Optimizing Assortments in Two-sided Markets: An Application to Dating

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We study how assortment composition and history affect users' behavior in a dating platform. We find that the average quality of other users in the assortment and the number of recent matches reduce the probability that a given profile is liked. Based on these results, we formulate a dynamic two-sided assortment optimization problem to find the optimal assortment to offer to each user in each period to maximize the expected number of matches. We show that the problem is NP-complete, and derive properties of the optimal solution in some special cases. We use these insights to propose simple and efficient heuristics, and through simulations we show that these can significantly improve the number of matches generated by the platform.

Key words: two-sided markets, platforms, matching.

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1. Introduction

Matching platforms are becoming an essential part of the economy. Examples include freelancing platforms such as TaskRabbit or UpWork, ride-sharing apps such as Blablacar, accommodation companies such as Airbnb, and dating platforms such as Hinge and Bumble, among others. A common feature of many of these marketplaces is their two-sided design, in which both sides of the market must mutually agree to generate a transaction. More specifically, many of these platforms start eliciting the preferences of agents on both sides of the market. For instance, Airbnb requires guests to report the location and dates of their trips, and allows them to add filters in relation to the price, type of place, and so on. Similarly, the platform allows hosts to set the price, the minimum number of nights, as well as other preferences. After collecting this information, these platforms often display a limited set of alternatives that agents can screen before starting their interaction with the other side of the market. Depending on the setting, this interaction can be either one-directional, with one side of the market sending an initial request/message/like and the other side of the market either accepting or rejecting it; alternatively, it can be bi-directional, with both sides of the market being able to screen alternatives and start their interaction with each other. Airbnb and Blablacar are examples of the former, while dating and some freelancing platforms are examples of the latter. Finally, on all these platforms a transaction takes place if, and only if, both sides of the market mutually accept/like each other.

As the previous discussion illustrates, one of the primary roles of the platform is to select the subset of alternatives—or, from now on, the *assortment*—to display, considering the preferences and characteristics of the agents on both sides of the market. We refer to this problem as a *two-sided assortment problem*. This is similar to the classic (one-sided) *assortment optimization problem*, in which a retailer must decide the subset of products to maximize the expected revenue obtained from a series of customers. In both cases, the assortment offered may play an important role as the probability that each alternative is chosen may depend on the other options available. Indeed, there is ample evidence in the literature that the *independence of irrelevant alternatives* (IIA) axiom does not hold in many settings, as there may be substitution and complementarity relationships between the options. In addition, the literature on context-dependent preferences shows that preferences are affected by the addition of extreme, intermediate, or similar options (Huber et al. (1982), Simonson and Tversky (1992), Tversky and Simonson (1993)). More generally, we refer to the change in the probability that an option is chosen due to the other alternatives offered as the *assortment effect*. Another element that is common to both one-sided and two-sided assortment problems is that agents may repeatedly interact with the platform, and may contact/like and match with more than one option. As a result, the history of past usage may affect the way that agents make decisions in the future. We refer to this as the *history effect*. For instance, a customer that has recently purchased detergent may not purchase it again during his next visit to the store. Similarly, guests that have booked listings with certain characteristics in the past may be more likely to rent similar listings in the future.

The assortment and history effects are present in both the one-sided and the two-sided assortment problems. However, a distinctive feature of the latter is the two-sidedness, which may play an important role. For instance, it may no longer be optimal to choose an assortment solely based on the preferences of the user being served and the availability of users on the other side of the market, as the latter may not accept their requests and thus a match may not be found. Hence, platforms must carefully balance these three elements—the assortment and history effects and the two-sidedness—when choosing which subset of alternatives to show. The present paper aims to investigate how platforms should make this decision. More specifically, the goal of this paper is twofold: (1) to study the effects of assortment and history on how agents make decisions on two-sided platforms; and (2) to show how these two effects and the two-sidedness of the market can be incorporated into the assortment optimization framework.

To accomplish these goals, we partnered with a major dating app,[1] which offered us the chance to study and answer these questions. Dating is an exciting market on its own. Since the launch of Tinder in 2012, hundreds of dating services have emerged worldwide, making this industry worth between \$2 and \$3 billion in the U.S. and \$12 billion worldwide.[2] As Rosenfeld et al. (2019) discuss, meeting online has become the most common way for couples to meet, replacing more traditional methods such as meeting through friends or co-workers. Indeed, using a nation-wide survey, they show that 39% of heterosexual couples that met in 2017 did so online, and this number raises to 65% for same-sex couples. Overall, approximately one out of five couples today met online.[3]

The size and relevance of the dating market highlight the need to make these platforms more efficient, and finding answers to our research questions may be the first step. To do this, we conduct an observational study to provide evidence that both assortment and history matter as a starting point. We find a strong negative correlation between the attractiveness of the other profiles in the assortment and the probability that a given profile is liked. Also, our observational study shows that the probability of liking new profiles is negatively correlated with the number of matches obtained by users in the recent past. These results suggest that assortment composition and history may play a significant role, although these effects cannot be taken as causal. To make a causal claim, we use a quasi-experiment to study the history effect. The quasi-experiment introduced an exogenous variation in the number of matches that some users obtained, allowing us to quantify the impact of recent matches in future like behavior. Our estimates show that each additional match reduces the probability of a new like between 1% and 2%.

Based on these findings, we introduce a stylized model of dynamic two-sided assortment optimization, in which the platform must decide, in each period, what subset of profiles to show to each user to maximize the overall expected number of matches. We show that the general problem is NP-complete, even in the case with a single user and no history effect. Given this complexity result, we focus on the individual user problem, where the platform must decide the sequence of assortments to show to a single user to maximize his expected number of matches, and we derive properties of the optimal solution for some particular cases. We then use these insights to design

[1] We keep the name of the app undisclosed as part of our NDA.

[2] Source: https://www.toptal.com/finance/business-model-consultants/online-dating-industry

[3] For example, the Pew Research Center reports that 15% of adults in the U.S. used online dating sites in 2016.

heuristics to solve the market-level problem, in which the platform jointly decides the assortments to show to each user in each period. Using simulations on real-data, we show that the proposed heuristics outperform relevant benchmarks, improving the overall match rate by at least 60% in relation to our partner's current algorithm and 35% relative to a Greedy algorithm that does not consider the assortment and history effects. Finally, we show that these benchmarks can be improved by introducing a sampling stage before choosing which assortments to show.

Contributions. Our paper contributes to the literature in several ways. First, our paper is the first to show the effect of the assortment composition and the history of past usage on the choice behavior of users when they are allowed to make more than one choice. These findings contribute to the literature on context-dependent preferences (see Tversky and Simonson (1993)), which mostly focuses on settings where only one choice can be made, and also to the emerging empirical literature on online platforms. Second, we contribute to the general assortment optimization literature in many ways. First, we introduce a dynamic two-sided assortment problem, where assortments must be chosen for each side of the market and matches take place if both users mutually like each other. Second, our model allows users to select as many alternatives as they want, while in most assortment optimization settings consumers are limited to one choice. Third, most of the literature assumes that arrivals are independent across time, whereas in our case we assume that users have repeated interactions that may affect their future behavior. Indeed, our paper is the first to show empirically that there is an effect of the history of previous usage in the behavior of users. Finally, given the complexity of this problem, we propose simple heuristics that scale to solve real instances of the market-level problem, and we show that these heuristics may considerably improve the platform's outcomes.

Managerial Implications. Our results provide useful insights that can be applied when designing online marketplaces. In particular, platforms should take into account the effects of assortment and history when making their assortment decisions. Doing so is nowadays possible considering all of the information that platforms collect every-time they interact with a user, and it applies to both one-sided and two-sided online markets. Notably, our results show that considering the two-sidedness of the market can lead to substantial improvements in the match rate. Hence, platforms should incorporate the probability of users being accepted/liked back when making their recommendations. Finally, while we focus on the dating context, we believe our findings apply more generally in other settings where the platform offers a subset of alternatives and users make repeated transactions with the platform, such as freelancing, accommodation, ride-sharing, among others.

Structure. The remainder of this paper is organized as follows. In Section 2, we review the most relevant related literature. In Section 3 we describe how the platform works, its main features, and we provide preliminary evidence of the existence of the assortment and history effects. In Section 4 we describe a quasi-experiment that supports the existence of the history effect. In Section 5, we describe our model, and in Section 6 we present our heuristics and simulation results. Finally, in Section 7 we conclude and provide directions for future work.

2. Literature

Our work sits at the intersections of several streams of literature. First, our paper contributes to the large literature in assortment optimization. Most of this literature assumes that a sequence of independent customers arrives over time and that a decision maker must decide which subset of products to offer in order to maximize the expected profit. Talluri and van Ryzin (2004) introduce a general version of this problem, and later focus on two particular cases: (i) independent demands, where the probability that each product is chosen is independent of the assortment offered; and (ii) the multinomial logit model. The authors show that the optimal policy is a nested allocation policy by fare order. More recent papers have extended this model to include capacity constraints (Rusmevichientong et al. (2010)), different choice models—including the nested logit model (Davis et al. (2014)), the mixed logit model (Rusmevichientong et al. (2014)), Markov chain based models (Blanchet et al. (2016)), non-parametric models (Farias et al. (2009)), among others—and also adding search (Wang and Sahin (2018)). We refer to Kök et al. (2015) for an extensive review of the current state of the assortment planning literature.

Another strand of the assortment optimization literature allows to dynamically change the assortment offered depending on the information available. Some papers focus on the problem of learning preferences based on the assortments offered and the purchasing decisions observed in the past. Examples include the papers by Caro and Gallien (2007), Rusmevichientong et al. (2010) and Sauré and Zeevi (2013). Other papers assume that the decision maker observes characteristics of the arriving customers, and uses this information to personalize the assortments to show. For instance, Berbeglia and Joret (2015) assume that customer preferences can be represented using a mixed logit model, and that the decision maker observes the segment of each arriving customer. Golrezaei et al. (2014) consider a more general choice model, and propose a family of inventory-balancing algorithms that minimize the asymptotic worst-case gap relative to an upper bound.

We contribute to the general assortment optimization literature in several ways. First, our paper is the first to analyze a two-sided assortment problem, where assortments must be chosen for each side of the market and matches take place if two users mutually like each other. Second, our model allows users to select as many alternatives as they want, while in most assortment optimization settings consumers are limited to one choice. Third, most of the literature assumes that arrivals are independent across time, whereas in our case we assume that users have repeated interactions with the platform that may affect their future behavior.

The second stream of literature that our paper is related to is on matching platforms. Starting with the seminal work of Rochet and Tirole (2003), this literature has focused on participation, competition, and pricing, highlighting the role of cross-side externalities. In the dating context, Kanoria and Saban (2017) study how the search environment can impact users' welfare and the performance of the platform. They find that simple interventions, such as limiting what side of the market reaches out first or hiding quality information, can considerably improve the platform's outcomes. Halaburda et al. (2018) show that two platforms can successfully coexist charging different prices by limiting the set of options offered to their users. They show that, depending on their outside option, users must balance two effects when choosing a larger platform: (1) a choice effect, whereby users are more likely to find a partner that exceeds their outside option; and (2) a competition effect, whereby agents on the other side of the market are less likely to accept a request as they have more options. All these models consider a stylized matching market, where users interact with the other side of the market and leave the platform upon getting a match. Chen et al. (2018) provides an overview of recent work in online matching platforms, and list several open research questions. We contribute to this literature by modeling more closely how some dating platforms work, where users can accumulate matches and do not necessarily leave the platform once they get matched.

Our paper is also related to the growing empirical literature on matching platforms. Many of these papers focus on the negative impact of search frictions. For instance, Fradkin (2018) shows that most rejections on Airbnb are due to stale vacancy and hosts' screening, and estimates a model of guests' and hosts' behavior. Using this model, the author shows that better tracking listings' availability and including host preferences can significantly reduce rejections and improve match rates. Similar findings are presented in Horton (2014, 2017, 2018) in the context of online labor. The author shows that allowing workers to signal their availability and prioritizing higher probability matches can significantly improve the platform's match rate.

More closely related to our paper is the empirical literature on dating platforms. Part of this literature focuses on understanding mate preferences. Using a speed dating experiment, Fisman et al. (2006, 2008) show that there are differences across genders, with women putting more weight on intelligence and race, while men focusing more on physical attractiveness. Similar results are reported in Hitsch et al. (2010). Inspired by Adachi (2003)'s search model, and using data from an online dating site, the authors model and estimate the decision of sending a first contact email using a threshold-crossing rule, whereby a message is sent if the expected utility derived from a potential match exceeds the outside option of not responding to a specific profile. Hitsch et al. (2010) confirm that users have strong same-race preferences, that women have a stronger preference than men for income over physical attributes, and they also find no evidence for strategic behavior.

In a complementary paper, Hitsch et al. (2013) note the existence of strong assortative patterns, and also show that the observed matches are similar to those that running the Deferred Acceptance algorithm (see Gale and Shapley (1962)) would generate using the estimated preferences. Other papers empirically show the impact of design decisions and information on matching outcomes. Lee and Niederle (2014) show that dating platforms can increase the number of matches they generate by allowing users to signal their preferences using data from a field experiment. Yu (2018) shows that users' beliefs about the market size affect their behavior, making them more selective if they believe there are more users on the other side of the market, while making them less selective if they believe they have more competition. We contribute to this literature by empirically showing that the history of usage and the assortments offered affect like behavior of users, and we propose a dynamic assortment optimization to leverage these findings.

Finally, our paper is also related to the behavioral economics literature on context-dependent preferences. This literature has established that changes in the choice set can affect the way choices are made. A classic example of this is the attraction effect, first studied by Huber et al. (1982). Also known as the asymmetric dominance effect, the attraction effect states that the addition of a third alternative that is strictly dominated by an existing option and undominated by another can increase the probability that the dominant option is chosen. This effect has been extensively documented on different settings, including partner selection. Using a lab experiment, Sedikides et al. (1999) show that there is a significant attraction effect in mate selection, whereby the desirability of a potential partner over another depends on the existence of a third option that is dominated. Bateson and Healy (2005) provide an overview of the empirical evidence of comparative evaluation in mate selection for different animals, and argue that context-dependent preferences may offer new insights to better understand mate choices.

3. Empirical Setting

We have partnered with a major dating app based in the United States. This platform is suitable to study assortment and history effects for two reasons. First, it allows us to observe the exact assortment offered to each user in each period, including all observable characteristics of the profiles involved. Hence, we can thoroughly characterize the assortments offered, and include this information in the estimation. In addition, we have access to the full history of interactions between each user and the platform, so we can completely describe the history of each user in each period.

Every user in the platform has a profile that includes name, age, location, education, profession, and interests. Also, each profile includes pictures that users can browse before making a decision. Users can also define their preferences regarding age, height, distance, education, ethnicity, and religion. Using this information, the platform computes a set of *potential* partners—or *potentials*—for each user, given by the intersection of users' preferences and the characteristics of the profiles on the other side of the market.[4]

Every day a user logs-in, the platform offers them a limited number of profiles taken from their set of *potentials*. Free users are offered only one subset of profiles—or *assortment*—each day, which contains three to four profiles.[5] Users can explore all the profiles offered, i.e., they can check their information, look at their pictures, and so on. Then, for each profile in their assortment, users can make one of three possible decisions: like, dislike, or skip. To illustrate this, suppose that Alice is evaluating Bob's profile. If Alice likes Bob's profile and Bob has not seen Alice's profile yet, then Alice is stored in Bob's *backlog*, which contains the set of all users that have liked Bob in the past but have not been shown to him yet. On the other hand, if Bob is in Alice's *backlog* and Alice likes him, then a match automatically takes place. In that case, the platform displays a notification and creates a new screen that enables both users to start a conversation.[6] In case that Alice dislikes Bob, then she will never see Bob's profile again, and also her profile is removed from Bob's set of potentials. Finally, if Alice skips Bob's profile, then the latter remains in the set of potentials of the former, and thus Alice may see Bob's profile again in the future.

Based on these evaluations, the platform computes an attractiveness score—or simply *score*—for each user. The score is computed as the fraction of likes over the number of evaluations received, all multiplied by 10. The platform then divides each side of the market in quintiles of attractiveness score, which are used as part of its algorithm to choose which profiles to show.

3.1. Hypotheses

We are interested in studying the effect of the assortment composition and the past usage history on the probability that a user likes or dislikes a given profile. As discussed in previous sections, there is evidence in the literature that the quality of a potential mate cannot be measured in absolute terms, as it depends on the other alternatives with whom he or she is compared. As a result, we expect that the set of users included in the assortment affects the perceived quality of each individual profile, affecting their probability of being liked. In particular, we expect that a given profile is less likely to be liked if it is shown along with other profiles that are more attractive. We formalize this in the next hypothesis.

HYPOTHESIS 1 **(Effect of Assortment)**. *The attractiveness of the other profiles in the assortment has a negative effect on the probability that a given profile is liked.*

[4] The set of potentials is automatically updated when users change their profiles or preferences.

[5] Paid users can get more profiles per assortment, ranging from 5 to 20. Users can also pay to obtain an extra assortment on a given day.

[6] Bob is notified of this match as soon as he logs-in again.

Regarding history, many dimensions could affect choice behavior. For instance, we expect that the probability that a profile is liked is negatively affected by the number of matches that a user has had in the recent past. A potential explanation is that users may have a limited capacity to deal with their matches, and therefore they would be more prone to dislike profiles to avoid getting new matches if they have had many in the recent past. Another potential mechanism by which history could matter is through the outside option. For example, users may become more selective if they have many current matches they believe could end up in a longer-term relation. These two mechanisms suggest that the current number of matches may have a negative effect in the like rate of new profiles, as stated in the next hypothesis.

HYPOTHESIS 2 **(Effect of History)**. *The number of matches in the recent past has a negative effect on the probability that a profile is liked.*

There are undoubtedly other dimensions of history that may affect behavior. For instance, an exchange of phone numbers could lead to a formal relationship, which would make users leave the platform. Nevertheless, we focus on the effect of previous matches as a starting point in the study of history effects.

3.2. Observational Data

We start studying these two effects using fixed effects models on observational data. Although this analysis only produces correlations and its results cannot be interpreted as causal, it provides an initial idea of the magnitude of the two effects by controlling for unobserved heterogeneity.

To carry out this study, we construct a panel at the individual level combining three sources of information:

- Evaluations: for each profile shown in a given day as part of a given assortment, we observe the decision made by the user (like, dislike or skip), the IDs of the users involved, the ID of the assortment (to map which other users were shown in the same assortment), relevant time stamps, among others.
- Observable characteristics: for each user, we have his profile information (age, height, education, religion, and race) and his attractiveness score (i.e., $10 \cdot$ likes/evaluations received).
- Past usage metrics: for each user's session we compute a set of usage metrics in the recent past, including the number of days active, the number of matches obtained, the number of likes, dislikes and skips given and received, the number of conversations, among others. We compute each of these metrics for different time windows, including the last day, week, month, and quarter.

The analysis in this section considers a sample of heterosexual users that have made no purchases and have no memberships, taken from a single market located in the west-coast of the US.[7] We

[7] We focus on this market because it was suggested by our industry partner, as it balances size and representativeness of other markets in the platform. Overall, this market had 2255 active users during the time window considered, out of which 83 are not heterosexual and 366 had spent a positive amount in the platform.

Table 1 Descriptives

		Days Active		Profiles Seen		Like Rate		Match Rate	
	N	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Women	1.066	10.672	8.628	34.701	27.647	0.301	0.249	0.112	0.167
Men	756	15.396	9.742	49.749	31.477	0.525	0.282	0.073	0.113

Note: Summary statistics of main activity variables for the sample considered in observational study. Like rate is computed as the fraction of profiles liked over the total number of profiles seen. Match rate is computed as the number of matches obtained over the total number of profiles seen.

exclude observations for which the user skipped all the profiles in the assortment, as it may be the case that the user opened the app without the intention of evaluating new profiles.[8] The data spans for four weeks, from April 17th to May 14th, 2019.[9] As a result, our sample contains 1.822 users and 63,899profile evaluations. In Table 1 we report summary statistics for the number of days users were active, the number of profiles seen, the like and the match rates, separating by gender.

