The Effect of Background Contrast on Perceived Transparency Evaluated with Contrast Metrics

Bachelor Thesis

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Contents

1 Introduction

Everyday we see through transparent media. A transparent medium can be smoke, a glass or foggy air. The objects and surfaces are in a three-dimensional environment in the human visual world, which is not the case by the human visual system. The inputs to human visual systems are a two-dimensional arrays of light intensities, not three-dimensional objects. These inputs depend on many variables, such as the reflectance of the surface, the light intensity and other characteristics of the background (Singh & Anderson, 2002). Therefore, the perceived image projected in the retina of a surface can depend on its physical conditions, and two identical surfaces can be perceived different.

1.1 Human Visual System

The human visual system is able to distinguish two different layers in the same retinal area. When there is a medium with a transmittance in front of a background, the human visual system recognizes two surfaces: the background and the transparent medium in front of it. In Figure 1 the human visual system is sketched. As it can be seen, the retinal stimulus is generated by the transmission of the light (transmittance) by the illumination (luminance) source on the reflectance. The perception of a stimulus depends on three factors: luminance, reflectance and transmittance. There are many combinations of these physical factors that can be perceived the same as the same retinal stimulus is generated although the physical characteristics of the stimuli are different Purves, Wojtach, and Lotto (2011).

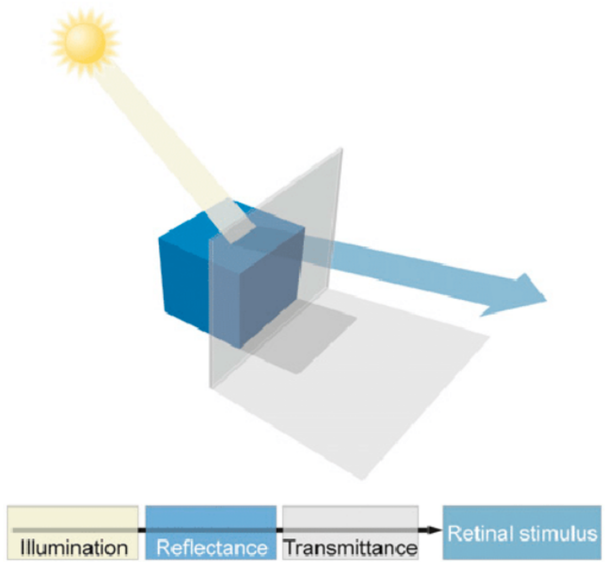


Figure 1: "The conflation of illumination, reflectance, and transmittance in retinal images". Taken from (Purves et al., 2011).

Moreover, there are also combinations where the reflectance is the same but the retinal stimulus is perceived different, because of the other factors: luminance and transmittance. This effect can be illustrated with the "Checker shadow illusion" by Edward H. Adelson. The Adelson checkerboard consists of light and dark squares, which part of them are shadowed by another object (Figure 3). Although the checker A and B have the identical brightness, the checker A appears to be darker than the checker B.

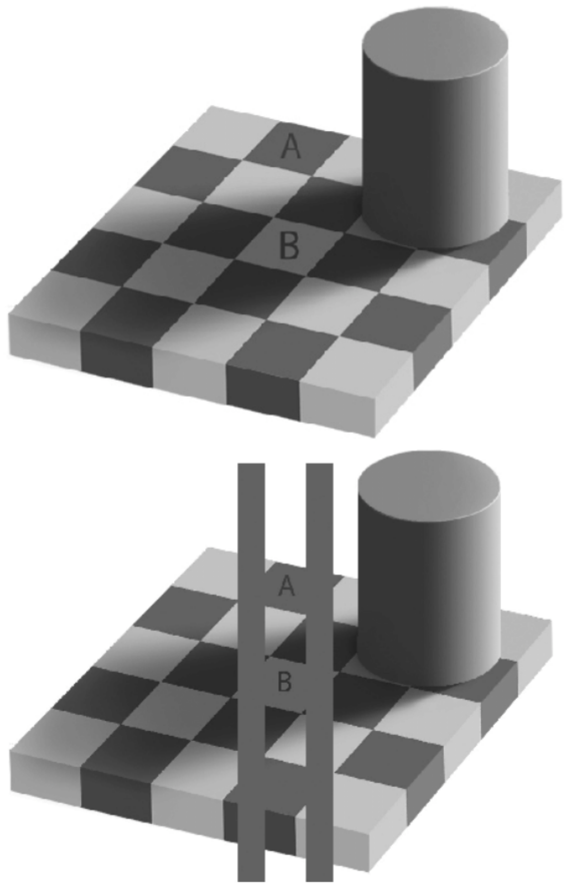


Figure 2: "Adelson´s Checker-shadow illusion. The squares A and B are exactly the same, but the brightness of both squares are perceived differently. Taken from Stahre Wästberg (2006).

