Recitation #1 (Section 03)

Irina Espejo (iem244@nyu.edu)

Center for Data Science

DS-GA 1014: Optimization and Computational Linear Algebra for Data Science

Subspaces: recall & practice

Subspace S

If V is a vector space, we say that S is a subspace $S \subseteq V$ if S is *closed* by the sum and multiplication by a scalar, i.e.

- if $u, w \in S$ then $u + w \in S$
- if $w \in S$ and $\lambda \in \mathbb{R}$ then $\lambda \cdot w \in S$

Exercise 1

Show that any subspace S contains the zero vector $\vec{0}$

Span: geometric interpretation

Span

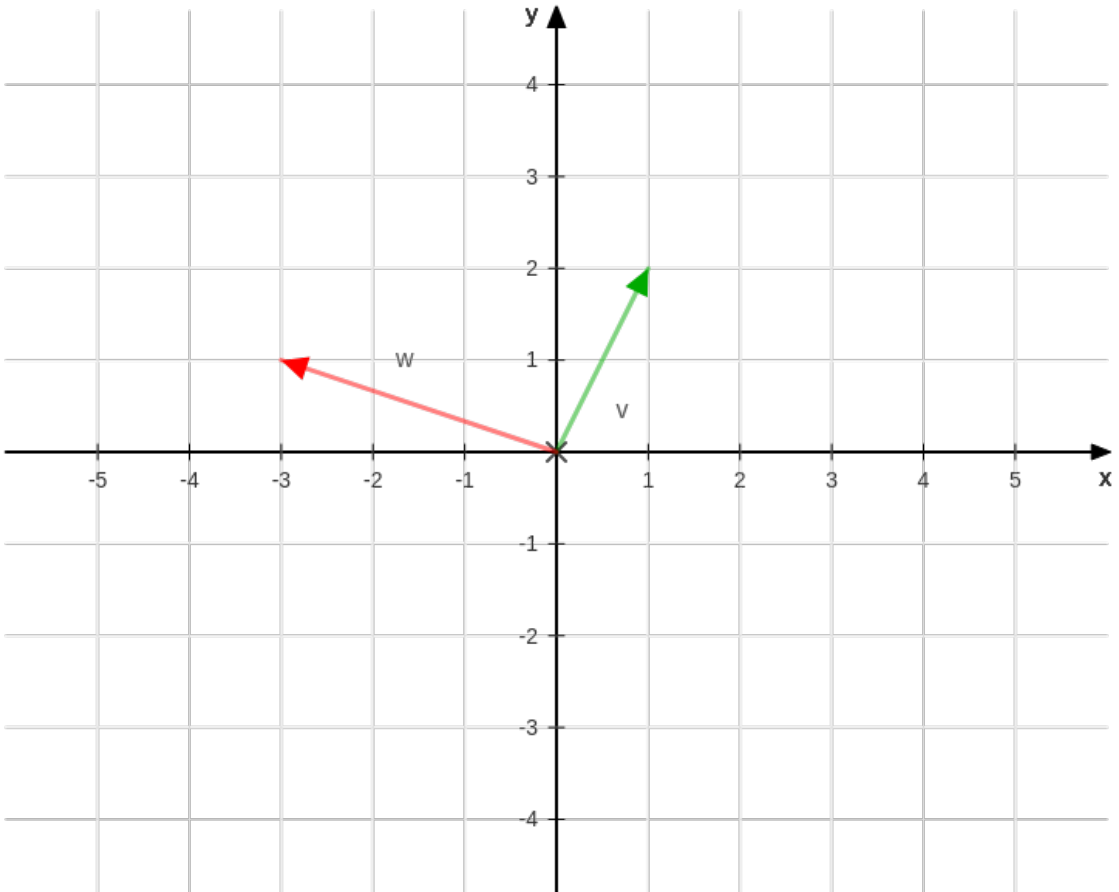
Let $v_1, v_2, ..., v_k \in V$ be elements of a vector space V. The $Span(v_1, v_2, ..., v_k)$ is a subspace that contains all possible linear combinations involving $v_1, v_2, ..., v_k$