To provide initial evidence on how the assortment and the history affect the like behavior of users, we estimate the following linear probability model with fixed effects:[10]

$$y_{ijt} = \alpha_i + \lambda_t + X'_{ij}\beta + \bar{X}'_{-j}\delta + M'_{it}\gamma + \epsilon_{ijt}. \tag{1}$$

The dependent variable, y_{ijt}, is binary and equal to 1 if user i likes user j in period t, and 0 otherwise. We control for users' unobserved heterogeneity including user fixed effects, α_i, and we also include period fixed effects, λ_t, to control for period-dependent unobservables. The third term on the right-hand side, $X'_{ij}\beta$, controls for observable characteristics of user j and also for their interaction with user i's observable characteristics. More specifically, let $X_i = (x_i, d_i)$ and $X_j = (x_j, d_j)$ capture users i and j observable characteristics, respectively.[11] As in Hitsch et al. (2013), we assume that $X'_{ij}\beta$ can be specified as:

$$X'_{ij}\beta = x'_j\theta + |x_j - x_i|_+^{a'}\kappa^+ + |x_j - x_i|_-^{a'}\kappa^- + \sum_{k,l} \mathbb{1}\{d_{ik} = 1,\, d_{jl} = 1\} \cdot \vartheta^{kl}, \tag{2}$$

where $(\theta, \kappa^+, \kappa^-, \vartheta)$ are parameters to be estimated. The first element in (2), $x'_j\theta$, is a linear valuation of user j's observable characteristics. The second and third elements capture the absolute difference between user i and user j's observable characteristics, as $|x_j - x_i|_+ = \max\{(x_j - x_i), 0\}$

[8] For instance, a user that opens the app to continue a conversation with a previous match may also get an assortment.

[9] We chose this time window because it is right before the time of the quasi-experiment presented in the next section, so we assume that there are no significant changes in users' behavior between this period and that of the quasi-experiment. Also, during this time window, there were no substantial changes to the algorithm, so it is a very stable period to analyze.

[10] We use a linear probability model to facilitate interpretation of the coefficients.

[11] x_i and d_i are continuous and categorical observable characteristics of user i, respectively.

and $|x_j - x_i|_- = \max\{(x_i - x_j), 0\}$ represent the positive and negative differences among observables. We consider $a=2$ to avoid the identification problems that arise when $a=1$ (see discussion in Hitsch et al. (2013)). The last term accounts for the interaction of categorical characteristics of users i and j. The fourth term in (1) captures the assortment effect, given by the average attractiveness score $\bar{X}_{-j}$ of the profiles in assortment S other than j.[12] The next term captures the history effect, given by the linear combination of the number of matches obtained by user i in different time windows in the past, encoded in the vector M_{it}. Finally, ϵ_{ijt} is an idiosyncratic shock.

In Table 2 we report the estimation results. In all models we consider user fixed effects, observable characteristics of the profiles evaluated and their interaction with those of the user evaluating as shown in (2). We also control for the number of profiles shown in the assortment. Column (2) includes controls for race interactions,[13] and column (3) adds measures of previous activity, including the number of likes and dislikes, the number of days active and the number of conversations, for the same time windows as for the number of matches..

Table 2 Regression Results - Observational Study

	Dependent variable: Liked		
	(1)	(2)	(3)
Assortment Effect	-0.004^{***}	-0.004^{***}	-0.004^{***}
	(0.001)	(0.001)	(0.001)
Matches	-0.013^{***}	-0.013^{***}	-0.016^{***}
$\{t-1\}$	(0.004)	(0.004)	(0.005)
Matches	-0.015^{***}	-0.015^{***}	-0.014^{***}
$[t-2, t-7]$	(0.002)	(0.002)	(0.002)
Matches	-0.005^{***}	-0.005^{***}	-0.008^{***}
$[t-8, t-14]$	(0.002)	(0.002)	(0.002)
Matches	-0.004^{**}	-0.004^{**}	-0.001
$[t-15, t-21]$	(0.002)	(0.002)	(0.002)
Matches	-0.007^{***}	-0.007^{***}	-0.007^{***}
$[t-22, t-28]$	(0.002)	(0.002)	(0.003)
User Fixed Effects	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes
Demographics	Yes	Yes	Yes
Extra Race	No	Yes	Yes
Extra History	No	No	Yes
Observations	63,899	63,899	63,899
R^2	0.220	0.222	0.225

Note: Linear probability models with fixed effects. Standard errors reported in parenthesis. All three models include demographics and period fixed effects. Column (2) controls for the interaction between users' races. Column (3) adds additional history variables (for same time windows than Matches), including likes, dislikes, days active, among others. Significance reported: *p<0.1; **p<0.05; ***p<0.01

[12] We focus on the effect of other users' attractiveness because we observe that this is the most important driver of users' decisions. However, we could extend our model to include other average observables, such as age, height, among others.

[13] We add a dummy variable for each pair of races: white, black, asian, hispanic, and other.

First, we observe that the assortment effect is negative and significant for all models considered. In particular, an assortment whose profiles are one point more attractive reduces the probability that a specific profile is liked by 0.4%. As the scale of the attractiveness score goes from 0 to 10, this suggests that the average effect is approximately 2%, and that an extreme assortment, where every profile has the highest score, can lead to a drop of 4% in the probability that a given profile is liked.[14] These results provide evidence sustaining Hypothesis 1.

Second, we observe that all the variables related to the number of matches in the past are negative and significant. For instance, the coefficient of the number of matches in the previous day is -0.016 (second row in column (3)), which implies that an extra match in the previous day reduces the like rate by 1.6%. Also, we observe that the magnitude of the coefficient is decreasing in the longevity of matches for most cases. This result is very intuitive, as we expect that older matches are less likely to convert to a long-term relationship and thus should have a smaller effect on current decisions. Overall, we conclude that Hypothesis 2 is also supported by our results.

3.2.1. Endogeneity Although linear fixed-effect models account for unobserved time-invariant confounders, the results in Table 2 cannot directly be interpreted causally. The reason is that, as in most dynamic panel models, there may be a lagged effect of the dependent variable on itself, and therefore the error terms may not be independent of the model predictors. More concretely, our estimates will be biased if the assortment and history depend on users' decisions in the past, or if they are affected by confounders correlated with their current choices, observed by the users, but unobserved by the researcher.

Since we are controlling for user and time fixed-effects, these unobservables can be either profile dependent or user-time dependent. In the former case, conditional on observable characteristics of the profiles shown, the assortments are random. In addition, since the market is large enough and the platform does not use any learning mechanism to provide better assortments based on users' past decisions,[15] we know that the assortments observed by the users are independent of their previous decisions. Hence, profile dependent and time-varying unobservables should not affect the assortments, and therefore we believe that our estimates of the assortment effect do not suffer from omitted variable bias.

On the other hand, the number of matches collected by user i in the recent past depends on three main elements: (1) user i's previous like behavior, (2) the like behavior of users on the other side of the market, and (3) the frequency by which i logs-in.[16] Therefore, any unobservable time-varying

[14] In Appendix D we report the results separating by gender.

[15] Whether a given user i sees a profile j depends on (1) whether user j is in user i's backlog, (2) user j's backlog, (3) user j's score, and (4) how active user j has been in the recent past.

[16] The latter not only affects how many profiles user i evaluates—and potentially gets matched with—but also influences how frequently the platform shows his profile on the other side of the market.

variable that causes a correlation between shocks and relates past and current like behavior—for example, how lonely a user feels—introduces an omitted variable bias.

There are different ways to address this potential endogeneity problem. One approach is to use lagged variables as instruments, and estimate the model using the Generalized Method of Moments (GMM). Another approach is to leverage exogenous variations in the variables of interest to identify the parameters. We adopt the latter approach, and in the next section, we report the results of a quasi-experiment that helped us to identify the history effect.

4. Effect of History: Evidence from a Quasi-Experiments

As discussed in the previous section, our main concern when estimating the effect of history is the potential endogeneity between past matches and future like behavior. In this section, we describe a quasi-experiment that exogenously changed the probability of getting new matches for some users, and we use it to estimate the causal effect of an extra match in like behavior.

4.1. Background

As described in Section 3, the assortments may include profiles taken from users' backlogs. The algorithm used by our industry partner prioritizes (to some extent) profiles taken from the backlog, as they immediately convert into a match if users like them. We refer to queries involving profiles taken from the backlog as *backlog queries*.

Depending on the number and attractiveness score of the users in their backlogs, free users can get up to three backlog queries each day. Before May 17th, backlog queries could only involve active users, i.e., users that were last seen within 45 days before the creation of the assortment. Starting on May 17th this constraint was removed, so all inactive users in the backlogs were eligible to be used as part of one of these queries. As a result, some users experienced a change in the number of backlog queries they received relative to the previous days, which increased their probability of getting new matches. Users did not know that this change in the algorithm was implemented, and it is not possible to differentiate a profile that is active from another that is not. Hence, being affected by the change in the algorithm can be used as an instrument for the number of matches obtained, and the causal effect of previous matches on like behavior can be estimated through an instrumental variables approach. Using instrumental variables is a standard approach to deal with endogeneity, as it provides consistent estimators and it is robust to multicollinearity issues.[17]

[17] See Angrist and Krueger (2001) and Wooldridge (2002) for more details.

4.2. Treatments

We know that the set of potentially treated users are those who observed inactive profiles within a short time window after the change in the algorithm, increasing their chances of getting a match. However, many of these users would have also observed similar assortments if no change in the algorithm was introduced. Hence, to guarantee that *treated* users are those who were directly affected by the change in the algorithm, we only consider as treated a user satisfying two conditions:

1. The user received an inactive profile as part of a backlog query.
2. The composition of the assortment shown (in terms of backlog queries) is different from what the user would have seen with no change in the algorithm.

The first condition guarantees that users in the treatment group experienced an exogenous increment in the probability of getting a match. The second condition ensures that these users would not have obtained the same assortment composition if the change in the algorithm was not implemented.

Based on this definition, there are three important periods in which we will focus our attention. We define by t_i^1 the time of the first assortment in which user i saw an inactive profile as part of a backlog query (due to change in the algorithm), and thus experienced an increment in the probability of getting a match. In addition, let t_i^0 and t_i^2 be the time of the last assortment seen before t_i^1 and the time of the first assortment seen after t_i^1, respectively.[18] To rule out possible aggregated market effects due to the change in the algorithm,[19] we restrict the analysis to a short time window around t_i^1. As most users got treated either on May 17th or May 18th, we consider a time window of three days around that, i.e., we add the constraints that t_i^0 is no sooner than June 14th and t_i^2 is no later than June 20th.[20] Since the average number of days between a first like and the corresponding backlog query (if any) on the other side of the is more than 3 days for approximately 80% of users,[21] considering 3 days around the change in the algorithm guarantees that the two-sidedness of the market will not considerably affect the perception of the assortments obtained and the users' like behavior. Finally, we define the control group as the set users that are not part of the treatment group, that had at least one backlog query prior to t_i^2,[22] and that logged in between June 14th and June 20th at least once before and after the change in the algorithm.

[18] When clear from the context, we will sometimes drop the dependence on i and simply write t^0, t^1 and t^2. In addition, to simplify notation we will sometimes write $t = \tau$ to denote $t = t^\tau$, where $\tau \in \{0, 1, 2\}$.

[19] For example, showing more inactive profiles reduces the number of active profiles shown, which in turn reduces the number of users added to the backlogs of other users, changing the number of backlog queries in future periods.

[20] The results remain relatively unchanged if we consider a time window of 2 or 4 days.

[21] This is computed using the data described in Section 3, i.e., prior to the change in the algorithm.

[22] Users with no backlog queries between May 8th and t_i^2 had no backlog before period t_i^2. We exclude these users to make the treatment and control groups more comparable. In Appendix D we report the results considering all users.

Table 3 Summary Statistics by Treatment and Period

Period	Treatment	N	Num. Batch Mean	Num. Batch Std.	Like Rate Mean	Like Rate Std.	Match Rate Mean	Match Rate Std.
t^0	0	116097	3.662	0.606	0.388	0.487	0.101	0.301
	1	11249	3.614	0.570	0.378	0.485	0.094	0.292
t^1	0	115205	3.633	0.601	0.397	0.489	0.112	0.315
	1	11346	3.640	0.544	0.360	0.480	0.181	0.385
t^2	0	113228	3.570	0.598	0.388	0.487	0.088	0.284
	1	11196	3.594	0.549	0.368	0.482	0.110	0.313

Note: Summary statistics of main activity variables, separating by period ($t \in \{t_0, t_1, t_2\}$) and treatment. We exclude users that are not part of the second stage regression. Num. Batch represents the number of profiles in a batch. Like Rate is computed as the fraction of profiles liked over the total number of profiles seen. Match Rate is computed as the number of matches obtained over the total number of profiles seen.

As shown in Table 3, users in the treatment group got a significantly higher probability of getting matched in period t^1 compared to users in the control group. On average, treated users got 0.66 $(= 0.182 \cdot 3.640)$ matches in t^1, while users in the control group got 0.31 $(= 0.086 \cdot 3.572)$ matches. From Table 3 we can also directly obtain an estimate of the ATT of the change in the algorithm, by taking the difference between the change in the like rate of the treated and control groups between t^0 and t^2. In this case, $ATT = (0.368 - 0.378) - (0.388 - 0.388) = -0.010$, which suggests that users that were affected by the change in the algorithm decrease their like rate by 1.0% in period t_2 relative to period t_0.

4.3. Estimation

To use our treatment variable as instrument we need to check two assumptions: (i) relevance, and (ii) exogeneity. The first condition requires that our instrument is a relevant predictor for the endogenous variable of interest, which in this case is the number of matches obtained by user i in period t_i^1. As shown in Table 3, being treated is positively correlated with the number of matches obtained in period t^1, so they are likely to satisfy the relevance condition. On the other hand, the exogeneity condition requires that the instrument is uncorrelated with the error term. We claim that being treated is unlikely to be correlated with the error term for two reasons: (1) the change in the algorithm was not announced, and (2) users cannot distinguish between active and inactive profiles, so it is unlikely that they noticed that there was a change in the algorithm. Hence, our treatment variable is also likely to satisfy the exclusion restriction, so we conclude that this variable is a potentially valid instrument. Hence, we employ the following 2SLS estimation procedure:
Step 1. Estimate using OLS and data corresponding to periods t^0 and t^1:

$$M_{it} = \alpha + \lambda_t + \theta \cdot W_i + \gamma \cdot D_{it} + X_i'\beta + \epsilon_{it}, \tag{3}$$

where M_{it} is the number of matches obtained by user i in period t; λ_t is a dummy variable for whether $t = t^1$; W_i is a dummy variable equal to 1 if user i is in the treatment group, 0 otherwise;

D_{it} is a dummy variable that takes value 1 if user i is in the treatment group and the period is after the change in the algorithm; and X_i are observable characteristics of user i, including gender, age, height, attractiveness score, education, region, and race dummies.[23] The instruments in this model are the treatment variable, W_i, and its interaction with the period, D_{it}. Finally, considering the estimated parameters of this model, we compute the predicted values of the endogenous variable, i.e., $\hat{M}_{i1}$.

Step 2. Estimate using OLS and robust standard errors:

$$y_{ij2} = \alpha + X'_{ij}\beta + \hat{M}_{i1}\gamma + \epsilon_{ij2}, \tag{4}$$

where y_{ij2} is 1 if user i likes user j in period $t = t^2$, 0 otherwise; and X_{ij} is a matrix of observable characteristics that includes X_i and also observable characteristics of user j and their interaction with those of user i, similar to the model described in Section 3.

In Table 10 (in Appendix D) we present the estimation results for the first stage. The coefficient of the variable D_{it} is positive and significant in all specifications, while the coefficient of the variable W_i is negative and significant. In addition, the F-statistic is greater than 10 in all specifications. Thus, we conclude that the treatment variables (W_i and D_{it}) are relevant instruments. For the second stage we use the estimates from the second specification, which controls for observable characteristics of user i.[24]

Table 4 presents the estimation results for the second stage. Column (1) reports the results of OLS, while columns (2) and (3) present the results of the 2SLS approach under different specifications. We observe that the coefficient of interest ranges from -0.015 to -0.019 according to the 2SLS approach, which is consistent with the results presented in Table 2. In other words, an extra match in period t^1 reduces the like rate in t^2 by at least 1.5%. This result suggests that our previous model with user fixed effects handles to a great extent the endogeneity problem. In addition, from column (1) we observe that not considering the endogeneity of the number of matches can lead to serious bias in the estimates.

One potential concern regarding this quasi-experiment is that we only consider users that received at least one backlog query before period t_i^2. In Table 12 (in Appendix D), we report the results without excluding these users. We observe that the results are directionally the same, but the magnitude of the history effect is slightly smaller. Nevertheless, the results in Table 12 are consistent with those in Table 13, where we report the results of a model similar to that in Section 3 but

[23] We add dummy variables for being white, black, asian, hispanic, and an extra dummy variable that covers all other races.

[24] In particular, we control for gender, age, height, education, score and race. The results are similar if we consider the other specification.

Table 4 Effect of History - Second Stage Results

	Dependent variable: y_{ij}		
	OLS	*IV - 2SLS*	
	(1)	(2)	(3)
Matches	0.093^{***}	-0.019^{***}	-0.015^{***}
(in t^1)	(0.002)	(0.006)	(0.003)
Constant	0.209^{***}	0.169^{***}	0.119^{***}
	(0.064)	(0.002)	(0.007)
Demographics and Scores	Yes	Yes	Yes
Assortment Effect Fixed	No	No	Yes
Observations	124,401	124,401	124,401

Note: Second stage regression results. Robust standard errors reported in parenthesis, clustered by date. The dependent variables y_{ij}, which is equal to 1 if user i likes j, 0 otherwise. Column (1) reports OLS estimates without addressing the potential endogeneity issue. Columns (2) and (3) report 2SLS estimates. All models control for observable characteristics and scores of both users i and j. Columns (3) fixes the assortment effect to be equal to $\delta = -0.004$. Significance reported: *p<0.1; **p<0.05; ***p<0.01

replacing the number of matches in the previous day by the number of matches in the previous session within 3 days, matching the specification used in the analysis of the quasi-experiment. These results confirm that the assortment and history effects are negative and significant, and also that a linear probability model with fixed effects captures to a great extent the effects of interest.

Another potential issue is that all users that were affected by the change in the algorithm have been on the platform for at least 45 days. Hence, our results do not directly translate to less experienced users. To address this point, in Appendix E we describe and analyze another quasi-experiment that focuses on new users, and we find that an extra match during the first week in the platform reduces users' average like rate in the second week by at least 2.0% (see Table 18), which is in line with our previous results.

Overall, our results confirm that there is an effect of the assortment composition and the recent history on the behavior of users. In the next section, we introduce a model of dynamic two-sided assortment optimization that takes these findings into account.

5. Model

Consider a two-sided market mediated by a platform. We refer to the two sides of the market as $\mathcal{I} = \{1, \ldots, I\}$ and $\mathcal{J} = \{1, \ldots, J\}$. Users are indexed by i and j, respectively. Without loss of generality, we describe the main elements of the model for a given user $i \in \mathcal{I}$; analogous definitions apply for $j \in \mathcal{J}$.

Let $\mathcal{T} = \{1, \ldots, T\}$ be the set of periods, indexed by t, and let $\mathscr{P}_i^t$ be the set of *potential partners*—or simply potentials—available for user i at time t. We assume that $\mathscr{P}_i^1$, the initial set of potentials, is given as an input for each user i.

The goal of the platform is to select a subset of potentials—or *assortment*—to show to each user in each period, in order to maximize the total expected number of matches in the entire horizon. Let $S_i^t \subseteq \mathscr{P}_i^t$ be the assortment offered to user i in period $t \in \mathcal{T}$. To mimic our industry partner's practice, we assume that the size of the assortments is fixed and equal to K, i.e. $|S_i^t| = K$ for some $K \ll \min\{I, J\}$.

