Recitation #1 (Section 03)

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DS-GA 1014: Optimization and Computational Linear Algebra for Data Science

Subspaces: recall & practice

Subspace S

If V is a vector space, we say that S is a subspace $S \subseteq V$ if S is *closed* by the sum and multiplication by a scalar, i.e.

Subset

belongs

- if $u, w \in S$ then $u + w \in S$
- if $w \in S$ and $\lambda \in \mathbb{R}$ then $\lambda \cdot w \in S$

Exercise 1

Show that any subspace S contains the zero vector $\vec{0}$

Goal $\vec{0} \in S$

$u \in S$, $\lambda = 0 \in \mathbb{R} \rightarrow 0 \cdot u = \vec{0} \in S$

Span: geometric interpretation

Span

Let $v_1, v_2, ..., v_k \in V$ be elements of a vector space V. The *Span*$(v_1, v_2, ..., v_k)$ is a subspace that contains all possible linear combinations involving $v_1, v_2, ..., v_k$

$\alpha_1 v_1 + \ldots + \alpha_k v_k \quad \alpha_1, \ldots \alpha_k \in \mathbb{R}$

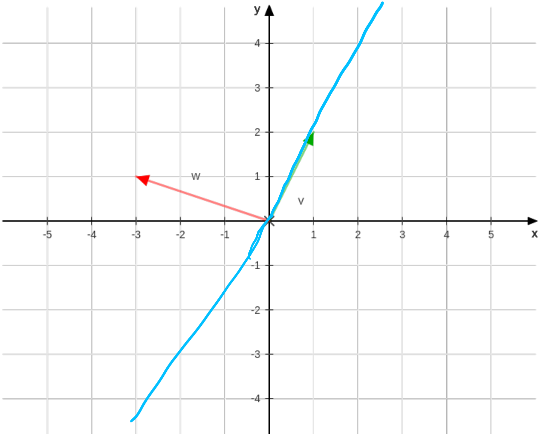
Span: geometric interpretation

Exercise 2

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $Span(v)$ "=" αV , $\alpha \in \mathbb{R}$

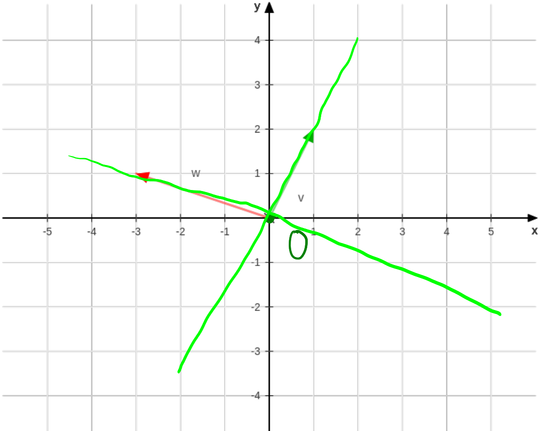


Span: geometric interpretation

Exercise 3

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

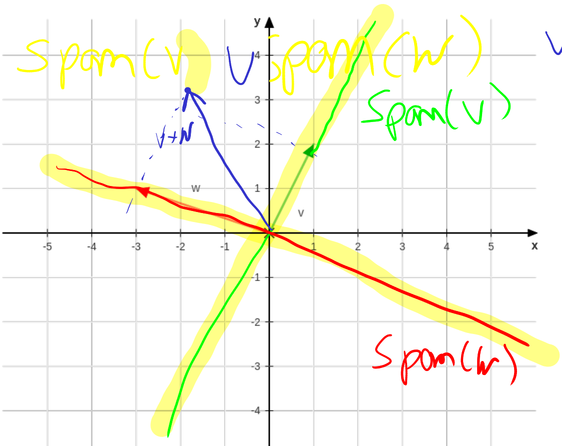
- $Span(v) \cap Span(w)$ (intersection)



Exercise 4

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $Span(v) \cup Span(w)$ (all of them)



