Team 3

Quantum Computation (18CSE310J)

20 November 2022

The CHSH Inequality

INTRODUCTION

The evolution of quantum mechanics is marked by heated debates about the nature of reality and the extent to which quantum mechanics can describe it. Given the phenomenal empirical success of quantum mechanics, it seemed obvious that people would not abandon it just because some of its components were difficult to reconcile with intuition.

At the heart of these divergent perspectives was the issue of measurement's nature. We know that quantum measurements contain an element of randomness, but is this true? Exists a devious method by which the Universe has already determined in advance what value a certain measurement would return in the future? This idea served as the foundation for other hidden variable hypotheses. However, these ideas had to explain randomness beyond the level of the individual particle. In addition, they needed to explain what occurs when various observers measure different components of a multicomponent entangled system! This transcended hidden variable theory. In order to reconcile the facts of quantum physics with a universe in which local reality is true, a local hidden variable theory was required.

What is regional actuality? In a universe where locality holds, two systems should be able to be separated so far apart in space that they cannot communicate. The idea of reality is associated with the question of whether a measured quantity has a specific value in the absence of future measurement.

THE CHSH INEQUALITY

In physics, the CHSH inequality could be used to prove Bell's theorem, which claims that local hidden-variable theories cannot recreate certain implications of entanglement in quantum mechanics. The experimental confirmation of the inequality violation is viewed as evidence that such theories cannot adequately represent nature. In 1969, John Clauser, Michael Horne, Abner Shimony, and Richard Holt described CHSH in a widely referenced publication.

They derived the CHSH inequality, which, like John Stewart Bell's original inequality, is a limitation on the statistical occurrence of "coincidences" in a Bell test that must hold if there are underlying local hidden variables, an assumption commonly referred to as local realism. Quantum physics experiments conducted in the present day consistently break the inequality.

THE CHSH STATEMENT

The CHSH Statement is given by

$$|S| \leq 2,$$

Where,

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b')$$

a, a′ - the detector settings on side A

b, b’ - the detector settings on side B,

The detector settings are evaluated in four distinct sub experiments. The terms E(a, b) represent the quantum correlations of the particle pairs, where the quantum correlation is defined as the expectation value of the product of the "outcomes" of the experiment, i.e. the statistical average of A(a) B(b), where A and B are the individual outcomes, using the coding +1 for the '+' channel and 1 for the '' channel. Clauser et al's derivation was geared toward the use of "two-channel" detectors, and this is the most common use. However, under their technique, only +1 and -1 were feasible results. In order to adapt to real settings, which at the time required polarized light and single-channel polarisers, '' had to be interpreted as "non-detection in the '+' channel," i.e. '' or nothing. In the original publication, they did not describe how the two-channel inequality might be implemented in actual trials with genuine flawed detectors; nonetheless, it was afterwards demonstrated that the inequality itself was true. The presence of zero outcomes, however, makes it less evident how to estimate the values of E from the experimental data.

The mathematical formalism of quantum mechanics predicts a maximum value for S of $2\sqrt{2}$ (Tsirelson's bound), which is larger than 2, and the theory of quantum mechanics consequently predicts CHSH violations.

DERIVATION

The original derivation from 1969 is not shown here since it is difficult to understand and assumes that all outcomes are either +1 or 1, never 0. Bell's 1971 derivation is more broad and quite simple to comprehend relatively. He assumes the "Objective Local Theory" that Clauser and Horner eventually adopted. Any hidden variables linked with the detectors themselves are believed to be independent on both sides and may be averaged out from the outset. Clauser and Horne's 1974 work provides another derivation of interest, which begins with the CH74 inequality.

From both of these subsequent derivations, it would appear that the only assumptions required for the inequality itself (as opposed to the estimation method for the test statistic) are that the distribution of possible states of the source remains constant and that the detectors on both sides act independently.

BELL’S THEOREM AND THE CHSH INEQUALITY

Bell's Theorem is only applicable to theories that violated Heisenberg's Uncertainty Principle since it implicitly assumed that all measurements happened simultaneously. By explicitly including time into our derivation of Bell's theorem, we find that an additional factor relating to the time-ordering of real measurements weakens the upper bound of the inequality.

We find that only classical measurement-order independent local hidden variable theories are confined by Bell's inequality; time dependent, non-classical local theories (i.e.

theories obeying Heisenberg's Uncertainty Principle) can meet this new restriction while surpassing Bell's limit. Nonlocality is only anticipated for Bell parameters between $2\sqrt{2}$ and 4. This weakening of Bell's inequality is observed as an additional element involving the commutators of local measurement operators for the quantum Bell operator (squared). We see that a factorizable second-quantized wave function may match experimental observations. Since such wave functions permit local de Broglie-Bohm hidden variable modeling, we have more evidence that violation of Bell's inequality does not need admission of non-locality.

