Guidelines for Using State-of-the-Art Methods to Estimate Propensity Score and Inverse Probability of Treatment Weights When Drawing Causal Inferences

Session 4b. Sensitivity Analysis

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No Unobserved Covariates

- **Causal effect estimates are unbiased only if there are no unobserved confounders**
- **We *assume* treatment assignment is ignorable given the observed pretreatment variables used in the modeling**

Checking the Assumption

- We cannot test for ignorability
- Instead we conduct sensitivity analysis
- Try to assess if estimated treatment effect could plausibly be explained by an omitted variable

Exploring Possible Omitted Variables

- **What are the two characteristics of omitted variables required for creating bias in estimated treatment effects?**
  (Hint: these are the two characteristics we used to select variables to include in propensity score modeling)

Exploring Possible Omitted Variable

- **What are the two characteristics of an omitted variable required for creating bias in estimated treatment effects? (Hint: these are the two characteristics we used to select variables to include in propensity score modeling)**
  1. **It must be related to treatment assignment, that is its distribution differs across treatment and control groups and the more it differs, the greater the potential for impact**
  2. **It must be related to (Y_0, Y_1) and the stronger the relationship, the the greater the potential for impact**

Two Approaches

- Find how strong the relationship between the omitted variable and treatment or the outcome must be to have a consequential impact inferences about treatment
  - Rosenbaum (2002), Ridgeway (2006)
- Estimated adjusted treatment effects that allow for uncertainty due to potential
  - Greenland (2001), VanderWeele and Arah (2011)

Impact of an Omitted Variable (1)

- **Concern is that there exists an omitted variable which differs between the treatment and control groups even after weighting by the observed covariates X**
- **Let U denote the omitted variable**
- **If distribution of U differs in the treatment and control groups, even after weighting for X, then we have**
  $Pr(Z = 1 \mid X) \neq Pr(Z = 1 \mid X, U)$
  - $Pr(Z = 1 \mid X) \neq Pr(Z = 1 \mid X, U)$ **could be the definition of an omitted variable**

Impact of an Omitted Variable (2)

- **To make things concrete we will focus on ATT**
- **If** $Pr(Z = 1 \mid X) \neq Pr(Z = 1 \mid X, U)$ **then**

$$\frac{Pr(Z=1|X)}{1-Pr(Z=1|X)} \neq \frac{Pr(Z=1|X,U)}{1-Pr(Z=1|X,U)} \Rightarrow \left(\frac{Pr(Z=1|X)}{1-Pr(Z=1|X)}\right) a = \frac{Pr(Z=1|X,U)}{1-Pr(Z=1|X,U)}$$

  for some $a \neq 1$

- **The further** a **is from 1 (in either direction) the stronger the effect of** U **on treatment or the more separated the distributions of** U **for treatment and control**
  - $f(U \mid X, Z = z) = \frac{Pr(Z=z|U,X)f(U|X)}{Pr(Z=z|X)f(X)}$**, for** $z = 0, 1$
  - $\frac{f(U|X,Z=1)}{f(U|X,Z=0)} = \frac{Pr(Z=1|U,X)/Pr(Z=0|U,X)}{Pr(Z=1|X)/Pr(Z=0|X)} = a$
- **Doesn't matter if** U **is single omitted variable or a group of omitted variables**

Simulating the impact of the omitted variable

- Let $w_i = \Pr(Z = 1|X_i)/(1\text{- }\Pr(Z = 1|X_i)) =$ our ATT weight
- Let $a_i = \frac{\Pr(Z = 1|U_i, X_i)/(1-\Pr(Z = 1|U_i, X_i))}{\Pr(Z = 1|X_i)/(1-\Pr(Z = 1|X_i))}$ = the distortion to the weight due to the omitted variable for observation *i*
- Let $w_i^* = a_i \times w_i$ = the correct weight for observation *i*
- We can approximate the $E[Y_0 \mid Z = 1]$ by $\sum_{i \in Z_i = 0} w_i^* Y_i$
- We derived this relationship between the a_i's and $U_i's$
  - $a_i = \pi * (1 + \exp(-ES * U_i))$ where $\pi = \Pr(Z = 1)$ and ES denotes the mean effect size difference in U for treatment and control
- So, if we simulating values for the $U_i's$ we can simulate the "true" $E[Y_0]$

