### I. Numbers and Arithmetic Operations

/'nΛmbə(r)/ /ə'riΘmətik/ /opə'ræisyen/

#### Numbers

Two kinds of activity made our ancestors develop numbers (**cardinal** and **ordinal numbers**). The first for comparing their things (which one has more elements), and the second for creating order.

# A. Cardinal Numbers (Counting Numbers) / 'ka: dinl 'n\nb\(\text{p}(r)\)/

/kaunting  $n\Lambda mb\theta(r)$ /

Example:

1	one	/wʌn/
2	two	/tu:/
3	three	/θri:/
4	four	/fɔː/
5	five	/faɪv/
6	six	/sīks/
7	seven	/'sevən/
8	eight	/eɪt/
9	nine	/naIn/
10	ten	/ten/
11	eleven	/I'levən/
12	twelve	/twelv/
13	thirteen	/θ3:'ti:n/
14	fourteen	/fɔːˈti:n/
15	fifteen	/fɪfti:n/
16	sixteen	/sɪkstˈi:n/
17	seventeen	/seven'ti:n/
18	eighteen	/eɪˈti:n/
19	nineteen	/naɪnˈtiːn/
20	twenty	/'twentI/
21	twenty-one	/twentI'w^n/
22	twenty-two	/twentɪ'tu:/
23	twenty-three	/twentɪˈθriː/

24	twenty-four	/twentɪˈfɔː/	
25	twenty-five	/twentɪˈfaɪv/	
26	twenty-six	/twenti'siks/	
30	thirty	/ˈθɜ:tɪ/	
40	forty	/ˈfɔːtɪ/	
50	fifty	/'fɪftɪ/	
60	sixty	/ˈsɪkstɪ/	
70	seventy	/'sevəntɪ/	
80	eighty	/'eɪtɪ/	
90	ninety	/'naɪntɪ/	
100	a hundred; one hundred	/ə 'hʌndrəd/ /wʌn 'hʌndrəd/	
101	a hundred and one	/ə 'hʌndrəd ən wʌn/	
110	a hundred and ten	/ə 'h∧ndrəd ən ten/	
120	a hundred and twenty	/ə 'h∧ndrəd ən 'twentı/	
200	two hundred	/tu: 'h∧ndrəd/	
300	three hundred	/θri: ˈhʌndrəd/	
900	nine hundred	/naɪn 'hʌndrəd/	
1 000	a thousand, one thousand	/ə θ'aʊzənd/ /wʌn 'θaʊzənd/	
1 001	a thousand and one	/ə ˈθaʊzənd ən wʌn/	
1 010	a thousand and ten	/ə ˈθaʊzənd ən ten/	
1 020	a thousand and twenty	/ə 'θαυzənd ən 'twentɪ/	
1 100	one thousand, one hundred	/wʌn ˈθαʊzənd wʌn ˈhʌndrəd/	
1 101	one thousand, one hundred and one	/wʌn ˈθaʊzənd wʌn ˈhʌndrəd ən wʌn/	
9 999	nine thousand, nine hundred and ninety- nine	/naɪn ˈθαʊzənd naɪn ˈhʌndrəd ən ˈnaɪntɪ ˈnaɪn/	
10 000	ten thousand	/ten 'θαυzənd/	
15 356	fifteen thousand, three hundred and fifty six	/ˈfɪfti:n ˈθaʊzənd θri: ˈhʌndrəd ən ˈfɪftɪ sɪks/	

100 000	a hundred thousand	/ə 'hʌndrəd 'θaʊzənd/
1 000 000	a million	/ə ˈmɪljən/
1 000 000 000	a billion	/ə ˈbɪljən/
1 000 000 000 000	a trillion	/ə ˈtrɪljən/

# B. Ordinal Numbers/Place Numbers / 'ərdinəl 'n $\Lambda$ mbə(r)/ Example:

1st	first	/f3:st/
2nd	second	/'sekənd/
3rd	third	/θ3:d/
4th	fourth	/fɔ:θ/
5th	fifth	/fɪfθ/
6th	sixth	/sɪksθ/
7th	seventh	/ˈsevənθ/
8th	eighth	/eɪtθ/
9th	ninth	/naɪnθ/
10th	tenth	/tenθ/
11th	eleventh	/ɪˈlevənθ/
12th	twelfth	/'twelfθ/
13th	thirteenth	/θ3:'ti:nθ/
14th	fourtheenth	/fɔːˈti:nθ/
15th	fidteenth	/fɪfˈti:nθ/
16th	sixteenth	/sɪksˈti:nθ/
17th	seventeenth	/seven'ti:nθ/
18th	eighteenth	/eɪˈti:nθ/
19th	nineteenth	/naɪnˈti:nθ/
20th	twentieth	/'twentɪəθ/
21st	twenty-first	/twentɪˈfɜ:st/
22nd	twenty-second	/twentɪˈsekənd/
23rd	twenty-third	/twentɪˈθɜːd/
24th	twenty-fourth	/twentɪˈfɔ:θ/
25th	twenty-fifth	/twentI'fIfθ/

26th	twenty-sixth	/twentI'sIksθ/
27th	twenty-seventh	/twentI'sevənθ/
28th	twenty-eighth	/twentI'eItθ/
29th	twenty-ninth	/twentɪˈnaɪnθ/
30th	thirtieth	/θsːtɪəθ/
31st	thirty-first	/θ3ːtɪˈf3:st/
40th	fortieth	/ˈfɔ:tɪəθ/
50th	fiftieth	/ˈfɪftɪəθ/
100th	hundredth	/ˈhʌndrədθ/
1 000th	thousandth	/ˈθaʊzəndθ/
1 000 000th	miilionth	/ˈmɪljənθ/

#### Natural Numbers

/'næt∫ral 'n∧mbə(r)/

1,2,3,... one, two, three, and so forth (without end).

1,2,3,..., 10 one, two, three, and so forth up to ten.

Natural numbers can be divided into two sets:

**Odd Numbers** /pd ' $\text{n}\Lambda\text{mb}\theta(r)$ / and **Even Numbers** /'i:vn ' $\text{n}\Lambda\text{mb}\theta(r)$ /

> Whole Numbers /həʊl 'n∧mbə(r)/

Natural Numbers + 0 zero/o/nought. /'ziərəu/ /nə:t/

➤ **Integers** / intəjər/

 $\dots, -2, 1, 0, 1, \dots$   $\dots$   $\dots$  negative two, negative one, zero, one,  $\dots$ 

- > Rational numbers /'ræ∫nəl 'n∧mbə(r)/ are numbers that can be expressed as fraction.
- ► Irrational Numbers /i'ræ∫nəl 'n $\Lambda$ mbə(r)/ are numbers that cannot be expressed as fraction, such as  $\sqrt{2}$ , $\pi$ .
- ➤ **Real Numbers** /riəl 'n/mbə(r)/ are made up of rational and irrational numbers.
- ➤ **Complex Numbers** /'kompleks 'n\nbə(r)/
  Complex numbers are numbers that contain **real** and **imaginary** part.

- 2 + 3i 2 is called the <u>real part</u>, 3 is called the <u>imaginary part</u>, and i is called <u>imaginary unit</u> of the complex number.
- $\blacktriangleright$  **A Digit** /'dɪd3ɪt/ is any one of the ten numerals 0,1,2,3,4,5,6,7,8,9.

#### Example:

3 is a single-digit number, but 234 is a three-digit number.

In 234, 4 is the units digit, 3 is the tens digit, and 2 is hundreds digit.

➤ **Consecutive** /kənˈsekjʊtɪv/ **numbers** are counting numbers that differ by 1.

#### Examples:

83, 84, 85, 86, and 87 are 5 consecutive numbers.

84, 85, 86, ... are successor /sək-'ses-ə(r)/ of 83.

84 is the immediate successor of 83.

1, 2, ..., and 82 are predecessor /'predə-ses-ə(r)/of 83.

82 is the immediate predecessor of 83.

36, 38, 40, and 42 are 4 consecutive even numbers.

#### **Operation on Numbers**

Addition (+), Subtraction (-), Multiplication (x), Division(:) /ə'diln/ /sab'træksyən/ /'maltəplə'keisyen/ /di'vi3n/

# Symbols in Numbers Operation

+	added by/plus/and /ædid bai/ /pl/\s/ /ənd/	
-	subtracted by/minus/take away /səb'træktid bai/ /'mainəs/ /teik ə'wei/	
±	plus or minus /plAs o:(r) 'mainəs/	
×	multiplied by/times /'mAltiplaid bai/ /taimz/	
:	divided by/over /di'vaidid bai/ /'əuvə(r)/	

#### Symbols for Comparing /kəm'peə(r)ing / Numbers

=	is equal to/equals/is /iz "i:kwəl tu:/ /"i:kwəlz/ /iz/
<b>≠</b>	is not equal to/does not equal /iz not "i:kwəl tu:/ /'dΛznt "i:kwəl/
<	is less than/is smaller than /iz les thən/ /iz smõlər thən/
>	is greater than/is more than /iz greitər thən/ /iz mə:(r) thən/
≤	is less than or equal to /iz les thən o:(r) "i:kwəl tu:/
≥	is more/greater than or equal to /iz mə:(r)/ /greitər thən o:(r) "i:kwəl tu:/
≅	is approximately equal to /iz ə'proksimatli "i:kwəl tu:/

The mathematical sentences that use symbols "=" are called **equation**, and the mathematical sentences that use symbols "<", ">", "≤", or "≥" are called inequalities.

#### **Examples**

ax + b = 0 is a linear equation.  $ax^2 + bx + c = 0$  is a quadratic equation.  $3x^3 - 2x^2 + 3 = 0$  is a cubic equation.

 $\frac{a+b}{2} \ge \sqrt{ab}$  is called AM-GM inequality.

#### Examples

$$\geq$$
 2 + 3 = 5

two	is added by	three	is equal to	five.
	plus		equals	
	and		is	

2 and 3 are called **addends** or **summands**, and 5 is called **sum**. /sΛm/

$$\rightarrow$$
 10 – 4 = 6

	is substracted by		is equal to	
Ten	minus	four	equals	six.
	take away		is	

10 is the **minuend**, 4 is the **subtrahend**, and 6 is the

difference/'difrans/

 $\rightarrow$  7 × 8 = 56

	is multiplied by		is equal to	
Seven	is multiplied by	eight	equals	fifty-six
	times		is	

7 is the **multiplicator**/'mAltəplə'kətwr/, 8 is the

multiplicand/'m\ltapla'kand/, and 56 is the product/'prodakt/.

**>** 45 : 5 = 9

fortu fino	is divided by	fino	is equal to	nino
forty-five	over	five	is	nine.

45 is the **dividend**, 5 is the **divisor** /də'vaizə(r)/, and 9 is the **quotient**/'kwəu∫nt/.

#### **Practice**

1. Read out the following operations, and for every operations name each number's function.

a. 
$$1,209 + 118 = 1,327$$

b. 
$$135 + (-132) = 3$$

c. 
$$2 - (-25) = 27$$

d. 
$$52 - 65 = -13$$

e. 
$$9 \times 26 = 234$$

f. 
$$-111 \times 99 = -10,989$$

g. 
$$36:9=4$$

- 2. Fill the blank spaces with the right words.
  - a. The \_\_\_\_\_ of three and seven is twenty-one.
  - b. The operation that uses symbol ":" is called \_\_\_\_\_\_.
  - c. 14 is the \_\_\_\_\_\_of 13, and the predecessor of 13 are \_\_\_\_\_.
  - d. The result of division is called \_\_\_\_\_.
  - e. Three multiplied \_\_\_\_\_\_ five equals \_\_\_\_\_.
  - f. In 123,456,789, the hundred thousands digit is \_\_\_\_, and 9 is the .

g.	We select a	number $htu$ , as $100h + 10t + u$ ,
	where <i>h</i> represents the	digit, t represents the
	digit, and $u$ represents the un	nits digit.
h.	When we two num	ibers, for example seven plus
	thirteen, the answer (twenty)	is called

# Fractions /frækln/

A **common (or simple) fraction** is a fraction of the form a/b where a is an integer and b is a counting number

Example: p/q

p is called the **numerator** /nyu:məreitə(r)/of the fraction

q is called the **denominator** /di'nomi'neitə(r)/ of the fraction

If the numerator < the denominator, then (p/q) is a **proper fraction** 

/propə(r) 'fræk∫n/

If the numerator > the denominator, then (p/q) is an **improper fraction** /im'prope(r) 'fræk $\ln$ /

3 ¼ is a <u>mixed numbers</u> /miksed 'nΛmbə(r)/ because it contains **number** part /'nΛmbə(r) pa:t/ and fractional part /ˈfræk∫nəl pa:t/

The fraction a/b is **simplified** ("in lowest terms") if a and b have no common factor other than 1

# **Saying Fraction**

1_	A/one half			
2	/ə/wΛn ha:f/			
<u>1</u>	A/one third			
3	/ə/wΛn θ3:d/			
<u>1</u>	A/one quarter			
4	/ə/wΛn 'kwɔ:tə(r)/			
<u>5</u> 6	Five sixths/Five over six			
$\frac{22+x}{7}$	Twenty-two plus x all over seven			
$13\frac{3}{4}$	Thirteen and three quarters			

0.3	Nought/zero/o point three	
3.056	Three point o five six	
273.856	Two hundred and seventy-three point eight five six	

#### **Practice**

1. Read out the following fractions

a. 
$$\frac{2}{5}$$

b. 
$$\frac{3}{4}$$

c. 
$$\frac{5}{8} \times \frac{1}{4} = \frac{5}{32}$$

d. 
$$2\frac{1}{2}:\frac{9}{10}=3\frac{2}{5}$$

e. 
$$\frac{1}{9} - \frac{1}{8} \neq \frac{1}{24}$$

h. 
$$6.9 \times 2.2 = 15.18$$

i. 
$$72.4 \times 61.5 = 4452.6$$

2. Fill the blank spaces with the right words.

a.	In the fraction seven ninths,	is the numerator, and
	is the	

b. The \_\_\_\_\_ of two thirds and a half is four over three.

c. An integer plus a fraction makes a \_\_\_\_\_\_.

# **Divisibility**

/di'vizəbl/

/m/ltipl/

/di'vaidz/

# 4 is a factor of 12

/'fæktə(r)/

```
15 is not divisible by 4.

If 15 divided by 4 then the quotient is 3 and the remainder is 3.

/thə ri'meIndə(r)/
0 is divisible by all integers
```

# **Prime numbers** /praim 'nΛmbə(r)z/

Every numbers is divisible by 1 and itself. These factors (1 and itself) are called **improper divisors**. /im'propə(r) də'vaizə(r)z/

Prime numbers are numbers that have only improper divisors.

#### Example:

5 is a prime number, but 9 is not a prime number or a **composite number**. /kompəzit 'nΛmbə(r)z/

# Common Divisors / 'komən də'vaizə(r)z/

# Example:

1,2,3,4,6, and 12 are divisors (factors) of 12.

1,3,5, and 15 are divisors of 15.

1 and 3 are common divisors of 12 and 15.

3 is the **greatest common divisor** /greitəst 'komən də'vaizə(r)/of 12 and 15.

The **g.c.d of** 12 and 15 is 3. gcd(12,15) = 3.

# Common Multiples / 'komən 'm\ltiplz/

# Example:

5,10,15,20,25, ... are multiples of 5.

4,8,12,16,20,24,... are multiples of 4.

5,10,15,20 are four first multiples of 5.

4,8,12,16,20 are five first multiples of 4.

20,40,60, ... are **common multiples** of 4 and 5.

20 is the  $\underline{least\ common\ multiple}$  /li:st 'komən 'm $\Lambda$ ltipl/ of 4 and 5.

The  $\underline{l.c.m of}$  4 and 5 is 20.

$$lcm(4,5) = 20.$$

#### **Practice**

- 1. Read the following conversation
  - A: I have two numbers, 36 and 42. Can you say their factors?
  - B: The factors of 36 are 1,2,3,4,6,9,12,24, and 36. 1,2,3,6,7,14,21, and 42 are factors of 42.
  - A: So, what are their common factors?
  - B: They are 1,2,3, and 6.
  - A: And what is the greatest common divisor of 36 and 42?
  - B: It's 6.
- 2. Make a small conversation about gcd or lcm of other numbers.

#### **Exercise**

Write down the spelling of these mathematical sentences

- $12 + 1/3 \le x 7$
- $3x \times 26 > 20 : y$
- $\boxed{1}$  x (2y + 3)  $\neq$  111.909
- $\boxed{1}$  (2 + x)/35 < 23/45

#### **Exercise**

Use the right words to complete these sentences.