For each potential j in S_i^t, user i must decide whether to like or dislike j.[25] We denote these decisions by $y_i^t = \left\{y_{ij}^t\right\}_{j\in S_i^t}$, where

$$y_{ij}^t = \begin{cases} 1 \text{ if } i \text{ likes } j \text{ in period } t, \\ 0 \text{ if } i \text{ dislikes } j \text{ in period } t. \end{cases}$$

Users can only evaluate profiles shown to them in their assortment, and once a user is shown to another, the former is removed from the set of potentials of the latter in all subsequent periods. As a result, users are shown to each other at most once. To capture this, let A_i^t and R_i^t be the sets of users that liked and disliked user i in period t, respectively,

$$A_i^t := \left\{j \in \mathscr{P}_i^t : i \in S_j^t,\, y_{ji}^t = 1\right\}, \quad R_i^t := \left\{j \in \mathscr{P}_i^t : i \in S_j^t,\, y_{ji}^t = 0\right\}.$$

Finally, let $\mathscr{B}_i^t$ be the backlog of user i at the beginning of period t, i.e., the subset of users that have liked user i before period t but have not been shown to him yet. Notice that $\mathscr{B}_i^t \subseteq \mathscr{P}_i^t$, so once a profile is chosen from the backlog to be part of S_i^t it is removed from both $\mathscr{P}_i^t$ and $\mathscr{B}_i^t$. More explicitly, the evolution of the set of potentials and the backlog is given by:

$$\mathscr{P}_i^{t+1} = \mathscr{P}_i^t \setminus (S_i^t \cup R_i^t) \quad \text{ and } \quad \mathscr{B}_i^{t+1} = (\mathscr{B}_i^t \cup A_i^t) \setminus S_i^t.$$

Recall that the objective of the platform is to generate matches between users. A match between users i and j takes place in period t if and only if

$$\left(i \in A_j^t \text{ and } j \in A_i^t\right) \quad \text{or} \quad \left(i \in A_j^t \text{ and } j \in \mathscr{B}_i^t\right) \quad \text{or} \quad \left(i \in \mathscr{B}_j^t \text{ and } j \in A_i^t\right).$$

The first condition implies that users i and j mutually like each other in period t. The second condition implies that user i likes j in period t and that user j did so in some period $\tau < t$, while the third condition captures the complementary case. Notice that these conditions cannot hold simultaneously, as users are shown to each other at most once, and thus we cannot simultaneously have that $i \in A_j^t$ and $i \in \mathscr{B}_j^t$.

Let μ_i^t be the number of matches obtained by user i in period t. We assume that each match obtained in period t expires in each subsequent period with probability p_ξ.[26] We refer to non-expired matches as *active matches*, and we denote by M_i^t the number of active matches that user

[25] For simplicity we assume that users can either like or dislike a profile, ruling out the skip option.

[26] In the platform we are collaborating with, users can expire their matches any time, and matches that have been inactive (no messages exchanged) are automatically expired after a given number of days.

i has at the beginning of period t. Then, the number of expired matches in period t, denoted by ξ_i^t, follows a binomial distribution with parameters (M_i^t, p_ξ), and the number of active matches in period $t+1$ can be computed as

$$M_i^{t+1} = M_i^t + \mu_i^t - \xi_i^t.$$

Users' Behavior. As discussed in Section 3.2, we assume that user i evaluates profile j based on its indirect utility, which we model as

$$U_i\,(j,M,S) = U(X_i,X_j,\theta) + A(j,S,\delta) + H(M,\gamma) + \epsilon_{ij} = u_{ij}(M,S) + \epsilon_{ij},$$

where $u_{ij}(M,S) := U(X_i,X_j,\theta) + A(j,S,\delta) + H(M,\gamma)$ is the deterministic component of the utility function given M active matches and an assortment S. Based on the results in Sections 3 and 4, we make the following assumption that we keep throughout the rest of the paper.

Assumption 1. *The assortment and history effects are linear in the constants δ,γ, i.e.,*

$$A(j,S,\delta) = \delta \cdot a(j,S) \quad \textit{and} \quad H(M,\gamma) = \gamma \cdot h(M),$$

where $\delta,\gamma \leq 0$, $a(j,S), h(M) \geq 0$, $a(j,\{j\}) \leq a(j,S)$ for all j and $S \ni j$, and $h(M)$ is non-decreasing in M.

This assumption captures our main findings from Sections 3 and 4.[27]

User i likes profile j if and only if $U_i(j,M,S) \geq \epsilon_{i0}$, where ϵ_{i0} represents the utility obtained from the outside option. We assume that the shocks $\{\epsilon_{ij}\}_{j\in\mathcal{J}\cup\{0\}}$ are i.i.d., and that the differences $\{\epsilon_{i0} - \epsilon_{ij}\}_{j\in\mathcal{J}}$ satisfy the following assumption:

Assumption 2. *The differences between idiosyncratic shocks, $\{\epsilon_{i0} - \epsilon_{ij}\}_{j\in\mathcal{J}}$, are independent and identically distributed according to a distribution F_i, with density f_i, that satisfies the* monotone likelihood ratio property *(MLRP).*

This assumption is fairly standard in the literature, and it is satisfied by common distributions such as the uniform, normal, logistic, and exponential.

Let $\phi_{ij}(M,S) := \mathrm{P}\left(y_{ij} = 1 \mid M,S\right) = F_i(u_{ij}(M,S))$ be the probability that user i likes j given M active matches and an assortment S. Similarly, $\phi_{ji}(M',S') := F_j(u_{ji}(M',S'))$ is the probability that user j likes i given M' and S'. We define $\beta_{ij}(M,M',S,S') := \phi_{ij}(M,S)\cdot\phi_{ji}(M',S')$ the probability of a match between users i and j given their number of active matches M, M' and their assortments

[27] Our estimation results also show that the number of profiles in the assortment has a negative effect on like probabilities, justifying the assumption that $a(j,\{j\}) \leq a(j,S)$ for all $S \ni j$. Notice that this assumption is similar to—but less restrictive than—the *substitutability* assumption in the assortment literature (see Golrezaei et al. (2014) for a definition), which holds for all choice models that satisfy the random utility maximization principle, including the multinomial logit, nested logit, d-level logit, among many others.

S, S'. Notice that we cannot compute this probability unless we know the assortments and the number of active matches of both i and j. Therefore, when assortments are chosen dynamically and users are not shown to each other in the same period, this probability cannot be computed. To handle this, let α_{ij}^t be an estimate of the probability that user i is liked by user j, computed as:

$$\alpha_{ij}^t = \begin{cases} 1 & \text{if } j \in \mathscr{B}_i^t, \\ \phi_{ji}(M_j^t, \{i\}) & \text{if } j \in \mathscr{P}_i^t \setminus \mathscr{B}_i^t, \\ 0 & \text{if } j \notin \mathscr{P}_i^t. \end{cases}$$

The first and the last cases are direct from our previous definitions. In the middle case, since j has not been shown to i and we do not know the assortment S_j^τ that will include i in period $\tau \geq t$ (if any), we estimate the probability that j likes i assuming no assortment effects—i.e., $S_j^\tau = \{i\}$—and using j's current number of active matches, M_j^t. Then, we denote by $\beta_{ij}^t(M,S) := \phi_{ij}(M,S) \cdot \alpha_{ij}^t$ the estimated probability of a match between users i and j in period t.

Using vector notation to refer to lists of sets or values for each user, e.g., $\vec{\mathscr{P}} = \{\mathscr{P}_i\}_{i \in \mathcal{I} \cup \mathcal{J}}$, the problem faced by the platform can be formulated as the following dynamic program:

$$\begin{aligned} V^t(\vec{\mathscr{P}}, \vec{\mathscr{B}}, \vec{M}) &= \max_{\substack{\vec{S} = \{S_i\}_{i \in \mathcal{I} \cup \mathcal{J}} \\ S_i \in \mathcal{S}_K(\mathscr{P}_i),\, \forall i \in \mathcal{I} \cup \mathcal{J}}} \left\{ \mathbb{E}\left[\sum_{i \in \mathcal{I} \cup \mathcal{J}} \mu_i^t(M_i, S_i) + V^{t+1}\left(\vec{\mathscr{P}} \setminus \left(\vec{S} \cup \vec{R} \right), \left(\vec{\mathscr{B}} \cup \vec{A} \right) \setminus \vec{S}, \vec{M} + \vec{\mu}^t - \vec{\xi}^t \right) \middle| \vec{M}, \vec{S}, \vec{\mathscr{B}} \right] \right\} \\ V^{T+1}(\vec{\mathscr{P}}, \vec{\mathscr{B}}, \vec{M}) &= 0, \ \forall \vec{\mathscr{P}}, \vec{\mathscr{B}}, \vec{M}, \end{aligned} \tag{5}$$

where $\mathcal{S}_K(\mathscr{P})$ is the set of assortments of size K that can be obtained from the set $\mathscr{P}$, i.e.,

$$\mathcal{S}_K(\mathscr{P}) = \{ S \subseteq \mathscr{P} : |S| = K \}.$$

As previously discussed, the objective of the platform is to maximize the expected number of matches, where the expectation is taken over the realized evaluations and the number of matches expired.[28] The state variables are, for each user, the set of potentials, the backlog, and the number of active matches at the beginning of the period. Finally, the controls are the assortments shown to each user in each period. In the remainder of the paper we denote by V^* the optimal value function in the initial period given the initial sets of potentials, backlogs and the initial number of active matches of each user.

5.1. Platform's Problem: Individual Level

According to Problem (5), the platform jointly chooses the assortments to show to each user in each period. However, the large scale of the problem may make this approach unfeasible. Instead, a common practice in many platforms, including our industry partner, is to make assortment

[28] To keep the notation simpler we do not condition the expectation on the state variables. In addition, operations involving vectors are element-wise.

decisions for each user independently.[29] For this reason, in this section we focus on the problem of finding the sequence of assortments to show to an individual user in order to maximize his expected number of matches. This is useful because it approximates the problem solved by the industry, and it also provides some insights that will be useful to solve the market level problem.

Consider an individual user $i \in \mathcal{I}$. The problem of finding the sequence of assortments to show to user i in each period t, so as to maximize the expected number of matches, can be formulated as a dynamic program whose value function can be computed recursively as follows: $\forall \mathscr{P}_i \subseteq \mathcal{J}, M_i \in \mathbb{N}_0^+$,

$$\begin{aligned} V_i^t(\mathscr{P}_i, M_i) &= \max_{S \in \mathcal{S}_K(\mathscr{P})} \left\{ \mathbb{E}_{\mu,\xi} \left[\mu_i^t(M_i, S) + V_i^{t+1}(\mathscr{P}_i \setminus S, M_i + \mu_i^t - \xi_i^t) \mid M_i, S, \mathscr{B}_i \right] \right\}, \forall t \in \mathcal{T}, \\ V_i^{T+1}(\mathscr{P}_i, M_i) &= 0, \; \forall \mathscr{P}_i \subseteq \mathcal{J}, M_i \in \mathbb{N}_0^+. \end{aligned} \tag{6}$$

In this case, the state variables are the set of potentials and the number of active matches, $(\mathscr{P}_i, M_i)$. As opposed to Problem (5), we do not consider the backlog as part of the state variables, since we focus on the problem involving user i only and we do not take into account the assortments and evaluations of the users on the other side of the market. Nevertheless, we use the initial backlog of each user to compute the estimated probability of a match, $\beta_{ij}(M, S)$. We denote by V_i^* the optimal value in the initial period given the initial set of potentials, backlog and the number of active matches of user i.

In Proposition 1 we characterize some properties of the value function $V_i^t(\mathscr{P}, M)$. All proofs are presented in Appendix B.

Proposition 1. *Suppose that Assumption 1 holds. Then, for each user i, the function $V_i^t(\mathscr{P}, M)$ is*

- *non-increasing in t,*
- *non-increasing in M,*
- *non-decreasing in $\mathscr{P}$, i.e. given $\mathscr{P} \subseteq \mathscr{P}'$, then $V_i^t(\mathscr{P}, M) \leq V_i^t(\mathscr{P}', M)$,*
- *neither concave nor convex in M.*

5.1.1. Particular Cases.

No Assortment nor History Effects. We start with the simplest case where there is neither assortment nor history effects, i.e., $\delta = \gamma = 0$. As a result, user i's utility becomes $U_i(j, M, S) = U(X_i, X_j, \theta) + \epsilon_{ij} = u_{ij} + \epsilon_{ij}$, and thus the probability of a match simplifies to $\beta_{ij}^t = \beta_{ij} = F_i(u_{ij}) \cdot F_j(u_{ji})$ for all $t \in \mathcal{T}$.[30]

[29] The only way in which assortment decisions are related in this platform is through a constraint that limits the maximum number of times that a given profile is shown on a given day. However, for the vast majority of users, this constraint is not binding, and thus it is reasonable to assume that assortment decisions are made independently.

[30] In a slight abuse of notation, in presence of assortment and history effects we make explicit the dependence of the functions $u_{ij}, \phi_{ij}^t, \beta_{ij}^t$ on the assortment S and the number of matches M. However, when clear from the context we omit these arguments and we also drop the time index to ease exposition. In addition, in absence of assortment effects we omit the second argument and write $u_{ij}(M)$ to represent $u_{ij}(M, \{j\})$ (same for ϕ_{ij} and β_{ij}). Similarly, when there is no history effect we drop the first argument and write $u_{ij}(S)$ to represent $u_{ij}(0, S)$ (same for ϕ_{ij} and β_{ij}). Finally, in absence of the two effects we omit both arguments and write u_{ij} to represent $u_{ij}(0, \{j\})$ (same for ϕ_{ij} and β_{ij}).

Lemma 1. *Suppose that $\delta = \gamma = 0$. Then, it is optimal to greedily select the $K \cdot T$ profiles with the highest values of β_{ij} and show them using any combination of feasible assortments.*

Lemma 1 implies that, when there is no assortment nor history effects, it is optimal to choose the set of profiles with the highest probability of generating a match, and show them in any order. This is not necessarily the case in the presence of assortment effects. For instance, profiles can benefit from being displayed with lower quality profiles, as the following example shows.

Example 1. Consider the case with $\delta = -0.25$, $\gamma = 0$, $J = 5$, $K = 2$, $T = 2$, $\{\epsilon_{i0} - \epsilon_{ij}\}_{j\in\mathcal{J}} \overset{\text{iid}}{\sim} \mathcal{N}(0,1)$, qualities $Q = \{0.5, 0, -0.8, -1.10, -2.25\}$, utilities $U_i(j, M, \{j, j'\}) = (1-\delta)q_j + \delta q_{j'} + \epsilon_{ij}$, and suppose for simplicity that $\alpha_{ij} = 1$ for all j.

A Greedy policy would choose profiles $\{1, 2, 3, 4\}$ and display them in any order. However, it can be shown that the optimal solution displays two assortments, $S_1^* = \{1, 3\}$ and $S_2^* = \{2, 5\}$. Profile $\{4\}$ is not shown because its probability of generating a match is not too different from that of $\{5\}$,[31] but $\beta_{ij}(\{j\} \cup \{5\}) > \beta_{ij}(\{j\} \cup \{4\})$ for any $j \in \{1, 2, 3\}$. In words, pairing any other profile $j \in \{1, 2, 3\}$ with $\{5\}$ leads to a much higher match probability β_{ij}, improving the overall expected number of matches. □

Only History Effect. Now suppose that $\delta = 0$ and $\gamma < 0$. Then, if Assumption 1 holds, the indirect utility of user i given a profile j can be written as

$$U_i(j, M, S) = U(X_i, X_j, \theta) + \gamma \cdot h(M) + \epsilon_{ij} = u_{ij} + \gamma \cdot h(M) + \epsilon_{ij},$$

where u_{ij} is the component of the utility that depends only on observable characteristics and not in the assortment or history.

Our next proposition shows that the order in which profiles are displayed matters, even in the case where assortments are of size $K = 1$. This case is interesting on its own because some platforms—including Tinder, one of the largest dating apps in the world—displays profiles sequentially, and thus can be thought of as showing assortments of size 1.

Proposition 2. *Suppose that Assumptions 1 and 2 hold, and that $p_\xi = 0$. Also, suppose that there is a fixed subset of profiles that must be displayed, $\tilde{\mathcal{J}} \subseteq \mathcal{J}$. Then, it is optimal to show the profiles in $\tilde{\mathcal{J}}$ in increasing order of u_{ij}.*

Proposition 2 states that, given a set of profiles to be shown, it is optimal to display them in increasing order of utility.[32] However, it is hard to know what is the optimal subset of profiles

[31] Unless they are shown in the same batch, which is clearly suboptimal.

[32] This result holds if $p_\xi = 0$. This case is especially relevant because most of expirations take place after two weeks, as Figure 5 in Appendix D shows. Example 2 in Appendix C.1 shows that this does not necessarily hold when $p_\xi > 0$.

to display, as it depends on the sequence of realized matches due to the history effect.[33] Nevertheless, we know that some profiles will never be part of the optimal subset. To formalize this, let $\mathscr{P}(M) \subseteq \mathscr{P}$ be the set of $K \cdot T$ profiles with the highest values of $\beta_{ij}(M)$. In addition, let $\underline{u}(M) = \min\{u_{ij} : j \in \mathscr{P}(M)\}$ and $\bar{u}(M) = \max\{u_{ij} : j \in \mathscr{P}(M)\}$.

LEMMA 2. *Let $\mathcal{J}^*$ the set of profiles that are displayed with positive probability in the optimal solution. Then, $\mathcal{J}^* \subseteq \bigcup_{M=0}^{K\cdot(T-1)} \mathscr{P}(M)$. Moreover,*

$$\mathcal{J}^* = \{j \in \mathcal{J} : \underline{u}(0) \leq u_{ij} \leq \bar{u}(K \cdot (T-1))\}.$$

Lemma 2 states that all profiles that could potentially be part of the optimal solution involve a utility that is in the interval $[\underline{u}(0), \bar{u}(K \cdot (T-1))]$. Also, notice that some of the profiles in $\mathcal{J}^*$ are shown with probability less than 1, as their optimality depends on the realized evolution of the number of matches M_i^t.

Only Assortment Effect. Now we focus on the case where $\delta < 0$ and $\gamma = 0$. As opposed to the case with history effects, our next lemma shows that the order in which assortments are shown is irrelevant when there is only assortment effects.

LEMMA 3. *Consider any sequence of assortments $\mathcal{S} = \{S_1, \ldots, S_T\}$, and suppose that there is no history effect. Then, the value generated by the sequence is independent of the order in which these assortments are shown.*

Based on Lemma 3, we know that the expected number of matches only depends on the assortments and not in the period they are shown. Hence, we can re-write Problem (6) as

$$\max_{S^1,\ldots,S^T} \left\{ \sum_{t=1}^{T} \mathbb{E}\left[\mu_i(S^t) \mid S^t, \mathscr{B}_i\right] : S^1, \ldots, S^T \in \mathcal{S}_K(\mathscr{P}_i), S^t \cap S^{t'} = \emptyset \; \forall t, t' \in \mathcal{T},\; t \neq t' \right\}, \tag{7}$$

which is equivalent to the following integer program:

$$\begin{aligned} \max_{x(S): S \in \mathcal{S}_K(\mathscr{P}_i)} \quad & \sum_{S \in \mathcal{S}_K(\mathscr{P}_i)} x(S) \cdot \mathbb{E}\left[\mu_i(S) \mid S, \mathscr{B}_i\right] \\ st. \quad & \sum_{S \in \mathcal{S}_K(\mathscr{P}_i)} \mathbb{1}_{\{j \in S\}} \cdot x(S) \leq 1, \quad \forall j \in \mathscr{P}_i \\ & \sum_{S \in \mathcal{S}_K(\mathscr{P}_i)} x(S) \leq T, \\ & x(S) \in \{0,1\}, \quad \forall S \in \mathcal{S}_K(\mathscr{P}_i). \end{aligned} \tag{8}$$

The first set of constraints guarantees that no profile $j \in \mathscr{P}_i$ is shown as part of more than one assortment. The next constraint ensures that no more than T assortments are shown.

Our next result shows that this problem is NP-complete.

[33] Indeed, there is not a unique ordering of profiles according to their probabilities of generating a match, as the order may change depending on M. See Example 3 in Appendix C.1.

Proposition 3. *Problem (7) is NP-complete.*

A direct consequence of this result is that the problem with assortment and history effects is also NP-complete, and so is the problem that jointly finds the assortments to show to each user in each period.

Corollary 1. *Problems (5) and (6) are NP-complete.*

5.1.2. Upper Bound. Given a set of potentials $\mathscr{P}_i$, a backlog $\mathscr{B}_i$, M_i active matches and τ periods left, let $z_i^P(\mathscr{P}_i, \mathscr{B}_i, M_i, \tau)$ be the optimal solution of the linear problem:[34]

$$
\begin{aligned}
z_i^P(\mathscr{P}_i, \mathscr{B}_i, M_i, \tau) := \max_{x(S): S \in \mathcal{S}_K(\mathscr{P}_i)} \quad & \sum_{S \in \mathcal{S}_K(\mathscr{P}_i)} x(S) \cdot \mathbb{E}\left[\mu_i(M_i, S) \mid S, \mathscr{B}_i\right] \\
st. \quad & \sum_{S \in \mathcal{S}_K(\mathscr{P}_i)} \mathbb{1}_{\{j \in S\}} \cdot x(S) \leq 1, \quad \forall j \in \mathscr{P}_i \\
& \sum_{S \in \mathcal{S}_K(\mathscr{P}_i)} x(S) \leq \tau, \\
& x(S) \geq 0, \quad \forall S \in \mathcal{S}_K(\mathscr{P}_i).
\end{aligned} \tag{9}
$$

Notice that $z_i^P(\mathscr{P}_i, \mathscr{B}_i, 0, T)$ is equivalent to the linear relaxation of Problem (8). Hence, $z_i^P(\mathscr{P}_i, \mathscr{B}_i, 0, T)$ provides a simple upper bound for Problem (6), as formalized in the next proposition.