Span: geometric interpretation

Exercise 2

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $Span(v)$

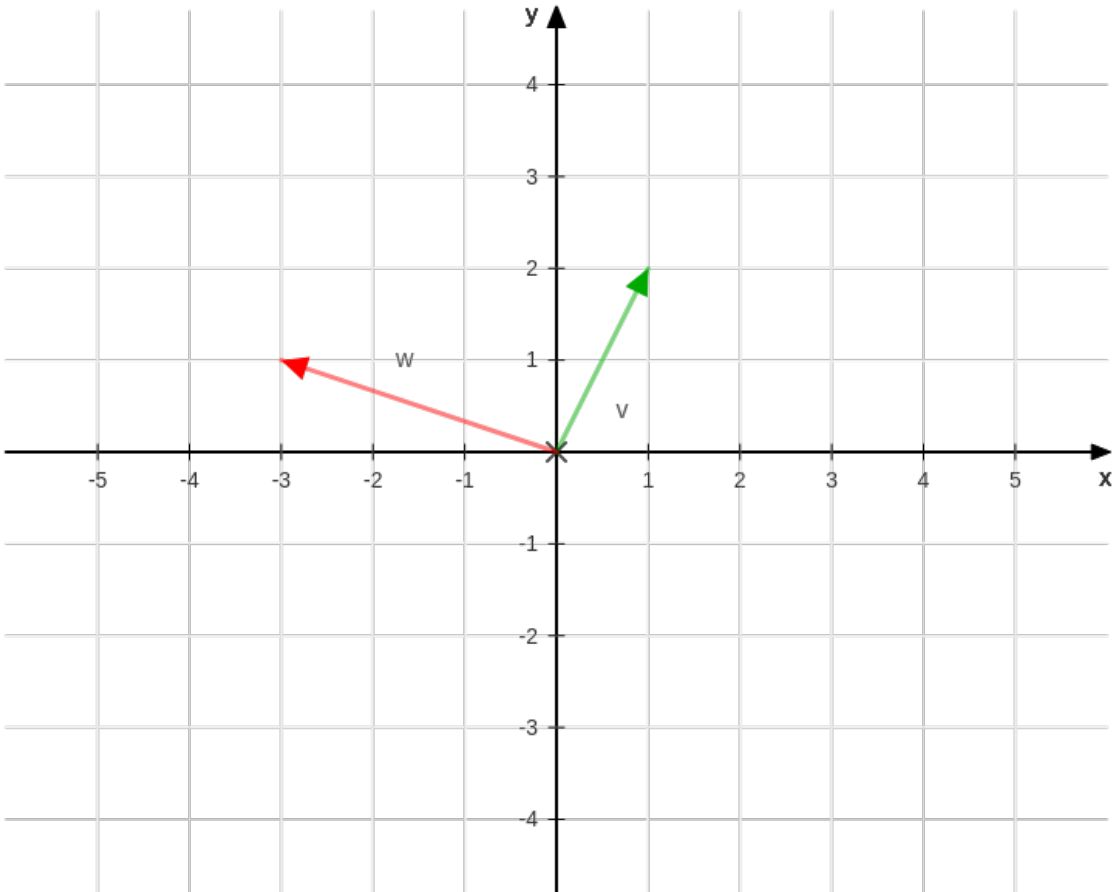


Span: geometric interpretation

Exercise 3

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $Span(v) \cap Span(w)$ (intersection)

