ACTSC372: Corporate Finance

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Arbitrage Pricing Theory (APT)

APT is a competing theory to CAPM, with multiple systematic factors F_i and multiple corresponding sensitivity measurements β_i

5.1 Recall CAPM Equation

$$\mu_i = r_f + \beta_i[\mu_M - r_f], \beta_i = \frac{Cov[R_i, R_M]}{Var[R_M]} = \frac{\sigma_{iM}}{\sigma_M^2}$$

5.2 Prerequisites

5.2.1 CAPM and Risk Decomposition

- CAPM formula also implies a special structure for the random return

$$R_i = r_f + \beta_i(R_M - r_f) + \epsilon_i$$

Note. *It is just a replacement of μ_i into R_i*

  - Assume $E[\epsilon_i] = 0$, $Cov[R_M, \epsilon_i] = 0$

- The variance of return is decomposed into two parts

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\epsilon i}^2$$

- Arbitrary opportunities: 0 initial cost, guaranteed future return

5.3 APT Models

- Actual Return = Expected Return + Unexpected Return (due to risk)
  - Risk = Systematic Risk + Idiosyncratic Risk

- APT model of asset returns:
$$R_i = \mu_i + m_i + \epsilon_i$$
where $m_i = \beta_{i1} + F_1 + \cdots + \beta_{ik}F_k$

  Note. *ϵ_i are unexpected risk factors and typically only affect the individual company, not all the companies.* ***Idiosyncratic risk can be eliminated through diversification.***

5.3.1 Factor Model for Systematic Risk

$$m_i = \beta_{i1}F_1 + \cdots + \beta_{ik}F_k$$

- R_i can be affected by k different factors, $F_1, ..., F_k$
- F_k can affect different assets to different extend ($\beta_{ik} \neq \beta_{jk}$)

  Note. *F_i are different systematic risk factors (random variables), which are macro factors that fluctuate and their fluctuations affect many stock prices. It affects the entire financial market, and* ***cannot be completely avoided.***

5.3.2 Factor Model: Definition and Special Cases

Assume <u>linear sensitivity</u> to risk factors

$$R_i = \mu_i + \beta_{i1}F_1 + \cdots + \beta_{ik}F_k + \epsilon_i$$

- β_{ij} measures sensitivity to risk factors
- F_j = j-th systematic risk factor = actual value - expected value
  - $E[F_j] = 0$
  - $Cov[F_i, F_j] = 0$ for $i \neq j$
- ϵ_i = idiosyncratic risk of asset i
  - $E[\epsilon_i] = 0$
  - $Cov[\epsilon_i, F_j] = 0$
  - $Cov[\epsilon, \epsilon_j] = 0$ for $i \neq j$

Single-Factor Model: simplest factor model

$$R_i = \mu_i + \beta_i F_1 + \epsilon_i$$

Note. *Usually used for proof of concept*

Market Model: market as the only risk factor

$$R_i = \mu_i + \beta_{iM}(R_M - \mu_M) + \epsilon_i$$

Note. *Coincides with the CAPM formula*

5.4 APT Formula

- Goals:
  - A simple but insightful model for the systematic return
  - Make some assumptions about the idiosyncratic return
  - Derive an expression for the expected return

$$\mu_i = r_f + \beta_{i_1}\gamma_1 + \cdots + \beta_{i_k}\gamma_k$$

- Linear structure
- Same $\gamma_1, ..., \gamma_k$ for all assets
- Want to show $\gamma_1, ..., \gamma_k$ exist and identify what they are

No arbitrage: same systematic risk $\Rightarrow$ same expected return

5.5 APT Formula Summary

- General factor Models
$$\mu_i = r_f + \beta_{i_1}\gamma_1 + \cdots + \beta_{i_k}\gamma_k$$
- Single-factor Model
$$\mu_i = r_f + \beta_i\gamma$$
- Market Model: $\gamma = \frac{\mu_i - r_f}{\beta_1}$ holds for any asset, including market portfolio
  - One can show that $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$
  - $\gamma = \frac{\mu_M - r_f}{\beta_M} = \mu_M - r_f$, because $\beta_M = 1$

    Note. *β_M is always 1*
    - γ is called the **excess market return** or **market premium**

5.6 CAMP vs. APT

- Similarities
  - Market Model APT formula coincides with the CAPM formula
  - Some common assumptions (e.g., frictionless & liquid market)

	CAPM	APT
Systematic Risk Factor	• Unique market portfolio • Derived mathematically	• Multiple risk factors • Selected by users
Return Model	• Known μ & Σ • Derived equilibrium return	• Model return statistically • Derived expected return μ
Asset prices	• All fairly priced assets should lie on the SML	• Assets can deviate from the SML due to idiosyncratic risk
Main challenge	• Estimation of μ & Σ	• Selection of systematic risk factors

Chapter 6

Risk, Return and Capital Budgeting

- Capital budgeting considers the value of a project given a discount rate
  - A project is a stream of cash flows
  - When discounted by r_f, cash flows are assumed risk-free
  - What if the cash flows are risky (e.g. capital gain of a stock)?
- CAPM and APT are ways computing the "fair" risky returns
  - "fair" return $\Leftrightarrow$ "appropriate" discount rate
  - Discount rate reflects the risk of in investment
  - Systematic risk of an investment is indicated by its β's
- Now we combines the two:
  - Find the appropriate discount rate
  - Capital budgeting for risky projects

6.1 Value of a Project

- Company Z is considering a project. The project company has two options:
  1. Invest in the project directly
  2. Pay dividends to the shareholders and let them invest
- What's the project's value under these two options?

6.1.1 Capital Budgeting Rule

The discount rate of a project should be the expected return on a financial asset of comparable risk

- Project depends on its cash flow pattern, not investor identity

  - Company and shareholders should use the <u>the same discount rate</u> for valuation

- The discount rate for a project is driven primarily by the **project beta**
  - Investor risk preference/appetite is irrelevant

6.1.2 Project Beta

- If the project "looks like" and extension of the company, use the **company's beta**
- If the project looks very different from the company, use the **industry beta**
- If the project P is a **combination** of n projects with known $\beta_1, ..., \beta_n$, then
  - $\beta_P = \sum_{i=1}^{n} w_i \beta_i$, where the weights sum to 1

Determinants of beta: Why do different companies/projects have different betas?