1.2 Physical Model of Transparency: the Episcotister Model

It is known that the physical transparency depends on the transmittance of the medium. This effect is observed with the episcotister model. However, the physical transparency is neither a necessary nor a sufficient condition for the perception of transparency (Metelli, 1985). The perceived transparency of a medium is not only dependent to the physical transmittance, but also dependent to the luminance of the medium, its reflectance.

The effect of transmittance and luminance on the perceived transparency can be better illustrated by considering the episcotister model of transparency, which was introduced by Metelli (1970, 1974). An episcotister is a disk with certain opening sector (its physical transmittance, α) and a certain luminance reflected by its surface (t). When it is rotated fast enough (approximately above 60Hz), it is perceived as a transparent disk as the visual system con not distinguish high temporal frequency (Figure 6B). Although the disk is opaque and therefore it is impossible to be transparent, the human visual system recognizes it as transparent. Thus it can be said that the perceived transparency of a medium depends on its transmittance α and luminance t. Transmittance is a ratio of the incident intensity of light (I_0) to the amount of intensity passes through the object (I)[1], basically, it is the ratio of light passing through the medium. The physical transmittance is calculated as follows:

$$T = \frac{I}{I_0} \tag{1}$$

In Figure 6B the transmittance α is shown as it represents the opening of the medium. Clearly, as the opening sector α increases, the disk will be perceived more transparent because the amount of light that passes through the disk is higher.

The other variable influencing perceived transparency is the luminance t. According to the episcotisters model, the luminance of the background seen through the transparencies P and Q on different backgrounds are calculated as follows:

$$P = \alpha \cdot a + \big(1 - \alpha\big) \cdot t \tag{2}$$

$$Q = \alpha \cdot b + \big(1 - \alpha\big) \cdot t \tag{3}$$

where a and b are the luminances of the background without the transparency, t is the luminance reflected from the medium and α is the opening sector. As dark surfaces reflect less light than brighter surfaces, the luminance t of the dark surfaces is lower than bright surfaces. Accordingly, the lower the luminance is, the darker is the medium.

[1] https://www.electrical4u.com/what-is-transmittance/

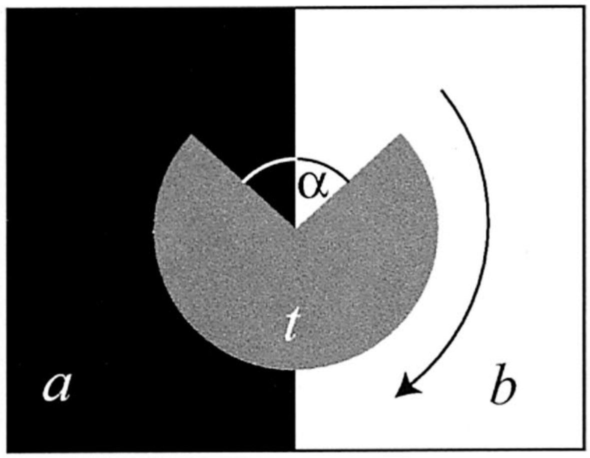


Figure 3: Metelli's episcotister model of transparency. Taken from Singh and Anderson (2002).

1.3 Perceived Transparency

When we assume that the episcotister model is a model of *perceived* transparency, it would predict that a light and a dark transparency would look equally transparent if they have the same transmittance α, because they would have the same amount of light passing through the medium. However, it is known that it is not the case.

The previous work showed that the perceived transparency of a medium depends on its background. Singh and Anderson (2002) observed that contrast is a "critical image variable that the visual system uses to assign perceived transmittance to transparent surfaces". As stimuli, they had a large disks that contain vertical lines and are placed on a black background. Inside the large disk, a smaller disk with lower contrast was placed. In their experiment, the observers were showed a target disk and asked to adjust the luminance of the small disk, until its perceived transmittance was equal that to the perceived transmittance of the target disk. Figure 4 shows an example of the procedure. Singh and Anderson (2002) found out that the perceived transparency of a transparent medium depends not only on its physical transmittance, but also on the mean luminance reflected from the background that the medium covers.

