In The Name Of Almighty GOD

Faculty Of Advanced Sciences & Tehcnologies

Fuzzy Decision Making Systems Course Final Project

۱۳۹۷ Spring Semester

Intuitionistic Fuzzy Numbers Calculator

By Mohammad Mirzanejad

Student ID: ۸۳۰۵۹۶۰۲۷

Under supervision of

Dr.Morteza Ebrahimi

Introduction:

Intuitionistic fuzzy set (IFS) was introduced as an extension of fuzzy set, which earlier proposed by Zadeh. Many researchers have confirmed the resourcefulness of IFSs in decision making problems due to its significance in tackling vagueness or the representation of imperfect knowledge.

Definition ١:Let X be a nonempty set. A fuzzy set A drawn from is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$

Where $\mu_A(x): X \longrightarrow [0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of memberships.

Definition ٢ : Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions

$$\mu_A(x), \nu_A(x): X \longrightarrow [0,1]$$

define respectively degree of membership and degree of non-membership of the element x £ Xto the set A, which is a subset of X, and for every element :

$$x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

Note that every fuzzy set is an intuitionistic fuzzy set, but the reverse is not true.

In this project I implemented a calculator which does some operations on two intuitionistic fuzzy numbers and plots the result. The program has been written in C# and Visual Studio ٢٠١٣.

Some basic operations on intuitionistic fuzzy sets :

For every two IFS's A and B the following operations and relations are valid:

[Inclusion] $A \subseteq B \leftrightarrow \mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)\ \forall x \in X$

[Complement] $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$

[Union]
$A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min\ (\nu_A(x), \nu_B(x))\rangle : x \in X\}$

[Intersection]
$A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max\ (\nu_A(x), \nu_B(x))\rangle : x \in X\}$

[Addition]
$A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)\rangle : x \in X\}$ [Multiplication]
$A \otimes B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle : x \in X\}$

Subtraction operation on IFSs :

Let us consider an equation of the type: C+B=A where the IFSs *A* and *B* are given, and the problem is to find the unknown IFS *C* which satisfies :

$$0 \leq \mu_C(x) + \nu_C(x) \leq 1$$

We know that

$$\mu_C(x) + \mu_B(x) - \mu_C(x) \cdot \mu_B(x) = \mu_A(x),$$
$$\nu_C(x) \cdot \nu_B(x) = \nu_A(x).$$

Then,

$$\mu_C(x) = \frac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)},$$

$$\nu_C(x) = \frac{\nu_A(x)}{\nu_B(x)}.$$

The membership degree of *C* must take values in the interval [٠,١], i.e.

$$0 \le \frac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)} \le 1$$

The right-hand side of above equation is valid because $\mu_A(x) \le 1$ but the left-hand side of that is not correct in the cases that $\mu_A(x) < \mu_B(x)$ or $\mu_B(x) = 1$

To fulfill the inequality, the below conditions are required:

$\mu_A(x) \ge \mu_B(x)$ and $\mu_B(x) \ne 1$

It is obvious that the left-hand side of equation below is correct

$$0 \le \frac{\nu_A(x)}{\nu_B(x)} \le 1$$

Because

$\nu_A(x), \nu_B(x) \ge 0$ but $\nu_B(x) \ne 0$

The satisfied condition of the right-hand side is tl $\nu_A(x) \le \nu_B(x)$, we know that A>=B.

Moreover, *C* is an IFS and thus

$$\mu_C(x) + \nu_C(x) = \frac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)} + \frac{\nu_A(x)}{\nu_B(x)} \le 1$$

Then,

$$\mu_A(x) \cdot \nu_B(x) - \mu_B(x) \cdot \nu_A(x) \le \nu_B(x) - \nu_A(x)$$

From the discussion of the above possible cases, we can now summarize the conditions for obtaining a solution for subtraction:

$$A \geq B\ ,\quad \mu_A(x)\cdot\nu_B(x)-\mu_B(x)\cdot\nu_A(x)\leq \nu_B(x)-\nu_A(x),\quad \mu_B(x)\neq 1 \text{ and } \nu_B(x)\neq 0$$

