LEXTRA LEARNING

ADVANCED LEVEL MATHEMATICS

A2 Differentiation and Integration – Year 13

Name: TIME ALLOWED: 1 HOUR

<u>INSTRUCTIONS TO CANDIDATES</u>

1. This examination paper contains **6** questions and all are compulsory.
2. Answer all questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT an OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

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Question 1. The curve C has equation $x^2 \tan(y) = 9$ for $0 < y < \frac{\pi}{2}$

(i) *(4 marks)* Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(ii) *(3 marks)* Show that the point of inflection of C lies at $x = \sqrt[4]{27}$

Question 2. *(7 marks)* Evaluate the following integral
HINT: Use substitution $\alpha = \ln(x)$, then integrate by parts

$$\int \sin(\ln(x))dx$$

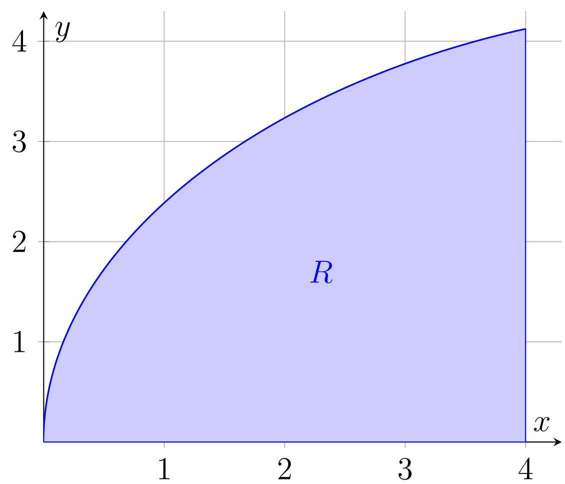

Figure 1: The curve C, with shaded region R

Question 3. For $0 \leq t \leq \dfrac{\pi}{2}$, the curve C has following parametric equations

$$\begin{aligned} x(t) &= 8\sin^2(t) \\ y(t) &= 2\sin(2t) + 3\sin(t) \end{aligned}$$

(i) *(5 marks)* Show that for region R bounded below the curve to x-axis from $x = 0$ to $x = 4$, as shown in Figure 1 has area given by

$$\int_0^a \left(8 - 8\cos(4t) + 48\sin^2(t)\cos(t)\right) dt$$

(ii) *(4 marks)* Hence, use algebraic integration to evaluate the area in region R as shown in Figure 1

Question 4. There exists a smooth and continuous function $y = f(x)$ which obeys the following differential equation for $0 < y < 1$

$$2\frac{dy}{dx} = y - y^2$$

(i) *(6 marks)* Using partial fractions or otherwise, find the general solution to the differential equation in the form $y = f(x)$

(ii) *(3 marks)* Evaluate the following limits

$$\lim_{x \to +\infty} f(x)$$
$$\lim_{x \to -\infty} f(x)$$

Question 5. *(8 marks)* Boyle's Law states that when a gas is kept at a constant temperature, its pressure P, which is measured in Newton per meter Nm^{-2}, is inversely proportional to its volume V, which is measured in cubic meters m^3.

When the volume of a certain gas is $80m^3$, its pressure is $5Nm^{-2}$ and the rate at which the volume is increasing is $10m^3s^{-1}$.

Find the rate at which the pressure is decreasing at this volume.

Question 6. A curve has equation $y = \dfrac{xe^{2x}}{x+k}$, where k is a constant and $k \neq 0$.

(i) *(5 marks)* Show that

$$\frac{dy}{dx} = \frac{e^{2x}\left(2x^2 + 2kx + k\right)}{(x+k)^2}$$

(ii) *(5 marks)* Given that y only has one stationary point, find the value of k and determine the nature of this stationary point.

END OF PAPER