Peer-to-Peer Energy Sharing: Effective Cost-Sharing Mechanisms and Social Efficiency

Sid Chi-Kin Chau, Jiajia Xu, Wilson Bow and Khaled Elbassioni

***Abstract*—P2P (peer-to-peer) energy sharing allows household users to share their local energy resources (e.g., rooftop PVs, home batteries) based on an agreed cost-sharing mechanism (e.g., implemented as a smart contract over a blockchain ledger). Sharing energy resources is becoming a new form of sharing economy. This not only promotes renewable energy adoption among household users but also optimizes their energy resources efficiently. However, household users are self-interested and incentive-driven. It is not clear how to motivate them to team up for energy sharing, and what proper economic mechanisms are to incentivize them to do so in a socially efficient way. This paper sheds light on the economic principles of cost-sharing mechanisms for P2P energy sharing. We investigate P2P energy sharing scenarios of direct connections and grid settlement with simple cost-sharing mechanisms (e.g., proportional-split, bargaining games), and the subsequent stable coalitions, such that no group of users will deviate to form other coalitions. We characterize the social efficiency of P2P energy sharing by the *strong price of anarchy* that compares the worst-case stable coalitions and a social optimum. We show that the strong price of anarchy is mild, both in practice (by an extensive data analysis on a real-world P2P energy sharing project) and in theory (by a small bound in general settings). This can hence bolster the viability of P2P energy sharing.**

***Index Terms*—Peer-to-Peer Energy Sharing; Sharing Economy; Cost-Sharing Mechanisms; Social Efficiency; Strong Price of Anarchy;**

I. Introduction

The energy sector is undergoing crucial transformative changes. First, as a global endeavor to mitigate climate change, ambitious plans of renewable energy integration are being introduced worldwide. As a result, a high level of renewable energy penetration will disrupt the management and control of traditional energy grids. Second, our society is embracing the notion of decentralization. End users can now gain better control of their own services in diverse sectors with increasing transparency and autonomy. The combination of accelerating renewable energy integration and decentralization leads to the paradigm of "transactive energy", which gives end users a higher degree of choices and control of how energy is generated, delivered and utilized [1]. An important realization of transactive energy is *P2P (peer-to-peer) energy exchanges* [2] that allow distributed coordination among local energy producers and consumers without centralized operators.

The concept of P2P energy exchanges is a disruptive revolution to the highly monopolized energy sector. The incumbent grid operators are often reluctant to incorporate user-led renewable energy because this will reduce their profit margin. Currently, household users can only sell electricity from their rooftop solar PVs to the grid at very low feed-in tariffs (sometimes as low as a fifth of the consumption tariffs). Low feed-in tariffs prolong the payback period of household renewable energy facilities and disincentivize renewable energy adoption. Furthermore, many household renewable energy facilities are often utilized inefficiently. For example, neighboring households do not coordinate their consumption usage to maximize the benefits of their mutual energy resources, despite of the temporal variations in their energy supplies and demands. Therefore, allowing household users to sell electricity among themselves in P2P energy exchanges and subsequently coordinate their usage not only incentivizes renewable energy adoption but also optimizes the overall energy efficiency.

There are various ways to realize P2P energy exchanges. On one hand, a possible approach is by a real-time trading process operating in short timescale to match suppliers and customers instantaneously [3], [4]. This, however, requires a challenging rapid control system in tandem with the trading process. The fast-adjusting trading process will induce reliability issues to power grid.

On the other hand, one can rely on a longer-term arrangement of energy sharing among household users by settling their supplies and demands based on an agreed cost-sharing mechanism. This will require minimum changes in a more reliable manner to the current power grid. We call this approach "*P2P energy sharing*". Note that a similar concept of cost-sharing is popular in diverse sectors (e.g., ride-sharing), giving rise to the trend of "sharing economy" [5], [6]. In keeping with this trend, sharing energy resources (e.g., rooftop PVs, batteries) is becoming a new form of sharing economy [7]. The cost-sharing mechanism can be implemented as a smart contract over a blockchain ledger [8].

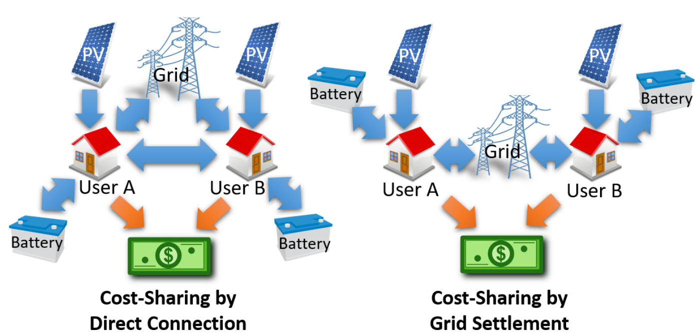

Fig. 1: Illustrations of P2P energy sharing scenarios.

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To motivate our study of P2P energy sharing, we next present two possible scenarios, as illustrated in Fig. 1.

Example 1 (*Cost-Sharing by Direct Connections*): Consider a community of household users who share their local energy resources (e.g., rooftop PVs, home batteries) by establishing direct electrical connections among themselves[1]. The users will coordinate the operations of their energy management systems to maximize the benefits of mutual energy resources for a relatively long period. The incurred operational cost will be split among the users in a fair manner based on individual consumption behavior and received benefits. A user will not participate in sharing, if he/she perceives disproportionate benefit.

Example 2 (*Cost-Sharing by Grid Settlement*): Consider a group of household users who are separated at a distance without direct connections. In this setting, the users may still be able to share energy by transferring the electricity through the grid among their energy resources in geographically dispersed locations. To enable P2P energy sharing, utility operators need to provide *billing settlement* among individual users, such that the instantaneous consumption of electricity from one user can be offset by the instantaneous export of electricity from another user. This can be achieved by smart meters that record and report instantaneous electricity flows. The users will first form a coalition to coordinate the operations of their energy management systems and underwrite a sharing agreement for a relatively long period on a separate platform (e.g., as a smart contract over a blockchain ledger). Then the agreed parties will notify the utility operators to settle their accounts on their behalf during the agreement period. Note that utility operators may impose an additional charge for the billing settlement service.

The aforementioned scenarios illustrate the promising potential of P2P energy sharing. To enable P2P energy sharing, there requires a proper *cost-sharing mechanism* among household users to split the cost among the involved parties in a desirable manner. Although cost-sharing mechanism in coalition formation has been studied in traditional economics [9], [10] and multi-agent systems [11], [12], the new context of P2P energy sharing exhibits a different set of characteristics and presents several challenges as follows:

1) **Decentralized Decision-Making:** There is no centralized planner, administrator or manager in P2P energy sharing. The operations are mostly carried out on P2P digital platforms on the basis of voluntary participations. Hence, the decision-making processes should preserve the autonomy and transparency of individual participants.
2) **Incentive-Driven Coalitions:** Every user acts according to his/her self-interested goals and economic purposes. The establishment of sharing agreement among users is solely motivated by self-interested behavior and incentives. In particular, the users will choose to join the best possible coalition that minimizes their individual costs.
3) **Presence of Uncertainty:** There is a high level of uncertainty regarding the future energy supplies and demands. The various sources of uncertainty make the decision-making process of P2P energy sharing highly challenging.

Because of these characteristics, it requires a proper understanding of cost-sharing mechanisms for P2P energy sharing. With such an understanding, new computational tools and platforms can be developed to assist P2P energy sharing in practice.

In this paper, we seek to address the following questions about P2P energy sharing. *Which cost-sharing mechanisms will motivate self-interested users to team up for sharing energy? Will they share their energy in a socially efficient way? If not, how can we reduce the gap from a socially efficient outcome?*

There are rich theories to be studied in P2P energy sharing. To model self-interested user behavior in P2P energy sharing, we formulate a coalition formation game [13], [14], [15], whereby users will split the cost associated with a potential coalition for an energy sharing agreement according to a given cost-sharing mechanism. We consider simple cost-sharing mechanisms, including proportional-split (that proportionally divides according to the users' original cost without coalitions) and bargaining games (based on bargaining game theory). The users will opt to join to a coalition that minimizes their individual costs. Thus, the likely outcome of self-interested user behavior will be a *stable coalition*, such that no group of users deviates to form another coalitions. A stable coalition is a *strong Nash equilibrium*, which is related to a collective deviation of a group of users, unlike typical Nash equilibrium that only considers a unilateral deviation of a single user.

To analyze the *social efficiency* of P2P energy sharing, we study the *strong price of anarchy* that compares the worst-case stable coalitions and a social optimum (i.e., the state with minimum total cost of all users). A desirable cost-sharing mechanism should have a small strong price of anarchy. By empirical evaluation and theoretical analysis, we show that the strong price of anarchy is mild for proportional-split and bargaining games. This can hence bolster the viability of P2P energy sharing.

In summary, this paper is structured as follows:

- (Sec. III) We formulate a general model of coalition formation for P2P energy sharing.
- (Sec. IV) We incorporate online control strategies to coordinate the operations of energy management systems in a coalition, which can cope with uncertain future demands and supplies.
- (Sec. V) We perform an extensive data analysis based on a real-world P2P energy sharing project. We examine the empirical ratios between stable coalitions and a social optimum.
- (Sec. VI) We derive the theoretical strong price of anarchy in general settings for several cost-sharing mechanisms.

[1] Besides of direction electrical connections, this also may require changes to the local metering infrastructure, in order to report the net aggregate electricity usage of household users, instead of individual usage, to the utility operator.

II. Related Work

This paper explores novel ideas of P2P exchanges among energy prosumers. There have been a number of studies investigating P2P energy trading. For example, see an extensive survey in [2], and the references therein. Among these studies, P2P energy trading has been applied to distributed energy resource management [4], [16], [17]. Moreover, the idea of shared pool of energy has been proposed in various energy sharing applications, such as virtual power plants [18], energy storage cloud [19], [7] and other multi-user energy systems [20], [21], [22]. P2P energy trading has been recently demonstrated in a real-world microgrid of the Brooklyn Microgrid Project [23]. However, a key difference between this work and previous studies is that we focus on the economic mechanisms enabling energy sharing in form of a long-term arrangement. We study coalition formation mechanisms of self-interested users and social efficiency in P2P energy sharing.

To model P2P energy sharing, we draw inspiration from the literature of sharing economy. For example, [13] studies ride-sharing, room-sharing, and pass-sharing applications. We adopt a similar coalition formation game with specific optimization problems applied to energy management systems. Furthermore, we extend the model in [13] to incorporate online decision-making with uncertainty arisen in P2P energy sharing.

Our coalition formation model belongs to the topic of network cost-sharing and hedonic coalition formation games [24], [14], [25], [26], [27], [15], [28], [29], [30], [31]. A study particularly related to our results is the price of anarchy for stable matching and the various extensions to K-sized coalitions [32], [33]. Our coalition formation game is a subclass of hedonic coalition formation games [14], [28] that allows arbitrary coalitions subject to a constraint on the maximum number of users per coalition. This model is useful in modeling practical applications of sharing economy. We also study simple cost-sharing mechanisms, such as egalitarian-split and Nash bargaining solution that are not considered in hedonic coalition formation games [32], [33].

III. Formulation and Models

P2P energy sharing is a new concept. We first propose a generic process for setting up P2P energy sharing. Then we will formulate the corresponding models and optimization problems. To arrange P2P energy sharing, it is likely to involve multiple steps from planning to actual operation as follows:

1) *Planning*: Each household needs to gather sufficient historical data of local demands and PV supplies. It will be used to plan the potential coalition-based optimization of energy management operations in P2P energy sharing.
2) *Matching*: The participating households will advertise the technical specifications of their energy resources (e.g., rooftop PVs and batteries) and profiles of past demands and PV supplies on an information repository. Automatic tools will be utilized to find and form a coalition of households for setting up a potential cost-sharing agreement.