Division Operation in IFSs:

Consider an equation of the following type:

D. B = A

If the IFSs *A* and *B* are given, then we can find the unknown IFS *D* which satisfies:

$$0\leq \mu_D(x)+\nu_D(x)\leq 1$$

We know that:

$$\mu_D(x)\cdot\mu_B(x)=\mu_A(x),$$
$$\nu_D(x)+\nu_B(x)-\nu_D(x)\cdot\nu_B(x)=\nu_A(x)$$

Then

$$\mu_D(x)=\frac{\mu_A(x)}{\mu_B(x)},$$
$$\nu_D(x)=\frac{\nu_A(x)-\nu_B(x)}{1-\nu_B(x)}$$

A solution for IFS *D* exists only if

$$0\leq \mu_A(x)/\mu_B(x)\leq 1\ ,\quad 0\leq (\nu_A(x)-\nu_B(x))/(1-\nu_B(x))\leq 1\ ,$$

And

$$\mu_A(x)/\mu_B(x)+(\nu_A(x)-\nu_B(x))/(1-\nu_B(x))\leq 1$$

From the first inequality, we obtain the conditions that

$$\mu_A(x)\leq\mu_B(x) \text{ and } \mu_B(x)\neq 0$$

Therefore, we summarize the conditions for obtaining an IFS solution:

$$A\leq B\ ,\quad \mu_A(x)\cdot\nu_B(x)-\ \mu_A(x)\cdot\nu_B(x)-\mu_B(x)\cdot\nu_A(x)\geq \mu_A(x)-\mu_B(x)$$

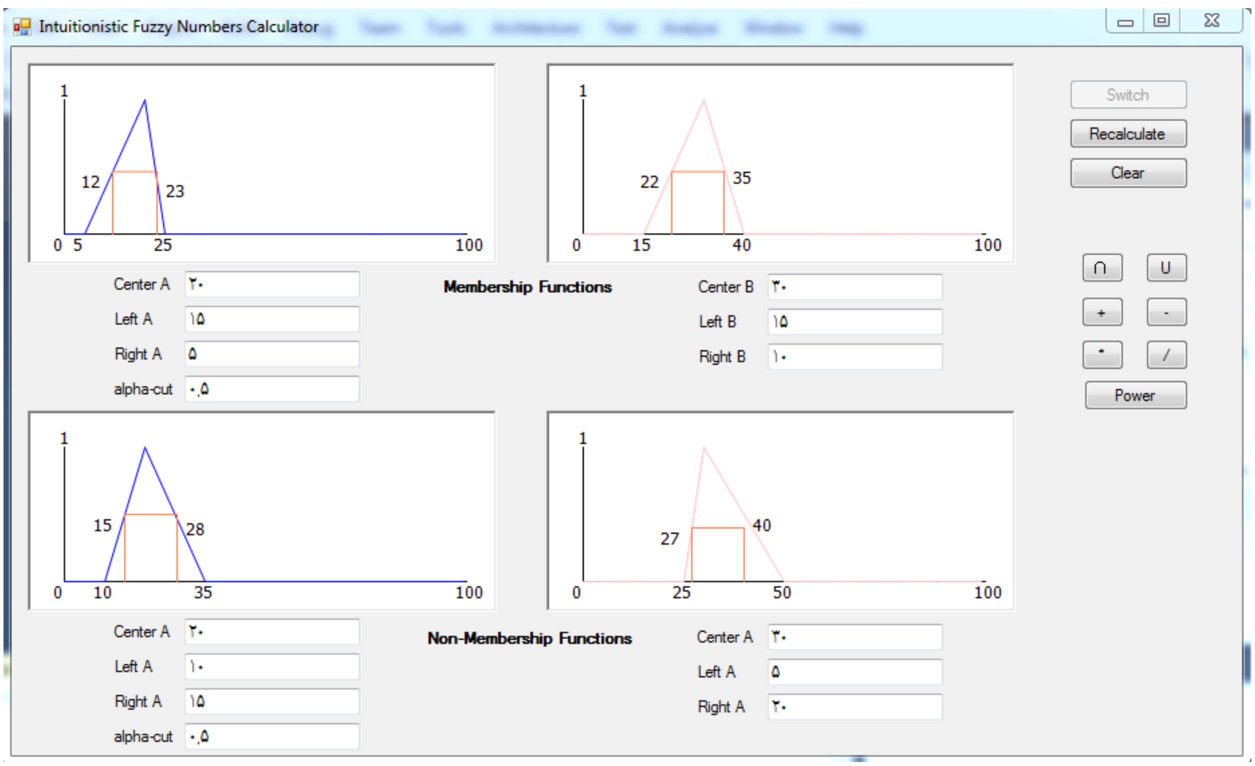
$$\mu_B(x) \neq 0 \quad \text{and} \quad \nu_B(x) \neq 1$$

