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In [1]:

```
import matplotlib.pyplot as plt
import numpy as np
```

1 Asymptotic notations

> **Algorithm** - is a set of steps to accomplish the task (c) Khan Academy

What makes good algorithm:

• Correctness

• Efficiency - is how well you are using your computer's resources to get a particular job done.

Complexity – is a measure of algorithm efficiency in terms of time usage.
Ways to approach tasks:

- Divide-and-concuer
- Gready method - the algorithm checks the problem one step at the time and focuses just on this step (often as a part of optimisation).

Asymptotic notations are the mathematical notations used to describe the running time of algorithms when the input tends towards a particular value or a limiting value. It helps to make comparison between different algorithms, especially on large inputs.
High level idea: to suppress constant factors (like system dependency) and lower terms (irrelevant for large inputs). So we simplify function in order to drop the less significant parts.
For example, if the function: $an^2 * bn + c$, we can say that our algorithm takes n^2, so we are dropping:
1) coefficient because it does not matter what coefficient used,
2) last part $bn + c$ because it is relatively small compare to first part n^2

Also it doesn't matter what base of the logarithm we use in asymptotic notation.

> The main idea of asymptotic analysis is to see how algorithms bahave on tail end, when input gets large. (c) Back To Back SWE. YouTube.

There are three main types of asymptotic notations:

```
- big-O - upper bound of an algorithm (the worst case)
- big-Θ or big-Theta - "tight" or "exact" bound. It is a combination of Big
(average case)
- big-Ω or big—Omega - lower bound of an algorithm (best case) or the faste
st the algorithm can go.
```

Examples of different complexity of big-O:
Constant complexity O(1) – the same time no matter how big input. Example:

In [2]:

```
def sum(a, b, c):
    return a + b + c
sum(3, 2, 1)
# takes the same time as:
sum(1000, 300, 111)
```

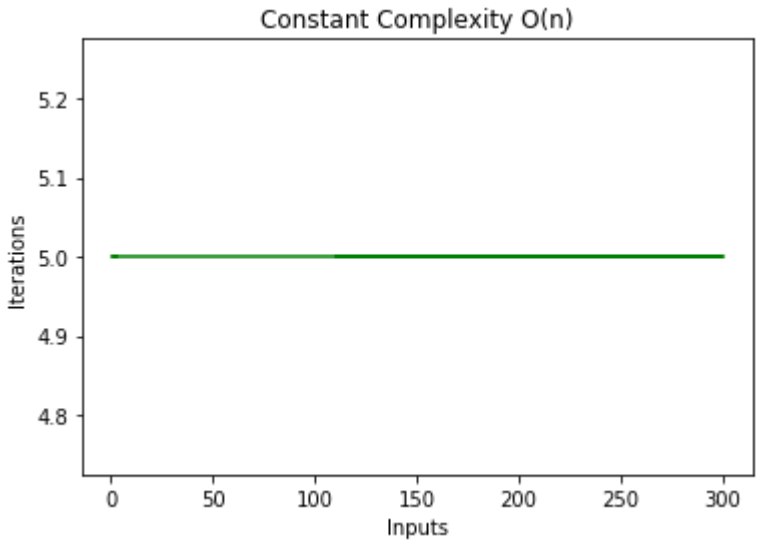
Out[2]:

```
1411
```

In [3]:

```
x = [3, 2, 1, 8, 300, 111] #input(n)
y = [5, 5, 5, 5, 5, 5] #number or iterations

plt.plot(x, y, 'g')
plt.title('Constant Complexity O(n)')
plt.xlabel('Inputs')
plt.ylabel('Iterations')
plt.show()
```



Linear complexity O(n) – the number of operations grows in the same rate as number of inputs. Example (linear search):

In [4]:

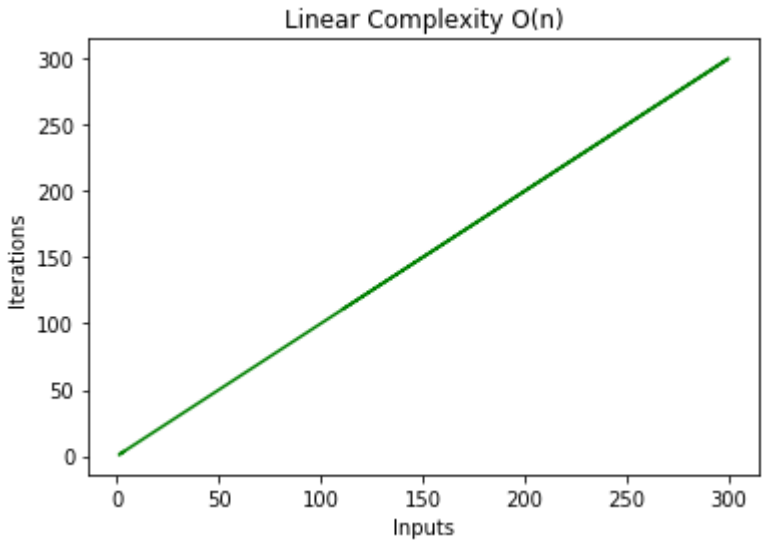
```
foodArray = ['pizza', 'burger', 'sushi', 'curry']
def findFoodInTheApp(arrayOfFood, desiredFood):
    for food in arrayOfFood:
        if food == desiredFood:
            return f'Here is your {food}'
print(findFoodInTheApp(foodArray, 'curry')) #To find a "curry" is taking O(n)
```

```
Here is your curry
```