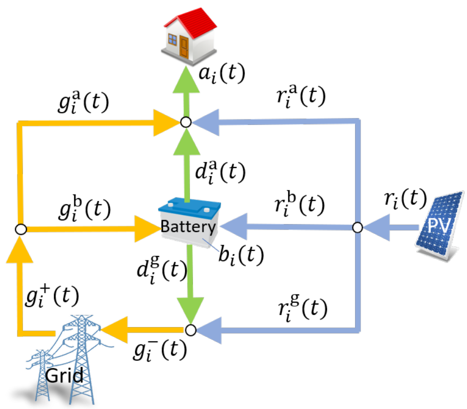

Fig. 2: Variables and parameters of the energy management system model.

3) *Agreement*: If the households agree on a cost-sharing mechanism and the corresponding coordinated optimization strategies of energy resources, they will underwrite a sharing agreement for a certain period on a database (e.g., a blockchain ledger). Then the agreed parties will notify the utility operator to settle their billing accounts on their behalf, if required. We consider a small number of participating households per each agreement because of the management complexity.
4) *Operation*: When the actual operation of P2P energy sharing is executed according to the agreement, each household will continuously monitor their benefits from P2P energy sharing. The households may renegotiate their agreement at the end of the agreed period.

Based on the above process, we will study the corresponding optimization and coalition formation processes in this paper. Next, we will present the models of energy management system, cost-sharing mechanisms and stable coalitions.

A. Energy Management System Model

We consider a set of n users, denoted by $\mathcal{N}$, who will form coalitions to share certain energy resources. There is a list of variables and parameters associated with the users and the management systems of their energy resources, as illustrated in Fig. 2.

- **Demand:** Each user $i \in \mathcal{N}$ is characterized by an energy demand function $a_i(t)$. The demand function can be constructed based on the prediction from historical data. In practice, there is uncertainty associated with future demand. An online decision-making process is required to manage energy resources with respect to uncertain future demand.
- **Rooftop PV:** Each user $i \in \mathcal{N}$ is equipped with rooftop PV, characterized by an energy supply function $r_i(t)$. The supply function $r_i(t)$ is divided into three feed-in rates: $r_i^{\rm a}(t)$ is for demand satisfaction, $r_i^{\rm b}(t)$ for charging battery, and $r_i^{\rm g}(t)$ for electricity feed-in to the grid.
- **Home Battery:** Each user $i \in \mathcal{N}$ is equipped with home battery, characterized by capacity B_i. The battery is constrained by charging efficiency $\eta_{\rm c} \leq 1$ and discharging

efficiency $\eta_{\rm d} \geq 1$, charge rate constraint $\mu_{\rm c}$ and discharge rate constraint $\mu_{\rm d}$. Let $b_i(t)$ be the current state-of-charge in the battery at time t, and $d_i^{\rm a}(t)$ be the discharge rate for demand satisfaction, whereas $d_i^{\rm g}(t)$ be the discharge rate for electricity feed-in to the grid.

- **Grid:** In addition to local rooftop PV and home battery, each user can also import electricity from the grid, if his demand is not entirely satisfied. Further, rooftop PV and home battery can inject excessive electricity into the grid, when feed-in compensation is offered by utility operator. Let $\mathsf{C}_{\rm g}^+$ be the per-unit cost by the grid on electricity consumption, and $\mathsf{C}_{\rm g}^-$ be the per-unit compensation on electricity feed-in. Let $g_i^+(t)$ be the electricity consumption rate of user i at time t, and $g_i^-(t)$ be the electricity feed-in rate. Let $g_i^{\rm a}(t)$ be the consumption rate for demand satisfaction, and $g_i^{\rm b}(t)$ be the consumption rate for charging battery.

Next, we will formulate the optimization problems that optimize the feed-in rates, discharge rates, and consumption rates.

1) Standalone Optimization Problem:

First, we focus on a standalone optimization problem with a single user, such that the user seeks to minimize the operational cost of electricity incurred by the grid, by optimizing the rooftop PV feed-in rates, battery charging/discharge rates, and grid consumption rates. We consider discrete timeslots $[1, T]$. The standalone optimization problem is formulated in `(P1)`. Cons. (2) states the state-of-charge update of battery system. Cons. (3) restricts the state-of-charge within the feasibility range of battery capacity, whereas Cons. (4)-(5) restrict the charging and discharge rates. Cons. (6) states the balance of energy for demand. There are some assumptions in `(P1)`. We assume that the demand and supply functions are known or predicted in advance for a sufficiently long period $[1, T]$. Let C_i be the optimal operational cost in `(P1)`.

$$
\begin{aligned}
\texttt{(P1)}\quad & C_i \triangleq \min \sum_{t=1}^{T} \big(\mathsf{C}_{\rm g}^+ g_i^+(t) - \mathsf{C}_{\rm g}^- g_i^-(t)\big) && (1)\\
\text{s.t. } & b_i(t+1) - b_i(t) = \eta_{\rm c}(r_i^{\rm b}(t) + g_i^{\rm b}(t)) \\
& \qquad\qquad - \eta_{\rm d}(d_i^{\rm a}(t) + d_i^{\rm g}(t)), && (2)\\
& 0 \leq b_i(t) \leq \mathsf{B}_i, b_i(0) = 0, && (3)\\
& g_i^{\rm b}(t) + r_i^{\rm b}(t) \leq \mu_{\rm c}, && (4)\\
& d_i^{\rm a}(t) + d_i^{\rm g}(t) \leq \mu_{\rm d}, && (5)\\
& d_i^{\rm a}(t) + g_i^{\rm a}(t) + r_i^{\rm a}(t) = a_i(t), && (6)\\
& r_i^{\rm a}(t) + r_i^{\rm b}(t) + r_i^{\rm g}(t) = r_i(t), && (7)\\
& g_i^{\rm a}(t) + g_i^{\rm b}(t) = g_i^+(t), && (8)\\
& d_i^{\rm g}(t) + r_i^{\rm g}(t) = g_i^-(t), && (9)\\
\text{var. } & b_i(t) \geq 0, d_i^{\rm a}(t) \geq 0, d_i^{\rm g}(t) \geq 0, \\
& r_i^{\rm a}(t) \geq 0, r_i^{\rm b}(t) \geq 0, r_i^{\rm g}(t) \geq 0, \\
& g_i^{\rm a}(t) \geq 0, g_i^{\rm b}(t) \geq 0, \forall t \in [1, T]
\end{aligned}
$$

Remarks: In this formulation, the rooftop PV provides free energy. Also, the feed-in tariff is assumed to be considerably less than the consumption tariff: $C_{\rm g}^- \ll \mathsf{C}_{\rm g}^+$. Hence, it will be more economical to use PV supply for demand satisfaction or battery charging, rather than exporting to the grid. Note that `(P1)` can be solved by standard linear programming, as it involves only linear constraints and a linear objective function.

2) Coalition-based Optimization Problem:

In this section, we study the problem of coalition-based optimization. We consider a group of users $G \subseteq \mathcal{N}$ forming a coalition to share their energy resources and minimize the total operational cost. The coalition-based optimization problem will extend the standalone optimization problem with coordination on the operations of energy management systems from multiple households.

We consider two P2P energy sharing scenarios as follows:

- (*Cost-Sharing by Direct Connections*): This scenario can be reduced to the standalone optimization problem by substituting local demand function by an aggregate demand function $a_G(t) = \sum_{i \in G} a_i(t)$ in `(P1)`. Also, multiple PV supply functions can be considered by an aggregate supply function $r_G(t) = \sum_{i \in G} r_i(t)$. Multiple battery systems can be considered as multiple constraints of a single battery system.
- (*Cost-Sharing by Grid Settlement*): If a group of household users choose to have settlement by the grid, there is per-unit service fee $\mathsf{C}_{\rm s}$. We also add two more variables: let $s_i^+(t)$ be the consumption rate under grid settlement, and $s_i^-(t)$ be the feed-in rate under grid settlement for user i, respectively. The coalition-based optimization problem is formulated in `(P2)`, which is similar to `(P1)`. But there are more variables from all the users in coalition G. Furthermore, Cons. (19) describes the balance of grid settlement.

$$
\texttt{(P2)}\quad C(G) \triangleq \min \sum_{t=1}^{T} \sum_{i \in G} \big(\mathsf{C}_{\rm g}^+ g_i^+(t) - \mathsf{C}_{\rm g}^- g_i^-(t) + \mathsf{C}_{\rm s} s_i^+(t)\big) \qquad (10)
$$

$$
\begin{aligned}
\text{s.t. } & b_i(t+1) - b_i(t) = \eta_{\rm c}(r_i^{\rm b}(t) + g_i^{\rm b}(t)) \\
& \qquad\qquad - \eta_{\rm d}(d_i^{\rm a}(t) + d_i^{\rm g}(t)), && (11)\\
& 0 \leq b_i(t) \leq \mathsf{B}_i, b_i(0) = 0, && (12)\\
& g_i^{\rm b}(t) + r_i^{\rm b}(t) \leq \mu_{\rm c}, && (13)\\
& d_i^{\rm a}(t) + d_i^{\rm g}(t) \leq \mu_{\rm d}, && (14)\\
& d_i^{\rm a}(t) + g_i^{\rm a}(t) + r_i^{\rm a}(t) = a_i(t), && (15)\\
& r_i^{\rm a}(t) + r_i^{\rm b}(t) + r_i^{\rm g}(t) = r_i(t), && (16)\\
& g_i^{\rm a}(t) + g_i^{\rm b}(t) = g_i^+(t) + s_i^+(t), && (17)\\
& d_i^{\rm g}(t) + r_i^{\rm g}(t) = g_i^-(t) + s_i^-(t), && (18)\\
& \sum_{i \in G} s_i^+(t) = \sum_{i \in G} s_i^-(t) && (19)\\
\text{var. } & b_i(t) \geq 0, d_i^{\rm a}(t) \geq 0, d_i^{\rm g}(t) \geq 0, \\
& r_i^{\rm a}(t) \geq 0, r_i^{\rm b}(t) \geq, r_i^{\rm g}(t) \geq 0, \\
& g_i^{\rm a}(t) \geq 0, g_i^{\rm b}(t) \geq 0, s_i^+(t) \geq 0, s_i^-(t) \geq 0, \forall t \in [1, T], \forall i \in G
\end{aligned}
$$

Let $C(G)$ be the optimal operational cost of coalition-based optimization problem with respect to coalition G.

Remarks: Cost-sharing by direct connections can also be modeled by `(P2)` by setting $\mathsf{C}_{\rm s} = 0$. In coalition-based optimization problem, we assume that there are certain coordination and communication operations among multiple energy management systems of the participating household users, such that battery charging and discharging operations can be synchronized with the consumption rates of different users. For example, it is possible to trigger the discharging from the battery of one user, when there is demand from another user. The coordination of separate energy management systems will need a data communication network interconnecting them. Note that there are commercial battery management systems supporting remote control via Internet connection [34], [35].

