$ , $y = 1$




(or)

$\cos \alpha = \frac{x}{\bar{a}} = \frac{1}{\sqrt{1^2+1^2}}$

$= \frac{1}{\sqrt{2}}$

$\alpha = \cos^{-1}(1/\sqrt{2})$

$= 45°$

→ $\vec{a} = 5i + 4j + 3k$

$\cos \alpha = \frac{5}{\bar{a}}$

$= \frac{5}{\sqrt{5^2+4^2+3^2}}$

$= 5/5\sqrt{2}$

$= 1/\sqrt{2}$

$\alpha = 45°$

→ The direction cosine of vector are $\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$

$\cos \alpha = \frac{3}{5\sqrt{2}}$ ; $\cos \beta = \frac{4}{5\sqrt{2}}$ ; $\cos \gamma = \frac{1}{\sqrt{2}} \times \frac{5}{5}$

$\bar{a} = 3i + 4j + 5k$

→ The directional cosine of vector are $\frac{1}{\sqrt{2}}$ & $\frac{1}{\sqrt{3}}$ with other

" " is ______

$\cos \alpha = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{6}}$ $x = \sqrt{3}$

$\cos \beta = \frac{1}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{6}}$ $y = \sqrt{2}$

$\cos \gamma = \frac{1}{\sqrt{6}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{1}{2} + \frac{1}{3} + x = 1$

$x = 1 - \frac{5}{6}$

$\cos^2\gamma = \frac{1}{6}$

$\cos \gamma = \frac{\sqrt{1}}{\sqrt{6}} = \frac{1}{\sqrt{6}}$

→ The resultant of two vectors $\bar{a}$ & $b$ is $c$ ...

reversed $\bar{a} - \bar{b} = \bar{d}$

$\bar{a} + \bar{b} = \bar{c} = \sqrt{a^2 + b^2 + 2ab}$

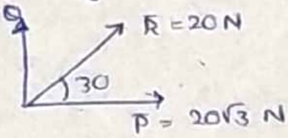
$c^2 = a^2 + b^2 + 2ab$

$\bar{d} = \sqrt{a^2 + b^2 + 2ab}$

$d^2 = a^2 + b^2 - 2ab$

$c^2 + d^2 = (a^2 + b^2) \times 2$

→ The Resultant of two forces is 20N which makes the angle 30° with the first force of $20\sqrt{3}$ N, then $F_2 = ?$




~~cos 30~~

~~$20\sqrt{3}$~~

$\cancel{R = P^2 + Q^2 + 2PQ\cos 30\sqrt{3}}$

$\cancel{= P^2 + Q^2 + PQ}$

$\tan 30 = \frac{F_2}{F_1}$

$\frac{1}{\sqrt{3}} = \frac{F_2}{20\sqrt{3}}$

$F_2 = 20$

$\bar{R} = \bar{F_1} + \bar{F_2}$

$F_2 = \bar{R} - \bar{F_1}$

$F_2 = \sqrt{20^2 + 20^2 \times 3 - 20 \times 20\sqrt{3} \times \sqrt{3}}$

$= \sqrt{20^2 + \cancel{20^2(3)} - \cancel{20^2(3)}}$

$= 20N$

$\tan 30° = \frac{F_2}{20\sqrt{3}}$

$F_2 = \frac{1}{\sqrt{3}} \times 20\sqrt{3}$

$F_2 = 20N$

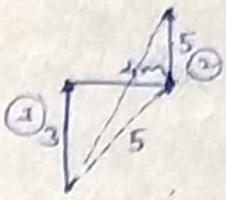
wrong bcoz $F_1$ is not ⊥ $F_2$

→ A Rectangular component of a force is 10N which makes angle of 60°, the other component is ______