In [5]:

```
x = [3, 2, 1, 8, 300, 111] #inputs(n)
y = [3, 2, 1, 8, 300, 111] #number or iterations

plt.plot(x, y, 'g')
plt.xlabel('Inputs')
plt.ylabel('Iterations')
plt.title('Linear Complexity O(n)')
plt.show()
```

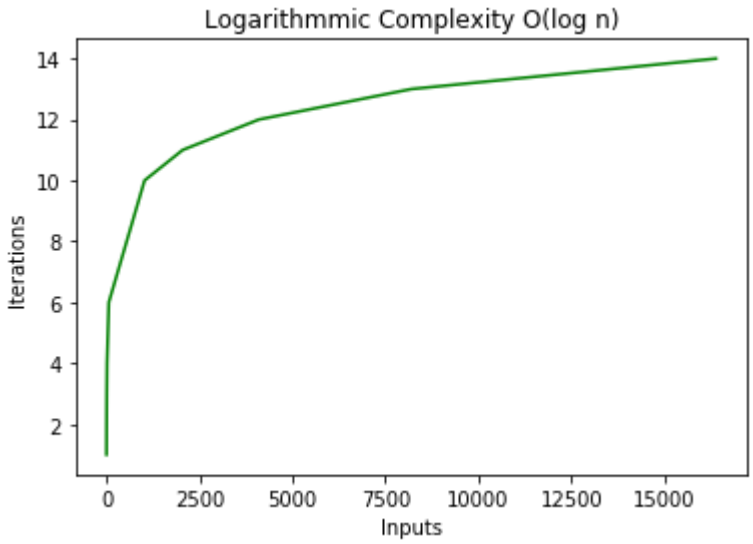


Logarithmic complexity O(log n) – the number of operation grows slower than the inputs. More efficient for larger inputs. Example: binary search (algorithms/searching/binary_search.py)

In [6]:

```
x = [1, 4, 8, 16, 64, 1024, 2048, 4096, 8192, 16384] #inputs grow faster
y = [1, 2, 3, 4, 6, 10, 11, 12, 13, 14] #number or iterations

plt.plot( x, y, 'g')
plt.xlabel('Inputs')
plt.ylabel('Iterations')
plt.title('Logarithmmic Complexity O(log n)')
plt.show()
```

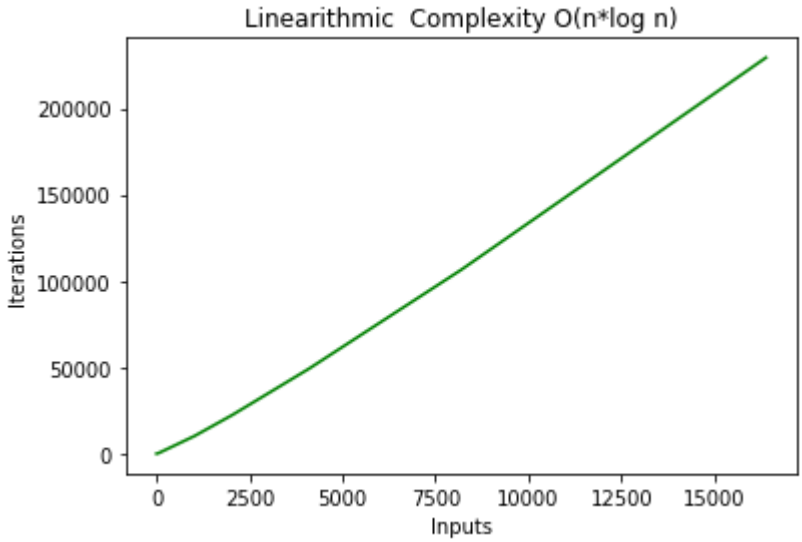


Linearithmic complexity O(n*log(n)) - slower then linear, but better then quadratic. Examples (merge sort, quick sort)

In [7]:

```
x = [1, 4, 8, 16, 64, 1024, 2048, 4096, 8192, 16384] #input(n)
y = [1, 4*2, 8*3, 16*4, 64*6, 1024*10, 2048*11, 4096*12, 8192*13, 16384*14] #number

plt.plot( x, y, 'g')
plt.xlabel('Inputs')
plt.ylabel('Iterations')
plt.title('Linearithmic  Complexity O(n*log n)')
plt.show()
```