Proposition 4. *Let V_i^* be the optimal solution of Problem (6) given an initial set of potentials $\mathscr{P}$ and M active matches. Then,*

$$V_i^* \leq z_i^P(\mathscr{P}_i, \mathscr{B}_i, 0, T).$$

We can also use the solution of the linear relaxation of the individual level problems $z_i^P(\mathscr{P}_i, \mathscr{B}_i, 0, T)$ to construct an upper bound for the optimal solution of the market-level problem (5), V^*, as the next proposition shows.

Proposition 5. *Let V^* be the optimal solution of Problem (5) given the initial sets of potentials $\vec{\mathscr{P}}$, backlogs $\vec{\mathscr{B}}$ and active matches $\vec{M}$. Then,*

$$V^* \leq \sum_{i \in \mathcal{I} \cup \mathcal{J}} z_i^P(\mathscr{P}_i, \mathscr{B}_i, 0, T).$$

Proposition 5 is important because it serves as a building block for the heuristics we propose for the market-level problem, described in the next section. However, one caveat of this upper bound is that it requires maximizing over the set of all possible assortments that can be formed from $\mathscr{P}_i$. For instance, if $|\mathscr{P}_i| = 1000$, we would need to consider $\binom{1000}{K} = \binom{1000}{3} > 10^8$ feasible assortments,

[34] In a slight abuse of notation, we also use $z_i^P(\mathscr{P}_i, \mathscr{B}_i, M_i, \tau)$ to represent the linear program (9) that considers as inputs the set of potentials $\mathscr{P}_i$, the backlog $\mathscr{B}_i$, M_i active matches and τ periods left.

which makes the problem computationally hard to solve. However, a simpler upper bound can be obtained from the problem with no assortment nor history effects. Indeed, if $\delta = \gamma = 0$ we can decouple the problem within and across periods, and thus the problem reduces to a many-to-many matching with the objective of maximizing the number of matches. This is formalized in the next proposition.

COROLLARY 2. *Let $\vec{\mathscr{P}}$ be the initial sets of potentials, $\vec{\mathscr{B}}$ be the initial backlogs and $\vec{M}$ be the vector of initial active matches. In addition, let*

$$\begin{aligned} \bar{z}(\vec{\mathscr{P}}, \vec{\mathscr{B}}, \vec{M}) \quad := \quad \max_x \quad & \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} x_{ij} \beta_{ij} \\ st. \quad & \sum_{i \in \mathcal{I}} x_{ij} \leq m \cdot T, \ \forall j \in \mathcal{J} \\ & \sum_{j \in \mathcal{J}} x_{ij} \leq m \cdot T, \ \forall i \in \mathcal{I} \\ & 0 \leq x_{ij} \leq 1, \ \forall (i,j) \in \mathcal{I} \times \mathcal{J}, \end{aligned} \tag{10}$$

where $\beta_{ij} = \phi_{ij}(M_i, \{j\}) \cdot \phi_{ji}(M_j, \{i\})$ if $i \in \mathscr{P}_j$ and $j \in \mathscr{P}_i$, and 0 otherwise. Then,

$$V^* \leq \bar{z}(\vec{\mathscr{P}}, \vec{\mathscr{B}}, \vec{M}).$$

6. Heuristics

A common approach in approximate dynamic programming is to solve the *certainty equivalent problem*, where a deterministic optimal control problem is solved at each stage replacing the uncertain quantities with their expected values.[35] From Proposition 4 we know that Problem (9) provides an upper bound for the individual-level problem, so one potential approach is to solve $z^P(\mathscr{P}_i^t, \mathscr{B}_i^t, M_i^t, \tau)$ (Problem (9)) for each user i in each period t considering their current set of potentials $\mathscr{P}_i^t$ and active matches M_i^t, their current backlog to compute the estimated probabilities of being liked back (α_{ij}^t), and a look-ahead length τ which is at most the number of periods left, i.e., $\tau \leq T - t + 1$. An alternative approach is to extend (9) to solve the joint problem of finding the assortments to show to each user in the platform. Then, the optimal solution of these problems can be used to set the assortments to show in period t.

One of the major challenges to scale these approaches is the large number of assortments that can be created as the number of profiles in the set of potentials becomes larger. However, many of these profiles will not be shown in the assortments selected, and thus there is no need to consider all of them. Following this logic, one way to reduce the size of the problem is to consider only a subset of potentials for each user, and use this subset to build the assortments for the following

[35] See Bertsekas (2017) for details on approximate dynamic programming in finite horizon problems.

days. Apart from reducing the input size and the complexity of the problem, sampling may also help to alleviate congestion, which may result in more matches.

There are certainly many ways to choose the sample of potentials to consider for each user. However, a natural starting point is to take a random sample, i.e., for each user i we consider a random subset of size R, which we denote by $\tilde{\mathscr{P}}_i^t \subseteq \mathscr{P}_i^t$. Then, the assortment for user i is built based on the profiles in $\tilde{\mathscr{P}}_i^t$. To dynamically take into account new profiles added to the backlog, we sample the subset $\tilde{\mathscr{P}}_i^t$ in each period, and we complement it with the set of profiles in user i's backlog.

These ideas motivate the two heuristics we propose, which we call *ICEC* and *MCEC*, respectively.

ICEC. This heuristic solves, for each user $i \in \mathcal{I} \cup \mathcal{J}$ and each period $t \in \mathcal{T}$, the problem $z_i^P(\tilde{\mathscr{P}}_i^t, \mathscr{B}_i^t, M_i^t, \min\{\tau, T-t+1\})$, i.e.,

$$\begin{aligned}
\max_{x(S): S \in \mathcal{S}_K(\tilde{\mathscr{P}}_i^t)} \quad & \sum_{S \in \mathcal{S}_K(\tilde{\mathscr{P}}_i^t)} x(S) \cdot \beta_{ij}^t(M_i^t, S) \\
st. \quad & \sum_{S \in \mathcal{S}_K(\tilde{\mathscr{P}}_i^t)} \mathbb{1}_{\{j \in S\}} \cdot x(S) \leq 1, \quad \forall j \in \tilde{\mathscr{P}}_i^t \\
& \sum_{S \in \mathcal{S}_K(\tilde{\mathscr{P}}_i^t)} x(S) \leq \min\{\tau, T-t+1\}, \\
& x(S) \geq 0, \quad \forall S \in \mathcal{S}_K(\tilde{\mathscr{P}}_i^t),
\end{aligned} \tag{11}$$

where $\tilde{\mathscr{P}}_i^t$ is the subset of size R sampled from the set of potentials $\mathscr{P}_i^t$, and $\beta_{ij}^t(M_i^t, S)$ is the estimated probability of a match between users i and j in period t, i.e.,

$$\beta_{ij}^t(M_i^t, S) = \begin{cases} \phi_{ij}^t(M_i^t, S) & \text{if } j \in \mathscr{B}_i^t, \\ \phi_{ij}^t(M_i^t, S) \cdot \phi_{ji}(M_j^t, \{i\}) & \text{if } j \in \mathscr{P}_i^t \setminus \mathscr{B}_i^t, \\ 0 & \text{if } j \notin \mathscr{P}_i^t. \end{cases}$$

Let x_i^* be the optimal solution of Problem (11) for user i. Then, the ICEC heuristic chooses the assortment involving the lowest average utility among the assortments shown with positive probability. Formally,

$$S_i^t = \arg\min_S \left\{ \sum_{j \in S} u_{ij} \,:\, S \in \mathcal{S}_K(\tilde{\mathscr{P}}_i^t),\, x_i^*(S) > 0 \right\}, \text{ for each } i \in \mathcal{I} \cup \mathcal{J}. \tag{12}$$

The idea to choose the assortment leading to the lowest utility follows from Proposition 2, where we show that in some special cases it is optimal to show profiles in increasing order of utility.

MCEC. This policy extends Problem (11) to jointly find the assortments to show to each user in the platform. This can be formulated as:

$$
\begin{aligned}
z^P(\vec{\tilde{\mathscr{P}}}^t, \vec{\mathscr{B}}^t, \vec{M}^t, \min\{\tau, T-t+1\}) = \max_{x_i(S): i\in\mathcal{I}\cup\mathcal{J}, S\in\mathcal{S}_K(\tilde{\mathscr{P}}_i^t)} \quad & \sum_{i\in\mathcal{I}\cup\mathcal{J}} \sum_{S\in\mathcal{S}_K(\tilde{\mathscr{P}}_i^t)} \sum_{j\in S} x_i(S)\cdot\beta_{ij}^t(M_i^t, S) \\
st. \quad & \sum_{S\in\mathcal{S}_K(\tilde{\mathscr{P}}_i^t)} \mathbb{1}_{\{j\in S\}}\cdot x_i(S) \le 1, \quad \forall j\in\tilde{\mathscr{P}}_i^t,\ i\in\mathcal{I}\cup\mathcal{J} \\
& \sum_{S\in\mathcal{S}_K(\tilde{\mathscr{P}}_i^t)} x_i(S) \le \min\{\tau, T-t+1\}, \quad \forall i\in\mathcal{I}\cup\mathcal{J} \\
& x_i(S)\ge 0, \quad \forall S\in\mathcal{S}_K(\tilde{\mathscr{P}}_i^t),\ i\in\mathcal{I}\cup\mathcal{J}
\end{aligned} \tag{13}
$$

As before, for each user i we consider a subset of potentials $\tilde{\mathscr{P}}_i^t \subseteq \mathscr{P}_i^t$. The objective function captures the overall expected number of matches considering both the assortment and history effects. The first constraint guarantees that each user $j\in\tilde{\mathscr{P}}_i^t$ is shown in at most one assortment to each user i, while the second constraint ensures that each user i is shown at most τ assortments, where τ is the number of periods of look-ahead. As in the previous case, let x^* be the optimal solution of Problem (13). Then, the assortment to show to each user in period t is chosen according to the same rule as in the ICEC heuristic, i.e., it is given by Equation (12). Although ICEC and MCEC are equivalent in terms of the assortments shown, ICEC has the advantage that each user's problem is solved separately, and thus the procedure can be parallelized to reduce computational time.

6.1. Benchmarks

We compare the aforementioned heuristics with three relevant benchmarks.

Greedy. For each user, this policy chooses the assortment that maximizes the expected number of matches in a single period, without taking into account the assortment and history effects. Formally,

$$
S_i^t = \arg\max_{S\in\mathcal{S}_K(\mathscr{P}_i^t)} \left\{\sum_{j\in S}\phi_{ij}(0,\{j\})\cdot\tilde{\alpha}_{ij}^t\right\}, \text{ for each } i\in\mathcal{I}\cup\mathcal{J},
$$

where

$$
\tilde{\alpha}_{ij}^t = \begin{cases} 1 & \text{if } j\in\mathscr{B}_i^t, \\ \phi_{ji}(0,\{i\}) & \text{if } j\in\mathscr{P}_i^t\setminus\mathscr{B}_i^t, \\ 0 & \text{if } j\notin\mathscr{P}_i^t. \end{cases}
$$

Naive. For each user, this policy chooses the set of profiles that maximizes the expected number of likes, without taking into account the assortment and history effects, i.e.,

$$
S_i^t = \arg\max_{S\in\mathcal{S}_K(\mathscr{P}_i^t)} \left\{\sum_{j\in S}\phi_{ij}(0,\{j\})\right\}, \text{ for each } i\in\mathcal{I}\cup\mathcal{J}.
$$

Partner. We implemented the algorithm currently used by the platform.[36] Although it does not have all the details and special cases,[37] our implementation provides a good approximation according to our industry partner, and this is confirmed by the simulation results reported in the next section.

6.2. Simulation Procedure

Our simulation procedure is summarized in Algorithm 1.

Algorithm 1 Simulation Procedure

Input. A policy π. For each $i \in \mathcal{I} \cup \mathcal{J}$: set of potentials, utilities and qualities.

Output. Sequence of assortments $\{S_i^1, \ldots, S_i^T\}$ for each $i \in \mathcal{I} \cup \mathcal{J}$ and total number of matches.

Initialization. Set $t=1$, $M_i^t = 0$ for all $i \in \mathcal{I} \cup \mathcal{J}$.

Main. For $t = 1, \ldots, T$ do:

Step 1. Set $S^t \subseteq \mathscr{P}^t$ according to policy π.

Step 2. Simulate active users, evaluations, expirations and new matches.

Step 3. Update potentials, backlogs and active matches: for each $i \in \mathcal{I} \cup \mathcal{J}$,

$$\mathscr{B}_i^{t+1} = (\mathscr{B}_i^t \cup A_i^t) \setminus S_i^t,$$

$$\mathscr{P}_i^{t+1} = \mathscr{P}_i^t \setminus (S_i^t \cup R_i^t),$$

$$M_i^{t+1} = M_i^t + \mu_i^t - \xi_i^t.$$

In Step 1 we define the assortment for each user in period t according to the desired policy. Based on the assortments chosen for period t, in Step 2 we simulate agents' decisions considering as like probabilities $\phi_{ij}(M_i^t, S_i^t)$, i.e., we assume that the true like probabilities are those computed by our model. In addition, we compute the number of matches expired and the number of new matches generated for each user $i \in \mathcal{I} \cup \mathcal{J}$. Finally, in Step 3 we update the set of potentials, the backlogs, and the number of active matches for each user.

6.3. Simulation Results

To perform our simulations we focus on the same market described in Section 3.2. We restrict the analysis to heterosexual users that have no purchases or memberships and that evaluated at least one profile in the four weeks considered. For each user in this sample, we compute a set of potentials by taking the union of all profiles they saw between January 1st and September 11th, 2019 and the set of potentials that have not seen yet. Finally, to guarantee that users have enough potentials

[36] Due to our NDA agreement we cannot disclose the exact elements of this algorithm.

[37] For instance, we do not consider queries related to new users or to paid users.

to build their assortments in our simulations, we exclude users with less than 50 potentials within the sample.

Once the market is defined, we need estimates for the probabilities that users will like each other. We compute these probabilities using the estimates of the linear probability model with fixed effects separating by gender, reported in Table 14 in Appendix D. In case that there are no observations for some users,[38] we compute their fixed effect by extrapolating a linear model of the fixed effect value on the attractiveness score of the user.[39] The distributions of the predicted like probabilities (for women and men) are presented in Figure 7 in Appendix D.

6.3.1. Market-Level Results In Table 5 we provide summary statistics on the number of potentials, scores, and also on the like and match rates for the sample considered.

Table 5 Summary Statistics - Sample Market

		Descriptives		Performance	
Gender	N	Num. Potentials	Score	Like Rate	Match Rate
Women	1164	188.337 (108.952)	5.107 (2.071)	0.321 (0.250)	0.126 (0.174)
Men	939	233.465 (131.070)	2.623 (1.557)	0.532 (0.275)	0.081 (0.118)

Note: Summary statistics of the sample market considered. The left part of the table describes some observable characteristics of the sample used for simulations. The right part of the table reports summary statistics for the like and match rates for users in the sample during the four weeks of data.

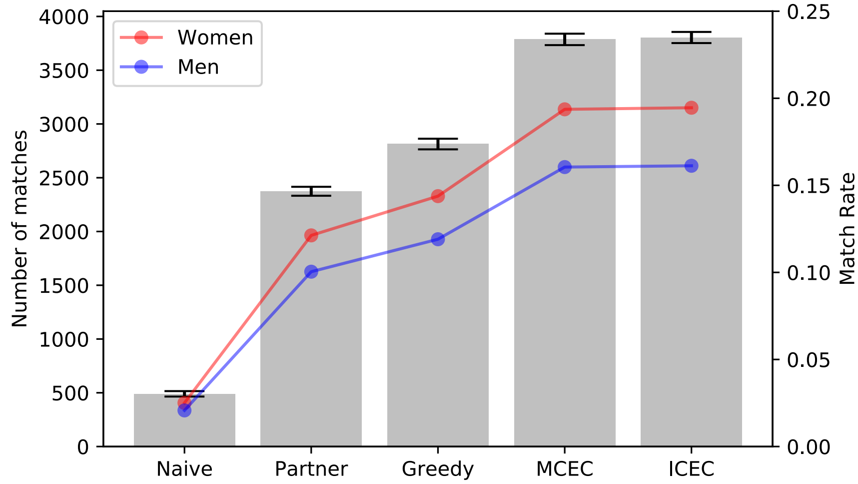
To perform our simulations, we consider an assortment size $K = 3$ and a time horizon of two weeks, i.e., $T = 14$. In addition, we assume that the values of the assortment and history effects are fixed and equal to $\delta = -0.005$ and $\gamma = -0.018$, respectively. We also assume that matches do not expire, and that men and women log-in each day with probability 0.6 and 0.4, respectively. Finally, for the ICEC and MCEC heuristics we consider a sampling size $R = 20$ and a look-ahead length $\tau = 2$.

In Figure 1 we report the results of 100 simulations. The bars represent the total number of matches generated by each policy (including error bars for one-standard deviations), while the lines depict the match rates for men (blue) and women (red).

[38] For instance, a user whose profile was added to the sample because it is in the set of potentials of another user may not have been observed in during the time window of four weeks, and thus we would not have an estimate for their fixed effect.

[39] Figure 6 in Appendix D show a scatter plot of the score and the fixed effect obtained from our linear probability model. In addition, it contains the fitted linear models by gender. The results of these regressions are reported in Table 15 in Appendix D. These results show a linear model on the score provides a good approximation for the estimated fixed effects.

Figure 1 Number of Matches and Match Rate - Market-Level



First, we observe that our implementation provides a good approximation for our partner's algorithm. As Table 5 shows, the average match rate observed in the data is 12.6% for women and 8.1% for men, while the average match rate in our simulations is 12.1% for women and 10.0% for men. Second, we observe that the performance of the ICEC and MCEC heuristics is relatively similar. As Table 6 shows, the total number of matches generated by ICEC is 3804.14, while MCEC produces 3786.56 matches on average. Finally, we observe that ICEC and MCEC considerably outperform the other heuristics considered. For instance, ICEC leads to an improvement of 60.23% relative to our industry partner's algorithm, and of 35.23% relative to the Greedy algorithm.

Table 6 Comparison Heuristics - Total Matches

		Naive	Partner	Greedy	MCEC	ICEC
Number of matches		490.17 (25.306)	2374.03 (41.601)	2813.33 (49.469)	3786.56 (53.156)	3804.14 (51.762)
Match Rate	Women	0.025 (0.001)	0.121 (0.002)	0.144 (0.002)	0.194 (0.002)	0.195 (0.002)
	Men	0.021 (0.001)	0.100 (0.002)	0.119 (0.002)	0.160 (0.002)	0.161 (0.002)

Note: Summary statistics for the number of matches and the match rates obtained by each policy. Standard errors reported in parenthesis.

6.3.2. Sensitivity Analysis In this section, we restrict the analysis to users with at least 100 potentials in the sample. In this way, we guarantee the number of users is not too large, reducing the computational time. In Table 7 we report summary statistics for the number of potentials and the scores of the users in the sample, and also for the like and match rates observed in the four weeks of data. Compared to Table 5, we observe that the number of users on each side of the market is reduced by a half, and that the match rates are slightly higher.

Table 7 Summary Statistics - Small Sample

Gender	N	Descriptives: Num. Potentials	Descriptives: Score	Performance: Like Rate	Performance: Match Rate
Women	590	183.129	5.155	0.331	0.135
		(74.341)	(2.116)	(0.209)	(0.154)
Men	520	207.781	2.795	0.533	0.095
		(72.987)	(1.598)	(0.255)	(0.117)

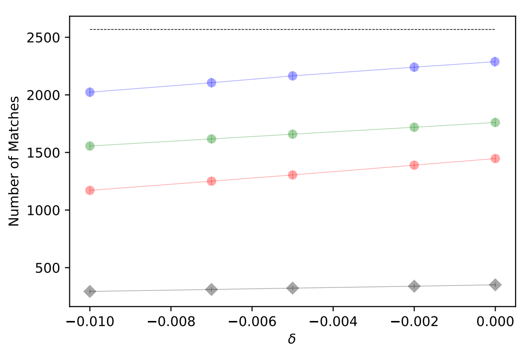
Note: Summary statistics of the sample considered. The left part of the table describes some observable characteristics of the sample used for simulations. The right part of the table reports summary statistics for the like and match rates for users in the sample during the four weeks of data.

