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$$D_+ = \frac{f(x+h) - f(x)}{h} \quad D_- = \frac{f(x) - f(x-h)}{h}$$

$$D_+^2 = \frac{f(x+h) - f(x)}{2h} - \frac{f(x) - f(x-h)}{2h} = \frac{f(x+h) - f(x-h)}{2h}$$

$$\frac{f(x+2h) - f(x)}{2h} - \frac{f(x) - f(x-2h)}{2h} = D_+^2 = \frac{f(x+2h) - 2f(x) + f(x-2h)}{4h^2}$$

$$\frac{f(x+3h) - 2f(x+h) + f(x-h)}{4h^2} - \frac{f(x+h) - 2f(x-h) + f(x-3h)}{4h^2}$$

$$D_+^3 = \frac{f(x+3h) - 3f(x+h) + 3f(x-h) - f(x-3h)}{8h^3}$$

$$\frac{f(x+4h) - 3f(x+2h) + 3f(x) - f(x-2h)}{8h^3} - \frac{f(x+2h) - 3f(x) + 3f(x-2h) - f(x-4h)}{8h^3}$$

$$D_+^4 = \frac{f(x+4h) - 4f(x+2h) + 6f(x) - 4f(x-2h) + f(x-4h)}{16h^4}$$

$$\left. \begin{aligned} 2h &= h' \\ x+2h' &= x_j + 2 \\ x+h' &= x_j + 1 \\ x &= x_j \\ x-h' &= x_j - 1 \\ x-2h' &= x_j - 2 \end{aligned} \right\}$$

$$D_+^4 f(x_j) = \frac{f(x_j+2) - 4f(x_j+1) + 6f(x_j) - 4f(x_j-1) + f(x_j-2)}{8h^4}$$

Aproximación orden — 4 $O(h^4)$