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On the Appropriate Use of Location Quotients in Generating Regional Input–Output Tables

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FLEGG A. T., WEBBER C. D. and ELLIOTT M. V. (1995) On the appropriate use of location quotients in generating regional input–output tables, *Reg. Studies* **29,** 547–561. This paper examines the conventional use of location quotients (LQs) to estimate regional input–output coefficients from national data. A new LQ-based adjustment formula is developed. By allowing for relative regional size, this new formula should overcome the tendency of conventional formulae to overstate regional multipliers. Furthermore, it is demonstrated that the conventional approach of first producing an aggregated regional model and *then* using LQs to adjust the coefficients is likely to produce biased estimates of regional multipliers; it is argued that it would be more sensible to adjust the data prior to aggregation. The arguments are illustrated using data for the English county of Avon.

Input–output analysis   Aggregation   Location quotients   Regional multipliers

FLEGG A. T., WEBBER C. D. et ELLIOTT M. V. (1995) L'emploi approprié des quotients de localisation afin d'engendrer des tableaux d'échanges intersectoriels, *Reg. Studies* **29,** 547–561. A partir des données nationales, cet article cherche à examiner l'emploi conventionnel des quotients de localisation afin d'estimer les coefficients régionaux relatifs aux entrées–sorties. Une nouvelle formule d'ajustement basée sur les quotients de localisation se voit développer. En tenant compte de la taille relative des régions, cette nouvelle formule devrait éviter de souligner les multiplicateurs régionaux, ce à quoi les formules conventionnelles ont tendance à faire. En outre, on cherche à démontrer que la façon conventionnelle – à savoir la construction dans un premier temps d'un modèle régional global suivie dans un deuxième temps d'un ajustement des coefficients à partir des quotients de localisation – risque de produire des estimations biaisées des multiplicateurs régionaux. Les arguments sont illustrés tout en se servant des données relatives au comté d'Avon en Angleterre.

Analyse des entrées–sorties   Agrégation   Quotients de localisation   Multiplicateurs régionaux

FLEGG A. T., WEBBER C. D. und ELLIOTT M. V. (1995) Zur angebrachten Verwendung von Standortquotienten bei der Aufstellung regionaler Input–Output-Tabellen, *Reg. Studies* **29,** 547–561. Dieser Aufsatz untersucht die konventionelle Verwendung von Standortquotienten (LQs) zur Schätzung regionaler Input–Output-koeffizienten von auf Nationalebene gewonnenen Daten. Es wird eine neue, sich auf LQ gründende Anordnungsformel entwickelt. Konventionelle Formen tendieren dazu, regionalen Multiplikatoren zuviel Gewicht beizumessen; dieser neuen Formel sollte es gelingen, dem zu begegnen, indem sie relative regionale Größe in Anschlag bringt. Darüber hinaus wird aufgezeigt, daß die konventionelle Handhabung des Problems in zwei Schritten: der Schaffung eines aggregierten Regionalmodells mit darauffolgender Verwendung von LQs zur Angleichung der Koeffizienten, durchaus voreingenommene Einschätzungen regionaler Multiplikatoren zeitigen kann; es wird die Ansicht vertreten, das es vernünftiger wäre, die Daten vor ihrer Zusammenfügung einander anzupassen. Die Argumente werden mit Hilfe von Daten für die englische Grafschaft Avon erläutert.

Input–Output-Analyse   Zusammenfügung   Standortquotient   Regionale Multiplikatoren

INTRODUCTION

The construction of a regional input–output model is fraught with difficulties. Ideally, one would want to carry out a detailed survey of the regional economy, to establish the strength of the interrelationships that exist among sectors. Two interesting examples of the survey-based approach are the study of North Staffordshire by PULLEN and PROOPS, 1983, and that of forestry by MCGREGOR and MCNICOLL, 1992. However, in most cases, such an exercise would be prohibitively expensive and time-consuming. Typically, therefore, one has to resort to adaptations of national input–output coefficients, using *location quotients* (LQs) derived from national and regional employment data. This approach was used, for example, by JOHNS and LEAT, 1986, in their study of the Grampian region of Scotland. Unfortunately, the resulting 'regional' coefficients are

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Table 1. Aggregated transactions table for the UK in 1984 (£ million)

	Agriculture	Energy	Manufacturing	Construction	Distribution	Transport	Services
Agriculture	2,304	0	8,707	6	330	8	38
Energy	611	17,065	6,477	631	2,880	1,835	1,496
Manufacturing	4,158	2,725	46,965	8,857	9,341	2,641	6,093
Construction	93	14	386	8,974	782	64	1,369
Distribution	375	674	6,565	930	1,656	900	842
Transport	224	1,078	5,045	512	2,782	1,975	1,100
Services	1,381	2,487	14,766	5,573	12,188	1,618	16,498
Total intermediate inputs	9,146	24,044	88,911	25,482	29,959	9,043	27,437
Imports	1,087	9,489	29,748	1,837	1,824	3,108	1,662
Sales by final demand	15	38	1,236	122	113	107	208
Taxes less subsidies	− 640	− 972	2,794	260	3,231	− 485	3,086
Income from employment	1,591	5,736	48,305	9,524	26,348	9,202	33,301
Gross profits etc.	3,920	21,348	9,471	6,625	8,969	2,635	20,740
Total inputs	15,119	59,682	180,466	43,850	70,444	23,609	86,433

Source: CENTRAL STATISTICAL OFFICE, 1988, p. 13.

bound to be inaccurate. Moreover, empirical studies by SMITH and MORRISON, 1974, and by HARRIGAN *et al.*, 1980a, indicate that this inaccuracy is systematic and that models based upon this approach tend to produce substantially overstated regional multipliers. This problem is compounded by the use of inappropriate methods of sectoral aggregation.

Given that funding for a full survey-based regional model is unlikely to be forthcoming in most instances, a plea has been made by JENSEN, 1990, for the logical foundations of non-survey methods to be examined. The major problem affecting the LQ method is clearly the overstatement of multipliers, which arises from the fact that conventional location quotients do not take sufficient account of interregional trade. A new adjustment formula is developed in this paper in an effort to overcome this problem. However, even if the systematic errors are removed, inaccuracies in individual coefficients are bound to remain. It is suggested here that a limited amount of survey work be undertaken to check the accuracy of key coefficients, but that the remaining coefficients should be accepted as reasonable approximations, given the finding by JENSEN and WEST, 1980, that errors in the smaller coefficients have little impact on the size of the estimated regional multipliers.

To gain a fuller understanding of the role of location quotients in generating regional input–output tables from national data, it may be helpful at this point to consider briefly the salient features of the input–output tables for the UK economy.

THE UK INPUT–OUTPUT TABLES

At the heart of an input–output model is a *transactions matrix*, which shows the extent to which each sector depends on itself and the other sectors for its inputs. A very detailed transactions table – containing 101 sectors – has been constructed by the Central Statistical Office (CSO) for the UK economy in 1984. However, for expositional purposes, the highly aggregated seven-sector matrix reproduced here in the top part of Table 1 should suffice.[1]

Each column of Table 1 shows the source of that particular sector's inputs and their value. It also reveals the division between *primary* inputs such as imports and income from employment and *intermediate* inputs purchased from other sectors. Each row of the transactions matrix shows, for a particular sector, the value of its sales to itself and to the other sectors. The remainder of its output is sold directly to consumers or to the government or ends up as exports, capital formation or additions to stocks.

The information in Table 1 is presented in a more convenient format in Table 2, where the inputs are shown as proportions. By way of illustration, the figures show that to produce, say, £1m of manufactured output requires approximately £260,000 worth of manufactured inputs, £82,000 worth of 'services', £36,000 worth of 'energy', £165,000 worth of imports and so on. The coefficients in the first seven rows of Table 2 can be described as ***national technical coefficients*** or as national coefficients, for short. Symbolically, the coefficient in row i and column j will be denoted by a_{ij}^n, this being the amount of input i needed to produce one unit of output j. It is worth emphasizing that only inputs sourced in the UK are included.