Service fee $\mathsf{C}_{\rm s}$ is an important factor to viability of coalition-based optimization. High service fee $\mathsf{C}_{\rm s}$ will deter coalition formation in P2P energy sharing. As long as $\mathsf{C}_{\rm s} < \mathsf{C}^{+}_{\rm g} - \mathsf{C}^{-}_{\rm g}$, it is still viable to transfer PV energy via grid settlement, because price difference $\mathsf{C}^{+}_{\rm g} - \mathsf{C}^{-}_{\rm g}$ that represents simultaneous importing and exporting electricity, is still higher than grid settlement at the cost $\mathsf{C}_{\rm s}$. Since $\mathsf{C}^{-}_{\rm g}$ is sometimes very low in practice, even $\mathsf{C}_{\rm s} \geq \mathsf{C}^{-}_{\rm g}$ is viable to coalition formation, as it pays more to grid operator for grid settlement than exporting to the grid.

B. Coalition Formation Model and Cost-Sharing Mechanisms

In this section, we adopt a general coalition formation model from [13] to model P2P energy sharing, which is derived from hedonic coalition formation games [14], [15]. Recall that a coalition of users for sharing energy is a subset $G \subseteq \mathcal{N}$. A *coalition structure* is a partition of $\mathcal{N}$ denoted by $\mathcal{P} \subset 2^{\mathcal{N}}$, such that $\bigcup_{G\in\mathcal{P}} G = \mathcal{N}$ and $G_1 \cap G_2 = \varnothing$ for any pair $G_1, G_2 \in \mathcal{P}$. A coalition structure represents a feasible state of coalition formation. For example, $\mathcal{P} = \{\{1, 2\}, \{3\}\}$, where users 1 and 2 form a coalition and user 3 is alone.

Let the set of all partitions of $\mathcal{N}$ be $\mathscr{P}$. Each element $G \in \mathcal{P}$ is a coalition (or a group). The set of singleton coalitions, $\mathcal{P}_{\rm self} \triangleq \{\{i\} : i \in \mathcal{N}\}$, is called the *default coalition structure*, wherein no one forms a coalition with others. We consider arbitrary coalition structures with at most K users per coalition. In practice, K is often much less than n. Let $\mathscr{P}_K \triangleq \{\mathcal{P} \in \mathscr{P} : |G| \leq K \text{ for each } G \in \mathcal{P}\}$ be the set of *feasible* coalition structures, such that each coalition consists of at most K users. We restrict our attention to a small value K, say $K = 2, 3$ because there will be high management complexity involving many users per each cost-sharing agreement to coordinate their energy management systems. Also, the grid operator may not permit billing settlement among many users.

Recall that the cost function of each coalition G is noted by $C(G)$. In this paper, we assume that most households are customers, rather than electricity exporters. Hence, C_i and $C(G)$ from `(P1)` and `(P2)` will be positive. There is an important property called *cost monotonicity*:

$$C(H) \leq C(G), \quad \text{if } H \subseteq G. \tag{20}$$

Namely, larger coalition should incur a large cost. If C_i and $C(G)$ are positive, then cost monotonicity will hold. Otherwise, it will violate the optimality of `(P2)`.

When $G = \{i, j\}$, we also denote $C_{i,j} \triangleq C(\{i, j\})$. We denote $C_i \triangleq C(\{i\})$ as the *singleton cost* for user i, when i does not form a coalition with any other.

1) Cost-Sharing Mechanisms:

By agreeing to form a coalition G, the users are supposed to share the cost $C(G)$. A cost-sharing mechanism is characterized by payment function $p_i(G)$, which is the shared cost of user $i \in G$, such that $\sum_{i\in G} p_i(G) = C(G)$. Cost-sharing mechanism $p_i(\cdot)$ is called *budget balanced*, if $\sum_{i\in G} p_i(G) = C(G)$ for every $G \subseteq \mathcal{N}$.

Given coalition G, let the *utility* of user $i \in G$ (i.e., the saving of joining coalition G) be

$$u_i(G) \triangleq C_i - p_i(G) \tag{21}$$

In this paper, we consider the following simple cost-sharing mechanisms (denoted by different superscripts):

1) **Equal-split Cost-Sharing**: The cost is split equally among all users: for $i \in G$,

$$p^{\rm eq}_i(G) \triangleq \frac{C(G)}{|G|} \tag{22}$$

2) **Proportional-split Cost-Sharing**: The cost is split proportionally according to the users' default costs: for $i \in G$,

$$p^{\rm pp}_i(G) \triangleq \frac{C_i \cdot C(G)}{\sum_{j\in G} C_j} \tag{23}$$

Namely, $u^{\rm pp}_i(G) = C_i \cdot \frac{(\sum_{j\in G} C_j) - C(G)}{\sum_{j\in G} C_j}$.

3) **Bargaining-based Cost-Sharing**: One can also formulate the cost-sharing problem as a bargaining game with a feasible set and a disagreement point. In this model, the feasible set is the set of utilities $(\hat{u}_i)_{i\in G}$, such that $\sum_{i\in G} \hat{u}_i \leq \sum_{i\in G} u_i(G)$ (namely, $\sum_{i\in G} p_i \geq C(G)$), and the disagreement point is $(\hat{u}_i = 0)_{i\in G}$, such that each user pays only the respective singleton cost. There are two common bargaining solutions in the literature [10]:

   a) **Egalitarian-split Cost-Sharing** is given by:

$$p^{\rm ega}_i(G) \triangleq C_i - \frac{(\sum_{j\in G} C_j) - C(G)}{|G|} \tag{24}$$

   Namely, every user in each coalition receives the same utility: $u^{\rm ega}_i(G) = \frac{(\sum_{j\in G} C_j) - C(G)}{|G|}$ for all $i \in G$.

   b) **Nash Bargaining Solution** is given by:

$$\big(p^{\rm ns}_i(G)\big)_{i\in G} \in \arg\max_{(\hat{p}_i)_{i\in G}} \prod_{i\in G} u_i(\hat{p}) \tag{25}$$

   subject to

$$\sum_{i\in G} \hat{p} = C(G)$$

   It can be shown that egalitarian-split cost-sharing is equivalent to Nash bargaining solution in this model [13].

Other possible cost-sharing mechanisms include Shapely value, which has a higher computational complexity. See [30], [36], [37].

Example: Considering $G = \{i, j\}$, we obtain:
- $p^{\rm eq}_i(G) = \frac{C_{i,j}}{2}$ and $u^{\rm eq}_i(G) = \frac{2C_i - C_{i,j}}{2}$.

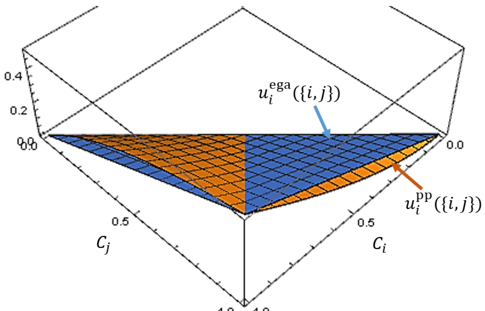

Fig. 3: Comparison of $u_i^{\rm ega}(\{i,j\})$ and $u_i^{\rm pp}(\{i,j\})$. If $C_j \ge C_i$, $u_i^{\rm ega}(\{i,j\}) \ge u_i^{\rm pp}(\{i,j\})$. Otherwise, $u_i^{\rm pp}(\{i,j\}) \ge u_i^{\rm eqa}(\{i,j\})$.

- $p_i^{\rm pp}(G) = \frac{C_i \cdot C_{i,j}}{C_i + C_j}$ and $u_i^{\rm pp}(G) = \frac{C_i\cdot(C_i+C_j-C_{i,j})}{C_i+C_j}$.
- $p_i^{\rm ega}(G) = p_i^{\rm ns}(G) = \frac{C_{i,j}+C_i-C_j}{2}$ and $u_i^{\rm ega}(G) = u_i^{\rm ns}(G) = \frac{C_i+C_j-C_{i,j}}{2}$.

While equal-split distributes the cost equally to every user regardless their singleton costs, proportional-split and egalitarian-split distributes the cost differently. When $G = \{i,j\}$, we plot $u_i^{\rm ega}(\{i,j\})$ and $u_i^{\rm pp}(\{i,j\})$ according to C_i and C_j in Fig. 3, assuming $C_{i,j} = 1$. If $C_j \ge C_i$, then $u_i^{\rm ega}(\{i,j\}) \ge u_i^{\rm pp}(\{i,j\})$. Otherwise, $u_i^{\rm pp}(\{i,j\}) \ge u_i^{\rm eqa}(\{i,j\})$. Namely, egalitarian-split favors smaller singleton costs, whereas proportional-split favors larger ones.

Sec. V-A will present a case study of different cost-sharing mechanisms from a real-world P2P energy sharing project.

C. Stable Coalition & Strong Price of Anarchy

Because users are self-interested and incentive-driven, they will join a coalition that maximizes his/her utility. Given certain costing-sharing mechanism $p_i(\cdot)$ and corresponding utility $u_i(\cdot)$, a coalition of users G is called a *blocking coalition* with respect to coalition structure $\mathcal{P}$ if all users in G can *strictly* reduce their shared costs (or increase their utilities) when they form a coalition G to share the cost instead. A coalition structure is called *stable coalition structure*, denoted by $\hat{\mathcal{P}} \in \mathscr{P}_K$, if there exists no blocking coalition with respect to $\hat{\mathcal{P}}$. Note that a stable coalition structure is also a strong Nash equilibrium[2]. In a stable coalition structure, the utility for every user is always non-negative $u_i(G) \ge 0$. Otherwise, the user will not join any coalition because of $u_i(\{i\}) = 0$.

Example: Considering $G = \{i,j,k\}$, we assume the following:

$$
\begin{aligned}
u_i(\{i,j\}) &> u_i(\{i,k\}) > u_i(\{i\}) \\
u_j(\{j,k\}) &> u_j(\{i,j\}) > u_j(\{j\}) \\
u_k(\{k\}) &> u_k(\{j,k\}) > u_k(\{i,k\})
\end{aligned}
$$

Then user k would prefer to stay to be alone in a singleton. Next, users i and j will form a stable coalition $\{i,j\}$, since there is no other better option.

[2]A strong Nash equilibrium is a Nash equilibrium, in which no group of players can cooperatively deviate in an allowable way that benefits all of its members.

There are efficient algorithms to find stable coalition structures [15], [14], [13]. In particular, when $K = 2$, it is known as the generalized stable roommates problem, and Irving algorithm can find a stable roommates configuration efficiently.

Given a coalition structure $\mathcal{P}$, let $u(G) \triangleq \sum_{i\in G} u_i(G)$ and $u(\mathcal{P}) \triangleq \sum_{G\in\mathcal{P}} u(G)$. We call $u(\mathcal{P})$ the social utility of $\mathcal{P}$. A *social optimum* is a coalition structure that minimizes the social utility of all users: $\mathcal{P}^* = \arg\max_{\mathcal{P}\in\mathscr{P}_K} u(\mathcal{P})$. A social optimum is a social efficient outcome. Define the utility-based *Strong Price of Anarchy* (SPoA) as the worst-case ratio between the social utility of a stable coalition structure and that of a social optimum:

$$
\mathsf{SPoA}_K^u \triangleq \max_{\hat{\mathcal{P}}, C(\cdot)} \frac{u(\mathcal{P}^*)}{u(\hat{\mathcal{P}})} \tag{26}
$$

Specifically, SPoA when using specific cost-sharing mechanisms are denoted by $\mathsf{SPoA}_K^{u,\rm eq}$, $\mathsf{SPoA}_K^{u,\rm pp}$, $\mathsf{SPoA}_K^{u,\rm ega}$, respectively. SPoA provides a natural metric of social efficiency.