$\tan 60° = \frac{F_y}{F_x} = \frac{F_y}{10}$

$F_y = 10\sqrt{3}$

→ An insect moves 3m north & 4m east and 5m vertically upwards



In ① st $\Delta^{le}$

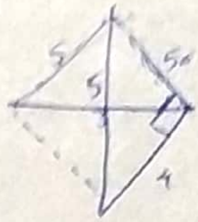
$R^2 = 3^2 + 4^2$

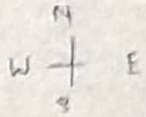
$R = 5$

In ② nd $\Delta^{le}$

$x = \sqrt{R^2 + 5^2}$

$= 5\sqrt{2}$

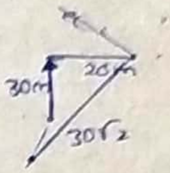





$\sqrt{9+16+25}$

$= 5\sqrt{2}$

→ A person moves 30m north, 20m east, $30\sqrt{2}$ m south-west

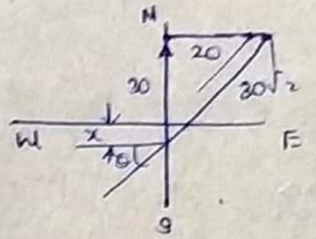


$30^2 + 20^2 + x^2 + 40x = 30^2(2)$

$\cancel{x^2 + 40x = 30^2(2-1) - 20^2}$

$\cancel{= 30^2 - 20^2}$

$\cancel{x^2 + 40x = 500}$




900
30×30
900
400

$(20 + x)^2 = 900$

$20 + x = 30$

$x = 10$

40×1
500/40

4)50(1265
48
20

$s_1 = 30i$

$s_2 = 20j$

$s_3 = -30\sqrt{2} \cos 45\, i$

$= -30\sqrt{2} \sin 45\, j$

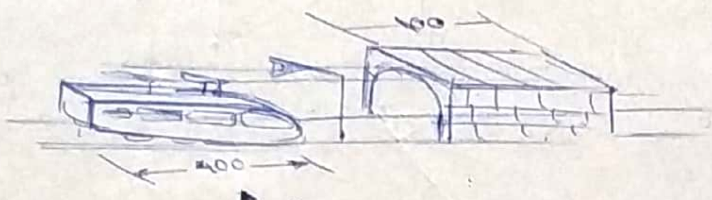
$\bar{s} = 30i - 30i + 20j - 30j$

$\bar{s} = -10j$

$\bar{s} = 10j$.

S = 10 is used

→ A train of length 400m running uniformly passes an electric pole in 20s, the time take to cross a bridge of length 100m is ______




$t = 20se$

400m = 20s
100m = x

$x = \frac{\cancel{20}^{5} \times 100}{\cancel{400}}$

~~x = 5 sec~~

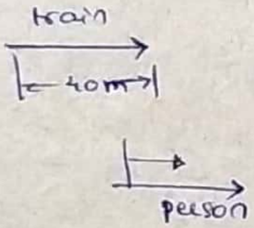
$V = \frac{dist}{t} = \frac{40\cancel{0}}{2\cancel{0}}$

$= 20m/s$

time to cross bridge (consideing train length)

$t = \frac{dist}{v} = \frac{40\cancel{0} + 10\cancel{0}}{2\cancel{0}}$

$= 25\ sec$




$t = 4se$

4sec x ~~20~~ X = 60m

A

4sec x 5 m/s = 20

(or)

$v = 5m/s$ $t = 4sec$

Relative velocity of train w.r.t $\boxed{\text{person}}$

R.V = V − 5

R.dist = 40

R.t = 4

$V - 5 = \frac{\cancel{40}^{10}}{\cancel{4}}$

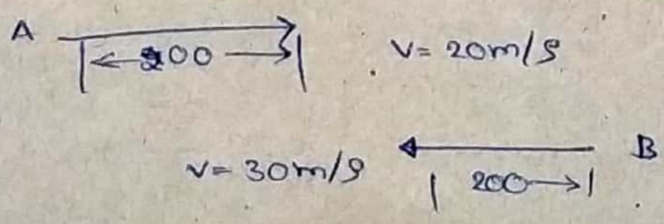
$V = 15\ m/s$

$x = 60/4$

$= 15$

---

→ Two trains of length 200m each running opp. with 20m/s & 30m/s the time taken to cross each