BELL'S DERIVATION

Bell (1971) demonstrated in his book "Speakable and Unspeakable in Quantum Mechanics" that all previous attempts to "establish" the completeness of QM were based on erroneous assumptions. In addition, he demonstrated that the interpretation proposed by de Broglie and subsequently developed by Bohm Bohm (1952) avoided all of these "proofs" but, when applied to two particles, entailed a disturbing characteristic: nonlocality or "activity at a distance." With the realization that the de Broglie-Bohm (dBB) interpretation resulted in nonlocal forces between particles, Bell developed a mathematical relationship that is believed to demonstrate that nonlocality is a fundamental aspect of reality, or at least a fundamental component of hidden variable theories. Experiments conducted over the past 35 to 40 years (e.g. Ou and Mandel (1988); Aspect et al. (1982); Weihs et al. (1998)) have conclusively demonstrated that this limit is breached.

We demonstrate that Bell's approach implicitly assumed measurement compatibility or simultaneity. This goes beyond assuming the simultaneous presence of hidden variables to further assume their simultaneous measurability; hence, his derivation contradicts Heisenberg's Uncertainty Principle (HUP). Our derivation eliminates this assumption by explicitly taking into account the fact that each measurement must occur at a distinct time, resulting in a weaker inequality that is not broken by Quantum Mechanics, experiment, or local non-classical hidden variable theories.

In Bell's derivation of the CHSH Inequality (1971), Bell starts by computing the difference between two (theoretical) averages or correlation coefficients.

$$\begin{aligned} \langle AB\rangle - \langle AB'\rangle &\equiv \int d\lambda\rho(\lambda)A(a,\lambda)B(b,\lambda) \\ &\quad - \int d\lambda\rho(\lambda)A(a,\lambda)B(b',\lambda) \end{aligned}$$

Where, $A,B,B'=\pm1$ - results of measurements which depend on the filters' orientation and might also depend on hidden variables like λ, which may have a distribution $\rho(\lambda)$.

The derivation begins with the integral being extracted from the difference

$$\langle AB\rangle - \langle AB'\rangle \qquad = \langle AB - AB'\rangle$$

With subsequent manipulations, we arrive at,

$$|\langle AB\rangle - \langle AB'\rangle| + |\langle A'B'\rangle + \langle A'B\rangle|$$

Which is the Bell's inequality.

Undistributing the integral in equation 2 requires us to assume that $B(b,\lambda)$ measurement result is known at the same (timeless) moment that $B(b',\lambda)$ measurement result is known, which is the theoretical equivalent of assuming that Heisenberg's Uncertainty Principle does not apply to these hidden variables or wavefunctions (i.e. that σz and σx can be known simultaneously). This undistribution is sometimes referred to as "counterfactual" because Bob cannot experimentally perform both observations at the same time — magnetic fields at 0 and 90 degrees simultaneously constitute a single field at 45 degrees, etc.

INFERENCES FROM BELL'S DERIVATION

The inherent assumption of simultaneous measurability or temporal order-independence of measurements at various orientations means that Bell's assertion of a locality bound is simultaneously a time independence or Heisenberg irrelevance restriction . Classical local hidden variable theories are precluded by experiment, but non-classical (i.e. non-commutative), dynamic local hidden variable theories are not subject to Bell's original limit, but to the weaker Cirel'son Cirel'son (1980) limit of $2\sqrt{2}$; nonlocality presumably only shows up with the violation of that weaker limit. The extra terms of equations 4 or 5 only contribute if non-classical effects occur locally at both sites; they do not need that a distant particle immediately influence the behavior of a close particle.

Insofar as the Copenhagen interpretation, i(ti) in the derivations and the de Broglie-Bohm interpretation, $\lambda i=\psi(ti)$+hidden variables produce the identical responses in all experimental scenarios, our conclusion demonstrates that neither interpretation must be non-local in order to adequately explain the results.

The de Broglie-Bohm interpretation of a single particle is a non-classical, dynamic hidden variable theory. It illustrates the evolution of variables in a way that clarifies why they cannot be examined simultaneously or get the same result if assessed in a different sequence. It is also a local HVT when applied to a factorizable wavefunction of many particles.

For entangled particles that violate Bell's inequality, the entanglement is the outcome of Alice and Bob's "post-selection" of just coincidences. The wavefunction of Ou and Mandel will create local dBB trajectories for each particle, demonstrating that dBB is not an issue even for numerous "entangled" particles.

Similar reasons should apply to studies like that of Weihs, et al.

Weihs et al. (1998), where the entanglement is “pre-selected” before being put into the optical fibers (throwing away the ~99% of the photons in the non-intersecting sections of the down-conversion cones).

Using a "post-selection" wavefunction, as is common in first quantization analyses, dBB will generate non-local forces along each trajectory, but each trajectory will contribute to a coincidence event; using a factored "pre-selection" wave function suitable for second quantization will generate local forces along each trajectory, but only a subset of trajectories will satisfy the coincidence conditions. A model may be nonlocal, but it doesn't have to be nonlocal to explain the facts. Computational efficiency does not appear to be a sufficient justification to abandon locality.

If Bell's inequality depends on ¬HUP and LOC, then we must accept either HUP or ¬LOC in order to violate his inequality. Nonlocality, however, is not an option, as our revised derivation, which presupposes HUP and LOC, provides a weaker inequality that is respected by all trials.