- Business Risk \ Operation Risk
  - Cyclicality of Revenue
  - Operating Leverage
- Financial Risk

  Note. *How much you earn and how much you are borrowing*

  - Financial Leverage

6.2 Cyclicality of Revenue

Note. *It is the fluctuations along with business cycle*

- **Highly cyclical stocks have high betas**
  - High-tech firms and retailers fluctuate with the business cycle
  - Utilities and funeral homes are less dependent upon the business cycle
- Cyclicality $\neq$ Variability

  Note. *This is more a qualitative idea than quantitative*

  - Stocks with high volatilities do not necessarily have high betas
    * Movie studios have revenues that are highly variable but low beta
      · Depends on movie quality but not the business cycle

6.3 Operating Leverage

6.3.1 Basic Idea

$$Operating\ Leverage = \frac{\%\ change\ in\ EBIT}{\%\ change\ in\ Sales}$$

- **High proportion fo fixed cost $\Rightarrow$ high OL $\Rightarrow$ high risk**
- Both qualitative and quantitative

High OL:

- $\Rightarrow$ profits are more sensitive to sales
- $\Rightarrow$ magnifies the effect of cyclicality
- $\Rightarrow$ cyclicality is a determinant of beta
- $\Rightarrow$ **High beta**
- $\Rightarrow$ **High WACC**

In calculation project:

$$r_A = r_f + \beta_A(\mu_M - r_f)$$
$$NPV = \frac{Cash\ inflows - variable\ cast}{r_A} - \frac{fixed\ cost}{r_f}$$

Note. *Fixed costs are risk-free*

$$w_{rev} = \frac{NPV_{rev}}{NPV},\ w_{VC} = \frac{NPV_{VC}}{NPV},\ w_{FC} = \frac{NPV_{FC}}{NPV}$$
$$\beta_A = w_{rev}\beta_{rev} + w_{VC}\beta_{VC} + w_{FC}\beta_{FC}$$

Note. $w_{FC} = 0$, *high fixed cost $\Rightarrow$ high beta $\Rightarrow$ high risk*

6.3.2 Reading Materials

Financial leverage and operating leverage are interrelated

- Decrease in OL $\Rightarrow$ Increase in FL

6.4 Financial Leverage

$$DE\ ratio = \frac{Debt}{Equity}$$

- Asset is a portfolio of Debt and Equity, based on the balance sheet equation

$$A = D + E \Rightarrow \beta_A = \frac{D}{A}\beta_D + \frac{E}{A}\beta_E$$

- Usually $\beta_E >> \beta_D \approx 0$, so

$$\beta_E \approx \beta_A(1 = \frac{D}{E})$$

  - Value of stock $\approx$ Value of equity
  - As D/E ratio increases, β_E increases, stock has higher risk

6.4.1 beta Computation in Practice

- Run a regression of stock returns against the market returns
  - Stock = Equity, so β_E is obtained from regression
- CAPM and APT computed the equity beta, not the asset beta
- Most companies/projects are financed by the company's asset
  - Asset is a mix of debt and equity
  - Need a "blended" discounted rate
- Asset beta:

$$\beta_E = \beta_A(1 + \frac{D}{E})$$

- Discount rate for asset: r_{WACC}

6.5 Weighted Average Cost of Capital (WACC)

$$r_{WACC} = \frac{E}{A}r_E + \frac{D}{A}r_D(1 - T_C)$$

- r_E: cost of equity, calculated by CAPM or APT
- r_D: cost of debt, promised interest rate on company's debt
- T_C: corporate tax rate

Note. *Cost of debt:* ***interest paid by a corporation is tax-deductible***

- Suppose $\$Xr_D$ interest is paid on principal $\$X$, tax-deductible

- Tax bill is reduced by $\$Xr_DT_C$
- After-tax interest paid is $\$Xr_D - \$r_DT_C = \$Xr_D(1-T_C)$
- After-tax interest rate is
$$\frac{\$Xr_D(1-T_C)}{\$X} = r_D(1-T_C)$$

6.5.1 Usages of WACC

- Investment rules, capital budgeting, CAPM/APT can all be connected
$$r_{WACC} = \frac{E}{A}r_E + \frac{D}{A}r_D(1-T_C)$$
- WACC can be viewed as the company's **overall return on assets**
- Subdivisions of a company may have different WACC of their own
    - Different divisions may have different risks, betas and r_E's
- IRR rules: accept investment project with
$$\text{IRR} > \text{WACC}$$
Note. *Some problems still exist; e.g. financing and mixed projects?*
- NPV/Capital budgeting: use WACC to discount overall EBIT
- CAPM/APT:
$$r_E = r_f + \beta_i(\mu_M - r_f)$$

Note. *We would use WACC to discount cash flows of projects that*

1. *Have the* ***same risk as the overall firm*** *AND*
2. *Are financed at the* ***same mixed of debt/equity as the overall firm***

6.5.2 Reducing the Cost of Capital

Suppose the EBIT remains constant in perpetuity
$$Asset\ Value = \frac{EBIT}{r_{WACC}}$$
Lower WACC ⇒ lower cost of capital ⇒ higher asset values

- Therefore, managers wish to have a low WACC
    - Liquidity
        - **High Liquidity ⇒ Low WACC**
    - Adverse selection
        - **Adverse selection ⇒ Low Liquidity ⇒ High WACC**

Also note that high operating leverage indicates a high WACC

6.5.3 Liquidity

All else equal, **stocks with high liquidity usually have a lower WACC**

- The **ease** and **cost** with which investors can trade a security
- Some transaction cost: (high transaction cost indicates high WACC)
  - Brokerage fees: price paid to brokers to complete trades
  - Bid-ask spread: price difference for purchasing and selling a stock
  - Market impact cost: price fluctuation due to large-volume trades

6.6 Reading Materials - EVA

EVA: Economic Value Added

$$EVA = EBIT(1 - T_C) - r_{WACC} \times \text{(Total Capital)}$$

EVA can simply be viewed as earnings after capital costs.

Chapter 7

Efficient Market Hypothesis (EMH)

An **efficient** capital market is one in which stock prices fully reflect available information.

- The EMH has significant implications for investors and firms
- Market is " because it knows much information
  - Beating the market requires knowing more than the market
- Information is reflected in security prices quickly
  - Knowing information after it is publicly released is pointless
- Firms should expect to receive the fair value for securities
  - Firm cannot profit from fooling investors about their stock prices

7.1 Types of Market Efficiency

7.1.1 Weak Form

- Security prices reflect all information **in past prices and volume**
- Technical Analysis does not work
  - look for the patterns in prices/volumes
- **Random stock price** movements support weak form efficiency.

