# Analysis of Suicide Trends

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#### INTRODUCTION

The aim of this project is to examine global trends in suicides in order to determine whether demographic variables are associated with suicide trends. The effectiveness of the developed models is assessed using the Root Mean Squared Error (RMSE).

The Suicides dataset contains global suicide rates from 1985 to 2016, as well as demographic and socio-economic variables. For purposes of this project, the explanatory variables of interest are: sex, age, generation, population, and continent. The Suicides dataset was divided into training and test sets, both of which contain 50% of the observations.

For the first part of this project, the features of the Suicides dataset are explored to determine possible trends in suicide rates. This is followed by the analysis section. Models were developed based on the assumption that the number of suicides  $(\hat{Y})$  is a function of demographic variables (X), where (X) includes sex, age, generation, population, and continent.

The project concludes with a summary of its findings, as well as a discussion of its limitation and possible extensions of the analysis.

#### EXPLORATORY DATA ANALYSIS

In this section, the Suicides dataset is explored and analyzed in order to determine the features of the data and to detect possible trends.

```
library(tidyverse)
```

```
## -- Attaching packages ----- tidyverse 1.3.0 --
## v ggplot2 3.2.1
                      v purrr
                               0.3.3
## v tibble 2.1.3
                      v dplyr
                               0.8.3
            1.0.0
## v tidyr
                      v stringr 1.4.0
            1.3.1
                      v forcats 0.4.0
## v readr
## Warning: package 'ggplot2' was built under R version 3.6.1
## Warning: package 'tidyr' was built under R version 3.6.1
## Warning: package 'readr' was built under R version 3.6.1
## Warning: package 'purrr' was built under R version 3.6.1
## Warning: package 'dplyr' was built under R version 3.6.1
## Warning: package 'forcats' was built under R version 3.6.1
## -- Conflicts ----- tidyverse conflicts() --
## x dplyr::filter() masks stats::filter()
                   masks stats::lag()
## x dplyr::lag()
library(caret)
## Warning: package 'caret' was built under R version 3.6.1
## Loading required package: lattice
```

```
##
## Attaching package: 'caret'
## The following object is masked from 'package:purrr':
       lift
##
library(lubridate)
## Warning: package 'lubridate' was built under R version 3.6.1
## Attaching package: 'lubridate'
## The following object is masked from 'package:base':
##
       date
library(countrycode)
## Warning: package 'countrycode' was built under R version 3.6.1
library(broom)
## Warning: package 'broom' was built under R version 3.6.1
library(car)
## Warning: package 'car' was built under R version 3.6.1
## Loading required package: carData
## Warning: package 'carData' was built under R version 3.6.1
## Attaching package: 'car'
## The following object is masked from 'package:dplyr':
##
##
       recode
## The following object is masked from 'package:purrr':
##
##
       some
library(randomForest)
## Warning: package 'randomForest' was built under R version 3.6.1
## randomForest 4.6-14
## Type rfNews() to see new features/changes/bug fixes.
## Attaching package: 'randomForest'
## The following object is masked from 'package:dplyr':
##
##
       combine
## The following object is masked from 'package:ggplot2':
##
       margin
```

```
library(ranger)
```

```
## Warning: package 'ranger' was built under R version 3.6.2
## Attaching package: 'ranger'
## The following object is masked from 'package:randomForest':
##
##
       importance
```

At the outset, the data should be downloaded and unzipped. The Suicides dataset can be found in Kaggle Suicide Rates Overview 1985 to 2016.

```
file <- unzip("./suicide-rates-overview-1985-to-2016.zip")
suicides <- read.csv("./master.csv")</pre>
```

This is followed by an examination of the dataset.

#### glimpse(suicides)

```
## Observations: 27,820
## Variables: 12
## $ i..country
                    <fct> Albania, Albania, Albania, Albania, Albania, Alb...
## $ year
                    <int> 1987, 1987, 1987, 1987, 1987, 1987, 1987, 1987, ...
## $ sex
                    <fct> male, male, female, male, male, female, female, ...
## $ age
                    <fct> 15-24 years, 35-54 years, 15-24 years, 75+ years...
                    <int> 21, 16, 14, 1, 9, 1, 6, 4, 1, 0, 0, 0, 2, 17, 1,...
## $ suicides_no
## $ population
                    <int> 312900, 308000, 289700, 21800, 274300, 35600, 27...
## $ suicides.100k.pop
                    <dbl> 6.71, 5.19, 4.83, 4.59, 3.28, 2.81, 2.15, 1.56, ...
## $ country.year
                    <fct> Albania1987, Albania1987, Albania1987, Albania19...
                    ## $ HDI.for.year
                    <fct> "2,156,624,900", "2,156,624,900", "2,156,624,900...
## $ gdp_for_year....
## $ generation
                    <fct> Generation X, Silent, Generation X, G.I. Generat...
```

#### summary(suicides)

```
##
          i..country
                                            sex
                              year
                                                                 age
##
   Austria
               :
                  382
                                :1985
                                        female:13910
                                                        15-24 years:4642
                        \mathtt{Min}.
## Iceland
                  382
                         1st Qu.:1995
                                        male :13910
                                                        25-34 years:4642
## Mauritius :
                  382
                        Median:2002
                                                        35-54 years:4642
   Netherlands:
                  382
                                :2001
                                                        5-14 years :4610
##
                        Mean
## Argentina :
                  372
                         3rd Qu.:2008
                                                        55-74 years:4642
## Belgium
                  372
                        Max.
                                :2016
                                                        75+ years :4642
##
   (Other)
               :25548
##
     suicides_no
                        population
                                          suicides.100k.pop
                                                                  country.year
##
                                                  : 0.00
  Min.
          :
                0.0
                      Min.
                            :
                                    278
                                          Min.
                                                             Albania1987:
                                                                             12
   1st Qu.:
                3.0
                      1st Qu.:
                                  97498
                                          1st Qu.: 0.92
                                                             Albania1988:
                                                                             12
               25.0
                                                             Albania1989:
                                                                             12
##
  Median :
                      Median :
                                430150
                                          Median: 5.99
                      Mean
##
    Mean
           : 242.6
                              : 1844794
                                          Mean
                                                 : 12.82
                                                             Albania1992:
                                                                             12
##
    3rd Qu.: 131.0
                      3rd Qu.: 1486143
                                          3rd Qu.: 16.62
                                                             Albania1993:
                                                                             12
           :22338.0
##
   Max.
                      Max.
                              :43805214
                                          Max.
                                                  :224.97
                                                             Albania1994:
                                                                             12
##
                                                             (Other)
                                                                        :27748
##
    HDI.for.year
                              gdp_for_year.... gdp_per_capita....
##
  Min.
           :0.483
                    1,002,219,052,968:
                                          12
                                               Min.
                                                       :
   1st Qu.:0.713
                    1,011,797,457,139:
                                          12
                                               1st Qu.:
                                                          3447
```

```
Median : 0.779
                      1,016,418,229
                                             12
                                                  Median:
                                                             9372
##
            :0.777
                                             12
##
    Mean
                      1,018,847,043,277:
                                                  Mean
                                                          : 16866
    3rd Qu.:0.855
                                                  3rd Qu.: 24874
##
                      1,022,191,296
                                             12
            :0.944
                                             12
##
    Max.
                      1,023,196,003,075:
                                                  Max.
                                                          :126352
##
    NA's
            :19456
                      (Other)
                                         :27748
##
               generation
##
    Boomers
                     :4990
    G.I. Generation: 2744
##
##
    Generation X
                     :6408
##
    Generation Z
                     :1470
    Millenials
                     :5844
##
    Silent
                     :6364
##
```

#### head(suicides)

## [1] 128

```
##
                                      age suicides_no population suicides.100k.pop
     i...country year
                         sex
## 1
        Albania 1987
                        male 15-24 years
                                                            312900
                                                    21
                                                                                 6.71
## 2
        Albania 1987
                        male 35-54 years
                                                     16
                                                            308000
                                                                                 5.19
##
   3
        Albania 1987 female 15-24 years
                                                     14
                                                            289700
                                                                                 4.83
## 4
        Albania 1987
                        male
                                75+ years
                                                     1
                                                             21800
                                                                                 4.59
                                                     9
## 5
        Albania 1987
                        male 25-34 years
                                                            274300
                                                                                 3.28
## 6
                                75+ years
                                                      1
                                                             35600
                                                                                 2.81
        Albania 1987 female
##
     country.year HDI.for.year gdp_for_year.... gdp_per_capita....
                                                                             generation
## 1
      Albania1987
                              NA
                                    2,156,624,900
                                                                    796
                                                                           Generation X
## 2
      Albania1987
                              NA
                                    2,156,624,900
                                                                    796
                                                                                 Silent
## 3
      Albania1987
                              NA
                                    2,156,624,900
                                                                    796
                                                                           Generation X
                                                                    796 G.I. Generation
## 4
      Albania1987
                              NA
                                    2,156,624,900
## 5
      Albania1987
                              NA
                                    2,156,624,900
                                                                    796
                                                                                Boomers
## 6
      Albania1987
                              NA
                                    2,156,624,900
                                                                    796 G.I. Generation
```

There are 27,820 observations with 12 variables. The dependent variable is number of suicides (suicides\_no). The explanatory variables of interest are: country, sex, age, generation, and population.

#### Trends in Number of Suicides

```
summary(suicides$suicides_no)

## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.0 3.0 25.0 242.6 131.0 22338.0

sd(suicides$suicides_no)

## [1] 902.0479

IQR(suicides$suicides_no)
```

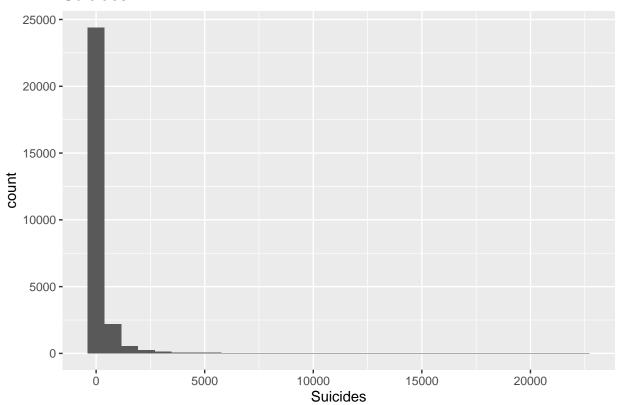
The value of the mean is significantly higher than the median. This shows that there are outliers in the data. Additionally, the standard deviation is 902.05. As the variance is larger than the mean, a Quasipoisson analysis might be worth considering.

The histograms below confirm the presence of outliers. Furthermore, the first histogram shows that the suicide numbers are heavily right skewed. Taking the log transformation of the number of suicides elicits a histogram which better illustrates the trends in the data.

```
suicides %>%
ggplot(aes(suicides_no)) + geom_histogram() +
  labs(title = "Suicides", x = "Suicides")
```

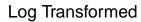
## `stat\_bin()` using `bins = 30`. Pick better value with `binwidth`.