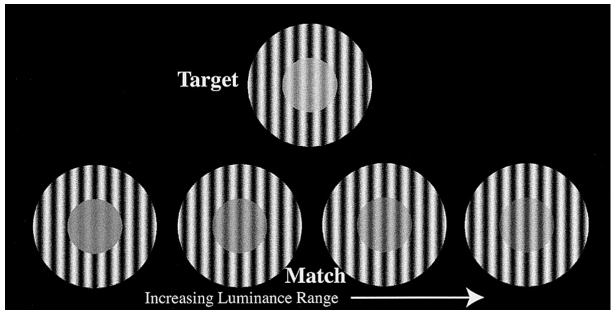
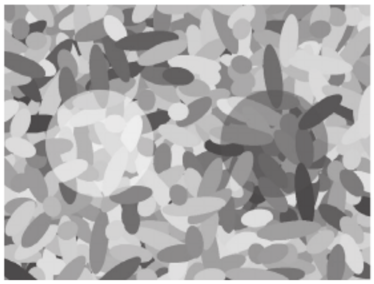


Figure 4: A demonstration of the transmittance-matching task of the experiment by Singh and Anderson (2002). The observers were asked to adjust the luminance of the small disk so that its perceived transparency matched the perceived transparency of the target disk. This was tested for different mean luminances.

Robilotto and Zaidi (2004) also investigated whether the perceived transparency depends on the background. An example of the stimuli they used can be seen in Figure 5. In the left side of each stimulus there is the standard filter, with fixed transmittance and reflectivity, which is the reflection rate of a filter. The right side of each display had the match filter, and one of the properties (transmittance and reflectivity) was adjusted by the observer throughout the experiment. Transmittance is the amount of light passes through the filter, whereas the reflectivity is the amount of light reflected from it, similarly to α and t from the episcotister model. In other words, if the transmittance of a filter is high enough, the filter appears transparent. Contrariwise, when the reflectance of the filter is high, the filter appears opaque. During the experiment, the observers were asked to adjust one property (transmittance or reflectivity) until the perceived transparency of the standard and match filters were equal.

The authors observed that decreasing the background's mean luminance or contrast decreases the perceived transparency of the filter and that the observers used the contrast when adjusting during the task until the perceived transparencies between the filters were equal. This shows that physically identical transparent media (filters) vary on the perceived transparency when they are located in front of backgrounds with different physical contrast and that the perceived transparency correlate with the perceived contrast of the medium. The results show that *reducing background contrast decreases the perceived transparency of the medium.*



(a) A uniform background

(b) Lower contrast background

Figure 5: Example of stimuli (Robilotto & Zaidi, 2004) used in their experiments. The left side of each stimuli contains the standard filter with a fixed transmittance and reflectivity values. The right side of each display contains the match filter and the observers adjusted one value while fixing the other one.

Aguilar and Maertens (2022) used maximum likelihood conjoint measurement (MLCM) to study the relationship between the perceived transparency, perceived contrast and the image luminance. The authors used variegated checkerboards with transparent media that varied in their transmittance α and luminance t. The transparent media were rendered stimulating the episcotister model, as it can be seen in Figure 6.

Consider the case of two disks with equal opening α but different luminance t, one dark and one light (Figure 6B). Then it is clear that when the disks are rotated, they would have equal physical transparency. However, their perceived transparency varies. The dark one with lower luminance t is perceived more transparent than the light one with higher t. Aguilar and Maertens (2022) observed that perceived transparency increases with increasing physical transmittance and decreases with increasing luminance. They found out that for two transparent media with equal transmittance, a dark one would always look more transparent than a bright one (Figure 6A).