Similarly, define cost-based SPoA as the worst-case ratio between the cost of a stable coalition structure and that of a social optimum:

$$
\mathsf{SPoA}_K^C \triangleq \max_{\hat{\mathcal{P}}, C(\cdot)} \frac{C(\hat{\mathcal{P}})}{C(\mathcal{P}^*)} \tag{27}
$$

Cost-based SPoA has been studied in prior work [13]. In this paper, we will relate utility-based SPoA to cost-based SPoA, and improve the extant results of cost-based SPoA in Sec. VI.

IV. Online Optimization

In the previous section, coalition formation uses the historical (or predicted) data to find the best possible coalitions. While offline optimization can be used in a-priori planning stage, *online* optimization is more useful in the operational stage, since predicted data may suffer deviation from the actual data. The strategies of not using future information are known as *online algorithms* [38], [39].

In this section, we present some heuristics for solving online standalone and coalition-based optimization problems (P1)-(P2). Evaluation studies comparing the performance of the online heuristics with offline optimization solutions will be provided in Sec. V.

A. Standalone Optimization

We present a heuristic for solving online standalone optimization problem (P1) in Online-Standalone (Algorithm 1). Note that operator $[\cdot]^+ \triangleq \max(\cdot, 0)$ represents truncation of negative numbers.

The basic idea of Online-Standalone is a greedy strategy, whereby the available PV supply will be first used to satisfy demand and charge battery as much as possible, before consuming any electricity from the grid, due to the fact that the feed-in tariff is well below the consumption tariff (namely, $\mathsf{C}_{\rm g}^- \ll \mathsf{C}_{\rm g}^+$). Unused PV supply will then be used for exporting to the grid. Since greedy strategy works without the knowledge of demand and supply functions, it can be applied to online optimization. Therefore, ONLINE-STANDALONE is a good online algorithm that can approximate the offline optimal solution. However, ONLINE-STANDALONE is not optimal because it does not consider the boundary case, namely, near the end of agreement period T. We will compare the performance of ONLINE-STANDALONE with the offline optimal solution by solving (P1) in the evaluation section. We note that if T is large, then ONLINE-STANDALONE is able to perform well as compared with the offline optimal solution.

B. Coalition-based Optimization

Although one can use a similar greedy strategy as ONLINE-STANDALONE independently at each household, this will suffer in performance because of the lack of coordination among the households. A better heuristic for solving online coalition-based optimization problem (P2) is presented in ONLINE-COALITION (Algorithm 2).

The coordination among households may involve multiple remote energy resources by transferring the electricity from one household to another by the following options:

1) (*Satisfy Demands from Remote PVs*): One household can transfer its electricity from its PVs, via grid settlement, to satisfy the demand of another household. Through grid settlement, the instantaneous consumption of electricity from one household can be offset by the instantaneous export of electricity from another household. This operation is the most preferable, particularly when one household runs out of local PV supply or depleted the local battery.
2) (*Charging Batteries from Remote PVs*): The electricity from local PVs can charge the battery in another household, via grid settlement. This allows storing the energy in another household for its later consumption. Hence, one can maximize the benefits of excessive PV supply and overcoming the limited battery capacity of one household by utilizing remote batteries. However, there is a disadvantage to this operation. Storing too much energy in battery from remote sources at a settlement service fee $\mathsf{C}_{\rm s}$ will reduce the opportunities of taking advantage of the free energy from local PVs.
3) (*Satisfy Demand from Remote Batteries*): Satisfying the demand from another household's battery can also save operational cost, because the consumption tariff $\mathsf{C}_{\rm g}^{+}$ is usually larger than settlement service fee $\mathsf{C}_{\rm s}$. However, if there are significantly more demands in a particular household, depleting the battery of such a household to satisfy other households may incur higher operational cost.
4) (*Charging Batteries from Remote Batteries*): This is the least preferable operation. In general, it is not optimal, even if there is excessive PV supply. Transferring energy among batteries will incur loss in energy conversion.

ONLINE-COALITION will use the first two options. First, we will satisfy local demands using PVs and batteries, and charge local battery using PVs from the same households. Then we will use the unused PV energy to satisfy remote demands. Next, we will use the remaining unused PV energy to charge remote batteries, but only up to a threshold $\theta \le 1$. Restricting the amount of stored energy from remote sources can take advantage of the potential future energy from local PVs. In the following, we set $\theta = 1$, when $\mathsf{C}_{\rm s} = 0$.

In Sec. V, we will present evaluation studies comparing the performance of the online heuristics with offline optimization solutions.

Algorithm 1 ONLINE-STANDALONE

Input: $\big(a_i(t), r_i(t), b_i(t-1)\big)$
Output: $\big(b_i(t), d_i^{\rm a}(t), d_i^{\rm g}(t), r_i^{\rm a}(t), r_i^{\rm b}(t), r_i^{\rm g}(t), g_i^{\rm a}(t), g_i^{\rm b}(t), g_i^{+}(t), g_i^{-}(t)\big)$

▷ *Satisfy demand from PV first, then battery, and finally the grid*
if $a_i(t) > 0$ **then**
 $r_i^{\rm a}(t) \leftarrow [a_i(t) - r_i(t)]^{+}$
end if
if $r_i^{\rm a}(t) < a_i(t)$ **then**
 $d_i^{\rm a}(t) \leftarrow \frac{1}{\eta_{\rm d}}[b_i(t-1) - \big(a_i(t) - r_i^{\rm a}(t)\big)]^{+}$
end if
if $r_i^{\rm a}(t) + d_i^{\rm a}(t) < a_i(t)$ **then**
 $g_i^{\rm a}(t) \leftarrow a_i(t) - r_i^{\rm a}(t) - r_i^{\rm g}(t)$
end if
▷ *Charge battery from PV*
$r_i^{\rm b}(t) \leftarrow \frac{1}{\eta_{\rm c}}[\mathsf{B}_i - b_i(t-1) - \big(r_i(t) - r_i^{\rm a}(t)\big)]^{+}$
$b_i(t) \leftarrow b_i(t-1) + \eta_{\rm c} r_i^{\rm b}(t) - \eta_{\rm d} d_i^{\rm a}(t)$
▷ *Feed-in to the grid from PV*
$r_i^{\rm g}(t) \leftarrow r_i(t) - r_i^{\rm a}(t) - r_i^{\rm b}(t)$
$d_i^{\rm g}(t) \leftarrow 0, g_i^{\rm b}(t) \leftarrow 0, g_i^{+}(t) \leftarrow g_i^{\rm a}(t), g_i^{-}(t) \leftarrow r_i^{\rm g}(t)$
return $\big(b_i(t), d_i^{\rm a}(t), d_i^{\rm g}(t), r_i^{\rm a}(t), r_i^{\rm b}(t), r_i^{\rm g}(t), g_i^{\rm a}(t), g_i^{\rm b}(t), g_i^{+}(t), g_i^{-}(t)\big)$

V. EVALUATION STUDIES

In this section, we evaluate our mechanisms and algorithms using empirical data from a real-world project. A field trial has been conducted on Bruny Island, Tasmania in Australia [40], [41], where approximately 31 batteries were installed with solar PV systems in the homes of selected residents of the island[3]. The batteries include customized management software that allows programmable control of the battery management operations. The participating households provide energy data for empirical studies. See Fig. 4 for photos of Bruny Island.

A. Case Study

First, we present a case study using empirical data to illustrate the effects of P2P energy sharing and cost-sharing mechanisms. We consider two particular households on the island. We collected the data of their winter consumption and PV supplies. Note that winter season has higher consumption but lesser PV supply than summer season. The battery capacity in the households was 9.8kWh. There were 4 kW solar PVs systems installed. We assume zero settlement service fee, or direct connections among the households.

[3]RepositPower provided anonymised historical load and PV data for these systems in a battery-coordination research trial on Bruny Island.

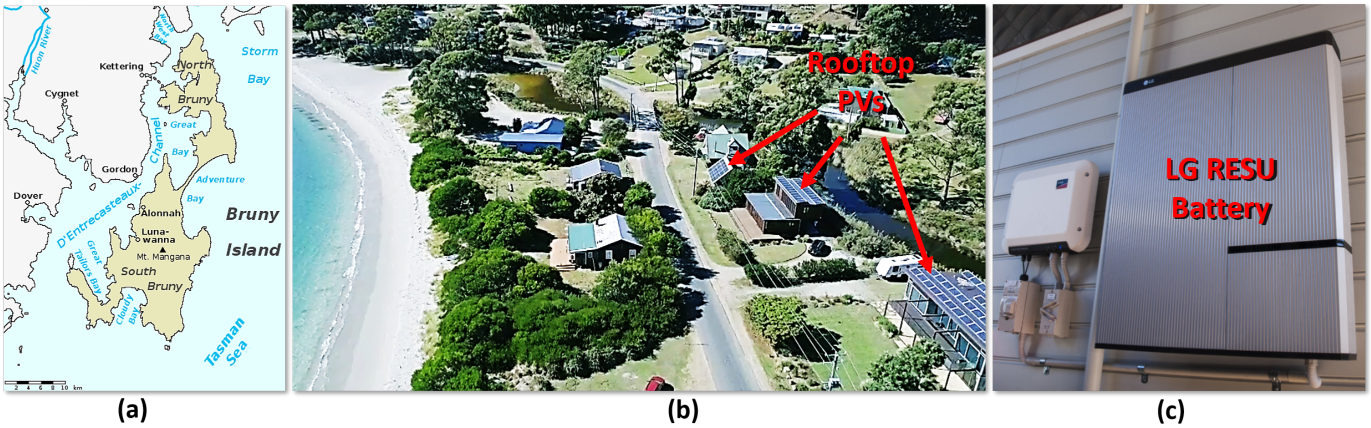

Fig. 4: (a) Map of Bruny Island, Tasmania in Australia. (b) Photo of typical households with rooftop PVs on Bruny Island. (c) Photo of the battery system of a household on Bruny Island.

In Fig. 6, we plot the data trace of the energy management system at user 1 running online heuristics. Figs. 6 (a)-(c) are for standalone optimization using ONLINE-STANDALONE (Algorithm 1), while Fig. 6 (d)-(f) are for coalition-based optimization using ONLINE-COALITION (Algorithm 2).

Comparing Fig. 6 (a) and Fig. 6 (d), we observe that user 1 is able to utilize the feed-in electricity from user 2 (i.e., $s_1^+(t) \geq 0$), when a coalition is formed between them. By leveraging the feed-in, user 1 can reduce electricity from the grid (i.e., $g_i^{\rm a}(t)$ is lesser in Fig. 6 (d)). As a result, the social utility due to coalition will be split between users 1 and 2 according to different cost-sharing mechanisms.

In Fig. 5 (a), we compare the utilities under different cost-sharing mechanisms. The first bar is the social utility, that is how much operational cost both users can save in a coalition from the total of singleton costs without coalition. In equal-split, user 2 is given negative utility, because he has a low singleton cost. Hence, user 2 will not participate in the coalition with user 1 under equal-split. In both proportional-split and egalitarian-split, users 1 and 2 can receive positive utilities. But user 2 is gaining an unfair proportion close to 0, whereas egalitarian-split divides the social utility equally among the users. In this case, users 1 and 2 are more likely to participate in a coalition for P2P energy sharing.

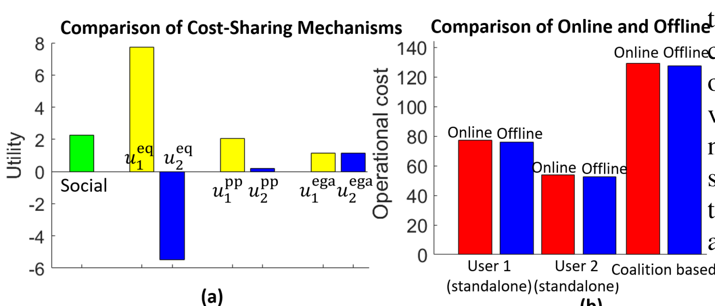

Fig. 5: (a) Comparison of cost-sharing mechanisms for a pair of households. (b) Comparison of operational cost of online heuristics and offline optimal solutions.