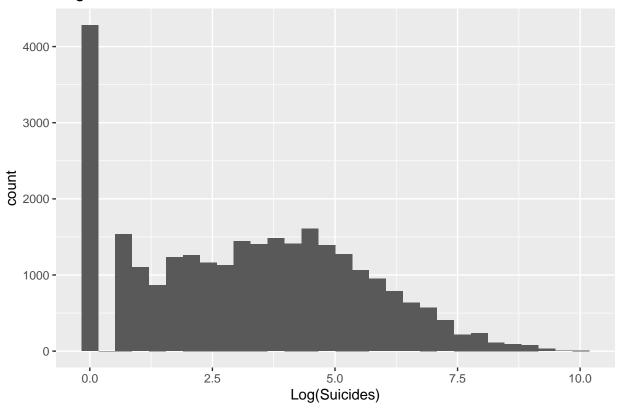
### Suicides



```
suicides %>%
ggplot(aes(log(suicides_no + 1))) + geom_histogram() +
   labs(title = "Log Transformed", x = "Log(Suicides)")
```

## `stat\_bin()` using `bins = 30`. Pick better value with `binwidth`.





# Trends in Suicides per Year

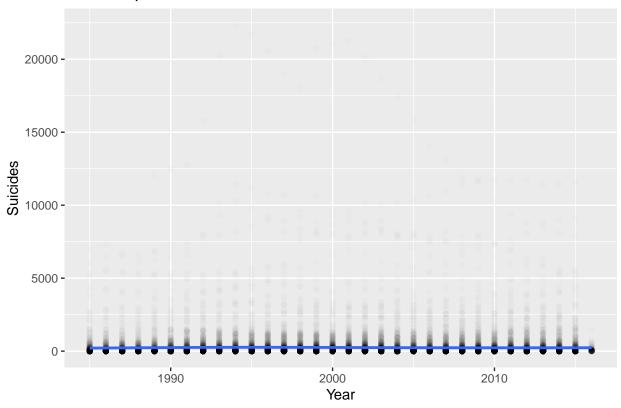
The plot below shows that there is no clear time trend in the number of suicides.

```
suicides$year <- as.Date(as.character(suicides$year), format = "%Y")
suicides$year <- year(suicides$year)

suicides %>%
   ggplot(aes(year, suicides_no)) +
   geom_point(alpha = 0.01) + geom_smooth() +
   labs(title = "Suicides per Year", x = "Year",
        y = "Suicides")
```

##  $geom_smooth()$  using method = 'gam' and formula 'y ~ s(x, bs = "cs")'

### Suicides per Year



# Trends per Country and Continent

```
colnames(suicides)[1] <- "country"</pre>
names(suicides)
   [1] "country"
                              "year"
                                                    "sex"
##
   [4] "age"
                              "suicides_no"
                                                    "population"
  [7] "suicides.100k.pop"
                              "country.year"
                                                    "HDI.for.year"
## [10] "gdp_for_year...."
                              "gdp_per_capita...." "generation"
levels(suicides$country)
                                          "Antigua and Barbuda"
##
     [1] "Albania"
##
     [3] "Argentina"
                                          "Armenia"
##
     [5] "Aruba"
                                          "Australia"
     [7] "Austria"
##
                                         "Azerbaijan"
##
     [9] "Bahamas"
                                          "Bahrain"
## [11] "Barbados"
                                          "Belarus"
                                          "Belize"
   [13] "Belgium"
##
                                         "Brazil"
##
   [15] "Bosnia and Herzegovina"
   [17] "Bulgaria"
                                          "Cabo Verde"
   [19] "Canada"
                                          "Chile"
##
    [21] "Colombia"
##
                                          "Costa Rica"
   [23] "Croatia"
                                          "Cuba"
##
  [25] "Cyprus"
                                          "Czech Republic"
```

```
[27] "Denmark"
                                          "Dominica"
##
    [29] "Ecuador"
                                          "El Salvador"
##
    [31] "Estonia"
                                          "Fiji"
   [33] "Finland"
##
                                          "France"
##
    [35] "Georgia"
                                          "Germany"
##
   [37] "Greece"
                                          "Grenada"
   [39] "Guatemala"
                                          "Guyana"
   [41] "Hungary"
                                          "Iceland"
##
    [43] "Ireland"
##
                                          "Israel"
   [45] "Italy"
                                          "Jamaica"
##
   [47] "Japan"
                                          "Kazakhstan"
                                          "Kuwait"
##
   [49] "Kiribati"
                                          "Latvia"
    [51] "Kyrgyzstan"
   [53] "Lithuania"
                                          "Luxembourg"
##
##
   [55] "Macau"
                                          "Maldives"
##
    [57] "Malta"
                                          "Mauritius"
##
   [59] "Mexico"
                                          "Mongolia"
   [61] "Montenegro"
                                          "Netherlands"
##
   [63] "New Zealand"
                                          "Nicaragua"
##
   [65] "Norway"
                                          "Oman"
                                          "Paraguay"
##
   [67] "Panama"
   [69] "Philippines"
                                          "Poland"
##
   [71] "Portugal"
                                          "Puerto Rico"
    [73] "Qatar"
                                          "Republic of Korea"
##
##
   [75] "Romania"
                                          "Russian Federation"
   [77] "Saint Kitts and Nevis"
                                          "Saint Lucia"
##
   [79] "Saint Vincent and Grenadines"
                                          "San Marino"
   [81] "Serbia"
                                          "Seychelles"
                                          "Slovakia"
##
  [83] "Singapore"
                                          "South Africa"
  [85] "Slovenia"
##
   [87] "Spain"
                                          "Sri Lanka"
##
   [89] "Suriname"
                                          "Sweden"
##
   [91] "Switzerland"
                                          "Thailand"
   [93] "Trinidad and Tobago"
                                          "Turkey"
##
                                          "Ukraine"
##
    [95] "Turkmenistan"
    [97] "United Arab Emirates"
                                          "United Kingdom"
##
   [99] "United States"
                                          "Uruguay"
## [101] "Uzbekistan"
```

The country variable is coded as a factor variable with 101 levels. To make the analysis more tractable, a continent variable will be added. The models will be using the continent variable, rather than the country variable.

```
## $ population
                       : int 312900 308000 289700 21800 274300 35600 278800 257200 137500 311000 ...
## $ suicides.100k.pop : num 6.71 5.19 4.83 4.59 3.28 2.81 2.15 1.56 0.73 0 ...
## $ country.year
                       : Factor w/ 2321 levels "Albania1987",..: 1 1 1 1 1 1 1 1 1 1 ...
## $ HDI.for.year
                        : num NA NA NA NA NA NA NA NA NA ...
## $ gdp_for_year.... : Factor w/ 2321 levels "1,002,219,052,968",..: 727 727 727 727 727 727 727 727 727
## $ gdp per capita...: int 796 796 796 796 796 796 796 796 796 ...
                       : Factor w/ 6 levels "Boomers", "G.I. Generation", ...: 3 6 3 2 1 2 6 1 2 3 ...
## $ generation
                        : chr "Europe" "Europe" "Europe" "...
## $ continent
suicides$continent <- factor(suicides$continent)</pre>
levels(suicides$continent)
## [1] "Africa"
                  "Americas" "Asia"
                                                   "Oceania"
                                        "Europe"
suicides %>%
   group_by(continent) %>%
    summarize(mean = mean(suicides_no),
              median = median(suicides_no),
              sd = sd(suicides_no),
              n = n()) \%
    arrange(desc(mean))
## # A tibble: 5 x 5
     continent mean median
               <dbl> <dbl>
                             <dbl> <int>
##
     <fct>
## 1 Europe
               299.
                         60 1061.
                                   11418
## 2 Asia
               271.
                         20 838.
                                    5366
## 3 Americas 194.
                          9 798.
                                    9214
## 4 Oceania
               87.3
                         23 148.
                                     972
## 5 Africa
                13.4
                          4
                              23.7
                                     850
Europe has the highest mean and median suicides, while Africa has the lowest.
suicides %>%
  group_by(country) %>%
  summarize(total = sum(suicides_no)) %>%
  arrange(desc(total)) %>%
  top_n(10)
## Selecting by total
## # A tibble: 10 x 2
##
      country
                           total
##
      <fct>
                           <int>
   1 Russian Federation 1209742
## 2 United States
                         1034013
## 3 Japan
                          806902
## 4 France
                          329127
## 5 Ukraine
                          319950
## 6 Germany
                          291262
## 7 Republic of Korea
                          261730
## 8 Brazil
                          226613
## 9 Poland
                          139098
## 10 United Kingdom
                          136805
```

The country with the highest total number of suicides is Russia, followed by the United States and Japan.

### Trends per Sex

```
levels(suicides$sex)

## [1] "female" "male"

suicides %>%
    group_by(sex) %>%
    summarize(total = sum(suicides_no))

## # A tibble: 2 x 2

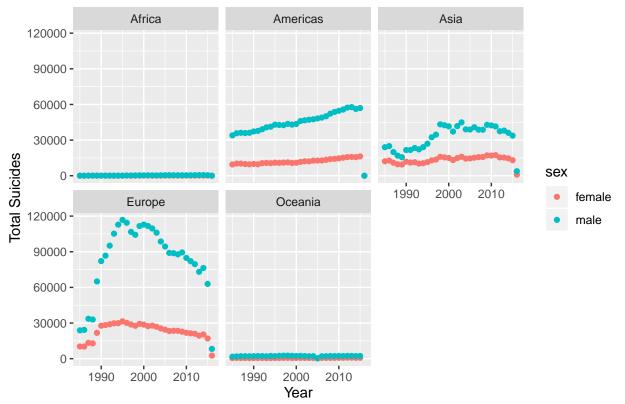
## sex total

## <fct> <int>
## 1 female 1559510

## 2 male 5188910
```

Males have a higher number of suicides than females. This trend can be seen in the Americas, Asia and Europe.

### Total Suicides, by Continent and Sex



### Trends per Age

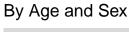
```
levels(suicides$age)
## [1] "15-24 years" "25-34 years" "35-54 years" "5-14 years" "55-74 years"
## [6] "75+ years"
```

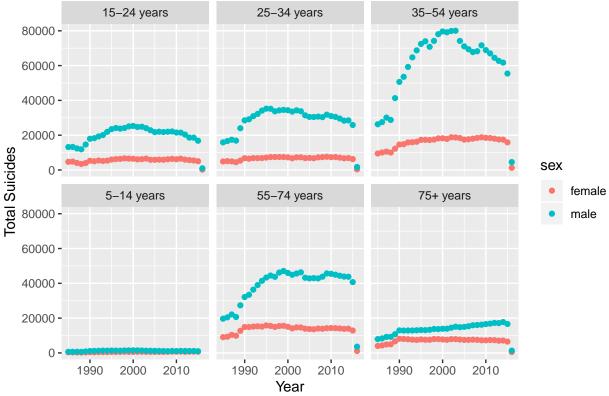
Age is encoded as a factor variable with 6 levels: 5-14 years, 15-24 years, 25-34 years, 35-54 years, 55-74 years, and 75+ years.

```
suicides %>%
  group_by(age) %>%
  summarize(total = sum(suicides_no)) %>%
  arrange(desc(total))
```

```
## # A tibble: 6 x 2
## age total
## <fct> <int>
## 1 35-54 years 2452141
## 2 55-74 years 1658443
## 3 25-34 years 1123912
## 4 15-24 years 808542
## 5 75+ years 653118
## 6 5-14 years 52264
```

The highest number of total suicides are in the age groups which correspond to adults, i.e., 25-74 years. The trend that suicide rates for males are higher can also be seen in the adult age groups.





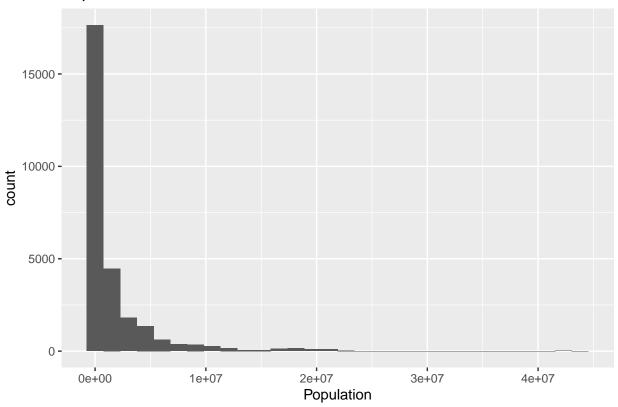
# Trends by Population

The population is also heavily right skewed. The log transformation of this variable can better show its distribution.

```
suicides %>%
ggplot(aes(population)) + geom_histogram() +
labs(title = "Population", x = "Population")
```

## `stat\_bin()` using `bins = 30`. Pick better value with `binwidth`.

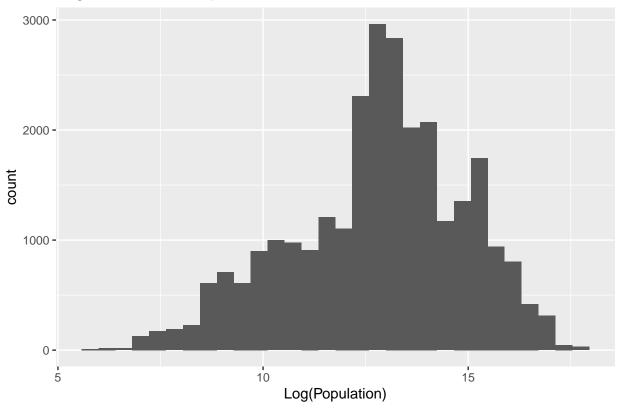
# Population



```
suicides %>%
ggplot(aes(log(population))) + geom_histogram() +
labs(title = "Log Transformed Population", x = "Log(Population)")
```

