Office Hours

$$x^{(t+1)} = P x^{(t)} \qquad P^{(t)} x \xrightarrow{t \to \infty} \mu$$

stochastic

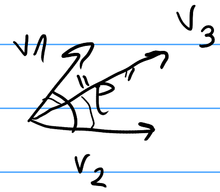
$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \end{pmatrix}$$

$$p_{ij} = P(i \to j)$$

$$i \to j$$

0.14

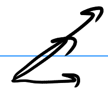
$$v_1, \ , v_k \in \mathbb{R}^n$$

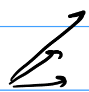


$\|v_i\| = 1$

$\langle v_i, v_j \rangle = \rho \quad i \neq j$

show $k \le n$





$\rho = 0$

$$\langle v_i, v_j \rangle = \cancel{\|v_i\|}^{1} \ \cancel{\|v_j\|}^{1} \underbrace{\cos\theta}_{\rho}$$

$$V^T V$$

$$V = \begin{pmatrix} | & & | \\ v_1 & \cdots & v_k \\ | & & | \end{pmatrix}$$

Assume $k > n$ $\qquad k = n+1$

$v_1, \dots, v_{n+1}$ $\qquad \langle v_{n+1}, v_j \rangle = \rho$

$\langle v_i, v_j \rangle = \rho \qquad i = 1, \dots, n \quad i \neq j$
$\qquad j = 1, \dots, n$

$\|v_i\| = 1 \qquad i = 1, \dots, n+1$

Goal $\langle v_{n+1}, v_1 \rangle \neq \rho \qquad v \in \mathbb{R}^n$

$v_{n+1} = \alpha_1 v_1 + \dots + \alpha_n v_n$

$\langle v_{n+1}, v_1 \rangle =$

$= \langle \alpha_1 v_1 + \dots + \alpha_n v_n, v_1 \rangle =$

$= \alpha_1 \cancel{\langle v_1, v_1 \rangle}^{1} + \alpha_2 \langle v_2, v_1 \rangle$

$+ \dots \alpha_n \langle v_n, v_1 \rangle$

$= \alpha_1 + \alpha_2 \rho + \dots + \alpha_n \rho$

$= \alpha_1 + \rho(\alpha_2 + \dots + \alpha_n) \quad = \rho$

$\langle v_{n+1}, v_2 \rangle = \alpha_2 + \rho(\alpha_1 + \dots + \alpha_n) = \rho$

$\vdots$

$\langle v_{n+1}, v_n \rangle = \alpha_n + \rho(\alpha_1 + \dots + \alpha_{n-1}) = \rho$

$$\alpha_1+\dots+\alpha_n + p(\alpha_1+\dots+\alpha_n)^{(n-1)} = p$$

$$A = \begin{pmatrix} \text{———} \; r_1 \; \text{———} \\ \text{————————} \\ \text{———} \; r_m \; \text{———} \end{pmatrix}$$

$n \times m$

row space $=$ Span$(r_1, \dots, r_m)$

$$A = \begin{array}{ccccc} c_1 & & & c_m \\ \left(\begin{matrix} \vert & \vert & \vert & \vert \end{matrix}\right. & & & \left.\right) \end{array}$$

