How to Use this Reviewer

Hello! This is a compilation of solved exercises for module 4 of MATH 51.4. All of these exercises are taken straight from Aldrich and Cisco's course notes, so you can expect tests to be very similar to the items given. Some items are bound to be a little bit trickier than others, so I'll note when these items show up.

Normal items will look like this:

1 A very easy math problem. What's $1 + 1$?

whereas difficult problems will be soulless, like this:

2 A very difficult math problem. Prove that $\binom{2n}{n} < 2^{2n-2}, \ \forall n \geq 5$ using induction.

I might also include warnings in my **Nerd Interjections!**

Nerd Interjection![a] These sections are for me to remind you of some necessary information to solve the problems, elaborate on something that I think isn't all that clear with just pure math symbols, give a helpful theorem, be an annoying piece of shit, anything, really! Just think of it as a tips and tricks section.

[a] Image from @Ellem__ on Twitter.

I also have another section called **Can we Prove it?** (unfortunately lacking a cute picture to go along with it; Mikh was nice enough to edit one up for me, but I haven't been able to format it in a way I like), where I include some interesting, not really necessary, but nonetheless relevant proofs. So far, these two are my only two gimmicks, but I might add more in the future.

Can we Prove it? This is just a random proof I yoinked from our homeworks.

Proof. ($\Longrightarrow$) Let $x \in (A \cap B) \setminus C$. Then, $x \in (A \cap B)$ and $x \notin C$.
Since $x \in (A \cap B)$, $x \in A$ and $x \in B$.
Since $x \in A$ and $x \notin C$, $x \in (A \setminus C)$.
Since $x \in B$ and $x \notin C$, $x \in (B \setminus C)$.
Thus, $x \in (A \setminus C) \cap (B \setminus C)$.

($\Longleftarrow$) Let $x \in (A \setminus C) \cap (B \setminus C)$. Then, $x \in (A \setminus C)$ and $x \in (B \setminus C)$.
Since $x \in (A \setminus C)$, $x \in A$ and $x \notin C$.
Since $x \in (B \setminus C)$, $x \in B$ and $x \notin C$.
Since $x \in A$ and $x \in B$, $x \in (A \cap B)$.
Thus, $x \in (A \cap B) \setminus C$.

Since both sides of the conditional are true, it holds that $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$. $\square$

Finally, there are blue boxes to indicate when instructions aren't obvious from the question itself, or if there are similar items that can be grouped together.

For items #7 to #12, we need to reevaluate our life decisions.

It's very important to note that this is a *work in progress!* I am human, and I will make mistakes, and I cannot finish doing all the exercises within the span of one day. If you spot anything wrong, please feel free to message me; I will correct it as soon as possible.

As a final note, these are not replacements for the modules/paying attention in class, these are supplements for them. I won't explain all the topics here, and I'll assume that you at least have read the basics, so don't treat these reviewers as your only source of information. Our teachers spends a lot of time on the handouts, they're really good! (except when they're wrong) With that, though, I think I've covered all pertinent points. Good luck, and happy studying!

4.1.1: Determinants

This is mostly pure computation, but there are some proving and show questions later on, which are a little bit interesting. Also, spoiler, items #10 and #11 are foreshadowing for eigenvalues. You'll see!

Find the determinants of the following matrices. In case they have elements with variables, solve for the determinants in terms of those variables.

1 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2 $\begin{bmatrix} 0 & 6 & 0 \\ 3 & 99 & -1 \\ -2 & 99 & 5 \end{bmatrix}$

3 $\begin{bmatrix} 0 & 2 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 3 \\ 0 & 4 & 0 & 5 & 6 \end{bmatrix}$

4 $\begin{bmatrix} 1 & -1 & -1 & -1 & -1 \\ 1 & 2 & -1 & -1 & -1 \\ 1 & 2 & 3 & -1 & -1 \\ 1 & 2 & 3 & 4 & -1 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$

5 $\begin{bmatrix} 0 & 0 & 0 & 0 & 5 & 5 \\ 0 & 0 & 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 6 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 4 & 4 & 4 \end{bmatrix}$

6 $\begin{bmatrix} a & 0 & a \\ 0 & a & 0 \\ a & 0 & -a \end{bmatrix}$

7 $\begin{bmatrix} b & b & c \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

8 $\begin{bmatrix} d & d & e & e \\ 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

9 $\begin{bmatrix} 1 & 1 & 1 \\ r & s & t \\ r^2 & s^2 & t^2 \end{bmatrix}$

10 $\begin{bmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix}$

11 $\begin{bmatrix} 1-\lambda & 1 & -1 \\ -1 & 2-\lambda & 15 \\ 1 & -18 & 3-\lambda \end{bmatrix}$