## `stat\_bin()` using `bins = 30`. Pick better value with `binwidth`.

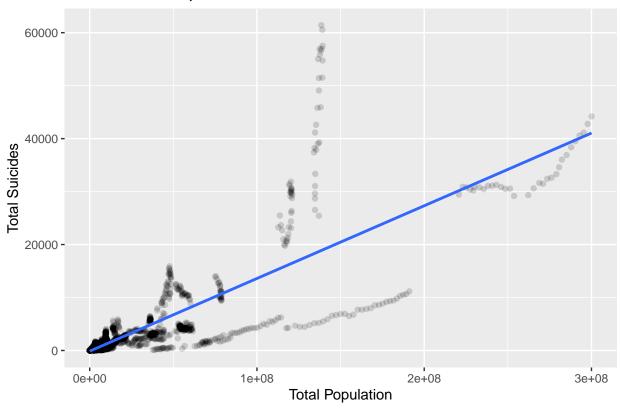
## Log Transformed Population



A cursory look at the data suggests that there is a positive association between population and suicide rates.

```
suicides %>%
  group_by(year, country) %>%
  mutate(total_pop = sum(population), total_suicides = sum(suicides_no)) %>%
  ggplot(aes(total_pop, total_suicides)) +
  geom_point(alpha = 0.01, position = "jitter") +
  geom_smooth(method = "lm", se = FALSE) +
  labs(title = "Suicides and Population",
        x = "Total Population",
        y = "Total Suicides")
```

### Suicides and Population



# Trends by Generation

## 3 Generation X

## 6 Generation Z

## 5 G.I. Generation 510009

## 4 Millenials

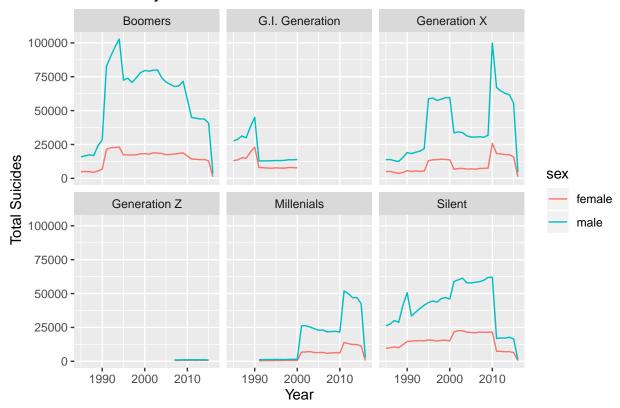
1532804

623459

15906

```
levels(suicides$generation)
## [1] "Boomers"
                           "G.I. Generation" "Generation X"
                                                                  "Generation Z"
## [5] "Millenials"
                           "Silent"
Generation is encoded as a factor variable with 6 levels: Boomers, G.I. Generation, Generation X, Generation
Z, Millenials, and Silent.
suicides %>%
    group_by(generation) %>%
    summarize(total = sum(suicides_no)) %>%
    arrange(desc(total))
## # A tibble: 6 x 2
     generation
                         total
     <fct>
                         <int>
## 1 Boomers
                      2284498
## 2 Silent
                       1781744
```

### Suicides by Generation and Sex



The Boomer generation has the highest number of total suicides, followed by the Silent generation and Generation X. Even when viewed according to generations, males still have a higher number of suicides than females.

# Trends by GDP

Before the trends can be analyzed, the variables must be cleaned.

```
colnames(suicides)[10] <- "gdp_per_year"</pre>
colnames(suicides)[11] <- "gdp_per_capita"</pre>
names(suicides)
                              "year"
                                                    "sex"
##
    [1] "country"
    [4] "age"
                              "suicides no"
                                                    "population"
##
    [7] "suicides.100k.pop"
                              "country.year"
                                                    "HDI.for.year"
   [10] "gdp_per_year"
                              "gdp_per_capita"
                                                    "generation"
  [13] "continent"
```

#### levels(suicides\$gdp\_per\_year)

```
"1,011,797,457,139"
      [1] "1,002,219,052,968"
                                                       "1,016,418,229"
##
##
      [4] "1,018,847,043,277"
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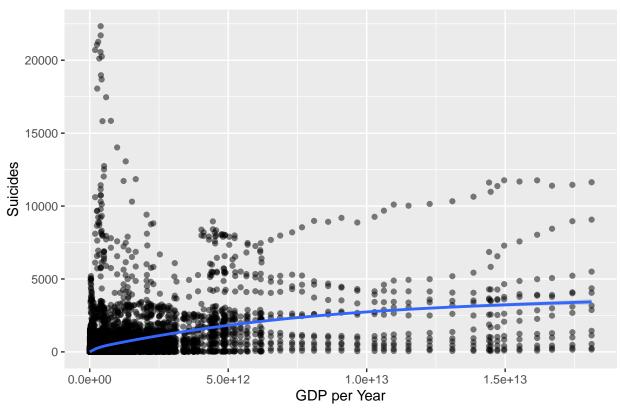
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                                "82,826,146,132"
                                                      "82,995,145,792"
                                                      "825,977,889"
## [2164] "823,837,141"
                                "824,880,550"
## [2167] "828,946,812,397"
                                "83,908,206,648"
                                                      "83,914,521,300"
## [2170] "830,158,778"
                                "832,072,450"
                                                      "832,523,681,194"
## [2173] "836,389,937,229"
                                "839,319,927"
                                                      "839,419,655,078"
## [2176] "84,952,360,922"
                                "842,620,111"
                                                      "847,397,850"
## [2179] "85,324,771,841"
                                "85,343,063,966"
                                                      "85,500,935,935"
## [2182] "85,707,636,233"
                                "850,426,432,992"
                                                      "851,962,785,585"
                                "857,932,759,100"
                                                      "859,796,872,794"
## [2185] "855,643,111"
## [2188] "86,142,018,069"
                                                      "86,283,126,844"
                                "86,186,158,685"
## [2191] "86,304,245,825"
                                "86,307,135,997"
                                                      "86,730,038,793"
## [2194] "860,630,923"
                                "863,723,411,633"
                                                      "866,680,000,367"
## [2197] "87,132,800,000"
                                "87,276,164,400"
                                                      "87,890,009,877"
## [2200] "87,924,544,000"
                                "870,179,739"
                                                      "871,860,600"
## [2203] "873,250,000"
                                "873,982,246,102"
                                                      "877,476,221,382"
## [2206] "879,635,084,125"
                                "88,250,885,550"
                                                      "88,416,668,900"
## [2209] "88,945,625,174"
                                "883,199,625,325"
                                                      "885,444,186"
                                                      "89,255,751,015"
## [2212] "888,667,913,419"
                                "89,242,382,961"
## [2215] "89,286,208,629"
                                "89,501,012,916"
                                                      "89,524,131,600"
## [2218] "891,000,000"
                                "891,630,175,813"
                                                      "892,164,394"
## [2221] "892,380,986,368"
                                "893,107,211"
                                                      "893,757,287,202"
## [2224] "897,031,250"
                                "898,137,194,716"
                                                      "9,008,273,721"
## [2227] "9,024,567,484"
                                "9,055,290,000"
                                                      "9,062,131,308"
## [2230] "9,062,906,915"
                                "9,089,168,000,000"
                                                      "9,128,843,109"
## [2233] "9,178,016,493"
                                "9,206,301,700"
                                                      "9,207,689,916"
                                "9,260,284,938"
## [2236] "9,209,559,296"
                                                      "9,298,839,655"
## [2239] "9,399,447,609"
                                "9,406,097,735"
                                                      "9,507,645,260"
## [2242] "9,537,297,507"
                                "9,546,441,564"
                                                      "9,586,327,800"
## [2245] "9,632,155,053"
                                "9,660,624,000,000"
                                                      "9,687,951,055"
## [2248]
          "9,694,169,757"
                                "9,697,416,974"
                                                      "9,750,161,053"
                                "9,788,391,733"
## [2251] "9,774,316,692"
                                                      "9,833,870,709"
## [2254] "9,836,200,000"
                                "9,870,494,000"
                                                      "9,965,225,497"
## [2257] "9,981,960,000"
                                "9,990,370,016"
                                                      "90,082,034,316"
## [2260] "900,045,362,045"
                                "901,934,953,365"
                                                      "906,853,273,138"
```

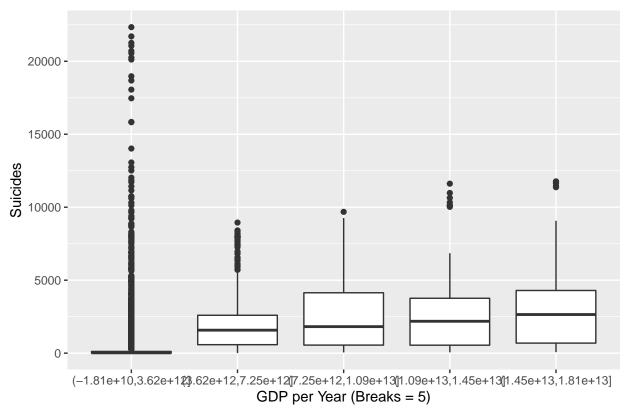
```
## [2263] "91,030,959,455"
                                "91,642,093,873"
                                                      "91,941,192,896"
## [2266] "910,122,732,124"
                                                      "919,103,255"
                                "911,481,481"
## [2269] "919,577,148"
                                "92,507,279,383"
                                                      "925,598,068,021"
## [2272] "926,448,240,318"
                                "926,884,816,754"
                                                     "929,607,500"
## [2275] "93,413,992,956"
                                "93,639,316,000"
                                                      "932,551,850"
## [2278] "934,173,305,686"
                                "934,185,915,467"
                                                     "936,228,211,513"
## [2281] "94,230,055,659"
                                "94,337,050,693"
                                                      "94,684,584,163"
## [2284] "945,000,000"
                                "95,019,103,603"
                                                      "95,129,659,000"
## [2287] "95,833,932,715"
                                "950,579,413,279"
                                                      "951,207,366"
                                "96,385,638,000"
## [2290] "96,045,645,026"
                                                      "96,403,758,865"
## [2293] "967,199,594"
                                "969,936,525"
                                                      "97,001,377,569"
                                "97,724,004,252"
## [2296] "97,160,109,278"
                                                      "97,798,351,648"
## [2299] "97,891,090,929"
                                "97,933,391,976"
                                                     "975,387,131,716"
## [2302] "979,850,000"
                                "98,203,546,156"
                                                      "98,234,695,722"
## [2305] "98,381,268,000"
                                "98,443,739,941"
                                                      "98,478,349,315"
## [2308] "98,585,185"
                                "98,691,849,563"
                                                      "984,297,589"
## [2311] "989,930,542,279"
                                "99,036,165,210"
                                                      "99,210,392,858"
## [2314] "99,290,381,000"
                                "99,627,140,274"
                                                      "99,697,566,668"
## [2317] "99,698,453,261"
                                "99,853,528,653"
                                                      "99,886,577,331"
## [2320] "990,374,050"
                                "997,007,926"
suicides$gdp_per_year <- gsub(",", "", suicides$gdp_per_year)</pre>
suicides$gdp_per_year <- as.numeric(suicides$gdp_per_year)</pre>
suicides %>%
  ggplot(aes(gdp_per_year, suicides_no)) +
  geom_point(alpha = 0.5, position = "jitter") +
  geom_smooth() +
 labs(title = "Suicides and GDP",
       x = "GDP per Year",
       y = "Suicides")
```

## `geom\_smooth()` using method = 'gam' and formula 'y ~ s(x, bs = "cs")'

# Suicides and GDP



### Suicides and GDP



There is no clear trend between suicides and GDP per year. The scatterplot suggests a slightly positive association. However, the boxplot hints that this might be due mainly to outliers.

#### MODELS AND RESULTS

In this section, different models are trained with the end goal of determining whether suicide rates are associated with demographic variables.