Table 2 indicates that approximately half of the total inputs into manufacturing are *intermediate* in the sense that they originate in manufacturing itself or are purchased from other sectors. The remaining inputs are termed *primary* and essentially represent wages and salaries, imports and gross profits. In contrast, the 'services' column shows that just over two-thirds of the inputs into services are primary. Thus one might expect the output and other multipliers to be larger for manufacturing than for services. It can also be discerned

Table 2. Technical coefficients for the UK in 1984

	Agriculture	Energy	Manufacturing	Construction	Distribution	Transport	Services
Agriculture	0·152	0·000	0·048	0·000	0·005	0·000	0·000
Energy	0·040	0·286	0·036	0·014	0·041	0·078	0·017
Manufacturing	0·275	0·046	0·260	0·202	0·133	0·112	0·071
Construction	0·006	0·000	0·002	0·205	0·011	0·003	0·016
Distribution	0·025	0·011	0·036	0·021	0·024	0·038	0·010
Transport	0·015	0·018	0·028	0·012	0·040	0·084	0·013
Services	0·091	0·042	0·082	0·127	0·173	0·069	0·191
Total intermediate inputs	0·605	0·403	0·493	0·581	0·425	0·383	0·317
Imports	0·072	0·159	0·165	0·042	0·026	0·132	0·019
Other primary inputs	0·323	0·438	0·342	0·377	0·549	0·485	0·663
Total inputs	1·000	1·000	1·000	1·000	1·000	1·000	1·000

Source: Derived from Table 1.

from the table that manufacturing purchased nearly a quarter of its inputs from other sectors in 1984, whereas only one eighth of the services sector's inputs came from elsewhere. This means, of course, that an expansion or contraction of manufacturing would have a larger impact on other sectors than would be true for an equivalent change in final demand for services.

LOCATION QUOTIENTS AND REGIONAL MODELS

In the absence of regional input and output figures, one often has to resort to roundabout methods of adjusting the national coefficients to produce a regional table. The major problem is that regional propensities to import are typically far higher than national propensities. The most common approach is to use location quotients. The rationale for using LQs is examined in RICHARDSON, 1972, chapter 6, and by MAYER and PLEETER, 1975. Alternative non-survey approaches are evaluated by ROUND, 1983.

The simplest type of LQ is defined as the ratio between the regional and national proportions of output or employment attributable to a particular industrial sector. For example, if sector X accounted for 40% of regional employment, but 60% of national employment, then this would produce a coefficient of two-thirds, indicating that X was under-represented in the regional economy. The principal diagonal of Table 3 shows in italics some simple LQs (SLQs) for the County of Avon in South West England, calculated using Census of Employment data for Great Britain in 1984, taken from the NOMIS database at the University of Durham. It is clear that energy and agriculture are substantially under-represented in Avon, whilst services is over-represented.

One could make use of the SLQs in Table 3 to produce a new set of input coefficients for Avon, that should be more representative of local conditions than those in Table 2. This would be done by reducing the size of the coefficients for those sectors under-represented in Avon, i.e. those with an SLQ < 1, whilst increasing the import coefficients by a corresponding amount. For instance, the figures in the manufacturing row of Table 2 would be multiplied by 0·94, on the assumption that the county's manufacturing sector would not be able to fulfil the whole of any increased demand for its products, with the gap having to be met by 'imports' from other regions. No adjustment would be made in the case of sectors with SLQs above one, as any increases in demand would not, by assumption, need to be met by imports.

A refinement: cross-industry location quotients

The use of simple LQs to adjust the national coefficients may, however, produce seriously misleading results. To explain why such distortions might occur, reconsider the case of manufacturing, for which the SLQ equals 0·94. As noted above, all figures in the manufacturing row of Table 2 would be multiplied by 0·94, in an effort to allow for the presumed lesser importance of manufacturing in Avon than nationally, and hence the greater reliance on imports to satisfy any increase in regional demand. This presupposes, however, that the discrepancy between the national and regional coefficients is the same, regardless of the sectors to which Avon's manufacturers are selling their output. This presumption cannot be sustained as it takes no account of the relative size of the sector providing the inputs (i.e. manufacturing in this example) and the sector purchasing them. Moreover, Avon's manufacturers may have specialized in fulfilling the needs of particular sectors and, in some cases, they may have no difficulty in satisfying local needs in full.

Cross-industry location quotients (CILQs) go some way towards overcoming the shortcomings of SLQs. An employment-based CILQ for sectors i and j is defined by the formula:

Table 3. Employment-based simple and cross-industry location quotients for Avon in 1984

	Agriculture	Energy	Manufacturing	Construction	Distribution	Transport	Services
Agriculture	*0·730*	1·058	0·777	0·737	0·702	0·702	0·629
Energy	0·945	*0·690*	0·734	0·697	0·664	0·664	0·595
Manufacturing	1·288	1·362	*0·940*	0·950	0·904	0·904	0·810
Construction	1·356	1·435	1·053	*0·990*	0·952	0·952	0·853
Distribution	1·425	1·507	1·106	1·051	*1·040*	1·000	0·897
Transport	1·425	1·507	1·106	1·051	1·000	*1·040*	0·897
Services	1·589	1·681	1·234	1·172	1·115	1·115	*1·160*

$$\frac{\text{regional/national employment in sector } i}{\text{regional/national employment in sector } j} \quad (1)$$

Sector i is presumed to be supplying inputs to j. The logic behind this formula is that, where the supplying sector is relatively small regionally compared to the purchasing sector (so that the CILQ is <1), some of the required inputs will have to be met by imports from outside the region. This means that the national coefficient will need to be adjusted downwards by multiplying it by the CILQ, with a corresponding upward adjustment being made to the relevant import coefficient. As in the case of the simple LQs, no adjustment is made if the CILQ is ≥ 1.[2]

It can easily be demonstrated (and verified from Table 3) that each CILQ is the ratio of the relevant SLQs, a fact that greatly simplifies the computations. An interesting issue arises when $i = j$ in the formula (1) above, since the CILQ = 1. This would entail making no adjustment to the national coefficients. SMITH and MORRISON, 1974, p. 66, have questioned this feature of the CILQ formula on the basis that it takes no account of the *size* of the local industry. They suggest that it would be preferable to use the SLQs to adjust the coefficients along the principal diagonal for this reason. Indeed, the practice of making no adjustment to the national coefficients where $i = j$ would only be valid if all *intra*sectoral trade at the national level were also intrasectoral at the regional level, yet it seems highly probable that a substantial part of such trade would, in fact, be interregional, particularly in cases where the regional industry is comparatively small and/or consists of a small number of specialist firms. Smith and Morrison's suggestion is a sensible one and it will be adopted here.

Table 4 shows the effects of adjusting the national technical coefficients, the a_{ij}^n, using SLQs along the principal diagonal and CILQs elsewhere.[3] It is noticeable that only 19 of the 49 coefficients have altered – a consequence of the fact that so many of the LQs in Table 3 are ≥ 1. Symbolically, the coefficient in row i and column j of Table 4 will be denoted by r_{ij}, this being the estimated amount of input i needed to produce one unit of output j. These coefficients will be referred to as ***intraregional input coefficients*** or as intraregional coefficients, for short. It is worth emphasizing that only inputs sourced in the County of Avon are included. It may be noted, in passing, that coefficients of the kind shown in Table 4 are often misleadingly described as regional technical coefficients. This issue of terminology is considered towards the end of the paper.

Another possible refinement: semi-logarithmic location quotients

The simple and cross-industry LQs discussed above provide alternative ways of estimating the relevant *trading coefficients*, which measure the proportion of any given commodity supplied from within the region (ROUND, 1978). In effect, trading coefficients measure the degree of self-sufficiency of a region. ROUND, 1978, has suggested that any trading coefficient, t_{ij}, where $0 \leq t_{ij} \leq 1$, will be a function of the following three variables:

- the relative size of the supplying sector i
- the relative size of the purchasing sector j
- the relative size of the region.