SIMULATION OF THE CHSH INEQUALITY IN QISKIT

Importing the required libraries

```
[3]: import qiskit
     from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister, transpile, Aer, IBMQ
     from qiskit.tools.visualization import circuit_drawer
     from qiskit.tools.monitor import job_monitor, backend_monitor, backend_overview
     from qiskit.providers.aer import noise

     import matplotlib.pyplot as plt
     import numpy as np
     import time

     provider = IBMQ.load_account()
```

Setting up devices

```
[4]: IBMQ.load_account()
     provider = IBMQ.get_provider('ibm-q')
     quito = provider.get_backend('ibmq_quito')
```

```
[5]: sim = Aer.get_backend('aer_simulator')
```

\

First, we will develop a function for generating CHSH circuits. We will pick, without sacrificing generality, that Bob always utilizes the computational (Z) and the X bases for his B and b measurements, respectively, whereas Alice will likewise use orthogonal bases, but whose angle with respect to Bob's bases will vary between 0 and 2π. This collection of angles will serve as the input to our CHSH circuit constructing function.

```
[6]: def makeCHSHcircuit(thetavec):

    chsh_circuits = []

    for theta in thetavec:
        obs_vec = ['00', '01', '10', '11']
        for el in obs_vec:
            qc = QuantumCircuit(2,2)
            qc.h(0)
            qc.cx(0, 1)
            qc.ry(theta, 0)
            for a in range(2):
                if el[a] == '1':
                    qc.h(a)
            qc.measure(range(2),range(2))
            chsh_circuits.append(qc)

    return chsh_circuits
```

Following this, we will create a function for predicting the amount CHSH. Two of these values can be defined: $\langle CHSH1\rangle=\langle AB\rangle-\langle Ab\rangle+\langle aB\rangle+\langle ab\rangle$ and $\langle CHSH2\rangle=\langle AB\rangle+\langle Ab\rangle-\langle aB\rangle+\langle ab\rangle$. Once the correct measurement axes for both parties have been determined, each expectation value may be approximated by simply combining the counts from the output bit strings with the proper sign (plus for even terms 00 and 11, and minus for odd terms 01 and 10).

```
[7]: def compute_chsh_witness(counts):

    CHSH1 = []
    CHSH2 = []
    for i in range(0, len(counts), 4):
        theta_dict = counts[i:i + 4]
        zz = theta_dict[0]
        zx = theta_dict[1]
        xz = theta_dict[2]
        xx = theta_dict[3]

        no_shots = sum(xx[y] for y in xx)

        chsh1 = 0
        chsh2 = 0

        for element in zz:
            parity = (-1)**(int(element[0])+int(element[1]))
            chsh1+= parity*zz[element]
            chsh2+= parity*zz[element]

        for element in zx:
            parity = (-1)**(int(element[0])+int(element[1]))
            chsh1+= parity*zx[element]
            chsh2-= parity*zx[element]
```

```
        for element in xz:
            parity = (-1)**(int(element[0])+int(element[1]))
            chsh1-= parity*xz[element]
            chsh2+= parity*xz[element]

        for element in xx:
            parity = (-1)**(int(element[0])+int(element[1]))
            chsh1+= parity*xx[element]
            chsh2+= parity*xx[element]

        CHSH1.append(chsh1/no_shots)
        CHSH2.append(chsh2/no_shots)

    return CHSH1, CHSH2
```

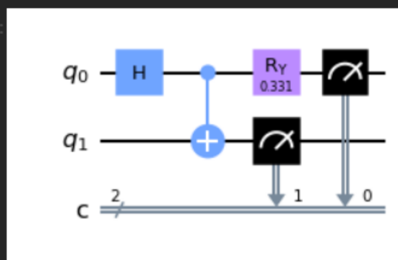
Lastly, we will divide the interval [0,2) into 15 angles and construct the matching CHSH circuits.

```
[9]: number_of_thetas = 20
theta_vec = np.linspace(0,2*np.pi,number_of_thetas)
my_chsh_circuits = makeCHSHcircuit(theta_vec)
```

Visualizing four CHSH circuits for a given value of theta.

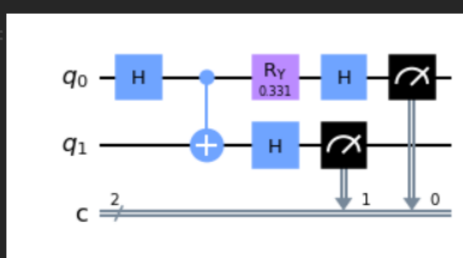
```
[10]: my_chsh_circuits[4].draw()
```

[10]:



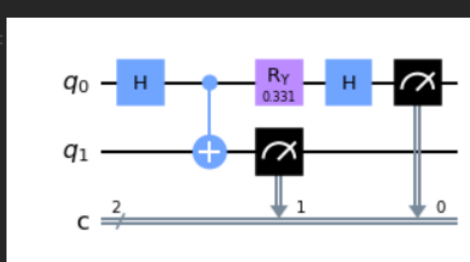

```
[11]: my_chsh_circuits[7].draw()
```

[11]:



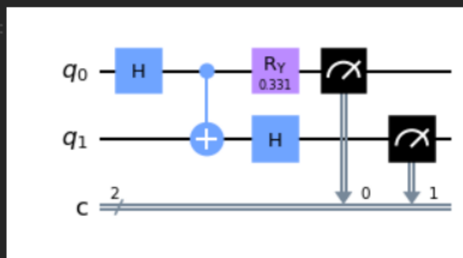

```
[12]: my_chsh_circuits[6].draw()
```

