Recitation #2 (Section 03)

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DS-GA 1014: Optimization and Computational Linear Algebra for Data Science

Linear transformations: recall & practice

Linear transformation L

A function $L : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation if

- for all $v \in \mathbb{R}^n$ and for all $\lambda \in \mathbb{R}$ it is true that $L(\lambda \cdot v) = \lambda \cdot L(v)$
- for all $v, w \in \mathbb{R}^n$ it is true that $L(w + v) = L(v) + L(w)$

Note that if L is a linear transformation *then* $L(\vec{0}_n) = \vec{0}_m$
(useful to quickly see if a function is NOT linear)

Exercise 1

Is the function f_1 linear? $f_1 : \mathbb{R}^2 \to \mathbb{R}^2$, $f_1(a, b) = (2a, a + b)$

Linear transformations: practice

Exercise 2

Is the function f_2 linear? $f_2 : \mathbb{R}^2 \to \mathbb{R}^3$,
$f_2(a, b) = (a + b, 2a + 2b, 0)$

Linear transformations: practice

Exercise 3

Is the function f_3 linear? $f_3 : \mathbb{R}^2 \to \mathbb{R}^3$,
$f_3(a, b) = (a + b, 2a + 2b, 1)$

Linear transformations: practice

Exercise 4

Is the function f_4 linear? $f_4 : \mathbb{R}^2 \to \mathbb{R}$, $f_4(a, b) = \sqrt{a^2 + b^2}$

Linear transformations: practice

Exercise 5

Is the function f_5 linear? $f_5 : \mathbb{R}^2 \to \mathbb{R}$, $f_5(a, b) = 5x + 3$

Linear transformations: practice

Exercise 6

If $v, w \in \mathbb{R}^n$ are linearly independent vectors, are $v, v + w \in \mathbb{R}^n$ also *linearly* independent?

Exercise 7

If $v, w \in \mathbb{R}^n$ are linearly independent vectors, are $v, \alpha w \in \mathbb{R}^n$ also *linearly* independent? $(\alpha \neq 0)$

Linear transformations: practice

Exercise 8

Let $v_1, ..v_m \in V \subseteq \mathbb{R}^n$ be linearly independent vectors. Show that if they do not span V (that is $V \neq span(v_1, .., v_m)$) then there is a vector $w \in V$ such that $v_1, .., v_m, w$ are linearly independent.