d) ***Introduction of a new decision variable***

The original problem becomes:

$$\max z = \sum_{j=1}^{n} c_j x_j + c_{n+1} x_{n+1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j + a_{i(n+1)} x_{n+1} \le b_i$$

$$x_j \ge 0$$

$$x_{n+1} \ge 0; \; j = 1, 2, ..., n; \; i = 1, 2, ..., m$$

The optimal solution of the original problem with $x_{n+1}=0$ (non-basic variable) is a feasible solution.

- Calculate $\mathbf{X}_{n+1} = B^{-1} P_{n+1}$
- Introduce this column in the tableau
- Calculate "$z_{n+1}-c_{n+1}$":
  - If ≥ 0, solution remains optimal
  - Otherwise, apply *simplex* algorithm (putting x_{n+1} in the basis) to determine the new optimal solution.

Example

Consider the previous example again (*page I-8*).

Suppose the company decided to analyze the implication of producing a new product: tables.

Studies of production conditions indicate that the production of a table requires 3 hours/machine in the SU and 2 hours/machine in the AFU, with no market limitation foreseen.

The estimated unitary profit for the tables is of 5 currency units (CU).

Consider the optimal *simplex* tableau (before the introduction of the tables):

x_B	c_B \ c_i / x_i	6 x_1	3 x_2	0 x_3	0 x_4	0 x_5	b
x_3	0	0	0	1	-1	2	160
x_2	3	0	1	0	1/4	-1	60
x_1	6	1	0	0	0	1	160
$z_j - c_j$		0	0	0	3/4	3	1140

$x_1 = 160$
$x_2 = 60$
$x_3 = 160$
$x_4 = 0$
$x_5 = 0$
$Z = 1140$

The formalization of the problem, already including the new product and in its augmented form, is:

maximize $z = 6x_1 + 3x_2 + 5x_6$

subject to

$$\begin{aligned} 2x_1 + 4x_2 + x_3 + 3x_6 &= 720 \\ 4x_1 + 4x_2 + x_4 + 2x_6 &= 880 \\ x_1 + x_5 &= 160 \\ x_i \geq 0;\ i = 1, 2, \ldots, 6 \end{aligned}$$

$$P_6 = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

$$X_6 = B^{-1}P_6 = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1/4 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ 0 \end{bmatrix}$$

The *simplex* tableau, after the introduction of x_6, is:

	c_i	6	3	0	0	0	5	
x_B	c_B \ x_i	x_1	x_2	x_3	x_4	x_5	x_6	b
x_3	0	0	0	1	-1	2	1	160
← x_2	3	0	1	0	1/4	-1	1/2 *	60
x_1	6	1	0	0	0	1	0	160
$z_j - c_j$		0	0	0	3/4	3	-7/2	1140
							↑	

The previous solution is no longer optimal, that is, it is advantageous to produce tables.

Simplex algorithm is applied until the new optimal is found.

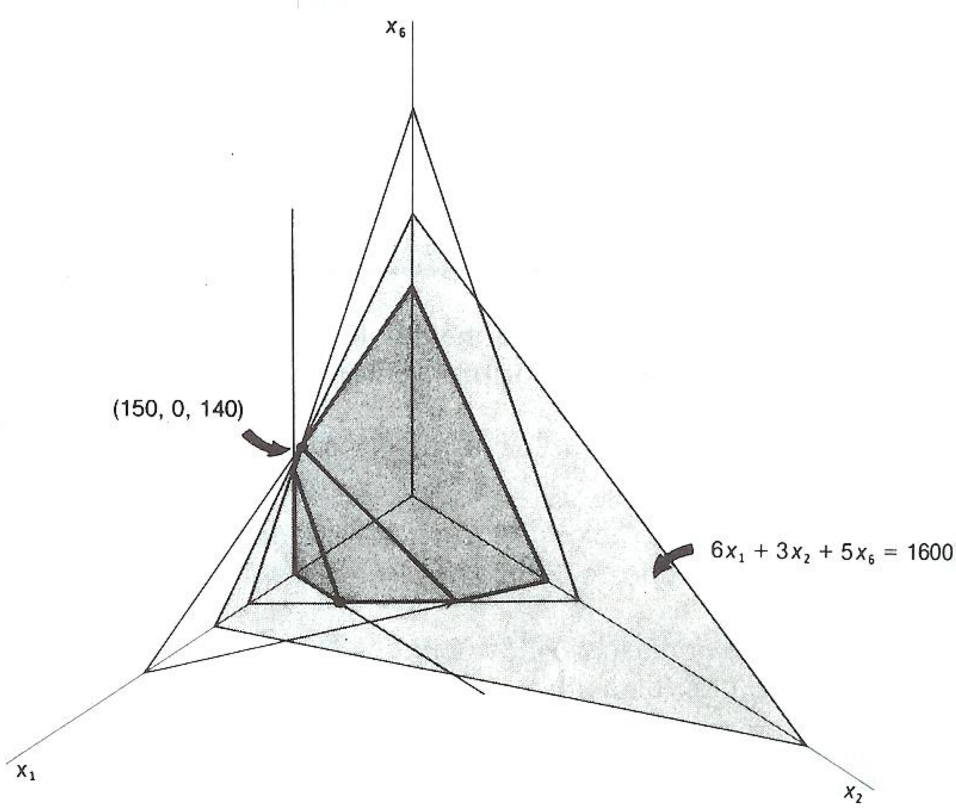
← x_3	0	0	-2	1	-3/2	4 *	0	40
x_6	5	0	2	0	1/2	-2	1	120
x_1	6	1	0	0	0	1	0	160
$z_j - c_j$		0	7	0	5/2	-4	0	1560
						↑		