How do we simulate the U's

- What are the 2 characteristics of an omitted variable required for creating bias in estimated treatment effects?
  1. It must be related to treatment assignment, i.e., its distribution differs across treatment and control groups and the more it differs, the greater the potential for impact
  2. It must be related to (Y_0, Y_1) and the stronger the relationship, the greater the potential for impact

Simulating the U's

- We want to simulate $U_i's$ to reflect
  1. Different levels for the strength of the relationship between U and Z
     - We capture this using effect size differences

$$ES_{U,Z} = \frac{(\bar{U}_{Z=1} - \bar{U}_{Z=0})}{sd(U)}$$

  2. Different levels for the strength of the relationship between U and Y_0 (we can restrict to Y_0 here since we are estimating ATT)
     - We capture this using the correlation between U and Y (denoted by $\rho_{U,Y}$)

Now we need to map the a's back to user-friendly quantities

- Recall we derived:

$$a = \pi(1 + \exp\{-ES \cdot U\})$$

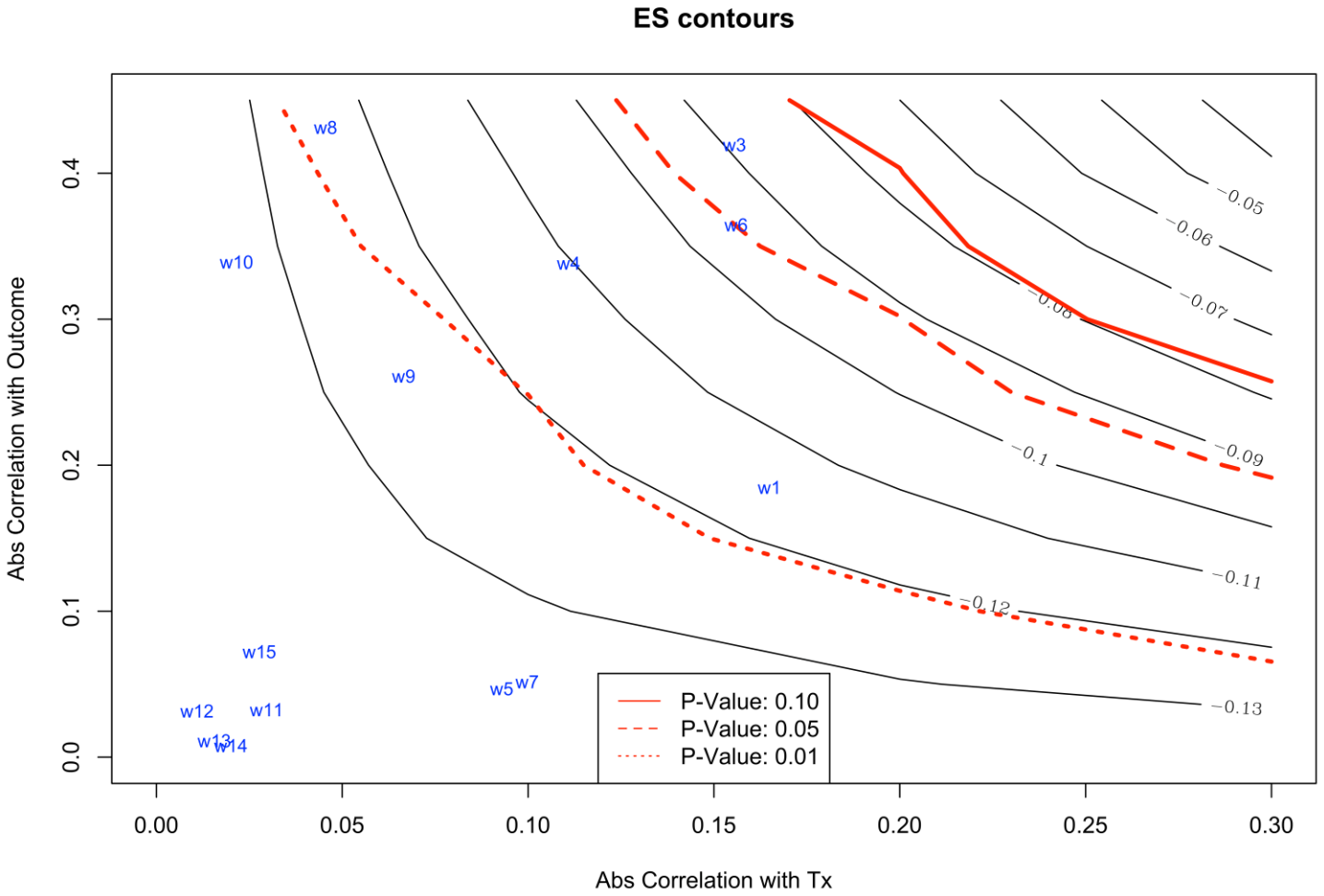
where $\pi = \Pr(Z = 1)$

  - So - once we generate U's based on a fixed $ES_{U,Z}$ and a fixed $\rho_{U,Y}$, we can compute the needed a's.
  - Then we can compute updated PSW and TE estimates