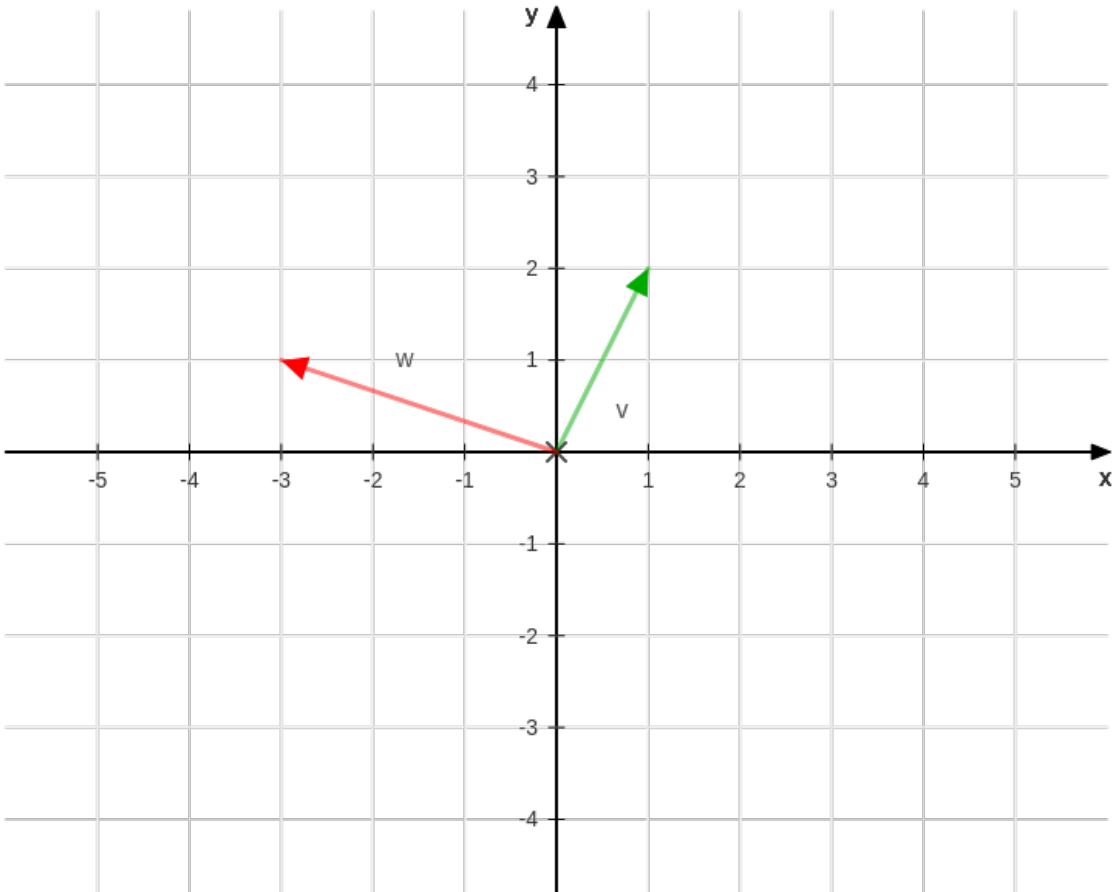


Span: geometric interpretation

Exercise 4

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $Span(v) \cup Span(w)$ (all of them)

