HW02-2

2022年3月28日 星期一　　下午8:07

- Proof of Beta-Binomial conjugation

likelihood : $P(X=m \mid N,p) = \binom{N}{m} p^m (1-p)^{N-m}$

, which is a binomial distribution with m success and N trials

prior : $B(\theta \mid a,b) = \theta^{a-1}(1-\theta)^{b-1}\frac{1}{\beta(a,b)}$

, which is a beta distribution with a success and a+b trials , $\frac{1}{\beta(a,b)} = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$

posterior = $P(\theta \mid event) = \frac{likelihood \cdot prior}{marginal}$

$= \frac{\binom{N}{m} p^m (1-p)^{N-m} \cdot p^{a-1}(1-p)^{b-1}\frac{1}{\beta(a,b)}}{\int_0^1 \binom{N}{m} \theta^m (1-\theta)^{N-m} \cdot \theta^{a-1}(1-\theta)^{b-1}\frac{1}{\beta(a,b)}\, d\theta}$, $\Gamma(x) = \begin{cases} 1 & , x=1,2 \\ (x-1)! & , otherwise \end{cases}$

$= \frac{p^{m+a-1}(1-p)^{N-m+b-1}}{\int_0^1 \theta^{m+a-1}(1-\theta)^{N-m+b-1} d\theta} = \beta(m+a, N-m+b)$

$= p^{m+a-1}(1-p)^{N-m+b-1}\frac{1}{\beta(m+a, N-m+b)}$

$= B(p \mid m+a, N-m+b)$, which is also a beta distribution

```python
import os
import argparse
import math

def parse_args():
    parser = argparse.ArgumentParser()
    parser.add_argument('--filename', default='./testfile.txt', type = str,)
    parser.add_argument("--a", default = 0, type = int)
    parser.add_argument("--b", default = 0, type = int)
    return parser.parse_args()

def C(N,m):
    c = math.factorial(N) / (math.factorial(m) * math.factorial(N-m))
    return c

def binomial(a, b):
    N = a + b + 0.0
    p = a / N
    bp = C(N,a) * (p**a) * (1.0-p)**b
    return bp

def counter(line):
    cnt_a = line.count('1')
    cnt_b = line.count('0')
    return cnt_a, cnt_b

if __name__ == '__main__':
```

```
28
29      args = parse_args()
30      f = args.filename
31      prior_a, prior_b = args.a, args.b
32      fp = open(f, "r")
33
34      lines = fp.readlines()
35      for i, _line in enumerate(lines):
36          line = _line.strip()
37
38          _a, _b = counter(line)
39          posterior_a, posterior_b = prior_a + _a, prior_b + _b
40          likelihood = binomial(_a , _b)
41
42          print('case', i, ':', line)
43          print('Likelihood:',likelihood)
44          print('Beta prior:\ta =',prior_a,'b =', prior_b)
45          print('Beta posterior: a =',posterior_a,'b =', posterior_b, end = '\n\n')
46
47          prior_a = posterior_a
48          prior_b = posterior_b
49
50      fp.close()
51
```