- 2367 is \_\_\_\_\_\_ by nine.
- 3 is \_\_\_\_\_ of 34.
- The \_\_\_\_\_ of three and four is twelve.
- Ē Eighteen subtracted \_ twenty equals \_\_\_\_.
- 3 is the \_\_\_\_\_ and 5 is the \_\_\_\_ of three fifths.

#### **Exercise**

- Write down five first multiples of 8.
- Write down all divisors of 18.
- Find all common divisors of eighteen and thirty-three.
- Write down the simplest form of 91/234
- Find the sum of the reciprocals of two numbers, given that these numbers have a sum of 50 and a product of 25.

- What is the product of the greatest common divisor of 9633 and 4693 and the least common multiple of the same numbers?
- Let x be the smallest of three positive integers whose products is 720. Find the largest possible value of x.
- If P represents the product of all prime numbers less than 1000, what is the value of the units digit of P?
- Find a positive integer that is eleven times the sum of its digits?
- What is the greatest common divisor of 120 and 49?
- The product of 803 and 907 is divided by the sum of 63 and 37. What is the remainder?
- The average of four consecutive even integers is 17. Find the largest of the four integers.
- When the six-digit number 3456**N**7 is divided by 8, the remainder is 5. List **both** possible values of the digit **N**.

#### Vocabularies of Chapter I

Words	Pronunciation	Indonesian	
Numbers	/'nΛmbə(r)z/	Bilangan	
Natural Numbers			
Odd Numbers	/ød 'nΛmbə(r)z/	Bilangan Asli Bilangan Ganjil	
Even Numbers	/ˈiːvn ˈnΛmbə(r)z/	Bilangan Genap	
Whole Numbers	/həʊl 'nʌmbə(r)/	Bilangan Cacah	
Integers	/'intəjərz/	Bilangan Bulat	
Rational numbers	/'ræ∫nəl 'n∧mbə(r)z/	Bilangan Rasional	
Irrational Numbers	/i'ræ∫nəl 'n∧mbə(r)z/	Bilangan Irrasional	
Real Numbers	/riəl 'nΛmbə(r)z/	Bilangan Real	
Complex Numbers	/'kompleks 'nΛmbə(r)z/	Bilangan Kompleks	
Digit	/'dɪd3ɪt/	Angka	
Consecutive numbers	/kənˈsekjʊtɪv 'n/\mbə(r)z/	Bilangan berurutan	
Prime numbers	/praim 'n/mbə(r)z/	Bilangan prima	
Composite numbers	/kompəzit 'n/mbə(r)z/	Bilangan komposit	
Addition /ə'di∫n/ Pe		Penjumlahan	
Subtraction	/sab'træksyən/	Pengurangan	
Multiplication	/'maltəplə'keisyen/	Perkalian	
Division	/di'vi3n/	Pembagian	
Equation	/ı'kweı∫n/	Persamaan	
Inequalities	/,ɪnɪ'kwɒləti/	Pertidaksamaan	
Difference			
Sum	/s/m/	Jumlah	
Multiplicator			
Multiplicand			
Product	/'m\ltəplə'kənd/ Yang dikali /'prodəkt/ Hasilkali		
Dividend			
Divisor /də'vaizə(r)/ Pembagi		Pembagi	

Quotient	/'kwəu∫nt/	Hasilbagi	
Fractions	/fræk∫n/	Pecahan	
Numerator	/nyu:məreitə(r)/	Pembilang	
Denominator	/di'nomi'neitə(r)/ Penyebut		
Proper fraction	/propə(r) 'fræk∫n/	Pecahan sejati	
Improper fraction	on /im'propə(r) 'fræk∫n/ Pecahan tak		
Mixed number	/miksed 'n\mbə(r)/	Pecahan campuran	
Numbert part	/'n\nbə(r) pa:t/ Bagian bilangan		
Fractional part	/'fræk∫nəl pa:t/	Bagian pecahan	

# II. Powers, Roots, and Logarithm

/'paʊə(r)z/ /ru:tz/ /'lɒgəriðəm/

**Powers/Indices** /'IndIsi:z/ is used when we want to multiply a number by itself several times.

$$a^b$$

In this term, a is called **base/basis** /beIs/'beIsəs/ and b is called **index/exponent** /Ik'spəunənt/. The word *power* sometimes also means the exponent alone rather than the result of an exponential /Ik'spəunənʃl/ expression.

#### **How to Say Powers**

$\mathbf{x}^2$	x squared /'skweə(r)d/			
<b>x</b> <sup>3</sup>	x cubed /kju:bd/			
X <sup>n</sup>	x to the power of n			
	x to the n-th power			
	x to the n			
	x to the n-th			
	x upper /'Λpə(r)/ n			
	x raised /reizd/ by n			
(x+y) <sup>2</sup>	x plus y all squared			
	bracket /'brækit/ x plus y bracket closed squared			
	x plus y in bracket squared			

#### **Practice**

- A. Read out the following terms and say their values.
  - 1. 26
  - $2. \left(\frac{2}{3}\right)^3$
  - 3.  $x^5 : x^2$
  - 4. (3ab)4

5. 
$$\left(\frac{x}{3y}\right)^3$$

6. 
$$(9x)^0$$

- B. Read these expressions and simplify them.
  - 1.  $5^3 \times 5^{13}$
  - 2. 814: 811
  - 3.  $(2^4)^3$
  - $4. \left(\frac{x^7}{x^3}\right)^2$

#### LAWS FOR POWERS

for equal exponents

> First Law for Power:

$$(ab)^n = a^n b^n$$

A product raised by an exponent is equal to product of factors raised by same exponent

$$(a/b)^n = a^n/b^n$$

For equal basis

> Second Law for Powers:

$$a^{m} \times a^{n} = a^{m+n}$$

- The product of two powers with equal basis equals to the basis raised to the sum of the two exponents
- When expressions with the same base are multiplied, the indices are added

How can we say this rule?

$$a^m : a^n = a^{m-n}$$

> Third Law for Powers:

$$(a^m)^n = a^{mn}$$

Exponentiating of powers equals to the basis raised to the product of the two exponents

#### **Practice**

Try to express in words these another rules of powers:

1. 
$$a^0 = 1$$
,  $a \ne 0$ .

2. 
$$a^{-n} = \frac{1}{a^n}, a \neq 0$$

3. 
$$(a/b)^n = a^n/b^n$$

4. 
$$a^m : a^n = a^{m-n}$$

# Roots and Radicals /rædiklz/

Root is inversion of exponentiation

$$\sqrt[n]{a} = b \leftrightarrow b^n = a$$

 $\sqrt[n]{a}$  is called **radical expression** (or **radical form**) because it contains a root.

The radical expression has several parts:

- the radical sign /sain/  $\sqrt{}$
- the radicand /rædīkən/: the entire quantity under the radical sign
- the **index**: the number that indicates the root that is being taken example:

$$\sqrt[3]{a+b}$$
 a + b is the radicand, 3 is the index.

The radical expression can be written in **exponential form** (**powers with** fractional exponents)

example:

$$\sqrt[n]{x} = x^{1/n}$$

So the law of powers can be used in calculating root

Examples:

$$\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}}b^{\frac{1}{n}} = \sqrt[n]{a}\sqrt[n]{b}$$

A number is said **perfect square** if its roots are integers. example:

9, 16, 36, and 100 are perfect squares, but 12 and 20 are not.

#### **How to Say Radicals**

$\sqrt{x}$	(square) root of x
<sup>3</sup> √y	cube root of y
<sup>n</sup> √z	n-th root of z
$\sqrt[5]{x^2y^3}$	fifth root of (pause) x squared times y cubed
	fifth root of x squared times y cubed in
	bracket

#### **Square Root**

The square root is in **simplest form** if:

- a. the radicand does not contain perfect squares other than 1.
- b. no fraction is contained in radicand.
- c. no radicals appear in the denominator of a fraction.

#### Example

 $\sqrt{24}$  is not a simplest form because we can write it as  $\sqrt{4 \times 6}$  where 4 is a perfect square. We can simplify the radical into  $2\sqrt{6}$ 

A radical and a number is called a **binomial** /baɪˈnəʊmɪəl/. The **conjugate** /ˈkɒndʒʊgeɪt/ of binomial is another binomial with the same number and radical, but the sign of second term is changed.

#### Example

 $2+\sqrt{6}$  is a binomial and its conjugate is  $2-\sqrt{6}$ 

# **Practice**

- a. Read out the following radical expressions and say theirs exponential notation.
  - 1.  $\sqrt{4x^4}$
  - 2.  $\sqrt[4]{m^3n^8}$
  - 3.  $\sqrt[5]{a^3}$
  - 4.  $\sqrt[3]{8x^6y^9}$
  - 5.  $\sqrt{x^2 + y^2}$
- b. Read out the following terms and say what their values are:
  - 1. 2431/5
  - 2. -4-2
  - 3.  $125^{1/3}$
  - 4.  $(-5)^{-1}$
  - 5. 3-3
- c. Simplify these radicals
  - 1.  $\sqrt{72}$
  - 2.  $\sqrt{234}$
  - 3.  $\frac{5}{2+\sqrt{3}}$
  - 4.  $\frac{\sqrt{3}}{\sqrt{6}-\sqrt{2}}$
- d. Find the conjugate of these binomials
  - 1.  $2+\sqrt{5}$
  - 2.  $6 \sqrt{4}$

# Logarithm

$$x = a^b \Leftrightarrow b = a \log x$$

In this term, a is also called base.

#### How to Say Logarithm

<sup>n</sup> log x	$\log / \log / x$ to the base of n			
	log base n of x			
ln 2	natural log of two			
	"L N" of two			
<sup>5</sup> log <sup>2</sup> 25	log squared of twenty-five to the base of			
	five			
	log base five of twenty-five all squared			

#### **Practice**

Read out the following terms:

- a. a xlog b
- b.  $\log a^2$
- c.  $2\log(1/6)$
- d.  $5\log(x^2+y)$
- e.  $(n\log x)^2$
- f.  $6\log^2 22 6\log x^2 1$

### Laws for Logarithm

#### > First Law for logarithm:

The logarithm of a product is equal to the sum of the logarithm of the factors

$$^{b} \log(xy) = ^{b} \log x + ^{b} \log y$$

# > Second Law for logarithm:

The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor

$$^{b}\log(x/y) = ^{b}\log x - ^{b}\log y$$

#### > Third Law for logarithm:

The logarithm of a power is equal to the exponent times the logarithm of the basis

$$^{b} \log(x^{a}) = a^{b} \log x$$

#### **More Examples**

	log base two of x plus y in bracket plus	
<sup>2</sup> log (x+y)+2 <sup>2</sup> log 4x >4	two times log base two of four x's is	
	greater than four	
2 1 .	x squared plus (pause) one over root of $x$	
$\mathbf{x}^2 + \frac{1}{\sqrt{\mathbf{x}}} = 1$	equals one	
	three upper x plus (pause) nine upper x	
$3^{x} + 9^{x-1} > 27$	minus one (pause) is more than twenty-	
	seven	
	nine to the x (pause) minus one is less	
9 <sup>x</sup> -1 < 2	than two	

#### **Some Algebraic Processes**

- 1. Expand (x-3)(x+2) into  $x^2-x-6$ .
- 2. Simplify (2x+2)/(x+1) into 2
- 3. Factorize  $x^3-2x^2+3x-2$  into (x-1)(x+1)(x-2)
- 4. Cancel (x+1) from (2x+2)/(x+1) to get 2
- 5. Add/subtract/multiply/divide both side

Examples: multiply both side of equation  $\frac{1}{2}x=4$  with 2 to get x=8

- 6. Subtitute y=4 into equation 2x+y=12
- 7. Collect (x+2) from  $(x+2)^3-2(x+2)(x+1)$  to get  $(x+2)[(x+2)^2-2(x+1)]$

#### **Example**

Find x that satisfy equation  $3^{x}-3^{x-1}=162$ .

Answer

First, we multiply both side with 3 to get  $3.3^{X}-3^{X}=486$ .

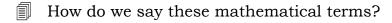
Then, we collect  $3^{X}$  and we have  $3^{X}(3-1)=486$ , which can be simplified into  $2.3^{X}=486$ .

Divide both side by 2, we get  $3^{X}$ =243.

We know that 243 is  $3^5$ , so we can write  $3^x=3^5$ .

According to the rule of powers, x must be equal to 5.

#### **Exercise**



1. 
$$(^3 \log x)^2 + ^3 \log x^2 = \sqrt[4]{4 - x^3}$$

2. 
$$x^{n \log(x+1)} = 0$$

$$3.\sqrt{2\sqrt{2}} = \log(\frac{x}{5})$$

Read the complete answers.

- 1. 132=...
  - 2. 29=...
  - 3. Every positive real numbers has ..... real-numbered square roots.
  - 4. The cube root of two hundred and sixteen is ...........
  - 5. If the root of eighty-one is raised by three, then we have ........
  - 6. 7 is the log base ten of .....

Solve this problem and try to explain it.

- 1. Which is greater,  $2^{95}+2^{95}$  or  $2^{100}$ ?
- 2. Which is the larger,  $10^{1/10}$  or  $2^{1/3}$ ?
- 3. Let A and B are real numbers greater than 1. If there is positive number  $C \neq 1$  such that  $2(A \log C + B \log C) = 9 AB \log C$ , then find the largest possible value for  $A \log B$ .
- 4. Given  $9 \log 20 = a$  and  $3 \log n = 4a$ . What is the value of n?
- 5. In March, the number of students was a perfect square. At the end of the semester, with 100 new students, the number of students became 1 more than a perfect square. At the end of the year with an additional 100 new students, the number of students is a perfect square. How many students were there in September?

Explain the process to find the solution x that satisfies each equation, inequality, or system of equations.

1. 
$$1-4x \le x+11$$

2. 
$$\frac{3}{2}y - \frac{5}{3} = \frac{4-2y}{5}$$

# **Vocabularies of Chapter II**

Words	Pronunciation	Indonesian	
Powers	/'paʊə(r)z/	Bilangan Berpangkat	
Indices	/'næt∫ral 'nΛmbə(r)z/	Bilangan Asli	
Base	/beis/	Bilangan Pokok	
Basis	/'beisəs/	Bilangan Pokok	
Exponent	/Ik'sp <b>ə</b> Unənt/	Pangkat	
Roots	/ru:tz/	Bentuk Akar	
Radicals	/rædɪklz/	Bentuk Akar	
Radical sign	/rædīklz saīn/	Tanda Akar	
Radicand	/rædīkən/	Bilangan yang diakarkan	
Perfect square	/'p3:fIkt skweə(r)/	Kuadrat Sempurna	
Binomial	/baɪˈnəʊmɪəl/	Binomial	
Conjugate	/ˈkɒnd3ʊgeɪt/	Sekawan	
ogarithm /ˈlɒgəriðəm/		Logaritma	
Expand	/ ik'spænd/	Uraikan/Jabarkan	
Simplify	/'sɪmplɪfaɪ/	Sederhanakan	
Factorize	/'fæktəraɪz/	Faktorkan	
Cancel	/'kænsl/	Coret/Hapus/Batalkan	
Substitute	/'sΛbstɪtju:t/	Substitusi	
Side	/said/	Ruas	
Left-hand side	/left hænd saɪd/	Ruas kiri	
Right-hand side	/raɪt hænd saɪd/	Ruas kanan	
Collect	/kə'ləkt/	Kumpulkan	
Eliminate	ninate /ɪ'lɪmɪneɪt/ Eliminasi		

#### III. Sequence, Series, and Trigonometry

/'si:kwəns/ /'sɪəri:z/ /trɪgə'nɒmətrɪ/

**Arithmetic** /əˈrɪθmətɪk/ **Sequence/Progression** /ˈprəˈgreʃn/

An arithmetic sequence is a sequence of the form

The number a is the **first term** /t3:m/, and d is the **common difference** /'kpmən 'dɪfrəns/ of the sequence.

The difference between two **consecutive** /k $\partial$ n'sekjUtIv/ **terms** is d

$$a_n - a_{n-1} = d$$

#### The nth Term of an Arithmetic Sequence

The **nth term** of the arithmetic sequence a, a+d, a+2d, ... is

$$a + (n-1) d$$

 $a_1$ ,  $a_2$ ,  $a_3$  are the first three terms.

 $a_{(n+1)/2}$  is called **middle** /'mɪdl/ **term**.

 $a_n$  and  $a_{n+1}$  are two consecutive terms

# Partial /'pa:/l/ sum of Arithmetic Sequence

For arithmetic sequence a, a+d, a+2d, ..., the nth **partial sum** 

$$S_n = (n/2)(2a + (n-1)d)$$

or

$$S_n = (n/2)(a + a_n)$$

#### Examples

In arithmetic sequence 1,4,7,10,13,..., the first term is 1, the difference is 3. So, the formula for n-th term is 1+(n-1)3=3n-2. Applying the formula we can say the 100-th term of the sequence is 298. The partial sum of the sequence is  $(n/2)(1+3n-2)=(3/2)n^2-(n/2)$ .