$R_v = V_A + V_B$

$= 50 m/s$

$R_d = 200 + 200$

$= 400 m$

$R_t = R_d / R_v$

$= 8 sec$

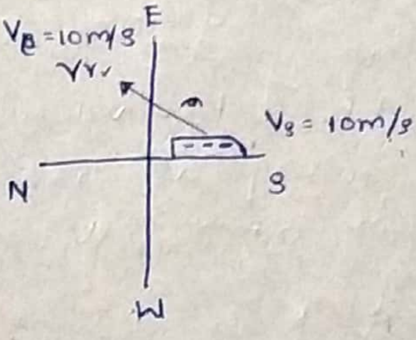
when they are in same direction

$R_t = 40 sec$

$R_v = \sqrt{10^2 + 10^2 - \cancel{2 \times 10 \times 10 \times \cos 90}}$

$= 10\sqrt{2}$

at direction N.E




→ A car is running level road with 3m/s it takes left turn by 60° moves 4m/s , ~~change~~ $\Delta V$ ____

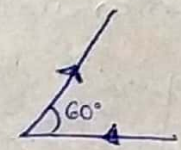
$\bar{\Delta V} = \bar{u} - \bar{v}$

$= \sqrt{U^2 + V^2 - 2UV\cos 60°}$

$= \sqrt{9 + 16 - \cancel{2} \times 3 \times 4 \times \frac{1}{\cancel{2}}}$

$= \sqrt{25 - 12} = \sqrt{13}$

$\bar{\Delta V} = \sqrt{13}$




↳ only for vectors

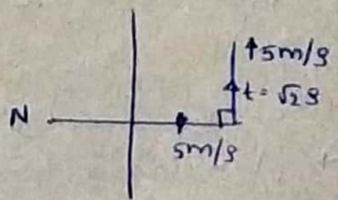
~~For scalars~~ →

→ Car is running towards south with 5m/s, it takes left by 90° in $\sqrt{2}$ s , the accl$^n$ during turn

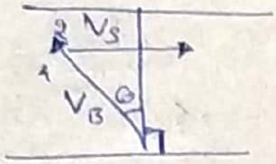
$a = \frac{\Delta V}{t} = \frac{\bar{U} - \bar{V}}{t} = \frac{\sqrt{U^2 + V^2 - 2UV\cos 90}}{\sqrt{2}}$

$= \frac{\sqrt{5^2 + 5^2}}{\sqrt{2}} = \frac{5\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$a = 5$



→ A boat has velocity 4 km/hr in still water (no movement), it crossing a river flowing at 2 km/hr, in the direction of the boat with the bank to cross the river along the shortest way




$\alpha = 90° + \theta$

$\sin\theta = \dfrac{V_s}{V_B}$

$\theta = (2/4)\sin^{-1}$

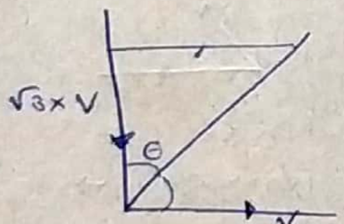
$= 30°$

$\alpha = 120°$

→ A boat has velocity 5 km/hr in still water, it is crossing at river of 1 km wide along shortest way in 15 min, Velocity of river is ______

A) 9

→ A man is walking on a level road, rain falling vertically down with $\sqrt{3}\times$ velocity of man, the dire of umbrella to cob

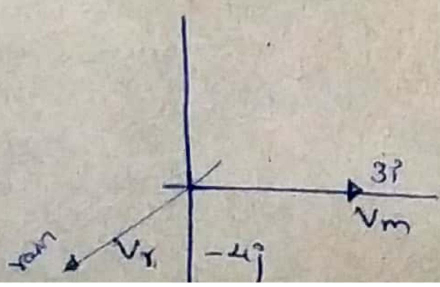



$\tan\theta = \dfrac{\cancel{v}}{\sqrt{3}\cancel{v}}$

$\theta = 30°$ with vertical

$\theta = 60$ " horizontal

→ A man is ~~walk~~ running with $3i$ m/s, rain falling with $-4j$ m/s, R. Velocity of rain w.r.t man ______




$R\bar{V} = V_r - V_s$

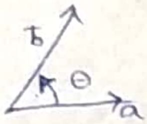
$\bar{V} = -4j - 3i$

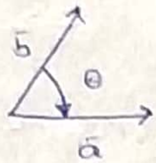
~~7 m/s~~

$V = \sqrt{16+9} = 5$ m/s

Multiplication of vector is [scalar product]