Implementation some operations on IFSs by C#:

This project implements a calculator which has ٧ operator including Intersection, Union, Addition, Multiplication, Division, Subtraction and Power. Formula for each operator is discussed above.

Program interface contains two fuzzy numbers which is specified by user and ٢ plot for each number (A or B) which upper is membership function of an intuitionistic fuzzy number and lower is non-membership function of that.

The program also calculates Alpha-Cut for each IFSs and shows it on the diagram.



The program is written with ٦ classes for better modularity.

- Solution 'Fuzzy' (1 project)
  - Fuzzy
    - Properties
    - References
    - FormFuzzy.cs
    - Function.cs
    - FuzzyNumber.cs
    - Operations.cs
    - Plot.cs
    - Program.cs
    - Render.cs

FormFuzzy.cs is main form of the program which contains UI and gets users input and plot the results of our calculator. This class performs graphical motions and calls some functions of other classes which will be explained, after each button of calculator is pressed.

Function.cs

```
public static void Calc(ref FuzzyNumber N)
{
    float x۱ = N.C - N.A, x۲ = N.C, x۳ = N.C + N.B, y۱ = ۰, y۲ = ۱, y۳ = ۰;
    float a۱ = ۱ / (x۲ - x۱) * (y۲ - y۱);
    float b۱ = y۲ - x۲ * a۱;
    float a۲ = ۱ / (x۳ - x۲) * (y۳ - y۲);
    float b۲ = y۳ - x۳ * a۲;

    for (int i = ۰; i < N.vect.Length; ++i)
    {
        if (i >= N.C - N.A && i < N.C + N.B)
            if (i < N.C)
            {
                // x۱ = c - a; x۲ = c; y۱ = pb.Height - offset; y۲ = offset;
                N.vect[i] = a۱ * i + b۱;
            }
            else
            {
                // x۱ = c; x۲ = c + b; y۱ = offset; y۲ = pb.Height - offset;
                N.vect[i] = a۲ * i + b۲;
            }
```

This class computes fuzzy number membership function with user’s input.

FuzzyNumbers.cs is responsible for holding intervals and center of a fuzzy number.

Operations.cs is core class of the program which is composed of functions implementing mentioned operators (+,*,-,/,U & ∩) on intuitionistic fuzzy numbers according to defined formulas.