#### Geometric /d31ə'mətrik/ Sequence

 $a, ar, ar^2, \dots$  is a geometric sequence

 $a_1$ =a is the **first term**.

 $a_n = ar^{n-1}$  is the *n***-th term** of the geometric sequence.

*r* is the **common ratio** /'reɪʃɪəʊ/.

# Partial Sum of Geometric Sequence

Let Sn be the partial sum of the geometric sequence. Then

$$Sn = a + ar + ar^2 + ... + ar^{n-1}$$

or 
$$Sn=a\frac{\left|r^{n}-1\right|}{\left|r-1\right|}$$

#### Examples

In arithmetic sequence 2,4,8,16,32,..., the first term is 2, the ratio is also 2. So, the formula for n-th term is  $2 \, 2^{n-1}=2^n$ . Applying the formula we can say the 10-th term of the sequence is 1024. The partial sum of the sequence is  $2(2^n - 1)/(2-1)=2^{n+1}-2$ .

The sum of finite /'faInaIt/or infinite /'InfInət/ sequence

$$\sum a_n = a_1 + a_2 + ...$$

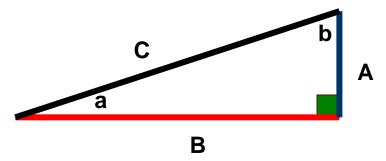
is called series.

#### **Practice**

- 1. Find the first 6 terms and the 300th term of the arithmetic sequence 13, 7, ...
- 2. The 10th term of an arithmetic sequence is 55 and the 2nd term is 7. Find the 1st term.
- 3. Find the sum of the first 40 terms of the arithmetic sequence 3, 7, 11, 15, ...
- 4. Find the 8th term of the geometric sequence 5, 15, 45, ...
- 5. The 3rd term of a geometric sequence is 63/4, and the 6th term is 1701/32. Find the 8th term.
- 6. Find the sum of the first 5 terms of the geometric sequence 1, 0.7, 0.49, 0.343, ...

# **Trigonometry**

- Trigonometry is the study of angle measurement.
- When you have a right triangle there are 5 things we can know about it: the lengths of the sides (A, B, and C), and the measures of the acute angles (a and b)



# Some terminology:

- The **hypotenuse**/haɪ'pɒtənyu:z/ will always be the longest side, and opposite from the right angle.
- The **opposite**/ppəzīt/ **side** is the side directly across from the angle you are considering (angle a).
- The **adjacent**/ə'd**3**eɪsnt/ **side** is the side next to angle you are considering.

#### The Trigonometric Functions

sin	sine	/saIn/
cos	cos; cosine	/kɒz/;/kɒzaɪn/
tan	tan; tangent	/tæn/;/tænd3ənt/
sec	sec;	/sek/
csc	cosec;	/ˈkəʊsek/
cot	cotangent	/ˈkəʊtænd3ənt /

- Sin /saın/ = Opposite side/Hypotenuse
- Tan /tæn; tænd3ənt/ x = Opposite side/Adjacent side
- Cot /'kəutændʒənt /a = adjacent side/opposite side
- Sec /sek/ a = hypotenuse/adjacent side
- Csc /'kəʊsek/ a = hypotenuse/opposite side

# Here is a way to remember how to make the 3 basic Trig Ratios

SOHCAHTOA is pronounced "Sew Caw Toe A" and it means

<u>Sin</u> is <u>Opposite over Hypotenuse,</u>

Cos is Adjacent over Hypotenuse,

and Tan is Opposite over Adjacent

We can use the mnemonic

Silly Old Hen, Cackles And Howls, Till Old Age.

This relationship can be summarized:

$$\sin \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$
  $\cos \theta = \frac{1}{\sec \theta}$   $\tan \theta = \frac{1}{\cot \theta}$ 

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$
 $\sec \theta = \frac{1}{\cos \theta}$ 
 $\cot \theta = \frac{1}{\tan \theta}$ 

$$\cot \theta = \frac{1}{\tan \theta}$$

#### **Practice**

Fill the table below and say the equation, such as  $\sin(0^{\circ})=...$ 

Degrees	Radians	Sin	Cos	Tan
/dɪˈgriːz/	/ˈrəɪdɪən/			
0	0			
30	π/6			
45	π/4			
60	π/3			
90	π/2			

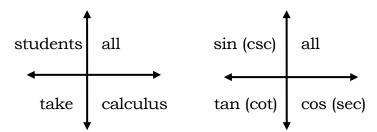
#### Periodicity

The trigonometric functions are periodic. The sine and cosine functions have the **period** /'piəriəd/  $2\pi$  (360°); the tangent and cotangent functions have the period  $\pi$  (180°).

#### Quadrant /'kwbdrənt/ relations.

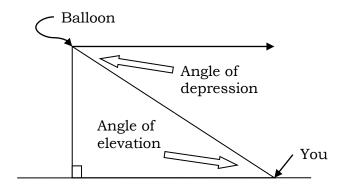
- All functions have positive values for angles in Quadrant I, Sine and Cosecant have positive values for angles in Quadrant II, Tangent and Cotangent have positive values for angles in Quadrant III, Cosine and Secant have positive values for angles in Quadrant IV.
- It will help to memorize by learning these words in Quadrants I IV: "All students take calculus"

And remembering reciprocal /rI'siprəkl/identities



Sometimes when we use right triangles to model real-life situations, we use the terms **angle of elevation** and **angle of depression**.

If you are standing on the ground and looking up at a hot air balloon, the angle that you look up from ground level is called the **angle of elevation**. If someone is in the hot air balloon and looks down to the ground to see you, the angle that they have to lower their eyes, from looking straight ahead, is called the **angle of depression**.



#### **Practice**

- Read the following expressions.
  - 1. (Formulas for Addition and Subtraction)

$$sin(A + B) = sin A cos B + cos A sin B$$

$$cos(A - B) = cos A cos B + sin A sin B$$

$$tan(A + B) = tan A + tan B / (1 - tan A tan B)$$

2. (Phytagorean Identities) 
$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

3. (Formula for Double Angle) 
$$\sin 2A = 2 \sin A \cos A$$

4. (Formula for half angle) 
$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

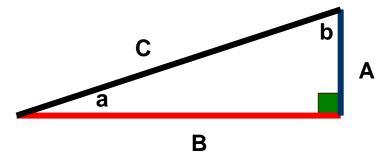
5. (Cosine Rule) 
$$a^2 = b^2 + c^2 - 2bc \cos A$$

- Explaining the solution of these problems
  - 1. Without using calculator, find cos(150).
  - 2. Find the values of x for which  $\sin 3x = 0.5$  if it is given that  $0 < x < 90^{\circ}$ .
  - 3. What is the equal value in degrees for x radians?

#### Exercise

Complete the sentences or give short answers.

- 1. Tangent has positive values for angles in \_\_\_\_\_\_, and \_\_\_\_\_ has positive values for angles in Quadrant IV.
- 2. The tangent and cotangent functions have the period \_\_\_\_\_\_
- 3. If  $\sin \alpha = 0.8$ , then the value of  $\sin (180-\alpha)$  is \_\_\_\_\_ and the value of  $\tan \alpha$  is \_\_\_\_\_
- 4. Without using calculator, find cos(150).
- 5. Find the values of x for which  $\sin 3x = 0.5$  if it is given that  $0 < x < 90^{\circ}$ .
- 6. What is the equal value in degrees for x radians?
- 7. Say anything about trigonometry of the picture below



For example: sin a is A/C.

**Greek Alphabets** 

Big Letters	Small Letters	Words	Pronunciation
A	α	alpha	/ˈælfə/
В	β	beta	/ˈbiːtə/
Γ	γ	gamma	/ˈgæmə/
Δ	δ	delta	/'deltə/
E	ε	epsilon	/'epsil <b>ə</b> n/
Z	ζ	zeta	/ˈziːtə/
Н	η	eta	/ˈiːtə/
Θ	θ	theta	/ˈθiːtə/

I	1	iota	/aɪˈəʊtə/
K	К	kappa	/ˈkæpə/
Λ	λ	lamda	/ˈlæmdə/
M	μ	mu	/'mjuː/
N	ν	nu	/'njuː/
Ξ	ξ	xi	/'ksaI/
О	О	omicron	/ˈəʊmɪkrən/
П	π	pi	/ˈpaɪ/
P	ρ	rho	/'rəʊ/
Σ	σ	sigma	/ˈsɪgmə/
Т	τ	tau	/'taʊ/
Y	υ	upsilon	/ˈjʊpsɪlən/
Φ	φ	phi	/ˈfaɪ/
X	x	chi	/ˈkaɪ/
Ψ	ψ	psi	/'psaɪ/
Ω	ω	omega	/ˈəʊmɪgə/

# **Vocabularies of Chapter III**

Words	Pronunciation	Indonesian
Sequence	/ˈsiːkwəns/	Barisan
Progression	/'prə'greln/	Barisan
Arithmetic sequence	/əˈrɪθmətɪk ˈsi:kwəns/	Barisan Aritmatika
Geometric	/d3ɪəˈmətrɪk ˈsi:kwəns/	Barisan Geometri
Difference	/'dɪfrəns/	Beda
Ratio	/ˈreɪ∫ɪəʊ/	Rasio
Series	/'sɪəri:z/	Deret
Finite	/ˈfaɪnaɪt/	Berhingga
Infinite	/'InfInət/	Tak Berhingga
Term	/t3:m/	Suku
First Term	/f3:st t3:m/	Suku Pertama
Middle Term	/ˈmɪdl tɜ:m/	Suku Tengah
Consecutive Terms	/kənˈsekjʊtɪv tɜ:mz/	Suku Berurutan
Partial sum	/'pa:∫l sAm/	Jumlah sebagian (suku)
Trigonometry	/trɪgəˈnɒmətrɪ/	Trigonometri
Adjacent side	/ə'd3eisnt said/	Sisi samping
Hypotenuse	/haɪˈpɒtənyu:z/	Sisi miring

Opposite side	/ppəzit said/	Sisi depan
Angle	/ˈæŋgl/	Sudut
Degrees	/dɪˈgriːz/	Derajat
Period	/ˈpɪərɪəd/	Periode
Quadrant	/ˈkwɒdrənt/	Kuadran
Radian	/ˈrəɪdɪən/	Radian
Reciprocal	/rɪˈsiprəkl/	Kebalikan

#### IV. Plane Geometry

Objects in plane geometry are points, lines, and two-dimensional figures.

A **point** /pɔɪnt/ is an exact location in space. A point has no size. Points are represented by dots, and named with capital letters.

A **line** /laɪn/ is a straight arrangement of points. Lines extend forever in opposite directions.

Lines are named using a lower case letter. For example: line m



A line could be **straight** or **curvy**. A straight line could be **horizontal**, **vertical**, or **oblique** /ə'bli:k/.

examples:

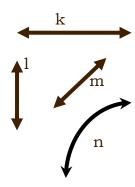
Line k is a horizontal line

Line l is a **vertical line**.

Line m is an **oblique line**.

Lines k, l, and m are **straight lines** 

Line n is a **curved line**. /k3:v/



#### Position of lines in a plane

Two lines in a plane could be **parallel** /pærəlel/, or **intersecting**. example:

Line 1 is parallel to line m.

(Lines I and m are parallel).

Line a **cuts** line b. Line a **intersects** line b.

Lines a and b are **intersected lines**.

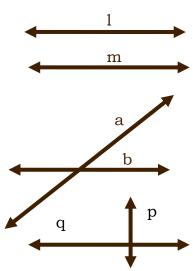
If two lines intersect, their intersection is a

point called the **point of intersection**.

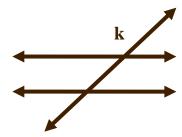
Line p is **orthogonal** to line q. (Line p is

perpendicular /p3:pən'dɪkjʊlə(r)/ to line q).

Lines p and q are orthogonal.



When we have more than two lines, this situation (see picture below) can happen



Line k is called a **transversal** /trænzv3:sl/ line, which is a straight line drawn across a set of two or more parallel lines.

### Line Segments

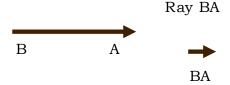
A line segment is part of a line. Line segments consist of 2 endpoints and all the points in between. Line segments are named using their endpoints.

example: line segment AB or AB

A B

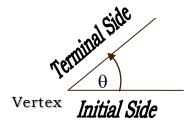
# Rays /rei/

A ray is a part of a line that has one **endpoint** and extends forever in one direction. Rays are named by writing the endpoint first, then another point on the ray. Example:



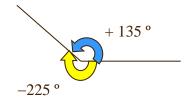
# Angle /'æŋgl/

An angle is formed by rotating a ray around its **end point**. One ray is fixed, and is called the **initial side**. The second ray is called the **terminal side**. The common end point is called the **vertex** /v3:teks/.



# Positive, Negative & Coterminal Angles

- ➤ A **positive** angle results from a **counter-clockwise** rotation.
- A negative angle results from a clockwise rotation.

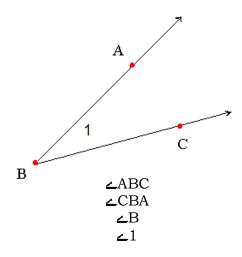


Two angles with the same terminal side are **coterminal**.

So + 135 ° and - 225 ° are coterminal!

# **Naming Angle**

- Angles can be named using letters or numbers.
- When using letters, the vertex may be named alone, or the vertex may be named between two other points on the angle.



# Angles according to their measures.

Measure of Angle $\theta$	Name
$0_0 < \theta < 30_0$	acute /ə'kju:t/angle
$\theta = 90_0$	right /raɪt/angle
900 < θ < 1800	obtuse /əb'tju:s/angle
$\theta = 180^{\circ}$	straight angle
1800 < θ < 3600	reflex angle
$\theta = 360^{\circ}$	full angle

#### Complementary & Supplementary Angles

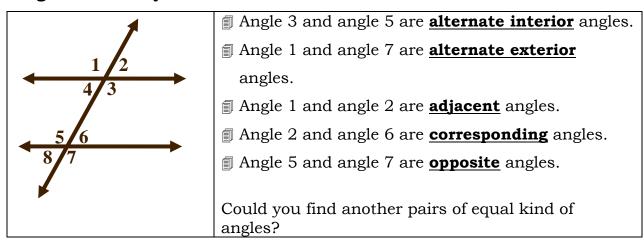
Two angles that have a sum of 90° are **complementary**.

If the measure of Angle A is 40° and Angle B is 50°, then we say Angle A and angle B are **complementary angles**.

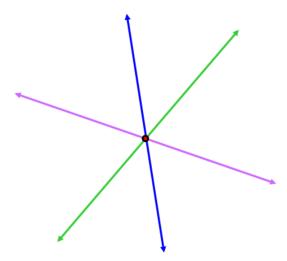
Two angles that have a sum of 180° are **supplementary**.

If the measure of Angle A is 120° and Angle B is 60°, then we say Angle A and angle B are **supplementary angles**.

#### Angles Formed by Transversal Line



**Concurrent lines** are three or more lines that intersect in one point. The point is called **point of concurrency**.



#### Two Dimensional Figures

There are two groups of two-dimensional figures: **Polygon** and **Circle**.

# Polygon

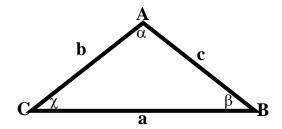
Polygon are two-dimensional figures which consists of n points and nearby points connected by straight lines which are called **sides**. They are named by the number of the sides.

#### example:

- TRIANGLE 3 sides
- QUADRILATERAL 4 sides
- PENTAGON 5 sides
- HEXAGON 6 sides
- **THEPTAGON 7 sides**
- OCTAGON 8 sides
- NONAGON 9 sides
- N-GON N sides

#### **Triangles**

A triangle is a three – sided figure . The three sides of a triangle meet at points called vertices (singular: vertex)



A triangle has three **vertices** (A,B,C), three **sides** (a,b,c), and three interior **angles** ( $\alpha$ , $\beta$ , $\chi$ ) whose sum is 180 degrees.

#### **Special Kind of Triangles**

Isosceles triangle is a triangle that has two equal sides.

The top vertex is called **apex** and the bottom side is called **base**.

- Equilateral triangle is a triangle that has three equal sides.
- **Right-angle triangle** (usually called **right triangle**) is a triangle that has **one right angle**. The side opposite the right angle is called **hypotenuse**.
- Scalene triangle is a triangle that has three different sides or all of its sides have different length.