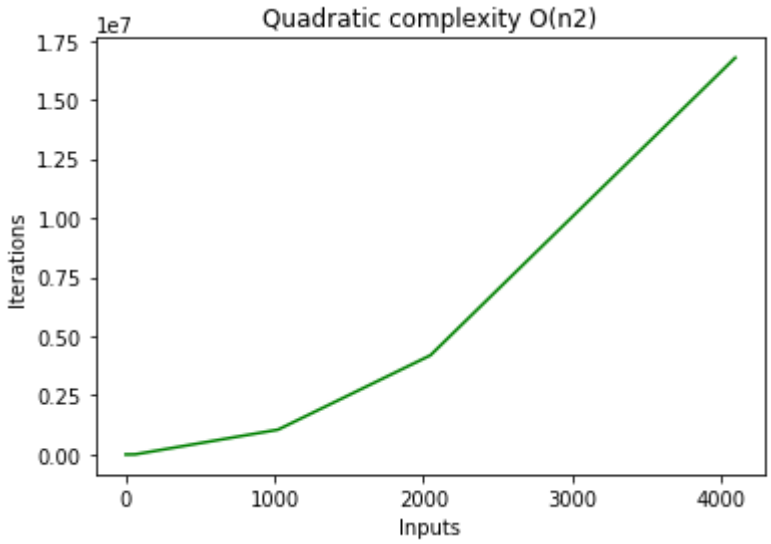
Quadratic complexity O(n^2) - operations grow as a square of the number of inputs. When we have nested iterations. Ex. bubble sort, selection sort, insertion sort

In [8]:

```
x = [1, 4, 8, 16, 64, 1024, 2048, 4096] #input(n)

y = [1, 16, 64, 256, 4096, 1048576, 4194304, 16777216] #number or iterations

plt.plot(x, y, 'g')
plt.xlabel('Inputs')
plt.ylabel('Iterations')
plt.title('Quadratic complexity O(n2)')
plt.show()
```



Exponential complexity O(2n) The algorithm takes twice the number of previous operations for every new element added. Example: recursive calculation of Fibonacci numbers.

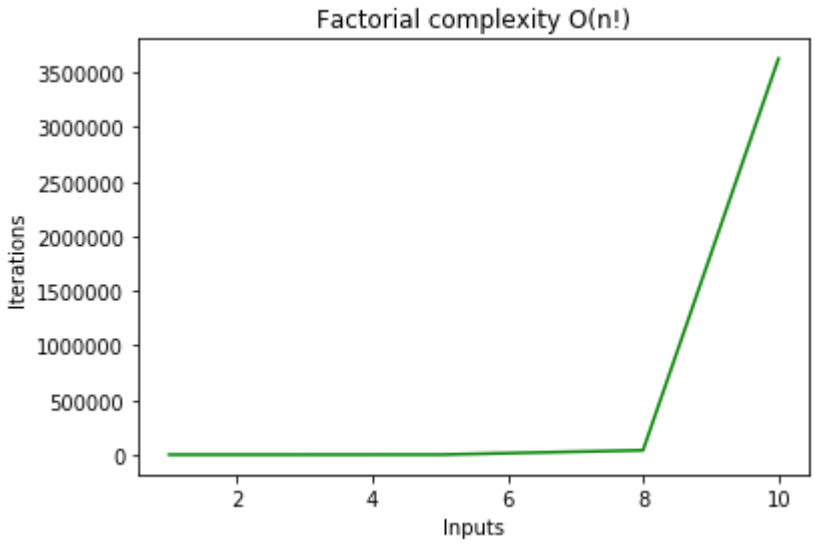
Factorial complexity O(n!) – very slow. Ex. traveling salesman

In [9]:

```
x = [1, 2, 3, 4, 5, 8, 10 ] #input(n)

y = [1, 2, 6, 24, 120, 40320, 3628800 ] #number or iterations

plt.plot(x, y, 'g')
plt.xlabel('Inputs')
plt.ylabel('Iterations')
plt.title('Factorial complexity O(n!)')
plt.show()
```



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2 Recursion

2.1 Description

Recursion is a method of solving problems that relies on the capability of a function to continue calling itself until it satisfies a particular condition.

> “Recursion is when function calls itself until it doesn't” (c)🙈 Fun Fun Functions

Recursion function consists of two main points:

1. Base case (exit condition, otherwise infinite loop)
2. The rule how to move through function (alter input parameter)

Recursion is similar to while loop, because we also keep going through some piece of code again and again till exit or break condition (when we need to stop).

2.2 Example

Find n!:
Base case: If n = 0, then n! = 1
The rule: If n > 0, then n! = n*(n-1)!

So factorial of 3 or 3! is

In [10]:

```
def factorial(n):
    if n == 0: #base case or when we need to stop
        return 1
    else:
        return n * factorial(n - 1) #how to move

factorial_3 = factorial(3)
print(factorial_3) #6
```

```
6
```

Steps:

1. $factorial(3)$
2. $3 * factorial(2)$
3. $3 * 2 * factorial(1)$
4. $3 * 2 * 1$
5. 6

We can see step by step logic of recursive function. We are not multiplying till we dig dipper and hit base case - factorial(1), then we move forward and multiply.