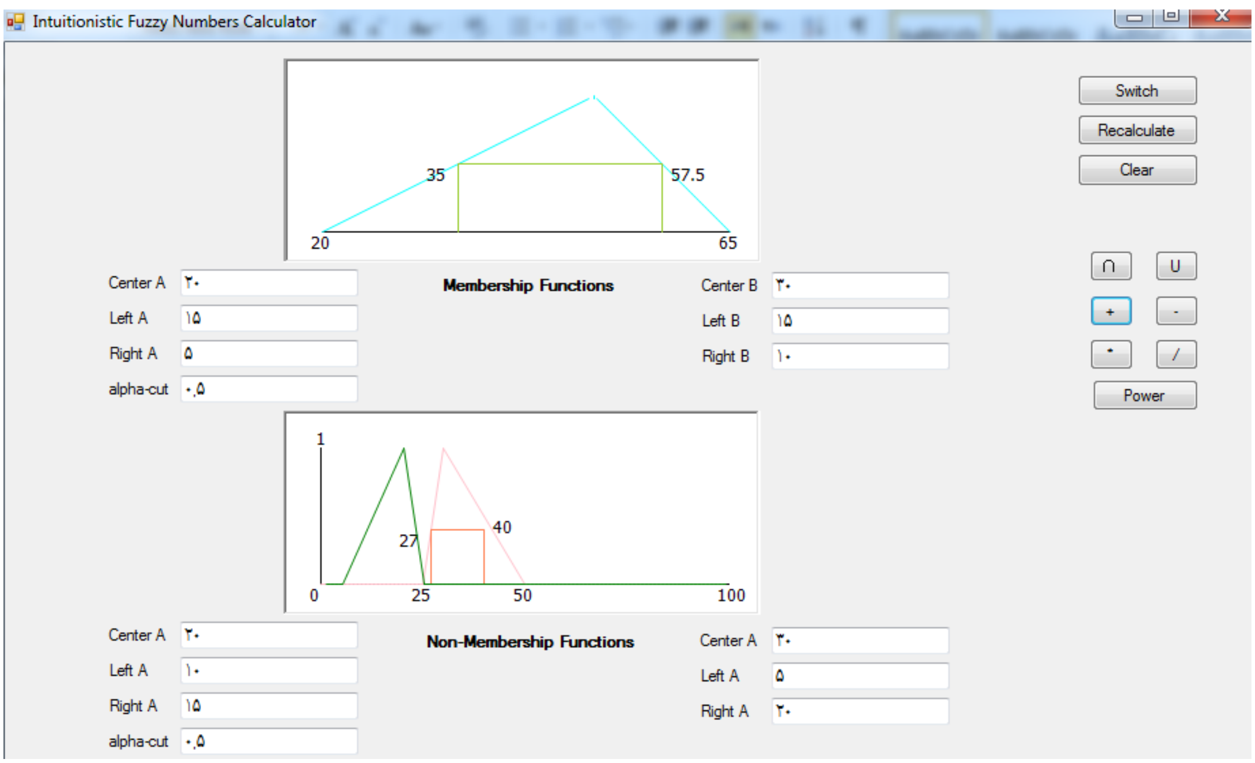
Plot.cs handles drawing vertical and horizontal lines of membership functions and related graphics.

Program.cs launches the main form of the program.

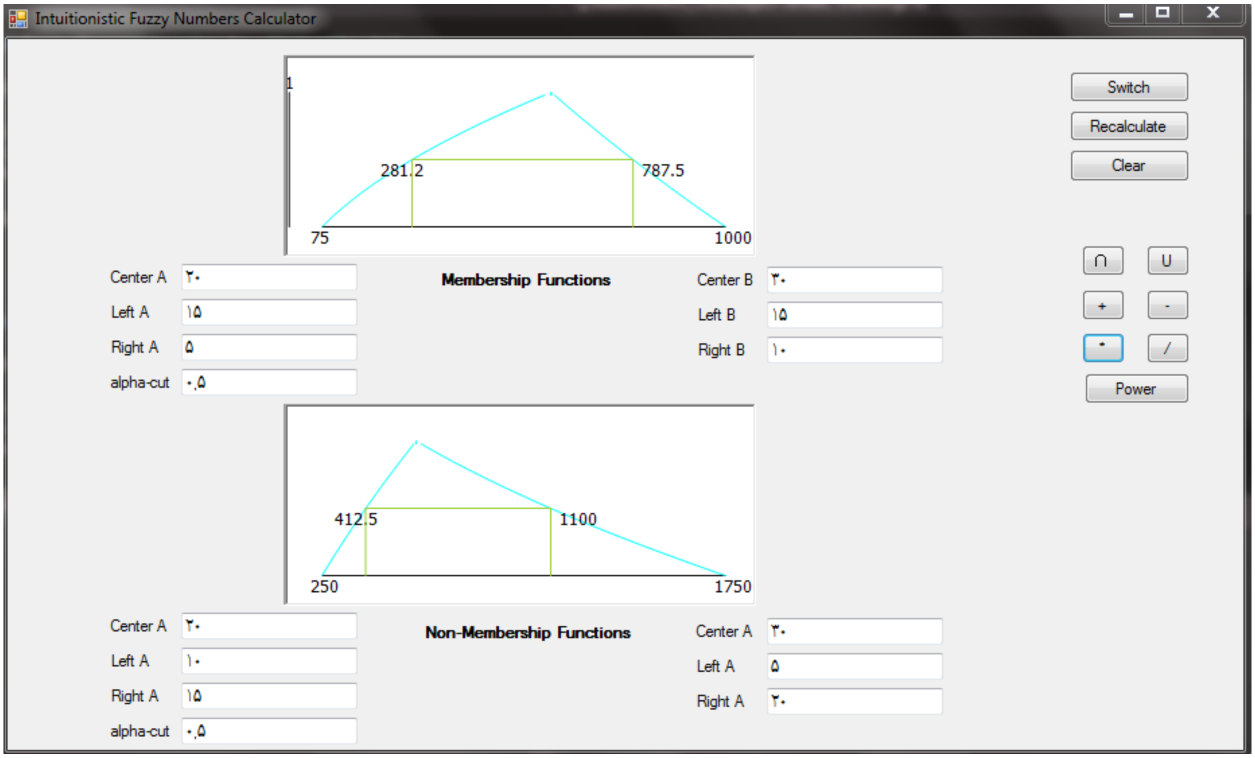
Render.cs performs some operations for calculating width and height of diagrams and determining their offsets and etc.