1. Dot product (scalar product)




$\bar{a} \cdot \bar{b}$

$= ab\cos\theta$

$b \cdot a \cos(-\theta)$

$= \cancel{a} \; b \cdot a\cos\theta$

Angle b/w two vectors

$\bar{a} \cdot \bar{b} = ab\cos\theta$

$\cos\theta = \dfrac{\bar{a} \cdot b}{ab}$

$\theta = \cos^{-1}\left(\dfrac{\bar{a} \cdot \bar{b}}{a \cdot b}\right)$

→ Dot product of two vectors

$\bar{a} = x_1 i + y_1 j + z_1 k$

$\bar{b} = x_2 i + y_2 j + z_2 k$

$\bar{a} \cdot \bar{b} = x_1 x_2 + y_1 y_2 + z_1 z_2$

Dot product of unit vectors

$i \times i = j \times j = k \times k = 1$

$i \times j = j \times k = i \times k = 0$

Ex:

$\bar{a} = i + j + k$

$\bar{b} = 2i + 2j + 2k$

$\bar{a} \cdot \bar{b} = 2 + 2 + 2 = 6$

$a = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$b = \sqrt{2^2 + 2^2 + 2^2} = 2\sqrt{3}$

$\cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{a \cdot b} = \dfrac{\cancel{3}}{2\cancel{\sqrt{3}}}$

$= 1/2$

$\theta = 60^\circ$

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→ Dot product of $\parallel^{le}$ vectore is maximum

$\bar{a}\cdot\bar{b} = ab\cos(\theta = 0^\circ)$

$= ab$

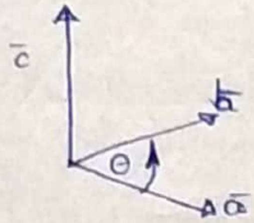
$\xrightarrow{\bar{a}}$
$\xrightarrow{\bar{b}}$

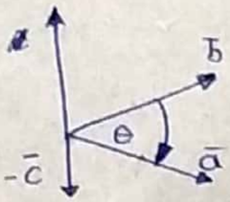
→ Dot product of $\perp^{le}$ vector is ~~an~~ zero

$\bar{a}\cdot\bar{b} = a.b\cos(\theta = 90^\circ)$

$= 0$

→ Cross product (vector product)




- Cross product of two vectors gives the resultant as vector
- Its direction is $\perp^{le}$ to the plane of two vector & follow R.H. thumb rule

$\bar{c} = \bar{a}\times\bar{b} = (ab\sin\theta)\,\hat{c}$

$|\bar{a}\times\bar{b}| = ab\sin\theta$

$\bar{b}\times\bar{a} = (ba.\sin(-\theta))\,\hat{c}$

$= ba.\sin(-\theta)$

$= -ba\sin\theta$

$\bar{b}\times\bar{a} = -(ab\sin\theta)$

$\bar{b}\times\bar{a} = -(\bar{c})$

$\bar{b}\times\bar{a} = -\bar{c}$

→ Cross product of two vectors:

$\bar{a} = xi + yj + zk$

$\bar{b} = x_2 i + y_2 j + z_2 k$

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ x_1 & y_1 & k_1 \\ x_2 & y_2 & k_2 \end{vmatrix}$$

$$= i(y_1k_2 - y_2k_1) - j(x_1k_2 - x_2k_1) + k(x_1y_2 - x_2y_1)$$

$\bar{a} = i + j + k$

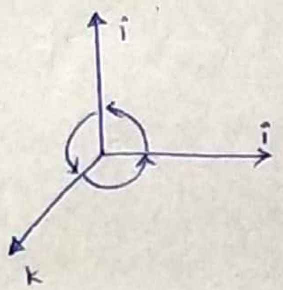
$\bar{b} = 2i + j + 2k$

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$\bar{c} = i[(2-1) - j(2-2) + k(1-2)$$

$$= i - k$$

→ Cross product of unit vector:




$i \times i = j \times j = k \times k = 0$

$i \times j = k$ ; $j \times i = -k$

$j \times k = i$ ; $k \times j = -i$

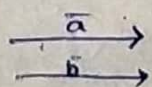
$k \times i = j$ ; $i \times k = -j$

$\Rightarrow \quad \bar{a} \times \bar{a} = 0$

$i \times j \times k = 0$

→ Cross product of $\parallel^{le}$ vectors is zero

$a \times b = ab \sin e = 0$




$$\left\{ \frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{k_1}{k_2} \right\}$$

$\bar{a} = i + j + k$

$\bar{b} = 2 + 2j + 2k$

$\Rightarrow \quad \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

$\Rightarrow \quad \bar{a}$ & $\bar{b}$ are $\parallel^{le}$

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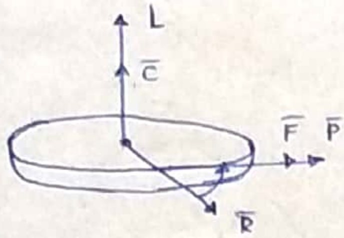
→ Cross product of a $\perp^{le}$ vectors is max

$\bar{a} \times \bar{b} = ab\sin(90^\circ) = ab$

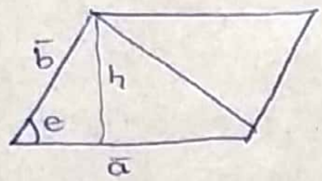
→




Torque $\tau = \bar{r} \times \bar{F}$

Ang$^{lr}$ momentum $\bar{L} = \bar{r} \times \bar{p}$

→ Area of $\|^{gm}$




area of $\Delta^{le}$ $= \frac{1}{2} \times \bar{a} \times h$ $\quad [h = b\sin\theta]$

$= \frac{1}{2} \times \bar{a} \times b\sin\theta$

$= \frac{1}{2} |\bar{a} \times \bar{b}|$

area of $\|^{gm}$ $= 2 \times \frac{1}{2} |\bar{a} \times \bar{b}|$

$= |\bar{a} \times \bar{b}|$

$$\boxed{\text{Area of parallelogram} = |\bar{a} \times \bar{b}|}$$

→ If $|\bar{a} \times \bar{b}| = |\bar{a} \cdot \bar{b}|$

$\cancel{ab}\sin\theta = \cancel{ab}\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \cdot$

$\tan\theta = 1$

$\theta = 45^\circ$

→ $\left|\frac{\bar{a} \times \bar{b}}{\bar{a} \cdot \bar{b}}\right| = \frac{ab\sin\theta}{ab\cos\theta}$

$\Rightarrow \tan\theta = 1$

$\theta = 45^\circ$

→ $|\bar{a} \times \bar{b}|^2 + |\bar{a} \cdot \bar{b}|^2 = \underline{a^2b^2}$

$a^2b^2\sin^2\theta + a^2b^2\cos^2\theta$

$a^2b^2(1)$

→ $(\bar{a} + \bar{b}) - (\bar{a} - \bar{b})$

$\cancel{\bar{a}\bar{a}}^{0} - \bar{a} \times \bar{b} + \bar{b}\bar{a} - \cancel{\bar{b}\bar{b}}^{0}$

$-(-\bar{b} \times \bar{a}) + \bar{b} \times \bar{a}$

$= +2|\bar{b} \times \bar{a}|$ or $-2|\bar{b} \times \bar{a}|$

Q) $\bar{a} = 2i - 6j + 2k$, $\bar{b} = xi + j + k$ } $\perp^{le}$ then $x =$ ______

~~use cross product~~

$\cos(90°) = 0$ so use dot. product

$\bar{a} \cdot \bar{b} = 2x - 6 + 2 = 0$

$\Rightarrow 2x = 4$

$x = 2$

Q) A force of $F = (3i + 2j + 2k)$ N displaces a body by $\bar{s} = (2i + 2j - k)$ m. the work done is ______

$\bar{F} \cdot \bar{s} = 6 + 4 - 2$

$= 8$ J

Q) A force of work $F = (3i + 2j + 2k)$ N displaces a body from the position $A(1,1,1)$ to the position $B(3,2,2)$ in 2 sec

Power is ______

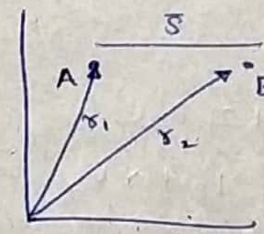
Sol:

$\bar{s} = \bar{r}_2 - \bar{r}_1$

$\bar{r}_2 = \cancel{3,2,} \; 3i + 2j + 2k$

$\bar{r}_1 = i + j + k$

$\bar{s} = (2i + j + k)$ m




$\bar{F} \cdot \bar{s} =$ $\bar{F} = (3i + 2j + 2k)$ N

$\bar{s} = (2i + j + k)$ m

$\bar{F} \cdot \bar{s} = (6 + 2 + 2)$ N.m

Power $= \dfrac{6 + 2 + 2 \;(\text{Nm})}{2 \text{ sec}}$

$= 5$ W

Q) $\bar{a} = (1)i + j$, $\bar{b} = (1)j + k$ the angle b/w $\bar{a}$ & $\bar{b}$ is ______

$\bar{a} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\bar{b} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\bar{a} \cdot \bar{b} = ab\cos\theta$

$\Rightarrow \cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{ab}$

$= \dfrac{0 + 1 + 0}{\sqrt{2} \times \sqrt{2}} = 1/2$

$\theta = 60°$.

Scanned by CamScanner

Q) $\bar{a} = i + 2j + k$

$\bar{b} = 2i + j + 2k$

$\theta = ?$

$\bar{a}.\bar{b} = ab\cos\theta$

$\cos\theta = \frac{\bar{a}.\bar{b}}{ab}$

$= \frac{2+2+2}{\sqrt{6} \times \sqrt{6} \times \sqrt{9}}$

$= \frac{6}{\sqrt{34}}$

$= \frac{\cancel{6}\,2}{\cancel{3}\sqrt{6}}$

$= \frac{2\times\sqrt{6}}{\cancel{6}\,3}$

$\cos\theta = \sqrt{6}/3$

$\theta = \cos^{-1}(2/\sqrt{6})$

Q) $\bar{a} = 2i + 3j + 4k$

$\bar{b} = 6i + 9j + xk$

$\Longrightarrow$ using cross product

$\bar{a}\times\bar{b} = a.b\sin(0)$
$= 0$

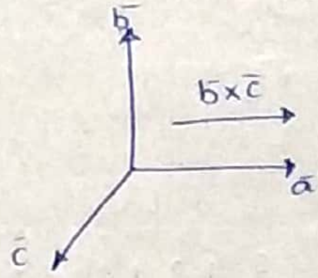
$\Rightarrow \frac{\cancel{3}}{\cancel{9}\,3} = \frac{4}{x}$

$x = 12$

Q) $\bar{a}.\bar{b} = 0$

$\bar{a}.\bar{c} = 0$

the vector $\|^{le}$ to $\bar{a}$ is ___




Ans: $\bar{b}\times\bar{c}$

Q) $\bar{a} = 3i + 4j$

$\bar{b}$ ?

$\bar{a}$ $\|^{le}$ to $\bar{b}$

$b = 10$

~~$\bar{a}.\bar{b}$~~

$\hat{a} = \frac{\bar{a}}{a}$

unit vector $= \frac{\bar{a}}{a}$

For $\|^{le}$ vectors unit vector is one

$\hat{a} = \hat{b}$

$\frac{\bar{a}}{a} = \frac{\bar{b}}{b}$

$\frac{3i+4j}{\sqrt{3^2+4^2}} = \frac{\bar{b}}{\cancel{10}\,2}$

$\bar{b} = 6i + 8j$

(or)

$\frac{3}{x_2} = \frac{4}{y_2}$

$b = 10$

$\sqrt{x_2^2 + y_2^2} = 10$

Guessing

$x_2 = 6 \quad y_2 = 8$

$\bar{b} = 6i + 8j$

Q) $\bar{a} = 2i + j + 2k$

$\bar{b} = i + 2j + k$

the unit vector $\perp^{le}$ a & b is ______

$\bar{c} = \bar{a} \times \bar{b}$

$$= \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} \Rightarrow i(1-4) - j(\cancel{2-2}) + k(4-1)$$