First, the variables of interest are selected from the main dataset.

```
suicides <- suicides %>%
    select(suicides_no, sex, age, population, generation, continent)

colnames(suicides)[1] <- "suicides"
names(suicides)

## [1] "suicides" "sex" "age" "population" "generation"

## [6] "continent"

Next, the training and validation sets are created.

set.seed(725, sample.kind = "Rounding")

## Warning in set.seed(725, sample.kind = "Rounding"): non-uniform 'Rounding'

## sampler used

y <- suicides$suicides

test_index <- createDataPartition(y, times = 1, p = 0.5, list = FALSE)</pre>
```

```
test_set <- suicides %>% slice(test_index)
train_set <- suicides %>% slice(-test_index)
```

The RMSE function, which will be used to assess the performance of the models, is defined next. The RMSE refers to the residual mean squared error of the predicted suicide rates against the actual rates. The aim is to minimize the RMSE.

```
RMSE <- function(actual_suicides, predicted_suicides){
   sqrt(mean((actual_suicides - predicted_suicides)^2))
}
actual_suicides <- test_set$suicides</pre>
```

### Using the Mean

The simplest model is to predict the same number of suicides for all continents, with random variation explained by an error term:

$$\hat{Y} = \mu + \varepsilon$$

The estimate that would minimize the RMSE is the average of all suicides.

```
mu <- mean(train_set$suicides)
mu</pre>
```

```
## [1] 237.4253
```

Using mu to predict the suicide rates, the RMSE is 913.738.

```
## # A tibble: 1 x 2
## Method RMSE
## <chr> <dbl>
## 1 Mean 913.738
```

The RMSE is quite high. It is higher than the standard deviation of the suicide rates. This ineffectiveness may be due to the presence of significant outliers in the data.

# Linear Regression

The next option is to develop a linear model which predicts the number of suicides  $(\hat{Y})$  based on the demographic variables (X), while at the same time minimizing the error.

As there are 5 explanatory variables, the linear model is selected using the nested models approach. Under this approach, models which add a new variable are compared to each other using the analysis of variance (ANOVA) method. The ANOVA results can show whether the added term is necessary for the model. The variance inflation factor (VIF) is also examined to determine whether the additional term significantly increases the variance of the model.

```
This nested model approach showed that it is not necessary to add generation as an explanatory variable.
```

```
fit1 <- lm(suicides ~ sex, train_set)</pre>
fit2 <- update(fit1, suicides ~ sex + age, train_set)</pre>
vif(fit2) ## to check the variance inflation ##
          GVIF Df GVIF<sup>(1/(2*Df))</sup>
## sex 1.000178 1
                         1.000089
                         1.000018
## age 1.000178 5
anova(fit1, fit2) ## model 2's terms are necessary over model 1
## Analysis of Variance Table
##
## Model 1: suicides ~ sex
## Model 2: suicides ~ sex + age
                  RSS Df Sum of Sq
   Res.Df
                                              Pr(>F)
## 1 13907 1.0801e+10
## 2 13902 1.0442e+10 5 358470902 95.449 < 2.2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
summary(fit2) ## Adj. R-squared = 0.05223
##
## Call:
## lm(formula = suicides ~ sex + age, data = train_set)
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
   -626.8 -260.4 -127.5
##
                              19.9 21079.2
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                    37.08
                               19.43
                                      1.909 0.05631 .
## sexmale
                   255.25
                               14.70 17.365 < 2e-16 ***
## age25-34 years
                   68.52
                               25.45
                                       2.692 0.00711 **
## age35-54 years
                               25.31 13.212 < 2e-16 ***
                   334.44
                  -154.84
## age5-14 years
                               25.48 -6.076 1.27e-09 ***
## age55-74 years 206.11
                                      8.113 5.34e-16 ***
                               25.40
## age75+ years
                   -31.89
                               25.42 -1.255 0.20959
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 866.7 on 13902 degrees of freedom
## Multiple R-squared: 0.05264, Adjusted R-squared: 0.05223
## F-statistic: 128.7 on 6 and 13902 DF, p-value: < 2.2e-16
fit3 <- update(fit1, suicides ~
                sex + age + generation, train_set)
vif(fit3) ## variance increased, for age and generation
##
                  GVIF Df GVIF<sup>(1/(2*Df))</sup>
## sex
              1.000294 1
                               1.000147
## age
             22.215400 5
                                 1.363532
## generation 22.213557 5
                                 1.363521
```

```
anova(fit2, fit3) ## model 2 over model 3, generation not necessary ##
## Analysis of Variance Table
##
## Model 1: suicides ~ sex + age
## Model 2: suicides ~ sex + age + generation
    Res.Df
                  RSS Df Sum of Sq
                                       F Pr(>F)
## 1 13902 1.0442e+10
## 2 13897 1.0436e+10 5 5737584 1.528 0.1773
summary(fit3) ## Adj. R-squared = 0.05241 ##
##
## Call:
## lm(formula = suicides ~ sex + age + generation, data = train_set)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
  -640.2 -260.8 -124.9
                             19.4 21067.8
##
## Coefficients:
##
                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                         37.689
                                                1.347
                              50.765
                                                         0.1780
## sexmale
                             254.833
                                         14.699 17.337 < 2e-16 ***
## age25-34 years
                              58.460
                                         27.952
                                                 2.091
                                                         0.0365 *
## age35-54 years
                             332.567
                                         36.734
                                                 9.053 < 2e-16 ***
                            -147.633
                                         28.934 -5.102 3.40e-07 ***
## age5-14 years
## age55-74 years
                             265.604
                                         48.703
                                                  5.453 5.02e-08 ***
                                                        0.4225
## age75+ years
                             42.862
                                         53.430
                                                0.802
## generationG.I. Generation -87.668
                                         44.523 -1.969
                                                        0.0490 *
## generationGeneration X
                              2.021
                                         30.547
                                                0.066
                                                        0.9472
## generationGeneration Z
                            -21.574
                                         52.338 -0.412
                                                         0.6802
                                         37.726 -0.691
                                                         0.4895
## generationMillenials
                             -26.075
## generationSilent
                             -88.680
                                         35.320 -2.511 0.0121 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 866.6 on 13897 degrees of freedom
## Multiple R-squared: 0.05316, Adjusted R-squared: 0.05241
## F-statistic: 70.93 on 11 and 13897 DF, p-value: < 2.2e-16
fit4 <- update(fit1, suicides ~
                  sex + age +
                  population, train_set)
vif(fit4) ## less variance
                 GVIF Df GVIF<sup>(1/(2*Df))</sup>
##
## sex
             1.000391 1
                                1.000196
## age
             1.034409 5
                                1.003389
## population 1.034483 1
                                1.017096
anova(fit2, fit4) ## model 4 over model 2 ##
## Analysis of Variance Table
## Model 1: suicides ~ sex + age
```

```
## Model 2: suicides ~ sex + age + population
## Res.Df
                  RSS Df Sum of Sq
                                             Pr(>F)
                                    F
## 1 13902 1.0442e+10
## 2 13901 6.3686e+09 1 4073551731 8891.6 < 2.2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
summary(fit4) ## Adj. R-squared = 0.4219 ##
##
## Call:
## lm(formula = suicides ~ sex + age + population, data = train_set)
## Residuals:
      Min
               1Q Median
                              3Q
## -3112.8 -172.2 -8.4
                           103.5 18713.8
## Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 -2.345e+02 1.544e+01 -15.187 < 2e-16 ***
## sexmale
                  2.711e+02 1.148e+01 23.609 < 2e-16 ***
## age25-34 years 7.904e+01 1.988e+01
                                        3.976 7.04e-05 ***
## age35-54 years 1.688e+02 1.985e+01
                                        8.503 < 2e-16 ***
## age5-14 years -1.516e+02 1.990e+01 -7.615 2.80e-14 ***
## age55-74 years 1.988e+02 1.984e+01 10.018 < 2e-16 ***
## age75+ years
                  1.477e+02 1.994e+01
                                       7.407 1.36e-13 ***
## population
                  1.448e-04 1.535e-06 94.295 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 676.9 on 13901 degrees of freedom
## Multiple R-squared: 0.4222, Adjusted R-squared: 0.4219
## F-statistic: 1451 on 7 and 13901 DF, p-value: < 2.2e-16
fit5 <- update(fit1, suicides ~
                sex + age +
                population + continent, train_set)
vif(fit5)
##
                 GVIF Df GVIF<sup>(1/(2*Df))</sup>
## sex
             1.000501 1
                               1.000250
## age
             1.036088 5
                               1.003552
## population 1.043113 1
                               1.021329
## continent 1.009527 4
                               1.001186
anova(fit4, fit5) ## model 5 over model 4 ##
## Analysis of Variance Table
## Model 1: suicides ~ sex + age + population
## Model 2: suicides ~ sex + age + population + continent
   Res.Df
                  RSS Df Sum of Sq
                                   F Pr(>F)
## 1 13901 6368559423
## 2 13897 6279289798 4 89269624 49.392 < 2.2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
## Call:
## lm(formula = suicides ~ sex + age + population + continent, data = train_set)
## Residuals:
##
       Min
                1Q
                    Median
                                 3Q
                                        Max
                              130.2 18618.6
                     -19.6
##
  -3040.9
            -163.7
##
## Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
##
                     -3.530e+02 3.479e+01 -10.147 < 2e-16 ***
## (Intercept)
## sexmale
                      2.707e+02
                                 1.140e+01
                                             23.736 < 2e-16 ***
## age25-34 years
                      8.243e+01
                                              4.175 3.00e-05 ***
                                 1.974e+01
## age35-54 years
                      1.690e+02
                                 1.971e+01
                                              8.573 < 2e-16 ***
## age5-14 years
                     -1.504e+02
                                 1.977e+01
                                             -7.610 2.91e-14 ***
## age55-74 years
                      1.973e+02
                                 1.970e+01
                                             10.013 < 2e-16 ***
## age75+ years
                                              7.608 2.97e-14 ***
                      1.507e+02
                                 1.981e+01
## population
                      1.458e-04
                                 1.531e-06
                                             95.193 < 2e-16 ***
## continentAmericas 1.588e+01
                                 3.331e+01
                                              0.477
                                                      0.6336
## continentAsia
                      1.372e+02
                                 3.440e+01
                                              3.988 6.69e-05 ***
## continentEurope
                      1.951e+02 3.300e+01
                                              5.911 3.47e-09 ***
## continentOceania
                      1.310e+02 4.382e+01
                                              2.989
                                                      0.0028 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 672.2 on 13897 degrees of freedom
## Multiple R-squared: 0.4303, Adjusted R-squared: 0.4299
## F-statistic: 954.3 on 11 and 13897 DF, p-value: < 2.2e-16
Using this linear model generated the RMSE of 691.42, which is better than the RMSE from simply using
the mean.
lm fit1 <- lm(suicides ~ sex + age +</pre>
               population + continent, train_set)
y_hat1 <- predict(lm_fit1, test_set)</pre>
RMSE_lm <- RMSE(actual_suicides, y_hat1)</pre>
rmse_results <- bind_rows(rmse_results,</pre>
                          tibble(Method = "Linear Model",
                       RMSE = RMSE_lm)
rmse_results
## # A tibble: 2 x 2
##
     Method
                     RMSE
##
     <chr>>
                    <dbl>
## 1 Mean
                  913.738
## 2 Linear Model 691.420
```