Examples :

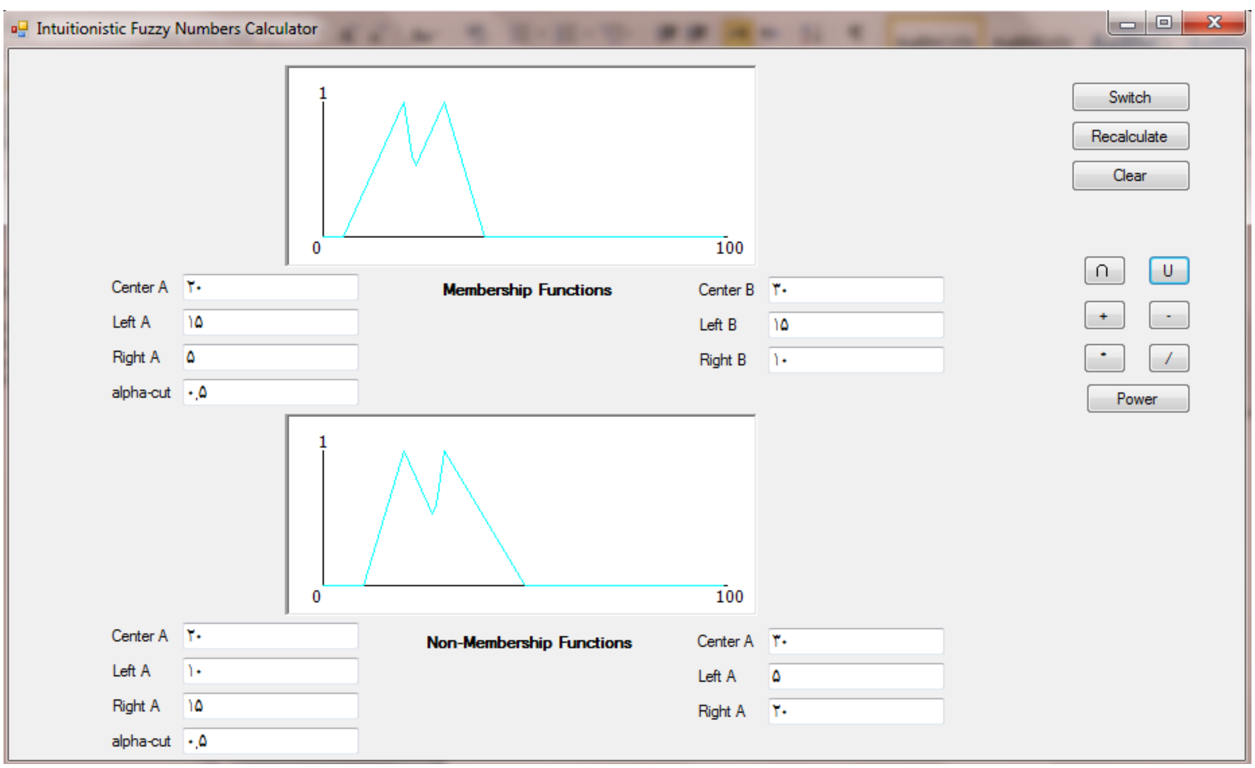
Sum :