[12]:



```
[13]: my_chsh_circuits[5].draw()
```

[13]:



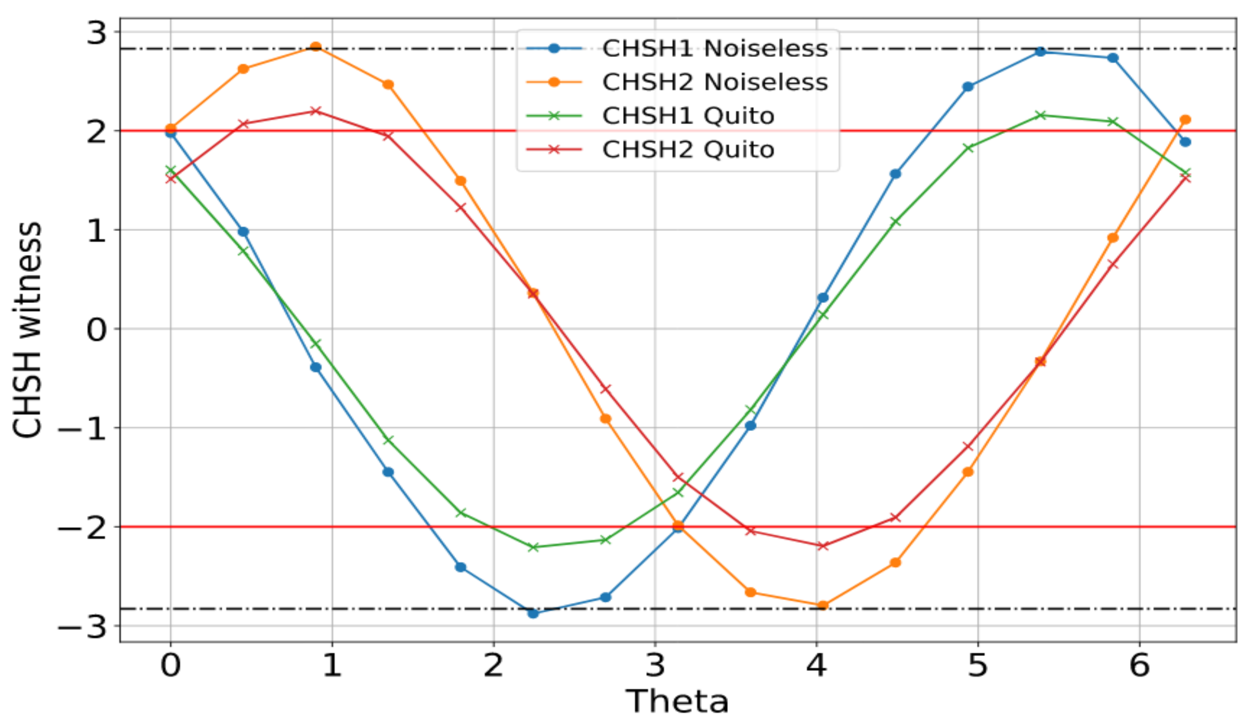
In these circuits, a Bell pair is created, and then each party is measured on a distinct basis. While Bob (q1) always measures in either the computational basis or the X basis, Alice's measurement basis rotates with regard to Bob's by the angle.

Executing and getting the counts

```
[*]: result_ideal = sim.run(my_chsh_circuits).result()

     tic = time.time()
     transpiled_circuits = transpile(my_chsh_circuits, quito)
     job_real = quito.run(transpiled_circuits, shots=8192)
     job_monitor(job_real)
     result_real = job_real.result()
     toc = time.time()

     print(toc-tic)
```

Plotting the gathered information

```
[ ]: plt.figure(figsize=(12,8))
     plt.rcParams.update({'font.size': 22})
     plt.plot(theta_vec,CHSH1_ideal,'o-',label = 'CHSH1 Noiseless')
     plt.plot(theta_vec,CHSH2_ideal,'o-',label = 'CHSH2 Noiseless')

     plt.plot(theta_vec,CHSH1_real,'x-',label = 'CHSH1 Quito')
     plt.plot(theta_vec,CHSH2_real,'x-',label = 'CHSH2 Quito')

     plt.grid(which='major',axis='both')
     plt.rcParams.update({'font.size': 16})
     plt.legend()
     plt.axhline(y=2, color='r', linestyle='-')
     plt.axhline(y=-2, color='r', linestyle='-')
     plt.axhline(y=np.sqrt(2)*2, color='k', linestyle='-.')
     plt.axhline(y=-np.sqrt(2)*2, color='k', linestyle='-.')
     plt.xlabel('Theta')
     plt.ylabel('CHSH witness')
```



CONCLUSIONS FROM THE SIMULATION

The fact that our actual technology violated the CHSH inequality is significant. A decade ago, such an experiment would have had a substantial influence. In the present day, quantum devices have vastly improved, and similar discoveries can be readily duplicated on modern technology. However, there are a number of loopholes that must be closed in order to say that either locality or realism has been disproven when the inequality is violated. These are the locality/causality loophole and the detection loophole (where our detector is flawed and fails to deliver relevant data) (where the two parts of the entangled system are separated by a distance smaller than the distance covered by the light in the time it takes to perform a measurement).

