Falling Forward : The Fermi paradox, supermassive black holes, and the likely flight of extraterrestrial intelligence toward extreme time dilation.

Chris Reiss

christopher.j.reiss@gmail.com

February 27, 2021

Abstract

It is argued that an advanced extraterrestrial intelligence has a compelling incentive to permanently occupy a reference frame where time is greatly slowed by relativistic effects. We then show that a near orbit around a supermassive black hole provides a viable - and perhaps only - possibility for such a migration. The expected character of radio emissions from such a civilization is discussed to suggest a method of potential detection.

1 Introduction

We suggest a resolution to the Fermi paradox, and propose a method which may be uniquely capable of detecting radio emissions from an Extra Terrestrial Intelligence (ETI). Suppose there exists an advanced ETI that, like us, is intrinsically curious. It seeks to explore the Cosmos, to discover life and to communicate with other ETIs. The form such a civilization might take has long been a topic of speculation. A familiar example is the Dyson Sphere hypothesis that an artificial shell is constructed around a star in order to harness its energy.

The Dyson Sphere presupposes that energy is a critical constraint to an interstellar life form. This is a reasonable assumption, but one which evokes a lingering suspicion that we - as early 21st century humans struggling to transition from chemical to nuclear energy - are guilty of mistaking our current problems for the more fundamental and enduring problems likely to confront a much more advanced civilization.

Before we speculate as to the possible form of an ETI civilization, we pause to consider : *What would their biggest problem necessarily be?* We seek the most universal limitations that would obstruct a civilization's ambition to explore and communicate. That is, limitations inherent to the laws of the space-time they share with us.

From this vantage, the cosmic speed limit of c appears the primary obstacle to a curious ETI. If an ETI on the other side of our Milky Way were to send a signal to Earth we would not receive it for 100,000 years. They would have to wait 200,000 years for the reply.

Special Relativity does, of course, allow for an explorer from that ETI to travel to Earth in a week, day, or any arbitrarily small duration as measured by that explorer. But 100,000 years will still have elapsed on Earth during the trip, and 200,000 will have elapsed on their home world upon their return. It has been suggested that a wormhole or other topological anomaly may be discovered or created in order to circumvent this universal speed limit. We propose another approach - rather surprising in its simplicity - which compensates for the c-limit but does not rely on exotic (and undiscovered) features of space-time.

2 Definition of a red-frame

While the phenomenon of time dilation is not at all novel, we typically think of it in terms of a traveller whose 'home time' is a planet or structure where clocks tick at roughly the same rate as Earth. In this paper we propose a drastic change in perspective where the 'home time' itself is untethered from its natural origin and deliberately slowed by orders of magnitude. The entire civilization has made a permanent transition and now experiences external events differently.

Such time-frames exist within our current understanding of General Relativity. They arise from motion at very high speeds (close to c), or from very intense gravity induced by a nearby massive object. For the purposes of this discussion,we shall refer to a reference-frame which is much slower than our own as a 'red-frame'. Frames with similar time-rates as ours we shall call 'blue-frames'.

We set practical considerations aside momentarily for the sake of a thought experiment in which we explore the prospects and perspective of a hypothetical red-frame civilization whose time is slowed by a factor of 100x.

3 Advantages of a red-frame : New prospects for exploration, communication and production

We assumed at the outset a race of beings who, like us, seek to explore distant solar systems. As seen from their new red-frame home, distant objects take on very different properties. External clocks are seen to run 100x faster and - most critically - external distances contract by 100x according to a local ruler. [1]

The red-frame sky becomes ablaze with the light of stars 100 times closer and blue-shifted due to time dilation.

It becomes evident that a red-frame civilization is vastly more capable of exploration. A radio signal can envelope the entire Milky Way in a mere 500 years; a response could be detected within 1,000. It is the same for physical exploration near-light speed. [2]

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[1]To clarify : They would observe a beam of light originating from their red-frame and reflected off a planet 500 light years away (by a blue-frame's ruler) to return in just 10 years local (proper) time. General Relativity cautions that this is not an illusory effect of time dilation; observations from the world-line of the red-frame have no less claim to reality or primacy than observations made from Earth.

[2]The advantage is not for the explorer travelling near C, but rather the society that sent them. The home world won't have to wait eons for their explorer to return.

Other advantages also emerge. The relative acceleration of time in any exterior blue-frame can be harnessed to the red-frame's benefit. Any productive process performed in a blue-frame becomes 100x faster. This accelerates many forms of work an ETI might wish to undertake: computation, harvesting energy, assembly of mega-structures, terra-forming of planets, and so on.

Such a blue-frame productive 'engine' can accelerate minds as well as machines. Some portion of this ETI race could 'drop down' to a nearby blue-frame in order to conduct scientific research and other intellectual labor. Their red-frame home would be the happy recipient of a century's progress delivered annually.

4   Orbit of a red-frame

We consider how our hypothetical red-frame might be actualized by an advanced civilization, by means of an artificial mega-structure (or natural body which is intentionally accelerated.)

General Relativity provides for an arbitrary degree of time dilation at very high speeds and in intense gravity. Intuition suggests that we maximize both by placing the red-frame in near orbit around a supermassive black hole (SMBH). In the *Discussion and Conclusions* section we justify this intuition, eliminating other possibilities in favor of such an orbit as the best - and possibly only - case to consider.

