2 Angle α is acute and $\cos\alpha = \frac{3}{5}$. Angle β is **obtuse** and $\sin\beta = \frac{1}{2}$.

(a) (i) Find the value of $\tan\alpha$ as a fraction. *(1 mark)*

(ii) Find the value of $\tan\beta$ in surd form. *(2 marks)*

(b) Hence show that $\tan(\alpha+\beta) = \dfrac{m\sqrt{3}-n}{n\sqrt{3}+m}$, where m and n are integers. *(3 marks)*

3 (a) Express $\dfrac{3+9x}{(1+x)(3+5x)}$ in the form $\dfrac{A}{1+x}+\dfrac{B}{3+5x}$, where A and B are integers.

(3 marks)

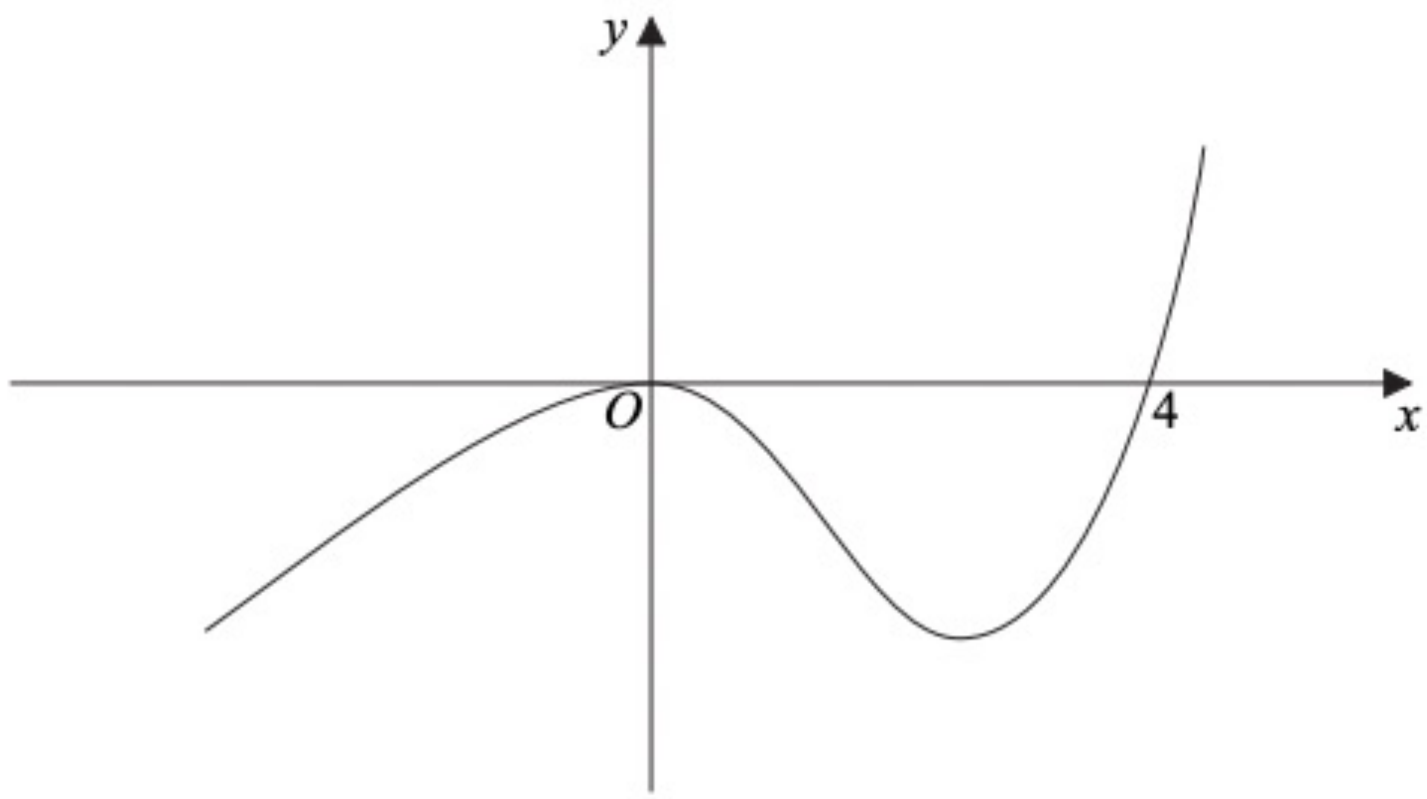
(b) Hence, or otherwise, find the binomial expansion of $\dfrac{3+9x}{(1+x)(3+5x)}$ up to and including the term in x^2.

(7 marks)

(c) Find the range of values of x for which the binomial expansion of $\dfrac{3+9x}{(1+x)(3+5x)}$ is valid.

