# Recitation #2 (Section 03)

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DS-GA 1014: Optimization and Computational Linear Algebra for Data Science



# Linear transformations: recall & practice

### Linear transformation L

A function  $L: \mathbb{R}^n \longrightarrow \mathbb{R}^m$  is a linear transformation if

- for all  $v \in \mathbb{R}^n$  and for all  $\lambda \in \mathbb{R}$  it is true that  $L(\lambda \cdot v) = \lambda \cdot L(v)$
- for all  $v, w \in \mathbb{R}^n$  it is true that L(w + v) = L(v) + L(w)

Note that if L is a linear transformation then  $L(\vec{0}_n) = \vec{0}_m$  (useful to quickly see if a function is NOT linear)

### Exercise 1

Is the function f linear?  $f: \mathbb{R}^2 \to \mathbb{R}^2$ , f((a,b)) = (2a, a+b)

# Linear transformations: practice

## Exercise 2

Is the function f linear?  $f: \mathbb{R}^2 \to \mathbb{R}^3$ , f((a,b)) = (a+b,2a+2b,1)

## Linear transformations: practice

## Exercise 3

Is the function f linear?  $f: \mathbb{R}^2 \to \mathbb{R}$ ,  $f((a,b)) = \sqrt{a^2 + b^2}$ 

# Linear transformations: practice

### Exercise 4.1

If  $v, w \in \mathbb{R}^n$  are linearly independent vectors, are  $v, v + w \in \mathbb{R}^n$  also *linearly* independent?

## Exercise 4.2

If  $v, w \in \mathbb{R}^n$  are linearly independent vectors, are  $v, \alpha w \in \mathbb{R}^n$  also linearly independent? ( $\alpha \neq 0$ )

## Linear transformations: matrices

### Recall

All linear transformations (synonym map)  $L: \mathbb{R}^n \to \mathbb{R}^m$  can be represented as a matrix  $\tilde{L}$  with respect to the basis  $\tilde{e}_1, ..., \tilde{e}_n$  of  $\mathbb{R}^n$ . If the selected basis of  $\mathbb{R}^n$  is the canonical basis  $e_1, ..., e_n$  then the matrix  $\tilde{L}$  is called *canonical matrix*.

## Matrices: practice

### Exercise 5

Given the linear transformtion  $R: \mathbb{R}^2 \to \mathbb{R}^2$  with  $R((x,y)) = (\frac{\sqrt{3}}{2}x - \frac{1}{2}y, \frac{1}{2}x + \frac{\sqrt{3}}{2}y)$  find the canonical matrix of R.

# Linear transformations: kernel and image

#### Kernel

Given a linear transformation  $L: mathbb{R}^n \to \mathbb{R}^m$ , the kernel  $Ker(L) = \{v \in \mathbb{R}^n | L(v) = \vec{0}_m\}$ 

#### **Image**

The image is  $Im(L) = \{ w \in \mathbb{R}^m | \text{ exists } v \in \mathbb{R}^n \text{ with } L(v) = w \}$ 

# Kernel and image: practice

## Exercise 5

Find the kernel of the linear transformation  $f:\mathbb{R}^2 o \mathbb{R}$ ,

$$f((a,b)) = 2a - 3b$$

Hint: The kernel is a subspace.

# Kernel and image: practice

## Exercise 6

Find the image of the linear transformation  $f: \mathbb{R}^2 \to \mathbb{R}$ , f((a,b)) = 2a - 3b

# Kernel and image: practice

## Exercise 7

Find the kernel of 
$$\tilde{L}=\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

# Questions