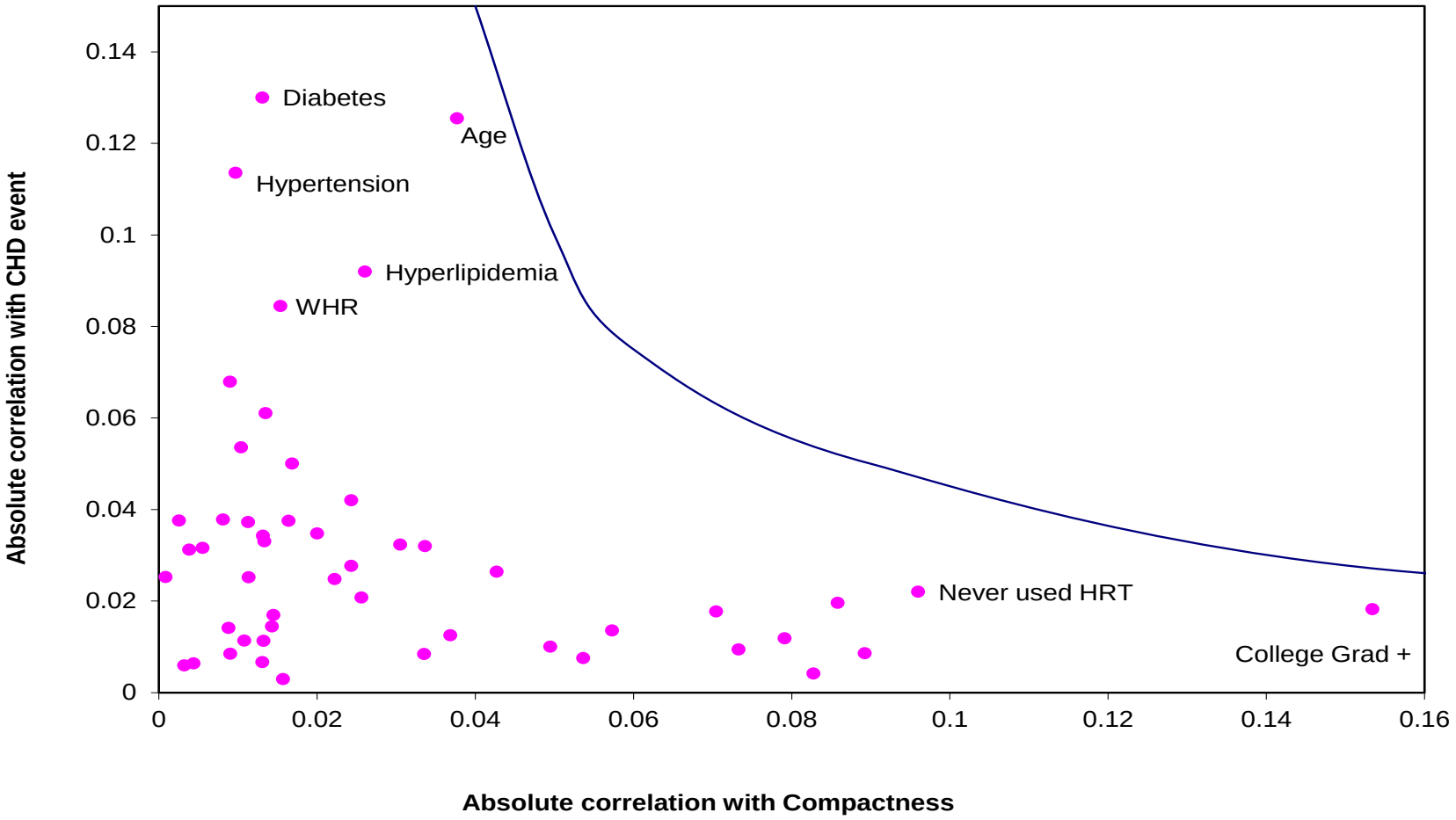
Our general approach

- State relationship between confounder and treatment status as an effect size
- State relationship between confounder and outcome as correlation
- Simulate unobserved confounder with exactly that ES and correlation within sample
- Translate U into needed a_i's and calculate updated propensity score weights (w_i^*)
- Re-run outcome models & capture adjusted p-value and treatment effect size
- Produce user-friendly graphics showing key findings

Sample graphic (work in progress): Results likely very sensitive



Another example: Results more robust



Generally two classes of omitted variable work in the field

- Class 1: Focuses on sensitivity analyses which show how sensitivity inferences are as a function of the relationship between U and Z and U and Y
- Class 2: Presents adjusted treatment effect (TE) estimates that directly adjust for the relationship between U and Z and U and Y

More recent work is promoting that all observational studies report an e-value: the minimum strength of a relationship that U would have to have with both Z or Y to wipe away a study's findings

(VanderWeele and Ding 2017)

Useful references

- Liu, W., et al. (2013). "An introduction to sensitivity analysis for unobserved confounding in nonexperimental prevention research." Prevention science **14**(6): 570-580.