Here we simply turn our attention to the supermassive black hole at the center of the Milky Way, Sagittarius-A* (Sgr A*.)

We make the simplifying assumption that Sgr A* is a Schwarzschild black hole (i.e., has no spin nor electric charge.) We take the mass to be 4.1 M Solar Masses [Schoeder et al. (2003)] yielding a Schwarzschild Radius $R_s = 12 \times 10^9$m

The Innermost Stable Circular Orbit (ISCO) is located at $R_{sco} = 36 \times 10^9$m from the center.

The photon sphere is at $R_{ph} = 18 \times 10^9$m

A satellite in the ISCO of a Schwarzschild black hole of any mass has a time dilation factor of $\gamma = 2/\sqrt{3}$ [see Eq. (1) below]. Since we seek a more extreme dilation of $\gamma = 100$, we must consider orbits that are much closer than the ISCO and are thus unstable.

These orbits would require constant minute corrections to their speed in order to hold their orbit. (The energy expenditure is arbitrarily small if orbital drift is detected quickly enough.) The observation of a satellite in steady circular orbit in the unstable region between R_{ph} and R_{sco} is suggestive of an ETI simply due to the inability of a natural satellite to hold such an orbit.

For a satellite in Keplerian orbit at radius r around a Schwartzschild black hole, the time dilation factor γ is found by multiplying the gravitational red-shift $1/\sqrt{g_{tt}}$ by the Lorentz boost $\dfrac{g_{tt}}{1 - 3M/r}$ where $g_{tt} = 1 - 2M/r$ to give

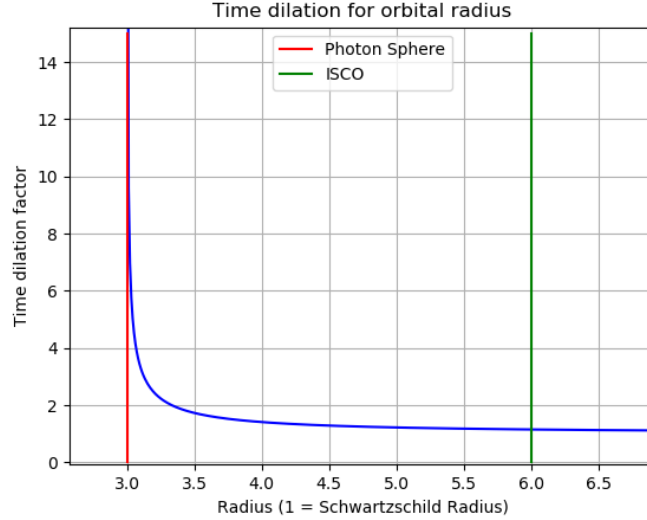$$\gamma = \sqrt{\frac{r - 2M}{r - 3M}} \tag{1}$$

[see, e.g., Opatrný et al. (2017)]

Solving for r gives $r = 3M + \dfrac{M}{\lambda^2 - 1}$ which we prefer to write as :

$$r = R_{ph} + \frac{R_{ph}}{3(\lambda^2 - 1)} \qquad (2)$$

A plot of Eq.(1) illustrates that orbits of significant time-dilation lie very near the photon sphere :

Figure 1

Time dilation for orbital radius

By way of example, we find the circular orbit around Sag A* where time dilation would slow clocks by a factor of $\gamma = 100$. Referring to $R_{ph} = 18$ M Km and Eq.(2), we find the corresponding orbit at $r = R_{ph} + 600$ Km, a mere 600 Km outside the photon sphere.

Although a point mass can rest comfortably in that orbit, a mega-structure is subject to tidal forces which are of considerable concern in such intense gravity. We need to calculate this force to ensure it doesn't tear the structure (or its inhabitants) apart.

The natural way to design a structure to minimize tidal forces is to extend it along an arc of the circular orbit :

We will set the maximum tidal acceleration we can tolerate and calculate the corresponding 'height' (radial extent) that it allows. This height is given by,

$\Delta r = \dfrac{A_t r^3}{3GM}$ where A_t is tidal acceleration. [A 7.1]

Figure 2: *Tidal forces for orbiting structure*
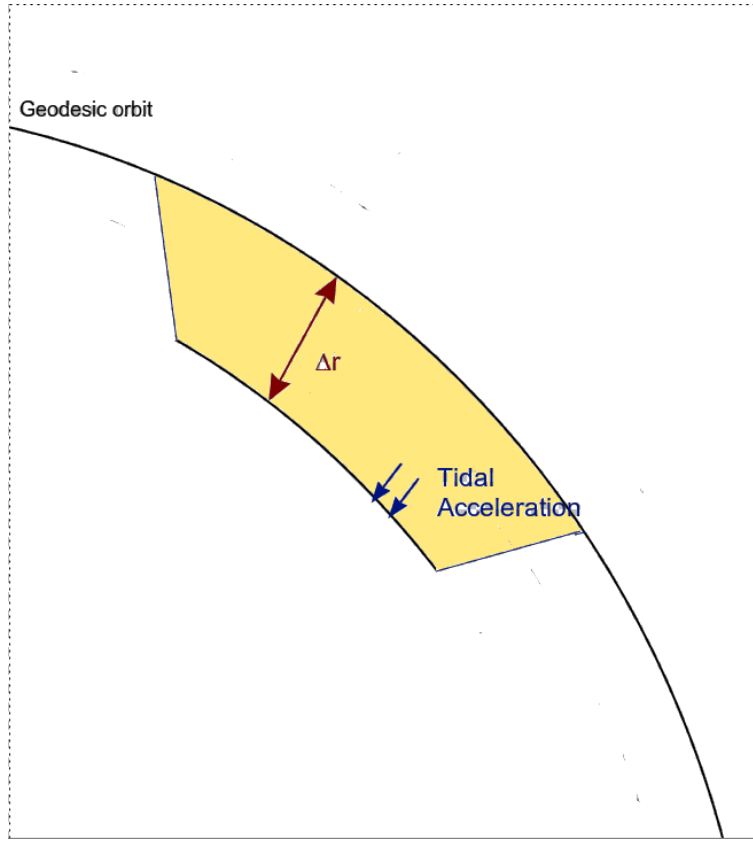
For our example of Sag A*, we set the tidal acceleration equal to the surface gravity of Earth and evaluate :