In Fig. 5 (b), we compare the operational cost between online heuristics (i.e., ONLINE-STANDALONE and ONLINE-COALITION) and offline optimal solutions by solving the linear programming problems of (P1)-(P2). We observe that in both standalone and coalition-based optimization, the online heuristics are able to approximate the offline optimal solutions well, and are effective in the practical operations of energy management systems.

Battery capacity (B_i,)	9.8 kWh
Consumption tariff ($\mathsf{C}_{\rm g}^+$)	\$0.20/kWh
Feed-in tariff ($\mathsf{C}_{\rm g}^-$)	\$0.10/kWh
Settlement service fee ($\mathsf{C}_{\rm s}$)	\$0.00/kWh
Charging efficiency ($\eta_{\rm c}$)	0.95
Discharging efficiency ($\eta_{\rm d}$)	1.05
Charge rate ($\mu_{\rm c}$)	5
Discharge rate ($\mu_{\rm d}$)	5

TABLE I: Default parameters used in evaluation.

B. Coalition Structures & Utility Distribution

Next, we evaluate the outcomes of coalition formation and cost-sharing mechanisms with 30 households. The evaluation is based on the default parameters in Table I. We consider coalitions with a maximum size of two (i.e., $K = 2$).

The coalition structures are visualized in Fig. 7 (a)-(d). For example, under the equal-split cost-sharing, users 1 and 14 will form a coalition. Each of the mechanisms successfully form either 14 or 15 coalitions, but the structures differ. The structure is the primary reason for differing total utilities. The total utility of a social optimum is \$558.02 and the egalitarian cost-sharing achieves \$536.19, which is very close to the social optimum. The distribution of utilities of individual users are visualized by the histogram in Fig. 7 (e). This identifies the number of households receiving different benefits. The equal-split is clearly skewed towards lower individual utilities while the egalitarian-split appears to favor most users by providing a benefits of over \$30 for more than half of the users.

C. Battery Capacity

Different battery capacities were considered to assess their effect on utility. Results suggest that larger batteries can result in greater utility. This analysis does not consider the investment cost of batteries to assess economically viability over the long term. The two batteries compared in Fig. 8 (a) have capacities of 13.2 kWh and 4.5 kWh, respectively. Under

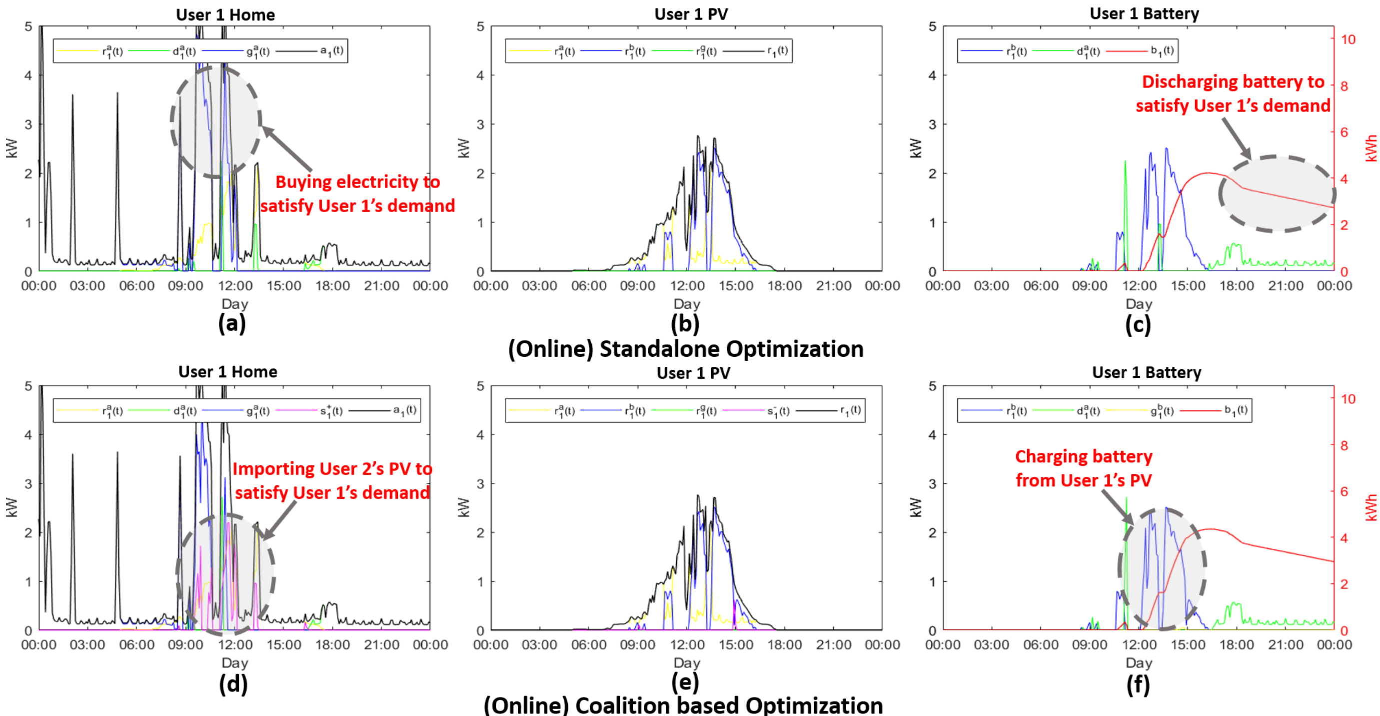

Fig. 6: Results of the data trace of the energy management system at user 1. (a)-(c): Standalone optimization using ONLINE-STANDALONE (Algorithm 1). (d)-(f) : Coalition based optimization using ONLINE-COALITION (Algorithm 2).

egalitarian-split, use of the large battery results in an aggregate utility of \$515 whereas the small battery has a utility of \$367 - rather close to the social optima of \$548 and \$381. Despite the larger battery having almost three times the capacity of the smaller battery, it has less than twice the utility. This suggests that battery capacity may not universally increase proportionally with utility.

D. Consumption Tariff

The consumption tariff is the cost per kWh of energy imported from the grid. The results suggest an increase in consumption tariff causes a decrease in the utility. This is expected because an increase in the cost of electricity will cause any potential savings to diminish. Two consumption tariffs were considered: a constant rate and time-of-use tariff. The constant rate assumed a consumption tariff of \$0.20/kWh and feed-in tariff of \$0.09/kWh. The time-of-use tariff was based on the battery trial with peak consumption tariff of \$0.2846/kWh, off-peak consumption tariff of \$0.1325/kWh and constant feed-in tariff of \$0.09/kWh. Both tariffs assumed no settlement service fee. From Fig. 8 (b), the constant tariff was much less favorable than the time-of-use tariff. The constant tariff had a utility of \$893 and the time-of-use tariff had a utility of \$481. This is explained by the offline algorithm's ability to schedule energy usage, taking advantage of off-peak pricing to charge the battery in preparation for its use during peak times.

E. Feed-in Tariff

The feed-in tariff is the cost per kWh of energy exported to the grid. Feed-in tariffs of \$0.10 and \$0.15 were used in the comparison. Feed-in tariff appears in Fig. 8 (c) to have little effect with egalitarian-split showing utilities of \$160 and \$154, respectively. This is a positive result, demonstrating that electricity feed-in is being minimized in favor of energy sharing. Under standalone optimization, greedy households would be inclined to maximize their energy export and a higher feed-in tariff provides greater utility to these households. Coalition forces are causing households to extract benefits by mutually reducing each other's consumption rate costs, rather than maximizing feed-in.

F. Settlement Service Fee

The settlement service fee is the rate per kWh charged by the grid operator in return for facilitating energy sharing over the grid. An increase in settlement fee indicates a lower utility. Higher settlement service fees act as a disincentive for frequent energy sharing events and add to the electricity bill when they do occur. Fig. 8 (d) demonstrates that egalitarian-split has a significant advantage over the equal and proportional splitting mechanisms. When the settlement service fee is \$0.02/kWh, the utility in the egalitarian-split is \$236. When the settlement fee is \$0.05/kWh, the utility is \$140. It reduces to approximately \$0.00 when the settlement service fee is too high at \$0.10. This suggests that settlement service fee is a critical factor to provide an incentive for energy sharing. In practice, it must be balanced between being sufficient to cover operational costs of the grid while not discouraging energy sharing.

VI. THEORETICAL ANALYSIS

We complement the evaluation studies with theoretical analysis of strong price of anarchy that can be applied to

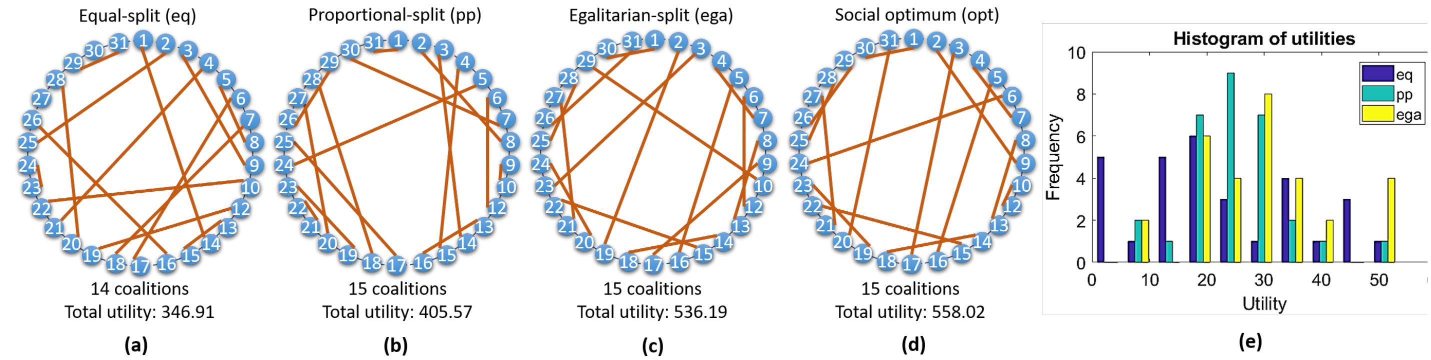

Fig. 7: Coalition structures under different cost-sharing mechanisms in (a)-(c) and social optimumin (d). (e) The distribution of utilities of individual users.

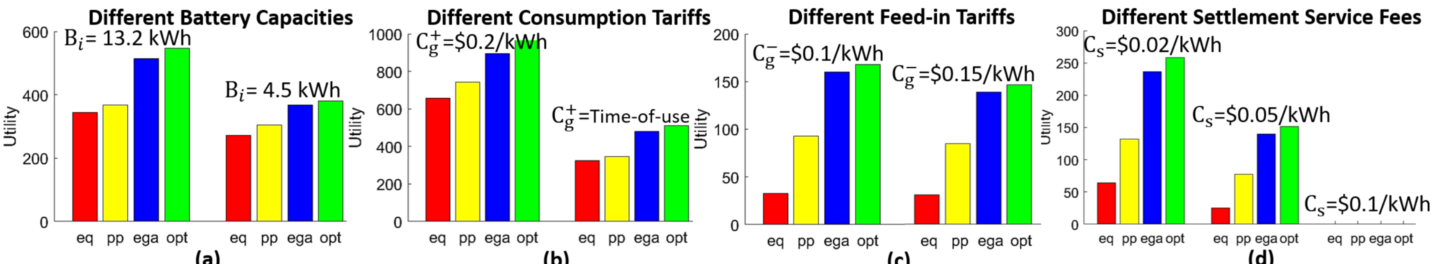

Fig. 8: Evaluation results under different settings

general settings. The full proofs are deferred to the appendix of the technical report.