Round points out that simple LQs incorporate the first and third of these variables, whereas cross-industry LQs embody the first two, but not the third. To demonstrate these characteristics formally, consider the formulae specified below:

$$SLQ_i \equiv (RE_i/TRE) \div (NE_i/TNE) \equiv (RE_i/NE_i) \times (TNE/TRE) \quad (2)$$

$$CILQ_{ij} \equiv (RE_i/NE_i) \div (RE_j/NE_j) \equiv SLQ_i \div SLQ_j \quad (3)$$

where RE_i and NE_i denote regional and national employment, respectively, in sector i. TRE and TNE are the respective regional and national totals. Given that the ratios RE_i/NE_i and RE_j/NE_j in equations (2) and (3) above measure the relative size of the supplying and purchasing sectors, respectively, whilst TNE/TRE captures the relative size of the region, it should be evident that the SLQ and CILQ formulae do have the properties noted by Round.

In order to capture all three desirable properties simultaneously, Round postulates the following semi-logarithmic adjustment formula:

$$RLQ_{ij} \equiv SLQ_i/[\log_2(1 + SLQ_j)] \quad (4)$$

It should be noted that the factor TNE/TRE does not

Table 4. Intraregional input coefficients for Avon in 1984

	Agriculture	Energy	Manufacturing	Construction	Distribution	Transport	Services
Agriculture	*0·111*	0·000	*0·038*	0·000	*0·003*	0·000	0·000
Energy	*0·038*	*0·197*	*0·026*	*0·010*	*0·027*	*0·052*	*0·010*
Manufacturing	0·275	0·046	*0·245*	*0·192*	*0·120*	*0·101*	*0·057*
Construction	0·006	0·000	0·002	*0·203*	0·011	0·003	*0·014*
Distribution	0·025	0·011	0·036	0·021	0·024	0·038	*0·009*
Transport	0·015	0·018	0·028	0·012	0·040	0·084	*0·011*
Services	0·091	0·042	0·082	0·127	0·173	0·069	0·191
Total intermediate inputs	*0·562*	*0·314*	*0·457*	*0·565*	*0·400*	*0·346*	*0·292*
	− 0·043	− 0·089	− 0·036	− 0·017	− 0·028	− 0·037	− 0·025
Foreign and interregional imports	*0·115*	*0·248*	*0·201*	*0·059*	*0·054*	*0·169*	*0·044*
	+ 0·043	+ 0·089	+ 0·036	+ 0·017	+ 0·028	+ 0·037	+ 0·025
Other primary inputs	0·323	0·438	0·343	0·377	0·549	0·485	0·663
Total inputs	1·000	1·000	1·000	1·000	1·000	1·000	1·000

Source: Derived by multiplying the cells in Table 2 by the respective location quotients in Table 3, where < 1. The numbers in italics represent values that are different from those given in Table 2.

cancel out in the above expression – as it does in the cross-industry quotient (3) – so that Round's formula does allow for the relative importance of the region. Also, via the inclusion of both SLQ_i and SLQ_j, it allows for the relative size of both sectors.

Round's formula is used by BATEY *et al.*, 1993, in an interesting study published in this journal. They do not, however, comment on why his measure was used in preference to the conventional cross-industry formula. SMITH and MORRISON, 1974, and HARRIGAN *et al.*, 1980s, carry out numerous calculations using Round's formula, in an attempt to ascertain its relative accuracy *vis-à-vis* other adjustment methods, but they do not examine its theoretical properties. Moreover, ROUND, 1978, does not do so either.[4] It seems appropriate, therefore, to attempt such an analysis here.

In fact, in Round's formula, the ratio TNE/TRE enters in a rather counterintuitive way. One might expect a relatively large region to be more self-sufficient than a relatively small region because the former would typically be able to produce a wider range of products than the latter. Also, many industrial sectors would not be present in a small regional economy. Thus the propensity to import is likely to decline with increases in regional size, yet Round's formula would yield a larger trading coefficient for the smaller region. To illustrate this point, consider two hypothetical regions, A and B, which account for 10% and 20% of national employment, respectively. In both cases, it is assumed that $RE_i/NE_i = 0{\cdot}08$ and $RE_j/NE_j = 0{\cdot}12$. As noted earlier, the CILQ is unaffected by the relative size of the region and it equals two-thirds in both cases. However, the RLQ equals 0·70 for A and 0·59 for B. Thus, if we were to use Round's formula to adjust the national coefficients, we would necessarily be assuming that a larger proportion of sector j's requirements would need to be met by imports in region B than in A, despite the fact that B had a larger share of national employment.

It may be appropriate, therefore, to reformulate Round's formula (4) as follows:

$$ELQ_{ij} \equiv [\log_2(1 + SLQ_i)]/SLQ_j \qquad (5)$$

In contrast to Round's formula, this one does yield a larger trading coefficient for larger regions. In the case of the hypothetical regions considered earlier, the larger region B would yield a quotient of 0·81, whilst that for the smaller region would be 0·71. Thus the analyst would be making a greater allowance for imports in the smaller region.

It is worth examining the relationship between the CILQ and ELQ formulae. For any given SLQ_j, these two measures coincide when $SLQ_i = 1$. First consider the case where $SLQ_i < 1$, i.e. the supplying sector is *under*-represented in the regional economy. In this case, ELQ > CILQ. Using the ELQ as the adjustment formula would thus entail making a *smaller* allowance for additional imports, so long as CILQ < 1. A cautious analyst who wished to avoid the danger of understating regional imports would find the ELQ unattractive in such circumstances. (As will become clear later, such caution would not be misplaced.)

Suppose now that $SLQ_i > 1$, i.e. the supplying sector is *over*-represented regionally. In this case, ELQ < CILQ, so that using the ELQ would mean making a *larger* adjustment for additional imports than indicated by the CILQ (so long as ELQ < 1). In this situation, given the relatively large size of the supplying sector, the ELQ would be less attractive than the CILQ. Moreover, the gap between the formula widens as SLQ_i goes increasingly above unity. It seems reasonable to conclude that, whilst the ELQ does adjust appropriately for regional size, its behaviour either side of $SLQ_i = 1$ renders it a theoretically unappealing adjustment formula *vis-à-vis* the CILQ.

Table 5. The behaviour of the function λ_r^β

TRE/TNE	0	0·01	0·02	0·05	0·1	0·2	0·5	0·8	0·9	1
λ_r	0·693	0·697	0·700	0·710	0·727	0·760	0·855	0·943	0·972	1
λ_r^2	0·480	0·485	0·490	0·505	0·529	0·578	0·731	0·890	0·945	1
λ_r^4	0·231	0·235	0·240	0·255	0·280	0·334	0·534	0·792	0·892	1
λ_r^5	0·160	0·164	0·168	0·181	0·203	0·254	0·456	0·747	0·867	1

THE FLQ FORMULA

The question now arises as to whether one could perhaps devise an adjustment formula which retained the merits of the ELQ and CILQ, whilst avoiding their shortcomings! Fortunately, this does appear to be possible. Consider the following formula:

$$FLQ_{ij} \equiv CILQ_{ij} \times \lambda_r^\beta \tag{6}$$

where $\lambda_r \equiv (TRE/TNE)/[\log_2(1 + TRE/TNE)]$.[5] TRE/TNE is the ratio of regional to national employment. The regional scalar, λ_r, has a range from $\log_e 2 \approx 0{\cdot}693$ to unity. It is assumed that $\beta \gtrsim 1$. Table 5 illustrates the sensitity of λ_r^β to variations in TRE/TNE.

It is evident that the value of the function λ_r^β increases monotonically with increases in regional size for all values of $\beta \gtrsim 1$. Clearly, the larger assumed value of β, the greater the implied adjustment for regional imports. The function has several characteristics which seem attractive on *a priori* grounds. It is asymptotic at both extremes and tends to an upper limit of unity as the region approaches national dimensions and to a non-zero lower limit as the region becomes increasingly smaller. The inclusion of the exponent β in the formula introduces an element of flexibility, which means that the lower asymptote does not have to be constrained to the arbitrary value of $\log_e 2 \approx 0{\cdot}693$.[6] As for the choice of value for β, this is considered to be an empirical matter – to be decided on the basis of case studies involving a comparison of survey-based intraregional coefficients with those derived via LQs from national data. Some evidence on this matter will be presented latter in this paper. A special case occurs when $\beta = 0$ as the FLQ and CILQ formulae then coincide.