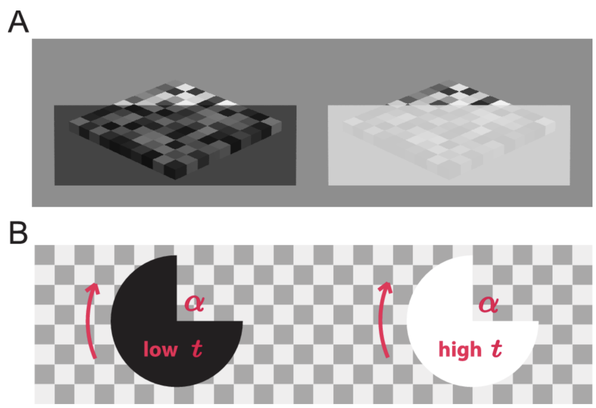


Figure 6: Episcotister Model
A: Two transparent media were rendered with varying luminance values t and equal transmittance α
B: Two episcotisters with identical openings α so that they have the same transmittance when rotated. The black one (low t) is perceived with higher transparency to the observers than the white one (higher t). t is the luminance reflected from the disk.
Taken from Aguilar and Maertens (2022)

Similar to (Singh & Anderson, 2002), (Robilotto & Zaidi, 2004) and (Aguilar & Maertens, 2022) also found out that perceived transparency seems to depend on perceived contrast.

The results mentioned above seam to indicate that *perceived contrast* is highly relevant for our perception of transparency. It is thus also relevant to review some aspects of what we know about contrast processing in the visual system.

1.4 Contrast

Contrast is what makes an object distinguishable from other objects and the background. In human visual perception, contrast is calculated by the difference of the luminance values of the pixels Maragatham and Roomi (2015). In Figure 7 the difference between an image with low contrast and high contrast can be seen.

Figure 7:
The image on the left side has a lower contrast than the image on the right side.
Taken from https://blog.roboflow.com/when-to-use-contrast-as-a-preprocessing-step/

One way of quantifying contrast is by the following equation, known as Michelson contrast:

$$MC = \frac{l_{min} - l_{max}}{l_{min} + l_{max}} \tag{4}$$

where l_{min} and l_{max} are the highest and lowest luminance. The Michelson contrast equation is used in visual perception research using simple stimuli, such as gratings.

Since only the highest and the lowest luminance values are considered in Michelson equation, it works better for images with two values of luminance, e.g. a checkerboard with black and white checkers. However, the Michelson contrast equation does not give the most accurate contrast value of an image consisting more than two luminance values. Consider two checkerboards, one with two luminance values [0.0, 1.0] and the other with three [0.0, 0.5, 1.0]. The Michelson contrast equation will calculate the same contrast for both images, yet the perceived contrasts of the checkerboards might differ. Figure 8 shows two images with different luminance values but equal contrasts according to Michelson contrast equation (6).

(a) A checkerboard with two luminance values [0.0, 1.0]

(b) A checkerboard with three luminance values [0.0, 0.5, 1.0]

Figure 8: Example of checkerboards with different luminance values but equal contrasts. Although both stimuli have the same luminance values, the stimulus b appears to have less contrast than the stimulus a.

Our perception of contrast is however more complex than the simple Michelson contrast equation. In fact, it is known that the background contrast also has an effect on the perceived contrast. In 1989 Charles Chubb and his colleagues observed the "contrast contrast illusion", where the perceived contrast of an object depends on the physical contrast of its background. They did an experiment with two physically identical disks with backgrounds with different contrasts. In Figure 9 it can be seen that the background of the left disk has the lowest contrast (zero-contrast) and the right disk has a higher contrast. The disks are identical, however Chubb, Sperling, and Solomon (1989) observed that the *left disk with lower background contrast appears to have a higher contrast than the right one with higher background contrast to all observers.*

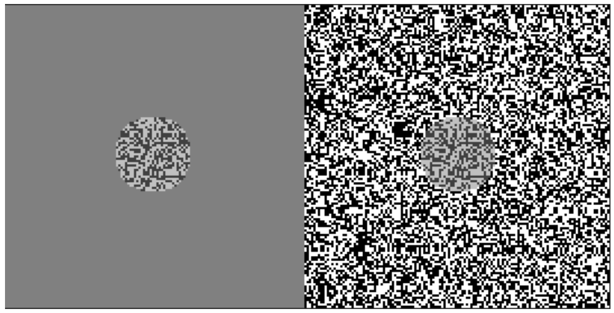


Figure 9: Two identical disks with different backgrounds. The background of the left disk has a lower contrast than the contrast of the background on the right. The one on the left appears to have higher perceived contrast. Source: (Chubb et al., 1989)

1.5 Specific Motivation and Research Question

Chubb and his colleagues found that a lower background contrast causes a higher perceived contrast of the central disk (Chubb et al., 1989). Contrarily, Robilotto and Zaidi (2004) observed that a lower background contrast leads to a lower perceived contrast and perceived transparency of the disk. These two findings seem incompatible with each other. Considering only the findings of Chubb et al. (1989), it is expected that a lower background contrast causes a higher perceived contrast of the medium, and accordingly a higher perceived transparency. In contrast, the findings of Robilotto and Zaidi (2004) would indicate otherwise; that a lower background contrast causes a lower perceived transparency of the medium, which would be associated with lower perceived contrast.