Theorem 1. *Consider a budget balanced cost-sharing mechanism $p_i(\cdot)$. Denote the cost-based and utility-based strong prices of anarchy by SPoA_K^C and SPoA_K^u, respectively. Then*

$$\mathsf{SPoA}_K^u \geq \frac{K-1}{K-\mathsf{SPoA}_K^C} \quad \mathsf{SPoA}_K^C \leq K - \frac{K-1}{\mathsf{SPoA}_K^u} \tag{28}$$

Theorem 2. *Consider proportional-split cost-sharing mechanism. When $K \leq 6$, we obtain*

$$\mathsf{SPoA}_K^{C,\mathrm{pp}} \leq \sum_{s=1}^{K} \frac{1}{s} \tag{29}$$

For general K, we obtain

$$\mathsf{SPoA}_K^{C,\mathrm{pp}} \leq \log K + 2 \tag{30}$$

Theorem 3. *For egalitarian-split and Nash bargaining cost-sharing mechanisms, the SPoA is upper bounded by*

$$\mathsf{SPoA}_K^{C,\mathrm{ega}} = \mathsf{SPoA}_K^{C,\mathrm{ns}} = O(\log K) \tag{31}$$

Here we improve the known results in [13]. By comparison, $\mathsf{SPoA}_K^{C,\mathrm{pp}} = O(\log K)$ and $\mathsf{SPoA}_K^{C,\mathrm{ega}} = O(\sqrt{K}\log K)$ were shown in [13]. Our theorems show that $\mathsf{SPoA}_K^{C,\mathrm{pp}}, \mathsf{SPoA}_K^{C,\mathrm{ega}}, \mathsf{SPoA}_K^{C,\mathrm{ns}}$ are within the order of magnitude of $\log K$. When $K = 2$, we obtain $\mathsf{SPoA}_K^C \leq \frac{3}{2}$ and $\mathsf{SPoA}_K^u \leq 2$ for these cost-sharing mechanisms, which are mild. In the previous evaluation studies, we observe that the empirical ratios between stable coalitions and a social optimum are well below the theoretical bound. In particular, the social utility under egalitarian-split cost-sharing is very close to that of a social optimum. Therefore, egalitarian-split cost-sharing is a highly socially efficient mechanism both in practice and in theory.

VII. Conclusion

In a decentralised smart grid, energy prosumers will form coalitions to share energy in a P2P fashion. But forming optimal coalitions requires proper tools to cope with complexity of energy systems and uncertainty of renewable energy. This paper sheds light on the principles of cost-sharing mechanisms for P2P energy sharing. We characterize the social efficiency of P2P energy sharing by the *strong price of anarchy* that compares the worst-case stable coalitions and a social optimum. We showed the strong price of anarchy is mild by a data analysis of a real-world P2P energy sharing project and by theoretical bounds. In particular, egalitarian-split is a highly socially efficient cost-sharing mechanism in practice and in theory.

In future, we will study a proper threshold in online heuristics for charging batteries from remote PVs, and its impact on the strong price of anarchy. We will also study random arrivals of users. This online decision problem shares certain similarity with ski rental and one-way trading problems [38], [39]. We will leverage the recent extensions of these problems in smart grid management [42], [43], [44].

Acknowledgment

The authors would like to thank Daniel Gordon and Reposit Power for sharing the data trace of Bruny Island field trial.

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Algorithm 2 ONLINE-COALITION

Input: $\big(a_i(t), r_i(t), b_i(t-1)\big)_{i \in G}$

Output: $\big(b_i(t), d_i^{\rm a}(t), d_i^{\rm g}(t), r_i^{\rm a}(t), r_i^{\rm b}(t), r_i^{\rm g}(t), g_i^{\rm a}(t), g_i^{\rm b}(t), s_i^{+}(t), s_i^{-}(t)\big)_{i \in G}$

▷ *Satisfy local demands and charge local batteries*

for $i \in G$ **do**
  if $a_i(t) > 0$ **then**
    $r_i^{\rm a}(t) \leftarrow [a_i(t) - r_i(t)]^+$
  end if
  if $r_i^{\rm a}(t) < a_i(t)$ **then**
    $d_i^{\rm a}(t) \leftarrow \frac{1}{\eta_{\rm d}}[b_i(t-1) - \big(a_i(t) - r_i^{\rm a}(t)\big)]^+$
  end if
  $r_i^{\rm b}(t) \leftarrow \frac{1}{\eta_{\rm c}}[\mathsf{B}_i - b_i(t-1) - \big(r_i(t) - r_i^{\rm a}(t)\big)]^+$
  $b_i(t) \leftarrow b_i(t-1) + \eta_{\rm c} r_i^{\rm b}(t) - \eta_{\rm d} d_i^{\rm a}(t)$
end for

▷ *Satisfy remote demands from PVs via grid settlement*

for $i \in G$ **do**
  if $r_i^{\rm a}(t) + d_i^{\rm a}(t) < a_i(t)$ **then**
    if $\exists j \in G \backslash \{i\}$ such that $r_j(t) > r_j^{\rm a}(t) + r_j^{\rm b}(t) + s_j^{-}(t)$ **then**
      $\Delta_1 \leftarrow r_j(t) - r_j^{\rm a}(t) - r_j^{\rm b}(t) - s_j^{-}(t)$
      $s_j^{-}(t) \leftarrow s_j^{-}(t) + [a_i(t) - r_i^{\rm a}(t) - d_i^{\rm a}(t) - \Delta_1]^+$
      $s_i^{+}(t) \leftarrow s_i^{+}(t) + [a_i(t) - r_i^{\rm a}(t) - d_i^{\rm a}(t) - \Delta_1]^+$
    end if
  end if
end for

▷ *Charge remote batteries from PVs via grid settlement*

for $i \in G$ **do**
  if $b_i(t) < \theta \mathsf{B}_i$ **then**
    if $\exists j \in G \backslash \{i\}$ such that $r_j(t) > r_j^{\rm a}(t) + r_j^{\rm b}(t) + s_j^{-}(t)$ **then**
      $\Delta_2 \leftarrow r_j(t) - r_j^{\rm a}(t) - r_j^{\rm b}(t) - s_j^{-}(t)$
      $g_i^{\rm b}(t) \leftarrow g_i^{\rm b}(t) + [\frac{1}{\eta_{\rm c}}\big(\theta \mathsf{B}_i - b_i(t)\big) - \Delta_2]^+$
      $s_j^{-}(t) \leftarrow s_j^{-}(t) + [\frac{1}{\eta_{\rm c}}\big(\theta \mathsf{B}_i - b_i(t)\big) - \Delta_2]^+$
      $s_i^{+}(t) \leftarrow s_i^{+}(t) + [\frac{1}{\eta_{\rm c}}\big(\theta \mathsf{B}_i - b_i(t)\big) - \Delta_2]^+$
      $b_i(t) \leftarrow b_i(t-1) + \eta_{\rm c}(r_i^{\rm b}(t) + g_i^{\rm b}(t)) - \eta_{\rm d} d_i^{\rm a}(t)$
    end if
  end if
end for
for $i \in G$ **do**
  if $r_i^{\rm a}(t) + d_i^{\rm a}(t) < a_i(t)$ **then**
    $g_i^{\rm a}(t) \leftarrow a_i(t) - r_i^{\rm a}(t) - r_i^{\rm g}(t)$
  end if
  $r_i^{\rm g}(t) \leftarrow r_i(t) - r_i^{\rm a}(t) - r_i^{\rm b}(t)$, $d_i^{\rm g}(t) \leftarrow 0, g_i^{\rm b}(t) \leftarrow 0$
  $g_i^{+}(t) \leftarrow g_i^{\rm a}(t) + g_i^{\rm b}(t) - s_i^{+}(t), g_i^{-}(t) \leftarrow d_i^{\rm g}(t) + r_i^{\rm g}(t) - s_i^{-}(t)$,
end for
return $\big(b_i(t), d_i^{\rm a}(t), d_i^{\rm g}(t), r_i^{\rm a}(t), r_i^{\rm b}(t), r_i^{\rm g}(t), g_i^{\rm a}(t), g_i^{\rm b}(t), s_i^{+}(t), s_i^{-}(t)\big)_{i \in G}$

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APPENDIX

A. Preliminaries

Lemma 4. *[13] Recall that $\mathcal{P}_{\rm self} \triangleq \big\{\{i\} : i \in \mathcal{N}\big\}$. A social optimum is denoted by $\mathcal{P}^* = \arg\max_{\mathcal{P}\in\mathscr{P}_K} u(\mathcal{P})$. Then we obtain*

$$K \cdot C(\mathcal{P}^*) \ge C(\mathcal{P}_{\rm self}) \tag{32}$$

A stable coalition structure is denoted by $\hat{\mathcal{P}} \in \mathscr{P}_K$. Consider a budget balanced cost-sharing mechanism $p_i(\cdot)$. Then we obtain

$$C(\mathcal{P}_{\rm self}) \ge C(\hat{\mathcal{P}}) \tag{33}$$

Hence, the cost-based strong price of anarchy $\mathsf{SPoA}_K^C \le K$.

Theorem 5. *Consider a budget balanced cost-sharing mechanism $p_i(\cdot)$. Denote the cost-based and utility-based strong prices of anarchy by SPoA_K^C and SPoA_K^u, respectively. Then*

$$\mathsf{SPoA}_K^u \ge \frac{K-1}{K-\mathsf{SPoA}_K^C} \tag{34}$$

$$\mathsf{SPoA}_K^C \le K - \frac{K-1}{\mathsf{SPoA}_K^u} \tag{35}$$

Proof. Since $p_i(\cdot)$ is budget balanced, we obtain $C(\mathcal{P}_K^{u*}) = C(\mathcal{P}_K^*)$. Consider a worst-case stable coalition structure $\hat{\mathcal{P}}$. Then

$$\mathsf{SPoA}_K^u \ge \frac{\sum_{G\in\mathcal{P}^*}\sum_{i\in G} u_i(p_i(G))}{\sum_{G\in\hat{\mathcal{P}}}\sum_{i\in G} u_i(p_i(G))} \tag{36}$$

$$= \frac{C(\mathcal{P}_{\rm self}) - C(\mathcal{P}^*)}{C(\mathcal{P}_{\rm self}) - C(\hat{\mathcal{P}})} = \frac{\frac{C(\mathcal{P}_{\rm self})}{C(\mathcal{P}^*)} - 1}{\frac{C(\mathcal{P}_{\rm self})}{C(\mathcal{P}^*)} - \frac{C(\hat{\mathcal{P}})}{C(\mathcal{P}^*)}} \tag{37}$$

$$= \frac{\frac{C(\mathcal{P}_{\rm self})}{C(\mathcal{P}^*)} - 1}{\frac{C(\mathcal{P}_{\rm self})}{C(\mathcal{P}^*)} - \mathsf{SPoA}_K^C} \tag{38}$$

Since $\frac{C(\mathcal{P}_{\rm self})}{C(\mathcal{P}^*)} \le K$ (by Lemma 4) and $\mathsf{SPoA}_K^C \ge 1$, the minimal is attained when $\frac{C(\mathcal{P}_{\rm self})}{C(\mathcal{P}^*)} = K$. □

Remarks: Theorem 5 allows us to lower (resp., an upper) bound the price of anarchy in terms of utility (resp., cost) from a lower (resp., an upper) bound on the price of anarchy in terms of cost (resp., utility). For instance, it is known from [32] that for $K = 2$, $\mathsf{SPoA}_2^{u,\rm eq} = 2$. From (34), we obtain that $\mathsf{SPoA}_2^{C,\rm eq} \le \frac{3}{2}$. Similarly we obtain that $\mathsf{SPoA}_K^{C,\rm ega} \le \frac{3}{2}$.