As in the previous section, we assume that $K = 3$, $T = 14$, and we fix the parameters of the assortment and history effects to $\delta = -0.005$ and $\gamma = -0.018$. In addition, we keep the assumptions that matches do not expire $p_\xi = 0$, and that each user logs in with probability 0.6 for men and 0.4 for women in each period.

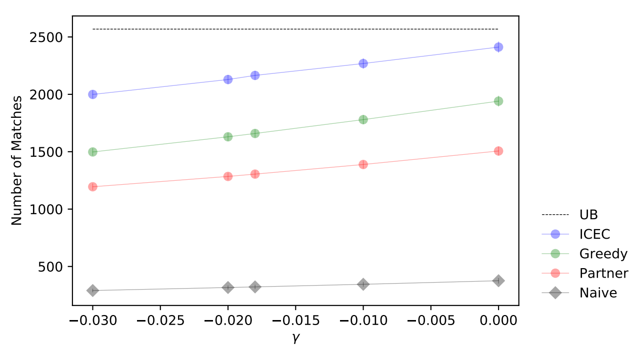
Sensitivity to Assortment and History Effects. In Figure 2 we report the average number of matches obtained for each policy varying the parameters for the assortment (δ) and history (γ) effects. Since MCEC and ICEC almost perfectly overlap, we only plot the results for the ICEC policy.

Figure 2 Sensitivity to Assortment and History Effects

(a) Assortment



(b) History



From Figure 2a we observe that the total number of matches increases as we decrease the magnitude of the assortment effect. This result was expected since the assortment effect reduces the probability that each profile is liked given a fixed assortment. Regarding the history effect, from Figure 2b we observe that the number of matches is strictly increasing in γ for all policies considered. This result is direct from the fact that previous matches can only decrease the chances of new profiles being liked. Finally, from both figures we observe that the results of the ICEC heuristic get relatively close to the upper bound.

Sensitivity to Parameters of Heuristic. In Table 8 we report summary statistics for the number of matches and match rates considering different values of the sampling size R and the look-ahead length τ.

Table 8 Sensitivity to Heuristic Parameters

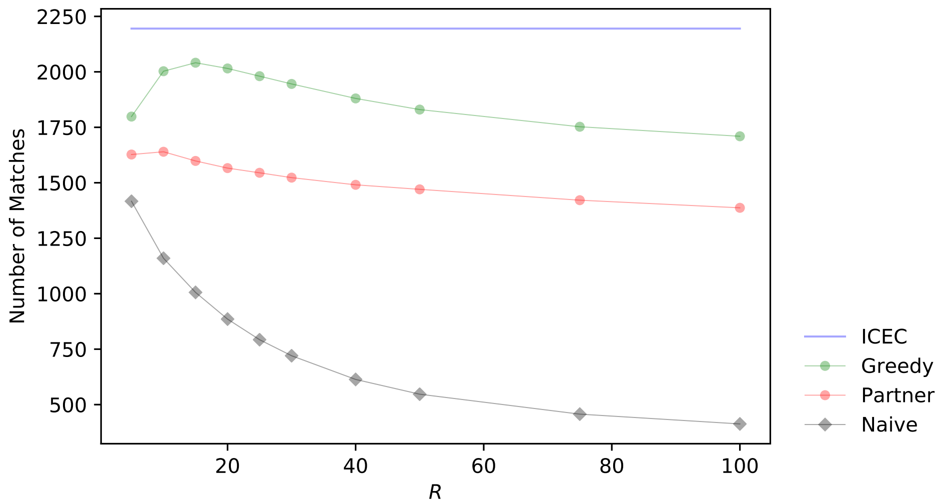
			MCEC			ICEC		
R			$\tau=1$	$\tau=2$	$\tau=3$	$\tau=1$	$\tau=2$	$\tau=3$
15	Number of Matches		2084.87 (37.412)	2108.42 (37.388)	1924.36 (38.252)	2088.5 (41.356)	2110.12 (39.285)	1930.66 (40.678)
	Match Rate	Women	0.21 (0.004)	0.213 (0.004)	0.194 (0.003)	0.211 (0.004)	0.213 (0.004)	0.195 (0.004)
		Men	0.16 (0.003)	0.161 (0.003)	0.147 (0.003)	0.16 (0.003)	0.162 (0.003)	0.148 (0.003)
20	Number of Matches		2084.12 (39.976)	2156.8 (42.263)	2036.31 (42.295)	2081.18 (41.9)	2164.97 (39.182)	2046.54 (38.61)
	Match Rate	Women	0.21 (0.004)	0.218 (0.004)	0.205 (0.004)	0.21 (0.004)	0.218 (0.004)	0.206 (0.004)
		Men	0.16 (0.003)	0.165 (0.003)	0.156 (0.003)	0.159 (0.003)	0.166 (0.003)	0.157 (0.003)
25	Number of Matches		2065.92 (39.744)	2185.25 (45.716)	2092.99 (44.733)	2057.66 (40.79)	2187.19 (41.267)	2101.76 (41.191)
	Match Rate	Women	0.208 (0.004)	0.22 (0.004)	0.211 (0.004)	0.208 (0.004)	0.221 (0.004)	0.212 (0.004)
		Men	0.158 (0.003)	0.167 (0.004)	0.16 (0.004)	0.158 (0.003)	0.167 (0.003)	0.161 (0.003)
30	Number of Matches		2040.98 (41.839)	2183.86 (44.153)	2120.2 (42.08)	2042.63 (41.26)	2191.91 (45.535)	2128.21 (42.622)
	Match Rate	Women	0.206 (0.004)	0.22 (0.004)	0.214 (0.004)	0.206 (0.004)	0.221 (0.004)	0.215 (0.004)
		Men	0.156 (0.003)	0.167 (0.004)	0.162 (0.004)	0.156 (0.003)	0.168 (0.004)	0.163 (0.003)

Note: Summary statistics for the number of matches and the match rate for the ICEC and MCEC policies over 100 simulations for different values of R and τ. Standard errors reported in parenthesis.

We observe that the performance of our heuristics is slightly better when we consider a look-ahead length $\tau=2$. In addition, we observe that the performance of our heuristics is increasing in R. For instance, comparing the results of ICEC with $R=15$ and $R=30$ (and $\tau=2$), we find that the match rates increase by 3.7% for both men and women.

6.3.3. Sampling in Benchmarks. One of the steps in the ICEC and MCEC heuristics is to sample a subset of potentials $\tilde{\mathscr{P}}_i$ that is later used to compute the assortments. As mentioned in Section 6, this may not only help to reduce the size of the problem, but it may also reduce congestion and potentially lead to more matches. Hence, sampling may be also useful for the other benchmarks considered.

Figure 3 **Number of Matches vs. Sampling Size - Benchmarks**



To test if sampling helps to improve the performance of our benchmarks, in Figure 3 we plot the number of matches obtained for different values of the sampling size R.[40] In addition, we include the number of matches generated by the ICEC policy when $R = 30$ and $\tau = 2$ as a reference.

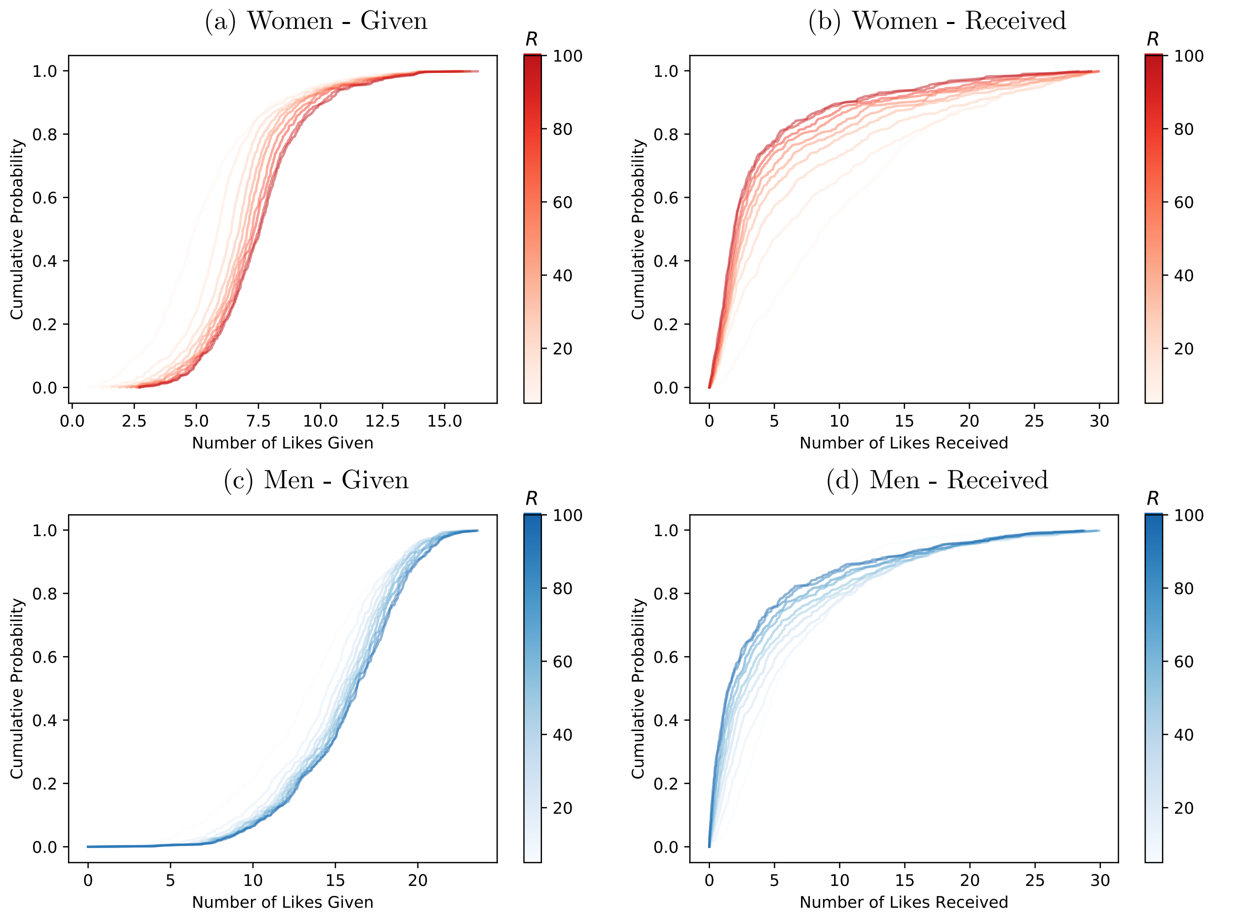
We observe that the number of matches generated by the Naive policy is strictly decreasing in R. This is not the case for neither our partner's algorithm nor for the Greedy policy. In the first case, we observe that the number of matches generated is slightly increasing in R when $R \leq 10$ (maximum of 1658.9 matches when $R = 10$), and then decreases for $R \geq 10$ (minimum of 1304.9 matches with no sampling). A similar pattern arises for the Greedy policy, as the number of matches is increasing in R when $R \leq 15$ and decreasing when $R \geq 15$. Overall, the maximum number of matches achieved by the Greedy policy is 2041.1 (when $R = 15$), while the minimum is equal to 1658.9 (with no sampling). Comparing the maximum number of matches generated by these benchmarks with that of the ICEC heuristic, we find that the overall improvement generated by the latter is 32.07% and 7.34% relative to our partner's algorithm and the Greedy with sampling, respectively.

To understand what is driving this result, in Figure 4 we plot the cumulative distribution of the average number of likes given and received by each user for different values of R under the Greedy policy, separating by gender.

We observe that two opposite effects take place as we increase the sampling size R. On the one hand, the distribution of the number of likes given by each user shifts towards the right. This is illustrated in Figures 4a and 4c. On the other hand, from Figures 4b and 4d we observe that the distribution of the number of likes received by each user shifts towards the left as we increase R. The reason for these effects is that, as R increases, the pool of profiles to create the assortments

[40] In Figure 8, Appendix D, we report a similar plot considering all the values of R, i.e., $R \in \{5, 10, 15, 20, 25, 30, 40, 50, 75, 100, 200, 500, 1000\}$.

Figure 4 Number of Likes Received and Given vs. Sample Size - Greedy



becomes larger, and thus higher quality profiles can be potentially added to the assortments. This implies that users will observe higher quality profiles, increasing the number of likes they give. However, this also implies that lower quality profiles will get less exposure, decreasing the number of likes they receive. These results are in line with the theoretical findings in Halaburda et al. (2018), where the authors show that increasing the number of potentials has a positive effect due to larger choice, but it also has a negative effect due to the higher competition between users on the same side of the market.

7. Conclusions

Motivated by the growing number of two-sided platforms that offer “curated” and personalized assortments, we study how users make decisions in these environments and how these platforms can influence users’ behavior through the options offered. Using data from an online dating platform, we identify two effects that have been mostly overlooked in previous literature: (i) the assortment effect, whereby the quality of the other alternatives in the assortment negatively affects the probability that each option is chosen; and (ii) the history effect, whereby the number of matches in the recent past reduces the probability that a user likes other profiles.

Using these findings, and taking into account the two-sidedness of the market, we introduce a stylized model of two-sided assortment optimization, where the platform takes into account the aforementioned effects and the probability that both users will mutually like each other when choosing the subsets of profiles to be shown to maximize the expected number of matches. We formulate this problem as a dynamic program, and we show that the problem is NP-complete, even in the case with a single user and no history effects. Given this result, we derive properties of the optimal solution for some particular cases, and we use these insights to propose heuristics to solve the problem in large markets. Our simulation results using data from a real-market show that our heuristics significantly outperform relevant benchmarks. In fact, our heuristics lead to an improvement of over 60% relative to the current algorithm used by our partner. This improvement is the results of considering the two-sidedness of the market, the assortment and history effects, and also the sampling procedure before choosing the assortments.

Managerial Implications. Our results provide valuable insights for platforms seeking to improve their search and recommendations systems. First, we show that taking into account the two-sidedness of the market and the behavior of users can lead to substantial improvement. Hence, platforms should focus on understanding what are the main drivers of users' behavior to make better recommendations. For instance, Airbnb realized that the preferences of hosts are also relevant when deciding which subset of listings to show, and by incorporating this into their search engine they were able to improve their booking conversion by almost 4%.[41] A second major finding is the existence of the assortment and history effects, which are present despite that users can like as many profiles as they want. Although in this paper we focus on two-sided markets, we think that these two effects may be also present in one-sided settings, so platforms such as e-commerce and food-ordering should take them into account when designing their recommendation systems.

Future work. We believe that there are many open directions for future research. One exciting direction is to analyze other potential objectives that the platform may consider, such as maximizing users' welfare or taking into account some fairness measures in the allocation of matches. In many cases, the number of matches may not be the best performance index, as there may be other goals that could help in other aspects such as retention and users' satisfaction. A second direction of future research is improving the scoring systems used by these platforms by taking into account the two-sidedness of the market. Recent literature has shown the existence of biases and distortions on rating systems (see Luca (2011), Besbes and Scarsini (2018)), and we believe that having a sound scoring system is critical to improve recommendations. Finally, it would be interesting to disentangle what is driving the assortment and history effects and provide micro-foundations that can translate to other settings.

[41] See Airbnb case on Medium.

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Appendix A: Summary of Notation

Table 9 Notation

Symbol	Description
$\mathcal{I}, \mathcal{J}, \mathcal{T}$	sides of the market and set of periods
I, J	number of agents on sides $\mathcal{I}, \mathcal{J}$
K	size of assortments to offer
T	number of periods
q_i	quality of agent i
Q	set of qualities
δ	weight of assortment effect
γ	weight of history effect
M_i^t	number of matches of user i in period t
S_i^t	assortment offered to user i in period t
$\mathscr{P}_i^t$	set of potentials of user i in period t
A_i^t	set of users that accepted user i in period t
R_i^t	set of users that rejected user i in period t
μ_i^t	set of users that matched with user i in period t
ξ_i^t	set of users whose match with user i expired in period t
$\mathscr{A}_i^t$	set of users that accepted user i before period t
$\mathscr{R}_i^t$	set of users that rejected user i before period t
$\mathcal{S}_K(\mathscr{P})$	set of assortments of size K that can be obtained from set of potentials $\mathscr{P}$
X_i, X_j	observable characteristics of users i and j
$U_i(j,S,M)$	IU that user i gets from profile j given assortment S and M matches
$U(X_i, X_j, \theta)$	part of IU that only depends on observable characteristics of users i and j
$A(j,S,\delta)$	part of IU that captures assortment effect
$H(j,M,\gamma)$	part of IU that captures history effect
$u_{ij}(M,S)$	deterministic part of IU that user i gets from profile j in assortment S with M current matches
ϵ_{ij}	idiosyncratic shock in IU of user i for profile j
$u_{i0} := \epsilon_{i0}$	outside option of user i
F_i, f_i	cdf and pdf of the difference in idiosyncratic shocks $\epsilon_{i0} - \epsilon_{ij}$
$\phi_{ij}(M,S)$	prob. that user i likes j given an assortment S and M current matches
α_{ij}^t	estimated prob. in period t that user i is liked back by j
$\beta_{ij}^t(M,S)$	estimated prob. in period t that user i will get matched with j
p_ξ	prob. that a match expires

Appendix B: Proofs

B.1. Proof of Proposition 1

First, since in every period a non-negative quantity is added, it is direct that V_i^t is non-increasing in t. Second, given two sets $\mathscr{P} \subseteq \mathscr{P}'$, solving the optimization problem considering $\mathscr{P}$ is equivalent to solving the problem with $\mathscr{P}'$ adding the constraint that $S \subseteq \mathscr{P}$ in every period. Then, since $V_i^t(\mathscr{P}', M)$ is a relaxation of this problem, it is direct that $V_i^t(\mathscr{P}, M) \leq V_i^t(\mathscr{P}', M)$. Next, we know that $\mathrm{P}\left(\mu_i^t \leq m \mid M, S\right) \leq \mathrm{P}\left(\mu_i^t \leq m \mid M+1, S\right)$ for all t, m, S and M, since by assumption $H(M,\gamma)$ is non-increasing in M and therefore $\phi_{ij}(M,S)$ is non-increasing in M for all S. Hence, it is direct that the expected number of matches generated by the former is greater than that generated by the latter, and thus we conclude that $V_i^t(\mathscr{P}, M)$ is non-increasing in M.

To see that $V_i^t(\mathscr{P}, M)$ is neither concave nor convex, consider the last period $t = T$ and suppose for simplicity that $H(M,\gamma) = \gamma \cdot M$ and that all profiles that are left like user i with probability 1, i.e., $\alpha_{ij}^T = 1$, $\forall j \in \mathscr{P}_i^T$. Since we are in the last period, we know that it is optimal to select the K profiles with the highest probability of generating a match, i.e.

$$V_i^T(\mathscr{P}, M) = \sum_{k=1}^{m} \phi_{ij}(M,S) = \sum_{k=1}^{K} F_i(u_{ij}(M,S)).$$

Then,

$$\frac{\partial V_i^T(\mathscr{P}, M)}{\partial M} = \sum_{k=1}^{m} f_i(u_{ij}(M,S)) \cdot \frac{\partial u_{ij}(M,S)}{\partial M} = \sum_{k=1}^{m} f_i(u_{ij}(M,S)) \cdot \gamma,$$

and

$$\frac{\partial^2 V_i^T(\mathscr{P}, M)}{\partial M^2} = \sum_{k=1}^{m} f_i'(u_{ij}(M,S)) \frac{\partial u_{ij}(M,S)}{\partial M} = \sum_{k=1}^{m} f_i'(u_{ij}(M,S)) \cdot \gamma^2.$$

The sign of the latter equation depends on the values of $u_{ij}(M,S)$ and the distribution F_i. For instance, if F_i is the standard normal distribution and $u_{ij}(M,S) > 0$ for all $j \in S$, then $\frac{\partial^2 V_i^T(\mathscr{P},M)}{\partial M^2} < 0$, i.e., the function is concave. In contrast, if $u_{ij}(M,S) < 0$ for all $j \in S$, then $V_i^T(\mathscr{P}, M)$ is convex.