summary(fit5) ## Adj. R-squared = 0.4299 ##

The figures above suggest that certain variables (i.e., population) might have a different effect on the number of suicides based on the values of another variable (i.e., continent, sex). As such, the possibility of interactions should also be accounted for.

```
The nested models approach is likewise used in selecting the linear regression model with interaction terms.
```

```
fit6 <- update(fit1, suicides ~ sex + age +
                population + continent + population:sex,
              train_set)
vif(fit6)
##
                     GVIF Df GVIF<sup>(1/(2*Df))</sup>
## sex
                 1.225158 1
                                    1.106868
## age
                 1.036587 5
                                    1.003600
## population
                 2.011642 1
                                    1.418324
## continent
                 1.009613 4
                                    1.001197
## sex:population 2.186076 1
                                    1.478538
anova(fit5, fit6) ## with interactions
## Analysis of Variance Table
##
## Model 1: suicides ~ sex + age + population + continent
## Model 2: suicides ~ sex + age + population + continent + sex:population
    Res.Df
                  RSS Df Sum of Sq
                                              Pr(>F)
## 1 13897 6279289798
## 2 13896 4859616789 1 1419673009 4059.5 < 2.2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
summary(fit6) ## adj. r2 = 0.5587
##
## Call:
## lm(formula = suicides ~ sex + age + population + continent +
##
      sex:population, data = train_set)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
## -4322.7 -120.8
                     -8.8
                            104.9 17123.5
##
## Coefficients:
##
                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                     -2.016e+02 3.070e+01 -6.569 5.26e-11 ***
## sexmale
                     -3.222e+01 1.110e+01 -2.902 0.003711 **
## age25-34 years
                      8.402e+01 1.737e+01
                                           4.837 1.33e-06 ***
## age35-54 years
                      1.685e+02 1.734e+01
                                             9.718 < 2e-16 ***
## age5-14 years
                     -1.523e+02 1.739e+01 -8.757 < 2e-16 ***
                      2.015e+02 1.733e+01 11.623 < 2e-16 ***
## age55-74 years
## age75+ years
                      1.697e+02 1.743e+01
                                           9.739 < 2e-16 ***
                      6.305e-05 1.871e-06 33.708 < 2e-16 ***
## population
## continentAmericas 1.187e+01 2.930e+01
                                           0.405 0.685531
                    1.338e+02 3.026e+01
## continentAsia
                                            4.422 9.86e-06 ***
## continentEurope
                      1.974e+02 2.903e+01 6.799 1.10e-11 ***
## continentOceania
                      1.288e+02 3.855e+01 3.341 0.000836 ***
## sexmale:population 1.681e-04 2.639e-06 63.714 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 591.4 on 13896 degrees of freedom
```

```
## Multiple R-squared: 0.5591, Adjusted R-squared: 0.5587
## F-statistic: 1469 on 12 and 13896 DF, p-value: < 2.2e-16
fit7 <- update(fit1, suicides ~ sex + age +
                population + continent + population:sex +
              population:age, train_set)
vif(fit7) ## increased variance with interaction
                      GVIF Df GVIF<sup>(1/(2*Df))</sup>
## sex
                 1.226332 1
                                    1.107399
## age
                  3.116415 5
                                     1.120380
## population
                  8.094232 1
                                     2.845036
                  1.017783 4
## continent
                                     1.002206
## sex:population 2.195574 1
                                    1.481747
## age:population 17.673750 5
                                     1.332701
anova(fit6, fit7)
## Analysis of Variance Table
## Model 1: suicides ~ sex + age + population + continent + sex:population
## Model 2: suicides ~ sex + age + population + continent + sex:population +
##
      age:population
##
    Res.Df
                  RSS Df Sum of Sq
                                             Pr(>F)
## 1 13896 4859616789
## 2 13891 3943719561 5 915897229 645.21 < 2.2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
summary(fit7) ## adj. r2 = 0.6418
##
## Call:
## lm(formula = suicides ~ sex + age + population + continent +
      sex:population + age:population, data = train_set)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -3926.1 -68.2 -14.4
                             80.2 16495.7
##
## Coefficients:
##
                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                            -9.055e+01 2.820e+01 -3.211 0.001326 **
                            -3.475e+01 1.001e+01 -3.473 0.000517 ***
## sexmale
## age25-34 years
                            -8.697e+00 1.770e+01 -0.491 0.623176
## age35-54 years
                            -3.366e+01 1.754e+01 -1.918 0.055087 .
                            1.188e+01 1.765e+01
## age5-14 years
                                                  0.673 0.500877
## age55-74 years
                            -1.973e+01 1.755e+01 -1.125 0.260754
                             3.386e+01 1.733e+01
                                                  1.954 0.050716 .
## age75+ years
## population
                            9.578e-06 3.381e-06 2.833 0.004617 **
## continentAmericas
                            1.845e+01 2.641e+01 0.699 0.484636
## continentAsia
                             1.200e+02 2.727e+01 4.402 1.08e-05 ***
## continentEurope
                             1.594e+02 2.618e+01 6.088 1.17e-09 ***
## continentOceania
                             1.003e+02 3.474e+01 2.886 0.003907 **
                             1.733e-04 2.383e-06 72.742 < 2e-16 ***
## sexmale:population
## age25-34 years:population 5.057e-05 4.638e-06 10.904 < 2e-16 ***
```

```
## age35-54 years:population 8.773e-05 3.698e-06 23.722 < 2e-16 ***
## age5-14 years:population -9.203e-05 4.493e-06 -20.483 < 2e-16 ***
## age55-74 years:population 1.198e-04 4.337e-06 27.634 < 2e-16 ***
                             1.257e-04 8.825e-06 14.240 < 2e-16 ***
## age75+ years:population
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 532.8 on 13891 degrees of freedom
## Multiple R-squared: 0.6422, Adjusted R-squared: 0.6418
## F-statistic: 1467 on 17 and 13891 DF, p-value: < 2.2e-16
fit8 <- update(fit1, suicides ~ sex + age +
                population + continent + population:sex +
                population:age + population:generation, train_set)
vif(fit8)
                              GVIF Df GVIF^(1/(2*Df))
##
## sex
                          1.226515 1
                                             1.107482
                          3.132044 5
                                             1.120941
## age
## population
                         18.969014 1
                                             4.355343
## continent
                         1.018870 4
                                             1.002340
## sex:population
                          2.198017 1
                                             1.482571
                        108.286476 5
## age:population
                                             1.597561
## population:generation 21.497319 5
                                             1.359059
anova(fit7, fit8)
## Analysis of Variance Table
## Model 1: suicides ~ sex + age + population + continent + sex:population +
      age:population
## Model 2: suicides ~ sex + age + population + continent + sex:population +
##
      age:population + population:generation
##
    Res.Df
                  RSS Df Sum of Sq
## 1 13891 3943719561
## 2 13886 3932559048 5 11160512 7.8816 2.013e-07 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
summary(fit8) ## adj. r2 = 0.6427
##
## Call:
## lm(formula = suicides ~ sex + age + population + continent +
##
      sex:population + age:population + population:generation,
##
      data = train_set)
##
## Residuals:
##
               1Q Median
                               3Q
      Min
                                      Max
## -3706.3
           -67.5 -13.7 80.6 16432.9
##
## Coefficients:
##
                                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                       -8.843e+01 2.817e+01 -3.139 0.001699 **
                                       -3.403e+01 9.995e+00 -3.405 0.000665 ***
## sexmale
## age25-34 years
                                       -8.536e+00 1.768e+01 -0.483 0.629226
```

```
-3.509e+01 1.753e+01 -2.002 0.045266 *
## age35-54 years
                                       1.172e+01 1.763e+01 0.665 0.506132
## age5-14 years
## age55-74 years
                                      -2.211e+01 1.755e+01 -1.260 0.207630
                                       3.252e+01 1.731e+01 1.879 0.060281 .
## age75+ years
                                       2.651e-05 5.169e-06 5.130 2.94e-07 ***
## population
## continentAmericas
                                      1.573e+01 2.638e+01 0.596 0.551127
## continentAsia
                                      1.182e+02 2.724e+01 4.337 1.45e-05 ***
                                      1.566e+02 2.615e+01 5.989 2.16e-09 ***
## continentEurope
                                      9.803e+01 3.470e+01 2.825 0.004735 **
## continentOceania
                                      1.730e-04 2.381e-06 72.653 < 2e-16 ***
## sexmale:population
## age25-34 years:population
                                     4.546e-05 4.915e-06
                                                              9.250 < 2e-16 ***
                                      7.445e-05 4.842e-06 15.376 < 2e-16 ***
## age35-54 years:population
                                    7.445e-05 4.842e-06 15.376 < 2e-16 ***
-9.127e-05 4.990e-06 -18.290 < 2e-16 ***
## age5-14 years:population
                                      9.690e-05 6.248e-06 15.508 < 2e-16 ***
## age55-74 years:population
## age75+ years:population
                                      9.487e-05 1.045e-05 9.075 < 2e-16 ***
## population:generationG.I. Generation 3.421e-05 8.233e-06 4.155 3.27e-05 ***
## population:generationGeneration X -1.474e-05 3.405e-06 -4.328 1.51e-05 ***
## population:generationGeneration Z
                                      -1.629e-05 7.789e-06 -2.091 0.036530 *
## population:generationMillenials
                                      -1.873e-05 5.154e-06 -3.633 0.000281 ***
## population:generationSilent
                                       4.980e-06 4.090e-06
                                                             1.218 0.223365
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 532.2 on 13886 degrees of freedom
## Multiple R-squared: 0.6432, Adjusted R-squared: 0.6427
## F-statistic: 1138 on 22 and 13886 DF, p-value: < 2.2e-16
fit9 <- update(fit1, suicides ~ sex + age +
                population + continent + population:sex +
                population:age + population:generation +
                population:continent, train_set)
vif(fit9) ## increased variance
                              GVIF Df GVIF^(1/(2*Df))
##
## sex
                          1.226573 1
                                            1.107508
                          3.180552 5
## age
                                            1.122665
## population
                       152.601126 1
                                           12.353183
                        3.904729 4
## continent
                                           1.185629
## sex:population
                          2.198875 1
                                            1.482861
## age:population
                       109.366630 5
                                            1.599147
## population:generation 21.754237 5
                                            1.360675
## population:continent 419.839471 4
                                            2.127578
anova(fit8, fit9)
## Analysis of Variance Table
##
## Model 1: suicides ~ sex + age + population + continent + sex:population +
      age:population + population:generation
## Model 2: suicides ~ sex + age + population + continent + sex:population +
##
      age:population + population:generation + population:continent
                  RSS Df Sum of Sq
    Res Df
                                      F
                                            Pr(>F)
## 1 13886 3932559048
## 2 13882 3410423709 4 522135339 531.33 < 2.2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
summary(fit9) ## adj. r2 = 0.69
##
## Call:
## lm(formula = suicides ~ sex + age + population + continent +
      sex:population + age:population + population:generation +
##
      population:continent, data = train_set)
##
## Residuals:
##
      Min
               10 Median
                               30
## -2944.8
            -29.6
                     24.9
                             70.7 14816.2
##
## Coefficients:
##
                                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                        1.148e+01 2.927e+01
                                                              0.392 0.695002
## sexmale
                                       -3.233e+01 9.310e+00 -3.473 0.000517 ***
## age25-34 years
                                       -7.692e+00 1.647e+01 -0.467 0.640435
## age35-54 years
                                       -6.333e+01 1.635e+01 -3.873 0.000108 ***
## age5-14 years
                                       1.291e+01 1.642e+01 0.786 0.431918
## age55-74 years
                                      -1.357e+01 1.636e+01 -0.830 0.406749
## age75+ years
                                       6.216e+01 1.615e+01 3.849 0.000119 ***
## population
                                       -6.396e-05 1.366e-05 -4.684 2.84e-06 ***
## continentAmericas
                                      -1.928e+01 2.791e+01 -0.691 0.489797
## continentAsia
                                      -4.537e+01 2.904e+01 -1.562 0.118286
## continentEurope
                                       -9.728e+01 2.788e+01 -3.489 0.000486 ***
## continentOceania
                                       9.299e-01 4.006e+01 0.023 0.981481
## sexmale:population
                                      1.749e-04 2.218e-06 78.854 < 2e-16 ***
## age25-34 years:population
                                                             8.317 < 2e-16 ***
                                      3.811e-05 4.582e-06
## age35-54 years:population
                                       5.986e-05 4.522e-06 13.237 < 2e-16 ***
## age5-14 years:population
                                      -9.138e-05 4.649e-06 -19.658 < 2e-16 ***
## age55-74 years:population
                                       7.516e-05 5.840e-06 12.871 < 2e-16 ***
## age75+ years:population
                                        8.497e-05 9.741e-06
                                                              8.723 < 2e-16 ***
## population:generationG.I. Generation 2.323e-05 7.674e-06
                                                              3.027 0.002474 **
## population:generationGeneration X
                                      -1.883e-05 3.174e-06 -5.933 3.05e-09 ***
## population:generationGeneration Z
                                       -1.452e-05 7.260e-06 -2.001 0.045464 *
## population:generationMillenials
                                       -2.186e-05 4.807e-06 -4.548 5.45e-06 ***
## population:generationSilent
                                       -6.280e-06 3.824e-06 -1.642 0.100592
## population:continentAmericas
                                                              5.452 5.07e-08 ***
                                      6.988e-05 1.282e-05
## population:continentAsia
                                      1.360e-04 1.312e-05 10.368 < 2e-16 ***
                                       1.965e-04 1.299e-05 15.126 < 2e-16 ***
## population:continentEurope
## population:continentOceania
                                       9.722e-05 3.134e-05
                                                              3.102 0.001928 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

The linear model which accounts for interactions generated an RMSE of 512.80, which is better than the linear model without interaction terms.

## Residual standard error: 495.7 on 13882 degrees of freedom

## F-statistic: 1192 on 26 and 13882 DF, p-value: < 2.2e-16

## Multiple R-squared: 0.6906, Adjusted R-squared:

##

```
population: continent,
               train_set)
y_hat2 <- predict(lm_fit2, test_set)</pre>
RMSE_lm_int <- RMSE(actual_suicides, y_hat2)</pre>
rmse_results <- bind_rows(rmse_results,</pre>
                            tibble(Method = "Linear Model, Interactions",
                                    RMSE = RMSE_lm_int))
rmse_results
## # A tibble: 3 x 2
##
     Method
                                      RMSE
     <chr>
##
                                     <dbl>
## 1 Mean
                                  913.738
## 2 Linear Model
                                  691.420
## 3 Linear Model, Interactions 512.796
```

# Poisson Regression

Since the Y in this case is count data with many zeros, a Poisson regression is a viable option. However, we earlier saw that the variance of the number of suicides is greater than its mean. As such, we should use Quasipoisson regression.

The quasipoisson model is developed in an analogous manner to how the linear model was developed. In this case, however, the pseudo R-squared values are compared in order to find which quasipoisson model better predicts the outcome.

