

Recitation #1 (Section 03)

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DS-GA 1014: Optimization and Computational Linear Algebra
for Data Science



Subspaces: recall & practice

Subspace S

If V is a vector space, we say that S is a subspace $S \subset V$ if S is *closed* by the sum and multiplication by a scalar, i.e.

- if $u, w \in S$ then $u + w \in S$
- if $w \in S$ and $\lambda \in R$ then $\lambda \cdot w \in S$

Exercise 1

Show that any subspace S contains the zero vector $\vec{0}$

Span: geometric interpretation

Span

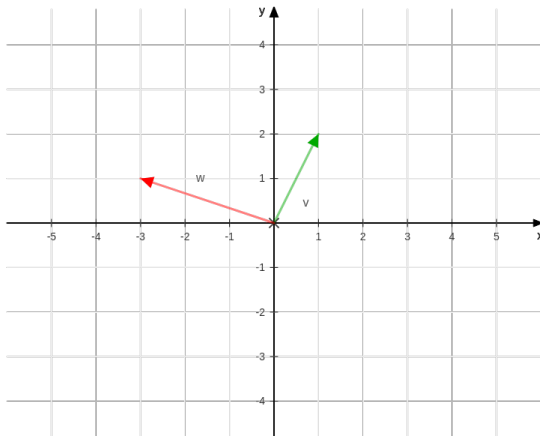
Let $v_1, v_2, \dots, v_k \in V$ be elements of a vector space V . The $\text{Span}(v_1, v_2, \dots, v_k)$ is a subspace that contains all possible linear combinations involving v_1, v_2, \dots, v_k

Span: geometric interpretation

Exercise 2

We will work in \mathbb{R}^2 . Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $\text{Span}(v)$



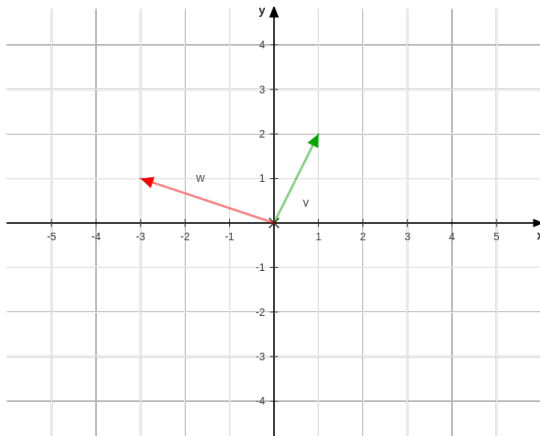
Span: geometric interpretation

Exercise 3

We will work in \mathbb{R}^2 . Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $\text{Span}(v) \cap \text{Span}(w)$ (intersection)



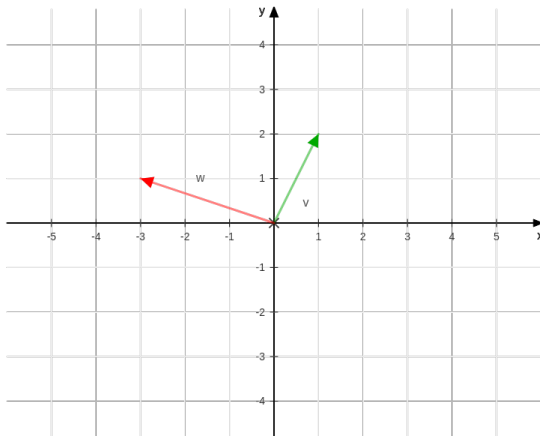
Span: geometric interpretation

Exercise 4

We will work in \mathbb{R}^2 . Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $\text{Span}(v) \cup \text{Span}(w)$ (all of them)



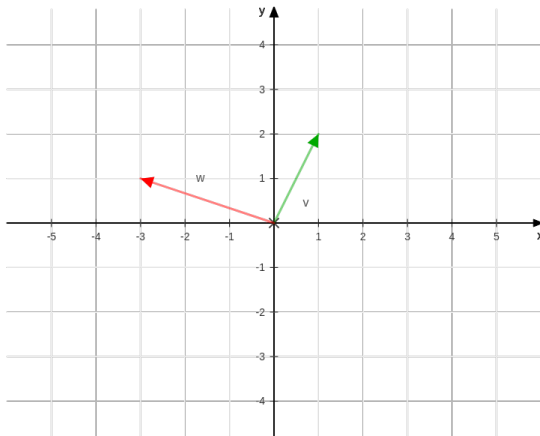
Span: geometric interpretation

Exercise 5

We will work in \mathbb{R}^2 . Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $\text{Span}(v, w)$