$$\Delta r = \frac{9.8(18 \times 10^9)^3}{3(6.67 \times 10^{-11})(8.15 \times 10^{36})} = 35 \text{ Km}$$

35 kilometers seems like quite a generous high ceiling - but we must take care to account for the dilation of space within the megastructure. In order to maintain the constancy of local measurements of **c**, space must dilate in proportion to the dilation of time. Given our $\gamma = 100$, the height is measured to be 350 meters within the structure. We can double this to 700 meters in order to create a two-layer structure as shown below.

This straddling of the orbit is necessary not only to maximize radial height; the tidal forces must be balanced to hold orbit. Interestingly, these tidal forces are seen to serve a positive role providing the inhabitants with a comfortable sense of gravity.

While 700 meters is admittedly a bit cramped for a civilization-bearing megastructure, this restriction in a single dimension does not appear fatal to our hypothesis. We note that the structure can extend without bound along the orbital path, yielding surface area greater than Earth's.

Figure 3: *Doubling the height and balancing the tidal force.*
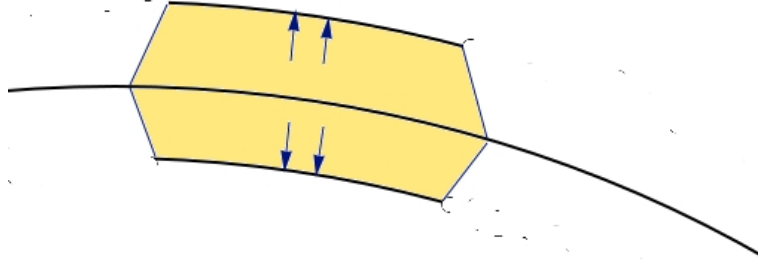
We caution that our calculation of the tidal force neglects relativistic effects; it is only intended as an illustrative approximation. For a full relativistic treatment see, for example, [Ydri (2017)]

The other force to be considered is the Coriolis effect. Within the structure, a massive ball suspended on a string will be seen to rotate once every orbital period. We need to calculate this period to ensure that the Coriolis effect isn't hazardous to such delicate biological structures as ourselves.

As seen from afar, the orbital period for any dilation is found by applying Kepler's Third Law to the radius given in Eq. (2) :

$$\text{orbital period} = \Omega/2\pi = \frac{1}{2\pi}\sqrt{\frac{GM}{[r_{ph}(1+\frac{1}{3\gamma^2})]^3}}$$

Again we need to correct for local space-time dimensions; within the satellite, time dilation reduces the observed local period to *period*$/\gamma$. For our example of Sag A* and $\gamma = 100$, the local orbital period is found to be 6.489 seconds.

Thus our ball suspended by string would rotate once every 6.489 seconds. While this effect is far from negligible, it also does not appear a fatal flaw to our hypothesis. The plot below shows the Coriolis effect is intense only for large time dilations.

Figure 4
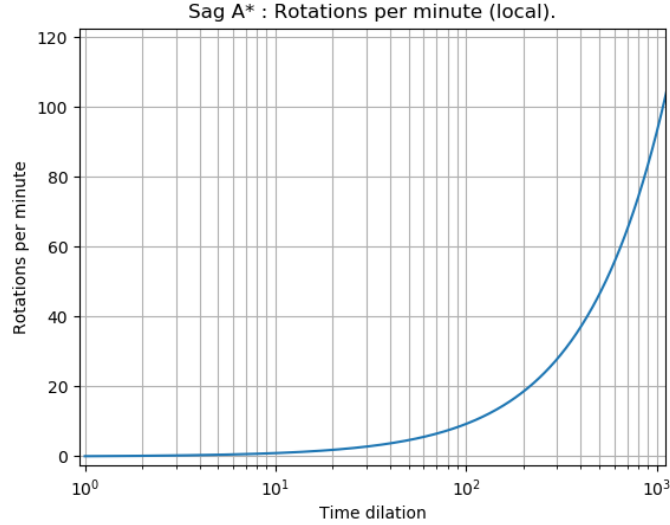
The other challenges of occupying such a hostile region of space, enduring the onslaught of both inbound matter and outbound radiation, certainly appear daunting. But these hazards are perhaps no more difficult than Apollo 11's defiance of heat, gravity, radiation and vacuum would have seemed to the Wright brothers.