(2 marks)

4 The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$.



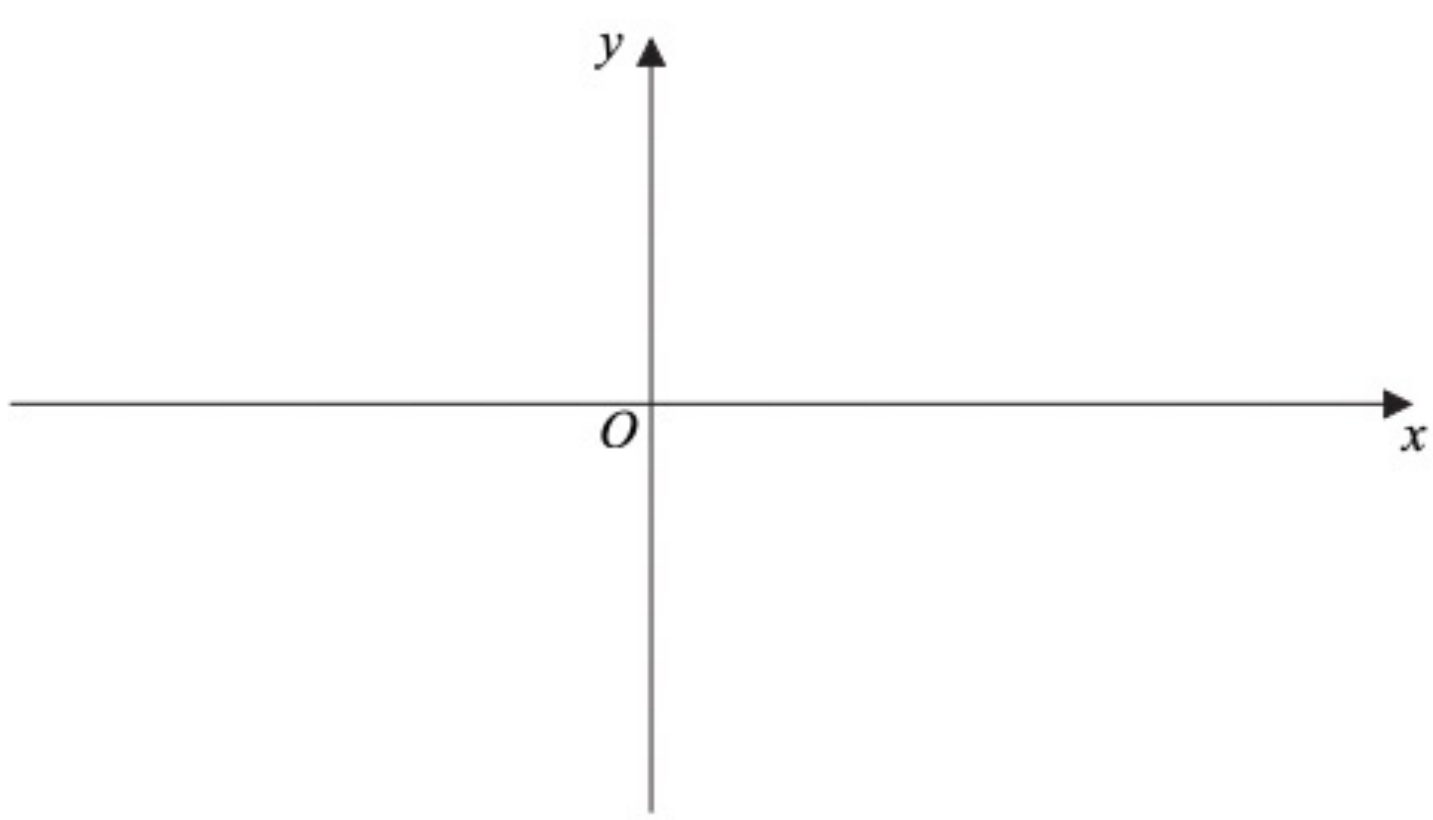
(a) On the axes below, sketch the curve with equation $y = |\mathrm{f}(x)|$. *(2 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(2x - 1)$. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 4

(a)



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5 The function f is defined by

$$f(x) = \frac{x^2 - 4}{3}, \quad \text{for real values of } x, \textbf{ where } \boldsymbol{x \leqslant 0}$$