An important difference between the FLQ and the other LQ-based formulae can be clarified with the help of the illustrative example shown in Table 6.[7] It can easily be established that sector 1 has the same SLQ in both regions and likewise for sector 2, notwithstanding the fact that region *B* is only one tenth the size of region *A*. Furthermore, because the CILQ, RLQ and ELQ are functions solely of SLQ_i and SLQ_j, they also yield identical values for each sector across regions. The reason why the SLQs remain constant across regions is that the ratio RE_i/NE_i and TNE/TRE in equation (2) above move in opposite directions in an exactly compensating way. Region *B* is simply a scaled-down version of Region *A*.

Table 6. Illustrative employment data for two hypothetical regions

	Sector 1	Sector 2	Sector 3	Sum
Region *A*	20,000	15,000	20,000	55,000
Region *B*	2,000	1,500	2,000	5,500
Nation	50,000	20,000	60,000	130,000

However, the FLQ *does* yield substantially different results for the two regions. To illustrate, suppose that $\beta = 5$, in line with some empirical evidence to be presented later in this paper; $\lambda_r^5 = 0{\cdot}3967$ for region *A* and $0{\cdot}1775$ for region *B*. Given a common $CILQ_{12}$ of $0{\cdot}5\dot{3}$, for example, we get an implied trading coefficient for region *A* of 0·2116 and a *substantially smaller* value of 0·0947 for region *B*. Moreover, this difference is in accordance with the authors' belief that the firms in region *B* would experience greater difficulty than their counterparts in region *A* in satisfying any increases in regional demand. Hence we would want to make a greater adjustment for imports in *B* than in *A*. Only the FLQ formula does this.

Clearly, one would not normally expect *all* of the industries in a particular region to be exactly proportional in size to those in another region, as is assumed here. What the example demonstrates is that the FLQ formula can be relied upon to give sensible results in a wide variety of situations.

The use of the FLQ is straightforward: in a region such as Avon whose employment constitutes about 2% of the national total, we would compute the intraregional input coefficients, the r_{ij}, from the corresponding national technical coefficients, the a_{ij}^n, using the formula:

$$r_{ij} = CILQ_{ij} \times 0{\cdot}7^\beta \times a_{ij}^n \tag{7}$$

unless we found that $0{\cdot}7^\beta \times CILQ_{ij}$ happened to exceed unity, in which case we would set $r_{ij} = a_{ij}^n$.

As noted earlier, SMITH and MORRISON, 1974, have questioned the practice, implicit in the CILQ formula, of setting $r_{ij} = a_{ij}^n$ for $i = j$, on the grounds that this takes no account of the *size* of the local industry. They suggest that it would be prefereable to use the SLQs to adjust the coefficients along the principal diagonal for this reason.

However, whilst Smith and Morrison's suggestion is a sensible one, the SLQs are open to the criticism that they incorrectly allow for regional size; as can be seen from equation (2) above, for a given RE_i/NE_i, SLQ_i would be smaller in the larger region. Hence it would seem sensible to multiply the SLQs by the scalar λ_r^β specified ealier. Thus, where $i = j$, the $CILQ_{ij}$ term in formula (7) would be replaced by SLQ_i. The figures for

r_{ii} generated for this formula could then be refined on the basis of the analyst's knowledge of the sector in question. Of special significance would be the number and size of firms, the diversity of their products and the degree of heterogeneity of the firms within the regional sector *vis-à-vis* the national one. It is worth emphasizing that there is no *a priori* reason why the r_{ii} should be adjusted by the same amount.

THE AGGREGATION PROBLEM

In reality, for various reasons, regional models typically contain far fewer sectors than the corresponding national models and the processes of aggregation adopted can have serious implications. The studies of SMITH and MORRISON, 1974, and HARRIGAN *et al.*, 1980a, can be taken as indicative of the normal procedure used in producing a non-survey-based regional input–output table. This approach involves taking a national matrix of dimension N and converting this into a regionalized coefficient matrix of dimension $R < N$ and *then* adjusting this latter matrix using LQs. Typically, before aggregation takes place, the national transactions matrix is scaled down to regional values by multiplying each *column* by the ratio RE_j/NE_j.

However, the view of the present authors is that this conventional approach is bound to introduce unnecessary errors in calcuating the intraregional coefficients, leading to errors in the multipliers. The following alternative procedure is, therefore, recommended:

1. Scale the national transactions matrix down to regional values by multiplying each column by the ratio RE_j/NE_j
2. Multiply each element in the regionalized matrix obtained in step 1 by the appropriate FLQ (where fractional), adjusting imports as necessary
3. Aggregate the cells of the matrix formed in step 2 to form a regional matrix of appropriate size
4. Calculate the intraregional input coefficients and hence multipliers

Lest the point be misunderstood, it is worth emphasizing that the approach recommended here goes beyond the well-known proposition that one should always convert a national matrix into a regional one of the same size and *then* aggregate; if the aggregation is done first, then one is merely obtaining a more aggregated version of the national industrial structure (RICHARDSON, 1972, p. 135). This study has taken Richardson's dictum one step further, by suggesting not only that one should scale the national transactions matrix down to regional values by applying a set of regional weights, but also that one should – *prior to aggregation* – adjust the derived transactions matrix for regional imports.

In Appendix A, the conventional approach to aggregation is contrasted with that recommended here, using a hypothetical three-sector model for illustrative purposes. (In a nutshell, the conventional approach is equivalent to making an adjustment for imports *after* aggregation has taken place.) A worked example, using the seven-sector model introduced earlier in this paper, is given in FLEGG *et al.*, 1994.

It is worth noting, finally, that the method of aggregation recommended in this paper requires a restatement of the adjustment formulae specified earlier. In particular, formula (7) above would only apply in cases where there was no aggregation. In the normal situation where such aggregation occurred, then the adjusted national *transactions* – rather than the national technical *coefficients* – would need to be multiplied by the FLQ (see step 2 above).

THE AVON ECONOMIC MODEL

To explore the consequences of using alternative adjustment formulae, a 32-sector input–output model was developed for the County of Avon.[8] At the outset, regional employment weights were applied to each column of the 101-sector national transactions matrix for 1984 (CENTRAL STATISTICAL OFFICE, 1988, Table 4) to scale down the matrix to regional values and to take account of differences in national and regional industrial structures.[9] The next step was to use employment-based LQs to adjust for regional imports, as discussed above. The 101 sectors were aggregated into 32 sectors and only then were intraregional coefficients calculated. For simplicity, the model examined here is an 'open' one, in which the household sector is treated as exogenous, i.e. part of final demand.

Some illustrative results

Table 7 shows three sets of 'type I' multipliers (see RICHARDSON, 1972, chapter 3; ARMSTRONG and TAYLOR, 1993, chapter 2). The insurance sector will be used for illustrative purposes. Consider first the multipliers based upon the conventional CILQ approach. The *output* multiplier indicates that a rise of, say, £100,000 in final demand for insurance would ultimately increase the output of the Avon economy by £146,000. The *income* multiplier suggests that an initial increase of £100,000 in the incomes of insurance workers would generate an additional £58,000 in incomes earned throughout Avon. Finally, the *employment* multiplier indicates that the employment of an extra 100 insurance workers would create another 67 new jobs in Avon, once the full multiplier effects had been felt.