```
glm_fit <- glm(suicides ~ sex, train_set,</pre>
               family = "quasipoisson")
summary(glm_fit)
##
## Call:
## glm(formula = suicides ~ sex, family = "quasipoisson", data = train_set)
## Deviance Residuals:
     Min
              1Q Median
                               3Q
                                      Max
## -26.92 -21.05 -13.86
                           -4.98 367.40
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.69949
                           0.05721
                                     82.14
                                             <2e-16 ***
                           0.06516
## sexmale
                1.19326
                                     18.31
                                             <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for quasipoisson family taken to be 2475.964)
##
##
      Null deviance: 11862120 on 13908
                                          degrees of freedom
## Residual deviance: 10874403 on 13907
                                          degrees of freedom
## AIC: NA
```

```
##
## Number of Fisher Scoring iterations: 7
glance(glm_fit) %>%
  summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
##
     pseudoR2
##
         <dbl>
## 1 0.0832665
 ## pseudoR2 = 0.0832665
glm_fit <- glm(suicides ~ sex + age, train_set,</pre>
               family = "quasipoisson")
summary(glm_fit)
##
## Call:
## glm(formula = suicides ~ sex + age, family = "quasipoisson",
##
      data = train_set)
##
## Deviance Residuals:
     Min
            1Q Median
                              3Q
## -39.19 -18.19 -9.85 -1.59 321.25
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
                  4.33492 0.07628 56.827 < 2e-16 ***
## (Intercept)
## sexmale
                  1.20482 0.05271 22.856 < 2e-16 ***
## age25-34 years 0.34338
                             0.08450
                                      4.064 4.85e-05 ***
## age35-54 years 1.10377
                             0.07433 14.849 < 2e-16 ***
## age5-14 years -2.69280
                             0.25879 -10.405 < 2e-16 ***
## age55-74 years 0.80964
                             0.07771 10.419 < 2e-16 ***
## age75+ years -0.20852
                             0.09635 - 2.164
                                             0.0305 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for quasipoisson family taken to be 1620.425)
##
##
       Null deviance: 11862120 on 13908 degrees of freedom
## Residual deviance: 9145589 on 13902 degrees of freedom
##
## Number of Fisher Scoring iterations: 6
glance(glm_fit) %>%
 summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
    pseudoR2
##
        <dbl>
## 1 0.229009
## pseudoR2 = 0.229009
```

```
glm_fit <- glm(suicides ~ sex + age +</pre>
                generation, train_set,
              family = "quasipoisson")
summary(glm_fit)
##
## Call:
## glm(formula = suicides ~ sex + age + generation, family = "quasipoisson",
      data = train_set)
##
## Deviance Residuals:
     Min 1Q Median
                             3Q
## -39.82 -18.15 -9.92 -1.55 319.81
##
## Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
                                       0.11401 38.847 < 2e-16 ***
## (Intercept)
                            4.42896
## sexmale
                            1.20340
                                       0.05261 22.873 < 2e-16 ***
## age25-34 years
                           0.27454
                                     0.09240 2.971 0.00297 **
## age35-54 years
                           1.03311 0.10347 9.985 < 2e-16 ***
                                     0.30668 -8.554 < 2e-16 ***
## age5-14 years
                           -2.62333
                                     0.12740
## age55-74 years
                           0.87345
                                               6.856 7.37e-12 ***
                           -0.11090 0.15142 -0.732 0.46396
## age75+ years
## generationG.I. Generation -0.18224 0.11861 -1.536 0.12446
                                               0.137 0.89079
## generationGeneration X 0.01012
                                     0.07368
## generationGeneration Z -0.21100 0.55529 -0.380 0.70396
## generationMillenials -0.18528 0.11709 -1.582 0.11359
                         -0.19767 0.07953 -2.485 0.01295 *
## generationSilent
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for quasipoisson family taken to be 1614.082)
##
##
      Null deviance: 11862120 on 13908 degrees of freedom
## Residual deviance: 9128759 on 13897 degrees of freedom
## AIC: NA
##
## Number of Fisher Scoring iterations: 6
glance(glm_fit) %>%
 summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
    pseudoR2
##
##
       <dbl>
## 1 0.230428
 ## pseudoR2 = 0.230428
glm_fit <- glm(suicides ~ sex + age +</pre>
                generation + population,
              train_set, family = "quasipoisson")
summary(glm fit)
```

##

```
## Call:
## glm(formula = suicides ~ sex + age + generation + population,
      family = "quasipoisson", data = train_set)
##
## Deviance Residuals:
                        Median
##
       Min
                  1Q
                                      3Q
                                               Max
## -191.185 -13.056 -6.882
                                  -0.498
                                           230.225
##
## Coefficients:
##
                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                             4.073e+00 6.345e-02 64.187 < 2e-16 ***
                             1.249e+00 2.924e-02 42.719 < 2e-16 ***
## sexmale
## age25-34 years
                             2.993e-01 5.142e-02
                                                   5.821 5.99e-09 ***
## age35-54 years
                            2.792e-01 5.972e-02
                                                   4.675 2.97e-06 ***
## age5-14 years
                            -2.643e+00 1.704e-01 -15.508 < 2e-16 ***
## age55-74 years
                             4.595e-01 7.098e-02
                                                   6.473 9.94e-11 ***
                            -2.546e-01 8.389e-02 -3.035 0.002409 **
## age75+ years
## generationG.I. Generation 2.272e-01 6.651e-02
                                                   3.417 0.000636 ***
## generationGeneration X -5.633e-02 4.035e-02 -1.396 0.162697
                            -2.117e-01 3.084e-01 -0.686 0.492494
## generationGeneration Z
                            -2.078e-01 6.465e-02 -3.214 0.001311 **
## generationMillenials
## generationSilent
                            1.874e-01 4.504e-02 4.160 3.20e-05 ***
                             1.188e-07 1.031e-09 115.145 < 2e-16 ***
## population
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for quasipoisson family taken to be 497.7751)
##
##
      Null deviance: 11862120 on 13908 degrees of freedom
## Residual deviance: 4717179 on 13896 degrees of freedom
## AIC: NA
##
## Number of Fisher Scoring iterations: 6
glance(glm_fit) %>%
 summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
##
    pseudoR2
       <dbl>
## 1 0.602333
 ## pseudoR2 = 0.602333
glm_fit <- glm(suicides ~ sex + age +</pre>
                generation + population +
                continent, train_set, family = "quasipoisson")
summary(glm_fit)
##
## Call:
## glm(formula = suicides ~ sex + age + generation + population +
##
      continent, family = "quasipoisson", data = train_set)
##
## Deviance Residuals:
##
       Min
                  1Q
                        Median
                                      3Q
                                               Max
```

```
## -199.064
             -11.172
                        -5.062
                                   0.783
                                            150.790
##
## Coefficients:
##
                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                             1.698e+00 2.421e-01
                                                    7.016 2.40e-12 ***
## sexmale
                             1.279e+00 2.434e-02 52.526 < 2e-16 ***
                                                    6.531 6.74e-11 ***
## age25-34 years
                             2.782e-01 4.259e-02
## age35-54 years
                             -1.448e-02 5.060e-02 -0.286 0.774696
## age5-14 years
                            -2.606e+00 1.417e-01 -18.387 < 2e-16 ***
## age55-74 years
                             1.889e-01
                                       6.025e-02
                                                    3.134 0.001725 **
## age75+ years
                             -4.289e-01 7.071e-02
                                                   -6.065 1.35e-09 ***
## generationG.I. Generation 3.676e-01 5.659e-02
                                                    6.496 8.51e-11 ***
## generationGeneration X
                            -1.259e-01 3.348e-02
                                                   -3.759 0.000171 ***
## generationGeneration Z
                            -3.212e-01 2.564e-01
                                                   -1.253 0.210363
## generationMillenials
                                                   -5.455 4.97e-08 ***
                            -2.919e-01 5.351e-02
## generationSilent
                             2.617e-01 3.901e-02
                                                    6.707 2.06e-11 ***
## population
                             1.539e-07 1.264e-09 121.760 < 2e-16 ***
## continentAmericas
                             1.174e+00 2.388e-01
                                                    4.916 8.94e-07 ***
                             2.644e+00 2.374e-01
## continentAsia
                                                   11.137 < 2e-16 ***
## continentEurope
                             2.829e+00 2.367e-01
                                                   11.950 < 2e-16 ***
## continentOceania
                             1.928e+00 2.517e-01
                                                    7.660 1.98e-14 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
  (Dispersion parameter for quasipoisson family taken to be 344.1494)
##
##
       Null deviance: 11862120
                                         degrees of freedom
                               on 13908
## Residual deviance: 3565530
                               on 13892
                                         degrees of freedom
## AIC: NA
##
## Number of Fisher Scoring iterations: 6
glance(glm_fit) %>%
  summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
##
     pseudoR2
##
        <dbl>
## 1 0.699419
  ## pseudoR2 = 0.602333
```

The quasipoisson model with the continent variable does not improve upon the pseudo R-squared of the model with just the sex, age, generation and population variables. As such, the latter model is the better one and it generates an RMSE of 1110.51.

```
RMSE = RMSE_glm))
rmse_results
```

As with the Linear Model, the Quasipoisson Model might be significantly improved by accounting for interaction terms. As such, interaction terms are added to the model and their respective pseudo R-squared values are compared to identify the best model.