(a) State the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

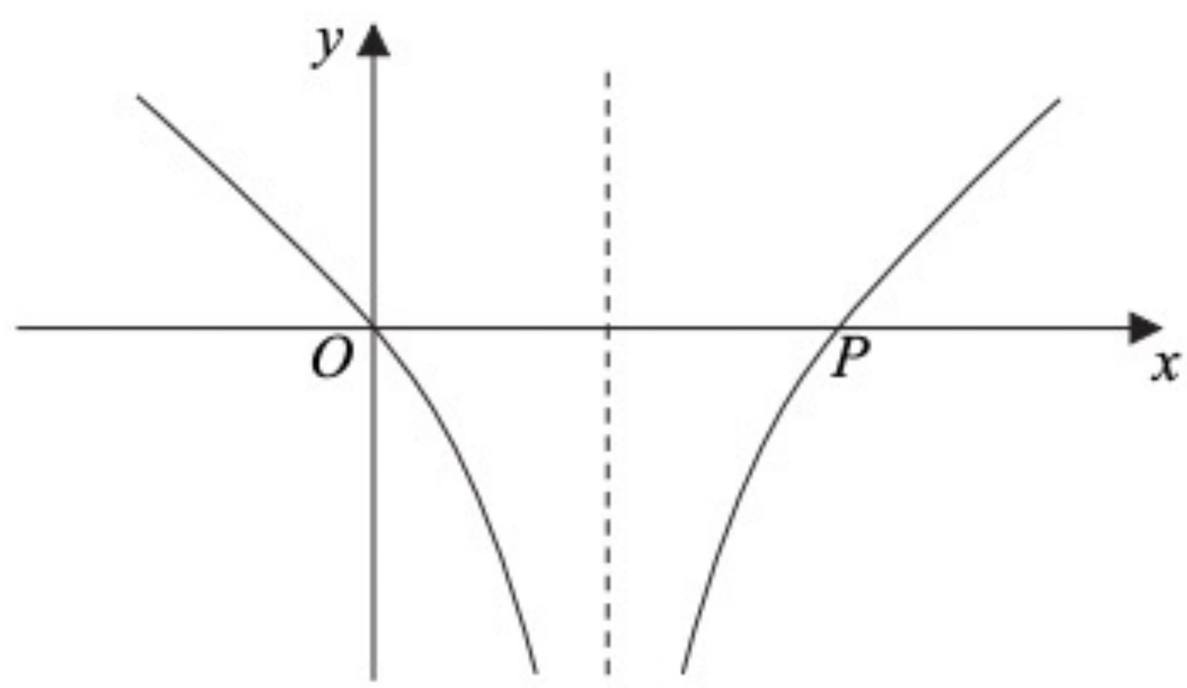
(i) Write down the domain of f^{-1}. *(1 mark)*

(ii) Find an expression for $f^{-1}(x)$. *(3 marks)*

(c) The function g is defined by

$$g(x) = \ln|3x - 1|, \quad \text{for real values of } x, \text{ where } x \neq \frac{1}{3}$$

The curve with equation $y = g(x)$ is sketched below.



(i) The curve $y = g(x)$ intersects the x-axis at the origin and at the point P.

Find the x-coordinate of P. *(2 marks)*

(ii) State whether the function g has an inverse. Give a reason for your answer. *(1 mark)*

(iii) Show that $gf(x) = \ln|x^2 - k|$, stating the value of the constant k. *(2 marks)*

(iv) Solve the equation $gf(x) = 0$. *(4 marks)*

6 (a) Show that

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)}$$

can be written as $\operatorname{cosec}^2 x$. *(3 marks)*

(b) Hence solve the equation

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)} = \operatorname{cosec} x + 3$$

giving the values of x to the nearest degree in the interval $-180° < x < 180°$. *(6 marks)*

(c) Hence solve the equation

$$\frac{\sec^2(2\theta - 60°)}{(\sec(2\theta - 60°) + 1)(\sec(2\theta - 60°) - 1)} = \operatorname{cosec}(2\theta - 60°) + 3$$

giving the values of θ to the nearest degree in the interval $0° < \theta < 90°$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 6

6 (a) A geometric series begins $420 + 294 + 205.8 + \ldots$.

(i) Show that the common ratio of the series is 0.7. *(1 mark)*

(ii) Find the sum to infinity of the series. *(2 marks)*

(iii) Write the nth term of the series in the form $p \times q^n$, where p and q are constants. *(2 marks)*

(b) The first term of an arithmetic series is 240 and the common difference of the series is -8.

The nth term of the series is u_n.

(i) Write down an expression for u_n. *(1 mark)*

(ii) Given that $u_k = 0$, find the value of $\sum_{n=1}^{k} u_n$. *(4 marks)*

5 (a) Wooden lawn edging is supplied in $1.8\,\text{m}$ length rolls. The actual length, X metres, of a roll may be modelled by a normal distribution with mean 1.81 and standard deviation 0.08.

Determine the probability that a randomly selected roll has length:

(i) less than $1.90\,\text{m}$;

(ii) greater than $1.85\,\text{m}$;

(iii) between $1.81\,\text{m}$ and $1.85\,\text{m}$.

[6 marks]

(b) Plastic lawn edging is supplied in $9\,\text{m}$ length rolls. The actual length, Y metres, of a roll may be modelled by a normal distribution with mean μ and standard deviation σ.

An analysis of a batch of rolls, selected at random, showed that

$$\mathrm{P}(Y < 9.25) = 0.88$$

(i) Use this probability to find the value of z such that

$$9.25 - \mu = z \times \sigma$$

where z is a value of $Z \sim \mathrm{N}(0, 1)$.

[2 marks]

(ii) Given also that

$$\mathrm{P}(Y > 8.75) = 0.975$$

find values for μ and σ.

[4 marks]