Optimization and Decision Support Methodologies
2021/2022

Practical work

Introduction

The objective of the proposed work is to complete the application developed in practical classes, so that it allows solving a pure integer linear programming (PILP) problem, that is, a problem in which all variables have integrality constraints. For this purpose, the Gomory algorithm should be used.

As it is known, in solving a PILP problem by this algorithm, the first step is to solve the associated LP problem by the *simplex* method.

If the optimal solution obtained respects the integrality constraints of the variables, then the optimal solution for the LP problem is also optimal for the PILP problem.

Otherwise, it is necessary to introduce a cut constraint and solve the new problem by the dual *simplex* method.

If the obtained solution still does not respect the integrality constraints, another cut constraint is introduced again and the new problem is solved by the dual *simplex* method and so on, successively, until reaching a solution in which the original variables of the problem are integers.

The format of the cut constraint is as follows:

$$\sum_{j \notin I_B} f_{sj}\, x_j \geq f_{s0}$$

Verifying that:

- x_{s0} is the value of column *b* of the final tableau of the *simplex*, corresponding to the s^{th} basic (non-integer) variable that can be decomposed in integer and fractional parts:

$$x_{s0} = [x_{s0}] + f_{s0} \ , [x_{s0}] \geq 0 \ , \ 0 \leq f_{s0} < 1$$

- x_{sj} is the element of row *s* column *j* of the *simplex* final tableau, where x_j is a non-basic variable. This can also be broken down into:

$$x_{sj} = [x_{sj}] + f_{sj}, 0 \ \leq f_{sj} < 1$$

Work to be performed

Use the application developed in practical classes and add one more option to the menu called "**5 - PILP problem**".

The goal is that, after solving a given LP problem (associated with a given PILP problem) by the *simplex* method using **option 1** of the menu, the user may choose the new **option 5** to ensure he gets an optimal solution with all original variables taking an integer value.

To this end, the **option 5** should call a function named ***PILP***, to be developed within the scope of this work, with the following header:

```
function [n,m,A,c,b,x,xB,cB,SBA,zjcj,z]=
                          PILP(n,m,A,c,b,x,xB,cB,SBA,zjcj,z)
```

The function should comprise the following steps:

1. Check whether the values of the original variables are integers or not
2. If all of them are integers, it should inform the user that the optimal solution of the associated LP problem is also an optimal solution of the PILP problem (and finish!)
3. Otherwise, it must inform the user of the need to introduce a cut constraint. Then, it has to:
   - Build the cut constraint
   - Make it a constraint of type "<="
   - Update *simplex* tableau with new constraint and new slack variable
   - Solve the new problem by the dual *simplex* method
   - Present the new optimal solution and the new optimal z value of the associated LP problem
   - Back to step 1

To demonstrate the good functioning of the developed function, you should test the program with several examples of PILP problems of your choice (at least three) and create a PDF file with the results of its complete resolution, from entering the data, to obtaining the optimal solution and optimal value of z.

Regulation

- The proposed work can be done individually or in groups of two students.
- This work is rated for 3 points (on a scale from 0 to 20).
- **There is no minimum score required** for this work.
- The deadline for delivery is Monday, January 3, 2022.
- The work will be subject to a defense in a date/time to be scheduled later.
- Assessment elements to submit in Moodle:

  A folder with:

  - The Matlab code files.
  - A .pdf file containing the results of running the program with the test examples you select.

  The folder name should follow the following code: word MOAD followed by the numbers of students belonging to the group, separated by '_'. Example: MOAD_20116214_20001738 (group formed by students numbers 20116214 and 20001738)

- ATTENTION! If copies of work are detected, all groups involved will be assigned a score of zero.