

Lecture 1: Introduction to Probability

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1 Introduction

Each time a concept is introduced, there will be an example that illustrates how this is useful in HRI

1.1 Probability

Why do we need probability? To understand how likely an outcome is!

Random variable: this is for cases when we can not predict exactly a specific outcome in advance, but we can know the probability. For example, a coin toss

Probability can also be used to predict human behavior.

Probability Distribution: A function that maps events to a number

$$P : events \rightarrow [0,1]$$

Example: Assume a robot is picking up a glass from a table. The human can choose whether or not to intervene. So, there are two human states: 1. trusting and 2. \neg trusting

	\neg Intervene	Intervene
Trusting	0.4	0.1
\neg Trusting	0.2	0.3

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/B) = \frac{P(B \cap A)P(A)}{P(B)}$$

So,

$$P(\text{Intervene} / \text{Trusting}) = 0.1 / 0.5 = 0.2$$

and

$$\begin{aligned} P(\text{Trusting} / \text{Intervene}) &= \frac{P(\text{Intervene} / \text{Trusting})P(\text{Trusting})}{P(\text{intervene})} \\ &= \frac{0.2 * 0.5}{0.4} = 0.1 / 0.4 = 0.25 \end{aligned}$$

We can compute $P(\text{Intervene})$ with conditioning.

Marginalization:

$$P(A) = P(A \cap B) + P(A \cap \neg B) = P(A/B)P(B) + P(A/B)P(\neg B)$$

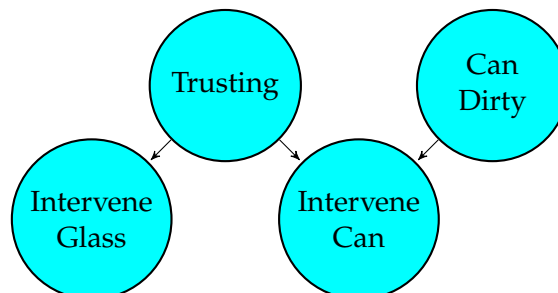
Therefore,

$$\begin{aligned} P(\text{intervene}) &= P(\text{intervene} \cap \text{trusting}) + P(\text{intervene} \cap \neg \text{trusting}) \\ &= P(\text{intervene} / \text{trusting})P(\text{trusting}) + P(\text{intervene} / \text{trusting})P(\neg \text{trusting}) \end{aligned}$$

Note: Trusting and Intervene are NOT independent.

Let's say that we know whether the person trusts the robot or not, perhaps through a survey. Then, whether the person picks up the can or glass is conditionally independent.

$$\begin{aligned} &P(\text{trusting} / \text{intervene} - \text{can}, \text{intervene} - \text{glass}) \\ &= \frac{P(\text{intervene} - \text{can}, \text{intervene} - \text{glass} / \text{trusting})P(\text{trusting})}{P(\text{intervene} - \text{can}, \text{intervene} - \text{glass})} \\ &= \frac{P(\text{Intervene} - \text{can} / \text{trusting})P(\text{intervene} - \text{glass} / \text{trusting})P(\text{trusting})}{P(\text{Intervene} - \text{can}, \text{intervene} - \text{glass})} \end{aligned}$$



Let's say that the robot goes for the can, and the human does not intervene. Therefore, our belief as to whether the can is dirty or not goes up. Our belief in the human's trust in the robot also goes up. Our belief in the human intervening in the glass goes down.

Let's say that the robot goes for the glass, and the human does not intervene in the glass. This increases our estimate in the trust. Then, our belief that the can is dirty goes down, BECAUSE: there are two possible causes for intervening in the can. Our belief in one went up, our belief in the other went down.

In general, if there are multiple causes and the belief in one goes up, then the belief in the other goes down.