The results in Table 7 reveal that Round's formula yields the largest multipliers and that these are, on average, about 2·5% larger than those based upon the conventional CILQ approach. The differences between the multipliers generated by the CILQ and FLQ formulae are much more marked; on average, the FLQ approach (for $\beta = 5$) yields multipliers which are approximately *three-quarters* of conventional values. It should be noted that this ratio would have increased

Table 7. *Type I multipliers based upon different adjustment formulae*

	Output			Income			Employment		
Sector	RLQ	CILQ	FLQ	RLQ	CILQ	FLQ	RLQ	CILQ	FLQ
Distribution and repairs	1·57	1·56	1·08	1·54	1·53	1·08	1·36	1·35	1·05
Business and real estate services	1·27	1·27	1·04	1·21	1·21	1·03	1·25	1·24	1·03
Other services	1·36	1·35	1·05	1·30	1·29	1·04	1·19	1·19	1·03
Hotels, catering and pubs	1·37	1·36	1·07	1·23	1·22	1·04	1·16	1·15	1·03
Construction	1·82	1·81	1·10	2·12	2·10	1·13	2·09	2·08	1·13
Aerospace, manufacturing and repair	1·20	1·12	1·06	1·18	1·11	1·05	1·22	1·13	1·06
Food, drink and tobacco	1·58	1·47	1·09	1·77	1·61	1·11	1·85	1·68	1·12
Banking and finance	1·53	1·51	1·08	1·74	1·72	1·10	1·83	1·81	1·12
Mechanical equipment	1·48	1·44	1·08	1·47	1·43	1·08	1·44	1·40	1·07
Roads and other inland transport	1·39	1·37	1·05	1·33	1·31	1·04	1·31	1·29	1·04
Insurance	1·60	1·46	1·06	1·76	1·58	1·08	1·89	1·67	1·09
Paper and plastics	1·38	1·31	1·06	1·41	1·34	1·06	1·47	1·38	1·07
Printing and publishing	1·56	1·54	1·07	1·50	1·49	1·07	1·52	1·50	1·07
Electrical and electronic equipment	1·47	1·44	1·11	1·47	1·44	1·11	1·42	1·40	1·10
Chemicals	1·44	1·36	1·11	1·68	1·56	1·15	1·66	1·54	1·15
Transport services	1·46	1·46	1·07	1·44	1·44	1·07	1·48	1·47	1·07
Telecommunications	1·23	1·22	1·03	1·21	1·20	1·03	1·25	1·24	1·03
Agriculture, forestry and fishing	1·85	1·76	1·21	2·90	2·74	1·43	2·03	1·93	1·22
Postal services	1·35	1·33	1·05	1·24	1·23	1·04	1·23	1·23	1·03
Motor vehicles and shipbuilding	1·50	1·49	1·09	1·52	1·51	1·09	1·63	1·62	1·11
Electricity and nuclear fuels	1·27	1·25	1·04	1·42	1·38	1·05	1·57	1·52	1·07
Timber products	1·57	1·54	1·08	1·57	1·54	1·08	1·50	1·47	1·07
Metal goods	1·51	1·50	1·12	1·48	1·47	1·11	1·50	1·48	1·11
Railways	1·38	1·38	1·06	1·13	1·13	1·02	1·17	1·16	1·02
Footwear	1·24	1·19	1·04	1·19	1·15	1·03	1·15	1·12	1·03
Building materials	1·60	1·58	1·12	1·79	1·76	1·15	1·89	1·87	1·17
Gas	1·16	1·15	1·02	1·26	1·24	1·03	1·33	1·30	1·04
Clothing and textiles	1·38	1·35	1·12	1·34	1·31	1·10	1·24	1·22	1·07
Extraction industries	1·38	1·32	1·13	2·00	1·90	1·37	1·64	1·57	1·23
Other manufacturing	1·46	1·45	1·11	1·43	1·43	1·10	1·40	1·39	1·09
Water	1·30	1·30	1·04	1·28	1·27	1·04	1·24	1·23	1·03
Industrial plant and steelwork	1·57	1·53	1·11	1·53	1·48	1·10	1·71	1·66	1·13
Weighted mean	1·47	1·44	1·07	1·51	1·47	1·08	1·46	1·42	1·07

Notes: The sectors are ordered by employment in 1984, 'distribution and repairs' being the largest sector. In calculating the weighted mean for each column, the sectoral multipliers were weighted by employment. In the CILQ and FLQ calculations, SLQs were used along the principal diagonal. The FLQs presume that $\beta = 5$.

to about 85% if $\beta = 2$ had been used. Clearly, the results are quite sensitive to variations in the assumed value of β.

The differences between the CILQ and FLQ results are even more striking if one makes the comparison in terms of *indirect* effects alone, which is a more sensible but less conventional approach. Table 7 shows that, on average, the FLQ method yields indirect effects (for $\beta = 5$) that are about *one-sixth* of the corresponding values for the CILQ method. On the same basis, the average differences for Round's formula are approximately 7% in the opposite direction. An interesting facet of the results which is not apparent from Table 7 is the relationship between the sectoral sums of intermediate input coefficients and the sectoral multipliers obtained under each method. This relationship is explored in Appendix B.

It should be noted that the income and employment multipliers in Table 7 are based upon some heroic assumptions, e.g. that the national direct income coefficients can be used at the regional level (see Appendix B). It should also be borne in mind that, if the household sector had been treated as endogenous, the resulting 'type II' multipliers would have been markedly larger (see HEWINGS, 1985, pp. 52–55, and RICHARDSON, 1972, chapter 3). These qualifications should not, however, invalidate the sensitivity analyses reported here.

Aggregation bias

As pointed out earlier, the conventional or 'old' approach to aggregation involves making adjustments for regional imports *after* a regional input–output table of the appropriate size has been derived, whereas the 'new' approach recommended in this paper entails making an adjustment for imports *prior* to aggregation. To gain some idea of the possible scale of the errors imparted by using the conventional approach, the CILQ results in Table 7 were calculated using this method. The results are presented in Table 8. This table shows that the output and income multipliers are overstated by approximately

Table 8. Type I multipliers based upon different methods of aggregation

Sector	Output multipliers New method	Output multipliers Old method	Income multipliers New method	Income multipliers Old method
Distribution and repairs	1·56	1·58	1·53	1·55
Business and real estate services	1·27	1·28	1·21	1·21
Other services	1·35	1·37	1·29	1·30
Hotels, catering and pubs	1·36	1·46	1·22	1·28
Construction	1·81	1·79	2·10	2·08
Aerospace manufacturing and repair	1·12	1·13	1·11	1·12
Food, drink and tobacco	1·47	1·68	1·61	1·94
Banking and finance	1·51	1·53	1·72	1·75
Mechanical equipment	1·44	1·60	1·43	1·58
Roads and other inland transport	1·37	1·37	1·31	1·30
Insurance	1·46	1·48	1·58	1·60
Paper and plastics	1·31	1·47	1·34	1·51
Printing and publishing	1·54	1·58	1·49	1·52
Electrical and electronic equipment	1·44	1·49	1·44	1·49
Chemicals	1·36	1·61	1·56	1·98
Transport services	1·46	1·45	1·44	1·43
Telecommunications	1·22	1·26	1·20	1·23
Agriculture, forestry and fishing	1·76	1·88	2·74	3·03
Postal services	1·33	1·36	1·23	1·25
Motor vehicles and shipbuilding	1·49	1·60	1·51	1·61
Electricity and nuclear fuels	1·25	1·30	1·38	1·43
Timber products	1·54	1·63	1·54	1·62
Metal goods	1·50	1·55	1·47	1·50
Railways	1·38	1·50	1·13	1·17
Footware	1·19	1·21	1·15	1·16
Building materials	1·58	1·65	1·76	1·85
Gas	1·15	1·21	1·24	1·31
Clothing and textiles	1·35	1·39	1·31	1·35
Extraction industries	1·32	1·36	1·90	2·01
Other manufacturing	1·45	1·52	1·43	1·48
Water	1·30	1·32	1·27	1·28
Industrial plant and steelwork	1·53	1·57	1·48	1·51
Weighted mean	1·44	1·49	1·47	1·53

4%, on average, as a result of using the 'old' method. More seriously, this average hides the fact that substantial increases occur in several multipliers, along with very small rises in a large number of the others. It is worth noting that the differences observed here are more striking if one compares the multipliers on the basis of the indirect effects alone, e.g. by comparing 0·49 with 0·44 rather than 1·49 with 1·44.

THE EVIDENCE FROM OTHER STUDIES

With the notable exceptions of the study of the small English town of Peterborough by SMITH and MORRISON, 1974, and the Scottish study by HARRIGAN *et al.*, 1980a, there have been few published investigations into the differences between LQ-based and survey-based regional multipliers. Peterborough and Scotland are particularly interesting case studies because they provide opposite ends of the spectrum in terms of the relevant size of regions one might wish to study in the UK and elsewhere, with the values for TRE/TNE of approximately 0·0015 and 0·1, respectively. If the value of β were found to be relatively constant over this very wide range, then this would add substantially to the potential usefulness of our formula.