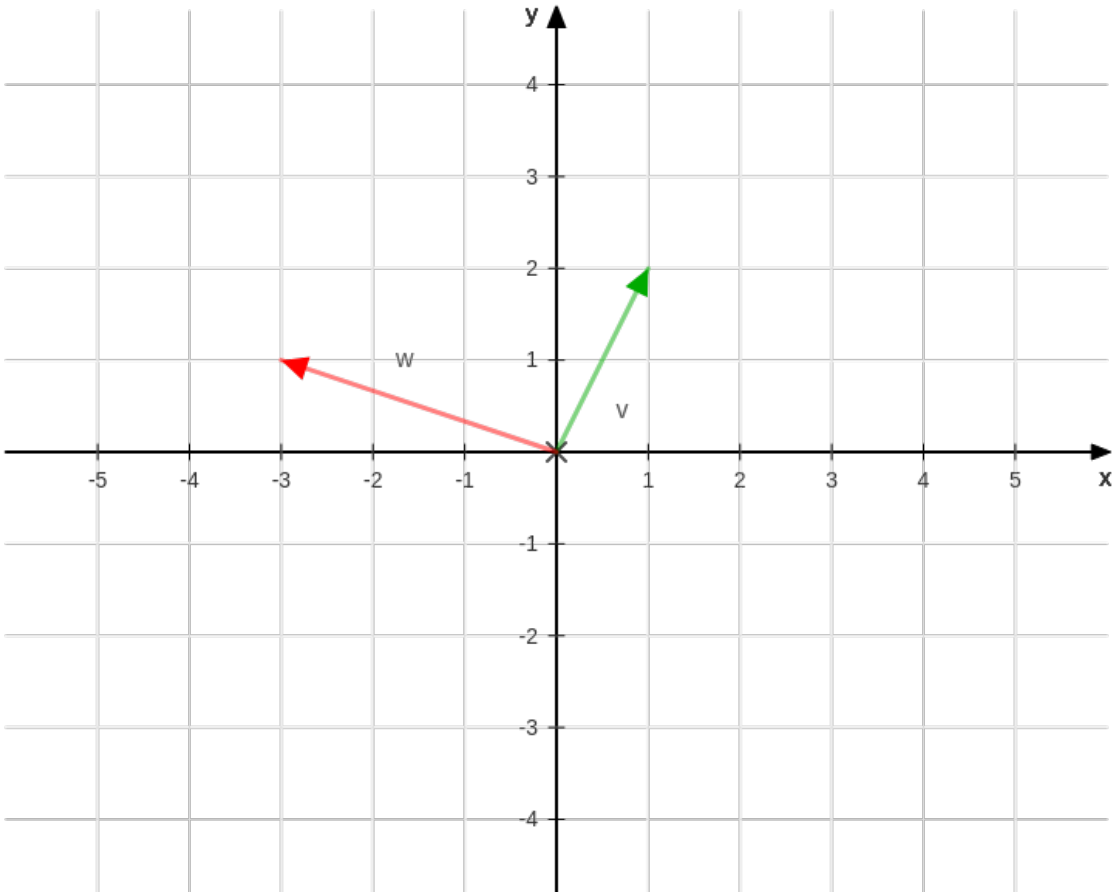


Span: geometric interpretation

Exercise 5

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $Span(v, w)$

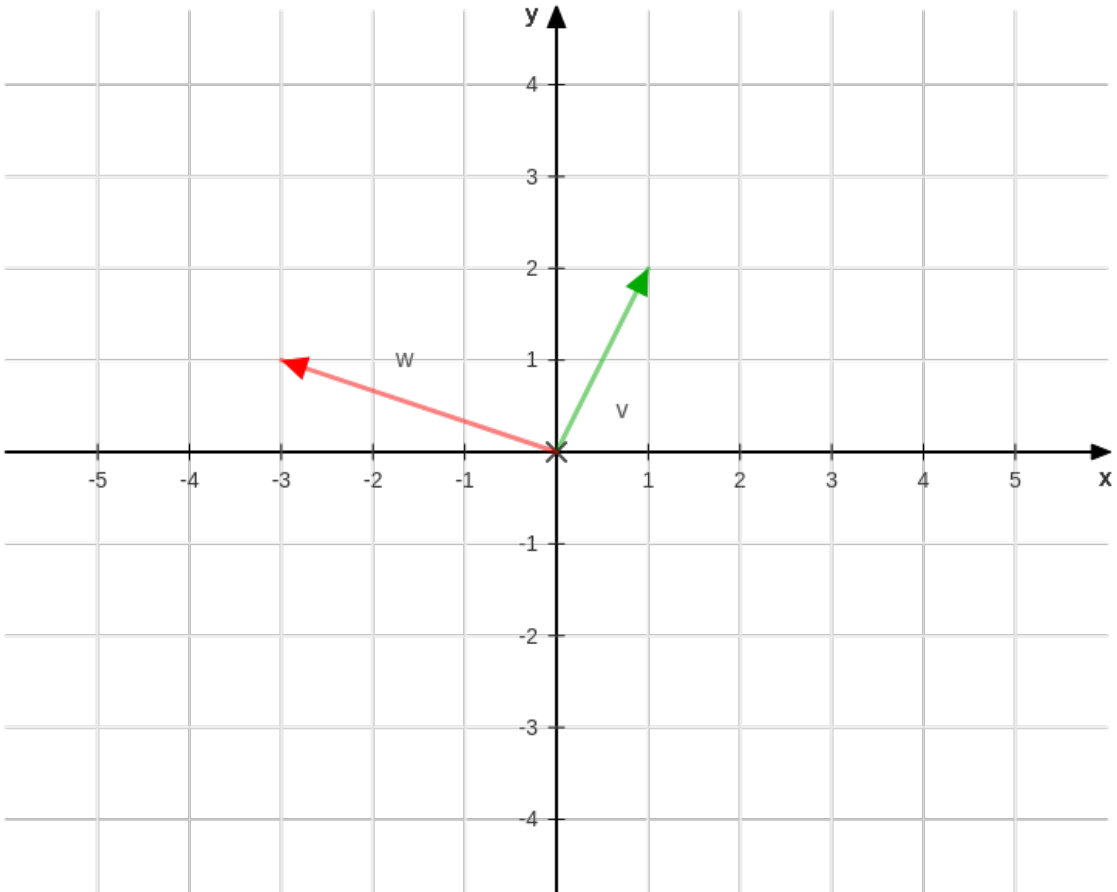


Span: geometric interpretation

Exercise 6