column space $=$ Span$(c_1, \dots, c_m)$

nullspace $=$ Kernel

$A: \mathbb{R}^n \to \mathbb{R}^m$

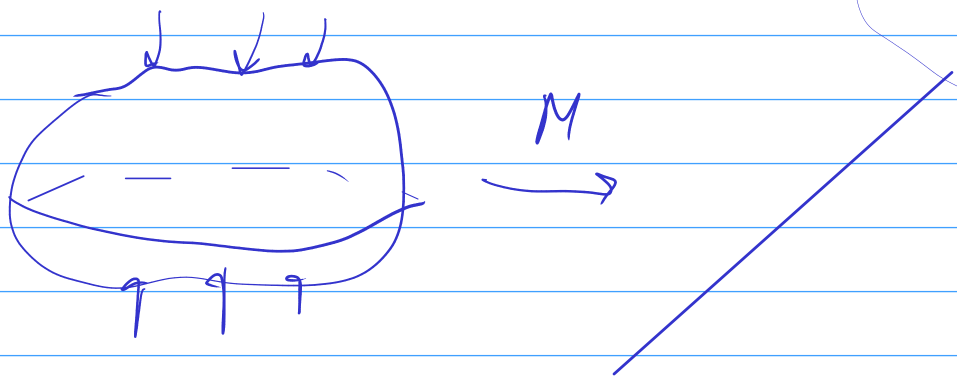
Kernel   Im

$Av = 0$

True or False

$$\boxed{\text{M is the matrix of an orthogonal proj, is M orthogonal ??}}$$

ORTHOGONAL PROJ $\quad M \sim \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 0 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}$



rank ≠ full

ORTHOGONAL MATRIX

has full rank

$\begin{pmatrix} | & | & | & | \\ | & | & | & | \end{pmatrix}$

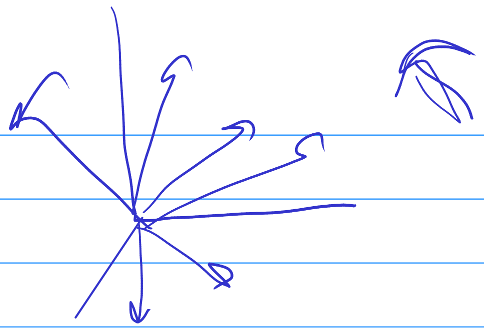
orthogonal vectors

↓

linearly independent

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$\mathbb{R}^3$ $\mathbb{R}^n$



$$\underset{n\times n}{A} = P \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 2 & & \\ & & & 3 & \\ & & & & 4 \end{pmatrix} P^{-1}$$

$\lambda = 1$ (multiplicity $= 2$)

multiplicity $\lambda \leq n$

A has $\lambda_1, \dots, \lambda_k$ $\quad \lambda_1 \neq \dots \neq \lambda_k$

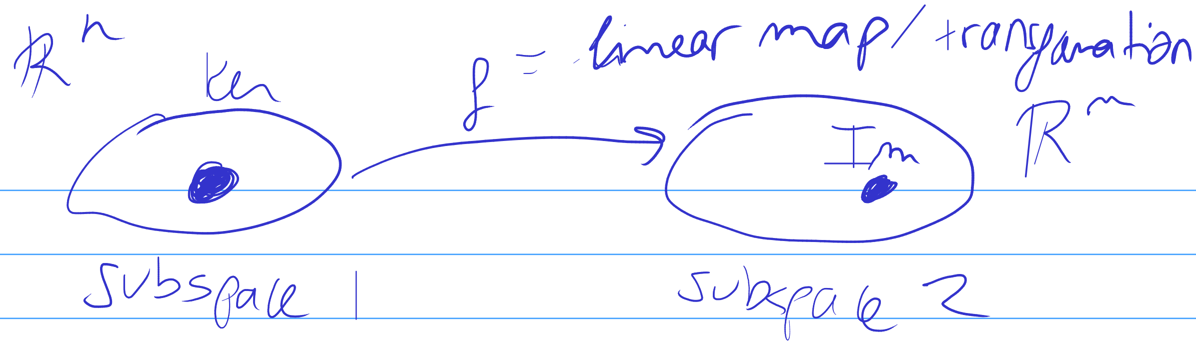
multiplicity $m_1, \dots, m_k$

$$\boxed{\sum_{i=1}^{k} m_i \leq n}$$

3×3 $\quad \lambda_1 = 1 \in \mathbb{R}$ $\quad m_1 = 1$

$1 \leq 3$

$\lambda_2 = i$, $\lambda_3 = -i$ $\quad x \in \mathbb{C}$, $\notin \mathbb{R}$



A

$c_1 \quad \dots \quad c_m$

$\mathrm{Ker}(A) = \{ v \in \mathbb{R}^n \mid Av = 0 \}$

$\mathrm{Ker}(A) = \{0\}$

$\Updownarrow$

A full rank

$\mathrm{span}(\square, \bigcirc, \triangle) = \alpha \square + \bigcirc \beta + \gamma \triangle$

$\mathrm{rank}_A(\bullet) = \dim(\mathrm{span}(\bullet))$

$A : \square \to \square$