

Recitation #5

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DS-GA 1014: Optimization and Computational Linear Algebra
for Data Science



recall: Orthogonal matrices

Orthogonal matrix

Let $A \in \mathbb{R}^{n \times n}$ be a matrix. A is an orthogonal matrix if its columns form an orthogonal family of \mathbb{R}^n (therefore linearly independent and basis of \mathbb{R}^n)

Alert!: orthogonal vectors given an inner product

Properties of Orthogonal matrices

The following are equivalent:

- ① A is orthogonal
- ② $AA^T = Id$
- ③ $A^T A = Id$

Exercise for home: re-prove it.

Exercise 1

Prove that the product of two orthogonal matrices is also an orthogonal matrix.

Exercise 2

Let $Q \in \mathbb{R}^{n \times n}$ be an orthogonal matrix and let $x, y \in \mathbb{R}^n$.

Show that $\langle Qx, Qy \rangle = \langle x, y \rangle$

recall: eigenvalues and eigenvectors

Eigenvalue, eigenvector

Let $A \in \mathbb{R}^{n \times n}$. A non-zero vector $v \in \mathbb{R}^n$ is an eigenvector of A if there exists $\lambda \in \mathbb{R}$ such that

$$Av = \lambda v$$

The scalar λ is the eigenvalue of A associated to v .

Eigenspace

The following set is called eigenspace of A associated to λ .

$$E_\lambda(A) = \{x \in \mathbb{R}^n \mid Ax = \lambda x\} = \ker(A - \lambda Id)$$

The dimension of the eigenspace is called multiplicity of λ .

Exercise 3

Show that the eigenspace $E_\lambda(A)$ is a subspace.

recall and practice: diagonalizable matrices

Diagonalizable matrix

A matrix $A \in \mathbb{R}^{n \times n}$ is diagonalizable if and only if

$$\sum_{\lambda} \dim(E_{\lambda}(A)) = n$$

(sum over all eigenvalues λ of A)

Exercise 4

Are all real matrices diagonalizable? Why?

✓True ✗False

Exercise 5

Let $Q \in \mathbb{R}^{n \times n}$ be an orthogonal matrix.

Prove that the eigenvalues of Q can only be -1 , $+1$.

Exercise 6

Suppose $D \in \mathbb{R}^{n \times n}$ is a diagonal matrix.

Give a vector $v \in \mathbb{R}^n$ with $\|v\| = 1$ such that $\|Dv\|$ is maximized.