I investigated the relationship between the background contrast and the perceived transparency of the medium. As mentioned above, it is known that perceived transparency is affected by its background, but so far no image metric or image statistic can predict our subjective perception. With that in mind, in my thesis I evaluate metrics based on contrast that could explain the perceptual effects. The goal is to find a metric that can predict the effect of the background contrast on the perceived transparency of the medium. In the following section, I present in detail the different contrast metrics used, based on the contrast literature. The evaluation of perceived transparency will be phenomenological, that is, by simply looking at the resulting stimuli.

A possible explanation for the contradictory results can be that Singh and Anderson (2002) observed that the ratio between the contrasts in the transparency region versus the background, can predict perceived transparency, calculated as follows:

$$R = \frac{C_{TM}}{C_{BG}} \tag{5}$$

where C_{TM} and C_{BG} are the contrast of the transparent medium and the contrast of the background.

In this thesis, the physical contrast of the transparent medium is kept the same for every stimuli that have the same α and t values. For this reason, as the background contrast increases, the ratio decreases. In such manner, for fixed α and t values, the ratio R_{HC} for high background contrast is lower than the ratio R_{LC} for low background contrast.

Considering all the findings mentioned above, it can be said that the ratio R, which is calculated in Equation 5, can be considered as the perceived transparency. Furthermore, I hypothesise that as the ratio R decreases, the perceived transparency increases.

2 Methodology

The procedure is as follows: First, different stimuli were rendered with varying background contrast c_bg, transmittance α and luminance t. Afterwards, I cut the transparent media with varying α and tau values from the highest background contrast stimuli, and pasted it on varying backgrounds. The aim for this step was to have the same transparent media for every background, so that the effect of the background on the perceived transparency could be judged. In Figure 12 the process of cutting the transparent medium from the background with highest background and pasting it on other backgrounds is sketched.

Following this step, for each rendered stimuli, I calculated the contrast of the background and the transparent medium with the same metric and repeated this step with different metrics. These metrics are Michelson contrast (MC), Space-Average Michelson contrast (SAM), Space-averaged logarithm of Michelson contrast (SAMLG) and Space-Average Whittle contrast (SAW). For these metrics, I calculated the ratio R between the contrast inside the transparent medium and the background contrast, and use the ratios to evaluate the metrics in order to find a metric that predicts the effect of the background contrast on the perceived transparency of the transparent medium better.

2.1 Initial Stimuli Creation

I generated variegated checkerboards with different physical contrasts in size of 22 x 22. In the middle of the checkerboard, a transparent medium with certain transmittance α and luminance t value is generated.

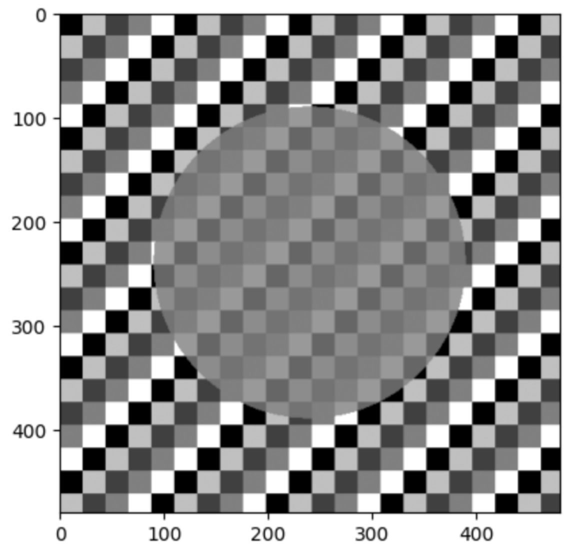


Figure 10: Example of stimuli with a variegated checkerboard with background contrast $c_{bg}(MC) = 1.0$, and a transparent medium with its transmittance $\alpha = 0.2$ and luminance $t = 0.5$ values.