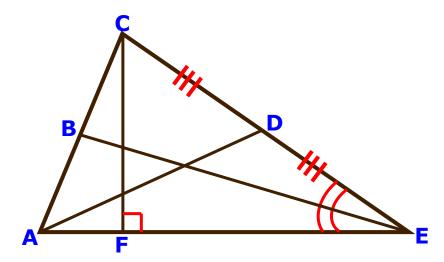
- **Acute triangle** is a triangle which all of its internal angles are **acute** angles.
- **Obtuse triangle** is a triangle that has an obtuse angle.

# 3 Important Lines of Triangles

Lines	Definition	
angle bisector	a segment which bisects an angle and connects a vertex and a point on the opposite side.	
median (bisector)	a segment that connects a vertex of the triangle and the midpoint of the opposite side	
altitude	a segment from the vertex of the triangle perpendicular to the line containing the opposite side	

#### **Practice**

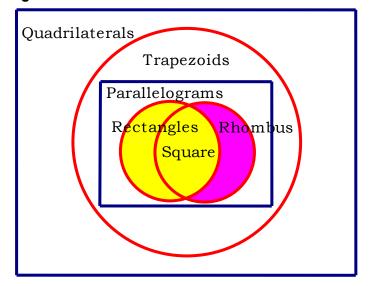
Which one is the angle bisector, median, or altitude of this triangle?



# Quadrilateral

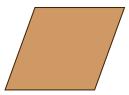
A quadrilateral is any 4 sided shape. The sum of all interior angles of any quadrilateral is 360 degrees.

# Classification of Quadrilaterals.



# Parallelogram

A parallelogram has 2 pairs of parallel sides.

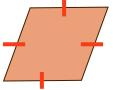


# **Types of Parallelogram**

A rectangle is a parallelogram with 4 right angles.



A rhombus is a parallelogram with 4 sides of equal length



A square is a parallelogram with 4 right angles and 4 sides of equal length.

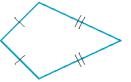


# **Another Types of Quadrilateral**

A <u>trapezoid</u> has exactly one pair of parallel sides.



A <u>kite</u> has exactly two pairs of congruent adjacent sides.



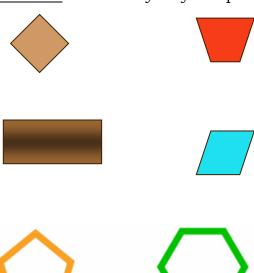
A <u>trapezium</u> has exactly one pair of parallel sides and two right angles.

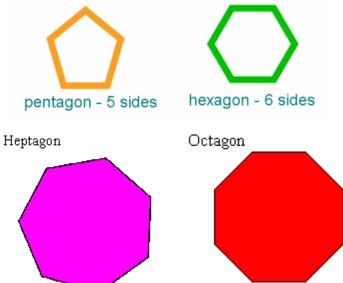


### **Practice**

Polygon

Classify the quadrilaterals in as many ways as possible.





### **Area and Perimeter**

- The formula of **area** of a triangle is
  ½ length of altitude × length of base
- The **perimeter** of a n-sided figure is the sum of all its side's lengths.

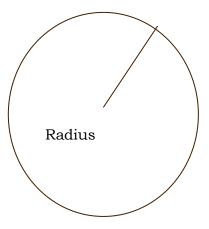
#### **Practice**

Find the formula for area of square, rectangle, rhombus, parallelogram, and trapezium.

#### Circle

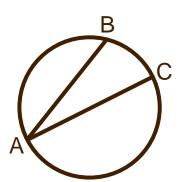
<u>Circle</u> is the set of all points in a plane that are a given distance from the center.

**Radius** (plural: **radii**) is a segment line that joins the center to a point on the circle.

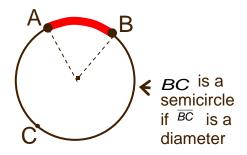


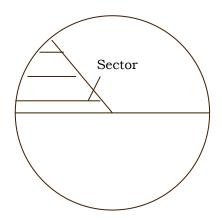
#### Parts of a Circle

- <u>Chord</u>: a line joining two points on a circle.
  - AC and AB are chords.
- <u>Diameter</u>: a chord that passes through the circles center.
  - AC is a diameter

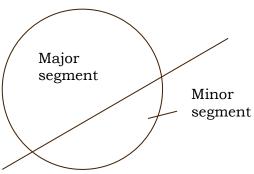


- <u>Arc</u>: two points on a circle and all the points needed to connect them.
  - minor arc AB or
  - major arc ACB or
- <u>Central Angle</u>: an angle whose vertex is at the center of the circle
- Sector: It is a region enclosed by two radii and arc of the circle.





Segment: It is a region enclosed by a chord and the arc joining the chord. The segment made my minor arc is called minor segment and segment by major arc as major segment.



<u>Circumference</u>: the perimeter of a circle, the distance around a circle.

$$C = d\pi$$
 or  $C=2 \pi r$  (d = diameter,

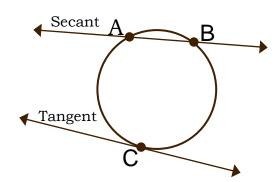
r = radius)

Area of a circle

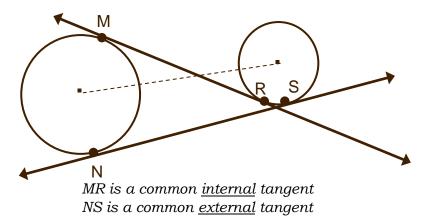
$$A = \pi r^2$$

### Line that Cuts the Circle

- Secant: a line that intersects a circle at exactly two points
- <u>Tangent</u>: a line that intersects a circle at exactly one point
  - The point of contact is called the **point of tangency** or point of contact.

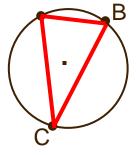


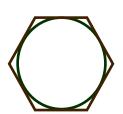
A <u>common tangent</u> is a line tangent to two circles (not necessarily at the same point)



### **Inscribed and Circumscribed Polygons**

- **Inscribed**: A polygon is inscribed in a circle (or another polygon) if all of its vertices lie on the circle (or another polygon).
  - The circle center is the incenter of the polygon
- **Circumscribed**: A polygon is circumscribed about a circle if each of its sides is tangent to the circle.
  - The circle center is the
     <u>circumcenter</u> of the polygon





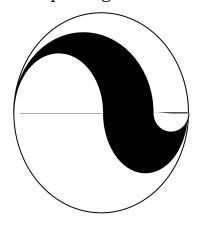
### **Practice**

Fill t	the blank with the right	words or	answer the que	estions.	
1.	consis	st of 2 end	points and all t	the points in	between.
2.	2. If each angle in a trian	ngle is less	s than $60^{\circ}$ , the	n the triang	le is called
	<del></del>		0		
3.	3. A line which meets an	other	_ at 90 is call	ed a	line.
4.	. If two angles of a triar	ngle are eq	ual to 45 <sup>0</sup> , the	n the triang	le is called
5.	5. If we a rig	ht angle, v	we will have two	o	_ angles of
	45 <sup>0</sup> .				
6.	6. If the measure of angl	e A is 130	<sup>0</sup> , then the		
			is -230°.		
7.	is called		o a side of trian	gles and thr	rough a vertex
8.	3. Each triangle has 3 po	oints, or _			
9.	)is a recta	ingle with	four congruent	sides.	
10	0. An octagon is		in a square if a	ll of its	lie
	on the square.				
11	1. A wi	th radius	10 m has		of 20π
	m.				
12	2. A quadrilateral which	ı only has	one pair of righ	nt angle can	be called
13	3	has se	ven vertices and	d sides	s.
14	4. If the		of a sector is 60	O degrees, th	nen the area of
	the sector is		of the circle's a	rea.	

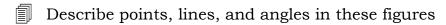
Answer these questions.

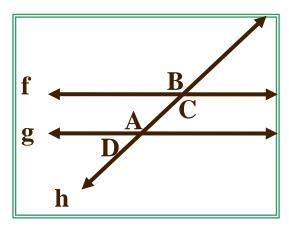
- 1. A right triangle has a hypotenuse of 6 and a perimeter of 14. Find the area of the triangle.
- 2. A regular hexagon is inscribed in a circle of radius 4 meters. What is the area of the hexagon?

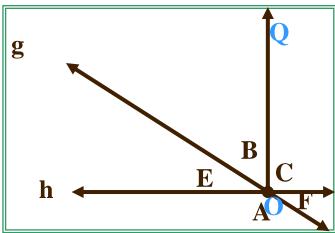
- 3. The total number of interior angles in two regular polygons is 17, and the total number of diagonals is 53. How many sides does each regular polygon have?
- 4. A triangle has sides of length 30, 40, and 50 meters. What is the length of the shortest altitude of this triangle?
- 5. A circle is inscribed in a triangle that has sides of lengths 60, 80, and 100 cm. Find the length of the radius of the circle.
- 6. We know that the vertices of a quadrilateral are 2, 3, 5, and 6 cm, respectively, from a point P. What is the largest possible area of this quadrilateral?
- 7. Five of the angles of an octagon have measures whose sum is 845°. Of the remaining three angles, two are complementary to each other and two are supplementary to each other. Find the measures of these three angles.
- 8. Gene wants to put a brick border around a tree. The border is to be placed 1.5m from the tree. If the circumference of the tree is 56.52 cm, what is the inner circumference of the brick border?
- 9. A hexagon is inscribed in a circle, which is inscribed in a square of side 10 cm. What is the length of each side of the hexagon?
- 10. Find the dimension of a rectangle of maximum area with a given perimeter P.
- 11. In how many different ways can you divide a square into four congruent shapes?
- 12. Find the area of the shaded region if the diameter of the circle is 24 cm and it is divided into four equal segments as shown.

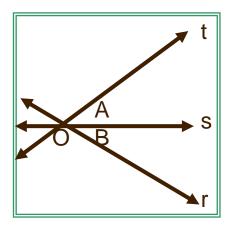


# **Practice**









# **Vocabularies of Chapter IV**

Words	Pronunciation	Indonesian	
Point	/pɔɪnt/	Titik	
Line	/laɪn/	Garis	
Horizontal line	/,hɒrɪ'zɒntl laɪn/	Garis datar	
Vertical line	/ˈvɜ:tɪkl laɪn/	Garis tegak	
Oblique line	/ə'bli:k laɪn/	Garis miring	
Straight line	/streIt laIn/	Garis lurus	
Curved line	/k3:v laɪn/	Garis lengkung	
Parallel	/pærəlel/	Sejajar	
Parallel line	/pærəlel laɪn/	Garis sejajar	
Cut	/kΛt/	Memotong	
Intersect	/,ɪntəˈsekt/	Memotong	
Intersected line	/,ɪntəˈsektɪd laɪn/	Garis berpotongan	
Point of intersection	/pɔɪnt əv ,ɪntəˈsekʃn/	Titikpotong	
Orthogonal	/,ɔ:θəˈgənəl/	Tegaklurus	
Perpendicular	/pɜ:pən'dɪkjʊlə(r)/	Tegaklurus	
Line segment	/laɪn ˈsegmənt/	Ruasgaris	
Endpoint	/endp <b>ɔɪ</b> nt/	Titik ujung	
Ray	/reI/	Sinar	
Angle	/ˈæŋgl/	Sudut	
Vertex	/v3:teks/	Titiksudut	
Vertices	/v3:tIsi:z/	Titiksudut-titiksudut	
Acute angle	/ə'kju:t ˈæŋgl/	Sudut lancip	
Right angle	/raɪt ˈæŋgl/	Sudut siku-siku	
Obtuse angle	/əb'tju:s 'æŋgl/	Sudut tumpul	
Straight angle	/streIt 'æŋgl/	Sudut lurus	
Full angle	/fʊl ˈæŋgl/	Sudut penuh	
Complementary angle	/,kømplɪ'mentrɪ 'æŋgl/	Sudut berpenyiku	
Supplementary angle	/ˈsΛplɪˈmentrɪ ˈæŋgl/	Sudut berpelurus	
Adjacent angle	/ə'd3eɪsnt 'æŋgl/	Sudut bersebelahan	
Corresponding angle	/,korī'sponding 'æŋgl/	Sudut sehadap	
Opposite angle	/ɒpəzɪt ˈæŋgl/	Sudut bertolakbelakang	
Alternate interior angle	/ɔ:l'tɜ:nət ɪn'tɪərɪə(r) æŋgl/	Sudut dalam berseberangan	
Alternate exterior angle	/ɔ:lˈtɜ:nət ɪkˈstɪərɪə(r) ˈæŋgl/	Sudut luar berseberangan	
Opposite-interior angle	/ppəzit in'tiəriə(r)	Sudut dalam sepihak	

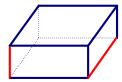
	ˈæŋgl/	
Opposite-exterior angle	/ppəzit ik'stiəriə(r) 'æŋgl/	Sudut luar sepihak
Polygon	/ˈpɒlɪgən/	Segibanyak
Triangle	/ˈtraɪæŋgl/	Segitiga
Quadrilateral	/,kwɒdrɪˈlætərəl/	Segiempat
Pentagon	/'pentəgən/	Segilima
Hexagon	/'heksəgən/	Segienam
Heptagon	/ˈheptəgən/	Segitujuh
Octagon	/ˈɒktəgən/	Segidelapan
Nonagon	/ˈnəʊnəgən/	Segisembilan
n-gon	/en' gən/	Segi n
Side	/said/	Sisi
Interior angle	/ɪnˈtɪərɪə(r) æŋgl/	Sudut dalam
Isosceles triangle	/aɪsɒsəli:z 'traɪæŋgl/	Segitiga samakaki
Equilateral triangle	/,ikwɪ'lætərəl 'traɪæŋgl/	Segitiga samasisi
Right triangle	/raɪt 'traɪæŋgl/	Segitiga siku-siku
Acute triangle	/əˈkju:t ˈtraɪæŋgl/	Segitiga lancip
Obtuse triangle	/əb'tju:s 'traɪæŋgl/	Segitiga tumpul
Scalene triangle	/ˈskeili:n ˈtraɪæŋgl/	Segitiga sembarang
Apex	/'eɪpeks/	Titikpuncak
Base	/beis/	Alas
Height	/haɪt/	Tinggi
Area of triangle	/ˈeərɪə əv ˈtraɪæŋgl/	Luas segitiga
Perimeter	/pəˈrɪmɪtə/	Keliling
Bisect	/bʌɪˈsɛkt/	Membagi dua sama
		besar
Median	/ˈmiːdɪən/	Garis berat
Angle bisector	/æŋglb bʌɪˈsɛktə(r)/	Garis bagi
Altitude	/ˈaltɪtjuːd/	Garis tinggi
Parallelogram	/_parəˈlɛləgram/	Jajargenjang
Square	/'skweə/	Persegi

Rectangle	/ˈrɛktæŋgl/	Persegipanjang
Rhombus	/ˈrɒmbəs/	Belahketupat
Kite	/kʌɪt/	Layang-layang
Trapezium	/trəˈpiːzɪəm/	Trapesium siku-siku
Trapezoid	/'trapizoid /	Trapesium
Circle	/ˈsəːk(ə)1/	Lingkaran
Center	/ˈsɛntə/	Titikpusat
Radius	/ˈreɪdɪəs/	Jari-jari
Radii	/ˈreɪdiʌɪ/	Jari-jari (jamak)
Circumference	/səˈkʌmfərəns/	Keliling (lingkaran)
Chord	/kɔːd/	Talibusur
Diameter	/dʌɪˈamɪtə/	Diameter
Arc	/ɒ:k/	Busur
Segment	/'segmənt/	Tembereng
Sector	/ˈsɛktə/	Juring
Secant	/ˈsiːkənt/	Garis memotong
		lingkaran
Tangent	/ˈtæŋʒənt/	Garissinggung
		lingkaran
Internal tangent	/ınˈtəːnəl ˈtæŋʒənt/	Garissinggung dalam
External tangent	/ɪkˈstəːnəl ˈtæŋʒənt/	Garisinggung luar
Inscribed	/ınˈskrʌɪb/	(segibanyak) Termuat
		(dalam lingkaran atau
		segibanyak)
Circumscribed	/ˈsəːkəmskrʌɪb/	(lingkaran) Memuat
		(segibanyak)

### V. Solid/Space Geometry

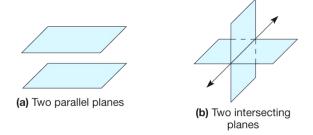
### Position of Lines in Space

In plane geometry, two lines can be parallel or intersect. But in space, there is another choice. Two lines can be skew. Skew Lines are nonintersecting lines that are not parallel.



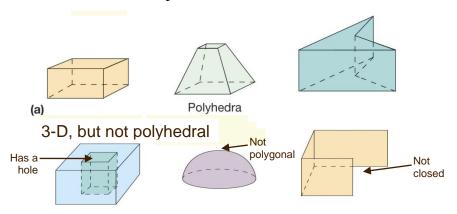
### Position of Planes in a Space

In three dimensions, planes are similar to lines in two dimensions. They can be parallel, or intersect.