These hazards might be mitigated by an orbit which is tilted with respect to the galactic plane in order to minimize interaction with matter inbound from the accretion disk. (Such a non equatorial orbit could make it easier to detect the structure; EM emissions would acquire a vertical polarization which stands apart from other sources in the galactic plane.)

There is also recent evidence to support that Sgr A* may be unusually 'tame' due its magnetic field which tends to hold the accretion disk a safe distance from the ISCO. [Bell et al.(2020)]

If we posit that such a red-frame satellite exists and is producing radio emissions, either intentionally or incidentally, we can make certain inferences about that signal. The prospects of detecting a radio signal emitted so close to Sgr A* do seem bleak due to the radio noise Sgr A* produces across a broad spectrum. However, the unique orbit of a red-frame satellite would lend a distinct signature to any radio signal it emits. We look to the combined effects of doppler beaming and gravitational lensing.

5   EM signature of a red-frame satellite

Consider a red-frame satellite of a SMBH which is emitting some EM signal. We examine how that signal would differ from those of natural bodies in stable orbit. We seek criteria that can uniquely identify and separate the red-frame signature from nearby natural sources.

Ideally, these criteria will be independent of the black hole mass (M) about which we may be

uncertain, even for Sag A*.

For a Schwartzschild black hole of any mass, the (local, proper) orbital speed of a test particle in stable orbit in the ISCO is fixed at $v_o = 1/2$ (see Eq.(3)) At this speed, the doppler shift will vary between some minimum F_{low} as the satellite recedes along the line-of-sight and $F_{high} = 3\,F_{low}$ as it directly approaches.

A red-frame satellite in unstable orbit below the ISCO will have a higher orbital velocity which induces a doppler shift cycling outside the range of $[F_{low}, 3F_{low}]$.

This feature alone, of course, is insufficient to classify a signal as a red-frame technosignature. To sharpen our search criteria, we examine more fully the effect of relativistic doppler beaming :

It is a little counter-intuitive that, despite the red-frame's perilous proximity to the SMBH, most of the time-dilation is due to motion and not gravity. Since the radius (for any time dilation) must be outside the photon sphere, this bounds the gravitational red-shift :

$$\text{Since } r > 3M, \text{ gravitational red-shift } \lambda_g = 1/\sqrt{g_{tt}} = 1/\sqrt{1 - \frac{2M}{r}} < \sqrt{3} = 1.732 \ .$$

Any time dilation beyond 1.732 is induced by orbital motion close to the speed of light. For any circular Keplerian orbit about a Schwartzschild BH, the local orbital velocity v_o is given by [see e.g., Opatrny et al., (2017)] :

$$v_o = \sqrt{\frac{M/r}{1 - 2M/r}} \tag{3}$$

For our purposes it is convenient to arrange this as:

$$v_o = \frac{1}{3\dfrac{r - R_{ph}}{R_{ph}} + 1}$$

Substituting for r via Eq. (2) gives simply,

$$v_o = \sqrt{1 - 1/\lambda^2} \tag{4}$$

For our example satellite with $\lambda = 100$, the orbital velocity is $v_o = 0.999950$.

At such speeds, the effect of relativistic doppler beaming becomes far more pronounced than for a body in natural orbit.

5.1   Doppler beaming

For a light source moving at speed v and an angle θ with respect to the sight-line of an observer, the observed frequency F_o varies from the source frequency F_s according according to [see e.g., Hartle (2003)],
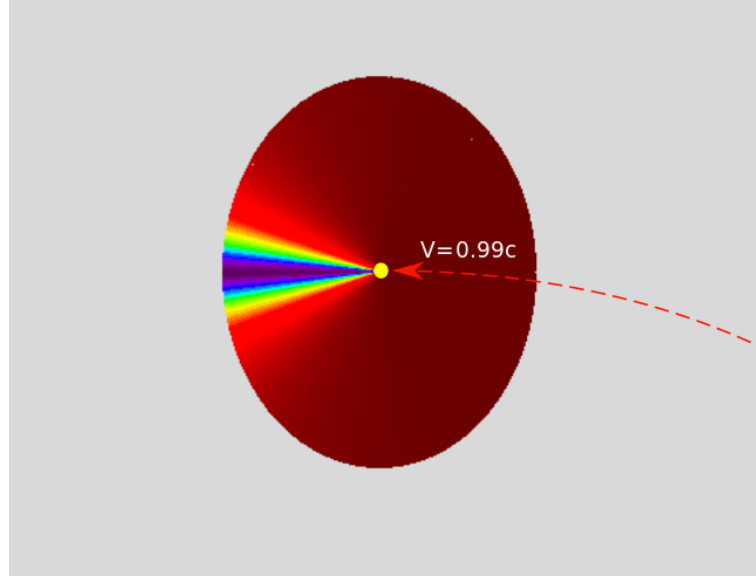
$$F_o = \frac{F_s\sqrt{1-v^2}}{1 - v\cos(\theta)} \tag{5}$$

Substituting for v via Eq. (4) gives :

$$F_o = \frac{F_s}{\lambda - \sqrt{\lambda^2 - 1}\cos(\theta)} \tag{6}$$

As velocity approaches the speed of light (and λ increases), this 'beams' almost all the signal energy into a narrowing cone as illustrated in Fig 5 :