Given an order of users $\mathcal{N} = \{i_1, ..., i_n\}$ and cost-sharing mechanism $p_i(\cdot)$, we define

$$\alpha\big(p_i(\cdot)\big) \triangleq \max_{\substack{C(\cdot),\, G\in\mathscr{P}_K \\ \forall s=1,\dots,k:\ p_{i_s}\left(H_s(G)\right)\ge 0}} \frac{\sum_{s\in\{1,\dots,n\}: i_s\in G} p_{i_s}\Big(H_s(G)\Big)}{C(G)}, \tag{39}$$

where

$$H_s(G) \triangleq \begin{cases} G\backslash\{i_1, ..., i_{s-1}\}, & \text{if } i_s \in G, \\ \varnothing, & \text{if } i_s \notin G \end{cases} \tag{40}$$

In the following, we made a slight but critical modification of a Lemma in [13], which allows us to improve the bound SPoA_K^C for egalitarian-split cost-sharing.

Lemma 6. *Consider budget balanced cost-sharing mechanism $p_i(\cdot)$, which assumes a non-negative value on any coalition belonging to a stable coalition structure. Given a stable coalition structure $\hat{\mathcal{P}}$ and any coalition structure $\mathcal{P} \subseteq \mathscr{P}_K$, we have the upper bound:*

$$\frac{C(\hat{\mathcal{P}})}{C(\mathcal{P})} \le \alpha\big(p_i(\cdot)\big) \tag{41}$$

Proof. Let $\mathcal{P} = \{G_1, ..., G_h\}$. Define $H_1^1 \triangleq G_1$. Then there exists a user $i_1^1 \in H_1^1$ and a coalition $\hat{G}_1^1 \in \hat{\mathcal{P}}_K$, such that $i_1^1 \in \hat{G}_1^1$ and $p_{i_1^1}(H_1^1) \ge p_{i_1^1}(\hat{G}_1^1) \ge 0$; otherwise, all the users in H_1^1 would form a coalition H_1^1 to strictly reduce their payments, which contradicts the fact that $\hat{\mathcal{P}}_K$ is a stable coalition structure.

Next, define $H_2^1 \triangleq H_1^1\backslash\{i_1^1\}$. Note that H_2^1 is a feasible coalition, because arbitrary coalition structures with at most K users per coalition are allowed in our model. By the same argument, there exists $i_2^1 \in H_2^1$ and a coalition $\hat{G}_2^1 \in \hat{\mathcal{P}}_K$, such that $i_2^1 \in \hat{G}_2^1$ and $p_{i_2^1}(H_2^1) \ge p_{i_2^1}(\hat{G}_2^1) \ge 0$.

Let $G_t = \{i_1^t, ..., i_{K_t}^t\}$, for any $t \in \{1, ..., h\}$. Continuing this argument, we obtain a collection of sets $\{H_s^t\}$, where each $H_s^t \triangleq \{i_s^t, ..., i_{K_t}^t\}$ satisfies the following condition:

> for any $t \in \{1, ..., h\}$ and $s \in \{1, ..., K_t\}$, there exists $\hat{G}_s^t \in \hat{\mathcal{P}}_K$, such that $i_s^t \in \hat{G}_s^t$ and $p_{i_s^t}(H_s^t) \ge p_{i_s^t}(\hat{G}_s^t) \ge 0$.

Because of budget balance, we have

$$\sum_{t=1}^{h}\sum_{s=1}^{K_t} p_{i_s^t}(\hat{G}_s^t) = C(\hat{\mathcal{P}}_K) \tag{42}$$

Hence, the SPoA, SPoA_K^C, with respect to $\{p_i(\cdot)\}_{i\in\mathcal{N}}$ is upper bounded by

$$\frac{C(\hat{\mathcal{P}}_K)}{C(\mathcal{P})} = \frac{\sum_{t=1}^{h}\sum_{s=1}^{K_t} p_{i_s^t}(\hat{G}_s^t)}{\sum_{t=1}^{h} C(G_t)} \tag{43}$$

$$\leq \frac{\sum_{t=1}^{h}\sum_{s=1}^{K_t} p_{i_s^t}(H_s^t)}{\sum_{t=1}^{h} C(H_1^t)} \tag{44}$$

$$\leq \max_{t\in\{1,\dots,h\}} \frac{\sum_{s=1}^{K_t} p_{i_s^t}(H_s^t)}{C(H_1^t)} \leq \alpha\big(\{p_i(\cdot)\}_{i\in\mathcal{N}}\big) \tag{45}$$

because $\alpha(\cdot)$ is non-decreasing in K. □

B. Proportional-split Cost-Sharing

Theorem 7. *Consider proportional-split cost-sharing mechanism. When $K \leq 6$, we obtain*

$$\mathsf{SPoA}_K^{C,\mathrm{pp}} \leq \sum_{s=1}^{K}\frac{1}{s} \tag{46}$$

For general K, we obtain

$$\mathsf{SPoA}_K^{C,\mathrm{pp}} \leq \log K + 2 \tag{47}$$

Proof. Applying Lemma 6 with $p_i = p_i^{\mathrm{pp}}$, we obtain

$$\mathsf{SPoA}_K^{C,\mathrm{pp}} \leq \max_{C(\cdot),\ H_1,\dots,H_{K'}} \frac{1}{C(H_1)}\sum_{s=1}^{K'} p_{i_s}^{\mathrm{pp}}(H_s)$$

Assume with loss of generality $K' = K$. Note that $p_{i_s}^{\mathrm{pp}}(H_s) = \frac{C_s C(H_s)}{\sum_{t=s}^{K} C_t}$. Hence,

$$\mathsf{SPoA}_K^{C,\mathrm{pp}} \leq \max_{\{C_s, C(H_s)\}_{s=1}^{K}} \frac{1}{C(H_1)}\sum_{s=1}^{K}\frac{C_s C(H_s)}{\sum_{t=s}^{K} C_t} \tag{48}$$

subject to monotonicity: $C(H_1) \geq ... \geq C(H_K)$ and $C(H_s) \geq \max\{C_s,...,C_K\}$, and sub-additivity: $C(H_s) \leq \sum_{t=s}^{K} C_t$. Without loss of generality, we assume that $C(H_1) = 1$, and hence, $C(H_K) \leq ... \leq C(H_1) = 1$.

For $K \leq 6$, it follows from Lemma 9 that

$$\mathsf{SPoA}_K^{C,\mathrm{pp}} \leq \sum_{s=1}^{K}\frac{1}{s} \tag{49}$$

For general K, let $\hat{s}$ be the least index, such that $\sum_{t=\hat{s}+1}^{K} C_t \leq 1$ (and hence $\sum_{t=s}^{K} C_t > 1$ for any $s \leq \hat{s}$). with loss of generality $\hat{s} \geq 1$. If $s \geq \hat{s}$, we apply sub-additivity to obtain $\frac{C_s C(H_s)}{\sum_{t=s}^{K} C_t} \leq C_s$. If $s < \hat{s}$, we use $\frac{C_s C(H_s)}{\sum_{t=s}^{K} C_t} \leq \frac{C_s}{\sum_{t=s}^{K} C_t}$. Hence,

$$\mathsf{SPoA}_K^{C,\mathrm{pp}} \leq \max_{\{C_s\}_{s=1}^{K}} \sum_{s=\hat{s}}^{K} C_s + \sum_{s=1}^{\hat{s}-1}\frac{C_s}{\sum_{t=s}^{K} C_t} \leq 1 + \sum_{s=1}^{\hat{s}-1}\frac{C_s}{\sum_{t=s}^{K} C_t} \tag{50}$$

$$\leq 1 + \log K \tag{51}$$

which follows from Lemma 8 and $C_{\hat{s}} \leq 1$. □

When $C(H_1) = ... = C(H_K)$ and $C_1 = ... = C_K$, $\frac{\hat{\mathcal{P}}_K^{\mathrm{pp}}}{\mathcal{P}_K^{*\mathrm{ks}}} = \sum_{s=1}^{K}\frac{1}{s} = \Theta(\log K)$. Hence, the upper bound of $\mathsf{SPoA}_K^{C,\mathrm{pp}}$ is tight.

Lemma 8. *[13] If $0 \leq C_s \leq 1$ for all $s = 1,...,K$ and $\sum_{s=\hat{s}}^{K} C_s \geq 1$ for some $\hat{s}$, we have*

$$\sum_{s=1}^{\hat{s}-1}\frac{C_s}{\sum_{t=s}^{K} C_t} \leq \log K \tag{52}$$

Lemma 9. *Let $b_s \triangleq C(H_s)$. Consider the following maximization problem:*

$$z^*(K) \triangleq \max_{\{C_s,b_s\}_{s=1}^{K}} C_K + \sum_{s=1}^{K-1}\frac{b_s C_s}{\sum_{t=s}^{K} C_t} \tag{53}$$

$$\text{s.t.} \qquad b_s \leq \sum_{t=s}^{K} C_t, \quad \text{for all } s = 1,\dots,K-1, \tag{54}$$

$$0 \leq C_s \leq b_s \leq 1, \quad \text{for all } s = 1,\dots,K. \tag{55}$$

We have $z^*(K) = \sum_{s=1}^{K}\frac{1}{s}$, when $K \leq 6$. The maximum $z^*(K)$ is uniquely attained at $C_K = C_s = b_s = 1$, for all $s \leq 6$.

Proof. The proof is by induction on $K \leq 6$. $K = 1$ is trivial. Let

$$f(b_1,\dots,b_{K-1},C_1,\dots,C_K) \triangleq C_K + \sum_{s=1}^{K-1}\frac{b_s C_s}{\sum_{t=s}^{K} C_t}, \tag{56}$$

and denote by $(b_1^*,\dots,b_{K-1}^*,C_1^*,\dots,C_K^*)$ a vector that maximizes $f(b_1,\dots,b_{K-1},C_1,\dots,C_K)$ subject to constraints (54)-(55). Note that $z^*(K) \geq \sum_{s=1}^{K}\frac{1}{s}$ as $b_1 = \cdots = b_{K-1} = C_1 = \cdots = C_K = 1$ is a feasible solution.

The proof follows from the following claims.

Claim 10. *$C_K^* = 1$ is the unique maximizer of the function $g_K(C_K) \triangleq f(b_1^*,\dots,b_{K-1}^*,C_1^*,\dots,C_{K-1}^*,C_K)$.*

Proof. For this, we show that the function $g_K(C_K)$ is convex in $C_K \in [0,1]$. This implies that the maximum of

$$f(b_1^*,\dots,b_{K-1}^*,1,C_1^*,\dots,C_{K-1}^*,C_K),$$

with respect to C_K, subject to the constraints (54)-(55) (where $b_1 = b_1^*$, ..., $b_{K-1} = b_{K-1}^*$, $b_K^* = 1$, and $C_1 = C_1^*$, ..., $C_{K-1} = C_{K-1}^*$, $C_K^* = C_K$) is attained at one of the end points of the interval $[\delta_K, 1]$, where

$$\delta_K \triangleq \max_{s=1,\dots,K-1}\left\{b_s^* - \sum_{t=s}^{K-1} C_t^*\right\} \tag{57}$$

Note that $\delta_K \geq 0$ by (55). It would be then enough to show that $g_K(\delta_K) < \sum_{s=1}^{K}\frac{1}{s}$, since that would imply that the only value that achieves $z^*(K) \geq \sum_{s=1}^{K}\frac{1}{s}$ is $C_K^* = 1$.