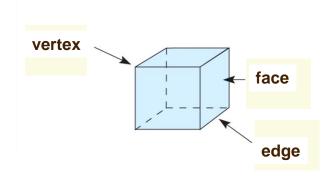
## Polyhedra (Bidang Banyak)

A **polyhedron** is the union of polygonal regions such that a finite region of space is enclosed without any holes in the interior.



### Parts of a Polyhedra

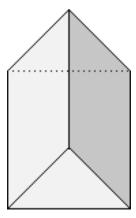
- The polygonal shapes that form the polyhedron are called its **faces/sides**.
- The line segments where two faces meet are its **edges**.
- A point where three or more edges meet is a **vertex**



# Types of a Polyhedra

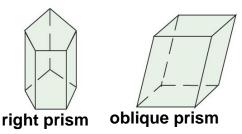
#### A. Prisms

- If a polyhedron has identical polygonal faces that are opposite each other, then it is a **prism**
- The segments that connect base side and top side is called <u>side-edges</u>. The others is called <u>based-edges</u>.
- A n-sided prism has n side-edges and 2n base-edges



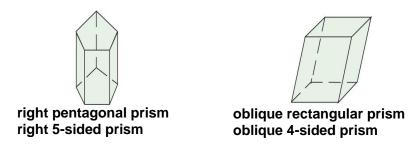
The opposite, identical side of a prism are called its **bases**. The other sides are parallelograms and are called the **lateral sides**.

If the lateral sides of a prism are rectangles, it is a **right prism**. If not, it is called an **oblique prism**.



## **Naming Prism**

When naming a prism we use two main descriptors. First, we say whether it is right or oblique, then we say what type of polygon form the prism's bases or the number of its lateral sides.



#### Formula for Volume and Surface Area of a Prism

Generally, the formula for volume of prism is

V = Area of its base × its height

The height in this formula is the real height not the slant height.

General formula for its surface area is

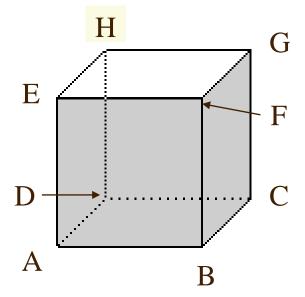
A = sum of all its sides area

## Special Kinds of (Rectangular) Prism

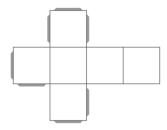
#### 1. Cube

#### Parts of a Cube

- flat sides of equal squares (ex: side ABCD)
- 12 edges of equal lengths (ex: edge AB)
- 8 vertices (ex: point A)
- 12 face-diagonals
  (ex: segment line AF)
- 4 space-diagonals (ex: segment line AG)
- for diagonal planes (ex: side ABGH)



### Net of a Cube



# Formula for Volume and Surface Area of a Cube

The formula for volume of cube is

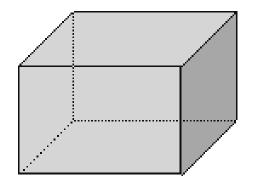
 $V = (length of edge)^3$ 

And for surface area is

 $A = 6 \times (length of edge)^2$ 

## 2. Cuboid (Rectangular Prism)

- A rectangular prism has 8 rectangular solid angles, 12 edges, equal and parallel in fours.
- It is bounded by three pairs of congruent rectangles lying in parallel planes.



#### Formula for Volume and Surface Area of a Cuboid

Suppose that we can name the three different edges of cuboid as length, width, and height. Then, the formula for volume of cuboid is

 $V = length \times width \times height$ 

And for surface area is

 $A = 2 \times [(length \times width) + (length \times height) + (width \times height)]$ 

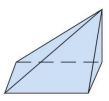
# **Pyramid**

A **pyramid** is a three-dimensional solid with one polygonal base and with line segments connecting the vertices of the base to a single point somewhere above the base.

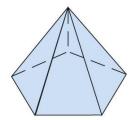


# **Naming Pyramid**

The lateral sides of a pyramid are triangles. If they are isosceles triangles, then it is a *right pyramid*, otherwise it is an *oblique pyramid*.



Oblique square pyramid



Right pentagonal pyramid

# Formula for Volume and Surface Area of a Pyramid

Generally, the formula for volume of prism is

 $V = (1/3) \times Area of its base \times its height$ 

The height in this formula also means the real height not the slant height.

General formula for its surface area also like prism which is

A = sum of all its sides area

#### The Platonic Solids

Only five possible regular polyhedra exist. These five solids are called PLATONIC SOLIDS



The tetrahedron
--4 triangular



The cube --6 square



The octahedron --8 triangular



The dodecahedron --12 hexagonal



The icosahedron --20 triangular

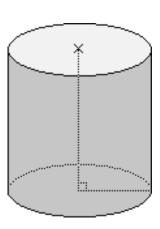
Regular Polyhedron	Number of Sides	Each Side is a	Number of Polygons at a vertex
Tetrahedron	4	Triangle	3
Octahedron	8	Triangle	4
Icosahedron	20	Triangle	5
Hexahedron	6	Square	3
Dodecahedron	12	Pentagon	3

# Cylinder

- A **cylinder** is a prism in which the bases are circles or ellips.
- The volume of a cylinder is the area of its base times its height  $V = \pi r^2 h$
- The surface area of a cylinder is  $A = 2\pi r^2 + 2\pi rh$

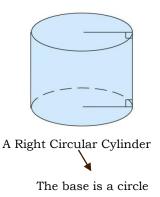
r = length of base's radius

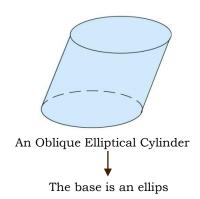
h = height of cylinder



# **Types of Cylinder**

A cylinder can be right or oblique, and cylinders are named in the same way as prisms and pyramids





#### Cone

- A cone is like a pyramid but with a circular base instead of a polygonal base.
- The volume of a cone is onethird the area of its base times its height:

$$V = \frac{1}{3}\pi r^2 h$$

 The surface area of a cone is base surface area + curved surface area:

$$A = \pi r^2 + \pi rs$$

or

r = length of base's radius

h = heigth of cone

s = slant height of cone

$$A = \pi r^2 + \pi r \sqrt{r^2 + h^2}$$



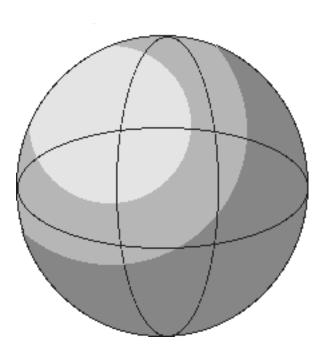
# **Sphere**

- Sphere is the mathematical word for "ball." It is the set of all points in space a fixed distance from a given point called the center of the sphere.
- A sphere has a radius and diameter, just like a circle does.
- The volume of a sphere is:

$$V=\frac{4}{3}\pi r^3$$

The surface area of a sphere is:

$$A = 4\pi r^2$$



# **Exercises**

A. An	swers these questions
1.	Intersection of the wall in a class can be described as
2.	Two planes in a space can or, but
	can'teach other.
3.	A have six vertices, six sides, and
	edges.
4.	If a cuboid has edges whose lengths are 8, 6, and 5 cm, then its surface
	area is
5.	A cylindrical oil tank can be filled with 7,700 liter gasoline. If its base's
	radial is 70 cm, then find its height.
6.	Draw a net of regular square pyramid and a cone.
7.	A prism is inscribed in a cube. The top of the pyramid is at the center of
	the top side of cube, while its base is the base side of cube. If the cube
	has edges of 9 cm, then volume of the pyramid is
8.	A three dimensional object that has 8 sides, 12 vertices, and 18
	is called
9.	A cone is inscribed in a cylinder. Their base side is equal, while the top
	of the cone is placed in the center of the top of cylinder. If the cylinder
	has radii of 7 cm and height of 24 cm, find the curved surface area of
	the cone.
10.	What is the diameter of the sphere inscribed in a cube that is inscribed
	in a sphere of diameter 10?
11.	The total surface area of a cube, expressed in square centimeters, is
	equal to the volume of the cube, expressed in cubic centimeters.
	Compute the length, in centimeters, of a side of the square.
12.	Two cylindrical water tank stand side by side. One has radius of 4
	meters and contains water to a depth of 12.5 meters. The other has a
	radius of 3 meters and is empty. Water is pumped from the first tank to
	the second tank at a rate of 10 cubic meters per minute. How long must
	the pump run before the depth of the water is the same in both tanks?
13.	The pyramid ABCDE has a square base and all four triangular faces are
	equilateral. The measure of the angle ABD is

B. Plan a presentation about one of three-dimensional objects that contains a description of the object, its main parts, the formula for its volume and surface area, and a problem with solution related to the object.

# Vocabularies of Chapter V

Words	Pronunciation	Indonesian
Skew Line	/skjuː laɪn/	Garis Bersilangan
Polyhedron	/ˌpɒlɪˈhɛdrən/	Bidang Banyak
Polyhedra	/ˌpɒlɪˈhɛdrə/	Bidang Banyak (jamak)
Face	/feis/	Sisi
Side	/saɪd/	Sisi
Edge	/ɛdʒ/	Rusuk
Vertex	/v3:teks/	Titiksudut
Vertices	/v3:tIsi:z/	Titiksudut-titiksudut
Side-edge	/saɪd ɛdʒ/	Rusuk tegak
Based-edge	/beisd ed3/	Rusuk alas
Base	/beis/	Alas
Lateral Side	/ <u>'latərəl</u> saɪd/	Sisi tegak
Prism	/ 'prɪz(ə)m/	Prisma
Right Prism	/raɪt ˈprɪz(ə)m/	Prisma tegak
Oblique Prism	/ə'bli:k ˈprɪz(ə)m/	Prisma miring
Volume	/ˈvɒljuːm/	Isi
Surface Area	/ˈsəːfɪs ˈɛːrɪə/	Luas Permukaan
Curved Surface	/kəːvd <u>'səːfɪs</u> /	Selimut (tabung/kerucut)
Height	/haɪt/	Tinggi
Slant Height	/sla:nt haɪt/	Panjang ruas garis Pelukis
Cube	/kju:b/	Kubus
Cuboid	/ˈkjuːbɔɪd/	Balok
Face-diagonal	/feis dai'ag(ə)n(ə)l/	Diagonal bidang
Space-diagonal	/speis dai'ag(ə)n(ə)l/	Diagonal ruang
Diagonal Plane	/dʌɪˈag(ə)n(ə)l pleɪn/	Bidang diagonal
Net	<u>/nɛt/</u>	Jaring-jaring
Pyramid	/ˈpɪrəmɪd/	Limas
Platonic Solid	/pləˈtɒnɪk ˈsɒlɪd/	Bangun padat Plato
Tetrahedron	/_tɛtrəˈ hɛdrən/	Tetrahedron
Hexahedron	/_hɛksəˈhɛdrən/	Heksahedron
Dodecahedron	/_dəudekə hedrən/	Dodekahedron
Icosahedron	/_ʌɪkɒsəˈhɛdrən/	Ikosahedron
Cylinder	/ˈsɪlɪndə/	Tabung
Cone	/kəʊn/	Kerucut
Sphere	<u>/sfiə/</u>	Bola

## VI. Logic and Set

### The Logic Vocabularies

Pernyataan	statement/proposition
nilai kebenaran	truth value
Benar	true
Salah	false
tabel kebenaran	truth table
Negasi	negation
pernyataan majemuk	compound statement
Konjungsi	conjunction
Disjungsi	disjunction
Implikasi	impication
Anteseden	antecedent
Konsekuen	consequent
Biimplikasi	biimplication
Deduksi	deduction
Argumen	argument
Premis	premise
konklusi/kesimpulan	conclusion

#### **Practice**

							answer		

- 1. \_\_\_\_\_ is a sentence that is either \_\_\_\_\_ or false.
- 2. If a statement is true, then its \_\_\_\_\_ is false.
- 3. A compound statement that use the word "or" is called \_\_\_\_\_
- 4. A compound statement that is true only when statements that make it are true is called \_\_\_\_\_
- 5. If the antecedent is true, and the \_\_\_\_\_\_ is false, then an \_\_\_\_\_ is false.
- 6. What is the name of compound statement which is true if the two statements that combined have the same truth value?
- 7. Given two statements P and Q. P is true. What can you say about the truth value of their disjunction and implication?
- 8. An \_\_\_\_\_ contains \_\_\_\_ and a conclusion.

# Symbols in Logic

12 111 1705	gic .						
$\forall$	For all/for every						
3	There exists/For some						
Э	Such that						
¬ A	The negation of A/not A						
$A \wedge B$	The conjunction of $A$ and $B/A$ and $B$						
$A \vee B$	The disjunction of $A$ and $B/A$ or $B$						
$P \Rightarrow$	If P, then Q/P implies Q						

Q	P is a sufficient condition for Q
	Q is a necessary condition for P
A ⇔	A if and only if B
В	A is equivalent to B

#### **Practice**

Read out the following sentences:

- 1.  $S \Rightarrow (H \wedge U)$
- 2.  $(S \Rightarrow H) \wedge U$
- 3.  $((N \vee G) \wedge (\neg N) \Rightarrow (G \Rightarrow N)$
- 4.  $(P \Rightarrow Q) \land (Q \Rightarrow R) \Leftrightarrow (P \Rightarrow R)$
- 5.  $\forall x, Ax \Rightarrow Mx$
- 6.  $\exists x, \neg Cx \lor Dx$

#### Set

Set is collection of objects which have same or equal characteristic.

The objects in a set are called elements or members of the set

 $t \in A$  t is an element of A

t is a member of A

t belongs to A

 $u \notin A$  u is not an element of A

u is not a member of A

u does not belong to A

# Special Kind of Set

- **Empty set** is a set which has no member.
- **Power set** of set S is collection of all subsets of the set S.

## Cardinality of Set

- The number of distinct objects in a set is called the **cardinality** of the set.
- The cardinality of empty set is zero.
- ➤ If a set has finite member, we called the set **finite set**. Otherwise, we called the set **infinite set**.

# Comparing two sets

 $\triangleright$  A = B A is equal to B

 $ightharpoonup A \subset B$  A is subset of B

A is contained in B

ho A  $\subseteq$  B A is subset of or equal to B

ho A  $\supset$  B A is superset of B

A contains B

#### Set operation

 $\triangleright$  A  $\cup$  B The union of A and B

A cup/joint B

ightharpoonup A  $\cap$  B The intersection of A and B

A cap/meet B

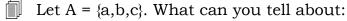
 $\triangleright$  A \ B The difference between A and B

A minus B

 $\triangleright$  A  $\times$  B The Cartessian product of A and B

A cross B

### **Exercise**



- 1. a and c
- 2. f
- 3.  $\{b,c\}$
- 4. {}
- 5. 8
- How do we say these mathematical terms?
  - 1.  $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$
  - 2.  $A \subseteq (B \setminus C) \cap E$
  - 3.  $A = A \setminus (A \setminus B)$
  - 4.  $A = (A \setminus B) \cup (A \cap B)$
  - 5. (D \ E)  $\cap$  (D  $\cap$  E) =  $\emptyset$

## VII. Probability and Statistics

### **Experiment**

A situation whose results depend on chance.

### Examples:

- rolling a dice,
- flipping a coin,
- choosing a card from a deck,
- selecting a domino piece.

### Events and Sample Space

- Event is the result of an experiment that can, but need not occur.
- ➤ Sample space is the set of all events

Example:

In one flipping coin experiment, the sample space is {head, tail}

# **Measures of Central Tendency**

#### > Mode

the value that occurs most often in the data

#### > Mean or the average

is the sum of the observations divided by the number of observations.

#### > Median

is the middle number (when data is odd) or average of two middle numbers (when data is even) when data are arranged from the smallest to the largest.

# **Measure of Dispersion**

- ranges: difference between greatest and smallest data
- variance and standard of deviation
- quartiles, deciles, and percentiles
- interquartile: difference between third and first quartiles.