For the background, randomly variegated checkerboards with five luminance values were rendered. To create various backgrounds with different contrasts, I created different vectors of five luminance values. The background contrast is calculated with Michelson Contrast equation (6) and background contrasts of the stimuli vary between 1.0 and 0.1. The luminance values used on the stimuli are [0.00001, 0.75, 0.25, 0.5, 1.0], [0.05, 0.725, 0.275, 0.5, 0.95], [0.1, 0.7, 0.3, 0.5, 0.9], [0.15, 0.675, 0.325, 0.5, 0.85], [0.2, 0.65, 0.35, 0.5, 0.8], [0.25, 0.625, 0.375, 0.5, 0.75], [0.3, 0.6, 0.4, 0.5, 0.7], [0.35, 0.575, 0.425, 0.5, 0.65], [0.4, 0.55, 0.45, 0.5, 0.6] and [0.45, 0.525, 0.475, 0.5, 0.55]. The background contrast decreases as the difference between the minimum and maximum luminance values l_{min}, l_{max} decreases.

Every possible combinations for the transparent medium were designed with three values of α $(0.1, 0.2, 0.5)$ and four values of t $(0.0, 0.2, 0.5, 1.0)$. The background contrast varies between 0.1 and 1.0. In the initial stimuli creation phase, 120 stimuli were rendered in total.

2.1.1 Choice of Transmittance α

The transparent medium decreases the contrast of the area it covers and it is required that the physical contrast of the transparent medium is lower than the background contrast, otherwise the transparent medium would never be perceived transparent. Figure 11a shows an example of such unwanted condition.

As the contrast inside the transparent medium is a function of $f(\alpha, t)$, therefore depends on the transmittance α and luminance t, it was important to select α and t carefully, to prevent the situation where the contrast inside the transparent medium being higher than the background contrast, otherwise there would be no transparency to judge. I observed that this issue occurred while stimuli were rendered with high transmittance α values. For every background, the contrast inside the transparent medium with $\alpha = 0.9$ was higher than the background contrast, regardless the luminance t. In Figure 11b the occurred problem can be seen. In order to prevent this, I excluded α values whereby a very high transparency is rendered, in this case α values higher than 0.5. Hence, 0.1, 0.2 and 0.5 were chosen for transmittance α to have a minimal to middle transparency.

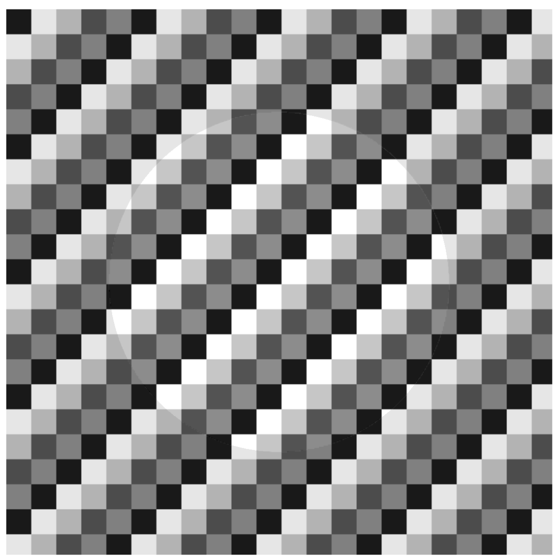


Figure 11: Example of stimuli where the background contrast is lower than the contrast inside the transparent medium. In case of the background contrast is lower than the transparent medium contrast, there is no transparency to be perceived.
Transmittance α is 0.9, luminance t is 1.0 and the background contrast according to Michelson Contrast (MC) metric is 0.8.

2.1.2 Choice of Luminance t

As observed before, (Aguilar & Maertens, 2022) mentioned that the "most observers perceive the light transparent medium as less transparent than the dark one, although both of them have the same physical transmittance". Thus it is expected that a transparent medium with low luminance t, e.g 0.0, would be perceived more transparent than the medium with high luminance value for instance 1.0. As it is necessary to examine the both extreme situations, along with two medium luminance values, I decided to design the transparent media using 0.0, 0.1, 0.5 and 1.0 for the luminance.