Figure 5: *Illustration of doppler beaming close to **c***

For our example of $\lambda = 100$, 67% of the energy is contained within 2 degrees of the sight-line [A2] . The direction of the beam keeps time with the revolution of the satellite. So for a distant observer, the signal frequency varies with time according to $F_o(\omega t) = F_o(\theta)$ where ω is orbital angular velocity:

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Figure 6: *Frequency at a distant receiver. Right side: zooming in on pulse.*
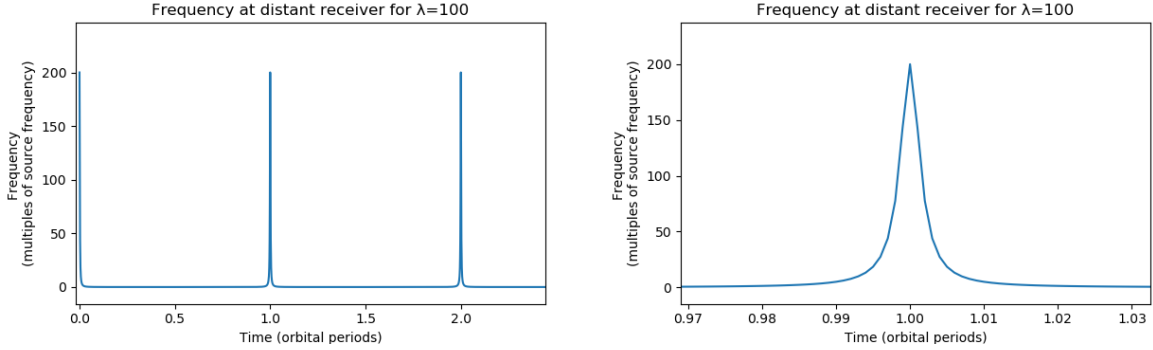
At near-light speed, Doppler beaming channels the signal energy into a series of sharp, symmetric pulses. Between pulses the signal may be so weak as to be inseparable from radio noise.

Thus far in our discussion, the signal is difficult to distinguish from a variety of natural periodic sources.

However, our prospects improve when we account for gravitational lensing. Since the red-frame orbit is close to the photon sphere, gravitational lensing is intense and introduces a dramatic effect.

5.1 Gravitational lensing

In any gravitational field, photons follow a null geodesic path x(s) according to [see, e.g., Lambourne (2010)]

$$g_{\mu v}dx^\mu dx^v = 0 \ , \ \frac{d^2 x^\mu}{ds^2} + \Gamma^\mu{}_{v\sigma}\frac{dx^v}{ds}\frac{dx^\sigma}{ds} = 0 \text{ for affine parameter s.}$$

For g the Schwartzschild metric about a black hole without spin or electric charge, in spherical coordinates chosen so the path lies in the equatorial plane, this path also uniquely satisfies [Bruneton (2020)][Weinberg (1972)] :

$$\frac{d^2(u)}{d\theta} = u^2 R_{ph} - u \text{ where } u = 1/r$$

Since this differential equation admits no analytic solution, we resort to numerical methods. The light paths are calculated using the fourth order Runge-Kutta method.

Accounting for lensing in this way, the light paths from doppler beaming (Fig 5) are bent in a distinctive way :

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Figure 7: *Lensing of the doppler beam near the photon sphere where $\lambda = 100$*
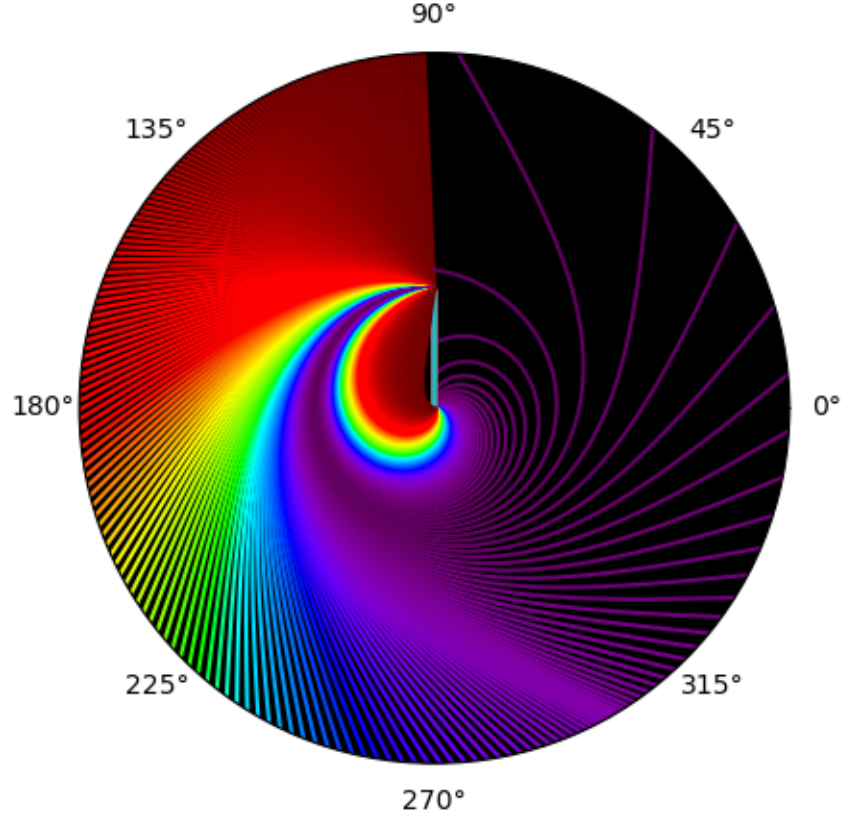
We see that lensing has sharply broken the time-symmetry of our pulses. The left half of each pulse, where frequency rises toward its peak, has fallen into the black hole. (This accords with intuition since these angles dip below the photon sphere.)