# **Grouped Data**

Range of grouped data = Midpoint of highest class - Midpoint of lowest class

Lower boundary of a class interval =  $\frac{\text{Upper limit of the class before it + Lower}}{\text{limit of the class}}$ 

Upper boundary of a class interval = Upper limit of the class + Lower limit of the class after it

Size of class interval = Upper boundary - Lower boundary

Size of class interval =  $\frac{\text{Range of data}}{\frac{1}{2}}$ Number of classses

Midpoint of a class =  $\frac{\text{Lower Limit} + \text{Upper Limit}}{\text{Upper Limit}}$ 

 $Mean = \frac{Sum of (Midpoint \times Frequency)}{}$ Sum of frequencies

Modal class = Class interval with the highest frequency

# Example of a grouped data

Marks	Frequency	<b>Cumulative Frequency</b>	Lower Limit	Upper Limit	Lower Boundary	Upper Boundary	Midpoint
41-50	6	6	41	50	40,5	50,5	45,5
51-60	14	20	51	60	50,5	60,5	55,5
61-70	42	62	61	70	60,5	70,5	65,5
71-80	28	90	71	80	70,5	80,5	75,5
81-90	8	98	81	90	80,5	90,5	85,5
91-100	2	100	91	100	90,5	100,5	95,5

There are many types of graph that can be used to show information. It is important that every graph used is appropriate, accurate, and has a title, labels and key.

**To represent data** effectively, we can choose several form of diagrams like: tables, bar and compound bar graphs, histograms, frequency polygons, pie charts, dot plots, line and broken line graphs, or stem and leaf plots.

### > Table

Food stamp particip	pants, people in poverty,	and unemployed	people, 1980-2002

Year	Food stamp participants <sup>1</sup>	People in poverty <sup>2</sup>	Unemployed people 3	
		Millions		
1980	21.1	29.3	7.6	
1981	22.4	31.8	8.3	
1982	21.7	34.4	10.7	
1983	21.6	35.3	10.7	
1984	20.9	33.7	8.5	
1985	19.9	33.1	8.3	
1986	19.4	32.4	8.2	
1987	19.1	32.2	7.4	
1988	18.6	31.7	6.7	
1989	18.8	31.5	6.5	
1990	20.1	33.6	7.0	
1991	22.6	35.7	8.6	
1992	25.4	38.0	9.6	
1993	27.0	39.3	8.9	
1994	27.5	38.1	8.0	
1995	26.6	36.4	7.4	
1996	25.5	36.5	7.2	
1997	22.9	35.6	6.7	
1998	19.8	34.5	6.2	
1999	18.2	32.3	5.9	
2000	17.2	31.1	5.7	
2001	17.3	32.9	6.7	
2002	19.1		8.4	

Note: Fiscal 2002 data for FSP participation are preliminary. Poverty measures are based on questions about income in the previous year, so they are not yet available for 2002.

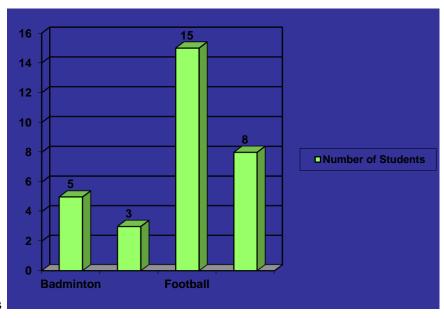
income in the previous year, so they are not yet available for 2002.

1/ Source: Food and Nutrition Service, USDA. Data as of April 24, 2003. Data refer to the average monthly number of participants during the fiscal year.

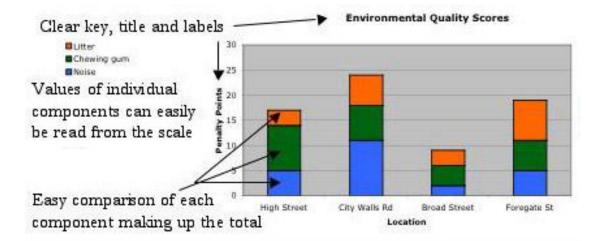
number of participants during the fiscal year.

2/ Source: U.S. Bureau of Commerce, Census Bureau. Data refer to the number of people in poverty during the calendar year.

the calendar year,
3/ Source: U.S. Department of Labor, Bureau of Labor Statistics. Data refer to the average monthly number
of unemployed people during the calendar year.

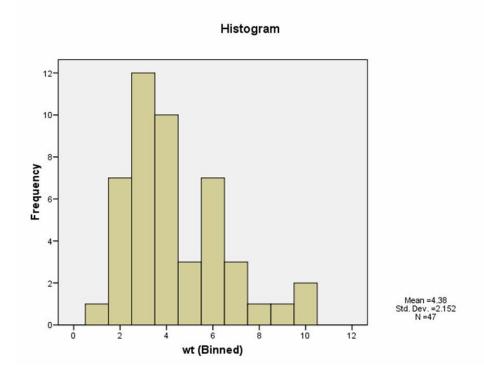


- Simple Bar Graphs
- Compound Bar Graphs



# > Histograms

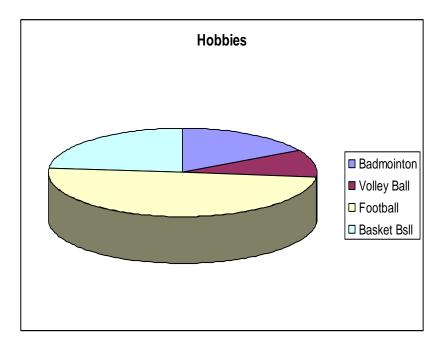
Histograms are similar to bar charts but a **continuous** scale divided into groups or "**classes**" is used (x axis). Vertical axis shows the frequency of each class



# ➤ Pie Charts

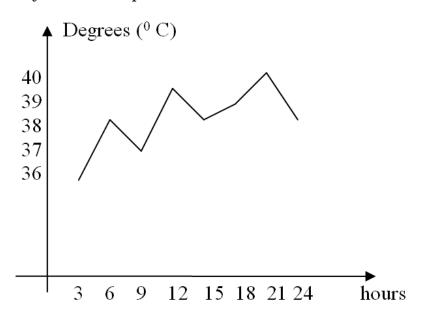
The pie chart is useful to show the total data divided into proportions. It often has good visual impact but can it is difficult to read the data accurately, particularly if there are several categories.

The segments should be drawn from the largest first and the smallest last unless there is an "others" category in which case that should be last regardless of its size. Segments should be shaded in different colours and a suitable key or labels added. The raw data and percentage figures can be added to the key if appropriate.



# Line Graphs

Line graphs show **changes** over time. All the points are joined up and the axes should normally begin at zero. Rates of change are shown well, although careful thought to the scale should be given. Unsuitable if there are only a few data points.



Stem-and-leaf Plot

The heights of 11 badminton players are (in inches): 56, 57, 58, 58, 59, 59, 60, 61, 61, 63

HEIGHT	IN INCHES
Stem	Leaves
5	6, 7, 8, 8, 9, 9
6	0, 1, 1, 1, 3

# Probability

Probability of an Event is number of outcomes in an event divide by number of outcomes in sample space

$$P(E) = \frac{n(E)}{n(S)}$$

# Some properties of probability

ightharpoonup P(E)=1 means the probability of an event that is certain to happen. So, P(S)=1

> P(E)=0 means the probability of an event that is certain not to happen

 $\triangleright$  0  $\leq$  P(E)  $\leq$  1

 $\triangleright$  P(A<sup>c</sup>)=1-P(A)

 $\triangleright$  P(A  $\cup$  B) = P(A) + P(B) - P(A  $\cap$  B)

Probability of A or B equals probability of A plus probability of B minus probability of A and B.

Two events are said to be **independent** of the happening of one event has no effect on the happening of the other.

Example: Choosing an apple and an egg. Let C be the event that a rotten apple is chosen.Let D be the event that a rotten egg is chosen. Then C and D are independent events.

➤ If there are 2 *independent events* A and B, we can calculate the probability of A and B by applying to Multiplication/Product Law:

$$P(A \cap B) = P(A) P(B)$$

➤ If there are 2 *independent events* A and B, we can calculate <u>the probability</u> of A or B by applying to Addition/Additive Law:

$$P(A \cup B) = P(A) + P(B)$$

#### **Permutations**

The product of the n first natural numbers is called n factorial and denoted by n!

$$n! = 1.2.3.4..(n-1).n$$

### Example

How many three-digit numbers that can be formed using the digits 1,2, and 4 if no digit is repeated.

Answer

3 digits can be placed for the unit digit.

2 digit left for the tens digit.

And there has been 1 remaining digit for the hundreds digit.

Thus, 1.2.3 = 6 numbers that can be formed

**Permutation** is number of arrangements of a number of discrete objects when order matters.

There are two kinds of permutation

a. of r different items

r!

b. of r items selected from n

$$_{n}P_{r} = P(n,r) = \frac{n!}{(n-r)!}$$

#### Example

Find the number of permutation of the letters in "mathematics".

**Combination** is number of possible arrangements when order is not important.

Formula for combination of r items from n where all are different or distinguishable.

$$_{n}C_{r} = C(n,r) = \frac{n!}{r!(n-r)!}$$

# Example

List all of the three digit combinations that can be formed from the number 2,3,4,5,6

#### Exercise

- 1. Give an example of an occasion when you would use a histogram rather than a bar graph, and say how you would construct it.
- 2. Two mathematics classes took the same test. The mean of the first class of twenty students was 90 percent. The mean of the second class was 83 percent. If the mean grade for the combined classes was 85 percent, how many students were in the second class?
- 3. In a football match, team A has a penalty kick. The coach is deciding which player to take that place. It is known that the goalkeeper will defend the left, the central and the right parts with probabilities of 0.3, 0.2 and 0.5 respectively. Find the probability that Ronaldo has a goal if the probability of kicking the ball to the left, central and right part is 0.2, 0.5 and 0.3.
- **4.** After the ace of spades is removed from an ordinary deck of cards, the cards are shuffled and the top card is discarded without its face value known. What is the probability that the next card is an ace?
- **5.** Both the mean and the median of a set of five distinct natural numbers are 7, and the range is 6. What numbers are included in the set?
- **6.** An apple, an orange, a banana, and a pear are laid out in a straight line. The orange is not at either end and is somewhere to the right of the banana. In how many ways can the fruit be laid out?
- **7.** If the chances of rain are 40 percent and 20 percent for the two days of a weekend, what is the chance that it will rain on at least one of the two days?
- **8.** A social club contains seven woman and four men. The club wants to select a committee of three members the represent it at state convention. How many of the possible committees that could be chosen contain at least one man?

- 9. A student has three mathematics books, two English books, and four science books. The number of ways that the books can be arranged on a shelf, if all books of the same subject are kept together, is:
- 10. Six peoples want to get in line in front of teller, and two people don't want to stand near each other. How much queuing patterns can be formed?

Vocabularies of Chapter VII

Words	Pronunciations	Indonesians
bar	/ba:(r)/	Batang
boundary	/ˈbaʊndrɪ/	Batas
chart	/t∫a:t/	Tabel/diagram
coin	/kɔɪn/	Koin
combination	/ˌkømbɪˈneɪ∫n/	Kombinasi
deviation	/ˌdi:vɪˈeɪʃn/	Simpangan/Deviasi
diagram	/ˈdaɪəgræm/	Diagram
dice	/daɪs/	Dadu
event	/ɪˈvənt/	Kejadian
experiment	/ɪkˈsperɪmənt/	Percobaan/Eksperimen
factorial	/fæk'tɔːrɪəl/	Faktorial
frequency	/ˈfri:kwənsɪ/	Frekuensi
graph	/ˈgra:f/	Grafik
histogram	/ˈhɪstəgræm/	Histogram
leaf	/li:f/	Daun
mean	/mi:n/	Rata-rata
median	/ˈmɪ:dɪən/	Median/nilai tengah
mode	/məʊd/	Modus
mutually exclusive	/ˈmjuːtʃʊəlli ɪkˈskluːsɪv/	Saling lepas
outcome	/ˈaʊtkʌm/	Hasil percobaan
permutation	/ˌp3mju:'teɪ∫n/	Permutasi
pie	/paɪ/	Lingkaran
plot	/plot/	Plot
probability	/prøbəˈbɪlətɪ/	Peluang
quartile	/kwɔ:taɪl/	Kuartil

range	/reInd3/	Jangkauan
sample	/'sa:mpl/	Sampel
standard	/'stændəd/	Standar/Baku
statistics	/stəˈtɪstɪks/	Stastik
stem	/sti:m/	Dahan/batang
variance	/'veərɪəns/	Variansi

### VIII. Calculus

Three Big Calculus Topics

- Limits
- Derivatives
- Integrals

#### Limit

Given real numbers c and L. If the values f(x) of a function approach or equal L as the values of x approach (but do not equal) c, then f has a **limit** L as x approaches c.

## How to say Limits

$\lim_{x\to\infty} f(x)$	The limit of f x as x approaches infinity The limit as x goes/tends to infinity of f x
$\lim_{x \to a^+} f(x)$	The limit of f x as x approaches a from above/right The right-hand limit of f x
$\lim_{x \to a^{-}} f(x)$	The limit of f x as x approaches a from below The left-hand limit of f x

# Examples:

Determine 
$$\lim_{x\to 10} \frac{x^2-100}{x-10}$$

Answer

Step 1 : Simplify the expression.

The numerator can be factorised.

$$\frac{x^2 - 100}{x - 10} = \frac{(x - 10)(x + 10)}{x - 10}$$

Step 2: Cancel all common terms

x - 10 can be cancelled from the numerator and denominator.

$$\frac{(x-10)(x+10)}{x-10} = x+10$$

Step 3: Let  $x \to 10$  and write final answer

$$\lim_{x \to 10} \frac{x^2 - 100}{x - 10} = \lim_{x \to 10} x + 10 = 20$$

#### **Exercise**

Find the limit of the following

$$\lim_{x \to 3} \frac{x^2 - 9}{x + 3}$$

$$\lim_{x \to 3} \frac{x + 3}{x^2 + 3x}$$

$$\lim_{x \to 2} \frac{3x^2 - 4x}{3 - x}$$

#### **Derivatives**

The derivative of f at point a with respect to x is the limit of changing rate of f near point a.

The derivative of a function f at a, denoted by f (a), is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

if this limit exists.

A function has a derivative at a point if and only if the function's right-hand and left-hand derivatives exist and are equal. In this case, we say the function is **differentiable**.

If the right-hand limit does not equal to the left-hand limit, then the limit does not exist and we said that f is **not differentiable** at a.

There are much rules about derivative (rules of differentiation)

- > Factor rule
- > Sum rule
- > Product rule
- Quotient rule
- Chain rule (for composite function)
- ➤ Rule for trigonometry function

#### **Exercise**

- 1. Find the derivative of the function  $f(x) = x^2-8x+9$  at a using definition of derivative.
- 2. If  $f(x) = 3x-2x^2$ , find f'(x) from definition and hence evaluate f'(4).

## How to say Derivatives

f (x)	f prime x/f dash x The (first) derivative of f with respect to x	
f`(x)	f double-prime x/f double-dash x The second derivative of f with respect to x	
f``(x)	f triple-prime x/f triple-dash x The third derivative of f with respect to x	
$f^{(IV)}x$	f four x/f four prime x the fourth derivative of f with respect to x	
$\frac{df}{dx}$	"D F D X" The derivative of f with respect to x	
$\frac{d^2f}{dx}$	"D" squared "F D X" squared the second derivative of f with respect to x	
$\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$	"D Y partially by X" The (first) partial derivative of y with respect to x Delta y by delta x	
$\frac{\partial^2 \mathbf{y}}{\partial \mathbf{x}^2}$	"D squared Y partially by X squared" The second partial derivative of y with respect to x Delta two y by delta x squared	

There are much rules about derivative

- > Factor rule
- > Sum rule
- Product rule
- Quotient rule
- > Chain rule (for composite function)
- > Rule for trigonometry function

#### Maxima and minima

- Find f'(x) and solve f'(x) = 0. This value of x (say  $x^*$ ) is the **stationary/extreme/critical point**; probably maxima or minima will occur at this point.
- Find f''(x). If f''(x) > 0 then x is a **local minima**. If f''(x) < 0 then x is a **local maxima**. We can say that x is a **maximum/minimum point**.

• If f''(x)=0, then x may be a **point of inflection**.

## Example

Michael wants to start a vegetable garden, which he decides to fence off in the shape of a rectangle from the rest of the garden. Michael only has 160 m of fencing, so he decides to use a wall as one border of the vegetable garden. Calculate the width and length of the garden that corresponds to largest possible area that Michael can fence off.

#### **Answer**

Step 1: Examine the problem and formulate the equations that are required

The important pieces of information given are related to the area and
modified perimeter of the garden. We know that the area of the garden
is:

$$A = W \times L$$
 (Equation 1)

We are also told that the fence covers only 3 sides and the three sides should add up to 160 m. This can be written as:

$$160 = W + L + L$$

However, we can use the last equation to write W in terms of L:

$$W = 160 - 2L \qquad (Equation 2)$$

Substitute Equation 2 into Equation 1 to get:

$$A = (160 - 2L)L = 160L - 2L^2$$
 (Equation 3)

Step 2: Differentiate

Since we are interested in maximizing the area, we differentiate Equation 3 to get:

$$A'(L) = 160 - 4L$$

Step 3: Find the stationary point

To find the stationary point, we set A'(L) = 0 and solve for the value of L that

maximizes the area.

$$A'(L) = 160 - 4L$$
 $0 = 160 - 4L$ 
 $4L = 160$ 
 $L = 40 \text{ metres}$ 

Substitute into Equation 2 to solve for the width.

$$W = 160 - 2L = 160 - 2(40) = 160 - 80 = 80m$$

Step 4: Write the final answer

A width of 80 m and a length of 40 m will yield the maximal area fenced off.

