Recitation #3 (Section 03)

Irina Espejo (iem244@nyu.edu)

Center for Data Science

DS-GA 1014: Optimization and Computational Linear Algebra for Data Science

recall: the Rank

Rank

Let $x_1, .., x_k \in \mathbb{R}^n$ we define the rank as

$$\boxed{\text{rank}(x_1, .., x_k) = \dim(Span(x_1, .., x_k))}$$

Informally: rank = "the number of linearly independent vectors among $x_1, .., x_k$"

practice: the Rank

Exercise 1

Calculate the rank of

$$A = \begin{pmatrix} 7 & 3 \\ 19 & 19 \\ -1 & 5 \end{pmatrix}$$

Exercise 2

Let $A, B \in \mathbb{R}^{n \times n}$ be two matrices then

$$rank(A + B) = rank(A) + rank(B)$$

True or false? ✓True ✗False

recall: invertible matrix

Invertible matrix

A matrix $M \in \mathbb{R}^{n \times n}$ is invertible if there exists another matrix $M^{-1} \in \mathbb{R}^{n \times n}$ such that

$$M\, M^{-1} = M^{-1}\, M = Id_n$$

Exercise 3

Let $M \in \mathbb{R}^{n \times n}$ be an invertible matrix, then its transpose M^T is also invertible.

practice: invertible matrix

Exercise 4

Prove that for an invertible matrix $A \in \mathbb{R}^{n \times n}$ then always $ker(A) = \{\vec{0}\}$

practice: invertible matrix

Exercise 5

Prove that for an invertible matrix $A \in \mathbb{R}^{n \times n}$ then always $rank(A) = n$

practice: invertible matrix

Note

With the last two exercises we have that: A being invertible, A having maximum rank and $ker(A) = \{0\}$ are equivalent.

Rank nullity theorem

Rank nullity theorem

Let $L : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation then

$$rank(L) + dim(ker(L)) = m$$

Exercise?

Prove that $rank(L) = dim(Im(L))$
In this course we take it as a definition.

Exercise 6

Which of the following are equivalent for $A \in \mathbb{R}^{m \times n}$?

1. The columns of A are linearly independent
2. rows of A are linearly independent
3. $rank(A) = m$
4. $rank(A) = n$
5. The equation $Ax = 0$ has one solution
6. The equation $Ax = b$ has at least one solution
7. $Im(A^T) = \mathbb{R}^n$
8. $Im(A) = \mathbb{R}^m$
9. The linear transformation corresponding to A is injective (one-to-one)
10. The linear transformation corresponding to A is exhaustive (onto)
11. $ker(A) = \{0\}$
12. The span of the columns of A is $\mathbb{R}^m$

Exercise 6

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