Let us make precise this relation of observed frequency to time : $F_o(t)$.

In the computation used to create Figure 7, a series of light rays are traced from the emitter (our orbiting satellite) at emission angle θ_{emit} to their impact on an outer circle O at angular location θ_{out}. The radius of O is taken to be large with respect to the photon sphere radius, so that the beam approximates one moving in a straight line from O's center at angle θ_{out} (That is, the photon crosses O at a point beyond the effects of lensing at a distance large compared to the ray's minimum distance to O's center.)

Figure 8: *Illustration of angles. Outer circle is greatly reduced for visual clarity; we calculate with circle of radius large enough that the satellite's orbital radius vanishes in comparison.*
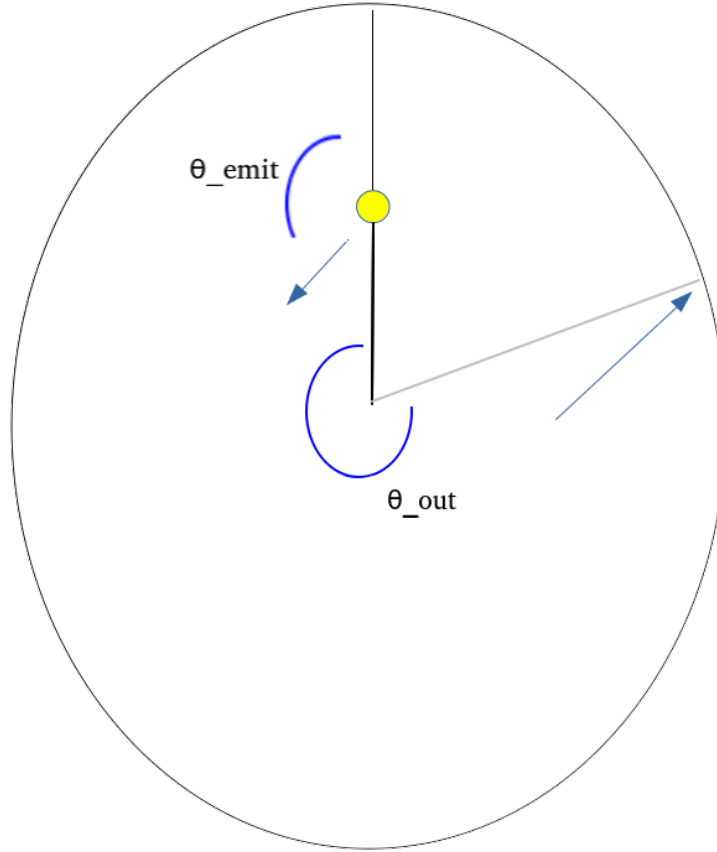
We seek to extract the relation between θ_{emit} and θ_{out}. (This relation depends only on the orbital radius, and thus the time dilation factor in Eq. (4). Subsequent calculations show that this relation does not vary significantly for any $\lambda > 20$.) We calculate $\theta_{out}(\theta_{emit})$ for $\lambda = 100$ and a range of values in $0 < \theta_{emit} < \pi/2$ and plot:

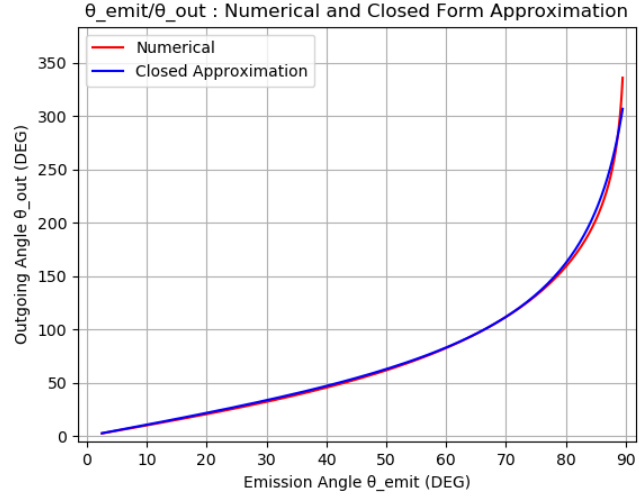**Figure 9**

Angle : Outgoing vs. Emission
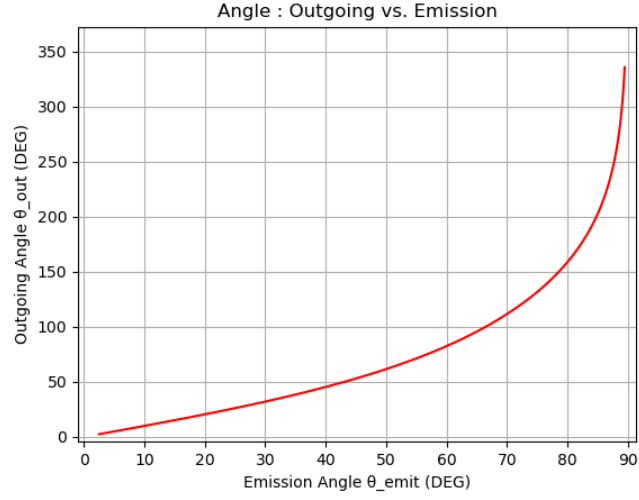
It is convenient to find a closed form function to closely approximate this curve. Some trial, error, and a little luck reveals :

$$\theta_{out} \approx \frac{\pi}{2}\sqrt{\sec\left(\frac{3}{\pi}\theta_{emit}\right) - 1}$$