```
##
## Call:
## glm(formula = suicides ~ sex + age + generation + population +
##
       continent + population:age, family = "quasipoisson", data = train_set)
##
##
  Deviance Residuals:
##
                   1Q
                         Median
                                       3Q
       Min
                                                Max
                         -3.194
##
  -103.115
               -8.454
                                    1.714
                                            136.226
##
## Coefficients:
##
                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                              1.165e+00 2.045e-01
                                                     5.696 1.25e-08 ***
## sexmale
                              1.367e+00 2.102e-02 65.007 < 2e-16 ***
## age25-34 years
                              2.755e-01 4.941e-02
                                                     5.576 2.51e-08 ***
## age35-54 years
                              9.899e-01
                                        4.953e-02
                                                   19.988 < 2e-16 ***
## age5-14 years
                             -2.588e+00 1.600e-01 -16.171 < 2e-16 ***
## age55-74 years
                             3.948e-01 6.006e-02
                                                     6.574 5.08e-11 ***
## age75+ years
                             -5.656e-01 6.888e-02
                                                   -8.212 2.37e-16 ***
## generationG.I. Generation 6.873e-01
                                        4.728e-02
                                                    14.536 < 2e-16 ***
                                                   -2.412
## generationGeneration X
                             -6.789e-02 2.815e-02
                                                            0.01588 *
## generationGeneration Z
                             -2.386e-01 2.152e-01
                                                   -1.109
                                                            0.26756
## generationMillenials
                             -2.733e-01 4.507e-02
                                                   -6.065 1.35e-09 ***
## generationSilent
                              3.326e-01 3.180e-02
                                                    10.460 < 2e-16 ***
## population
                                                    76.718 < 2e-16 ***
                              2.619e-07 3.413e-09
## continentAmericas
                              6.334e-01 2.009e-01
                                                     3.153 0.00162 **
                                                    12.206
## continentAsia
                              2.431e+00 1.991e-01
                                                            < 2e-16 ***
## continentEurope
                                                    13.635
                              2.707e+00
                                        1.986e-01
                                                            < 2e-16 ***
## continentOceania
                              1.968e+00 2.111e-01
                                                     9.324 < 2e-16 ***
## age25-34 years:population 6.460e-10
                                        4.065e-09
                                                     0.159
                                                           0.87374
## age35-54 years:population -1.234e-07
                                         3.312e-09 -37.265
                                                           < 2e-16 ***
## age5-14 years:population -3.225e-10
                                        1.259e-08
                                                   -0.026
                                                            0.97956
## age55-74 years:population -6.131e-08 3.578e-09 -17.134
                                                           < 2e-16 ***
## age75+ years:population
                              2.970e-07 8.625e-09 34.435 < 2e-16 ***
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
  (Dispersion parameter for quasipoisson family taken to be 241.9741)
##
##
      Null deviance: 11862120 on 13908 degrees of freedom
## Residual deviance: 2474451 on 13887 degrees of freedom
## AIC: NA
##
## Number of Fisher Scoring iterations: 6
glance(glm fit) %>%
 summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
##
    pseudoR2
##
       <dbl>
## 1 0.791399
  ## pseudoR2 = 0.791399
glm_fit <- glm(suicides ~ sex + age +</pre>
                generation + population +
                 continent + population:age +
                population:continent, train_set, family = "quasipoisson")
summary(glm_fit)
##
## Call:
## glm(formula = suicides ~ sex + age + generation + population +
      continent + population:age + population:continent, family = "quasipoisson",
##
      data = train_set)
##
## Deviance Residuals:
      Min
                10
                    Median
                                  30
                                          Max
## -68.657
           -8.189
                    -3.453
                               1.141
                                       89.002
## Coefficients:
                                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                                8.222e-01 2.667e-01
                                                     3.082 0.002059 **
## sexmale
                                1.399e+00 1.766e-02 79.207 < 2e-16 ***
## age25-34 years
                               2.487e-01 4.228e-02
                                                      5.883 4.13e-09 ***
                               6.419e-01 4.344e-02 14.778 < 2e-16 ***
## age35-54 years
                               -2.605e+00 1.362e-01 -19.124 < 2e-16 ***
## age5-14 years
                                3.002e-01 5.153e-02
                                                      5.827 5.77e-09 ***
## age55-74 years
                               -4.324e-01 5.835e-02 -7.410 1.34e-13 ***
## age75+ years
                               5.895e-01 3.998e-02 14.744 < 2e-16 ***
## generationG.I. Generation
                               -1.276e-01 2.353e-02 -5.421 6.01e-08 ***
## generationGeneration X
## generationGeneration Z
                               -2.336e-01 1.806e-01 -1.293 0.195927
## generationMillenials
                               -3.020e-01 3.759e-02 -8.033 1.02e-15 ***
                                2.645e-01 2.725e-02
                                                      9.708 < 2e-16 ***
## generationSilent
## population
                                4.686e-07 7.234e-08 6.478 9.64e-11 ***
## continentAmericas
                                2.005e+00 2.646e-01
                                                     7.578 3.73e-14 ***
## continentAsia
                                2.531e+00 2.647e-01 9.562 < 2e-16 ***
                                2.869e+00 2.638e-01 10.877 < 2e-16 ***
## continentEurope
                                1.552e+00 2.869e-01 5.412 6.35e-08 ***
## continentOceania
## age25-34 years:population
                                1.305e-09 3.526e-09 0.370 0.711397
```

```
## age35-54 years:population
                               -1.001e-07 2.896e-09 -34.582 < 2e-16 ***
## age5-14 years:population
                              -2.216e-09 1.086e-08 -0.204 0.838310
                               -5.181e-08 3.149e-09 -16.452 < 2e-16 ***
## age55-74 years:population
## age75+ years:population
                                2.545e-07 7.180e-09 35.442 < 2e-16 ***
## population:continentAmericas -2.602e-07 7.229e-08
                                                     -3.599 0.000321 ***
## population:continentAsia -1.677e-07 7.232e-08
                                                     -2.319 0.020389 *
## population:continentEurope -1.684e-07 7.229e-08 -2.329 0.019870 *
## population:continentOceania 4.982e-07 9.294e-08
                                                     5.360 8.44e-08 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
  (Dispersion parameter for quasipoisson family taken to be 170.5721)
##
##
##
      Null deviance: 11862120
                               on 13908 degrees of freedom
## Residual deviance: 1931383 on 13883 degrees of freedom
## AIC: NA
##
## Number of Fisher Scoring iterations: 6
glance(glm_fit) %>%
 summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
##
    pseudoR2
       <dbl>
## 1 0.837181
 ## pseudoR2 = 0.837181
glm_fit <- glm(suicides ~ sex + age +</pre>
                generation + population +
                continent + population:age +
                population:continent + population:sex,
              train_set, family = "quasipoisson")
summary(glm_fit)
##
## Call:
## glm(formula = suicides ~ sex + age + generation + population +
##
      continent + population:age + population:continent + population:sex,
##
      family = "quasipoisson", data = train_set)
##
## Deviance Residuals:
                10 Median
##
      Min
                                  3Q
                                          Max
## -70.961
           -8.440
                    -3.595
                               0.926
                                       89.686
##
## Coefficients:
##
                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                9.320e-01 2.658e-01
                                                     3.506 0.000456 ***
                                1.266e+00 2.498e-02 50.662 < 2e-16 ***
## sexmale
## age25-34 years
                                2.474e-01 4.206e-02
                                                      5.883 4.13e-09 ***
## age35-54 years
                                6.351e-01 4.324e-02 14.690 < 2e-16 ***
## age5-14 years
                               -2.605e+00 1.355e-01 -19.226 < 2e-16 ***
## age55-74 years
                                2.922e-01 5.130e-02
                                                      5.696 1.25e-08 ***
                               -4.386e-01 5.809e-02 -7.550 4.62e-14 ***
## age75+ years
## generationG.I. Generation
                               5.928e-01 3.980e-02 14.893 < 2e-16 ***
```

```
## generationGeneration X
                               -1.277e-01 2.342e-02 -5.452 5.07e-08 ***
                               -2.325e-01 1.797e-01 -1.294 0.195831
## generationGeneration Z
## generationMillenials
                               -3.020e-01 3.741e-02 -8.073 7.41e-16 ***
                                2.693e-01 2.713e-02 9.924 < 2e-16 ***
## generationSilent
## population
                                4.582e-07 7.200e-08 6.364 2.03e-10 ***
## continentAmericas
                                2.005e+00 2.633e-01
                                                      7.613 2.84e-14 ***
## continentAsia
                                                     9.603 < 2e-16 ***
                                2.529e+00 2.634e-01
                                2.865e+00 2.625e-01 10.916 < 2e-16 ***
## continentEurope
## continentOceania
                                1.551e+00 2.855e-01
                                                       5.433 5.63e-08 ***
## age25-34 years:population
                               1.311e-09 3.507e-09
                                                      0.374 0.708564
## age35-54 years:population
                               -9.960e-08 2.880e-09 -34.576 < 2e-16 ***
## age5-14 years:population
                               -2.298e-09 1.080e-08 -0.213 0.831476
## age55-74 years:population
                               -5.106e-08 3.134e-09 -16.296 < 2e-16 ***
## age75+ years:population
                                2.536e-07 7.163e-09 35.395 < 2e-16 ***
## population:continentAmericas -2.602e-07 7.194e-08
                                                      -3.616 0.000300 ***
## population:continentAsia
                               -1.675e-07
                                           7.197e-08
                                                      -2.328 0.019947 *
                                                      -2.329 0.019852 *
## population:continentEurope
                               -1.676e-07 7.194e-08
## population:continentOceania
                               4.996e-07 9.250e-08
                                                      5.401 6.74e-08 ***
                                1.245e-08 1.724e-09
                                                     7.225 5.27e-13 ***
## sexmale:population
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for quasipoisson family taken to be 168.8917)
##
      Null deviance: 11862120
                               on 13908
                                         degrees of freedom
## Residual deviance: 1922293 on 13882 degrees of freedom
## AIC: NA
## Number of Fisher Scoring iterations: 6
glance(glm_fit) %>%
 summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
##
    pseudoR2
        <dbl>
##
## 1 0.837947
 ## pseudoR2 = 0.837947
glm_fit <- glm(suicides ~ sex + age +</pre>
                 generation + population +
                 continent + population:age +
                population:continent + population:sex +
                population:generation, train_set, family = "quasipoisson")
summary(glm fit)
##
## Call:
## glm(formula = suicides ~ sex + age + generation + population +
##
      continent + population:age + population:continent + population:sex +
      population:generation, family = "quasipoisson", data = train_set)
##
##
## Deviance Residuals:
      Min
                10
                     Median
                                  3Q
                                          Max
## -84.250
           -8.304
                     -3.606
                               0.898
                                       84.814
```

```
##
## Coefficients:
##
                                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                       8.836e-01 2.647e-01
                                                              3.338 0.000848 ***
## sexmale
                                       1.270e+00 2.489e-02 51.031 < 2e-16 ***
## age25-34 years
                                       2.881e-01 4.465e-02
                                                              6.452 1.14e-10 ***
## age35-54 years
                                       7.394e-01 5.110e-02 14.469 < 2e-16 ***
## age5-14 years
                                      -2.620e+00 1.472e-01 -17.794 < 2e-16 ***
## age55-74 years
                                       6.261e-01 6.238e-02 10.035
                                                                     < 2e-16 ***
## age75+ years
                                       8.850e-02 7.159e-02 1.236 0.216425
## generationG.I. Generation
                                       2.449e-03 5.529e-02
                                                              0.044 0.964667
## generationGeneration X
                                      -1.284e-01 3.554e-02 -3.612 0.000304 ***
## generationGeneration Z
                                      -1.819e-01 2.570e-01 -0.708 0.478928
## generationMillenials
                                      -2.075e-01 5.557e-02 -3.734 0.000189 ***
                                      -1.043e-01 3.974e-02 -2.624 0.008691 **
## generationSilent
## population
                                       4.573e-07 7.119e-08
                                                              6.424 1.37e-10 ***
## continentAmericas
                                       1.983e+00 2.603e-01
                                                             7.617 2.77e-14 ***
## continentAsia
                                       2.530e+00 2.603e-01
                                                              9.720 < 2e-16 ***
## continentEurope
                                      2.865e+00 2.594e-01 11.043 < 2e-16 ***
## continentOceania
                                       1.601e+00 2.819e-01
                                                              5.678 1.39e-08 ***
## age25-34 years:population
                                      -3.767e-09 3.963e-09 -0.951 0.341830
## age35-54 years:population
                                      -1.086e-07 4.002e-09 -27.134 < 2e-16 ***
## age5-14 years:population
                                      -2.207e-10 1.296e-08 -0.017 0.986413
## age55-74 years:population
                                      -7.768e-08 4.565e-09 -17.018 < 2e-16 ***
## age75+ years:population
                                       2.028e-07 8.148e-09 24.893 < 2e-16 ***
## population:continentAmericas
                                      -2.523e-07 7.109e-08 -3.549 0.000389 ***
## population:continentAsia
                                      -1.643e-07 7.112e-08 -2.311 0.020869 *
## population:continentEurope
                                      -1.621e-07 7.109e-08 -2.280 0.022638 *
## population:continentOceania
                                       4.755e-07 9.113e-08
                                                            5.218 1.83e-07 ***
## sexmale:population
                                       1.287e-08 1.732e-09
                                                             7.431 1.14e-13 ***
## generationG.I. Generation:population 7.165e-08 5.150e-09 13.914 < 2e-16 ***
## generationGeneration X:population
                                       1.682e-10 2.035e-09
                                                              0.083 0.934126
## generationGeneration Z:population
                                       -6.549e-09 2.232e-08 -0.293 0.769246
## generationMillenials:population
                                      -1.118e-08 4.462e-09 -2.506 0.012234 *
## generationSilent:population
                                       3.074e-08 2.529e-09 12.155 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for quasipoisson family taken to be 165.074)
##
##
      Null deviance: 11862120 on 13908 degrees of freedom
## Residual deviance: 1880745 on 13877 degrees of freedom
##
## Number of Fisher Scoring iterations: 6
glance(glm_fit) %>%
 summarize(pseudoR2 = 1 - (deviance/null.deviance)) ## equivalent to R2 ##
## # A tibble: 1 x 1
    pseudoR2
##
       <dbl>
## 1 0.841450
```

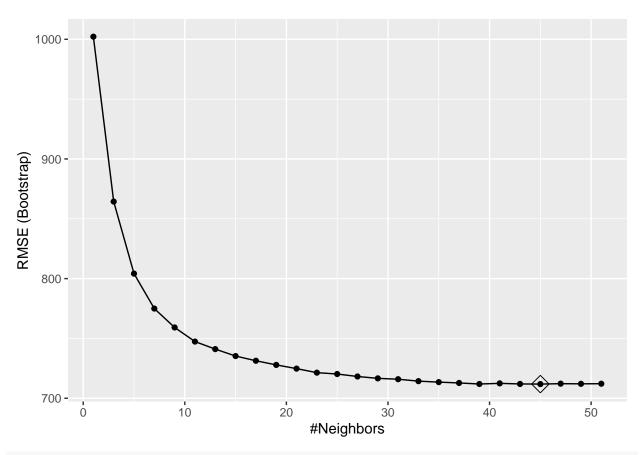
```
## pseudoR2 = 0.841450
```

The final model, with 4 interaction terms, generated the highest pseudo R-squared. This quasipoisson model generated the RMSE of 407, which is a significant improvement from both the first Quasipoisson Model and the Linear Model with Interactions.