- Carnegie, N. B., et al. (2016). "Assessing sensitivity to unmeasured confounding using a simulated potential confounder." Journal of Research on Educational Effectiveness **9**(3): 395-420.

- Schneeweiss, S. (2006). "Sensitivity analysis and external adjustment for unmeasured confounders in epidemiologic database studies of therapeutics." Pharmacoepidemiology drug safety **15**(5): 291-303.

- Blackwell, M. (2014). "A selection bias approach to sensitivity analysis for causal effects." Political analysis **22**(2): 169-182.

- VanderWeele, T. J. and P. Ding (2017). "Sensitivity analysis in observational research: introducing the E-value." Annals of Internal Medicine **167**(4): 268-274.

Other useful packages

- tipr – Tipping Point Analysis - D'Agostino-McGowan dissertation
  - tells you tipping points at which results will lose significance
  - only get simple stats; 1 value for one specific scenario
- treatSens – Bohme Carnegie, Harada and Hill 2016 JREE
  - Much closer to what we are trying for
  - Simulate U and then produce graphics like we have discussed, including use of observed covariates
  - BART can be used to estimate PS weights and/or outcome model
  - No user-friendly tutorial which would've been appreciated given complexity; the example code helped but ran into a bug which didn't have bandwidth to debug

Sample graphic from treatSens