Given that we can construct entangled pairs with great precision and that every measurement provides a result (i.e., no observed particle is "lost"), we have closed the detection gap in our previous studies. Given the distance between our qubits (a few millimeters) and the time required to do a measurement (on the order of picoseconds), we cannot claim to have closed the causality loophole. The fact that we violated the CHSH inequality in our actual device carries a great deal of weight and importance. Only ten years ago, such an experiment would have had a significant bearing on the world. These days, quantum devices are noticeably better than they were in the past, and the results can be easily replicated in cutting-edge hardware thanks to these advancements. In order to claim that either locality or realism have been disproven, it is necessary to close a number of loopholes when violating the inequality. However, there are a number of loopholes that must be closed.

BELL TESTS WITH NO NON-DETECTIONS

Take, for instance, the thought experiment that David Bohm developed, in which a molecule disassembles into two atoms that have opposing spins. Assume that this spin can be modeled using a true vector that can point in any direction. In our model, it will be referred to as the "hidden variable." If we consider it to be a unit vector, then any conceivable value of the hidden variable may be found as a point on the surface of a unit sphere if we make that assumption.

Consider the possibility that the spin is going to be measured in the direction a. Given that all atoms are detected, the natural assumption that can be made is that all atoms the projection of whose spin in the direction an is positive will be detected as spin-up (coded as +1), while all atoms whose projection is negative will be detected as spin-down (coded as 1) Given that all atoms are detected, the natural assumption that can be made is that all atoms the projection of whose spin in the direction an is positive will be detected as spin

On the surface of the sphere, a great circle will be drawn in the plane that is perpendicular to a, creating a dividing line between two distinct regions: one for +1 and one for -1. Assuming for the sake of simplicity that an is perpendicular to the horizontal, which would correspond to the angle a with respect to any acceptable reference direction, the dividing circle will be in a plane perpendicular to the horizontal. Up to this point, we have only modeled the A side of our experiment.

Now to model side B. Let's assume that b is horizontal as well, which would equate to the angle b. On the same sphere, a second large circle will be made, and on one side of that circle, we will have +1, and on the other side, we will have -1 for particle B. After this, the circle will be in a vertical plane once more.

The surface of the sphere is segmented into four different areas by the two circles. The zone in which each particle's hidden variable is located will define the type of "coincidence" (++, --, +-, or -+) that is detected for any given pair of particles. This region can be positive or negative. Assuming that the source is "rotationally invariant" (meaning that it produces all possible states with the same probability), the probability of a specific type of coincidence will undoubtedly be proportional to the area associated with it, and the size of these areas will change in a linear fashion depending on the angle that exists between a and b. (To further understand this, picture an orange cut into segments.) Roughly speaking, the amount of peeling space allotted to a given number of segments is proportionate to that number. To be more precise, it is directly proportional to the angle that is subtended at the center.)

The formula that was presented earlier has not been employed directly since it is scarcely important in a case that is totally deterministic like this one. The issue might be rethought in terms of the functions presented in the formula, with serving as a constant and the probability functions taking the form of step functions. The idea that underpins has, in point of fact, been put to use, although in an intuitive capacity.

The prediction made by realists for quantum correlation (shown by solid lines) when there are no non-detections. The curve with dots represents the prediction made by quantum mechanics.

Therefore, the local hidden-variable prediction for the chance of a coincidence is proportional to the angle that exists between the detector settings (b minus a). The expected value of the total number of possible outcomes is what is meant when one talks about the quantum correlation, and this is what it is.

THE CHSH GAME

The CHSH game is a thought experiment in which two parties are separated by a great distance (enough to preclude classical communication at the speed of light), and each of these parties has access to one half of an entangled two-qubit pair. The distance between the parties is great enough to preclude classical communication at the speed of light. The results of this analysis demonstrate that no traditional local hidden-variable theory is capable of explaining the correlations that might emerge as a consequence of entanglement. The fact that this game is certainly feasible from a physical standpoint provides compelling evidence that classical physics is fundamentally incapable of understanding some quantum occurrences, at least in a "local" sense.

Alice and Bob are the two players that will be working together in the CHSH game, while Charlie will serve as the referee. These three agents will be referred to using the abbreviations A, B, and C, in that order. After then, Alice and Bob are responsible for providing Charlie with a response that includes the bits a, b in displaystyle a,b in the values 0 and 1, respectively. Now, when Charlie has received the replies that Alice and

Bob have given him, Charlie will check to see if $a \oplus b = x \wedge y$. Should this equality be maintained, Alice and Bob will emerge victorious; otherwise, they would be deemed unsuccessful.

It is also needed that Alice's and Bob's answers can only depend on the bits that they see. This means that Alice and Bob are prohibited from directly interacting with one another about the values of the bits that were supplied to them by Charlie. Charlie sent the bits to Alice and Bob. However, Alice and Bob are permitted to discuss and agree upon a shared plan prior to the start of the game.