---

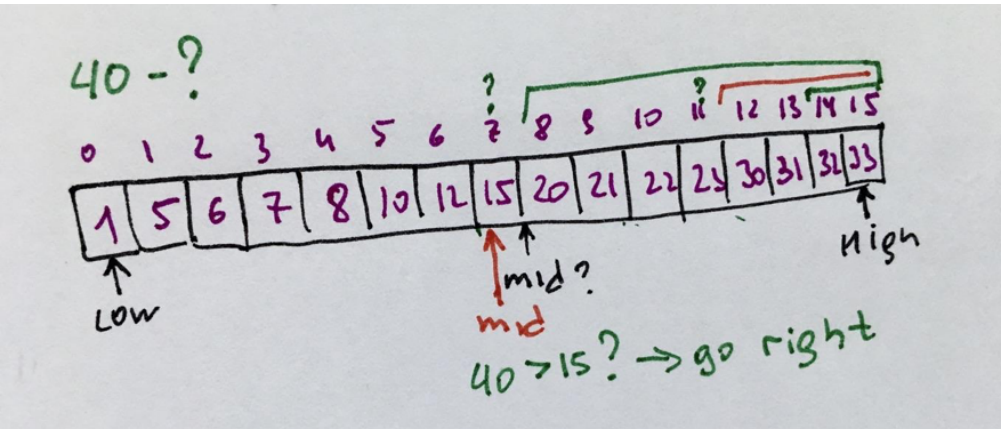
3 Binary Search

3.1 Description

> The goal of **binary search** is finding an item from a **sorted** list of items. It works by repeatedly dividing in half the length of the list that could contain the item, until it has narrowed down the possible locations to just one.(c) Khan Academy

3.2 Example

Searching for number 40 (Picture 1).



Picture 1. Binary Search example

Steps:

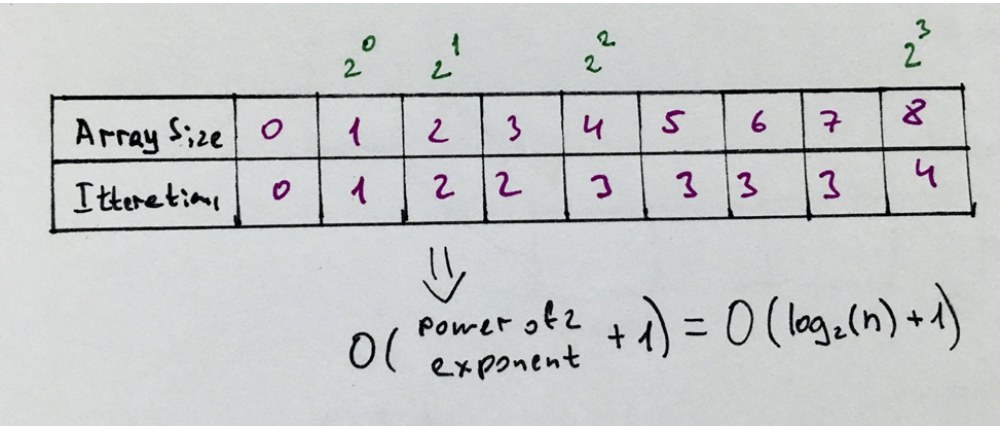
1. the lowest number = 1 and the highest number = 33
2. middle in array with length = 16 so the middle is 8, and the element with index (8-1) is 15.
3. 15 became the new lowest number and the higherst number is the same = 33. 15 < 33
4. 40 is bigger then 15 so we move to the right part of the array and repeat same steps.
5. In the end we compare the number to 32 and the last one 33.
6. Then we can conclude that there is no such a number in the current sorted array.

Termination condition: while the lowest number smaller or equal to the hignest number.

Criteria that algorithms is correct: The array must be ordered either in increasing or in decreasing order according to the comparison operator above. It takes an array and element which has to be found and returns the index of the element (in case it is in the array)

3.3 Time complexity

Efficiency is not going to be as big as Linear O(n). Let's consider how many iterations we have with different sizes of array (assumption from Udacity example).



Picture 2. Finding a pattern with binary search from Data Structures & Algorithms in Python. Udacity.

Logarithms are the inverse of exponentials, so that if log(n)=x then $n = 2^x$. We can see patterns and logic behind of the examples (Picture 2), it follows as that efficiency of the binary search could be:

$$O(log(n) + 1$$

From above explanation of asymptotic notation, we can conclude that adding one does not change efficiency much, so we get

$$O(log(n)$$

Best case of binary search is $O(1)$ (when searched element is in the middle position)

The logarithm function grows very slowly. So O(log n) is faster than O(n), but it gets a lot faster as the list of items grows.