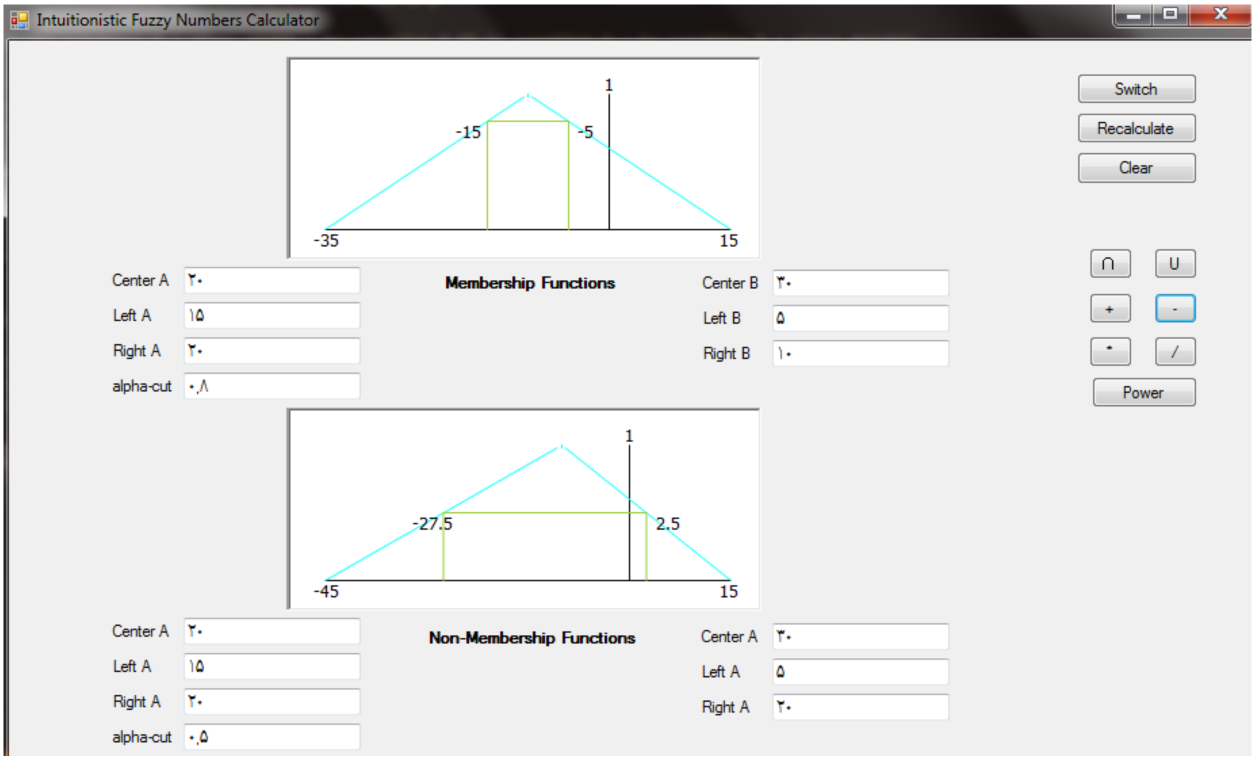
Multiply :


Intuitionistic Fuzzy Numbers Calculator
1
281.2
787.5
75
1000
412.5
1100
250
1750
Switch
Recalculate
Clear
Power
Center A ٢٠
Left A ١٥
Right A ٥
alpha-cut ٠,٥
Membership Functions
Center B ٣٠
Left B ١٥
Right B ١٠
Center A ٢٠
Left A ١٠
Right A ١٥
alpha-cut ٠,٥
Non-Membership Functions
Center A ٣٠
Left A ٥
Right A ٢٠