#### **Exercises**

- 1. The sum of two positive numbers is 20. One of the numbers is multiplied by the square of the other. Find the numbers that make this products a maximum
- 2. After doing some research, a transport company has determined that the rate at which petrol is consumed by one of its large carriers, travelling at an average speed of x km per hour, is given by P(x) = (55/2x) + (x/200) litres per kilometre.
  - i. Assume that the petrol costs Rp.4,000 per litre and the driver earns Rp.18,000 per hour (travelling time). Now deduce that the total cost, C, in Rupiahs, for a 2,000 km trip is given by:

$$C(x) = (256000)/x + 40x$$

ii. Hence determine the average speed to be maintained to effect a minimum cost for a 2,000 km trip.

#### **Integral**

Integral is could be described as the set of all antiderivatives, because if the derivative of F is f, then the integral of f is F plus a constant.

$$\int f dx = F + c$$

The process of finding F is called **integration**, the function f is called **integrand**, and the differential dx indicates that x is **the variable of integration**.

If the **bounds or limits** of integral is given, we said the integral as **definite integral**, but when the integral has no bounds nor limits, we said the integral as **indefinite integral**.

## How to Say Integrals

$\int_{a}^{b} f(x) dx$	The integral of f x from a to b  The integral from a to b of f x	
∫°.	integral from zero to infinity	
$\int f(x) dx$	The indefinite integral of f x	

There are some technique for integration, such as

- Integration by subtitution
- Integration by parts or partial integration

### **Tasks**

- Write 2 problems and complete solution about calculus. At least, one of the problem is applied problem.
- Next week, the results will be presented by selected students.

# **Vocabularies of Chapter VIII**

Words	Pronunciations	Indonesians
antiderivative	/,æntɪdɪˈrɪvətɪv/	
approach	/əˈprəʊt∫/	
dash	/dæ∫/	
definite	/'defɪnət/	
derivative	/dI'rIvətIv/	
differential	/ˌdɪfəˈrenʃl/	
extreme	/Ik'stri:m/	
function	/ˈfʌŋk∫n/	
infinity	/ɪnˈfɪnətɪ/	
inflection	/In'flek∫n/	
integral	/'IntIgrəl/	
interval	/'Intəvl/	
limit	/'limit/	
maximize	/'mæksɪmaɪz/	
maximum	/'mæksɪməm/	
monotone	/ˈmɒnətəʊn/	
prime	/praIm/	
respect	/'rIspekt/	
stationary	/ˈsteɪʃənrɪ/	
tend	/tend/	

### IX. Sentence Analysis

### PART I: Jenis-jenis Kalimat (Sentence Types)

Kalimat dibentuk dari **Klausa** (Clause), yaitu kelompok kata yang memiliki **Subject** (S) dan **Predicate** (P)

### Example:

- 1. **We** entered the room. (**S**,P)
- 2. Arif and Alvin chased the ball. (CS, P)
- 3. **Beni** caught the ball and threw it to Ilham. (S, CP)
- 4. The **dean** and the **students** stood and sang the national anthem (**CS**,CP)

Ada dua macam klausa: **Klausa Bebas** (Independent Clause/IC) dan **Klausa Terikat** (Dependent Clause/DC)

IC adalah klausa yang dapat berdiri sendiri (tetap memiliki makna)

#### Examples:

- 1. I nearly **score a goal**. (with adverb modifier)
- 2. **The building** near the river caught fire. (with phrase modifier)
- 3. **They bought the book** that sold over a million copy around the world. (with clause modifiers)

DC adalah klausa yang tidak dapat berdiri sendiri (kehilangan makna) dan biasanya mengikuti Kata Penghubung (Conjunction: as, since, because, etc) atau (Relative Pronoun: that, which, who, etc)

#### Examples:

- 1. Adjective Clause: menerangkan noun/pronoun
  - This is the **car** that broke the speed record.
  - **Anybody** who is tired may leave.
  - Malaysia is the **nation** (that) we made the treaty with.
- 2. Adverb Clause: menerangkan verb/adjective/adverb
  - The child **cried** when the dentist appeared.
  - I am **sorry** (that) he is sick.
  - He thinks more **quickly** than you do.

- Noun clause: berfungsi sebagai noun (subjek, predikat, objek dari verb/kt. depan)
  - What John wants is a better job
  - This is where we came in.
  - Please **tell** him (that) I will be late.
  - He has no interest **in** what he is reading.

Jenis-jenis Kalimat	IC	DC
Kalimat Sederhana	1	
(Simple Sentence)	1	-
Kalimat Majemuk	≥ 2	
(Compound Sentence)	2 4	-
Kalimat Kompleks	1	> 1
(Complex Sentence)	1	≥ 1
Kalimat Majemuk Kompleks		
(Compound-complex	≥ 2	≥ 1
Sentence)		

### Examples:

- 1. The wind **blew**. (Simple)
- 2. The wind **blew** and the leaves **fell**. (Compound)
- 3. When the wind **blew**, the leaves **fell**. (Complex)
- When the sky darkened, the wind blew and the leaves fell.
   (Compound-complex)

#### Catatan:

- A. Simple sentence may have two or more subjects and predicates.
  - 1. <u>Addition</u> and <u>scalar multiplication</u> **are** two well-defined operations in vector space.
  - 2. We **left** the class and **went** to library.
  - 3. Bambang and Susilo **watched** the movie and **ate** popcorns.
- B. IC joined with a **coordinating conjunction** (and, but, for, or, nor, so, etc) by a **comma** or an **adverb** (then) by **semicolon**.
  - 1. I **like** geometry, <u>but</u> I **like** algebra better.
  - 2. Jarwo ran into the house, and he locked the door.
  - 3. You **first** read the problem; then you **write** the solution.

### **Exercise**

- 1. This is the student who got the highest grade in calculus.
- 2. Algebra is the subject (that) we thought very difficult.
- 3. Four scholarships have been offered, and I hope that you will apply for one.
- 4. My cousin has applied for three colleges, and the application that she sent to UNP has just been accepted
- 5. What Riski wants is a better grade.
- 6. This is where we came in.
- 7. He has no interest in what he is reading.

### PART II: Pola-pola Kalimat (Sentence Patterns)

- 1. Subject + Intransitive Verb
- 2. Subject + Transitive Verb + Direct Object
- 3. Subject + Linking Verb + Subject Complement
- 4. Subject + Transitive Verb + Indirect Object + Direct Object
- 5. Subject + Transitive Verb + Direct Object + Object Complement

## **Examples**

Subject	Intransitive Verb	
The Students	argued	(clearly)
Ani	walks	(to the campus)
Bambang	sat	(quietly in mosque)

Subject	Transitive Verb	Direct Object
The Students	began	final examination
Ani	underlined	the keywords
We	shall discuss	the property of natural numbers

		Subject C	omplement
Subject	Linking Verb	Predicate Substantive	Predicate Adjective
Ani	may become	a president	
The winner	is	me	
The perfume	smells		good
The book	seem		obscene
His face	went		white
Bambang	became		very angry

Subject	Transitive Verb	Indirect Object	Direct Object
The lecturer	gave	him	an A
Не	showed	us	the proof
The suspect	told	the interrogator	the truth

Subject	Transitive Verb	Direct Object	Object Complement
The senate	elected	him	chairman
We	thought	algebra	difficult
They	made	the lecturer	angry

## Dua perubahan dari Pola Dasar

#### **Passive**

Subject	Passive Verb	Original Subject
The solution	is presented	(by Budi)
Не	was called dummy	(by Jono)
Victory	was bought to Indonesia	(by Andik)

## **Expletive**

Expletive	Verb	Complement	Subject
It	is		time to leave
It	is	doubtful	that they will arrive
There	are		forty students
There	was		rain yesterday

## **Appositive**

Appositive adalah keterangan tambahan (additional explanation) yang biasanya ditulis diantara dua koma atau dua strip (-). Appositive ini dapat diabaikan karena kehilangannya tidak mempengaruhi arti.

### Examples

- 1. This chapter, <u>an excellent summary of the real number properties</u>, presents some basic concepts in real analysis.
- 2. The notion of absolute value, which is based on order properties, is discussed later.

### PART III: Empat Langkah dalam Menganalisa Kalimat

- **1. Menemukan Penghubung (Connector)** jika ada. (sekaligus mengetahui jenis kalimat)
  - Diantara dua IC (sometimes between two DC)
     and, but, for, or, nor, so (following a comma).
     then (following a semicolon).
  - Di depan DCas, since, because, when, etc.that, which, who, etc.

## 2. Pisahkan Klausanya

Example:

Two sets are said to be disjoint **if** their intersection is empty.

The connector: if

IC: Two sets are said to be disjoint.

DC: Their intersection is empty.

#### 3. Temukan Predicate

Predicate ada dua macam, yaitu: **verbs** dan **linking verb** Verbs dapat dikenali melalui:

> Kemampuannya berubah dari bentuk infinitive ke bentuk:

berakhiran **-s**: I run → he runs

berakhiran **-ed** he walks → he walked

berakhiran -ing she talks → she is talking

- Ditambah -er bisa menjadi pelaku: walker, receiver, admirer, lover, hanger.
- Ditambah awalan dis- menjadi negatif: disagree, disapprove, dismiss, dislike.
- Karakteristik posisinya setelah kata-kata seperti let's, please, dan simple noun.

let's **play**, please **go**, Joko **hit** Kadir

#### 4. Temukan Subject

Subject biasanya ada di depan Predicate, kecuali dalam bentuk Expletive.

### **Examples**

- (1) Four scholarship have been offered, and I hope that you will apply for one
- > Connectors : and, that
- Clauses:
  - 1. Four scholarship have been offered. (IC)
  - 2. I hope (something) (IC)
  - 3. You will apply for one. (DC)
- > Verbs
  - 1. have been offered
  - 2. hope
  - 3. will apply
- > Subjects
  - 1. Four scholarship
  - 2. I
  - 3. You
- (2) An angle at the circumference has a unique arc, but an arc subtends infinitely many angles at circumference.
  - > Connectors: and, that
  - Clauses:
    - 1. an angle at the circumference has a unique arc. (IC)
    - 2. an arc subtends infinitely many angles at circumference. (IC)
  - > Verbs
    - 1. has
    - 2. subtends
  - Subjects
    - 1. an angle at the circumference
    - 2. an arc

### **Exercise**

Find the connectors, clauses, subjects, and verbs in these sentences

- ➤ Any finite field is of finite characteristic.
- ➤ Cardinal and ordinal numbers have developed in close interconection and form the two aspects of natural numbers.

- ➤ Points, lines, and planes are the foundation stones of elementary geometry in three-dimensional space.
- ➤ The matrices operation is discussed in the last chapter.
- We shall use the notation (a,b) for the greatest common divisor of a and b.
- > Sequences and functions are the last concept that we will be studying.
- > Every natural number has exactly one immediate successor.
- Our goal is to gain an understanding of the theory and to be able to solve applied problems
- An event is the result of a trial that can, but need not occur.
- > Prime numbers are numbers that have only improper divisors.

### X. Paragraph Analysis and Mathematical Writing

### **PART I: Analisis Kalimat (Paragraph Analysis)**

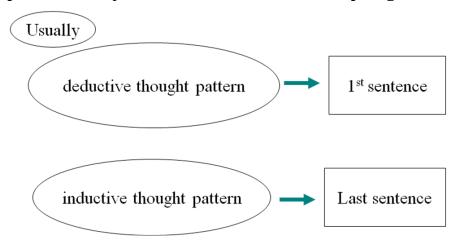
Dalam menganalisis paragraf, perlu diingat bahwa

- Suatu paragraf memuat:
  - 1 topik (**topic**)
  - 1 kalimat utama (main idea/sentence),

Dan beberapa kalimat penjelas/pendukung (subordinate sentences).

• Kalimat utama biasanya terletak di awal paragraf (**first sentence**), tetapi kadang-kadang juga terletak pada akhir paragraf (**last sentence**) atau di tengah paragraf.

Teks matematika sebagian besar menggunakan pola berfikir deduktif sehingga pada umumnya kalimat utama ada di awal paragraf.



Jadi, langkah dalam analisis kalimat pada teks matematika adalah:

- 1. Temukan kalimat utama (sebagian besar adalah kalimat pertama).
- 2. Temukan Subjek (adakalanya Objek) dari kalimat tersebut. Ini adalah topik dari paragraf.

### Example

In arithmetic, it is never possible to add <u>unlike quantities</u>. For example, we should not add inches and gallons and expect to obtain a sensible answer. Neither should we attempt to add volts, amperes, farads, ohms, etc. So it goes through algebra – we can never add quantities unless they are expressed in the same units.

Topic: Adding (unlike) quantities

Main Sentence: In arithmetic, it is never possible to add unlike quantities

### Example

The study of mathematics may be likened to the study of a language. In fact, mathematics is a language of number and size. Just as the rules of the grammar must be studied in order to master English, so must certain concepts, definitions, rules, terms, and words be learned in the pursuit of mathematical knowledge. These form the vocabulary or structure of the language. The more language is studied and used, the greater becomes the vocabulary; the more mathematics is studied and applied, the greater becomes its usefulness.

Topic: Study of mathematic

Main sentence: The study of mathematics may be likened to the study of a

language

### Exercise

Find the topic and main sentence of paragraphs below

- 1. So far, we have described the properties of the real number system relative to addition, multiplication, and order. A system which satisfies all the postulates that we have stated so far is called an ordered field. We repeat the definition of an ordered field as follows
- 2. Applied mathematics should read like good mystery, with an intriguing beginning, a clever but systematic middle, and a satisfying resolution at the end. Often the resolution of one mystery opens up a whole new problem, and the process starts all over. For the applied scientist, there

is the goal to explain or predict the behavior of some physical situation. One begins by constructing a mathematical model which captures the essential features of the problem without masking its content with overwhelming detail. Then comes the analysis of the model where every possible tool is tried, and some new tools developed. Finally, one must interpret and compare these results with real world facts. Sometimes this comparison is quite satisfactory, but most often one discovers that important features of the problem are not adequately accounted for, and the process begins again.

## PART II: Kelompok Pernyataan Matematika (Mathematical Structure)

#### 1. Definitions

Definition is used to state/prescribe the meaning of a word, phrase or expression.

In math, definitions usually use term: is/are, if, if and only if

Example:

- A matrix is a rectangular array of numbers.
- $\odot$  A is contractive with respect to the norm || || if ||A|| < 1.
- An integer n is called even if n is divisible by two.

#### 2. Theorems

- a. A theorem is a mathematical statement (major result) that has been proved.
- b. An auxiliary result is called lemma.
- c. A minor result is called a proposition.
- d. A immediate consequences of a previous result is called corollary.
- e. A result which has been accepted to be true and need not a proof is called axiom.

#### Example:

• For all real numbers *a*, *b*, the following identity holds

$$(a+b)(a-b) = a^2-b^2$$

A natural number n is divisible by 9 if and only if the sum of it digits is divisible by9.

#### 3. Proofs

A proof of a mathematical statement is an argument that shows that the statement is correct

### Example:

If S=T, then, by definition, S and T have precisely the same elements. In particular, this means that  $x \in S$  implies that  $x \in T$  and also  $x \in T$  implies that  $x \in S$ . That is, S $\subset$ T and T $\subset$ S.

### PART III: Menulis Teks Matematika (Mathematical Writing)

### Aturan Kecil (Small Rules):

Digunakan untuk kalimat:

Pemisahan, Gaya Penulisan, Aturan Simbol, Penggunaan Singkatan, dan Pemilihan "that" atau "which".

### A. Pemisahan (Breaking up)

Aturan 2-3-4 (2-3-4 rule)

Pertimbangkan unttuk memisahkan

- 1. Setiap kalimat yang melebihi 2 baris
- 2. Setiap kalimat yang memiliki lebih dari 3 verb
- 3. Setiap paragraf yang memiliki lebih dari 4 kalimat "panjang"

### Example

- $\bullet$  If  $\Delta = b^2 4ac \ge 0$ , then the roots are real. (bad)
- **(a)** Set  $\Delta = b^2 4ac$ . If  $\Delta \ge 0$ , then the roots are real. (good)

### B. Gaya Penulisan (Narrative Style)

1. Lebih baik menulis kalimat aktif.

### example:

- Gaussian elimination gave the answer to sixteen decimal places.
   (active)
- The answer <u>was given</u> to sixteen decimal places by Gaussian elimination. (passive)

2. Kata "we" ("penulis dan pembaca bersama-sama") berguna untuk menghindari penggunaan kalimat pasif. Hindari penggunaan "I".