$v + w \notin$ subset

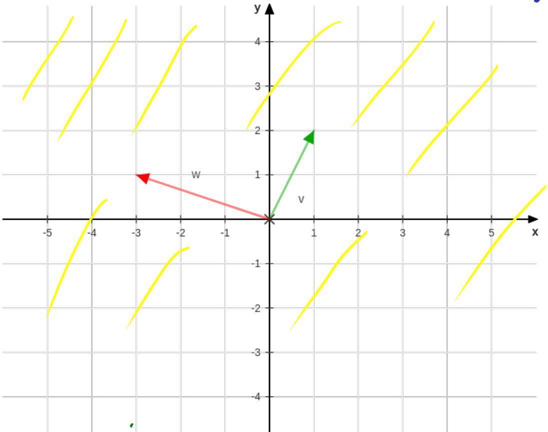
Span: geometric interpretation

Exercise 5

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $Span(v, w)$ "=" $\alpha v + \beta w$ = any vector in $\mathbb{R}^2$ with suitable α, β



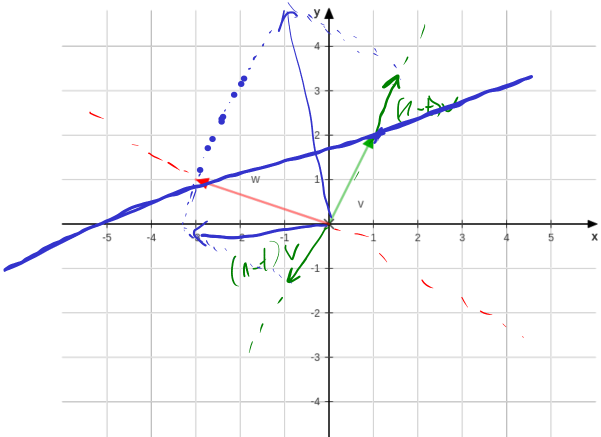
Span: geometric interpretation

Exercise 6

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $\{(1-t) \cdot v + t \cdot w \mid t \in \mathbb{R}\}$



Exercise 7

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

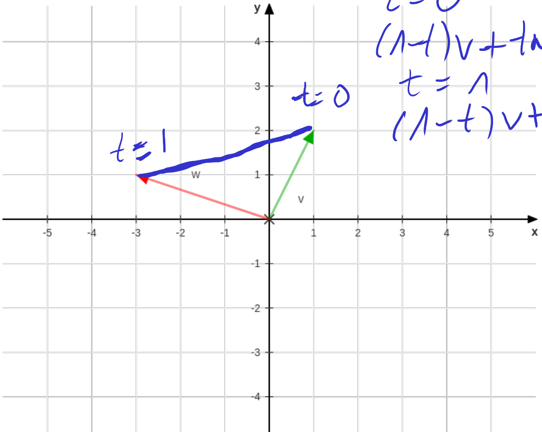
- $\{(1-t) \cdot v + t \cdot w \mid t \in [0, 1]\}$

$t = 0$

$(1-t)v + tw = v$

$t = 1$

$(1-t)v + tw = w$



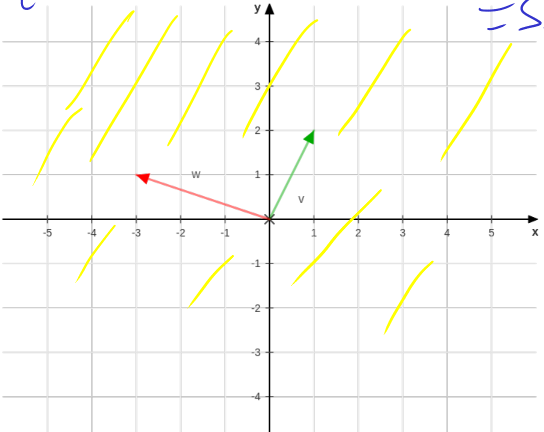
Span: geometric interpretation

Exercise 8

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $\{\alpha \cdot v + \beta \cdot w \,|\, \alpha, \beta \in \mathbb{R}\}$ = Span (v, w)

1-t t



= Span(v,w)

Exercise 9

Subspaces are closed by linear combinations.

Linear combination : $v_1, \dots, v_k \in S \subset V$

S subspace $\boxed{\alpha_1 v_1 + \dots + \alpha_k v_k}$ $\alpha_1, \dots, \alpha_k \in \mathbb{R}$

linear comb

By the definition of subspace

$\alpha_1 \cdot v_1 \in S$

$\alpha_k \cdot v_k \in S$

So the summation $\alpha_1 v_1 + \dots + \alpha_k v_k \in S$ $\square$