7.1.2 Semi-Strong Form

- Security prices reflect all **publicly** available information
- Fundamental Analysis does not work
  - use public information to select investments

7.1.3 Strong Form

- Security prices reflect all information **public and private**
- Insider Trading does not work

7.2 Long-Term Financing

7.3 Equity

7.3.1 Tobin's Q as Performance Measures

market value of assets / replacement value of assets

- $Q > 1$: the management has done well and selected good projects
- $Q < 1$: the management destroyed value

Note. *Value investors prefer companies with low Tobin's Q: their stocks are "cheap"*

7.3.2 Shareholders' rights

Chapter 8

Capital Structure

8.1 The Pie Theory

- A firm's value is the sum of its debt and equity

$$V = D + E$$

- D/E ratio is also known as Financial Leverage
  - $D/E = 0 \Leftrightarrow D = 0$ called ***all-equity*** or ***unlevered*** firm

8.2 M&M Proposition I (No Taxes)

8.2.1 Prerequisites

- Assumptions:
  - EMH, frictionless market
  - Cash flows of the firm are perpetual
  - Same interest rate for individuals and corporations

8.2.2 M&M Proposition I (No Taxes)

- Then the value of the firm is unaffected by financial leverage

$$V_L = V_U$$

- Key point: $V = PV(future\ CF)$, with appropriate discount rate
- Firm L and Firm U are two otherwise identical firms
  - Same business, different capital structure
  - Same business, same discount rate

8.2.3 Proof of M&M Proposition I (No Taxes)

Direct proof: Compare cash flows received by two firms that are otherwise identical except capital structures (i.e. different D/E ratios)

- Shareholders in an unlevered firm

$$CF_U^S = EBIT$$

- Shareholders and bondholders in a levered firm

$$CF_L^S = EBIT - r_D D$$

$$CF_L^B = r_D D$$

- Clearly

$$CF_L^S + CF_L^B = CF_U^S$$

  , so same as annual cash flows

- Firms are otherwise identical, so overall risk is the same
- Therefore the same present value

$$V_L = V_U$$

8.2.4 Homemade (un)Leverage

Proof via contradiction via "homemade leverage"

- Firm L has debt D_L and has equity E_L

$$V_L = D_L + E_L$$

- Firm U is all equity and $V_U = E_U$
- Interest rate (cost of debt, Yield to Maturity (YTM)) is r_D
- Firm L and Firm U are other wise identical except capital structures

Which investment is better, buying $X\%$ of Firm L or $X\%$ of Firm U?

- Main ides:
  - Start by assuming $V_U < V_L$
  - Match future cash flows, from shareholders perspective
  - Two strategies must have the same initial costs, otherwise arbitrage

Homemade Leverage

- Strategy: Buy $X\%$ of Firm L

  - Future Cash Flows = $(X\%)(EBIT - r_D D_L) = (X\%)EBIT - (X\%)r_D D_L$
  - Initial Cost = $(X\%)E_L$

- Strategy: Buy $X\%$ of Firm U, also borrow $(X\%)D_L$ at rate r_D
  - Future Cash Flows = $(X\%)EBIT - (X\%)r_D D_L$
  - Initial Cost = $(X\%)E_U - (X\%)r_D D_L = (X\%)(V_U - D_L)$

- Two strategies have the same future cash flows, but
  - $E_U = V_U < V_L = E_L + D_L$, then $(X\%)(V_U - D_L) < (X\%)E_L$
  - $E_U = V_U > V_L = E_L + D_L$, then $(X\%)(V_U - D_L) > (X\%)E_L$

  In both cases, arbitrage exist, contradicting with EMH, so $V_U \not< V_L$ & $V_U \not> V_L$

- Intuition: "lever up" Firm U by borrowing at r_D (so homemade leverage)

Homemade Unleverage We can arrive the same conclusion by ""homemade unleverage

- Strategy: Buy $X\%$ of Firm U
  - Future Cash Flows = $(X\%)EBIT$
  - Initial Cost = $(X\%)E_U$

- Strategy: Buy $X\%$ of Firm L, also depositing $(X\%)D_L$ at rate r_D
  - Future Cash Flows = $(X\%)(EBIT - r_D D_L) + (X\%)r_D D_L = (X\%)EBIT$
  - Initial Cost = $(X\%)E_L + -(X\%)D_L = (X\%)V_L$

- Proof via contradiction to show $V_N \not< V_L$ & $V_N \not> V_L$

- Intuition: "unlever" Firm L by depositing at $(X\%)D_L$

8.2.5 Interpretation of MM I (No Taxes)

- The value of a firm is independent of its capital structure
  - Total size of the pie is unaffected by how the pie is sliced

- Management should not worry about the financing decisions of a project
  - Don't worry about issuing new debt or new equity
  - Management should still identify positive NPV projects

- The value of a firm equals to its discounted future CFs

$$V = V_L = V_U = \frac{EBIT}{WACC}$$

  - If V is independent of capital structure, then so is $WACC$ ($EBIT$ is assumed to be fixed)

8.3 M&M Proposition II (No Taxes)

8.3.1 Financial Leverage and WACC

- Effect of leverage (intuitively)
  - Cost of debt is usually low, more debt may lower WACC
  - Increased debt makes equity more risky, so higher WACC
- MM II says both are true and
  - The two effects offset each other exactly
  - So WACC is independent of capital structure
- The increased risk of remaining equity exactly offsets the higher proportion of low-cost debt

8.3.2 M&M Proposition II (No Taxes)

$$r_L = r_U + (\frac{D}{E}()r_U - r_D)$$

- r_L = cost of equity for levered firm
- r_U = cost of equity for all-equity (unlevered) firm (also its WACC)
- r_D = cost of debt

Note. *$r_{WACC} = r_U$ is constant regradless of D/E ratio*

8.3.3 Proof of M&M Proposition II (No Taxes)

Idea: WACC is constant.

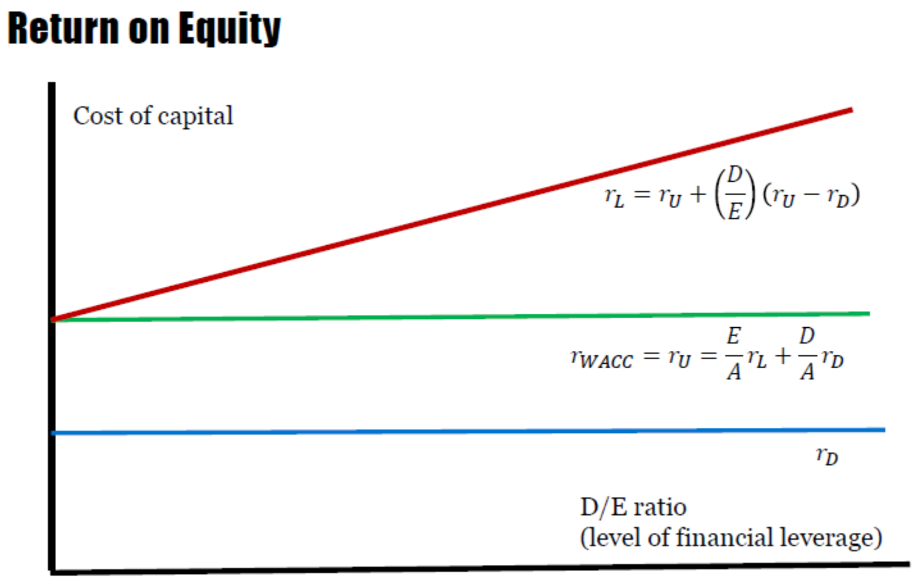
$$r_U = r_{WACC} = \frac{E}{D+E} r_L + \frac{D}{D+E} r_D$$

$$\Rightarrow r_L = r_U + \frac{D}{E}(r_U - r_D)$$

8.3.4 Interpretation of MM II (No Taxes)

- Projects' cash flows should be appropriately discounted
  - The return on alternative investments that bear the same risks
  - Use this return as the discount rate to compute the NPV
- Project has the same risk as a firm's asset $\Rightarrow$ Use WACC for discounting
- Project has the same risk as a firm's equity $\Rightarrow$ Use cost of equity