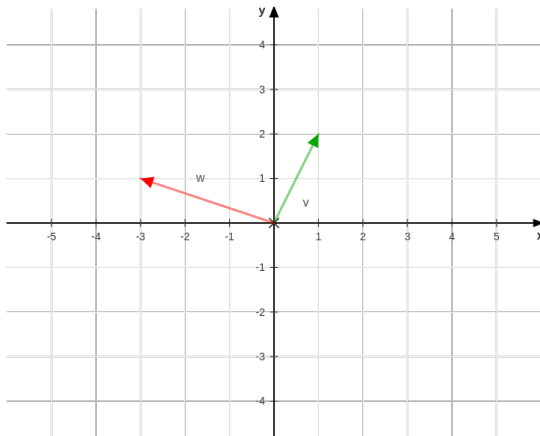
Span: geometric interpretation

Exercise 6

We will work in \mathbb{R}^2 . Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $\{(1 - t) \cdot v + t \cdot w \mid t \in \mathbb{R}\}$



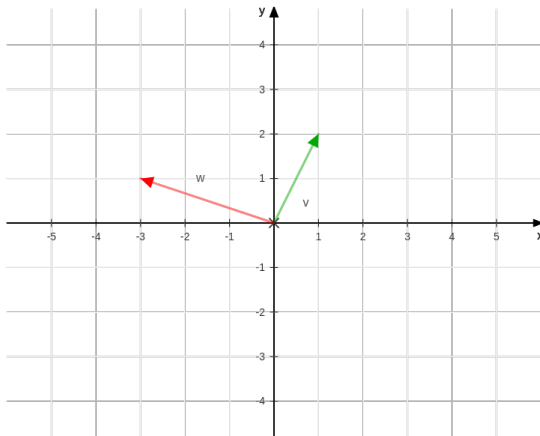
Span: geometric interpretation

Exercise 7

We will work in \mathbb{R}^2 . Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $\{(1 - t) \cdot v + t \cdot w \mid t \in [0, 1]\}$



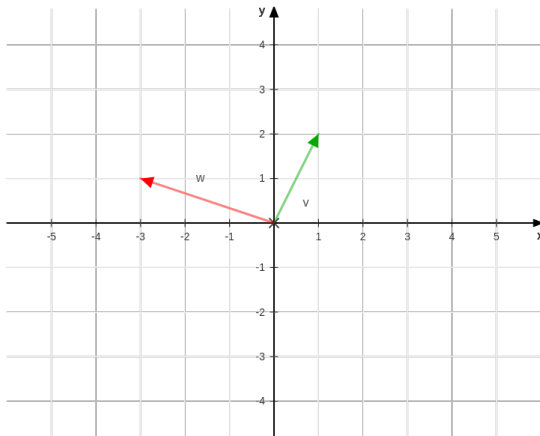
Span: geometric interpretation

Exercise 8

We will work in \mathbb{R}^2 . Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $\{\alpha \cdot v + \beta \cdot w \mid \alpha, \beta \in \mathbb{R}\}$



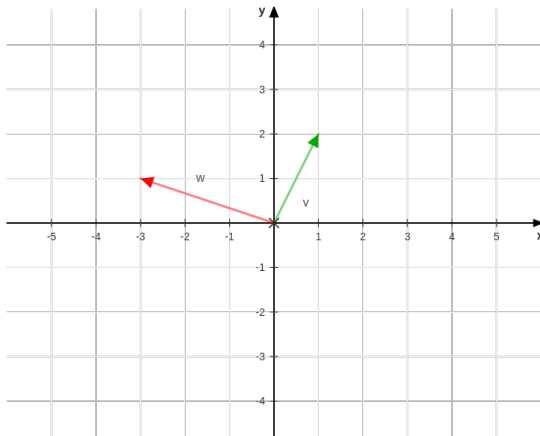
Span: geometric interpretation

Exercise 9

We will work in \mathbb{R}^2 . Let $v = (1, 2)$ and $w = (-3, 1)$

Sketch the following sets and identify which are subsets

- $\{(1 - t) \cdot v + t \cdot w \mid t \in [0, 1]\}$



Exercise 10

Subspaces are closed by linear combinations.