x_5	0	0	-1/2	1/4	-3/8	1	0	10
x_6	5	0	1	1/2	-1/4	0	1	140
x_1	6	1	1/2	-1/4	3/8	0	0	150
$z_j - c_j$		0	5	1	1	0	0	1600

The optimal solution to the problem, after introducing the new variable, is:

$$\mathbf{x}^* = (150, 0, 0, 0, 10, 140) \quad \text{with} \qquad z^* = 1600$$

In other words, 150 desks and 140 tables must be produced, with no shelves being produced, resulting in a total profit of 1600 CU.



e) Introduction of a new constraint

It does not change the objective function, but it may restrict the feasible region.

The first step is to check whether the optimal solution to the original problem satisfies the additional constraint:

- If it satisfies, the optimal solution remains the same;
- Otherwise, there is a new optimal solution that needs to be determined.
  - Introduce a row (corresponding to the constraint) and a column (slack and/or artificial variable) in the optimal tableau
  - Do the necessary condensation operations (to reconstruct the identity matrix). If the obtained solution is not feasible, perform a new iteration (by the dual *simplex* method) to determine the new optimal solution.

Example

Return to the previous example (*page I-8*).

Market studies show that shelves production must be at least of 100. Add constraint $x_2 > 100$.

Consider the optimal tableau:

x_B	c_B \ c_i / x_i	6 x_1	3 x_2	0 x_3	0 x_4	0 x_5	b	
x_3	0	0	0	1	-1	2	160	$x_1 = 160$
x_2	3	0	1	0	1/4	-1	60	$x_2 = 60$
x_1	6	1	0	0	0	1	160	$x_3 = 160$
$z_j - c_j$		0	0	0	3/4	3	1140	$Z = 1140$

This solution does not satisfy the new constraint because $x_2 = 60$.

Transforming $x_2 \geq 100$ in $-x_2 \leq -100$ and adding a slack, you get:

$$-x_2 + x_6 = -100$$

The new enlarged tableau will be:

x_B	c_B \ c_i, x_i	6	3	0	0	0	0	
		x_1	x_2	x_3	x_4	x_5	x_6	b
x_3	0	0	0	1	-1	2	0	160
x_2	3	0	1	0	1/4	-1	0	60
x_1	6	1	0	0	0	1	0	160
x_6	0	0	-1 ⇐	0	0	0	1	-100
$z_j - c_j$		0	0	0	3/4	3	0	1140

Performing the necessary condensation operations:

x_3	0	0	0	1	-1	2	0	160
x_2	3	0	1	0	1/4	-1	0	60
x_1	6	1	0	0	0	1	0	160
← x_6	0	0	0	0	1/4	-1 *	1	-40
$z_j - c_j$		0	0	0	3/4	3 ↑	0	1140

The tableau is no longer optimal due to the appearance of a negative value in column *b*. The dual *simplex* method has to be applied:

x_3	0	0	0	1	-1/2	0	2	80	
x_2	3	0	1	0	0	0	-1	100	
x_1	6	1	0	0	1/4	0	1	120	$x_1 = 120$
x_5	0	0	0	0	-1/4	1	-1	40	$x_2 = 100$
$z_j - c_j$		0	0	0	3/2	0	3	1020	Z = 1020

The introduction of the new restriction meant that the previous optimal solution was no longer feasible and the optimal was reached at another point:

$x^* \rightarrow x'^* = (120, 100, 80, 0, 40, 0)$

with $z^* = 1020$.