- Project has the same risk as a firm's debt $\Rightarrow$ Use cost of debt
- Project has no risk $\Rightarrow$ Use risk-free rate



8.4 M&M With Corporate Taxes

- Surprisingly, the situation changes when we include the effects of corporate taxes
- **Interest on debt is tac deductible**
  - Increased debt has two effects:
    * Higher proportion of low-cost debt (offset exactly by increased equity risk, as before)
    * Higher tax deductions
  - The additional tax reduction has value

8.4.1 M&M I (With Taxes)

Firm values **increases** with financial leverage

$$V_L = V_U + DT_C$$

Proof. Let r_U = discount rate for the after-tax EBIT, then

$$V_U = \frac{EBIT(1 - T_C)}{r_U}$$

Total cash flows to Firm L's shareholders and bondholders are

$$(EBIT - r_D D)(1 - T_C) + r_D D = EBIT(1 - T_C) + r_D DT_C$$

Discounted the cash flows by the **appropriate rates**

- Discount first term by r_U, PV is V_U
- Discount second term by r_D, PV is DT_C (PV of Interest Tax Shield)

Total cash flows to Firm L's shareholders and bondholders are

$$(EBIT - r_D D)(1 - T_C) + r_D D = EBIT(1 - T_C) + r_D DT_C$$

Discount two cash flows with the appropriate rates

$$V_L = \frac{EBIT(1 - T_C)}{r_U} + \frac{r_D DT_C}{r_D}$$

$$V_L = V_U + DT_C$$

□

8.4.2 M&M II (With Taxes)

Cost of equity increases as a result of the increase of debt

$$r_L = r_U + (\frac{D}{E})(1 - T_C)$$

Cost of equity increase, but

- This increase is offset by the increased proportion of low-cost debt
- Then further offset by the interest tax shield

Proof. Consider the market value balance sheet for a levered firm

Asset (MM I with taxes)	Debt and Equity
Unlevered Assets = V_U	Debt = D
Tax Shield = $T_C D$	Equity = E

The company does not retain values, cash in = cash out

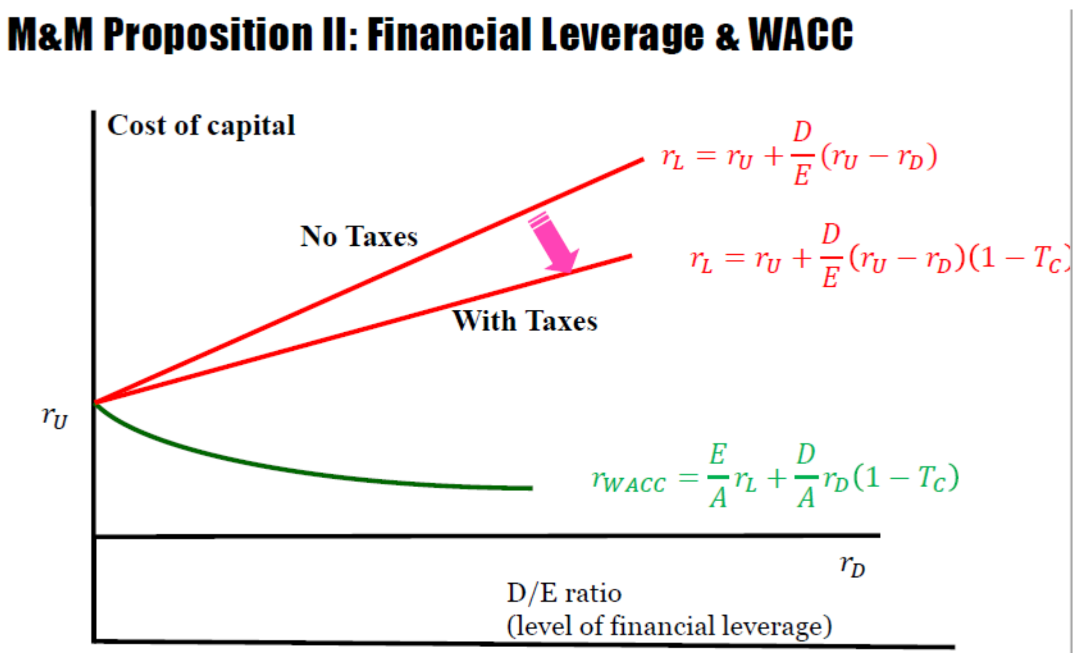
$$r_U V_U + r_D T_C D = r_D D + r_L E$$

- LHS: $EBIT(1 - T_C) + r_D DT_C$ = business earning + tax shield
  - Also the total cash flows to bondholders and shareholders
- RHS: $r_D D + r_L E$ = expected returns for bondholders and shareholders

Solve for r_L with the help of MM I.

$$
\begin{gathered}
r_U V_U + r_D T_C D = r_D D + r_L E \\
V_U + DT_C = V_L = D + E \\
\Rightarrow V_U = E + D(1 - T_C) \\
\Rightarrow r_D D + r_L E = r_U(E + D(1 - T_C)) + r_D T_C D \\
\Rightarrow r_L = R_u + \frac{D}{E}(1 - T_C) r_U + \frac{D}{E} r_D T_C - \frac{D}{E} r_D \\
\Rightarrow r_L = R_u + \frac{D}{E}(1 - T_C) r_U - \frac{D}{E}(1 - T_C) r_D \\
\Rightarrow r_L = R_u + \frac{D}{E}(1 - T_C)(r_U - r_D)
\end{gathered}
$$

□



8.5 Summary

	Without Taxes	With Taxes
M&M Prop I	$V_L = V_U$	$V_L = V_U + T_C D$
M&M Prop II	$r_L = r_U + \frac{D}{E}(r_U - r_D)$	$r_L = r_U + \frac{D}{E}(r_U - r_D)(1 - T_C)$
Unlevered Firm Value	$V_U = \frac{EBIT}{r_U} = \frac{EBIT}{r_{WACC}}$	$V_U = \frac{EBIT(1 - T_C)}{r_U}$
WACC	$r_{WACC} = r_U = \frac{E}{A} r_L + \frac{D}{A} r_D$	$r_{WACC} = \frac{E}{A} r_L + \frac{D}{A} r_D (1 - T_C)$
Equity Value	# shares outstanding × price per share	
Comments	$V = D + E$, taxes no included in V Interest are tax deductible: more debt ➔ less tax ➔ higher firm value	