This formula is in close agreement ($< 1\%$) across the range of θ_{emit}:

Figure 10

θ_emit/θ_out : Numerical and Closed Form Approximation

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Since we wish to find $F_o(t)$ and given that $\omega t = \theta_{out}$, we invert $\theta_{out}(\theta_{emit})$ to $\theta_{emit}(\theta_{out})$:

$$\theta_{emit} = \frac{\pi}{3} \sec^{-1}[(\frac{2}{\pi}\theta_{out})^2 + 1]$$

We can now substitute ωt for θ_{out} to get :

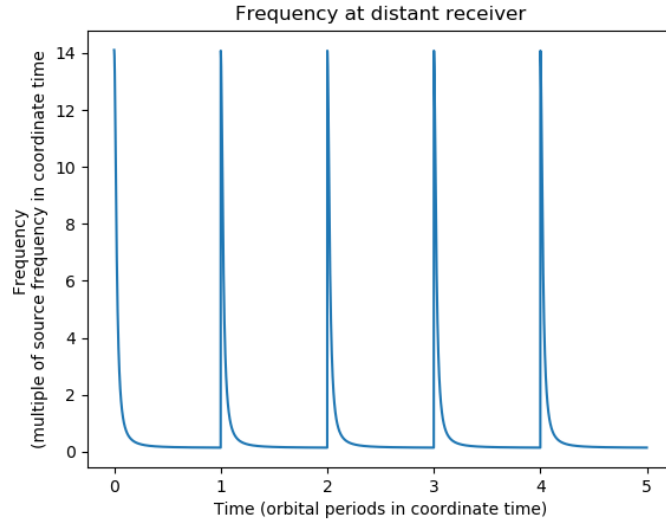$$\theta_{emit} = \frac{\pi}{3} \sec^{-1}[(\frac{2}{\pi}\omega t)^2 + 1].$$

Substituting θ_{emit} into the Doppler beaming formula Eq. (6) yields the frequency-time dependence for a distant receiver :
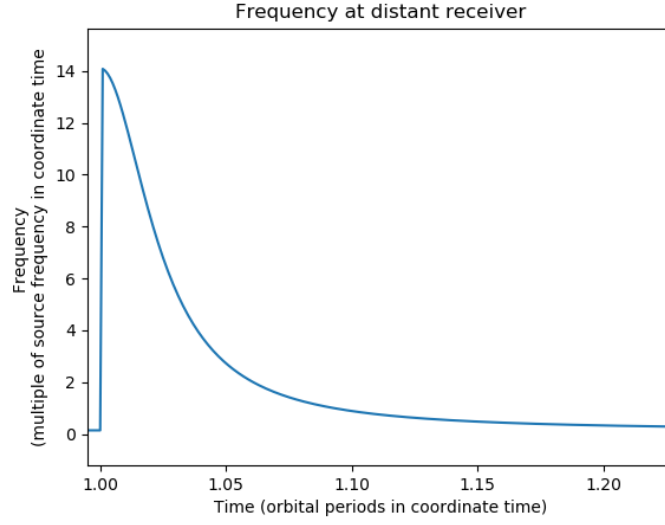
$$F_o(t) = F_s \frac{1}{\lambda - \sqrt{\lambda^2 - 1}\ \cos(\frac{\pi}{3} \sec^{-1}[(\frac{2}{\pi}\omega t)^2 + 1])} \tag{7}$$

This gives a series of time-asymmetric pulses as plotted in Fig. 11 ($\lambda = 100$).

Figure 11

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Figure 12: *Zooming in to a pulse from Fig 11*

For a Schwartzschild SMBH, a periodic series of pulses of this form is suggestive of an artificially guided satellite in unstable orbit very close to the speed of light.

This curve bears a striking congruence with the flux density measurements from Fast Radio Burst FRB110220 (see Fig. 1 from [Thorton et al. (2013)]) :

Figure 13: *Overlay of figure 12 with [flux density/time] for FRB110220*

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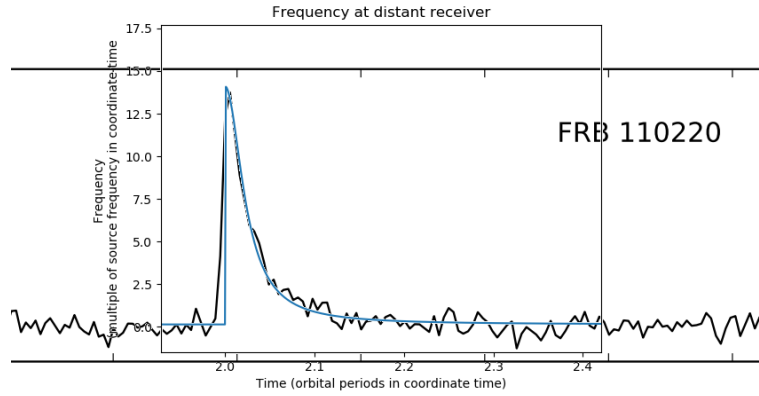
6 Discussion and Conclusions

We've shown that it is possible for an ETI to overcome the obstacle imposed by the speed of light by permanently occupying a reference-frame where time is dilated (slowed) by orders of magnitude. One possible location of such a reference-frame is shown to be in near-orbit around a SMBH between the photon sphere and Innermost Stable Circular Orbit.