2.2 New Background

As the goal of the experiment is to investigate the effect of the background contrast on the perceived transparency, I generated different backgrounds to a fixed transparent medium with a fixed transmittance and luminance. In order to achieve this, various transparent media with different α and t values were created on the background with highest contrast 1.0, cut out and pasted on variegated checkerboards with different background contrasts. The intention of this process is to keep the transparent medium the same while changing the background, thus preventing the change of contrast inside the medium so that the perceived transparency of the medium would be evaluated by changing the backgrounds. Transparent media were cut out from the backgrounds with highest contrast and pasted onto other backgrounds with decreasing contrasts, because the transparent medium decreases the contrast of the area it covers and I wanted to keep the contrast in this area as high as possible. Keeping the decreased contrast in this area high was necessary because if the decreased contrast inside the transparent medium is higher than the background contrast, there would be no transparency to judge (Figure 11).
QUESTION: wouldn't it make more sense if I cut and paste the TM from a stimuli with lower background contrast? this way the chance of the contrast of TM being lower than BG contrast would be higher.

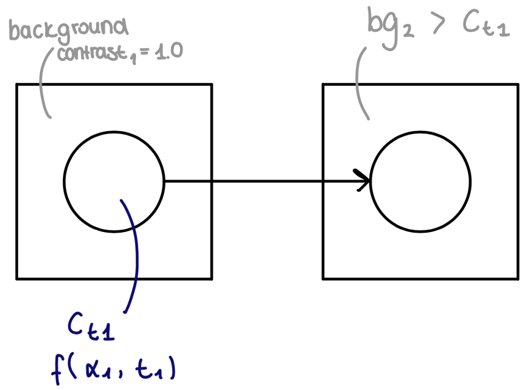


Figure 12: A sketch to explain how the transparent medium is cut and pasted on different backgrounds.
On the left side a stimulus is rendered where the background has the highest contrast bg_1 and the transparent medium has the contrast c_{t1} that is a function of $f(\alpha, t)$.
On the right side a stimulus with a lower background contrast bg_2 can be seen. The transparent medium of the stimuli with the highest background contrast is cut out and pasted on the background with lower contrast.

2.3 Contrast Metrics and Ratio Calculation

For each stimulus, I calculated different contrast metrics for the inside of the transparent medium and for the background. I then calculated ratios between the transparent medium contrast and the background contrast for each contrast metric. The aim of this step is to evaluate the contrast metrics and to find a metric that predicts the effect of the background contrast on the perceived transparency, better.

To calculate the contrasts inside the transparent medium and in the background separately, I created a mask with the exact shape and coordinates of the transparent medium. I then applied the mask to the numpy array, which consists of the luminance values for each checker of the stimulus ($22 * 22 = 484$), and created two arrays: one for luminance values inside the transparent region and one for the luminance values in the background. The metrics are calculated using these arrays in order to calculate the transparent medium contrast and background contrast separately.

I calculated four contrast metrics: Michelson contrast (MC), Space-averaged Michelson contrast (SAM), Space-averaged logarithm of Michelson contrast (SAMLG) and Space-averaged Whittle contrast (SAW), which are taken from Aguilar and Maertens (2022). The metrics are calculated as follows, whereby l_i refers to the luminance value of a checker and n to the number of luminance values in the area, in this case, 5:

1) Michelson contrast (MC)

$$MC = \frac{l_{min} - l_{max}}{l_{min} + l_{max}} \tag{6}$$

The following metrics (SAM, SAMLG and SAW) are space averaged metrics, meaning that the contrast calculation is done between all possible unique pairs of luminance values ($\forall i \neq j$).

2) Space-averaged Michelson contrast (SAM)

$$SAM = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| \frac{l_i - l_j}{l_i + l_j} \right| \tag{7}$$

3) Space-averaged logarithm of Michelson contrast (SAMLG)

$$SAMLG = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} log \left| \frac{l_i - l_j}{l_i + l_j} \right| \tag{8}$$

4) Space-averaged Whittle contrast (SAW)

$$SAW = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| \frac{l_i - l_j}{min(l_i, l_j)} \right| \tag{9}$$

After the calculation of different metrics for each rendered stimulus, I calculated the ratio R between the contrast inside the transparency region and the contrast of the background. By this means, there were four ratios for each stimulus, R_{MC}, R_{SAM}, R_{SAMLG} and R_{SAW}. The ratio R is calculated as follows:

$$R = \frac{c_t}{c_{bg}} \tag{10}$$

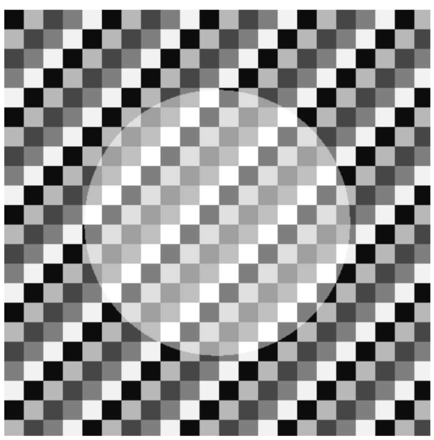
where c_t and c_{bg} are calculated with the same contrast metric.