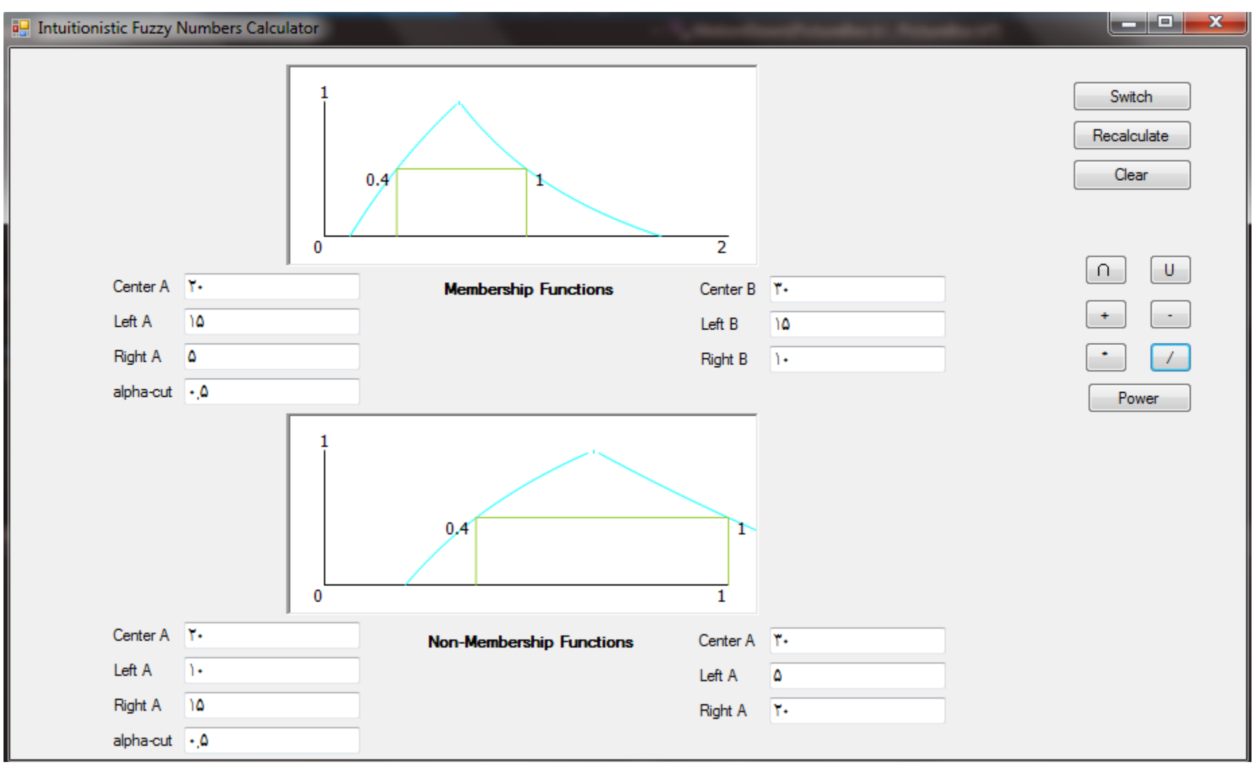
Union :


Intuitionistic Fuzzy Numbers Calculator
1
0
100
1
0
100
Switch
Recalculate
Clear
Power
Center A ٢٠
Left A ١٥
Right A ٥
alpha-cut ٠,٥
Membership Functions
Center B ٣٠
Left B ١٥
Right B ١٠
Center A ٢٠
Left A ١٠
Right A ١٥
alpha-cut ٠,٥
Non-Membership Functions
Center A ٣٠
Left A ٥
Right A ٢٠