Remark. *The return we get from CAPM and APT can either be r_U or r_L, it depends on whether the β given in question is for levered or unlevered firm.*

Chapter 9

Limits to the Use of Debt

- With taxed, $M\&M$ properties suggests that debt lowers WACC
  - The more debt the better?
  - Nothing wrong with the math and logic, but something is missing
- 100% debt = bankruptcy, so that can't be right
  - **The *possibility*** of bankruptcy has negative effect on firm value
    - Not the risk of bankruptcy itself (shareholders and bondholders are fairly compensated by higher cost of equity/debt capital)
    - But the actual costs of bankruptcy (a.k.a., cost of financial distress)

9.1 Costs of Financial Distress

- Direct Costs
  - Legal, accounting, accounting, administrative costs
  - Could be large in mangnitude ($2.2 bil for Lehman Brothers)
  - Tend to be a small percentage of firm value
- Indirect Costs
  - Lost sales (e.g. lost brand value, lost customer loyalty, etc.)
  - Agency costs due to conflicts of interests

9.1.1 Agency Costs

- Shareholders is the "agent" of bondholders in owning the company
- Management is the "agent" of shareholders in managing the company
- Conflicts of interest cause agency costs thus decrease firm value

- Shareholders' selfish investment strategies (near bankruptcy)
  1. Incentive to take large risks
  2. Incentive toward underinvestment
  3. "Milking the company"

Incentive to take large risks

- When a firm is close to bankruptcy:
  - Shareholder's viewpoint: *there is nothing to lose anyway*
  - So why not go for a BIG gamble

Incentive toward underinvestment

Milk the Company

- Sell a building for cash.

9.1.2 Managing/Reducing costs of debt

- Bondholders are aware of these issues (i.e., agency cost by shareholders)
- Fair compensation ⇒ Bondholders will often demand high interest rate
- This increase in the costs of debt is bad for shareholders
- Managing costs of debt is actually better for everyone

Protective Covenant

- "Covenants" are terms or conditions that the borrower agrees to
  - Written into the loan documents (or "loan indenture")
- Two main categories:
  - Positive covenants – things the borrower (company) must do
    * Insure assets
    * Maintain good conditions of assets
    * Provide audited financial statements
    * Allow redemption in the event of a manager, spinoff or other major corporate change
  - Negative covenants – things the borrower (company) cannot do
    * Not to pay dividend (above a limit)
    * Not to issue more debt (above a limit)

    * Not to issue more senior debt (above a limit)
    * Not to sell assets without paying bondholders with proceeds

- Covenants are usually in the interests of the bondholders

9.1.3 Agency Costs between Management and Shareholders

- Often management has little ownership and has different incentives than S/H
- Agency cost of equity: ”wasting for personal pleasure”
  - High salaries
  - Corporate jets
  - Office art work and collectibles
- Free cash flow hypothesis
  - Management tend to waste more money when there are ”free cash” around
  - Issue more debt to ”dry out” some free cash
    * Negative covenants prevents wasting to some extend

9.1.4 Tradeoffs between Tax Benefit and Cost of Financial Distress

- Debt brings tax shield advantage and costs of financial distress
  - Considering this tradeoff, what is the optimal D/E ratio?

Chapter 10

Capital Budgeting for Levered Firms

- The Adjusted Present Value (APT) method
  - Firms total risk = business risk + financial risk
  - Add unlevered NPV by the present value of financing effects
- The Weighted Average Costs of Capital (WACC) method
  - Same formula as before, but with adjusted discount rate
  - Use WACC to discount cash flows
- The Flow Equity (FTE) method
  - NPV of a project increases the NPV of equity value
  - Discount levered equity cash flows by levered equity cost

10.1 Review of Capital Budgeting

- Given a project, we calculate its NPV as

$$\text{NPV} = -\text{Initial Cost} + \text{PVATOCF} + \text{PV}_{\text{Salvage}}$$

  - PVATOCF: Present value of after-tax operating cash flows (rent, cost, etc.)
  - PVCCATS: Present value of Capital Costs Allowance (formula)
  - PVs calculated using given discount rates (real or nominal)
- Project was (implicitly assumed) financed **entirely by equity**
  - Discount rate calculated from CAPM/APT
  - Capital structure was ignored
- In this chapter, projects are financed by debt and equity

  Remark. *In this case, the return we generated from CAPM/APT is* r_L

10.2 The APV Method

10.2.1 Perpetual EBIT

- Given a debt level, the APV of a project is

$$NPV_{APV} = NPV_{unlevered} + PV_{financing}$$

- Compare this to MM Proposition I (with taxes)

$$V_L = V_U + T_C D$$

- Find the NPV_{APV} of the previous example

$$NPV_{unlevered} = C_0 + \frac{EBIT(1 - T_C)}{r_U}$$

$$PV_{financing} = D - \frac{r_D D(1 - T_C)}{r_D} = T_C D$$
$$NPV_{APV} = NPV_{unlevered} + PV_{financing}$$

Remark. C_0 *is negative*

10.2.2 More Complicated Case

$NPV_{unlevered}$

- n years of project life (not perpetuity)
  - $NPV_{unlevered}$ changes

$$NPV_{unlevered} = C_0 + EBIT(1 - T_C)\left(\frac{1 - (1 + r_U)^{-n}}{r_U}\right)$$

  - $PVCCATS$ appears with CCA rate d

$$PVCCATS = \frac{CdT_C}{r_U + d} \times \frac{1 + 0.5r_U}{1 + r_U} - \frac{SdT_C}{r_U + d} \times \frac{1}{(1 - r_U)^n}$$

  - $PV_{Salvage}$ appears with salvage S

$$PV_{Salvage} = \frac{S}{(1 + r_U)^n}$$

$PV_{financing}$

$$PV_{financing} = D - r_D D(1 - T_C)\frac{1-(1+r_D)^{-n}}{r_D} - \frac{D}{(1+r_D)^n}$$

- Debt amount
- Present value of after-tax interest payments
- present value of principal repayment

Remark.