If c_t and c_{bg} are calculated with MC, I referred to the ratio between these contrasts as R_{MC}.

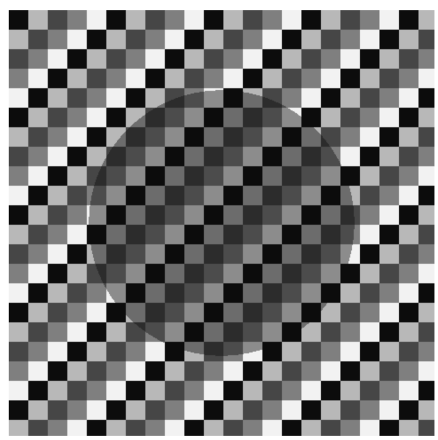
2.4 Comparison with Subjective Perception

The final step is the comparison of the stimuli with my subjective impression. I observed two stimuli side by side and judged the perceived transparency by answering the question: which of two transparent media looked more transparent. Figure 13 shows an example of one trial of comparing two stimuli.

The idea is to compare two stimuli at once visually, and then to compare their ratios for every contrast metric to evaluate if a contrast metric can predict the effect of the background on the perception of the transparency, and if the answer is yes, which metric predicts the effect better.



(a) Example of stimuli with background contrast $c_{bg}(MC) = 1.0$, and a transparent medium with its transmittance $\alpha = 0.5$ and luminance $t = 1.0$ values.



(b) Example of stimuli with background contrast $c_{bg}(MC) = 1.0$, and a transparent medium with its transmittance $\alpha = 0.5$ and luminance $t = 0.1$ values.

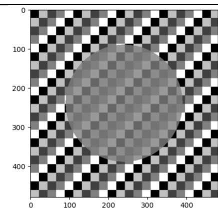
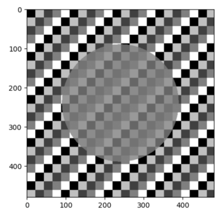
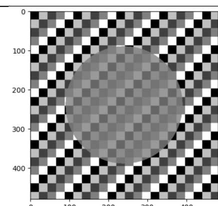
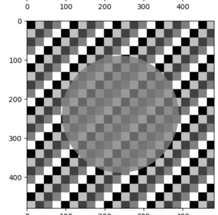
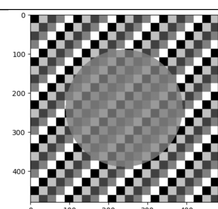
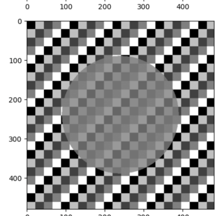
Figure 13: Example of one trial of comparing two stimuli visually.

In each trial I judged the perceived transparencies between the stimuli and used five-level Likert scale to judge, which is as follows:

1. The first stimulus appears to have much more transparency.
2. The first stimulus appears to have much more transparency.
3. They seem the same.
4. The second stimulus appears to have much more transparency.
5. The second stimulus appears to have much more transparency.

After each judgment of the stimuli on their perceived transparency, I compared their ratios with different metrics, R_{MC}, R_{SAM}, R_{SAMLG} and R_{SAW}. The goal of this comparison was to evaluate the metrics and to find out which metric predicts the effect between the background contrast and the transparency perception better. In this step, I expected the metrics to correlate with my answers for the five-level Likert scale. For instance, if I answered 1. in a trial, I expected the difference between the contrast metrics on the stimuli to be very high. On contrary, if I judged the stimuli to appear to have the same transparency and gave the answer 3., I expected the difference between the metrics to be very low, almost zero.

3 Results

	$\alpha = 0.1$	Ratio	Difference	Subjective Impression
$t = 0.0$	 	0.1 0.4	0.3	They look the same
$t = 0.5$	 	0.1 0.4	0.3	They look different
$t = 1.0$	 	0.1 0.4	0.3	They look the same

4 Discussion

5 Conclusion

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