SMITH and MORRISON, 1974, p. 73, found that the type I output multipliers based on the SLQ method were, on average, 17·2% higher than those obtained from a survey of firms. Round's formula yielded multipliers which were 23·2% higher. The CILQ method produced multipliers that were 24·9% higher, but this gap was reduced to 19·8% when SLQs were placed along the diagonal.

In contrast to Smith and Morrisons's findings, HARRIGAN *et al.*, 1980a, found that the SLQ method produced type I output multipliers which were 25% higher than their survey-based equivalents, whereas the CILQ multipliers (with SLQs along the principal diagonal) were 18·1% higher. Round's formula produced multipliers which were 22·5% higher. It is worth noting that the somewhat larger errors found in Peterborough are consistent with the view that smaller regions tend to have a higher propensity to import from other regions.

The major cause of the large discrepancies identified above beween LQ-based and survey-based multipliers is indubitably that the conventional LQ formulae understate regional propensities to import, with the error increasing as the relative size of the region decreases. This is the shortcoming that the FLQ formula posited earlier is designed to correct. In addition, survey-based coefficients are bound to contain errors, although in the two studies examined above there was no suggestion that the overall bias had been in any particular direction. It should also be recognized that UK input–output tables are far from perfect. Finally, differences in technology and errors of aggregation are bound to introduce further discrepancies.

Putting to one side for the moment other possible causes of discrepancies in the calculated multipliers, an interesting question can now be posed: if we were to apply the FLQ formula, what value(s) of β would be needed to reproduce the *survey-based* multipliers found in the above-mentioned studies?

Luckily, we can address this question directly in the case of Smith and Morrison's study, as the authors provide simple LQs for Peterborough, a set of survey-based multipliers and a regionalized matrix of national technical coefficients obtained by applying regional employment weights to the provisional UK transactions table for 1968 and then aggregating this to the required

Table 9. Mean proportional percentage differences between simulated and survey-based type I output multipliers for Peterborough in 1968

	Value of β							
	2	3	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	$5\frac{1}{4}$	$5\frac{1}{2}$
Unweighted	10·8	7·5	4·4	2·7	1·9	1·1	0·3	−0·3
Weighted by employment	10·9	7·3	3·9	2·0	1·1	0·3	−0·4	−1·0

size.[10] From this information, we were able to obtain the illustrative results shown in Table 9. (For reasons which should be obvious from the later discussion, one of the sectors used in Smith and Morrison's simulations – public administration – was excluded from the analysis here.)

Smith and Morrison (and also Harrigan *et al.*) used simple unweighted mean proportional differences to measure the degree of accuracy (or rather inaccuracy!) of the various LQ-based formulae. On this basis, the optimal value for β is approximately 5·4. However, this measure takes no account of the importance of the errors in different sectors. For instance, there was an enormous discrepancy between the simulated and survey-based multipliers for the chemicals sector, yet this sector employed only 0·4% of Peterborough's workers in 1968.[11] It seems sensible, therefore, to weight the proportional differences by sectoral employment.[12] This has the effect of reducing the required value of β to approximately 5·1.[13] Furthermore, given the remarkable similarity of the findings of Smith and Morrison and those of Harrigan *et al.*, it seems reasonable to suppose that a value for β in the neighbourhood of five would be appropriate for Scotland too. Before drawing any definite conclusions about the value of β, however, it is necessary to consider two additional and potentially important causes of discrepancies between the simulated and survey-based multipliers: technological differences; and aggregation bias.

Regional versus national technology

Since the phrase 'technical coefficient' is sometimes used rather loosely in the literature, a digression on terminology is in order. RICHARDSON, 1972, p. 114, distinguishes between a 'regional technical coefficient' and a 'regional coefficient', denoted by a_{ij}^r and r_{ij}, respectively. The former refers to regional technical requirements per unit of output of sector j, whereas the latter describes the proportion of required inputs supplied by firms located within the region. Thus $r_{ij} \equiv a_{ij}^r - m_{ij}$, where m_{ij} denotes the proportion of sector j's inputs imported from other regions, as well as from abroad. The national input–output coefficients (as in Table 2) are commonly described as 'technical' coefficients, a description also used by Richardson. For consistency with Richardson's regional terminology, however, they should instead be described as national technical coefficients *net of national imports*.

This inconsistency in the national and regional terminology is most unsatisfactory and, in an effort to clarify the situation, the new phrase 'technological coefficient' has been adopted here to describe the amount of an input needed to produce one unit of gross output, regardless of source. Our terminology is as follows:

$$a_{ij}^n \equiv ntc_{ij} - m_{ij}^n \tag{8}$$

$$r_{ij} \equiv rtc_{ij} - m_{ij}^r \tag{9}$$

$$m_{ij}^r \equiv m_{ij}^{rr} + m_{ij}^{ra} \tag{10}$$

$$a_{ij}^r \equiv r_{ij} + m_{ij}^{rr} \tag{11}$$

where:

ntc_{ij} and rtc_{ij} = the respective national and regional *technological* coefficients

a_{ij}^n and a_{ij}^r = the respective national and regional *technical* coefficients

m_{ij}^n and m_{ij}^r = national and regional imports per unit of output of sector j

r_{ij} = the proportion of required inputs supplied by firms located within the region (*intraregional* input coefficient)

m_{ij}^{rr} = the proportion of sector j's inputs imported from other regions

m_{ij}^{ra} = the proportion of sector j's inputs imported from abroad.

HARRIGAN *et al.*, 1980b, made a detailed comparison of technical coefficients for Scotland and the UK, using information derived from the 1973 UK input–output tables and from a contemporaneous Scottish survey. They found that, in general, the Scottish technical coefficients were smaller than their UK equivalents, reflecting a lower ratio of intermediate to gross output in the Scottish economy.[14] The implied output multipliers for most sectors were higher in the UK than in Scotland. Using a 46-sector model, HARRIGAN *et al.* quantified these differences by regressing the survey-based Scottish technical coefficients, the a_{ij}^s, on the UK technical coefficients, the a_{ij}^u. The result was:

$$\hat{a}_{ij}^s = 0{\cdot}0004 + 0{\cdot}8818\ a_{ij}^u \tag{12}$$

where $R^2 = 72{\cdot}7\%$. In a joint F test at the 1% level, the intercept and slope were found to be significantly different from their theoretical values of zero and unity, repectively. Equation (12) shows the extent to which the use of the UK technical coefficients would tend to produce upwardly biased estimates of the corresponding Scottish coefficients.

From these results, it would appear that the overstatement of the Scottish multipliers by the LQ method is, to a considerable extent, the result of technological differences between the UK and Scotland. In this regard, it is interesting to observe that Smith and Morrison obtained a remarkably similar regression equation (intercept = 0·0008, slope = 0·8712 and $R^2 = 77{\cdot}5\%$), yet they chose to interpret this as indicating the degree of similarity between the technical coefficients for Peterborough and the UK, rather than as evidence of dissimilar technology.[15]

However, Smith and Morrison's regression does not provide a satisfactory basis for testing for differences in technology. This is because the national and regional technical coefficients for the household sector were computed from identical data and likewise for public administration. Fortunately, it was possible to fit the following new regression to data for the remaining 18 sectors:

$$\hat{a}_{ij}^{p} = -0{\cdot}0006 + 0{\cdot}9393\, a_{ij}^{u} \qquad (13)$$

where $R^2 = 43{\cdot}8\%$, and the respective t ratios for the null hypothesis $\alpha = 0$ and $\beta = 1$ are $-0{\cdot}25$ and $-1{\cdot}024$.[16]

Equation (13) provides little evidence of the existence of systematic technological differences between Peterborough and the UK. Indeed, by applying the F test used by HARRIGAN *et al.*, it can be established that the probability of wrongly rejecting the null hypothesis of identical technology is under 1%. Even so, it is of some interest as a sensitivity analysis to see what difference it would make if we were to accept the average 6% difference in technical coefficients suggested by equation (13). Accordingly, the a_{ij}^{u} were scaled down by a factor of 0·94 to conform with the assumed technology in Peterborough. A new optimal value of β of approximately 4·8 was then determined. As a further test, a 12% reduction – equivalent to that indicated by the Scottish regression (12) – was applied. This reduced the required value of β to 4·6.