To show that $g_K(C_K)$ is convex, we take the first and second derivatives:

$$g_K'(C_K) = \frac{\partial f}{\partial C_K} = 1 - \sum_{s=1}^{K-1}\frac{b_s^* C_s^*}{\left(\sum_{t=s}^{K-1} C_t^* + C_K\right)^2}, \tag{58}$$

$$g_K''(C_K) = \frac{\partial^2 f}{\partial^2 C_K} = 2\sum_{s=1}^{K-1}\frac{b_s^* C_s^*}{\left(\sum_{t=s}^{K-1} c_t^* + C_K\right)^3}. \tag{59}$$

Since $g_K''(C_K) \geq 0$ for all $C_K \geq 0$, $g_K(C_K)$ is convex for $C_K \in [\delta_K, 1]$.

Now we show that $g_K(\delta_K) < \sum_{s=1}^{K} \frac{1}{s}$. Indeed, if $p \in \{1,\ldots,K-1\}$ is such that $\delta_K = b_p^* - \sum_{t=p}^{K-1} C_t^*$, then from $b_p^* - \sum_{t=p}^{K-1} C_t^* \geq b_s^* - \sum_{t=s}^{K-1} C_t^*$ for all $s = 1,\ldots,K-1$, we get, when setting $C_K = \delta_K$, that $b_s^* \leq C_K + \sum_{t=s}^{K-1} c_t^*$, for all $1 \leq s \leq K-1$. Thus,

$$
\begin{aligned}
g_K(\delta_K) =& C_K + \sum_{s=p}^{K-1} \frac{b_s^* C_s^*}{C_K + \sum_{t=s}^{K-1} c_t^*} + \sum_{s=1}^{p-1} \frac{b_s^* C_s^*}{C_K + \sum_{t=s}^{K-1} c_t^*} \\
\leq& (C_K + \sum_{s=p}^{K-1} C_s^*) + \sum_{s=1}^{p-1} \frac{b_s^* C_s^*}{(C_K + \sum_{t=p}^{K-1} C_t^*) + \sum_{t=s}^{p-1} c_t^*} \\
=& \bar{c} + \sum_{s=1}^{p-1} \frac{b_s^* C_s^*}{\bar{c} + \sum_{t=s}^{p-1} c_t^*}, \qquad (60)
\end{aligned}
$$

where $\bar{c} \triangleq C_K + \sum_{t=p}^{K-1} C_t^* = b_p^* \leq 1$. It follows that the vector $(b_1^*,\ldots,b_{p-1}^*, C_1^*,\ldots,C_{p-1}^*, \bar{c})$ is feasible for (54)-(55) with K replaced by $p \leq K-1$. Thus by induction, and (60), $g_K(\delta_K) < \sum_{s=1}^{K} \frac{1}{s}$. □

Claim 11. *$b_s^* = 1$, for all $s = 1,\ldots,K-1$.*

Proof. When we set $C_K = C_K^* = 1$, the constraints (55) become redundant, and the only constraint on b_s comes from (54). Since the function f is strictly monotonically increasing in b_s, it is uniquely maximized when $b_s = 1$. □

With the above two claims, the only constraint that remain to be satisfied is $C_s \leq 1$, for $s = 1,2,\ldots,K-1$.

Claim 12. *$C_1^* = 1$.*

Proof. For this, we note that the function $g_1(C_1) \triangleq f(1,\ldots,1,1,C_1,C_2^*,\ldots,C_{K-1}^*,1)$ is (strictly) monotone increasing in $C_1 \in [0,1]$. This implies that setting $C_1 = 1$ would (uniquely) maximize $g_1(C_1)$. □

We show next by induction on $p = 1,2,\ldots,K-1$ that $C_p^* = 1$ and no other value of C_p would be optimal for (53). Assume this is true for $s = 1,2\ldots,p-1$ (we call this the second induction hypothesis with the base case shown in step 2). Then we need to show the following claim for $2 \leq p \leq K-1 = 5$.

Claim 13. *$C_p^* = 1$.*

Proof. Let $g(C_p) \triangleq f(1,\ldots,1,C_p,C_{p+1}^*,\ldots,C_{K-1}^*,1)$. Then by our assumptions

$$
\begin{aligned}
g_p(C_p) =& 1 + \sum_{s=p+1}^{K-1} \frac{C_s^*}{1 + \sum_{t=s}^{K-1} c_t^*} + \frac{C_p}{1 + x_p + C_p} \\
&+ \sum_{s=1}^{p-1} \frac{1}{1 + p - s + x_p + C_p}, \qquad (61)
\end{aligned}
$$

where $x_p \triangleq \sum_{t=p+1}^{K-1} C_t^*$.

We first note by the (first) induction hypothesis that $g_p(0) < \sum_{s=1}^{K} \frac{1}{s}$. Thus it remains to show that the maximum of $g_p(C_p)$ is achieved uniquely at one of the end point $C_p \in [0,1]$. For this it is enough to show that $g_p(C_p)$ does not have an extreme point in the interval $[0,1]$. we write the the derivative of $g(C_p)$ with respect to C_p as $g_p'(C_p) = h(C_p,x_p) - t(C_p,x_p)$, where

$$
h(C_p,x_p) \triangleq \frac{1+x_p}{(1+x_p+C_p)^2}, \quad t(C_p,x_p) \triangleq \sum_{s=1}^{p-1} \frac{1}{(1+p-s+x_p+C_p)^2} \qquad (62)
$$

□

Then it is enough to show that, for all $C_p \in [0,1]$ and $x_p \geq 0$, $h(C_p,x_p) > t(C_p,x_p)$, that is,

$$
1 + x_p > \sum_{s=1}^{p-1} \left(\frac{1+x_p+C_p}{1+p-s+x_p+C_p} \right)^2 \qquad (63)
$$

Since the R.H.S. of (63) is monotone increasing in $C_p \in [0,1]$, it would be enough to show that

$$
1 + x_p > w(x_p) \triangleq \sum_{s=1}^{p-1} \left(\frac{2+x_p}{2+p-s+x_p} \right)^2, \qquad (64)
$$

for all $x_p \geq 0$ and $p = 5$. We note that $w(x_p)$ is strictly monotone increasing and concave as

$$
w'(x_p) = \sum_{s=1}^{p-1} \frac{2(p-s)(2+x_p)}{(2+p-s+x_p)^3} > 0, \qquad (65)
$$

$$
w''(x_p) = \sum_{s=1}^{p-1} \frac{2(p-s)(p-s-2x_p-4)}{(2+p-s+x_p)^4} \leq 0 \qquad (66)
$$

Thus, we would be done if we show that $w(0) < 1$ and $w'(0) < 1$ (as the tangent of the function $w(x_p)$ at $x_p = 0$ will be below and parallel to the line $1 + x_p$). But this is a routine calculation:

$$
w(0) = \left(\frac{2}{3}\right)^2 + \left(\frac{2}{4}\right)^2 + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{6}\right)^2 = 0.9655... \qquad (67)
$$

$$
w'(0) = 4\left(\frac{4}{6^3} + \frac{3}{5^3} + \frac{2}{4^3} + \frac{1}{3^3}\right) = 0.44322... \qquad (68)
$$

This completes the proof. □

C. Egalitarian-split Cost-Sharing

Given cost function $C(\cdot)$, we define a truncated cost function $\tilde{C}(\cdot)$ as follows:

$$
\tilde{C}(G) \triangleq \begin{cases} C(G), & \text{if } C(G) \leq \sum_{j \in G} C_j \\ \sum_{j \in G} C_j, & \text{if } C(G) > \sum_{j \in G} C_j \end{cases} \qquad (69)
$$

Note that $\tilde{C}(G) \leq \sum_{j \in G} C_j$ for any G.

As mentioned earlier, egalitarian-split cost-sharing is equivalent to Nash bargaining solution. Thus,

$$
\mathsf{SPoA}_K^{C,\mathrm{ega}} = \mathsf{SPoA}_K^{C,\mathrm{ns}}
$$

Lemma 14. *[13] For egalitarian and Nash bargaining solutions, we have*

$$
\mathsf{SPoA}_K^{C,\mathrm{ega}}(C(\cdot)) = \mathsf{SPoA}_K^{C,\mathrm{ega}}(\tilde{C}(\cdot)) \qquad (70)
$$

Lemma 15. *5 [13] For Nash bargaining solution, given a stable coalition structure $\hat{\mathcal{P}} \in \mathscr{P}_K$ and $G \in \hat{\mathcal{P}}$, then every user has non-negative payment: $p_i^{\mathrm{ns}}(G) \geq 0$ for all $i \in G$.*

Lemma 16. *[13] Let $b_s \triangleq C(H_s)$. Consider the following maximization problem:*

$$(\mathrm{M1})\ y^*(K) \triangleq \max_{\{C_s,b_s\}_{s=1}^K} \sum_{s=1}^{K} \Big(C_s - \frac{(\sum_{t=s}^K C_t) - b_s}{K-s+1} \Big) \tag{71}$$

subject to

$$b_s \le \sum_{t=s}^{K} C_t, \text{ for all } s = 1, ..., K-1, \tag{72}$$

$$0 \le C_s \le b_s \le b_{s+1} \le 1, \text{ for all } s = 1, ..., K, \tag{73}$$

$$b_1 + KC_s - \sum_{t=1}^{K} C_t \ge 0, \text{ for all } s = 1, ..., K \tag{74}$$

The maximum of $(\mathrm{M1})$ is upper bounded by $y^*(K) \le 1 + \mathcal{H}_{K-1} = O(\log K)$.

Theorem 17. *For egalitarian-split cost-sharing mechanism, the SPoA is upper bounded by*

$$\mathsf{SPoA}_K^{C,\mathrm{ega}} = \mathsf{SPoA}_K^{C,\mathrm{ns}} = O(\log K) \tag{75}$$

Proof. First, by Lemma 14, it suffices to consider egalitarian bargaining (or Nash bargaining) solution with cost function satisfying $C(G) \le \sum_{j\in G} C_j$ for any G.

Next, by Lemma 15, for any stable coalition structure $\hat{\mathcal{P}}$, we have $p_i^{\mathrm{ns}}(G) \ge 0$ for all $i \in G \in \hat{\mathcal{P}}$. Hence, by Lemma 6, $\mathsf{SPoA}_K^{C,\mathrm{ega}} \le \alpha\big(\{p_i^{\mathrm{ega}}(\cdot)\}\big)$.

Let $H_s = \{i_s, ..., i_K\}$, with the default costs denoted by $\{C_s, ..., C_K\}$. Recall that egalitarian bargaining solution is given by

$$p_{i_s}^{\mathrm{ega}}(H_s) = C_s - \frac{(\sum_{t=s}^K C_t) - C(H_s)}{K-s+1}$$

subject to $C(H_1) \ge ... \ge C(H_K)$ and $C(H_s) \ge \max\{C_s, ..., C_K\}$ (by monotonicity), and $C(H_s) \le \sum_{t=s}^K C_t$ (by Lemma 14). Finally, it follows that $\mathsf{SPoA}_K^{\mathrm{eqa}} \le O(\log K)$ by Lemma 16. $\square$