B.2. Proof of Lemma 1

If there is no assortment nor history effects, then the assortments are independent over time and that there is no interaction among the profiles chosen. Hence, Problem (6) is equivalent to solve

$$\max_{S^1,\ldots,S^T} \left\{ \sum_{t=1}^{T} \sum_{j \in S} \beta_{ij}(0,\{j\}) \,:\, S^1,\ldots,S^T \in \mathcal{S}_K(\mathscr{P}_i),\, S^t \cap S^{t'} = \emptyset,\, \forall t,t' \in \mathcal{T}, t \neq t' \right\},$$

which is equivalent to

$$\max_{S} \left\{ \sum_{j \in S} \beta_{ij}(0,\{j\}) \,:\, S \subseteq \mathscr{P}_i,\, |S| = K \cdot T \right\}.$$

Then, it is clear that the optimal solution is to choose the subset of $K \cdot T$ profiles with the maximum values of $\beta_{ij}(0,\{j\})$, which is equivalent to what a greedy policy would do.

B.3. Proof of Proposition 2

We start showing a couple of results that will be used in the proof of Proposition 2.

LEMMA 4. *Suppose that a distribution F (with density f) satisfies the MLRP. Then, given a constant $c > 0$, the function*

$$\rho(x) = \frac{F(x)}{F(x-c)}$$

is non-increasing in x.

Proof: Taking derivative of $\rho(x)$ with respect to x,

$$\rho'(x) = \frac{d\rho(x)}{dx} = \frac{f(x) \cdot F(x-c) - F(x) \cdot f(x-c)}{F(x-c)^2}.$$

Since f satisfies MLRP, we know that for $\theta_1 > \theta_0$

$$\frac{F_{\theta_1}}{F_{\theta_0}}(x) \le \frac{f_{\theta_1}}{f_{\theta_0}}(x)$$

and therefore

$$f_{\theta_0}(x) \cdot F_{\theta_1}(x) - f_{\theta_1}(x) \cdot F_{\theta_0}(x) \le 0.$$

Notice that the numerator of $\rho'(x)$ is equivalent to this considering $\theta_0 = 0$ and $\theta_1 = c$. Therefore, the numerator of $\rho'(x)$ is non-positive, and thus we conclude that $\rho'(x) \le 0$. □

A direct corollary of this is the following:

COROLLARY 3. *Suppose that Assumptions 1 and 2 hold, and that there is no assortment effect, i.e., $\delta = 0$. Let $\rho_{ij}(M) = \phi_{ij}(M)/\phi_{ij}(M+1)$, and suppose that $u_{ij_1} \ge u_{ij_2} \ge \ldots \ge u_{ij_J}$. Then,*

$$\rho_{ij_1}(M) \le \rho_{ij_2}(M) \le \ldots \le \rho_{ij_J}(M), \; \forall M.$$

Proof: This is direct recalling that $\phi_{ij}(M) = F(u_{ij} + \gamma \cdot h(M))$, $\gamma \le 0$ and Lemma 4.

LEMMA 5. *Consider $K = 1$. Then, the function $g(m) = m + V_i^t(\mathscr{P}, M+m)$ is non-decreasing in m for all t, $\mathscr{P}$ and M.*

Proof: We proceed by induction. The base case when $t = T$ is direct, since

$$m + 1 + V_i^{T+1}(\mathscr{P}, M+m+1) = m+1 > m = m + V_i^{T+1}(\mathscr{P}, M+m)$$

and therefore our claim holds in the last period. Next, suppose that our claim holds in period $t+1$, i.e.

$$m + 1 + V_i^{t+1}(\mathscr{P}, M+m+1) \ge m + V_i^{t+1}(\mathscr{P}, M+m) \Leftrightarrow V_i^{t+1}(\mathscr{P}, M+m+1) - V_i^{t+1}(\mathscr{P}, M+m) + 1 \ge 0.$$

We want to show that this holds also holds in period t. Let $V_i^t(\mathscr{P}, M \mid S)$ be the continuation value if assortment S is shown in period t and then the DP is solved for all subsequent periods, i.e.,

$$V_i^t(\mathscr{P}, M \mid S) = \sum_{l=0}^{K} \left[l + V_i^{t+1}\left(\mathscr{P} \setminus S, M+l\right)\right] \cdot \mathrm{P}\left(\mu_i^t = l \mid M, S\right),$$

and let $S^* = \{j^*\}$ be the optimal assortment to show given the set of potentials $\mathscr{P}$ and that user i has $M' = M + m$ active matches. Then

$$
\begin{aligned}
& V_i^t(\mathscr{P}, M'+1) - V_i^t(\mathscr{P}, M') + 1 \\
& \geq V_i^t(\mathscr{P}, M'+1 \mid S^*) - V_i^t(\mathscr{P}, M' \mid S^*) + 1 \\
& = \beta_{ij^*}(M'+1) + \beta_{ij^*}(M'+1) \cdot V_i^{t+1}(\mathscr{P} \setminus S^*, M'+2) + (1 - \beta_{ij^*}(M'+1)) \cdot V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1) \\
& - \beta_{ij^*}(M') - \beta_{ij^*}(M') \cdot V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1) - (1 - \beta_{ij^*}(M')) \cdot V_i^{t+1}(\mathscr{P} \setminus S^*, M') + 1 \\
& = 1 + \beta_{ij^*}(M'+1) - \beta_{ij^*}(M') + V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1) - V_i^{t+1}(\mathscr{P} \setminus S^*, M') \\
& + \beta_{ij^*}(M'+1) \cdot \left[V_i^{t+1}(\mathscr{P} \setminus S^*, M'+2) - V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1)\right] \\
& - \beta_{ij^*}(M') \cdot \left[V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1) - V_i^{t+1}(\mathscr{P} \setminus S^*, M')\right] \\
& = 1 + V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1) - V_i^{t+1}(\mathscr{P} \setminus S^*, M') \\
& + \beta_{ij^*}(M'+1) \cdot \left[1 + V_i^{t+1}(\mathscr{P} \setminus S^*, M'+2) - V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1)\right] \\
& - \beta_{ij^*}(M') \cdot \left[1 + V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1) - V_i^{t+1}(\mathscr{P} \setminus S^*, M')\right] \\
& = (1 - \beta_{ij^*}(M')) \cdot \left[1 + V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1) - V_i^{t+1}(\mathscr{P} \setminus S^*, M')\right] \\
& + \beta_{ij^*}(M'+1) \cdot \left[1 + V_i^{t+1}(\mathscr{P} \setminus S^*, M'+2) - V_i^{t+1}(\mathscr{P} \setminus S^*, M'+1)\right] \\
& \geq 0
\end{aligned}
$$

The first inequality follows because S^* is a feasible solution but may not be the maximizer of $V_i^t(\mathscr{P}, M'+1)$, and therefore $V_i^t(\mathscr{P}, M'+1) \geq V_i^t(\mathscr{P}, M'+1 \mid S^*)$, while $V_i^t(\mathscr{P}, M') = V_i^t(\mathscr{P}, M' \mid S^*)$ by optimality of S^*. The last inequality follows from the induction hypothesis and the fact that $\beta_{ij}(M) \in [0,1]$ for all $j \in \mathcal{J}$. □

B.3.1. Proof of Proposition 2. Now we proceed to the proof of Proposition 2. We argue by induction on the size of the assortment K.

Base Case. We start with the case where $K = 1$, i.e., only one profile is shown each period. Suppose that there is a fixed subset of $K \cdot T$ profiles, $\tilde{\mathcal{J}} \subseteq \mathcal{J}$, that must be displayed. Consider two possible sequences of assortments obtained from $\tilde{\mathcal{J}}$:

$$
\mathcal{S} = \{\{j_1\}, \ldots, \{j_{\tau-1}\}, \{j\}, \{k\}, \{j_{\tau+2}\}, \ldots, \{j_T\}\}
$$

$$
\mathcal{S}' = \{\{j_1\}, \ldots, \{j_{\tau-1}\}, \{k\}, \{j\}, \{j_{\tau+2}\}, \ldots, \{j_T\}\}
$$

i.e. $\mathcal{S}'$ is obtained by interchanging $\{j\}$ and $\{k\}$ in $\mathcal{S}$. Since the assortments shown in the first $\tau - 1$ periods are the same, the distribution of the number of matches at the beginning of period τ, M^τ, is the same for both sequences. Hence, we only need to compare the number of matches generated by these sequences starting from period τ onwards. Let $\mathcal{S}_t$ be the subsequence of $\mathcal{S}$ starting from period t onwards, i.e., given a sequence $\mathcal{S} = \{S^1, \ldots, S^T\}$, then $\mathcal{S}_t = \{S^t, \ldots, S^T\}$. Also, let $M^{\tau,T}(M, \mathcal{S}_\tau)$ be the number of matches generated between periods τ and T by the sequence $\mathcal{S}_\tau$ given that $M^\tau = M$. Then,

$$
\begin{aligned}
\mathbb{E}_{M^\tau, M^{\tau,T}}\left[M^{\tau,T}(M^\tau, \mathcal{S}_\tau)\right] &= \mathbb{E}_{M^\tau}\left[\beta_{ij}(M^\tau) + \beta_{ij}(M^\tau) \cdot \beta_{ik}(M^\tau + 1) + (1 - \beta_{ij}(M^\tau)) \cdot \beta_{ik}(M^\tau)\right] \\
&+ \mathbb{E}_{M^\tau}\left[(1 - \beta_{ij}(M^\tau)) \cdot (1 - \beta_{ik}(M^\tau)) \cdot \mathbb{E}_{M^{\tau+2,T}}\left[M^{\tau+2,T}(M^\tau, \mathcal{S}_{\tau+2})\right]\right] \\
&+ \mathbb{E}_{M^\tau}\left[(1 - \beta_{ij}(M^\tau)) \cdot \beta_{ik}(M^\tau) \cdot \mathbb{E}_{M^{\tau+2,T}}\left[M^{\tau+2,T}(M^\tau + 1, \mathcal{S}_{\tau+2})\right]\right] \\
&+ \mathbb{E}_{M^\tau}\left[\beta_{ij}(M^\tau) \cdot (1 - \beta_{ik}(M^\tau + 1)) \cdot \mathbb{E}_{M^{\tau+2,T}}\left[M^{\tau+2,T}(M^\tau + 1, \mathcal{S}_{\tau+2})\right]\right] \\
&+ \mathbb{E}_{M^\tau}\left[\beta_{ij}(M^\tau) \cdot \beta_{ik}(M^\tau + 1) \cdot \mathbb{E}_{M^{\tau+2,T}}\left[M^{\tau+2,T}(M^\tau + 2, \mathcal{S}_{\tau+2})\right]\right].
\end{aligned}
$$

Similarly,

$$\begin{aligned}\mathbb{E}_{M^\tau,M^{\tau,T}}\left[M^{\tau,T}(M^\tau,\mathcal{S}'_\tau)\right] &= \mathbb{E}_{M^\tau}\left[\beta_{ik}(M^\tau)+\beta_{ik}(M^\tau)\cdot\beta_{ij}(M^\tau+1)+(1-\beta_{ik}(M^\tau))\cdot\beta_{ij}(M^\tau)\right]\\ &+\mathbb{E}_{M^\tau}\left[(1-\beta_{ik}(M^\tau))\cdot(1-\beta_{ij}(M^\tau))\cdot\mathbb{E}_{M^{\tau+2,T}}\left[M^{\tau+2,T}(M^\tau,\mathcal{S}'_{\tau+2})\right]\right]\\ &+\mathbb{E}_{M^\tau}\left[(1-\beta_{ik}(M^\tau))\cdot\beta_{ij}(M^\tau)\cdot\mathbb{E}_{M^{\tau+2,T}}\left[M^{\tau+2,T}(M^\tau+1,\mathcal{S}'_{\tau+2})\right]\right]\\ &+\mathbb{E}_{M^\tau}\left[\beta_{ik}(M^\tau)\cdot(1-\beta_{ij}(M^\tau+1))\cdot\mathbb{E}_{M^{\tau+2,T}}\left[M^{\tau+2,T}(M^\tau+1,\mathcal{S}'_{\tau+2})\right]\right]\\ &+\mathbb{E}_{M^\tau}\left[\beta_{ik}(M^\tau)\cdot\beta_{ij}(M^\tau+1)\cdot\mathbb{E}_{M^{\tau+2,T}}\left[M^{\tau+2,T}(M^\tau+2,\mathcal{S}'_{\tau+2})\right]\right].\end{aligned}$$

Then, after some algebra we know that

$$\begin{aligned}&\mathbb{E}_{M^\tau,M^{\tau,T}}\left[M^{\tau,T}(M^\tau,\mathcal{S}_\tau)\right]-\mathbb{E}_{M^\tau,M^{\tau,T}}\left[M^{\tau,T}(M^\tau,\mathcal{S}'_\tau)\right]\\ &=\mathbb{E}_{M^\tau}\left[(\beta_{ij}(M^\tau)\cdot\beta_{ik}(M^\tau+1)-\beta_{ik}(M^\tau)\cdot\beta_{ij}(M^\tau+1))\cdot(1-\mathbb{E}_{M^{\tau+2,T}}\left[M_1(M^\tau)\right]+\mathbb{E}_{M^{\tau+2,T}}\left[M_2(M^\tau)\right])\right]\end{aligned} \tag{14}$$

where $M_1(M^\tau)=M^{\tau+2,T}(M^\tau+1,\mathcal{S})$ and $M_2(M^\tau)=M^{\tau+2,T}(M^\tau+2,\mathcal{S})$. Notice that this holds because the sequences $\mathcal{S}$ and $\mathcal{S}'$ are the same starting from period $\tau+2$ onwards, and therefore $M^{\tau+2,T}(\mathcal{S},M)=M^{\tau+2,T}(\mathcal{S}',M)$ for all M.

From Lemma 5 we know that $g(M)=M+V_i^t(\mathscr{P},M)$ is weakly increasing in M. Hence,

$$g(M+1)-g(M)=1-V_i^t(\mathscr{P},M)+V_i^t(\mathscr{P},M+1)\geq 0$$

which directly implies that $(1-\mathbb{E}[M_1(M^\tau)]+\mathbb{E}[M_2(M^\tau)])\geq 0$ for all M^τ. In addition, Corollary 3 implies that the sign of

$$\beta_{ij}(M^\tau)\cdot\beta_{ik}(M^\tau+1)-\beta_{ik}(M^\tau)\cdot\beta_{ij}(M^\tau+1)$$

does not change with M^τ. Hence, for Equation (14) to be non-negative we must have that

$$\frac{\beta_{ij}(M)}{\beta_{ij}(M+1)}\geq\frac{\beta_{ik}(M)}{\beta_{ik}(M+1)},$$

which holds for all M (by Corollary 3). Recalling that $\beta_{ij}(M)=\phi_{ij}(M)\cdot\alpha_{ij}$, this becomes

$$\frac{\phi_{ij}(M)}{\phi_{ij}(M+1)}\geq\frac{\phi_{ik}(M)}{\phi_{ik}(M+1)}.$$

This result implies that it is optimal to show profiles in decreasing order of $\rho_{ij}(M)$, which is equivalent to show profiles in increasing order of u_{ij} by Corollary 3.

Inductive Step. It is direct that this result also holds for $m>1$, recalling that the order of $\rho_{ij}(M)$ does not change with M. Hence, in a given period, it is optimal to show the same subset of K, regardless of the realized number of matches in previous periods. □

B.4. Proof of Lemma 2

From Lemma 1 we know that if there are no assortment nor history effects, then it is optimal to greedily display the profiles with the highest probability of generating a match. Having no history is equivalent to assume that $M_i^t=0$ for all $t\in\mathcal{T}$. The proof of Lemma 1 can be easily extended to account for the case where $M_i^t=M$ for all $t\in\mathcal{T}$ for any fixed M. Hence, since there is no assortment effect, it is optimal to show the subset of $K\cdot T$ profiles with the highest values of $\beta_{ij}(M,\{j\})$, i.e., $\mathscr{P}(M)$. Finally, since any path of realized matches has $M_i^t\in[0,K\cdot(t-1)]$ for all $t\in\mathcal{T}$, we conclude that $\mathcal{J}^*\subseteq\bigcup_{M=0}^{K\cdot(T-1)}\mathscr{P}(M)$.

To show the second part, the following lemma is useful.

Lemma 6. *Suppose that Assumptions 1 and 2 holds. Then,* $\underline{u}(M)$ *is non-decreasing in* M*.*

Proof: To find a contradiction, suppose there exists $\tilde{M}$ such that $\underline{u}(\tilde{M}) > \underline{u}(\tilde{M}+1)$. Then, we know $\exists j \in \mathscr{P}(\tilde{M})$, $k \in \mathscr{P}(\tilde{M}+1)$ such that $j,k \notin \mathscr{P}(\tilde{M}) \cap \mathscr{P}(\tilde{M}+1)$ and $u_{ij} > u_{ik}$. Since $j \in \mathscr{P}(\tilde{M})$ and $k \in \mathscr{P}(\tilde{M}+1)$, we know that $\beta_{ij}(\tilde{M}) > \beta_{ik}(\tilde{M})$ and $\beta_{ij}(\tilde{M}+1) < \beta_{ik}(\tilde{M}+1)$. We can re-write the first inequality as

$$\beta_{ij}(\tilde{M}) > \beta_{ik}(\tilde{M}) \Leftrightarrow F_i(u_{ij} + \gamma \cdot h(\tilde{M})) \cdot \alpha_{ij} > F_i(u_{ik} + \gamma \cdot h(\tilde{M})) \cdot \alpha_{ik} \Leftrightarrow \frac{F_i(u_{ij} + \gamma \cdot h(\tilde{M}))}{F_i(u_{ik} + \gamma \cdot h(\tilde{M}))} > \frac{\alpha_{ik}}{\alpha_{ij}}.$$

From Lemma 4 we know that the function

$$\rho(M) = \frac{F_i(u_{ij} + \gamma \cdot h(M))}{F_i(u_{ij} + \gamma \cdot h(M) - (u_{ij} - u_{ik}))} = \frac{F_i(u_{ij} + \gamma \cdot h(M))}{F_i(u_{ik} + \gamma \cdot h(M))}$$

is non-decreasing in M as $\gamma \leq 0$ and $u_{ij} - u_{ik} > 0$. Therefore, we have that

$$\frac{F_i(u_{ij} + \gamma \cdot h(\tilde{M}+1))}{F_i(u_{ik} + \gamma \cdot h(\tilde{M}+1))} > \frac{\alpha_{ik}}{\alpha_{ij}}$$

which is equivalent to

$$\beta_{ij}(\tilde{M}+1) > \beta_{ik}(\tilde{M}+1),$$

which leads to a contradiction. □

So, from Lemma 6 we know that $\underline{u}(M)$ is non-decreasing in M. Hence, we know that profiles $j \notin \mathscr{P}(0)$ such that $u_{ij} < \underline{u}(0)$ will never be part of the optimal subset $\mathcal{J}^*$, as their rank will only decrease as we increase M. Similarly, we know that the total number of matches before the last period of the horizon cannot exceed $K \cdot (T-1)$. Then, we know that $j \notin \mathscr{P}(K \cdot (T-1))$ with $u_{ij} > \bar{u}(M)$ will never be part of of the top $K \cdot T$ profiles for an attainable M, and thus will never be part of $\mathcal{J}^*$.

