STAT540-HW2-Problem1 & 2

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Problem1

a)

$g(z) = \frac{1}{1+e^{-z}}$ Therefore,
$\frac{\partial(g(z))}{\partial(z)} = \frac{0-(-e^{-z})}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}}(\frac{1+e^{-z}-1}{1+e^{-z}}) = g(z)(1-g(z))$

b)

$NLL(\theta) = -logP(D \mid \theta) = \sum_{i=1}^{n}[-y^{(i)}logh_\theta(x^{(i)}) - (1-y^{(i)})log(1-h_\theta(x^{(i)}))]$
$h_\theta(x^{(i)}) = g(\theta^T x^{(i)}) = \frac{1}{1+e^{-\theta^T x^{(i)}}}$
Let $z = \theta^T x^{(i)}$
Then $\frac{\partial(NLL(\theta))}{\partial(\theta)} = \frac{\partial(NLL(\theta))}{\partial(z)}\frac{\partial(z)}{\partial(\theta)}$
$\frac{\partial(z)}{\partial(\theta)} = \frac{\partial(\theta^T x^{(i)})}{\partial(\theta)} = x^{(i)}$
$\frac{\partial(NLL(\theta))}{\partial(z)} = \sum_{i=1}^{n}(-y^{(i)}\frac{(g(z)^{'})}{g(z)} - (1-y^{(i)})\frac{(1-g(z))^{'}}{1-g(z)}) = \sum_{i=1}^{n}(-y^{(i)}\frac{g(z)(1-g(z))}{g(z)} - (1-y^{(i)})\frac{g(z)(g(z)-1)}{(1-g(z))}) =$
$\sum_{i=1}^{n}(-y^{(i)} + y^{(i)}g(z) - y^{(i)}g(z) + g(z)) = \sum_{i=1}^{n}(h_\theta(x^{(i)} - y^{(i)})$
$\frac{\partial(NLL(\theta))}{\partial(\theta)} = \sum_{i=1}^{n}(h_\theta(x^{(i)} - y^{(i)})x^{(i)}$