### <u>example</u>:

- $\odot$  We solve the equation to get x = 0. (good)
- The equation was solved to get x = 0. (bad)
- $\odot$  I solved the equation and got x = 0. (bad)
- 3. Penggunaan Present Tense selalu OK Kalau ragu, pilihlah present tense.

### example:

- This is proved in Theorem 3 below. (present)

  This will be proved in Theorem 3 below. (future)
- Banach shows that the conclussion is correct. (present)
   Banach showed that the conclussion is correct. (past)
- We have proved that f(x) was equal to g(x). (bad) We have proved that f(x) is equal to g(x). (good)

# C. Aturan Penulisan Kalimat yang Memuat Simbol Matematika.

1. Pisahkan simbol-simbol dari rumus berbeda dengan kata-kata.

## example:

- lacktriangle Consider  $x_k$ , k = 1, ..., n. (bad)
- O Consider  $x_k$  for k = 1, ..., n. (good)
- 2. Jangan memulai kalimat dengan simbol.

#### example:

 $\bullet$   $x^n$  – a has n distinct zeroes. (bad)

The polynomial  $x^n$  – a has n distinct zeroes. (good)

- f is a continuous function. (bad)The function f is continuous. (good)
- m is the negative reciprocal of -3/4. (bad)

  The slope m is the negative reciprocal of -3/4. (good)
- 3. Jangan gunakan simbol-simbol  $\therefore, \Rightarrow, \Leftrightarrow, \forall, \exists, \flat, \in$  kecuali dalam rumus logika. Gantilah dengan kata yang sesuai (that is/therefore, then/implies, if and only if, for all, there exist, such that, elemen/in, etc).

## example:

Therefore, if x is a real number, then there is a natural number n such that n > x. (good)

4. Gunakan kalimat lengkap sebelum definisi atau teorema.

## <u>example</u>:

• We now have the following.

**Theorem 1**. H(x) is continuous. (bad)

• We can now prove the following result.

**Theorem 1.** The function H(x) defined by formula (2.1) is continuous. (good)

5. Bilangan yang kecil ditulis secaa lengkap, kecuali sebagai nama.

## example:

- Method 2 recquires 2 steps. (bad)
- Method 2 recquires two steps. (good)
- 6. Nama ditulis dalam huruf besar.

### example:

Theorem 1, Lemma 2, Definition 3, Equation 4.

### D. Penggunaan Singkatan secara Tepat

#### 1. Yang diterima

i.e	id est	that is (to say) in other words
e.g.	exempli gratia	for example
etc.	et cetera	and so forth and the rest

## Example:

- However, the converse is not true; i.e., bounded sequences are not necessarily convergent.
- If a function is Riemann-integrable, e.g. if it is continuous on a closed finite interval, then the integral is the limit of the Riemann sums.

#### 2. Yang tidak diterima

В

Singkatan berikut tidak formal, hanya bisa digunakan pada penulisan di kertas "buram".

• iff if and only if

• s.t. such that

• w.r.t. with respect to

without loss of generality

### E. Penggunaan "that" atau "which"

1. Kata "assume" dan "suppose" sebaiknya diikuti oleh kata "that"

## example:

- Assume A is a group. (bad)
- Assume that A is a group. (good)

But, never write

- $\bullet$  We have that A = B. (bad)
- $\bullet$  We have A = B. (good)
- 2. Gunakan "which" jika didahului oleh koma atau kata depan, atau secara interogatif.
- 3. Bereksperimenlah dengan bunyi untuk mendapatkan yang lebih enak didengar.

### Example:

- Don't use symbols which aren't necessary.
- Don't use symbols that aren't necessary.

The last is better.

#### **Exercise**

Write a math problem with solution that follows the rules of mathematical writing.

### **Exercise**

- Find two definitions and two theorems, and try to analyze the sentence; then try to translate it.
- Translate the next two texts using paragraph ad sentence analysis.

#### First Text.

### **Egyptian fractions**

Fractions and the notation to represent them were developed in the <u>Middle Kingdom of Egypt</u>, (between 2055 BC and 1650 BC). Five early texts in which fractions appear were: the <u>Egyptian Mathematical Leather Roll</u>, the <u>Moscow Mathematical Papyrus</u>, the <u>Reisner Papyrus</u>, the <u>Kahun Papyrus</u> and the <u>Akhmim Wooden Tablet</u>. A later text, the <u>Rhind Mathematical Papyrus</u>, introduced improved ways of writing Egyptian fractions.

The Rhind papyrus was written by <u>A'h'mes</u> and dates from the <u>Second Intermediate</u> <u>Period</u> (1650–1550 BC); it includes a <u>table of Egyptian fraction expansions for rational numbers 2/n</u>, as well as 84 <u>word problems</u>.

#### 1. Notation

To write the unit fractions used in their Egyptian fraction notation, in hieroglyph script, the Egyptians placed the <u>hieroglyph</u>



(translation: " one among" or possibly "re", mouth) above a number to represent the <u>reciprocal</u> of that number. Similarly in hieratic script they drew a line over the letter representing the number. For example:

$$\bigcap_{1} = \frac{1}{3} \quad \bigcap_{1} = \frac{1}{10}$$

The Egyptians had special symbols for 1/2, 2/3, and 3/4 that were used to reduce fractions larger that 1/2.

$$=\frac{1}{2}$$
  $\Rightarrow =\frac{2}{3}$   $\Rightarrow =\frac{3}{4}$ 

After subtracting one of these special fractions, the remaining number was written using as a sum of distinct unit fractions.

The reason of this representation is not clear; certainly it's not because it makes the calculation with fractions simpler.

But it is easier to compare two Egyptian fractions than it is to compare two regular fractions because unit fractions compare easily.

Egyptian fraction notation continued to be used in Greek times and into the Middle Ages despite complaints by <u>Ptolemy</u> (AD 90 – AD 168)) about the clumsiness of the notation compared to alternatives such as the <u>Babylonian base-60 notation</u>.

### 2. Egyptian fraction

In number theory, an **Egyptian fraction** is the sum of **distinct** <u>unit fractions</u>, such as  $\frac{1}{2} + \frac{1}{3} + \frac{1}{16}$ . The value of an expression of this type is a <u>positive rational number</u> a/b; for instance the Egyptian fraction above sums to 43/48.

Sums of this type, and similar sums also including 2/3 and 3/4 as <u>summands</u>, were used as rational numbers by the ancient Egyptians, and continued to be used by other civilizations into medieval times. In modern mathematical notation, Egyptian fractions have been superseded by <u>regular fractions</u> and <u>decimal</u> notation. However, Egyptian fractions continue to be an object of study in modern <u>number theory</u> and <u>recreational mathematics</u>, as well as in modern historical studies of ancient mathematics.

### 3. Fibonacci's greedy algorithm for Egyptian fractions

An important text of medieval mathematics, the <u>Liber Abaci</u> (1202) of <u>Leonardo of Pisa</u> (more commonly known as Fibonacci), provides some insight into the uses of Egyptian fractions in the Middle Ages, and introduces topics that continue to be important in the number theory. He also provides *a greedy algorithm* to represent every proper reduced fraction (i.e., fractions that are <1 and are such that the denominator and numerator do not have common factors) into an Egyptian fraction. *A greedy algorithm* is an <u>algorithm</u> that makes a locally optimal choice at each stage of the problem solving process with the hope of finding a global optimum. Fibonacci's algorithm for transforming <u>rational numbers</u> into <u>Egyptian fractions</u> represents the first published systematic method for constructing such expansions, and it is described in the Liber Abaci (1202).

The general principle behind the algorithm it is the following. At each step we fix d, and we chose the expansion

$$\frac{x}{y} = \frac{1}{d} + \frac{xd - y}{yd},$$

We use this algorithm to prove the following

**Theorem:** Every proper reduced fraction with numerator n can be written as an Egyptian fraction with  $\leq n$  terms.

The proof is by induction. **Suppose first that n=2.** We consider the fraction 2/m, with m>2 odd (if m is even the fraction can be simplified).

We can chose d so that 2/m = 1/d + 1/(md); after computing the common denominator, we have the equation 2d=m+1, which gives d=(m+1)/2, m is odd, and so m+1 is even.

When n=3, we shall consider two cases:

a) **if the remainder of the division of m by 3 is 2**, then we can chose d so that 3/m = 1/d + 1/(md); after computing the common denominator, we have the equation

3d=m+1, which gives d=(m+1)/3. This is an integer because by assumption m=3p+2, and so m+1=3p+3 is divisible by 3. In this case, we only need 2 fractions to represent 3/m.

b) **if the remainder of the division of m by 3 is 1**, then we can chose d so that 3/m = 1/d + 2/(md); after computing the common denominator, we have the equation 3d=m+2, which gives d=(m+2)/3. In this case, 3/m=1/d+2/(md). If either d or m are even, we can simplify the fraction 2/(md). and we are done. If that is not the case, we have shown how to write this fraction as sum of 2 unit fractions. So, 3/m can be represented as the sum of 3 fractions.

The proof for n=3 applies well to prove this proposition for general n's.

We assume that when the denominator is  $\leq$  n, the proper reduced fractions of the form of n/m can be written as a sum of  $\leq$  n fractions.

Suppose that  $k \ge 1$  is the remainder of the division of m by n. Then we chose d so that n/m = 1/d + (n-k)/(md).

That implies nd = m + (n-k), and d = (m + n-k)/n, which is an integer because m = np+k.

By assumption, the fraction (n-k)/(md) can be written as an Egyptian fraction with n-k terms, and so n/m can be written as an Egyptian fraction with n-k+1 terms or less. Since k>=1, we can write n/m as an Egyptian fraction with n terms or less. This concludes the proof.

**Example.** Write 5/13 as an Egyptian fraction

**Solution.** The remainder of the division of 13 by 5 is 3, so we find d such that

5/13 = 1/d + 2/(13d). So, 5d=13+2=15, which gives d=3.

We have

5/13 = 1/3 + 2/39. We need to decompose 2/39 in Egyptian fraction. We chose d so that

2/39 = 1/d + 1/(39 d)

That gives 2d = 39 + 1 = 40, and d = 20.

So, 2/39 = 1/20 + 1/780, and

5/13=1/3+1/20+1/780

This method is effective and easy to apply, but when the numerator is large, it can produce fractions with a very large denominator. Other methods are able to produce Egyptian fractions with smaller denominators, although they may have a large number of terms. In other cases, we get less terms and smaller denominators with a "smart" choice of d. For example,

$$\begin{split} \frac{5}{121} &= \frac{1}{25} + \frac{1}{757} + \frac{1}{763309} + \frac{1}{873960180913} + \frac{1}{1527612795642093418846225}, \\ \text{But also} \\ \frac{5}{121} &= \frac{1}{33} + \frac{1}{121} + \frac{1}{363}. \end{split}$$

Natural questions that arise are: what is the minimum number of (distinct) unit fraction that are necessary to represent a given rational number? And what is the smallest denominator of one such representations?

### D. The Erdős-Straus conjecture

The **Erdős-Straus conjecture** states that for all <u>integers</u>  $n \ge 2$ , the <u>rational</u> <u>number</u> 4/n can be expressed as the sum of **three** <u>unit fractions</u>. <u>Paul Erdős</u> and <u>Ernst G.</u> <u>Straus</u> formulated the conjecture in 1948; It is one of many <u>conjectures by Erdős</u>.

More formally, the conjecture states that, for every integer  $n \ge 2$ , there exist positive integers x, y, and z such that

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}.$$

These unit fractions form an Egyptian fraction representation of the number 4/n. For instance, for n = 5, there are two solutions:

$$\frac{4}{5} = \frac{1}{2} + \frac{1}{4} + \frac{1}{20} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10}.$$

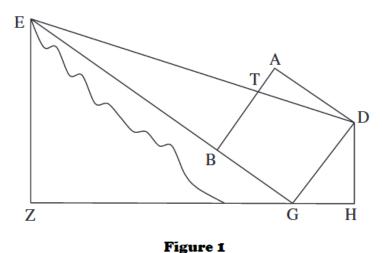
The restriction that x, y, and z be positive is essential to the difficulty of the problem, for if negative values were allowed the problem could be solved trivially. Also, if n is a composite number, n = pq, then an expansion for 4/n could be found immediately from an expansion for 4/p or 4/q. Therefore, if a counterexample to the Erdős–Straus conjecture exists, the smallest n forming a counterexample would have to be a <u>prime number</u>. Computer searches have verified the truth of the conjecture up to  $n \le 10^{14}$ , but proving it for all n remains an open problem.

#### **Second Text**

#### Al-Biruni's measurement of the earth

For showing how history may help us to find a suitable example relating to the real world, here we bring the method that Al-Biruni measured the earth circumstance; actually it is an elegant application of elementary trigonometry in real world.[2] Al- Biruni introduces his method,"Here is another method for the determination of the circumference of the earth. It does not require walking in deserts." Since the method assumes one knows how to determine the height of the mountain, al-Biruni first explains how to do that. The problem is nontrivial since a mountain is not a pole and therefore we cannot easily measure the distance from us to the point within the mountain where the perpendicular from its summit hits ground level.

To measure the height of a mountain al-Biruni first requires that we prepare a square board ABGD whose side AB is ruled into equal divisions and which has pegs at the corner B,G. Then ,at D, we must set a ruler, ruled with the same divisions as the edge AB and free to rotate around D. It should be as long as the diagonal of the square. Set the apparatus as in Fig.2 so that the board is perpendicular to the ground and the line of sight from G to B just touches the summit of the mountain. Fix the board there and let H be the foot of the summit of the perpendicular from D. Also, rotate the ruler around D until, looking along it, the mountain peak is sighted along the ruler's edge DT. Now AD is parallel to EG,  $\angle ADT = \angle DEG$ , and therefore the right triangles ADT and GED are similar. Thus TA: AD = DG: GE, and since of the four quantities in this proportion only GE is unknown we may solve for GE = AD.DG/TA. However, since both  $\angle EGZ + \angle DGH$  and  $\angle EGZ + \angle GEZ$  are equal to right angles it follows that  $\angle DGH = \angle GEZ$ , and thus the two right triangles DGH and GEZ are similar, so that GE: EZ = DG: GH. This means that we may solve for the single unknown EZ = GE.GH/DG, which is desired height.



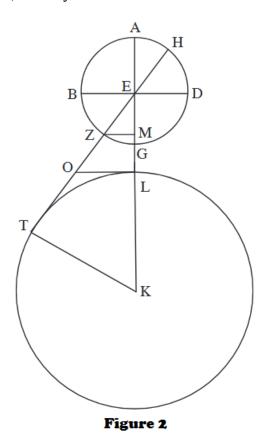
Then al-Biruni uses this method of measuring the height of a mountain to determine the circumference of the earth as following, illustrated in Fig.2:

Let KL be the radius of the earth and EL the height of the mountain. Let ABGD be a large ring whose edge is graduated in degrees and minutes, and let ZEH be a rotatable ruler, along which one can sight, which runs through E, the center of the ring. An astrolabe, would be perfectly suitable for this, using the ruler and scale of degree on the back of this instrument.

Now move the ruler from a horizontal position *BED* until you can see the horizon, at T, along it. The angle BEZ is called the dip angle, d. From L on the earth, imagine LO drawn so LO is tangent to the earth at L. By the law of sines applied to (ELO)

$$EL: LO = Sin(O): Sin(E) = Sin(A): Sin(90^{\circ} - d)$$

Since the two angles, as well as EL, the height of the mountain, are known, we may determine LO; but, TO=LO, since both are tangets to a circle from a point o outside it. Also, since EL and LO are known, it follows from the Pythagorean Theorem that  $EO = \sqrt{EL^2 + LO^2}$  is known, and hence ET=EO+OT is known. Again, by the law of Sines,  $ET : KT = Sin(d) : Sin(90^\circ - d)$ . Since KT, the radius of the earth, is the only unknown quantity in this proportion, we may solve for KT and so find the radius of the earth.



As al-Biruni said, he tried the method on a mountain near Nandana in India where the height, EL, was 652;3,18 cubits and the dip angle was 34'.(Note the very small angles and the rather optimistically accurate height.)These give for the radius of the earth 12,803,337;2,9 cubits. Al-Biruni takes the value  $3\frac{1}{7}$  for  $\pi$  and arrives at the value 80,478,118;30,39 cubits for the circumference of the earth, which, upon division by 360 yields the value of 55;53,15 miles/degree on a meridian of the earth.