B.5. Proof of Lemma 3

Consider a sequence of assortments $\mathcal{S} = \{S^1, \ldots, S^T\}$. As any permutation of $\mathcal{S}$ can be achieved through a finite number of pairwise swaps, it is enough to show that the value obtained from this sequence remains the same when we swap two adjacent assortments in the sequence. In particular, let $\mathcal{S}' = \{S^1, \ldots, S^{t-1}, S^{t+1}, S^t, \ldots, S^T\}$ be the sequence of assortments that starts from $\mathcal{S}$ and swaps assortments S^t and S^{t+1}. Since there is no history effect—i.e., $\gamma = 0$—we can drop M from the argument of the value function and simply write $V_i^t(\mathscr{P}, M) = V_i^t(\mathscr{P})$. Given any sequence of T assortments $\mathcal{S} = \{S^1, \ldots, S^T\}$, let $\mathcal{S}_t$ be the sub-sequence starting from period t onwards, i.e., $\mathcal{S}_t = \{S^t, \ldots, S^T\}$. In addition, let $V_i^t(\mathscr{P} \mid \mathcal{S}_t)$ be the value to go function starting from period t onwards for a fixed sequence of assortments $\mathcal{S}_t = \{S^t, \ldots, S^T\}$. Finally, to keep the notation simpler, let $\tilde{\mu}_i^t = \mu_i(S^t)$ for all $t \in \mathcal{T}$. Then,

$$
\begin{aligned}
&V_i(\mathscr{P} \mid \mathcal{S}) \\
&= V_i^1(\mathscr{P} \mid \mathcal{S}_1) \\
&= \sum_{m_1=0}^{K} \left[m_1 + V_i^2(\mathscr{P} \setminus S^1 \mid \mathcal{S}_2)\right] \mathrm{P}\left(\tilde{\mu}_i^1 = m_1\right) \\
&= \sum_{m_1=0}^{K} \left[m_1 + \sum_{m_2=0}^{m} \left[m_2 + V_i^3(\mathscr{P} \setminus \bigcup_{\tau=1}^{2} S^\tau \mid \mathcal{S}_3)\right] \mathrm{P}\left(\tilde{\mu}_i^2 = m_2\right)\right] \mathrm{P}\left(\tilde{\mu}_i^1 = m_1\right) \\
&= \vdots \\
&= \sum_{m_1=0}^{K} \cdots \sum_{m_t=0}^{K} \sum_{m_{t+1}=0}^{K} \left[m_1 + \left[\cdots + \left[m_t + \left[m_{t+1} + V_i^{t+2}(\mathscr{P} \setminus \bigcup_{\tau=1}^{t+1} S^\tau \mid \mathcal{S}_{t+2})\right] \mathrm{P}\left(\tilde{\mu}_i^{t+1}) = m_{t+1}\right)\right] \mathrm{P}\left(\tilde{\mu}_i^t = m_t\right)\right] \cdots\right] \mathrm{P}\left(\tilde{\mu}_i^1 = m_1\right) \\
&= \sum_{m_1=0}^{K} \cdots \sum_{m_t=0}^{K} \sum_{m_{t+1}=0}^{K} \left[m_1 + \cdots + m_t + m_{t+1} + V_i^{t+2}(\mathscr{P} \setminus \bigcup_{\tau=1}^{t+1} S^\tau \mid \mathcal{S}_{t+2})\right] \prod_{\tau=1}^{t+1} \mathrm{P}\left(\tilde{\mu}_i^\tau = m_\tau\right) \\
&= V(\mathscr{P} \mid \mathcal{S}')
\end{aligned}
$$

where the last equality comes form the commutative properties of the sum and the product.

B.6. Proof of Proposition 3

Consider the decision problem of whether there exists a sequence of assortments $\mathcal{S} = \{S^1, \ldots, S^T\}$ such that the expected number of matches is at least W. First, it is easy to see that this problem is in NP, since it only requires to evaluate T expectations (one for each S^t) and this can be done in polynomial time. To show that the problem is NP-hard, we show that a special case of it reduces to a known NP-hard problem. First, notice that Problem (7) can be re-written as:

$$
\max_{S^1,\ldots,S^T} \left\{ \sum_{t=1}^{T} \sum_{j \in S} \beta_{ij}(S^t) \,:\, S^1, \ldots, S^T \in \mathcal{S}_K(\mathscr{P}_i),\ S^t \cap S^{t'} = \emptyset,\ \forall t, t' \in \mathcal{T}, t \neq t' \right\}.
$$

Consider the special case where $T = 1$. Then, our problem becomes:

$$
\max_{S} \left\{ \sum_{j \in S} \beta_j(S) \,:\, S \in \mathcal{S}_K(\mathscr{P}) \right\}. \tag{15}
$$

where we removed the dependence on i to facilitate notation.

We claim that our problem reduces to the maximum weight K-clique problem, which is known to be NP-hard (see Wu and Hao (2015)). To see this, consider an instance of the maximum weight K-clique problem, i.e. a graph $G = (V, E)$ and weights $\{w(v)\}_{v \in V}$. Given a constant W, the decision problem is whether there exists an K-clique of weight at least W in G. Now we show that we can start from this instance and construct an instance of Problem (15) in polynomial time. Let $S \subseteq V$ be a subset of nodes, and let $G_S = (S, E_S)$ the induced subgraph, and let $cl_K(G) = 1$ if graph G is an K-clique, 0 otherwise. Define $u_j = 0$, $\alpha_j = 1$ for all $j \in V$, $\delta = 1$ and

$$
a(j, S) = \begin{cases} 0 & \text{if } j \notin S \text{ or } cl_K(G_S) = 0 \\ F^{-1}\left(\frac{\sum_{k \in S} w(k)}{K}\right) & \text{otherwise} \end{cases}.
$$

Therefore, since $\beta_j(S) = \phi_j(S) \cdot \alpha_j = F(u_j + a(j, S))$, we have that

$$
\beta_j(S) = \begin{cases} 0 & \text{if } j \notin S \text{ or } cl_K(G_S) = 0, \\ \frac{\sum_{k \in S} w(k)}{K} & \text{if } j \in S \text{ and } cl_K(G_S) = 1. \end{cases}
$$

Notice that if $cl_K(G_S)=1$, then $S\in\mathcal{S}_K(V)$. Then,

$$\begin{aligned}\max_S\left\{\sum_{k\in S}w(k)\,:\,S\subseteq V,\,cl_K(G_S)=1\right\}&=\max_S\left\{\sum_{j\in S}\sum_{k\in S}\frac{w(k)}{K}\,:\,S\subseteq V,\,cl_K(G_S)=1\right\}\\&=\max_S\left\{\sum_{j\in S}\beta_j(S)\,:\,S\subseteq V,\,cl_K(G_S)=1\right\}\\&=\max_S\left\{\sum_{j\in S}\beta_j(S)\,:\,S\in\mathcal{S}_K(V)\right\}\\&=\max_S\left\{\sum_{j\in S}\beta_j(S)\,:\,S\in\mathcal{S}_K(\mathscr{P})\right\}\end{aligned}$$

Hence, maximizing the weight among all K-cliques in graph G is equivalent to maximize the sum of $\beta_j(S)$ among subsets of size K. Therefore, given an instance of the max-weight K-clique problem in which we must check if the objective is at most W, we can reduce it to an instance of our assortment optimization problem and check if the expected revenue is at least W. Therefore, the decision version of our problem is also NP-complete.

B.7. Proof of Proposition 4

To simplify the notation we will omit the dependence on the backlog, $\mathscr{B}$. In a slight abuse of notation, let $V^t_{i,\gamma}(\mathscr{P},M)$ be the value function of Problem (6) in period t parametrized by γ. In addition, let $V^*_{i,\gamma}(\mathscr{P},M)$ be the optimal value with history effect γ, and let $V_{i,\gamma}(\mathscr{P},M\mid\mathcal{S})$ be the expected total number of matches achieved by a policy $\mathcal{S}$ given an initial set of potentials $\mathscr{P}$, M active matches and a fixed value of γ. Finally, let $\mathcal{S}_\gamma=\left\{S^t_\gamma\right\}_{t\in\mathcal{T}}$ be the optimal policy for the problem with history effect γ, i.e., $S^t_\gamma:2^{\mathcal{J}}\times\mathbb{N}^+_0\to 2^{\mathcal{J}}$ is such that $S^t_\gamma(\mathscr{P},M)$ represents the optimal assortment to show in period t given a set of potentials $\mathscr{P}$ and M active matches. Then, we know that

$$V_{i,0}(\mathscr{P},M\mid\mathcal{S}_\gamma)\le V^*_{i,0}(\mathscr{P},M)=z^{IP}_i(\mathscr{P},0,T)\le z^P_i(\mathscr{P},0,T),\tag{16}$$

where $z^{IP}_i(\mathscr{P},0,T)$ is the optimal value of Problem (8). The first inequality follows from $\mathcal{S}_\gamma$ being a feasible policy for Problem (6) with no history. The equality comes from the equivalence between Problem (6) with no history and the integer program (8). The last inequality comes from Problem (9) being a linear relaxation of Problem (8).

On the other hand, since $\phi_{ij}(M,S)$ is non-increasing in M for all $j\in\mathcal{J}$, $S\in\mathcal{S}_K(\mathscr{P})$, we know that $\mathrm{P}\left(\mu^t_i(0,S)\le m\right)\le\mathrm{P}\left(\mu^t_i(M,S)\le m\right)$ for all $m,M\in\mathbb{N}^+_0$. Hence,

$$\mathbb{E}\left[\mu^t_i(M,S)\right]\le\mathbb{E}\left[\mu^t_i(0,S)\right],\,\forall S\in\mathcal{S}_K(\mathscr{P}),\,M\in\mathbb{N}^+_0,\,t\in\mathcal{T}.\tag{17}$$

Then, we know that

$$V^*_{i,\gamma}(\mathscr{P},M)=V_{i,\gamma}(\mathscr{P},M\mid\mathcal{S}_\gamma)\le V_{i,0}(\mathscr{P},M\mid\mathcal{S}_\gamma),\tag{18}$$

where the first equality is by optimality of $\mathcal{S}_\gamma$ in the problem with history effect γ, and the inequality is a direct consequence of (17). Finally, combining Equations (16) and (18) we obtain

$$V^*_i=V^*_{i,\gamma}(\mathscr{P},M)\le V_{i,0}(\mathscr{P},M\mid\mathcal{S}_\gamma)\le z^P_i(\mathscr{P},0,T),$$

concluding the proof.

B.8. Proof of Proposition 5

We start showing that

$$z^P(\vec{\mathscr{P}},\vec{\mathscr{B}},\vec{0},T) \le \sum_{i\in\mathcal{I}\cup\mathcal{J}} z^P(\mathscr{P}_i,\mathscr{B}_i,0,T).$$

Consider a fixed user i. Let $\{S_i^1,\ldots,S_i^T\}$ be a feasible sequence of assortments, and let $S_i = \bigcup_{t=1}^T S_i^t$. Let $B_i = \bigcup_{t=1}^T \{j \in S_i^t \,:\, j \in \mathscr{B}_i^t\}$ be the set of profiles shown to user i taken from the backlog, and let $B_i^C = S_i \setminus B_i$. Then, the contribution of user i to the objective function of the market level problem z^P can be written as

$$\sum_{t=1}^T \sum_{j\in S_i^t} \beta_{ij}^t(S_i^t) = \sum_{t=1}^T \sum_{j\in S_i^t\cap B_i} \phi_{ij}(S_i^t)\cdot\phi_{ji}(S_j^{\tau_{ij}}) + \sum_{t=1}^T \sum_{j\in S_i^t\cap B_i^C} \phi_{ij}(S_i^t)\cdot\phi_{ji}(\{i\}),$$

where τ_{ij} is the period where user j evaluated user i.[42]

If the assortment effect is negative, we know that $\phi_{ij}(S) \le \phi_{ij}(\{j\})$ for any $S \ni j$. Hence,

$$\sum_{t=1}^T \sum_{j\in S_i^t\cap B_i} \phi_{ij}(S_i^t)\cdot\phi_{ji}(S_j^{\tau_{ij}}) \le \sum_{t=1}^T \sum_{j\in S_i^t\cap B_i} \phi_{ij}(S_i^t)\cdot\phi_{ji}(\{i\})$$

and therefore

$$\sum_{t=1}^T \sum_{j\in S_i^t\cap B_i} \phi_{ij}(S_i^t)\cdot\phi_{ji}(S_j^{\tau_{ij}}) + \sum_{t=1}^T \sum_{j\in S_i^t\cap B_i^C} \phi_{ij}(S_i^t)\cdot\phi_{ji}(\{i\}) \le \sum_{t=1}^T \sum_{j\in S_i^t} \phi_{ij}(S_i^t)\cdot\phi_{ji}(\{i\}).$$

Notice that the right-hand side of the the last expression is equal to the value function of the individual level problem evaluated at the sequence $\{S_i^1,\ldots,S_i^T\}$. Then, we know that, for any sequence of assortments, the contribution of user i to the value function of the market level problem is less than or equal to the value function of the individual level problem evaluated at the same sequence. Since this holds for each user $i \in \mathcal{I}\cup\mathcal{J}$, we conclude that

$$z^P \le \sum_{i\in\mathcal{I}\cup\mathcal{J}} z_i^P.$$

It remains to show that

$$V^* \le z^P(\vec{\mathscr{P}},\vec{\mathscr{B}},\vec{0},T).$$

To see this, notice first that $V^* \le V_0^*$, where V_0^* is the optimal value function of the problem with no history effect. This is direct since the history effect is negative, and that any feasible solution for the problem with history is also feasible for the problem without history effects. Next, let h be a realized history (or sample path) of the problem with no history effect. Then, we can write V_0^* as

$$V_0^* = \mathbb{E}_h\left[\sum_{i\in\mathcal{I}\cup\mathcal{J}} \sum_{j\in\mathscr{P}_i} \mu_{ij}(h)\right]$$

[42] Since $j \in \mathscr{B}_i^t$, we know that $\tau_{ij} < t$.

where $\mu_{ij}(h)=1$ if a match between users i and j takes place under path h, and 0 otherwise. Let $S_i(h)=\{S_i^t(h)\}_{t=1}^T$ denote the (optimal) sequence of assortments shown to user i in the path h, and let $t_{ij}(h)$ the time that user i observes profile j under path h. Then,

$$
\begin{aligned}
\mathrm{P}\left(\mu_{ij}(h)=1 \mid \vec{S}(h)\right) &= \mathrm{P}\left(\mu_{ij}(h)=1 \mid S_i(h), S_j(h)\right) \\
&= \mathrm{P}\left(y_{ij}^{t_{ij}(h)}=1,\, y_{ji}^{t_{ji}(h)}=1 \mid S_i^{t_{ij}(h)}(h), S_j^{t_{ji}(h)}(h)\right) \\
&= \mathrm{P}\left(y_{ij}^{t_{ij}(h)}=1 \mid S_i^{t_{ij}(h)}(h)\right)\cdot \mathrm{P}\left(y_{ji}^{t_{ji}(h)}=1 \mid S_j^{t_{ji}(h)}(h)\right) \\
&\leq \mathrm{P}\left(y_{ij}^{t_{ij}(h)}=1 \mid S_i^{t_{ij}(h)}(h)\right)\cdot \mathrm{P}\left(y_{ji}^{t_{ji}(h)}=1 \mid \{i\}\right) \\
&= \phi_{ij}\left(0, S_i^{t_{ij}(h)}(h)\right)\cdot \phi_{ji}(0,\{i\})
\end{aligned}
$$

Then,

$$
\mathbb{E}_h\left[\sum_{i\in\mathcal{I}\cup\mathcal{J}}\sum_{j\in\mathscr{P}_i}\mu_{ij}(h)\right] \leq \max_h \sum_{t=1}^T \sum_{i\in\mathcal{I}\cup\mathcal{J}} \sum_{j\in S_i^t(h)} \mathbb{E}\left[\mu_{ij}(h)\right] \leq \sum_{t=1}^T \sum_{j\in S_i^t(h^*)} \sum_{i\in\mathcal{I}\cup\mathcal{J}} \phi_{ij}\left(S_i^t(h^*)\right)\cdot\phi_{ji}(\{i\}) \leq z^P,
$$

where the last inequality comes from the fact that $\vec{S}(h^*)$ provides a feasible solution for Problem (13), whose optimal solution is z^P, concluding our proof.

Corollary 2 follows directly from this proof, since

$$
\sum_{t=1}^T \sum_{j\in S_i^t(h^*)} \sum_{i\in\mathcal{I}\cup\mathcal{J}} \phi_{ij}\left(S_i^t(h^*)\right)\cdot\phi_{ji}(\{i\}) \leq \sum_{t=1}^T \sum_{j\in S_i^t(h^*)} \sum_{i\in\mathcal{I}\cup\mathcal{J}} \phi_{ij}(\{j\})\cdot\phi_{ji}(\{i\}) \leq \bar{z}
$$

where the first inequality is due to the negative assortment effect, and the last inequality follows since we can construct a feasible solution for Problem (10) by taking $x_{ij}=1$ if $i\in\bigcup_{t=1}^T S_j^t(h^*)$ and $j\in\bigcup_{t=1}^T S_i^t(h^*)$, and 0 otherwise.

Appendix C: Examples

C.1. Only History

The following example shows that Reversed-Greedy may not be optimal if $p_\xi > 0$.

Example 2. Consider two profiles $\{1,2\}$, two periods, $K=1$, and $\alpha_{i1}=\alpha_{i2}=0.1$. In addition, suppose that $u_{i1}=1$, $u_{i2}=0$, $\gamma=-0.1$ and $p_\xi=0.5$. Then,

- $\beta_{i1}(0)=\Phi(u_{i1})\cdot\alpha_{i1}=0.0841$
- $\beta_{i2}(0)=\Phi(u_{i2})\cdot\alpha_{i2}=0.05$
- $\beta_{i1}(1)=\Phi(u_{i1}-0.1)\cdot\alpha_{i1}=0.0815$
- $\beta_{i2}(1)=\Phi(u_{i2}-0.1)\cdot\alpha_{i2}=0.0460$
- $\beta_{i1}(2)=\Phi(u_{i1}-0.2)\cdot\alpha_{i1}=0.0788$
- $\beta_{i2}(2)=\Phi(u_{i2}-0.2)\cdot\alpha_{i2}=0.0420$

If user i starts with zero matches, then we can show that it is optimal to show $S^1,S^2=\{\{1\},\{0\}\}$, leading to an expected value of 0.1340 (compare to 1.337 obtained showing profiles in the reversed order). However, if user i starts with one match, then it is optimal to show $S^1,S^2=\{\{0\},\{1\}\}$ (value of 0.1292 vs 0.1287 obtained from the reversed order). Hence, it is not always the case that Reversed-Greedy is optimal when $p_\xi > 0$.

Example 3. Consider the same setup as in Example 1, but now $q_{i1}=1, q_{i2}=0.7, \alpha_{i1}=0.9, \alpha_{i2}=1$ and $\gamma=-0.1$. Then,

- $\beta_{i1}(0)=\Phi(q_{i1})\cdot\alpha_{i1}=0.8413\cdot 0.9=0.7572$
- $\beta_{i2}(0)=\Phi(q_{i2})\cdot\alpha_{i2}=0.7580\cdot 1=0.7580$
- $\beta_{i1}(1)=\Phi(q_{i1}-0.1)\cdot\alpha_{i1}=0.8159\cdot 0.9=0.7343$
- $\beta_{i2}(1)=\Phi(q_{i2}-0.1)\cdot\alpha_{i2}=0.7257\cdot 1=0.7257$

Hence, we have that $\beta_{i1}(0)<\beta_{i2}(0)$ but $\beta_{i1}(1)>\beta_{i2}(1)$, which illustrates how the ordering of profiles may change depending on M.

C.2. Upper Bound

The following example shows that the optimal solution of the linear relaxation of the problem with no history may not be integral.

Example 4. Consider three profiles $\{1,2,3\}$ and $K=2$. Then, the set of feasible assortments is $\{(1,2),(1,3),(2,3)\}$, and therefore the matrix that captures the constraints of LP is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

which is not totally unimodular as erasing the first row results in a square non-singular matrix whose determinant is equal to -2.