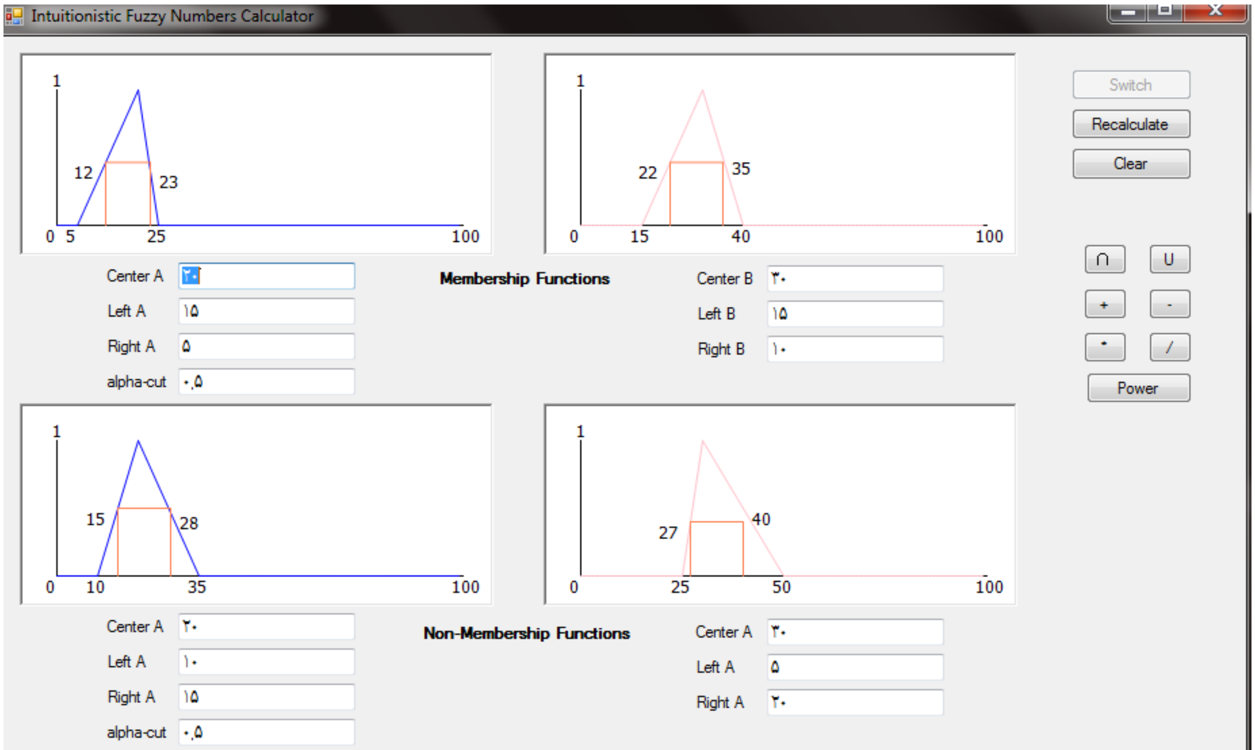
Subtraction :



Division :



Intersection :



Regarding to values of memebership and nonmemberships functions , intersection is zore or empty.

References :

[١] K.T. Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia, ١٩٨٣.

[٢] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems ٢٠ (١) (١٩٨٦) ٨٧-٩٦.

[٣] K.T. Atanassov, Review and new results on intuitionistic fuzzy sets, Preprint IM- MFAIS-١-٨٨, Sofia, ١٩٨٨.

[٤] K.T. Atanassov, More on intuitionistic fuzzy sets, Fuzzy Sets and Systems ٣٣ (١) (١٩٨٩) ٣٧-٤٦.

[٥] K.T. Atanassov, Temporal intuitionistic fuzzy sets, C.R. Acad. Bulgare. Sci. ٤٤ (٧) (١٩٩١) ٥-٧.

[٦] K.T. Atanassov, Remark on the intuitionistic fuzzy sets, Fuzzy Sets and Systems ٥١ (١) (١٩٩٢) ١١٧-١١٨.

[٧] K.T. Atanassov, New operations defined over Intuitionistic fuzzy sets, Fuzzy Sets and Systems ٦١ (٢) (١٩٩٤) ١٣٧-١٤٢.

[٨] K.T. Atanassov, Intuitionistic fuzzy sets past, present, and future, CLBME-Bulgarian Academy of Science, Sofia, ٢٠٠٣.