$\bar{c} = -3i + 3k$

$c = \sqrt{(-3)^2 + 3^2}$

$= \sqrt{18}$ $\quad \sqrt{9} \times \sqrt{2}$

$= 3\sqrt{2}$ $\quad 3\sqrt{2}$

$\left\{ |\bar{a}| = a = \sqrt{x^2 + y^2 + z^2} \right.$

unit vector $\left.\right\}$ $\hat{c} = \dfrac{-3i + 3k}{3\sqrt{2}}$

$\dfrac{\cancel{3}(-i + k)}{\cancel{3}\sqrt{2}}$

$\hat{c} = \dfrac{-i}{\sqrt{2}} + \dfrac{k}{\sqrt{2}}$

$c = \sqrt{\left(\frac{-1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}}$

$c = |\hat{c}| = 1$

Q) $\bar{a} = 3i + 2j + 2k$

$\bar{b} = i - j + 2k$ $\Big\}$ Area of $\parallel^{gm}$ is ______ formed by $\bar{a}$ & $\bar{b}$

area of $\parallel^{gm}$ of $\bar{a}$ & $\bar{b}$ $= |\bar{a} \times \bar{b}|$

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 3 & 2 & 2 \\ 1 & -1 & 2 \end{vmatrix}$$

$= i(+3+3) - j(4+2) + k(-3-2)$

$\bar{a} \times \bar{b} = 6i - 6j - 5k$

$|\bar{a} \times \bar{b}| = \sqrt{6^2 + 6^2 + 5^2}$

$= \cancel{6\sqrt{2}+} \sqrt{77}$

Area of $\Delta^{le} = \frac{1}{2} |\bar{a} \times \bar{b}|$

$= \frac{1}{2}\sqrt{77}$

Q) Area of $\parallel^{gm}$ formed by $\bar{a}$ & $\bar{b}$ is $\frac{ab}{2}$, the angle b/w a & b is ______

Area $= |\bar{a} \times \bar{b}| = ab \sin\theta$

$\frac{ab}{2} = ab \sin\theta$

$\theta = 30^\circ$

Q) The scalar product & vector product of two vectors have magnitudes $48\sqrt{3}$ & 144, the angle b/w the vectors is ______

$\bar{a} \cdot \bar{b} = 48\sqrt{3}$

$ab\cos\theta = 48\sqrt{3}$ — (1)

$\bar{a} \times \bar{b} = 144$

$ab\sin\theta = 144$ — (2)

(1) % (2)

$\frac{ab\cos\theta}{ab\sin\theta} = \frac{48\sqrt{3}}{144}$

$\tan\theta = \frac{144}{48\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

Q) $\bar{a} = i + j + k$

$\bar{b} = 2i + j + k$

$\bar{c} = i + 2j + k$

$\bar{a} \times \bar{b} \times \bar{c}$ is ______

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & +1 & 1 \end{vmatrix}$$

$$0-(-j)+k(-1)$$

$$\bar{a}\times\bar{b} = j-k$$

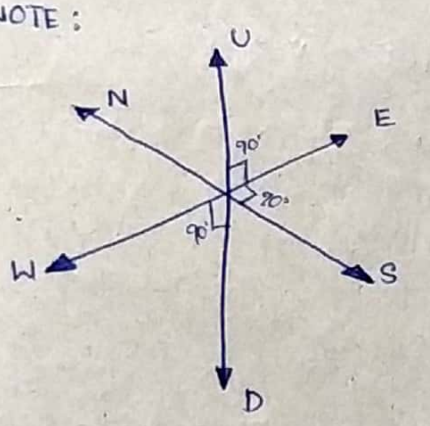
$$\bar{c} = i+2j+k$$

$$(\bar{a}\times\bar{b})\times\bar{c} = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$i(3)-j(1)+K(-1)$$

$$\Rightarrow\ 3i-j-k$$

NOTE :




Using R.H. Thumb Rule

$\bar{E}+\bar{S} = \overline{SE}$

$\bar{E}-\bar{S} = \bar{E}+\bar{N} = \overline{NE}$

$\bar{S}\times\bar{E} = \bar{U}$

$\bar{E}\times\bar{S} = \bar{D}$

$\bar{E}\times\bar{U} = \bar{S}$

$\bar{E}\times\bar{D} = \bar{N}$

$\bar{D}\times\bar{E} = \bar{S}$