```
glm fit <- glm(suicides ~ sex + age +
                  generation + population +
                  continent + population:age +
                 population:continent + population:sex +
                 population:generation, train_set,
               family = "quasipoisson")
y_hat4 <- predict(glm_fit, test_set, type = "response")</pre>
RMSE_glm_int <- RMSE(actual_suicides, y_hat4)</pre>
rmse_results <- bind_rows(rmse_results,</pre>
                           tibble(Method = "Quasipoisson Model, Interactions",
                                  RMSE = RMSE_glm_int))
rmse_results
## # A tibble: 5 x 2
##
     Method
                                            RMSE
##
     <chr>
                                           <dbl>
## 1 Mean
                                         913.738
## 2 Linear Model
                                         691.420
## 3 Linear Model, Interactions
                                         512.796
## 4 Quasipoisson Model
                                        1032.60
## 5 Quasipoisson Model, Interactions 406.995
```

# K-Nearest Neighbors

The next model is developed using K-Nearest Neighbors. Under this model, the suicide rate estimates are based on the average of its k closest points. This approach is suitable here, considering that K-Nearest Neighbors is a non-parametric approach which models complex non-linear situations well.



train\_knn\$bestTune ## k = 45

## k ## 23 45

The KNN model generated an RMSE of 718.76, which is not better than either the Linear Model with Interactions or the Quasipoisson Model with Interactions.

```
## # A tibble: 6 x 2
##
     Method
                                           RMSE
     <chr>>
##
                                          <dbl>
## 1 Mean
                                        913.738
## 2 Linear Model
                                        691.420
## 3 Linear Model, Interactions
                                        512.796
## 4 Quasipoisson Model
                                       1032.60
## 5 Quasipoisson Model, Interactions 406.995
## 6 K-Nearest Neighbors
                                        718.762
```

### Random Forest

## 6 K-Nearest Neighbors

The final method uses the random forest approach. This approach is a meta-estimator which takes the average of multiple decision trees, thereby improving accuracy without overfitting.

The first random forest model is developed using the ranger package. This package works well with larger datasets and is faster than the randomForest package.

```
set.seed(123, sample.kind = "Rounding")
## Warning in set.seed(123, sample.kind = "Rounding"): non-uniform 'Rounding'
## sampler used
fit_rf1 <- ranger(suicides ~ sex + age + generation +
                    population + continent,
                  data = train_set,
                    num.trees = 500,
                  respect.unordered.factors = "order",
                  seed = 1234)
print(fit_rf1)
## Ranger result
##
## Call:
  ranger(suicides ~ sex + age + generation + population + continent,
##
                                                                              data = train_set, num.trees
                                      Regression
## Type:
## Number of trees:
                                      500
## Sample size:
                                      13909
## Number of independent variables:
                                      5
## Mtry:
## Target node size:
## Variable importance mode:
                                      none
## Splitrule:
                                      variance
## 00B prediction error (MSE):
                                      79700.62
## R squared (00B):
                                      0.899433
y_hat6 <- predict(fit_rf1, test_set)$predictions</pre>
RMSE_rf_ranger <- RMSE(actual_suicides, y_hat6)
rmse_results <- bind_rows(rmse_results,</pre>
                           tibble(Method = "Random Forest - Ranger",
                                  RMSE = RMSE_rf_ranger))
rmse_results
## # A tibble: 7 x 2
    Method
                                           RMSE
     <chr>
##
                                          <dbl>
## 1 Mean
                                        913.738
## 2 Linear Model
                                        691.420
## 3 Linear Model, Interactions
                                        512.796
## 4 Quasipoisson Model
                                       1032.60
## 5 Quasipoisson Model, Interactions 406.995
```

718.762

```
## 7 Random Forest - Ranger 276.148
```

The first Random Forest model generated an RMSE of 276.15, which is better than Quasipoisson Model with Interactions and the Linear Model with Interactions. Under this model, the population variable is the most important as it has the highest variable importance value.

```
fit_rf1$variable.importance
```

#### ## NULL

The Random Forest model can be tuned in order to generate better results. For example, the mtry value can be optimized. Mtry refers to the number of variables which are selected at each split.

The randomForest package is used to develop this second tuned Random Forest mode. First, we examine the un-tuned Random Forest model.

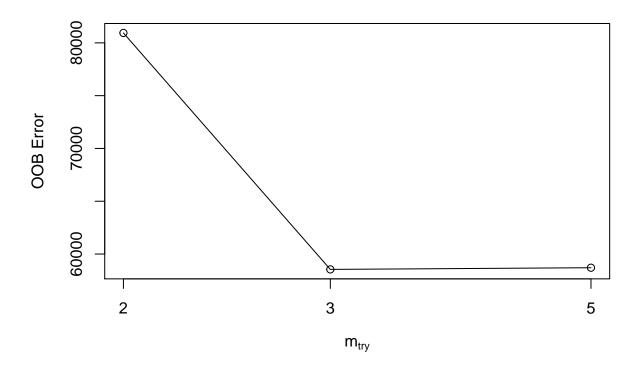
```
set.seed(1234, sample.kind = "Rounding")
## Warning in set.seed(1234, sample.kind = "Rounding"): non-uniform 'Rounding'
## sampler used
fit_rf2 <- randomForest(suicides ~ sex + age + generation +</pre>
                           population + continent,
                         data = train_set,
                         ntree = 500)
which.min(fit_rf2$mse)
## [1] 23
print(fit_rf2)
## Call:
    randomForest(formula = suicides ~ sex + age + generation + population +
                                                                                     continent, data = trai:
                  Type of random forest: regression
##
                         Number of trees: 500
## No. of variables tried at each split: 1
##
##
             Mean of squared residuals: 283131.4
                        % Var explained: 64.27
y_hat7 <- predict(fit_rf2, test_set)</pre>
RMSE_rf_1 <- RMSE(actual_suicides, y_hat7)</pre>
rmse_results <- bind_rows(rmse_results,</pre>
                           tibble(Method = "Random Forest - Not Tuned",
                                  RMSE = RMSE_rf_1))
rmse results
## # A tibble: 8 x 2
##
    Method
                                            RMSE
##
     <chr>
                                           <dbl>
## 1 Mean
                                         913.738
## 2 Linear Model
                                         691.420
## 3 Linear Model, Interactions
                                         512.796
## 4 Quasipoisson Model
                                        1032.60
## 5 Quasipoisson Model, Interactions 406.995
## 6 K-Nearest Neighbors
                                         718.762
```

```
## 7 Random Forest - Ranger 276.148
## 8 Random Forest - Not Tuned 543.567
```

## -0.002713164 0.01

This un-tuned Random Forest model generated an RMSE of 543.57, which is higher than that generated by the un-tuned Random Forest - Ranger model.

Next, the Random Forest model is tuned in order to determine the optimal mtry value. We start with an mtry of 3 (mtryStart), which is increased by a factor of 2 (stepFactor) until the out-of-bag (OOB) error stops improving by 1%.



```
best_m <- mtry[mtry[,2] == min(mtry[,2]), 1]
print(mtry)
     mtry OOBError
## 2
        2 80939.13
## 3
        3 58544.36
## 5
        5 58703.20
print(best_m)
## [1] 3
Here, optimal mtry = 3 as it has the lowest OOB error. Next, we develop the model using the best mtry
set.seed(123, sample.kind = "Rounding")
## Warning in set.seed(123, sample.kind = "Rounding"): non-uniform 'Rounding'
## sampler used
fit_rf3 <- randomForest(suicides ~ sex + age + generation +</pre>
                           population + continent,
                         data = train_set,
                         mtry = best_m,
                         importance = TRUE,
                         ntree = 500)
print(fit_rf3)
##
## Call:
## randomForest(formula = suicides ~ sex + age + generation + population +
                                                                                     continent, data = trai:
##
                   Type of random forest: regression
                         Number of trees: 500
##
## No. of variables tried at each split: 3
##
##
             Mean of squared residuals: 57907.79
##
                        % Var explained: 92.69
y_hat8 <- predict(fit_rf3, test_set)</pre>
RMSE_rf_2 <- RMSE(actual_suicides, y_hat8)</pre>
rmse_results <- bind_rows(rmse_results,</pre>
                           tibble(Method = "Random Forest - Tuned",
                                  RMSE = RMSE_rf_2))
rmse_results
## # A tibble: 9 x 2
##
    Method
                                            RMSE
##
     <chr>
                                           <dbl>
## 1 Mean
                                         913.738
## 2 Linear Model
                                         691.420
## 3 Linear Model, Interactions
                                         512.796
## 4 Quasipoisson Model
                                        1032.60
## 5 Quasipoisson Model, Interactions 406.995
## 6 K-Nearest Neighbors
                                        718.762
```

276.148

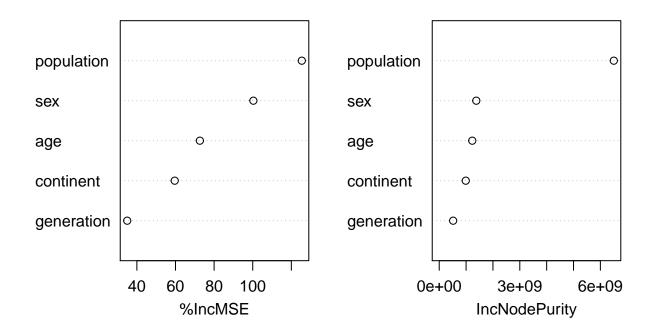
## 7 Random Forest - Ranger

```
## 8 Random Forest - Not Tuned 543.567
## 9 Random Forest - Tuned 219.898
```

The tuned Random Forest model generated the lowest RMSE of 219.90. As with the first Random Forest model, the population variable is the most important in the Tuned Model.

```
varImpPlot(fit_rf3, main ="Variable Importance")
```

## Variable Importance



#### varImp(fit\_rf3)

```
## Sex 100.34178
## age 72.63087
## generation 35.00050
## population 125.43243
## continent 59.60981
```

#### SUMMARY AND CONCLUSIONS

This project aimed to examine global suicide trends using machine learning models in order to determine whether demographic variables were associated with suicide rates. The effectiveness of the models were assessed using the RMSE.

K-Nearest Neighbors is among the least effective of the approaches, as the KNN model generated a high RMSE.

We found that linear models did not adequately capture the relationship between suicide rates, on the one hand, and demographic variables on the other. This suggests that the relationship between suicide rates and

these variables are non-linear in nature. Nevertheless, the adjusted R-squared values of the linear models improved when (1) the population variable and (2) population's interaction with the other variables were included.

The Quasipoisson Model with Interactions fared marginally better. However, there seems to still be a significant lack of fit considering the residual deviance of 1,880,745 with 13,877 degrees of freedom.

The Random Forest:Ranger and Random Forest:Tuned Models performed best, generating an RMSE of 276.15 and 219.90 respectively. These models show that the population variable is the most important, followed by sex.

This analysis was limited to an exploration of the association between suicide rates and the available demographic variables. The models developed here suggest that suicide rates are associated with population and sex. Specifically, males are more prone to suicides.

The relationship between population and suicides, however, is more complex, due to the interaction effect of other variables. Thus, this project can be expanded upon by looking into the interaction of population with other, possibly, confounding variables.