A method is proposed for detecting an artificial mega-structure in such an orbit: EM radiation from such a source is shown to beam into intense pulses of declining frequency according to Eq. (7)

Our assumption of a Schwartzschild black hole neglects the role of black hole angular momentum and electric charge. It remains to more fully develop this line of reasoning using the Kerr-Newman metric.

We have focused exclusively on a SMBH near-orbit as a means to extreme time dilation. General Relativity does, however, allow for other possibilities. We examine each of these in turn and justify setting them aside as candidates for a red-frame megastructure.

- **Against Uniform Motion**: One may question why we've bypassed the apparently simplest case of uniform motion near C. Here the motivating advantages of time dilation are lost as the red-frame loses the ability to maintain contact with blue-frame colonies or star systems of interest. For these purposes, a red-frame must remain confined to a region of space whose dimensions are not large relative to interstellar distances

- **Against gravity alone**:(I.e., stationary on the surface of a massive object.) We can quickly dispense with this possibility as a time dilation factor = 2 would place the red-frame on a body so dense it would collapse into a black hole, pulling the red-frame contents into the event horizon.

- **Against thrust alone**:We might consider a circular path maintained by constant thrust rather than gravity. Here we rely on a mechanical force - thrust - to provide the inward acceleration to trace a circular path. At the speed required to achieve a 100x dilation - if we fix the radius to be that of Pluto's orbit, the red-frame experiences crushing (local, proper) centripital acceleration of the order of 100,000 G. If we fix the (local,proper) centripital force to be 1G, the required radius balloons up to the order of 10,000 light years. This violates our requirement that the red-frame is spacially constrained. In addition, there would be a large and constant energy expenditure.

- **Against stellar black holes** A stellar black hole has at most (with rare exceptions), 15 solar masses. This gives a Schwartzchild radius of 40 Km, about 1000 times smaller than that of a SMBH. This puts two critical values for a red-frame satellite out of apparently plausible range for a civilization-bearing mega-structure: For a time-dilation factor = 100 the local (proper) orbital frequency will rise to 500 Hz, making coriolis effects intense. The radial 'height' will constrict to about 1 meter for a tidal force of 1g.

A free-fall orbit near a SMBH appears the most feasible - and perhaps only - possibility. This case has virtually no sustained (theoretical) energy requirement and exerts minimal forces within the red-frame structure. Rather counter-intuitively, it is not the gravitational time dilation (whose overall contribution is small) that matters, but the more familiar gravitational attraction that serves to constrain such terrific speed to a small region of space. (We leave unaddressed the possibility of

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a hybrid approach where the red-frame is outside the ISCO, and kept in orbit with some assistance by constant thrust.

In closing, our hypothesis suggests an explanation for the Fermi paradox : An ETI which is advanced to the point of interstellar travel will migrate at the first opportunity to a red-frame megastructure in order to escape the speed-of-light limitation. By this reasoning, the stars may only appear lifeless because advanced intelligent life has abandoned them in its migration to the galaxy's center. We therefore suggest directing some of SETI's efforts toward the center of the Milky Way and other galaxies.

7 Appendix

[7.1] Calculation of tidal acceleration

Let A_g = Difference in gravitational acceleration over radial distance Δr

$$A_g = \frac{GM}{(r - \Delta r)^2} - \frac{GM}{r^2}$$

$$\approx \frac{GM}{r(r - 2\Delta r)} - \frac{GM}{r^2} \qquad r >> \Delta r$$

$$= GM/r[\frac{1}{r - 2\Delta r} - \frac{1}{r}]$$

$$\approx GM/r[\frac{2\Delta r}{r^2}] \qquad r >> \Delta r$$

$$\approx \frac{2GM}{r^3}\Delta r$$

Let A_c = Difference in centrifugal pseudo-acceleration over Δr

$$A_c = \frac{GM}{r^2}\frac{\Delta r}{r} = \frac{GM}{r^3}\Delta r$$

Let A_t = Net tidal acceleration.

$$A_t = A_g + A_c \approx \frac{3GM}{r^3}\Delta r$$

$$\boxed{\therefore \Delta r \approx \frac{A_t r^3}{3GM}}$$

[7.2] Calculation of energy concentration near sight-line of Doppler beam

$$e = \left[\int \frac{F_s}{\lambda - \sqrt{\lambda^2 - 1}\cos(\theta)} d\theta \right]^2 = \left[\frac{\arctan\left(\sqrt{(\lambda^2 - 1)} + \lambda\right) * \tan(\theta/2)}{\pi} \right]^2$$

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