- *$PV_{financing}$ is the interest tax shield*
- *$PV_{financing} = 0$ when $T_C = 0$*

Loan Subsidize

Suppose the government subsidizes the debt by offering a low interest rate $r_D < r_D$

$$PV'_{financing} = D - r'_D D(1 - T_C)\frac{1-(1+r_D)^{-n}}{r_D} - \frac{D}{(1+r_D)^n}$$

- Company's **risk** to repay debt remains unchanged (r_D)
- The **amount** of interest paid is less (r'_D)
- Higher interest tax shield due to government subsidy

The value of the subsidy:

$$PV'_{financing} - PV_{financing} = (r_D - r'_D)D(1 - T_C)\frac{1-(1+r_D)^{-n}}{r_D}$$

10.3 The WACC Method

- Use r_{WACC} to discount unlevered future cash flows

$$NPV_{WACC} = C_0 + \frac{EBIT(1-T_C)}{r_{WACC}}$$

  - C_0 is negative
  - $r_L = r_U + \frac{D}{E}(r_U - r_D)(1 - T_C)$
  - $r_{WACC} = \frac{E}{D+E}r_L + \frac{D}{D+E}r_D(1 - T_C)$

- In order to calculate r_{WACC}, D/E ratio is needed
  - What are debt and equity of project
  - After accepting the project, firm value increases by NPV_{APV}
  - After raising money needed, firm value increases by initial cost

10.4 The FTE Method

- Main idea:
$$NPV_{proj} = NPV_{S/H} + NPV_{B/H}$$
  - B/H are fairly compensated, i.e. $NPV_{B/H} = 0$
  - Can calculate NPV_{proj} by calculating $NPV_{S/H}$
- From S/H's perspective: What is S/H cash flows (not the company)
  1. Find the levered cash flows to shareholders
  $$CF_{0,E} = C_0 + D$$
  $$CF_E = (EBIT - r_D D) \times (1 - T_C)$$
  2. Find the cost of levered equity
  $$r_L = r_U + \frac{D}{E}(r_U - r_D)(1 - T_C)$$
  3. Discount the levered cash flows by cost of levered equity
  $$NPV_{FTE} = CF_{0,E} + \frac{CF_E}{r_L}$$

10.5 Summary

- When cash flows are **perpetual**, the three methods yield the same result
- In other situations, they often don't

	Initial Cost	Future Cash Flows	Discount Rate
APV	ALL	$(EBIT)(1 - T_C)$ & Tax Shield	r_U
WACC	ALL	$(EBIT)(1 - T_C)$	r_{WACC}
FTE	Equity Portion	S/H Cash Flows	r_L

- WACC and FTE have the same "formula" as the $NPV_{unlevered}$
- APV has an extra adjustment term $PV_{financing}$

10.5.1 Which Method to Use?

- APV method needs the debt level in dollars
- WACC method needs the D/E ratio
- FTE method needs both

- WACC method is the most common method by far
- APV method is flexible when considering additional complications
  - Finite project life: Perpetuity CFs become annuities
  - Salvage values
  - PVCCATS
  - Complicated financing (subsidized loan)
    - Benefit of loan subsidies
    - Repayment of debt principal

Chapter 11

Dividends

Chapter 12

Utility Theory

12.1 Part I: Axioms of Cardinal Utility

12.1.1 Motivation: Decision Under Uncertainty

- How do people make decision under uncertainty?
  - The NPV rule: appropriately discount future cash flows
  - Expected gain is the <u>the basis</u> for decision
- It is (somewhat) unsatisfying to point to a number and say "you *should* do this"
  - Theoretical conclusions are correct under <u>all</u> the assumptions
  - Mathematical predictions are valid if <u>all</u> relevant factors are considered
- Another way to solve the problem is to observe then explain
  - Observe how human *actually* make decision under uncertainty
  - Develop a mathematical model to explain "*why* do you make that decision"
- The model should be
  - General enough To explain most (if not all) human decisions
  - Flexible enough to explain individual differences in decision making
- People usually don't make decisions based on the expected value principle
  1. Don't want to play fair game
  2. Don't even want to play favourable games
  3. Won't pay much to play games with HUGE expected gain
     - St. Petersburg Paradox
  4. For the same game, people will pay different amount to play
- We need an alternative principle to explain human preference:
  - **The (expected) utility theory**

12.1.2 Utility Theory - Terminology

Definition. Utility Function $u(\cdot)$
*$u(x)$ is the **utility** of consuming wealth \$$x$, unit of utility is called **utile***

- *1 utile has no real meaning*
- *the relative magnitude is important*

Definition. Lottery / Gamble
*A **lottery** or **gamble** is a probability distribution $(50\% - 50\%)$ over a set of outcomes (head-tail)*

Definition. Simple Lottery
*A **simple lottery** is denoted by $L(x, y; p)$, meaning payoff x with prob. p and payoff y with prob $1 - p$*

Definition. Compound Lottery
*A **compound lottery** can be as $L(X, Y; p)$ where X and Y are themselves lotteries (simple or compound)*

12.1.3 Utility Theory - Utility Function

- The more the better, so $u(\cdot)$ is increasing
  - For two amounts $\$x > \$y \Rightarrow u(x) > u(y)$
  - If $u(\cdot)$ is differentiable, then $u' > 0$
- Decreasing marginal utility of wealth, so $u(\cdot)$ is concave
  - If an investor is wealthier, each additional dollar has less utility
  - If $u(\cdot)$ is twice differentiable, then $u'' < 0$
- Instead of maximizing expected value $E(X)$, we may maximize expected utility $E[u(X)]$

12.1.4 About Decision Making

- Preference relations: used to rank investment opportunity
  - $X \succ Y$: X is strictly preferred to Y
  - $X \succcurlyeq Y$: X is weakly preferred to Y
  - $X \sim Y$: X is indifferent to Y
- As it turns out (no proof in this class)
  - $u(x)$ exists if and only if some axioms hold for preference relations
- Rational individual make preferences by maximizing the expected utility

Remark.

1. *Expected value is a special case of expected utility*
   - *For utility function* $u(x) = x \Rightarrow E[u(X)] = [X]$
2. *Individuals can make different decisions because their utility functions may be different.*

12.1.5 Axioms of Cardinal Utility

Axiom 1: Completeness / Comparability

- Given any two lotteries X and Y, exactly one of the following holds:
  - $X \succ Y$
  - $X \prec Y$
  - $X \sim Y$
- Decision maker can rank all investment opportunities
- One can always make decision on two alternatives
- Counterexample: Hesitation, you hesitate in making decisions not because of $X \sim Y$, but because of you are not informed about what X and Y

Axiom 2: Transitivity

- Given any three lotteries X, Y and Z, the following relationship must hold
  - $X \succ Y \,\&\, Y \succ Z \Rightarrow X \succ Z$
  - $X \sim Y \,\&\, Y \sim Z \Rightarrow X \sim Z$
- Preference cannot be cyclical (acyclicity)
  - What will happen if this is not true? (money pump)
- The self-torturer paradox:
  - A torturing device has 1000 levels, from no pain to excruciating pain/death
  - Turning up 1 level has negligible difference , but you will get $1000 for it
  - What's your preference between two consecutive levels $k \,\&\, k+1$
  - What's your preference between level 1 and level 1000?

Axiom 3: Continuity

- If $X \succ Y \succcurlyeq Z$ or $Z \succcurlyeq Y \succ X$, the there exists a unique probability p between 0 and 1 such that
  - $Y \sim L(X, Z; p)$
- "there is always something in between"
- Is this really true?
  - What is p if X receiving \$1000 and Z is immediate death?