Worse case	Average case	Best case
$(Olog(n)$	$\Theta(log(n))$	Ω(1)

3.4 Code example

[binary_search.py (algorithms/searching/binary_search.py)]

---

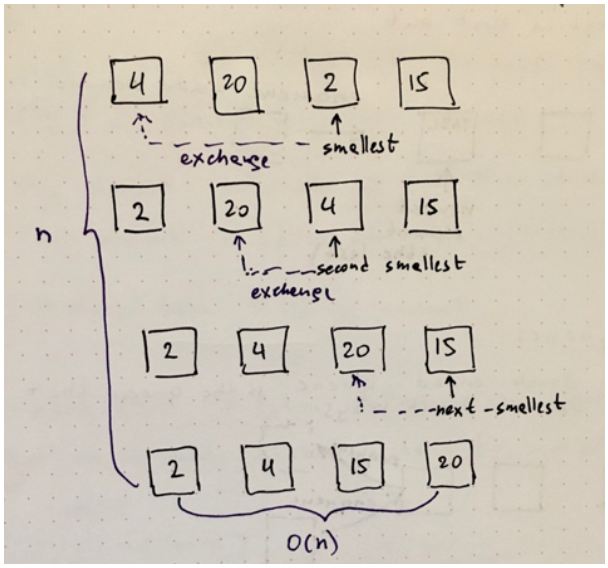
4 Selection Sort

4.1 Description

Selection sort – is the simplest sorting algorithm when we select the next smallest element and exchange it into place again and again.

4.2 Example

Sort an array [4, 20, 2, 15]



Picture 3. Selection sort example.

Steps:

1. Find the smallest number which is 2 and exchange it with the first number so we get [2, 20, 4, 15].
2. Find the second-smallest number (4) and exchange it with the second number of the array, so we get [2, 4, 20, 15]
3. Do over and over again finding the next-smallest number, and exchanging it into the correct position until the array is sorted.

Termination condition: If the next-smallest is the smallest element of the array.

Criteria that algorithms is correct: Input: array of n elements. Output is the same length array sorted in increasing or decreasing order.

4.3 Time complexity

We have n steps and on each step we do n comparisons. So the time complexity in the worst case is $O(n^2)$

Even if get sorted array, we still need to go through each level that's why the best case is $O(n^2)$.

Worse case	Average case	Best case

Worse case	Average case	Best case
$O(n^2)$	$\Theta(n^2)$	$\Omega(n^2)$

4.4 Code example

selection_sort.py (algorithms/sorting/selection_sort.py)

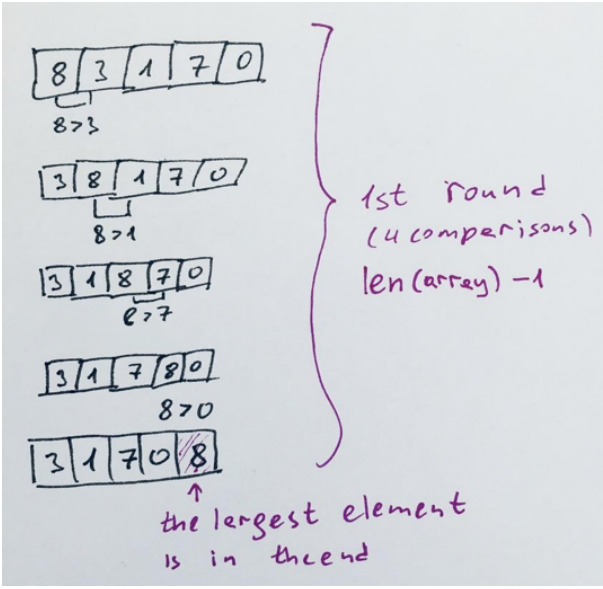
5 Bubble Sort

5.1 Description

Bubble sort – is an approach when in each iteration the largest element in the array will bubble up to the top. The goal of the algorithm is by checking two items at a time, rearrange those not already in ascending order (in the same array) from left to right.

5.2 Example

Sort an array [8, 3, 1, 7, 0] The array = [8, 3, 1, 7, 0] has length(n) equal 5, so n= 5



Picture 4. Bubble sort example.

Steps:

1. First round took 4 comparisons and we get [3, 1, 7, 0, 8]
2. Second round took 4 comparisons and we get [1, 3, 0, 7, 8]
3. Third round took 4 comparisons and we get [1, 0, 3, 7, 8]
4. Forth round took 4 comparisons and we get the sorted array [0, 1, 3, 7, 8]

Termination condition: No more swaps needed - array is sorted.

Criteria that algorithms is correct: Input: array of n elements. Output is the same length array sorted in increasing or decreasing order.

5.3 Time complexity

It took n-1 rounds to sort the array,each round took n-1 comparison steps.