It will be shown in the following sections that it is impossible for Alice and Bob to win with a probability greater than 75% if they only use traditional strategies that involve their local information (and possibly some random coin tosses). This will be demonstrated by showing that it is impossible for them to do so. If, on the other hand, Alice and Bob are permitted to share a single entangled qubit pair, then it is possible to devise a strategy that would allow them to be successful with an approximate chance of 85%.

DEMONSTRATION OF THE CHSH GAME

Games of quantum are comparable to those of probability that are played in casinos. In most cases, a number of players will work together in order to compete against the casino (sometimes called referee). The players are presented with a number of unpredictable inputs, and they are tasked with deciding which of their outputs to hand over to the casino. Because it is forbidden for them to talk to one another during the game, they have to come up with a plan ahead of time in order to give themselves the best opportunity to come out on top. In order to emerge victorious from the competition, the outputs that are provided by the participants must fulfill a specific criterion that is unique to the game that is being competed.

Players participating in these games are permitted to employ either classical or quantum strategies in their play. When we really get into the game, we'll have a better idea of what is meant by "classical strategy" and "quantum strategy." For the time being, you should only think of it as the types of manipulations that they are permitted to undertake. They are only permitted to modify their inputs in a classical manner, which is with the assistance of objects whose behavior can be explained using the principles of classical physics, while adopting a classical strategy. To modify the inputs that are provided to them in quantum strategies, players are permitted to make use of quantum objects, which are governed by the principles of quantum mechanics.

Importing Qiskit

```
[1]:
import qiskit
qiskit.__qiskit_version__
```

The game that will be played is known as the CHSH game, and it was created by John Clauser, Michael Horne, Abner Shimony, and Richard Holt. Although it is connected to the CHSH inequality, it was not created by the same people, and its place of genesis appears to have been forgotten in the annals of quantum computing history.

Alice and Bob are the two players that compete against one other and the house in this game. Alice and Bob are allowed to consult one another and decide on a plan before the game begins; but, once it has begun, communication between the two of them is prohibited. A referee will produce two random bits, x and y, which will then be given to Alice and Bob, respectively, to use as inputs. Alice and Bob are unaware of the inputs made by the other. They are free to alter their own input in any way they see fit, in accordance with the approach they have chosen, and then they can select some information to output. Alice will send out bit a, while Bob will send out bit b.

In the event that the following condition is met, Alice and Bob will emerge victorious over the casino:

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{x} \wedge \mathbf{y}$$

```
[2]: import numpy as np
     import random as rand
     from qiskit import BasicAer
     from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister, execute
```

```
[3]: from IPython.display import HTML
     HTML('''
     code_show=true;
     function code_toggle() {
      if (code_show){
      $('div.input').hide();
      } else {
      $('div.input').show();
      }
      code_show = !code_show
     }
     $( document ).ready(code_toggle);

     ''')
```

```
[3]:
('div.input').hide();
 } else {
 $('div.input').show();
 }
 code_show = !code_show
code_show=true; function code_toggle() { if (code_show){ } ( document ).ready(code_toggle);
```

Let's get started by playing the game with a more traditional, deterministic approach.

It is important to keep in mind that this indicates that Alice and Bob are unable to interact with one another and can only formulate a strategy based on the value of the input bit that corresponds to their respective inputs.

The following are some potential courses of action:

- output = input
- output = NOT(input)
- always output = 1
- always output = 0

Coding down the potential courses of action

```
def cplayer_output(strategy, inp):
    if(strategy == 1):
        return inp

    elif(strategy == 2):
        return abs(inp-1)

    elif(strategy == 3):
        return 1

    elif(strategy == 4):
        return 0

    else:
        print("INVALID choice")
        return 100
```

Beginning the game by picking the classical strategies for both Alice and Bob

```
[5]: A_st = int(input('select the classical strategy for Alice, input 1,2,3 or 4 to pick one of the strategies listed above '))

     B_st = int(input('select the classical strategy for Bob, input 1,2,3 or 4 to pick one of the strategies listed above '))

     select the classical strategy for Alice, input 1,2,3 or 4 to pick one of the strategies listed above  1
     select the classical strategy for Bob, input 1,2,3 or 4 to pick one of the strategies listed above  2
```

Simulating the game and printing the results:

Setting parameters of the quantum run of the game

```
[6]: shots = 1 # set how many times the circuit is run, accumulating statistics about the measurement outcomes
     backend = BasicAer.get_backend('qasm_simulator') # set the machine where the quantum circuit is to be run

     #fixes the numbers of games to be played
     N=100

     # initializes counters used to keep track of the numbers of games won and played by Alice an Bob
     cont_win = 0 # counts games won
     cont_tot = 0 # counts games played
```

Here,

Count_win - number of games won

Count_tot - total number of games plates

Looping till 100 games:

```
for i in range(N):

    q = QuantumRegister(2, name='q')

    c = ClassicalRegister(2, name='c')


    game = QuantumCircuit(q, c, name='game')


    game.h(q[0])
    game.cx(q[0],q[1])


    random_num1 = rand.random()
    random_num2 = rand.random()

    if(random_num1 >= 1/2):
        x = 0
    else: x = 1

    if(random_num2 >= 1/2):
        y = 0
    else: y = 1
```

Gates used: Hadamard on qubit 1, CNOT on both the qubits.

```
theta = qAlice_output(qA_st, x)
phi = qBob_output(qB_st, y)

game.ry(theta,q[0])
game.ry(phi,q[1])

game.measure(q[0], c[0])
game.measure(q[1], c[1])

result = execute(game, backend=backend, shots=shots).result()

data = result.get_counts('game')


for outcomes in data.keys():
    out = outcomes
```

The most important aspect of Alice and Bob's quantum strategy is that each of them must set a unique rotation angle for their qubit based on the input values of x and y.