Axiom 4: Independence / Substitution

- Given two lotteries X and Y, any lottery Z and any probability $0 < p < 1$
  - $X \succ Y \iff L(X, Z; p) \succ L(Y, Z; p)$
  - $X \sim Y \iff L(X, Z; p) \sim L(Y, Z; p)$
- Mix each of the two lotteries (X & Y) with a common lottery (Z)
  - The preference ordering of the resulting mixtures is independent of the third lottery Z & the mixing probability p
- Very useful in establishing preferences between more complex alternative based on preferences between simpler alternatives
- You've already seen two examples (without noticing them?)

Axiom 5: Monotonicity

- Given four lotteries X, Y, Z and W, and probabilities p_1 and p_2
- Suppose $X \succcurlyeq Y succcurlyeq Z$ and $X \succcurlyeq W succcurlyeq Z$
- $Y \sim L(X, Z; p_1)$ and $W \sim L(X, Z; p_2)$, then
  - $p_1 > p_2 \iff Y \succ W$
  - $p_1 = p_2 \iff Y \sim W$
  - $p_1 < p_2 \iff Y \prec W$
- These are "if and only if" statements
  - In proofs, need to argue both directions

12.1.6 Preference and Utility

Theorem 1 (no proof)

Preference relation $\succcurlyeq$ that satisfies Axioms 1-5 (and other technical assumptions) if and only if there exists a utility function $u(x)$ such that

$$E[u(X)] \geq E[u(Y)] \iff X \succcurlyeq Y$$

Remark. *All axioms can be restated/proved using utility function* $u(x)$

- Individuals may have different utility functions
  - How to assess your own utility function?
  - Utility functions for people who make similar decisions?

Theorem 2

$u(x)$ is unique up to an affine transformation, e.g.

$$u^*(x) = a \cdot u(x) + b, \;\; a > 0$$

express exactly the same preferences as $u(x)$ does.

12.1.7 Use Theorem 1 to Prove Axioms

Axiom 1: Completeness/Comparability

- Given any two lotteries X and Y, exactly one of the following holds:
  - $X \succ Y$, $X \prec Y$, $X \sim Y$
- Proof Sketch:
  - For any two lotteries X and Y (r.v.'s) and given utility function $u(\cdot)$
  - $E[u(X)]$ and $E[u(Y)]$ are numbers so **exactly one** of the following holds:
    * $E[u(X)] > E[u(Y)]$, $E[u(X)] < E[u(Y)]$, $E[u(X)] = E[u(Y)]$
  - By Theorem 1, exactly one of the following holds:
    * $X \succ Y$, $X \prec Y$, $X \sim Y$

Axiom 2: Transitivity

- Given any three lotteries X, Y and Z, the following relationship must hold
  - $X \succ Y \,\&\, Y \succ Z \Rightarrow X \succ Z$
  - $X \sim Y \,\&\, Y \sim Z \Rightarrow X \sim Z$
- Proof Sketch:

- Apply Theorem 1 to translate preferences into expected utilities (r.v.s' to numbers)
$$X \succ Y \,\&\, Y \succ Z \Rightarrow E[u(X)] > E[u(Y)] > E[u(Z)]$$
- Apply Theorem 1 again to translate expected utilities into preferences
$$E[u(X)] > E[u(Z)] \Rightarrow X \succ Z$$
- Similar proof for the "$\sim$"

Axiom 3: Continuity

- If $X \succ Y \succcurlyeq Z$ or $Z \succcurlyeq Y \succ X$, the there exists a unique probability p between 0 and 1 such that
  - $Y \sim L(X, Z; p)$
- Proof Sketch:
  The key is to show (1). $0 \leq p \leq 1$ and (2). uniqueness of p
  - Apply Theorem 1 to translate given conditions/final goal
$$X \succ Y \succcurlyeq Z \Rightarrow E[u(X)] > E[u(Y)] > E[u(Z)]$$
$$Y \sim L(X, Z; p) \Rightarrow E[u(Y)] = pE[u(X)] + (1-p)E[u(Z)]$$
  - The second equation implies uniqueness of p
$$p = \frac{E[u(Z)] - E[u(Y)]}{E[u(Z)] - E[u(X)]}$$
  - The first equation implies that $0 \leq p \leq 1$

Axiom 4: Independence / Substitution

- Given two lotteries X and Y, any lottery Z and any probability $0 < p < 1$
  - $X \succ Y \iff L(X, Z; p) \succ L(Y, Z; p)$
  - $X \sim Y \iff L(X, Z; p) \sim L(Y, Z; p)$
- Proof Sketch:
  For convenience, define $\tilde{X} \sim L(X, Z; p) \,\&\, \tilde{Y} \sim L(Y, Z; p)$
  - The expected utilities for $X \,\&\, Y$ are: $E[u(X)] \,\&\, E[u(Y)]$
  - The expected utilities for $\tilde{X} \,\&\, \tilde{Y}$ are:
$$E[u(\tilde{X})] = pE[u(X)] + (1-p)E[u(Z)]$$
$$E[u(\tilde{Y})] = pE[u(Y)] + (1-p)E[u(Z)]$$

    COMPLETE IT!

$\Rightarrow$ Since $X \succ Y$, $E[u(X)] > E[u(Y)]$, $E[u(\tilde{X})] > E[u(\tilde{Y})]$
$\Leftarrow$ Since $\tilde{X} \succ \tilde{Y}$, $E[u(\tilde{X})] > E[u(\tilde{Y})]$,

$$E[u(X)] = \frac{E[u(\tilde{X})] - (1-p)E[u(Z)]}{p} > \frac{E[u(\tilde{Y})] - (1-p)E[u(Z)]}{p} = E(Y)$$

Axiom 5: Monotonicity

- Given four lotteries X, Y, Z and W, and probabilities p_1 and p_2
- Suppose $X \succcurlyeq Y \succcurlyeq Z$ and $X \succcurlyeq W \succcurlyeq Z$
- $Y \sim L(X, Z; p_1)$ and $W \sim L(X, Z; p_2)$, then
  - $p_1 > p_2 \iff Y \succ W$
  - $p_1 = p_2 \iff Y \sim W$
  - $p_1 < p_2 \iff Y \prec W$
- Proof Sketch:
  Given that $X \succcurlyeq Z$, we have (by Theorem 1) $E[u(X)] \geq E[u(Z)]$
  - The expected utilities for Y & W are :

$$E[u(Y)] = p_1 E[u(X)] + (1-p_1)E[u(Z)]$$

$$E[u(W)] = p_2 E[u(X)] + (1-p_2)E[u(Z)]$$

COMPLETE IT!

$$\begin{aligned} E[u(Y)] - E[u(W)] &= (p_1 - p_2)E[u(X)] - (p_1 - p_2)E[u(Z)]) \\ &= (p_1 - p_2)(E[u(X)] - E[u(Z)]) \end{aligned}$$

12.1.8 Summary: Axioms of Cardinal Utility

- Prescribing "correct decision" via expected value principle is insufficient
  - **Expected utility theory** describes human decisions under uncertainty
  - **Expected utility theory** can be used for predictions as well
    - There exist counter-examples/paradoxes for the axioms
    - These predictions are not perfect either, but at least worth a try
- Define/Illustrate/Prove five **Axioms of Cardinal Utility**
  - There are alternative/equivalent axioms that gives Theorem 1 too
  - Illustration of paradoxes using real examples
  - "Prove" the axioms using Theorem 1

    * Axioms are the foundations for the expected utility theory (Theorem 1)

- Puzzle: We did the proofs without knowing what $u(\cdot)$ is
  - $u' > 0$ and $u'' < 0$ are unnecessary for a mathematically valid $u(\cdot)$
  - However, $u(\cdot)$ without these conditions lead to "weird" decisions.