$(n-1)(n-1) = n^2 - 2n + 1$ According asymptotic notation we can simplify to n^2, so we get the running time of bubble sorting is:

$$O(n^2)$$

Best case of bubble sort is $O(n)$ (when array is already sorted or there is only one element in the array)

Worse case	Average case	Best case
$O(n^2)$	$\Theta(n^2)$	$\Omega(n)$

Space complexity is O(1) – everything we do in the same array.

5.4 Code example

bubble_sort.py (algorithms/sorting/bubble_sort.py)

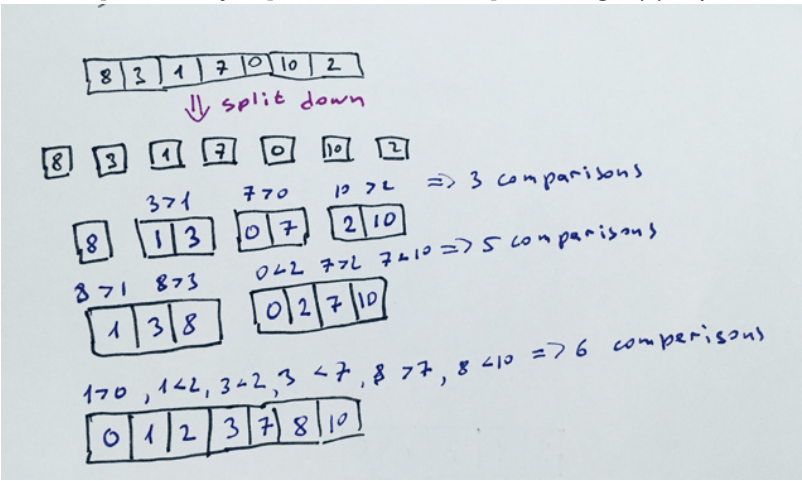
6 Merge sort

6.1 Description

Merge sort is an algorithm when we split the array down as much as possible and build it back up (merging) doing the comparisons and sorting on each step. Divide and Conquer principles here.

6.2 Example

Sort an array [8, 3, 1, 7, 0, 10, 2] The array = [8, 3, 1, 7, 0, 10, 2] has length(n) equal 7, so n= 7



Picture 5. Merge sort example.

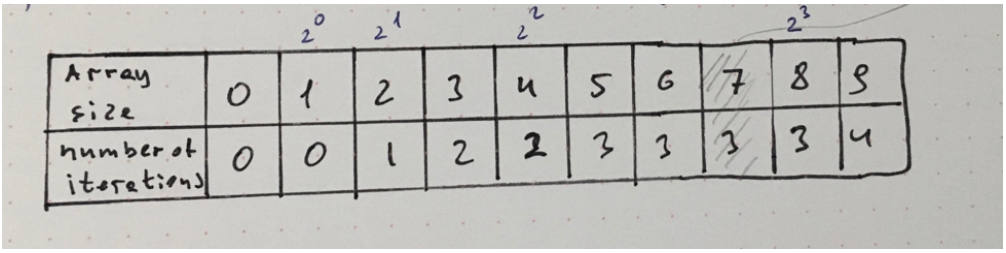
Steps:

1. Split array
2. Start building back, making comparisons which one is smaller (3 rounds)

Termination condition: it is recursive algorithm so we have the base case which is one.

If n is the size of the array that we are building, the number of comparisons is going to be n - 1. In our example we can say that approximately we are going to do 7 comparisons at each round (it is very approximately!). Now we need to find how many rounds we do. For array of 7 element we did 3 rounds.

If we draw array of result, we get the similar table as for binary search.



Picture 6. Merge sort iterations.

6.3 Time complexity

We get approx. $log(n)$ for number of rounds, and n comparisons on the each round, so the efficiency is:

$$O(n * log(n))$$

Time complexity is the same in all 3 cases (worst, average and best) as we always need to divide the array and then merge.

Worse case	Average case	Best case
$O(n * log(n))$	$\Theta(n * log(n))$	$\Omega(n * log(n))$

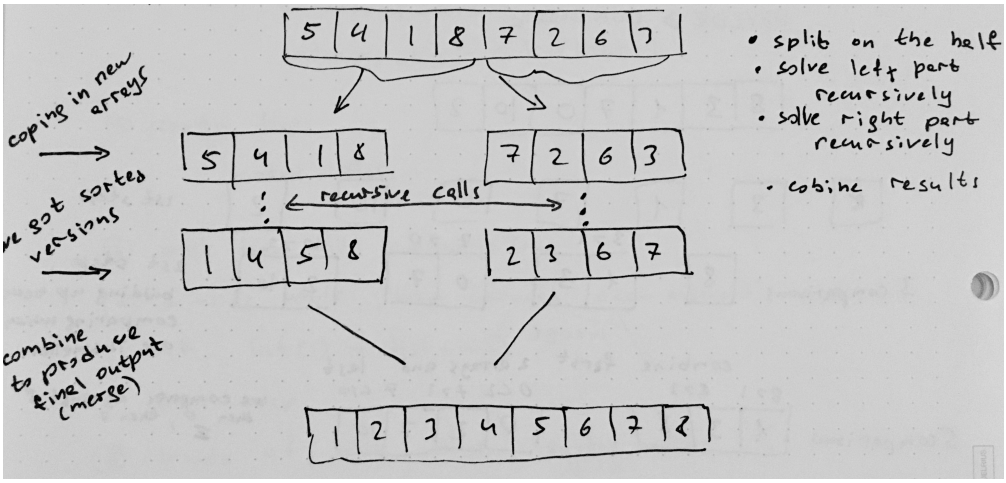
Space complexity is O(n) – on each step we need to create new arrays and copy into it.