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $\{(1 - t) \cdot v + t \cdot w \mid t \in \mathbb{R}\}$

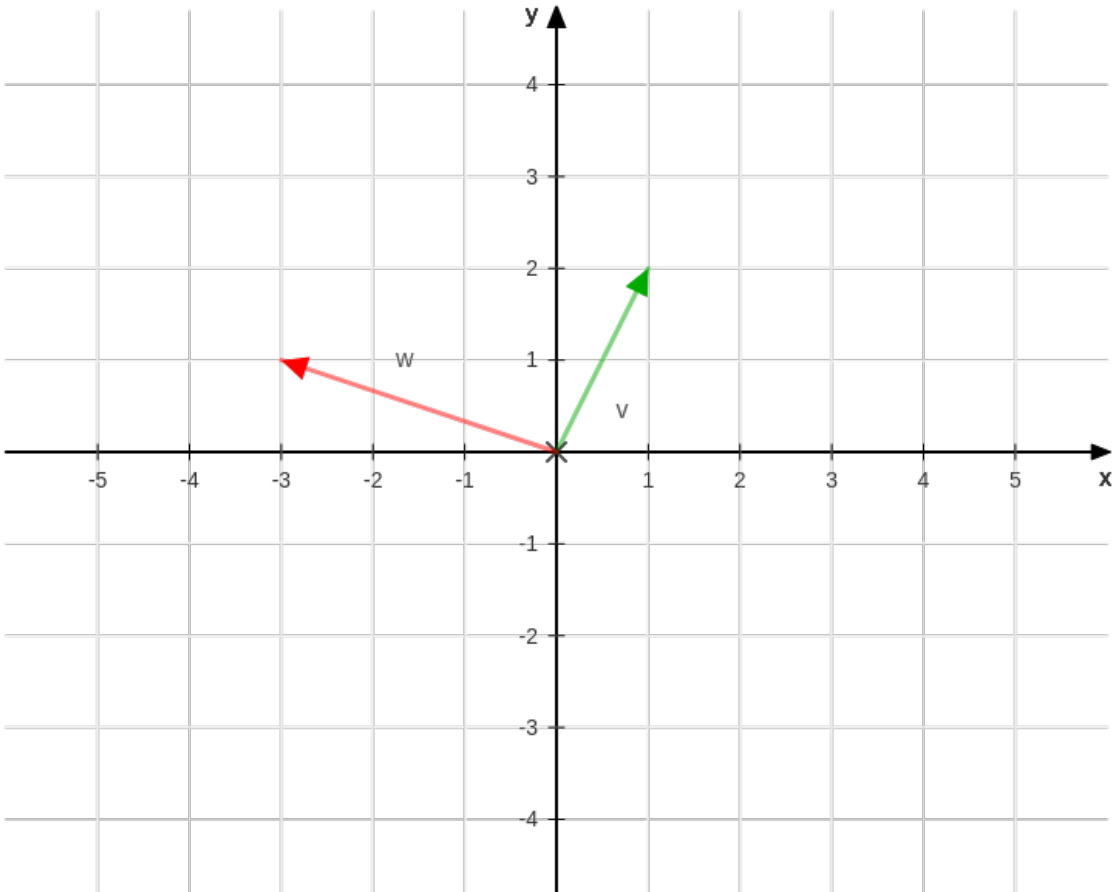


Span: geometric interpretation

Exercise 7

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $\{(1 - t) \cdot v + t \cdot w \mid t \in [0, 1]\}$

