Recitation #2 (Section 03)

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DS-GA 1014: Optimization and Computational Linear Algebra for Data Science

Linear transformations: recall & practice

Linear transformation L

A function $L : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation if

- for all $v \in \mathbb{R}^n$ and for all $\lambda \in \mathbb{R}$ it is true that $L(\lambda \cdot v) = \lambda \cdot L(v)$
- for all $v, w \in \mathbb{R}^n$ it is true that $L(w + v) = L(v) + L(w)$

Note that if L is a linear transformation *then* $L(\vec{0}_n) = \vec{0}_m$ (useful to quickly see if a function is NOT linear)

Exercise 1

Is the function f linear? $f : \mathbb{R}^2 \to \mathbb{R}^2$, $f((a, b)) = (2a, a + b)$

Linear transformations: practice

Exercise 2

Is the function f linear? $f : \mathbb{R}^2 \to \mathbb{R}^3$,
$f((a,b)) = (a+b, 2a+2b, 1)$

Linear transformations: practice

Exercise 3

Is the function f linear? $f : \mathbb{R}^2 \to \mathbb{R}$, $f((a, b)) = \sqrt{a^2 + b^2}$

Linear transformations: practice

Exercise 4.1

If $v, w \in \mathbb{R}^n$ are linearly independent vectors, are $v, v + w \in \mathbb{R}^n$ also *linearly* independent?

Exercise 4.2

If $v, w \in \mathbb{R}^n$ are linearly independent vectors, are $v, \alpha w \in \mathbb{R}^n$ also *linearly* independent? ($\alpha \neq 0$)

Linear transformations: matrices

Recall

All linear transformations (synonym map) $L : \mathbb{R}^n \to \mathbb{R}^m$ can be represented as a matrix $\tilde{L}$ with respect to the basis $\tilde{e_1}, ..., \tilde{e_n}$ of $\mathbb{R}^n$. If the selected basis of $\mathbb{R}^n$ is the canonical basis $e_1, .., e_n$ then the matrix $\tilde{L}$ is called *canonical matrix*.

Matrices: practice

Exercise 5

Given the linear transformtion $R : \mathbb{R}^2 \to \mathbb{R}^2$ with $R((x, y)) = (\frac{\sqrt{3}}{2}x - \frac{1}{2}y,\ \frac{1}{2}x + \frac{\sqrt{3}}{2}y)$ find the canonical matrix of R.

Linear transformations: kernel and image

Kernel

Given a linear transformation $L : \mathbb{R}^n \to \mathbb{R}^m$, the kernel $Ker(L) = \{v \in \mathbb{R}^n | L(v) = \vec{0}_m\}$

Image

The image is $Im(L) = \{w \in \mathbb{R}^m | \text{ exists } v \in \mathbb{R}^n \text{ with } L(v) = w\}$

Exercise 5

Find the kernel of the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}$,
$f((a, b)) = 2a - 3b$
Hint: The kernel is a subspace.

Kernel and image: practice

Exercise 6

Find the image of the linear transformation $f : \mathbb{R}^2 \to \mathbb{R}$, $f((a, b)) = 2a - 3b$

Kernel and image: practice

Exercise 7

Find the kernel of $\tilde{L} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$

Kernel and image: practice

Kernel and image: practice

Exercise 7: Another way of solving it

Note that $\tilde{L} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ is an invertible matrix!

Prove that for an invertible matrix A then always $ker(A) = \{\vec{0}\}$

Questions