Merge sort is often used for sorting a linked list.

Merge sort explanation by Coursera (more datailed).

Merge sort is recursive algorithm.

Example:



Picture 7. Merge sort algorithm.

Steps:

1. Split array on the half
2. Recursively sort 1st part of the array
3. Recursively sort 2nd part of the array
4. Merge two sorted sublists into one array.

We have two routines:

1. split
2. merge

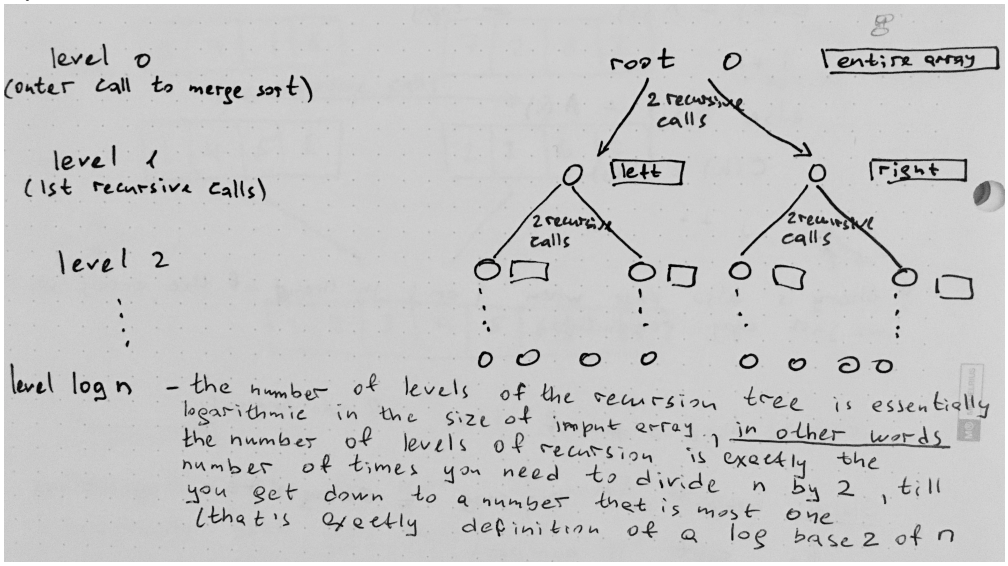
Pseudocode for merge routine:

1. "C" - is output array (length = n); A - is 1st sorted array (n/2); B - is 2nd sorted array (n/2)
2. Traversing two sorted sub-arrays, A and B in parallel. i - counter to traverse through A j - counter to traverse through B

```
 i = 1
j = 1

 for k = 1 to n:
    if A(i) < B(j): <-- check the smallest
        C(k) = A(i)    <--copy
        i++
    else B(j) < A(i):
        C(k) = B(j)
        j++
```

So let's see the split routine:



Picture 8. Merge sort algorithm split routine.

We can see from picture that at each level j = 0, 1, 2, ...log(n), there 2^j subproblems, each of size $n/2^j$. So if we have array of n = 8, then at level 2, there 2^2 subproblems, each of size $8/2^2$.

So as we can see from picture split routine takes log(n)

Also we remember that merge routine will execute at most 6 lines of code(see above pseudocode).

Let's count all operations (amount of work) at each level but independent of number leveles:

$$2^j * n/2^j * 6 = 6 * n,$$

where 2^j is number of level j sub-problems, $n/2^j$ is subproblem size at level j.

How many leveles we have? We have log(n) levels, but we have n-1 comparisons, cuz the last level is base case and no comparisosns there. So we need to count (log(n) -1) levels.

And total: $6n * (log(n) + 1) = 6nlog(n) + 6n$

Following asymptotic rules we drop unnecessary parts and get $nlog(n)$ or complexity:

$$O(n * log(n))$$

6.4 Code example

merge_sort.py (algorithms/sorting/merge_sort.py)

7 Quick Sort

7.1 Description

Quick sort is one of the most efficient sorting algorithms. Quick sort characteristics:

- Divide and conquer - splits the array into smaller arrays until it ends up with an empty array, or one that has only one element, before recursively sorting the larger arrays.
- In-place - doesn't create any copies of the array.