Appendix D: Additional Results

Table 10 First Stage Results

	Dependent variable: M_{it}	
	(1)	(2)
D_{it}	0.357***	0.357***
	(0.015)	(0.014)
W_i	−0.058***	−0.078***
	(0.010)	(0.010)
λ_t	0.055***	0.055***
	(0.004)	(0.004)
Constant	0.213***	−0.118
	(0.003)	(0.119)
Demographics	No	Yes
Observations	71,515	71,515
R^2	0.018	0.044
F Statistic	432.935*** (df = 3; 71511)	46.508*** (df = 71; 71443)

Note: First stage regression results. Standard errors reported in parenthesis. We pool data for periods t_0 and t_1. Significance reported: *p<0.1; **p<0.05; ***p<0.01

Table 11 First Stage Results - Full Sample

	Dependent variable: M_{it}	
	(1)	(2)
D_{it}	0.376***	0.375***
	(0.012)	(0.012)
W_i	0.013	−0.033***
	(0.009)	(0.009)
λ_t	0.037***	0.037***
	(0.003)	(0.003)
Constant	0.142***	−0.237***
	(0.002)	(0.075)
Demographics	No	Yes
Observations	104,150	104,150
R^2	0.023	0.068
F Statistic	803.405*** (df = 3; 104146)	106.129*** (df = 71; 104078)

Note: First stage regression results. Standard errors reported in parenthesis. We pool data for periods t_0 and t_1. Significance reported: *p<0.1; **p<0.05; ***p<0.01

Table 12 Impact of History on Experienced Users - Full Sample

	Dependent variable: y_{ij}		
	OLS	*IV - 2SLS*	
	(1)	(2)	(3)
Matches	0.090^{***}	-0.010^{***}	-0.009^{***}
(in t_1)	(0.002)	(0.001)	(0.002)
Constant	0.104^{***}	0.054	0.026
	(0.016)	(0.071)	(0.075)
Demographics and Scores	Yes	Yes	Yes
Assortment Effect Fixed	No	No	Yes
Observations	178,217	178,217	178,217

Note: Second stage regression results. Robust standard errors reported in parenthesis, clustered by date. The dependent variables y_{ij}, which is equal to 1 if user i likes j, 0 otherwise. Column (1) reports OLS estimates without addressing the potential endogeneity issue. Columns (2) and (3) report 2SLS estimates. All models control for observable characteristics and scores of both users i and j. Columns (3) fixes the assortment effect to be equal to $\delta = -0.005$. Significance reported: *p<0.1; **p<0.05; ***p<0.01

Table 13 Regression Results - Observational Study with Last Session

	Dependent variable: Liked			
	(1)	(2)	(3)	(4)
Assortment Effect	-0.004^{***}	-0.004^{***}	-0.004^{***}	-0.004^{***}
	(0.001)	(0.001)	(0.001)	(0.001)
Matches	-0.009^{**}	-0.009^{**}	-0.011^{***}	-0.011^{***}
[last session]	(0.003)	(0.003)	(0.004)	(0.004)
Matches			-0.014^{***}	-0.014^{***}
$[t-2, t-7]$			(0.002)	(0.002)
Matches			-0.005^{***}	-0.008^{***}
$[t-8, t-14]$			(0.002)	(0.002)
Matches			-0.004^{**}	-0.0005
$[t-15, t-21]$			(0.002)	(0.002)
Matches			-0.007^{***}	-0.007^{**}
$[t-22, t-28]$			(0.002)	(0.003)
Demographics	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes	Yes
Extra Race	No	Yes	Yes	Yes
Extra Matches	No	No	Yes	Yes
Extra History	No	No	No	Yes
Observations	63,899	63,899	63,899	63,899
R^2	0.219	0.221	0.222	0.225

Note: Linear probability models with fixed effects. Standard errors reported in parenthesis. All four models include demographics and period fixed effects. Columns (2) to (4) control for the interaction between users' races. Column (3) and (4) add the number of matches obtained for different time windows in the past. Finally, column (4) controls adds additional history variables (same as in Table 2), including likes, dislikes, days active, among others. Significance reported: *p<0.1; **p<0.05; ***p<0.01

Figure 5 Distribution Days to Expiration

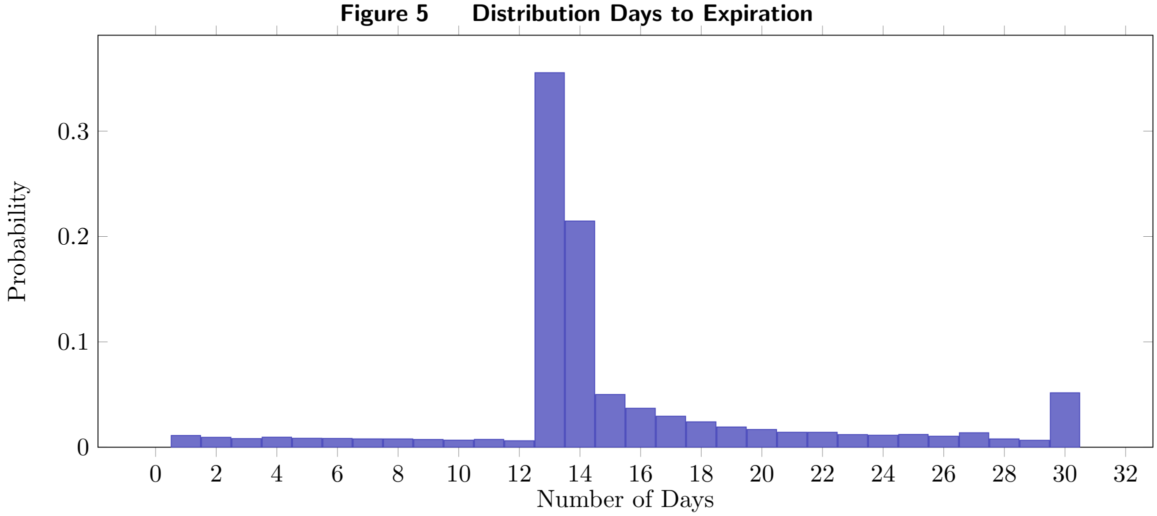


Table 14 Regression Results - Observational Study, by Gender

	Dependent variable: Liked			
	Women		Men	
	(1)	(2)	(3)	(4)
Assortment Effect	-0.004^{*}	-0.004^{*}	-0.003^{*}	-0.003^{*}
	(0.002)	(0.002)	(0.002)	(0.002)
Matches	-0.010^{*}	-0.010^{*}	-0.017^{***}	-0.017^{***}
$\{t-1\}$	(0.005)	(0.005)	(0.005)	(0.005)
Matches	-0.017^{***}	-0.017^{***}	-0.013^{***}	-0.013^{***}
$[t-2, t-7]$	(0.002)	(0.002)	(0.002)	(0.002)
Matches	-0.007^{***}	-0.007^{***}	-0.004^{**}	-0.004^{*}
$[t-8, t-14]$	(0.002)	(0.002)	(0.002)	(0.002)
Matches	-0.006^{**}	-0.006^{**}	-0.002	-0.002
$[t-15, t-21]$	(0.002)	(0.002)	(0.002)	(0.002)
Matches	-0.011^{***}	-0.011^{***}	-0.003	-0.004
$[t-22, t-28]$	(0.003)	(0.003)	(0.003)	(0.003)
Demographics	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes	Yes
Extra Race	No	Yes	No	Yes
Observations	30,144	30,144	33,755	33,755
R^2	0.169	0.172	0.269	0.271

Note: Linear probability models with fixed effects, separating by gender. Standard errors reported in parenthesis. All four models include demographics. Columns (2) and (4) add additional history variables (for same time windows than Matches), including likes, dislikes, days active, among others. Significance reported: *p<0.1; **p<0.05; ***p<0.01

Figure 6 Fixed Effect vs. Attractiveness - by Gender

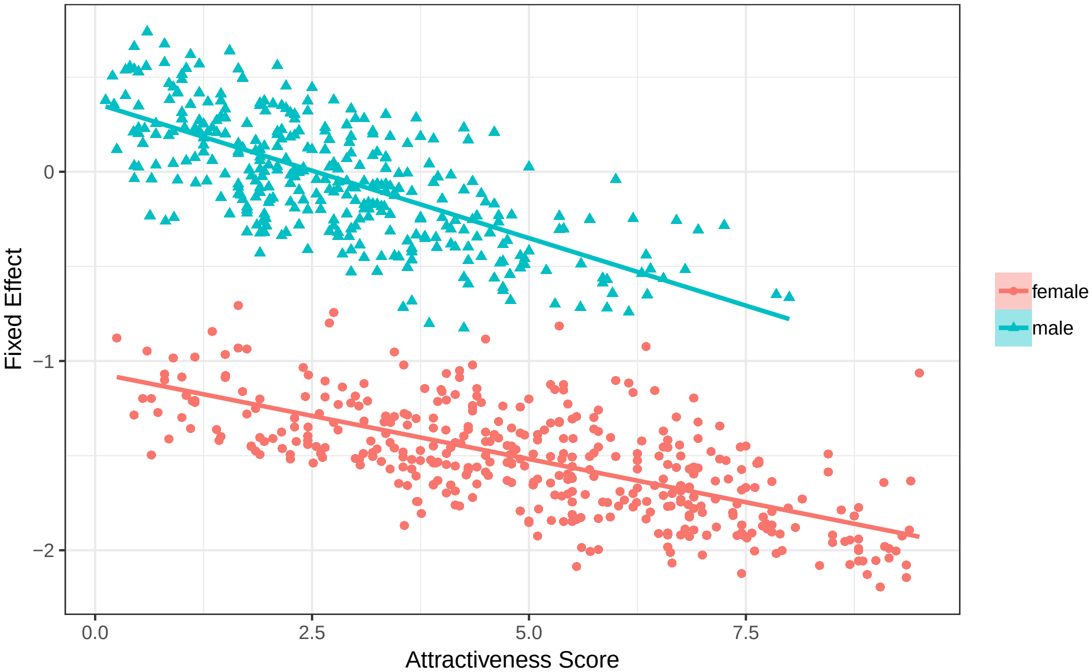


Table 15 Regression Results - Fixed Effect on Attractiveness Score

	Dependent variable: Fixed Effect	
	Women	Men
Score	-0.089^{***}	-0.144^{***}
	(0.004)	(0.005)
Constant	-1.071^{***}	0.355^{***}
	(0.024)	(0.017)
Observations	603	521
R^2	0.419	0.592

Note: Standard errors reported in parenthesis. Significance reported: *p<0.1; **p<0.05; ***p<0.01

Figure 7 Predicted Like Probabilities without Assortment and History Effects - by Gender

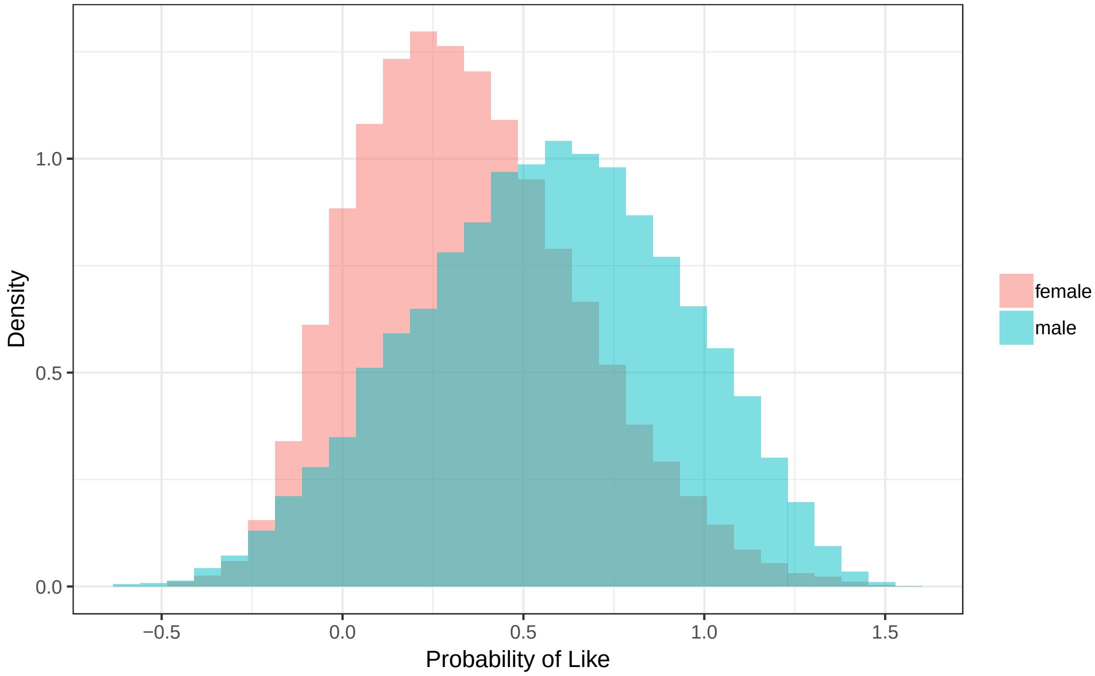


Figure 8 Number of Matches vs. Sampling Size - Benchmarks

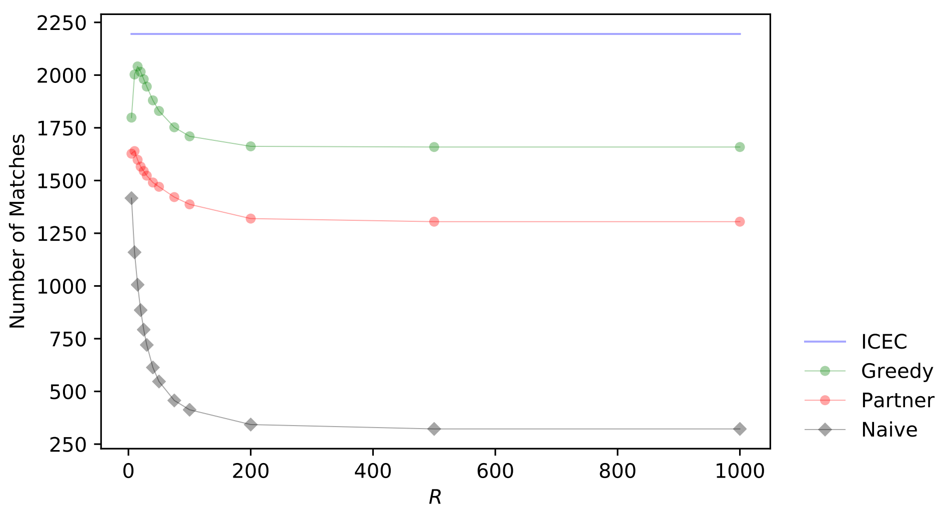
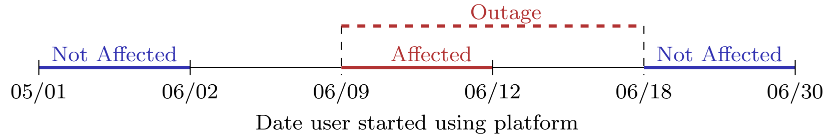


Figure 9 Treatment Assignment - History Effect on New Users



Appendix E: Effect of History on New Users

To study the effect of history on new users we rely on an outage in the search engine used by the algorithm. This generated an exogenous reduction in the number of matches, which we exploit to study history effects on new users.

E.0.1. Background To find which users are part of another user's backlog, the algorithm relies on a search engine that stopped working between June 9th and June 18th, 2019. As a result, likes that happened after June 9th were not considered and distributed by the search engine, and therefore, the probability of these been shown as part of a backlog query decreased considerably. This outage affected all users, making it hard to analyze what was the impact on the entire population. However, we can get a good estimate of how it affected new users by comparing the behavior of users that started using the app before, during, and after the outage. Those that were not affected experienced an average number of matches in their first week, while those who started using the app during the outage observed almost no matches. Hence, comparing the like behavior in their second week across these two groups we can obtain a precise estimate of what is the effect of the number of matches obtained in the past on the probability of liking new users.

E.0.2. Treatment The treatment assignment, in this case, is relatively simple. We define a user as treated if he/she started using the app between June 9th and June 12th, and the control group is formed by users that started using the app between May 1st and June 2nd or after June 18th.[43] In this way, users in the treatment group were affected by the outage in their entire first week, while users in the control group were not affected at all.[44] This is illustrated in Figure 9.

Table 16 shows the average like rate in the first and second week, and also the number of matches obtained in the first week, separating by treatment.[45] We observe that the number of matches obtained by users in the treatment group is significantly smaller ($p < 0.001$, rank-sum test), and we also observe that the change in the like rate of the treatment group ($0.472 - 0.541 = -0.069$) is significantly smaller than that in the control group ($0.410 - 0.533 = -0.123$). Taking the difference between the changes in like rate among treatment and control we can obtain an estimate of the average treatment effect on the treated (ATT), which in this case is equal to $ATT = -0.069 + 0.123 = 0.054$.

[43] The results are equivalent if we only include users who started using the app before June 2nd.

[44] For this reason, we exclude users that were drafted between June 3rd and June 8th, as these users were partially affected by the outage.

[45] The number of matches obtained by users in the treatment group is not equal to zero because there is still a chance that the algorithm showed them profiles from the backlog as part of a different query.

Table 16 Summary Statistics by Treatment

		Like Rate (1st. Week)		Like Rate (2nd. Week)		Num. Matches (1st. Week)	
Treated	N	Mean	Std.	Mean	Std.	Mean	Std.
0	14333	0.533	0.262	0.410	0.269	2.022	2.575
1	664	0.541	0.265	0.472	0.270	0.244	0.646

Note: Summary statistics of main activity variables, separating by treatment (1) and control (0) groups. Like rate is computed as the fraction of profiles liked over the total number of profiles seen. Num. Matches (1st week) represents the number of matches obtained by a user during his/her first week in the app.

E.0.3. Estimation As discussed in Section 4.3, to ensure that our treatment variable is a valid instrument we need to check two assumptions: (i) relevance, and (ii) exogeneity. As shown in Table 16, our treatment variable is highly correlated with the number of matches obtained in the first week, so it is likely to satisfy the relevance condition. In addition, being treated is unlikely to be correlated with the error term for two reasons: (1) the outage in the algorithm was not announced, and (2) since users were in their first week, they didn't have a clear idea of what is a "normal" number of matches to obtain in their first week, and thus couldn't infer that there was an outage. In fact, users' attrition was 87.36% in the treatment group and 85.30% in the control group, suggesting that treated users did not perceived the outage. Hence, our treatment variable is also likely to satisfy the exclusion restriction, so we conclude that it is a potentially valid instrument. Hence, we employ the following 2SLS estimation procedure:
Step 1. Estimate using OLS:

$$M_{i1} = \alpha + \theta Z_i + X_{i1}\beta_1 + \epsilon_{i1} \tag{19}$$

where M_{i1} is the number of matches obtained by user i in his first week; τ_i is 1 if user i is in the treatment group, 0 otherwise; and X_{i1} are observable characteristics of user i, including gender, age, height, attractiveness, and education. In addition, X_{i1} includes the average attractiveness of the profiles seen during the first week. Considering the parameters of this model, we compute the predicted values of the endogenous variable, i.e., $\hat{M}_{i1}$.

Step 2. Estimate using OLS and robust standard errors:

$$\Delta L_i = \alpha + \delta \hat{M}_{i1} + X_{i2}\beta_2 + \epsilon_{i2} \tag{20}$$

where $\Delta L_i = L_{i2} - L_{i1}$ is the change in the like rate of user i in the second week (L_{i2}) relative to week 1 (L_{i1}), and X_{i2} are observable controls in the second week (including the change on average attractiveness of the profiles seen relative to the previous week).

The results of the first stage are reported in Table 17 (in Appendix D). We observe that the treatment variable is negative, significant, and its p-value is 0 in all specifications. These results confirm that the treatment variable is a relevant instrument.

Table 18 shows the estimation results of the second stage. Column (1) reports the results of OLS without considering the potential endogeneity issue. Columns (2) to (4) follow the 2SLS procedure considering different sets of controls. Overall, we find that the 2SLS estimate is between -0.020 and -0.031 depending on the controls considered. This result suggests that an extra match in the first week reduces like rate in the next week by at least 2%.

Table 17 Impact of History on New Users - First Stage Results

	Dependent variable: M_{i1}		
		IV - 2SLS	
	(1)	(2)	(3)
Treated	-1.778^{***}	-1.864^{***}	-1.798^{***}
	(0.033)	(0.045)	(0.047)
Constant	2.022^{***}	-1.079^{**}	0.408
	(0.022)	(0.482)	(0.485)
Demographics	No	Yes	Yes
Scores	No	No	Yes
Observations	14,997	14,995	14,995
R^2	0.021	0.112	0.133

Note: First stage regression results. Robust standard errors reported in parenthesis. Column (1) only includes the treatment variable. Column (2) adds demographics of user i. Column (3) adds the average score of the profiles evaluated by i in the first week. Significance reported: *p<0.1; **p<0.05; ***p<0.01

Table 18 Impact of History on New Users

	Dependent variable: ΔL_i			
	OLS		*IV - 2SLS*	
	(1)	(2)	(3)	(4)
Matches	-0.012^{***}	-0.031^{***}	-0.029^{***}	-0.020^{***}
(1st week)	(0.001)	(0.005)	(0.005)	(0.004)
Constant	0.133^{***}	-0.062^{***}	-0.094^{**}	0.112^{**}
	(0.042)	(0.010)	(0.046)	(0.044)
Demographics	Yes	No	Yes	Yes
Scores	Yes	No	No	Yes
Observations	14,995	14,997	14,995	14,995
R^2	0.168	-0.018	0.010	0.160

Note: Second stage regression results. Robust standard errors reported in parenthesis. The dependent variables is the change in the like rate in the second week relative to the first week. Column (1) reports OLS estimates, while columns (2) to (4) reports 2SLS estimates. Column (2) only includes the treatment variable. Column (3) adds demographics of user i. Column (4) adds the difference in average score of the profiles evaluated by i in the second week relative to the first week. Significance reported: *p<0.1; **p<0.05; ***p<0.01