1.

① misclassification rates

$$A = \frac{100 + 100}{(300+100)+(100+300)} = 0.25 \qquad B = \frac{0 + 200}{(200+0)+(200+400)} = 0.25$$

② Entropy

$$A = \frac{1}{2}\times 2\left(-\frac{3}{4}\lg_2\frac{3}{4} - \frac{1}{4}\lg_2\frac{1}{4}\right)$$
$$= \left[-\frac{3}{4}(\lg_2 3 - 2) - \frac{1}{4}\cdot(-2)\right]$$
$$= 2 - \frac{3}{4}\lg_2 3 = 0.8112$$

$$B = \frac{1}{4}\cdot 0 + \frac{3}{4}\left(-\frac{1}{3}\lg_2\frac{1}{3} - \frac{2}{3}\lg_2\frac{2}{3}\right)$$
$$= \frac{3}{4}\left[\frac{1}{3}\lg_2 3 - \frac{2}{3}(1 - \lg_2 3)\right]$$
$$= \frac{3}{4}\left(\lg_2 3 - \frac{2}{3}\right) = \frac{3}{4}\lg_2 3 - \frac{1}{2} = 0.6887$$

③ Gini

$$A = \frac{1}{2}\left[1 - \left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] + \frac{1}{2}\left[1 - \left(\frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$$
$$= 1 - \frac{10}{16} = \frac{3}{8} = 0.3750$$

$$B = \frac{1}{4}\left[1 - (1)^2 - (0)^2\right] + \frac{3}{4}\left[1 - \left(\frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2\right]$$
$$= \frac{3}{4}\left(1 - \frac{5}{9}\right) = \frac{3}{4}\left(\frac{4}{9}\right) = \frac{1}{3} = 0.3333$$

2.

Let $J = E[e^{-ty(x)}|x] = P(t=1|x)e^{-y(x)} + P(t=-1|x)e^{y(x)}$

assume that $\frac{\delta J}{\delta y(x)} = -P(t=1|x)e^{-y(x)} + P(t=-1|x)e^{y(x)} = 0$

$$\Rightarrow P(t=1|x)e^{-y(x)} = P(t=-1|x)e^{y(x)}$$

Taking $\ln$ on both sides $\Rightarrow \ln P(t=1|x) - y(x) = \ln P(t=-1|x)\, y(x) = 0$

$$\Rightarrow \ln\frac{P(t=1|x)}{P(t=-1|x)} = 2y(x)$$

$$\therefore y(x) = \frac{1}{2}\ln\frac{P(t=1|x)}{P(t=-1|x)}$$

Alternative:

Let $J(y) = E[e^{-ty(x)}]$

suppose we have a current estimate $y(x)$

and seek an improved estimate $y(x)+f(x)$

by minimizing $J(e^{-ty(x)}e^{-tf(x)})$ at each x.

$$J(y(x)+f(x)) = E(e^{-ty(x)}e^{-tf(x)}|x)$$
$$= e^{-f(x)}E[e^{-ty(x)}I_{[t=1]}|x] + e^{f(x)}E[e^{-ty(x)}I_{[t=-1]}|x],$$

where $E[I_{[t=j]}|x] = P(t=j|x)$

Dividing this through by $E[e^{-ty(x)}|x]$ and setting its derivative w.r.t. $f(x)$ to zero

, we get $f(x) = \frac{1}{2}\ln\frac{E_w[I_{[t=1]}|x]}{E_w[I_{[t=-1]}|x]}$

$$= \frac{1}{2}\ln\frac{P_w[I_{[t=1]}|x]}{P_w[I_{[t=-1]}|x]}, \text{ where } w(x,t) = e^{-ty(x)}$$

Therefore, $y(x) = \frac{1}{2}\ln\frac{P(t=1|x)}{P(t=-1|x)}$