Clearly, where general rather than sector-specific technological differences between national and regional economies exist, the FLQ formula is required to adjust simultaneously for both: (1) interregional trade; and (2) differences in the national and regional technical coefficients. Thus any analyst wishing to use this formula would need to judge how far the national ratio of intermediate to primary inputs departed from that in the region being modelled, before deciding on an appropriate value of β. Fortunately, as indicated by the above results, even fairly large differences in technology do not appear to affect the required value of β by very much.

Aggregation bias

Smith and Morrison and Harrigan *et al.* used variants of what has been described here as the conventional or 'old' method of aggregation, whereby the LQ-based adjustments for interregional trade are made after aggregation of the transactions table.[17]

In Appendix A, it is demonstrated that the use of this approach should not introduce any bias so long as the sectors being aggregated have identical ratios of regional to national employment. It seems most unlikely that this condition would be satisfied (even approximately) in most cases. Moreover, in Peterborough, the aggregation was extensive, with 73 UK sectors being aggregated into a mere 19. Hence there are good reasons for suspecting the existence of bias. Unfortunately, it is impossible to establish the direction of such bias *a priori* and there is no basis for believing that the bias of 4% found in Avon (where 101 national sectors were aggregated into 32 regional sectors) would necessarily apply in Peterborough. Clearly, we need to be cautious in inferring too much from the Peterborough results. In contrast, the degree of aggregation in the Scottish study was far more modest (59 UK sectors were aggregated into 46), so that the potential for bias was much more limited.

CONCLUSION

This paper has identified reasons why the conventional approach to generating regional input–output tables from national data tends to produce excessively large regional multipliers. The major factor is the use of adjustment formulae which fail to take account of the relative size of regions and hence underestimate propensities to 'import' from other regions. Secondary factors may well be the use of inappropriate methods of sectoral aggregation and a failure to adjust for differences between regional and national technology.

In view of the theoretical and empirical shortcomings of both the conventional cross-industry location quotient and Round's semi-logarithmic quotient, a new formula – labelled the FLQ – was developed. This has the characteristic that greater allowance for regional imports is made, the smaller the region in question. A complicating factor is the need to specify the value of an exponent β. However, in the light of evidence from case studies of Peterborough and Scotland, it would seem that a value of at least 4·5 would be needed to offset the tendency for the conventional cross-industry location quotient to generate excessively large regional multipliers. It should be emphasized that the latter figure does not take account of possible aggregation biases, which were found to be substantial in the case of Avon. A new approach to aggregation has been developed in this paper in an effort to overcome this problem. Clearly, it would be helpful if more survey-based regional tables could be produced in order to provide more information on the appropriate value for β. It should be noted, finally, that it is necessary to adjust the value of β in instances where a significant difference between regional and national technology is thought to exist. Fortunately, the required adjustment may not be large in many cases.

Nevertheless, there are those who would argue that the LQ approach is too crude to be of any use and that one would be better off using informed guesses as first-stage estimates and then employing the RAS method to adjust these figures accordingly. However, informed guesses are not obtained cheaply or quickly and, in the light of the refinements of the LQ method developed in this paper, it may well be preferable to use this method to generate the initial estimates. Moreover, the RAS method requires sectoral observations on gross outputs and on intermediate inputs and outputs (DEWHURST, 1992; HEWINGS, 1985, pp. 50–52). Such data would not normally be easy or cheap to obtain. Thus, given its simplicity and cheapness, the LQ method clearly does have some advantages.

It should be emphasized that the LQ method cannot provide definitive answers. Indeed, it would be extraordinary if it did produce accurate intraregional coefficients, given the stringency of the underlying assumptions, e.g. identical national and regional *technical* coefficients and productivity of labour. The simulated intraregional coefficients would always need to be refined on the basis of informed judgement, surveys of selected industries, etc. However, one should recall that errors in the smaller coefficients have little impact on sectoral multipliers and that limited and selective survey information can yield considerable gains in terms of accuracy (JENSEN and WEST, 1980; WEST, 1981).

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NOTES

1. This transactions matrix is described by the CSO as a 'commodity by commodity domestic use matrix'. The 1990 tables were unavailable at the time of writing.
2. ROUND, 1978, has questioned the logic of the procedure whereby no allowance is made for the extent to which a CILQ is above one, yet a full adjustment is made when the quotient is below one.
3. There is an inconsistency here in that employment figures for Great Britain are being used to adjust a transactions matrix relating to the UK. This should not, however, have a material impact on the results.
4. SMITH and MORRISON, 1974, p. 453, refer to a personal communication from Round in 1971, in which he suggested using the semi-logarithmic formula (4). The formula apparently did not appear in a learned journal until 1978.
5. λ_r can also be computed using the formula $0{\cdot}30103(TRE/TNE)[\log_{10}(1 + TRE/TNE)]$.
6. As an alternative to our formula, one referee suggested estimating λ_r using the simple linear equation $K + (1 - K)(TRE/TNE)$, where $0 < K < 1$. This provides a close approximation to our own formula for values of β in the neighbourhood of 2 but diverges increasingly from it as β rises above 2. It is worth emphasizing that both our formula and the referee's suggested alternative require the specification of only one parameter, so that the informational requirements are identical.
7. We are grateful to one of the referees for providing this example.
8. Details are given in FLEGG *et al.*, 1994. A refined version of this model, using data for 1990, is currently being developed by the authors (FLEGG and WEBBER, 1995).
9. All employment figures were obtained via NOMIS from the 1984 Census of Employment; see also note 3.
10. SMITH and MORRISON, 1974, pp. 95, 97, 114.
11. For example, when $\beta = 2$, the FLQ formula yields a sectoral multiplier of 1·40, far higher than the survey-based value of 1·02. The chemicals sector consistently generated the largest proportional differences in multipliers. Another reason why one would want to minimize the influence of this sector is that there was a nil response in the survey and the authors had to use other information to estimate its input profile.
12. SMITH and MORRISON, 1974, p. 19.
13. Other possible criteria include the weighted mean of *squared* proportional errors and the weighted mean of *absolute* proportional errors. The former criterion was rejected because it would place undue emphasis on the avoidance of large errors. Using absolute values is seemingly more attractive but was also rejected because it conflicted with our objectives of finding a value of β that yielded the correct sectoral multipliers, on the average. Nonetheless, it is interesting that the absolute approach indicated a minimum mean weighted error of approximately 6%, for a value of β between 7 and 8.
14. HARRIGAN *et al.*, 1980b, pp. 804–6, suggest that the smaller degree of intermediation observed in Scotland in 1973 can be explained *inter alia* by the less extensive use of service inputs than in the UK, reflecting the production of more basic commodities, which require less elaborate distributional, informational and service back-up systems.
15. See SMITH and MORRISON, 1975, pp. 29–31. It could be argued that one should be comparing technological rather than technical coefficients when assessing differences in technology. For instance, suppose that national and regional technological coefficients were identical but the regional propensity to import from abroad exceeded the national propensity, then this would produce smaller regional than national technical coefficients. However, there is no reason, in general, why different propensities should mean differences in the underlying technology of production.
16. SMITH and MORRISON, 1974, pp. 94–95, give technical coefficients for 19 sectors including public administration but not households. The dramatic drop in R^2 can presumably be explained by the fact that the household sector's row is characterized by substantially larger coefficients than those for other sectors, so that the exclusion of households caused a sharp drop in the mean value of a_{ij}^p. This must have affected the total sum of squares far more than the residual sum of squares. This

hypothesis is borne out by the fact that the exclusion of public administration caused virtually no change in R^2.

