The Program

```
#include <iostream>
#include <cmath>   ← to use fabs

/* This program will approximate an integral by using more and more
points in the rectangle rule. It requires a function program for the
integrand, the interval (a,b) of integration, an initial number of points
n0 for the approximation, and a convergence parameter epsilon */

/** function to be integrated
      @param x  the value at which to evaluate f
      @return the value of f(x)
*/
double f(double x)
{
  return x / (1. + x*x);
}
```

$f(x) = \frac{x}{1+x^2}$

```
/** Computes the rectangle rule approximation to the integral
      of a function
      @param a  the left endpoint of the integral
      @param b  the right endpoint of the integral
      @param n  the number of rectangles used
      @return  the approximation of the integral
*/
double rectangle(double a, double b, int n)
{
  double h = (b – a)/(n + 1);
  double x = a;
  double value = f(x);
  for (int i = 0; i <= n; n++)
  {
    x = x + h;
    value = value + f(x);
  }
  return value * h;
}
```

```
int main()
{
  int n0;
  double a, b, epsilon;
  // prompt for input
  cout << "Please enter values for a, b, n0, and epsilon: ";
  cin >> a >> b >> n0 >> epsilon;
  // echo the input
  cout << "a = " << a << " b = " << b << endl;
  cout << "n0 = " << n0 << " epsilon = " << epsilon << endl;
  // compute the first two approximations to the integral
  double RC;  // will hold the current approximation
  RC = rectangle(a, b, n0);
```

```
  // double the number of rectangles
  int n = 2. * n0;
  double RN;   // will hold the next approximation of the integral
  RN = rectangle(a, b, n);
  // repeat integral approximations until converge
  while (fabs(RN – RC) > epsilon)
  {
    n = 2. * n;  // double n
    RC = RN;  // save current approximation
    RN = rectangle(a, b, n);  // compute next approximation
    // display result
    cout << "Integral = " << RN << " for n = " << n << endl;
  }
  return 0;
}
```

Test Run

Use $a = 0, b = 1, n_o = 2$, and $\epsilon = 0.001$

The exact solution to the integral is

$$\int_0^1 \frac{x}{1+x^2}\,dx = \int_1^2 \frac{\cancel{x}}{u}\frac{du}{2\cancel{x}} = \frac{1}{2}\int_1^2 \frac{1}{u}du$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

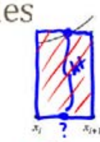
$$= \frac{1}{2}\ln|u| \Big|_1^2$$

$$= \frac{1}{2}\ln 2 - \frac{1}{2}\cancel{\ln 1}^{\,0}$$

$$= \frac{1}{2}\ln 2 \approx .3465736$$

Other Rules

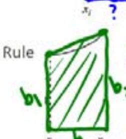
- Midpoint Rule



$ht = f\left(\frac{x_i + x_{i+1}}{2}\right) = f\left(\frac{2x+h}{2}\right) = f(x+h/2)$ with $x_i = x$, $x_{i+1} = x+h$

width $= h$

- Trapezoid Rule

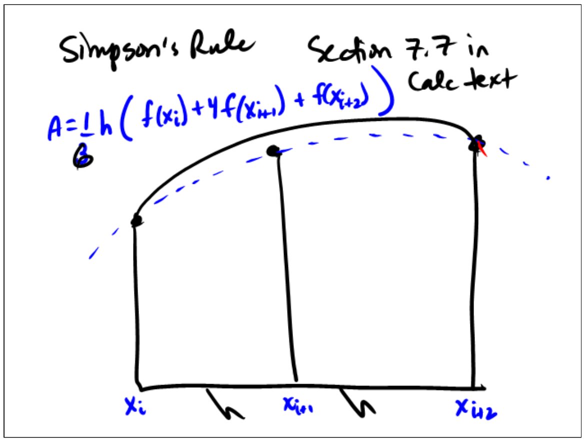


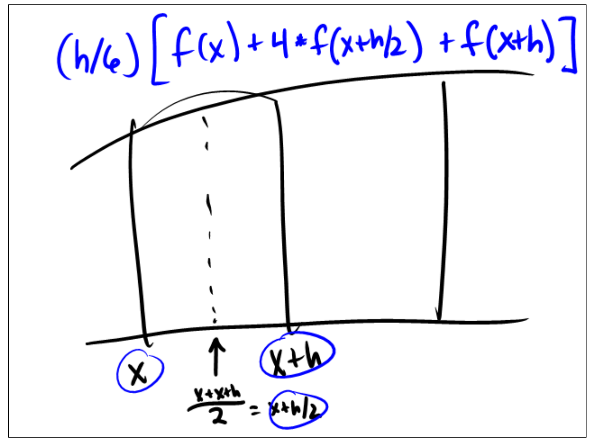
$A_{trap.} = \frac{1}{2}h(b_1 + b_2)$

$f(x_i) + f(x_{i+1})$

$f(x) \quad f(x+h)$

- Simpson's Rule


Simpson's Rule
Section 7.7 in Calc text
A = 1/6 h (f(x_i) + 4f(x_{i+1}) + f(x_{i+2}))
x_i
h
x_{i+1}
h
x_{i+2}


(h/6) [f(x) + 4*f(x+h/2) + f(x+h)]
x
x+h
(x+x+h)/2 = x+h/2