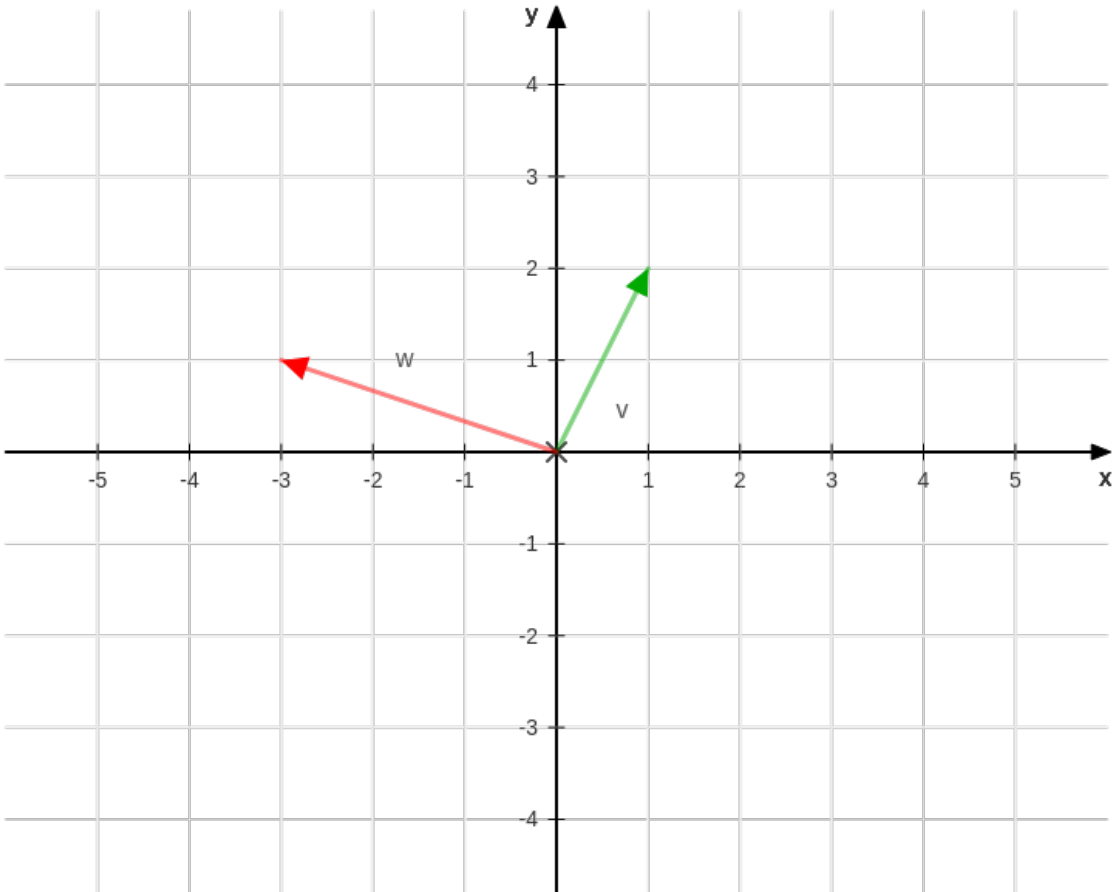


Span: geometric interpretation

Exercise 8

We will work in $\mathbb{R}^2$. Let $v = (1, 2)$ and $w = (-3, 1)$
Sketch the following sets and identify which are subsets

- $\{\alpha \cdot v + \beta \cdot w \,|\, \alpha, \beta \in \mathbb{R}\}$



Practice doing proofs

Exercise 9

Subspaces are closed by linear combinations.