17. See SMITH and MORRISON, 1974, pp. 23–24; and HARRIGAN *et al.*, 1980b, p. 809. Although seemingly different, essentially the same aggregation procedure appears to have been used in both studies, namely to weight each column of the national transactions matrix by the ratio RE_j/TRE prior to aggregation. In both cases, the adjustment for imports was made after aggregation.
18. Strictly speaking, under the 'old' method, the transactions would normally be converted into a *coefficient* matrix before multiplying by CILQs, SLQs, etc. (see, for example, SMITH and MORRISON, 1974, appendix A.) However, the results would not be affected in any way if this were done and, to simplify the discussion, this difference has been ignored.

APPENDIX A

Alternative approaches to sectoral aggregation

In this appendix, the 'old' method of aggregation is contrasted with the 'new' approach recommended in this paper. For illustrative purposes, consider the matrix in Table A1. The first step in constructing a regional matrix from Table A1 is to scale down the national matrix to regional values, by multiplying each column j by the ratio of regional to national employment, RE_j/NE_j. The results are shown in Table A2.

Suppose that we now decide to construct a two-sector regional model from Table A2, by combining sectors 1 and 2 into a new sector, 1★, and using cross-industry LQs (with ones down the principal diagonal) to adjust for regional imports. The results of applying the 'old' method of aggregation are presented in Table A3.

It can be seen from Table A3 that, under the 'old' method, the aggregation is carried out *before* applying the CILQ-based adjustments (see note 18 above). It must also be emphasized that the CILQs used under the 'old' method are based on employment data for the aggregated sector. However, if we had opted, instead, for the 'new' method of aggregation, then the regional transactions matrix would have been as in Table A4.

Since the size of each sectoral multiplier depends crucially on the sum of transactions in the appropriate

Table A1. *A hypothetical three-sector national transactions matrix*

T_{11}	T_{12}	T_{13}
T_{21}	T_{22}	T_{23}
T_{31}	T_{32}	T_{33}

Table A3. *A two-sector regional transactions matrix based on the 'old' method of aggregation*

$(R_{11}+R_{12}+R_{21}+R_{22})\times 1$	$(R_{13}+R_{23})\times CILQ_{1\star 3}$
$(R_{31}+R_{32})\times CILQ_{31\star}$	$R_{33}\times 1$

Table A4. *A two-sector regional transactions matrix based on the 'new' method of aggregation*

$R_{11}\times 1+R_{12}\times CILQ_{12}+R_{21}\times CILQ_{21}+R_{22}\times 1$	$R_{13}\times CILQ_{13}+R_{23}\times CILQ_{23}$
$R_{31}\times CILQ_{31}+R_{32}\times CILQ_{32}$	$R_{33}\times 1$

column, it is interesting to examine the circumstances in which Tables A3 and A4 would yield identical totals for corresponding columns. In fact, if the two sectors being aggregated exhibited identical ratios of regional and national employment, then it would not matter which method of aggregation one used. This is because, when $(RE_1/NE_1)=(RE_2/NE_2)$, $SLQ_1=SLQ_2$. Also, $CILQ_{12}=CILQ_{21}=1$, so that the corresponding diagonal elements of Tables A3 and A4 would be identical. Furthermore, given that $CILQ_{31}\equiv(RE_3/NE_3)/(RE_1/NE_1)$ and also that $CILQ_{32}\equiv(RE_3/NE_3)/(RE_2/NE_2)$, it follows that $CILQ_{31}$, $CILQ_{32}$ and $CILQ_{31\star}$ would all be equal when $(RE_1/NE_1)=(RE_2/NE_2)$. This fact would yield identical elements in the bottom left-hand corners of the tables. It can also be demonstrated that the elements in the top right-hand corners would be equal.

Clearly, the greater the intersectoral differences in the ratios of regional to national employment for the sectors being aggregated, the more pronounced will be the differences in the results obtained from the two methods of aggregation. It is interesting to see whether anything can be said *a priori* regarding the relative magnitude of the column sums in Tables A3 and A4. In general, we might reasonably expect either $CILQ_{12}$ or $CILQ_{21}$ to be <1, which would mean that the element in the top left-hand corner of Table A3 would be larger than the corresponding element in Table A4. ($CILQ_{12}$ and $CILQ_{21}$, being reciprocals, cannot both be <1 and a CILQ >1 is treated as equal to one.) Although it is easy to see why the elements in the bottom left-hand corners of the two tables are unlikely to be equal, there is no obvious reason why this difference should invariably (or even normally) be positive or negative. The same point applies to the top right-hand corner elements. However, the bottom right-hand corner elements are necessarily equal. It

Table A2. *A hypothetical national transactions matrix scaled down to regional dimensions*

$R_{11}\equiv T_{11}\times(RE_1/NE_1)$	$R_{12}\equiv T_{12}\times(RE_2/NE_2)$	$R_{13}\equiv T_{13}\times(RE_3/NE_3)$
$R_{21}\equiv T_{21}\times(RE_1/NE_1)$	$R_{22}\equiv T_{22}\times(RE_2/NE_2)$	$R_{23}\equiv T_{23}\times(RE_3/NE_3)$
$R_{31}\equiv T_{31}\times(RE_1/NE_1)$	$R_{32}\equiv T_{32}\times(RE_2/NE_2)$	$R_{33}\equiv T_{33}\times(RE_3/NE_3)$

Table B1. *OLS regressions showing the relationship between sectoral multipliers and sums of intraregional input coefficients for Avon in 1984*

	Type of multiplier								
	Output			Income			Employment		
Method	RLQ	CILQ	FLQ	RLQ	CILQ	FLQ	RLQ	CILQ	FLQ
Intercept	0·96 (93)	0·97 (111)	1·00 (2655)	0·68 (5·3)	0·77 (6·6)	0·94 (48)	0·84 (8·7)	0·90 (10)	0·99 (80)
Slope	1·60 (49)	1·55 (54)	1·10 (235)	2·72 (6·8)	2·45 (6·4)	2·17 (9·0)	2·13 (7·1)	1·91 (6·7)	1·31 (8·5)
R^2	0·988	0·990	0·999	0·607	0·580	0·728	0·625	0·598	0·709

Notes: $N = 32$ in all cases; $\beta = 5$ in the FLQ calculations; t ratios are in parentheses.

seems reasonable to conclude that the column sums obtained via the two methods are most unlikely to be the same, but there is no *a priori* reason for expecting the differences to be negative or positive. This point applies *a fortiori* in cases where SLQs are being used along the principal diagonal, since there is then no longer any obvious reason why the element in the top left-hand corner should be larger using the 'old' method.

APPENDIX B

Intermediate inputs and sectoral multipliers

It can be seen from Table B1 that the sectoral *output* multipliers are the most closely linked with the respective sums of intraregional input coefficients. This is hardly surprising when one considers the relevant formulae defining the type I multipliers for sector j:

$$K_j^{out} = \Sigma_i b_{ij} \qquad K_j^{inc} = (\Sigma_i b_{ij} y_i)/y_j \qquad K_j^{emp} = (\Sigma_i b_{ij} e_i)/e_j$$

where:

b_{ij} = the ijth element of the inverse Leontief matrix

y_i = the direct income coefficient for sector i (ratio of income from employment to gross output)

e_i = the direct employment coefficient for sector i (ratio of employment to gross output).

b_{ij} measures the direct and indirect input requirements of sector i per unit of gross output of sector j; it is a function solely of the intraregional input coefficients, the r_{ij} (see RICHARDSON, 1972, chapter 3). In the case of the income and employment multipliers, the b_{ij} are weighted by y_i *and* e_i, respectively. These multipliers are also seen to vary inversely with y_j and e_j.

The fact that the output multipliers are so closely linked with the sums of intermediate inputs means that small errors in individual intraregional coefficients may not matter, particularly if there are reasons for believing that such errors will be compensating. Clearly, large proportionate errors in the coefficients for the most important supplying sectors must be avoided. Where the intraregional coefficients have been derived via the FLQ method, there is an obvious role for surveys in verifying the values of the larger coefficients. It is interesting that the FLQ formula provides the best basis for predicting sectoral output multipliers from given (or assumed) sums of intermediate inputs.

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