Simulating the game storing the results and finding the win probability

```
    if(out == '00'):
        a = 0
        b = 0
    if(out == '01'):
        a = 1
        b = 0
    if(out == '10'):
        a = 0
        b = 1
    if(out == '11'):
        a = 1
        b = 1


    if(x*y == a^b):
        cont_win += 1

    cont_tot += 1\

qProb_win = cont_win/cont_tot

print('Alice and Bob won the game with probability: ', qProb_win*100, '%')
```

From the outcomes,

Alice and Bob won the game with probability: **85.0 %**

The **quantum strategy** gave Alice and Bob higher chances of winning

Analyzing the results for the best winning strategy:

1. Classical Strategy

   Find the best classical approach that will maximize the likelihood that Alice and Bob will win, and let's go from there. Taking into consideration the information in the following truth table:

x	y	x.y	.	.	.	.	a	b	a⊕b
0	0	0	.	.	.	.	0	0	0
0	1	0	.	.	.	.	0	1	1
1	0	0	.	.	.	.	1	0	1
1	1	1	.	.	.	.	1	1	0

   The table below shows the winning conditions

x	y	a⊕b
0	0	a=b
0	1	a=b
1	0	a=b
1	1	a=b⊕1

As a result, the odds are three-quarters in favor of Alice and Bob winning. Ignoring the input values and choosing the option "Always Output=0" as a strategy is one of the options available to Alice and Bob (strategy 4 for Alice and strategy 4 Bob). Another strategy would be for Bob to choose "Always Output=0," and for Alice to gamble by choosing "Input=Output." This would allow Alice to win the game if both of their inputs were equal. There is a 75% chance of success with one of these two approaches.

2. The Quantum Strategy

Quantum systems provide a degree of freedom that has no counterpart in the classical world. Alice and Bob have an 80% chance of coming out on top of the competition if, before it begins, they are already in an entangled bipartite state together.

The winning strategy is as follows:

1- Alice and Bob share two-qubit EPR

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

2-They define a measurement bases denoted by where:

$$|v_0(\theta)\rangle = cos(\theta)|0\rangle + sin(\theta)|1\rangle$$
$$|v_1(\theta)\rangle = -sin(\theta)|0\rangle + cos(\theta)|1\rangle$$

Alice and Bob each have a measuring basis based on the value that they enter. After making any necessary substitutions, the following are all conceivable winning probabilities.

Alice $x = 0$, Bob $y = 0$

$$P_{0,0} = |\langle_{AV0}(\theta_{A0}) \otimes \langle_{BV0}(\theta_{B0}) \otimes |\Psi\rangle_{AB}|^2 + |\langle_{AV1}(\theta_{A0}) \otimes \langle_{BV1}(\theta_{B0}) \otimes |\Psi\rangle_{AB}|^2$$
$$P_{0,0} = cos^2(\theta_{A0} - \theta_{B0})$$

Alice $x = 0$, Bob $y = 1$

$$P_{0,1} = |\langle_{AV0}(\theta_{A0}) \otimes \langle_{BV0}(\theta_{B1}) \otimes |\Psi\rangle_{AB}|^2 + |\langle_{AV1}(\theta_{A0}) \otimes \langle_{BV1}(\theta_{B1}) \otimes |\Psi\rangle_{AB}|^2$$
$$P_{0,1} = cos^2(\theta_{A0} - \theta_{B1})$$

Alice $x = 1$, Bob $y = 0$

$$P_{1,0} = |\langle_{AV0}(\theta_{A1}) \otimes \langle_{BV0}(\theta_{B0}) \otimes |\Psi\rangle_{AB}|^2 + |\langle_{AV1}(\theta_{A1}) \otimes \langle_{BV1}(\theta_{B0}) \otimes |\Psi\rangle_{AB}|^2$$
$$P_{1,0} = cos^2(\theta_{A1} - \theta_{B0})$$

Alice $x = 1$, Bob $y = 1$

$$P_{1,1} = |\langle_{AV0}(\theta_{A1}) \otimes \langle_{BV1}(\theta_{B1}) \otimes |\Psi\rangle_{AB}|^2 + |\langle_{AV1}(\theta_{A1}) \otimes \langle_{BV0}(\theta_{B1}) \otimes |\Psi\rangle_{AB}|^2$$
$$P_{1,1} = sin^2(\theta_{A1} - \theta_{B1})$$

CONCLUSIONS FROM THE GAME

If Alice and Bob adopt the strategy that allows them to "rotate to maximize winning probability" (strategy 3 for Alice and strategy 3 for Bob), they have a greater chance of winning while utilizing the quantum system as opposed to the classical system (85% > 75%)

INTERPRETING THE INEQUALITY VIOLATIONS

Since the very earliest experiments, deviations from the predictions of the Bell inequalities have been found. The initial configuration brought up a number of concerns regarding the appropriateness of the experimental setup. These concerns included the effectiveness of the detectors, the space-time separation of the detectors on both sides of the experiment, and a number of other issues.