12.2 Part II: Utility Functions and Applications

12.2.1 Preference and Utility

Theorem 1 (no proof)

Preference relation $\succcurlyeq$ that satisfies Axioms 1-5 (and other technical assumptions) if and only if there exists a utility function $u(x)$ such that

$$E[u(X)] \geq E[u(Y)] \iff X \succcurlyeq Y$$

Theorem 2

$u(x)$ is unique up to an affine transformation, e.g.

$$u^*(x) = a \cdot u(x) + b, \;\; a > 0$$

express exactly the same preferences as $u(x)$ does.

- Proof: Consider **any** lotteries X and Y. If $X \succ Y$ for $u(\cdot)$, then by Theorem 1, $E[u(X)] > E[u(Y)]$. If $u^*(x) = a \cdot u(x) + b$ and $a > 0$, then

$$E[u^*(X)] = E[a \cdot u(x) + b] = aE[u(X)] + b > aE[u(Y)] + b = E[u^*(Y)]$$

  Since $E[u^*(X)] > E[u^*(Y)]$, then $X \succ Y$ for $u^*(\cdot)$ by Theorem 1

If $u^*(x) = a \cdot u(x) + b$ and $a > 0$, then

$$E[u^*(X)] = E[a \cdot u(X) + b] = aE[u(X)] + b > aE[u(Y)] + b = E[u^*(Y)]$$

Since $E[u^*(X)] > E[u^*(Y)]$, then $X \succ Y$ for $u^*(\cdot)$

12.2.2 Utility Functions

In this course, assume $u'(X) > 0$ and $u''(X < 0)$ unless stated otherwise

Name	Formula	Wealth Range	Remark
Logarithmic	$u(x) = \log(x)$	$x > 0$	What is the base?
Quadratic	$u(x) = x - ax^2$	$x < \frac{1}{2a}$	$a > 0$
Exponential	$u(x) = -e^{-ax}$	$-\infty < x < \infty$	$a > 0$
Power	$u(x) = \frac{x^a - 1}{a}$	$x > 0$	$a < 1, a \neq 0$
Fractional Power	$u(x) = x^a$	$x > 0$	$0 < a < 1$

Remark. *We prefer the lotteries giving higher expected utility, and different utility functions allows for different preferences!*

12.2.3 Logarithmic Utility Function and Affine Transformation

- Change-of Base formula for logarithmic functions

$$u^*(x) = \log_p(x) = \frac{\log_q(x)}{\log_q(p)} = a \cdot u(x) + b$$

  - $u(x) = \log_q(x)$
  - $a = \frac{1}{\log_q(p)} = \log_p(q) > 0$ provided that $p, q > 0$ and $p, q \neq 1$
  - $b = 0$

12.2.4 Certainty Equivalent

Definition. Certainty Equivalent *Given lottery X, the fixed monetary amount with the same utility as X is called the **certain equivalent** of X, denoted by $CE[X]$.*

$$u(CE(X)) = E[u(X)]$$

Remark. *For the same X, different $u(\cdot)$ can result in different $CE(X)$*

- *maximize $E[u(X)] \iff$ maximize $u(CE(X))$*
- *If $u(x)$ is increasing, maximize $u(CE(X)) \iff$ maximize $CE(X)$*

In calculation:

$$CE(X) = u^{-1}(E[u(X)])$$

where $u^{-1}(\cdot)$ is defined such $u^{-1}[u(x)] = x$

Example: T/F: If $E[]$
false because we are not given u, but if u is increasing, then it is true

12.3 Risk Aversion

12.3.1 Motivation: Why do we need the utility theory?

- Gain understanding of some risk aversion behaviours
  - Most people are risk averse, don't want to play fair games
  - Some gamblers are risk seeking, don't prefer additional risk
  - Some people are neither, just neutral to risk
- How to characterize these groups of behaviours in one mathematical framework?
  - Warning: relaxing the sign of $u''(x)$
- Increasing **concave** utility describes "risk aversion" $(u''(x) < 0)$

- Common utility functions in general

  Remark. *Risk Averse*

- Increasing **linear** utility describes "risk neutral" ($u''(x) = 0$)
  - Expected value principle as a special case

    Remark. *Risk Neutral*

- Increasing **convex** utility describes "risk seeking" ($u''(x) > 0$)
  - These are utility functions for heavy gamblers

    Remark. *Risk Seeking*

12.3.2 Characterizations of Risk Aversion: Definitions

Definition. Risk Averse *Let X be a random payoff and $E[X]$ be its actuarial value. An investor is called* ***risk averse*** *if and only if*

$$u(E[X]) > E[u(X)] \text{ or equicalently, } E[X] > CE(X)$$

- Alternative definitions
  - $E[X]$ with certainty is strictly preferred to X
  - $u(x)$ is increasing and **concave**, i.e. $u'(x) > 0$ and $u''(x) < 0$
  - Can be shown using Jensen's Inequality
- Other risk appetite characterizations
  - Risk seeking: $u(E[X]) < E[u(X)]$, or equivalently, $E[X] < CE(X)$
  - Risk neutral: $u(E[X]) = E[u(X)]$, or equivalently, $E[X] = CE(X)$

12.3.3 Insurance Premiums: A Utility Viewpoint

- Utility theory explains why insurance premiums are higher than expected loss
  - Assume people wanting insurance are **risk averse**
- Idea:

$$\text{expected utility of having insurance} = \text{expected utility of not having insurance}$$

$$\Rightarrow E[u(x_0 - X_{\text{with insurance}} - P)] = E[u(x_0 - X_{\text{without insurance}})]$$
$$\Rightarrow P = ?$$

12.3.4 Measures of Risk Aversion

Absolute Risk Aversion (ARA)

$$ARA(x) = -\frac{u''(x)}{u'(x)}$$

Relative Risk Aversion (RRA)

$$RRA(x) = x \cdot ARA(x)$$

Remark.

- *Risk neutral* $\iff ARA = RRA = 0$
- *Risk loving* $\iff ARA < 0, RRA < 0$