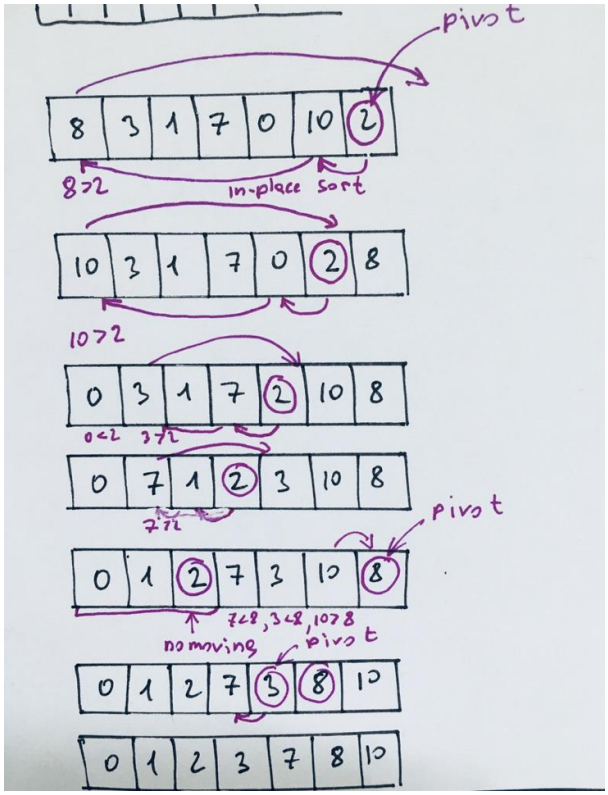
The goal of the algorithm is to use divide and conquer method to gain the same advantages as the merge sort, but without creating new arrays. In case if the the list may not be divided in half, it would be taking more time.

We have two subroutines:

- split function, which splits the input
- partitioning subroutine - the goal is choose the pivot

7.2 Example

Sort the list [8, 3, 1, 7, 0, 10, 2] using quick sort



Picture 9. Quick sort example.

Steps:

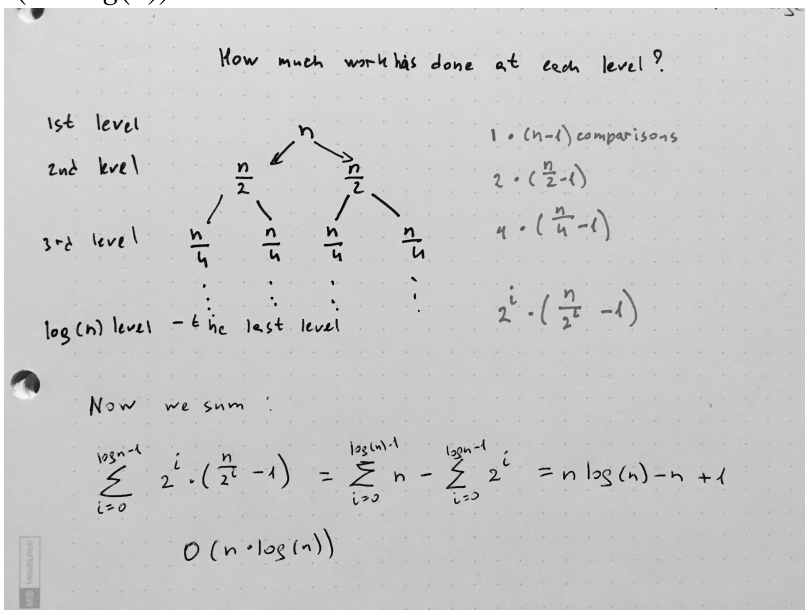
1. pick one of the values randomly (pivot is the random number)
2. move all values larger than it above it
3. move all values lower than it below it
4. continue on recursively picking a pivot in the upper and lower sections
5. sort them till whole array is sorted

Termination condition: the size of the array we are sorting recursively is one (the base case). Criteria that algoritm is correct: output is sorted array the same length as input

7.3 Time complexity

In the worst case, when we got array which near sorted, we need to compare each element in the array again and again, the same like in Bubble sort algorithm. So in the worst case everytime pivot is the greatest item or the least item. To compare each item on the first level takes (n-1) comparisons, then (n-2).....and so on. Using math, we can conclude that it take in the wors scenario $(n-1) * n/2$ If we drop constants, the worst case time complexity is $O(n^2)$.

In the best and average case, when pivot is moved to the middle (the pivot is median of partitioning space) so we will half the array every time, we get the stack size is O(log n), but still each level takes O(n) time. Therefore, the time complexity is $O(n * log(n))$.



Picture 10. Explanation of complexity.

> Quicksort is faster in practice because it hits the average case way more often than the worst case. (c) Grokking Algorithms.

Worse case	Average case	Best case
$O(n^2)$	$\Theta(n * log(n))$	$\Omega(n * log(n))$

Quicksort is used by Chrome (V8 engine) javascript sorting.

7.4 Code example

quick_sort.py (algorithms/sorting/quick_sort.py)

8 References

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