The majority of the initial criticisms have been addressed by using settings that are more sophisticated, which has also helped to close a large number of the loopholes that were described in the literature. The dispute about the veracity of the observed breach of Bell inequality is still going strong despite the incredible experimental inventiveness and astounding technology advancements that have been made. In what follows, we conduct an analysis of the implications on the presumption that the experiments do in fact demonstrate a breach of Bell's inequalities.

Throughout this conversation, it has been clear that there are several theories that can give rise to a variety of distinct disparities. The acceptance of the experimental violation indicates that some of the hypotheses need to be called into question for each and every one of them. Both the first and third inequalities may be derived from the same set of hypotheses. The vast majority of the research that has been done on this topic focuses on the concepts of location and independence.

As its name implies, locality is perceived through its relation with space, and it relates to the impossibility of reciprocal impact between occurrences in space-time whose gap is space-like. This interpretation is based on the fact that locality is read through its relation with space. Probability independence, also known as statistical independence, is often considered to mean independence from the physical circumstances.

However, context can also play a role in probability independence. The repercussions of their actions have been the subject of much discourse in a variety of exemplary writings.

We shall not focus excessively on them. When any of these three inequalities are violated, it calls into question any component of the probability space, whether it the sample space, the event space, or the probability measure. This interpretation is possible because each of these inequalities may be understood in its own unique way. First, we should investigate the possibility that the problem is within the sample space. To put it another way, the sample space that was evaluated does not accurately represent the experimental circumstance.

As a result of the fact that all sample spaces contain hidden variables, it is possible to fall prey to the temptation of concluding that the sheer existence of hidden variables ought to be disregarded, which would render the application of any of the sample spaces useless. This viewpoint is also rather close to the traditional interpretation of quantum physics. In particular, it is connected to the ongoing discussion on the exhaustiveness of quantum mechanics.

KOLMOGOROV'S AXIOMS

However, in light of the fact that there exist theories such as Bohmian mechanics that make use of non-local hidden variables and are able to effectively recreate the findings of quantum experiments, it is difficult to maintain this stance. One might make a case for or against the utilization of the sample spaces 2 and 3, respectively.

The simultaneous assignment of values for non-commuting observables has been denied ever since the early days of quantum physics; nonetheless, these sample spaces continue to practice the assignment of values in this manner. In the same vein, an instrumentalist stance will argue against these spaces on the grounds that there are values that cannot be seen simultaneously. This idea is summed up in the well-known statement that unperformed experiments have no findings.

In order to proceed, take into consideration that the error is in the event space. It is necessary to complete it using unions and intersections. It is feasible to accept the sample spaces 2 and 3, but to reject their event space. One may make the case that one should only take into account events that can be witnessed, and this would be consistent with this possibility. It is necessary to assign a probability to every occurrence and to ensure that Kolmogorov's axioms are satisfied. As a result, one may argue that the sample and event spaces actually apply to the scenario, despite the fact that not all occurrences have a probability associated with them.

FINAL REMARKS ON THE INEQUALITY VIOLATIONS

The presence of the joint probability for the four variables of interest, i.e. P(A, A′, B, B′), is a condition that is both required and sufficient to conclude Bell inequality 2. As a result, the violation of the inequality might be understood as a joint probability that does not exist. This line of reasoning may be traced back to Fine, where it was also investigated.

The idea that violations of Bell's inequalities invalidate the sample space, the event space, and/or the probability measure can be summed up with the statement that "Bell inequality violation just proves that there cannot be a reduction to one common probability space." This is one interpretation of what happens when Bell's inequalities are violated. The possibility that the Kolmogorov axioms can be applied is the very last hypothesis that has to be investigated. Accepting the validity of the probability space and calling into question the application of the Kolmogorov axioms (3) to this circumstance is still another method for making sense of the fact that the Bell inequalities have been violated.

It is guaranteed by the axioms that the probabilities have values between zero and one, that they add up to one, and so on. In the context of quantum mechanics, a phase space that includes non-commuting observables like position and momentum is commonly used as a probability space. However, the "probabilities" that are defined there do not fulfill the non-negativity Kolmogorov's postulate in the same way that the Wigner quasi-probability distribution does.

There are also distributions that do not have a negative value, but the marginals of these distributions, like the Husimi distribution, do not agree with the quantum mechanical probabilities found in position 15 of 19 or momentum. There has been research in fields other than physics that investigate the relationship between contextuality and negative probability.

CONCLUSION

We showed that a model with local hidden variables can be used to approximate the CHSH inequality in all of its specifics as well as the quantum correlation, and that this can be done to whatever level of desired precision that is required. This does not provide a convincing proof that locality and causality can be brought back into quantum theory. However, this casts doubt on any claims of conclusiveness about evidence that physical experiment can do away with locality and causation.

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