2023年度一般力学II
重心レポート

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目次

1 概要

今回は，扇形と長方形と三角形が結合した図形の重心を求める．

2 重心

2.1 長方形の重心

図1のように 4×6 の長方形の重心を求める

$$\begin{aligned} C\colon 0 \leqq x \leqq 2, 0 \leqq y \leqq 4 \\ dV = dxdy \\ V_c = 4 \times 6 = 24 \end{aligned}$$

とする．

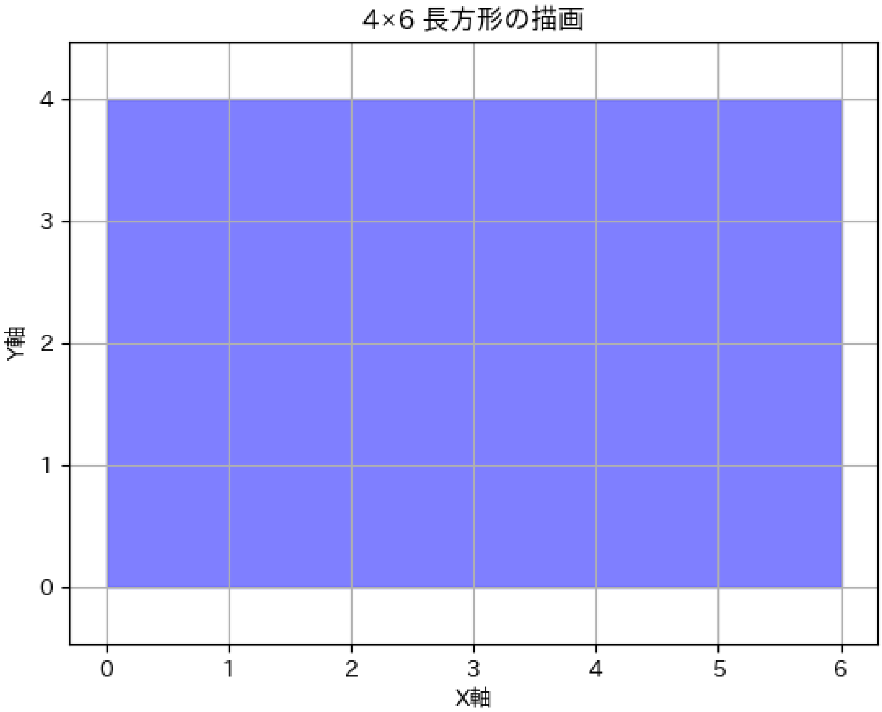

図 1　4 × 6 の長方形

$$
\begin{aligned}
x_g &= \frac{1}{V_c}\int_C x dV \\
&= \frac{1}{V_c}\int_0^6 \int_0^4 x dy dx \\
&= \frac{1}{V_c}\int_0^6 x(4) dx \\
&= \frac{1}{V_c} 4 \frac{1}{2} x^2 \Big|_0^6 \\
&= \frac{1}{V_c} 4 \frac{1}{2} \times 6^2 \\
&= \frac{1}{V_c} 72 \\
&= 3
\end{aligned}
$$

$$
\begin{aligned}
y_g &= \frac{1}{V_c}\int_C y dV \\
&= \frac{1}{V_c}\int_0^4\int_0^6 y dy dx \\
&= \frac{1}{V_c}\int_0^4 y(6) dy \\
&= \frac{1}{V_c} 6 \frac{1}{2} y^2 \Big|_0^4 \\
&= \frac{1}{V_c} 6 \frac{1}{2} \times 4^2 \\
&= \frac{1}{V_c} 48 \\
&= 2
\end{aligned}
$$

2.2　三角形の重心

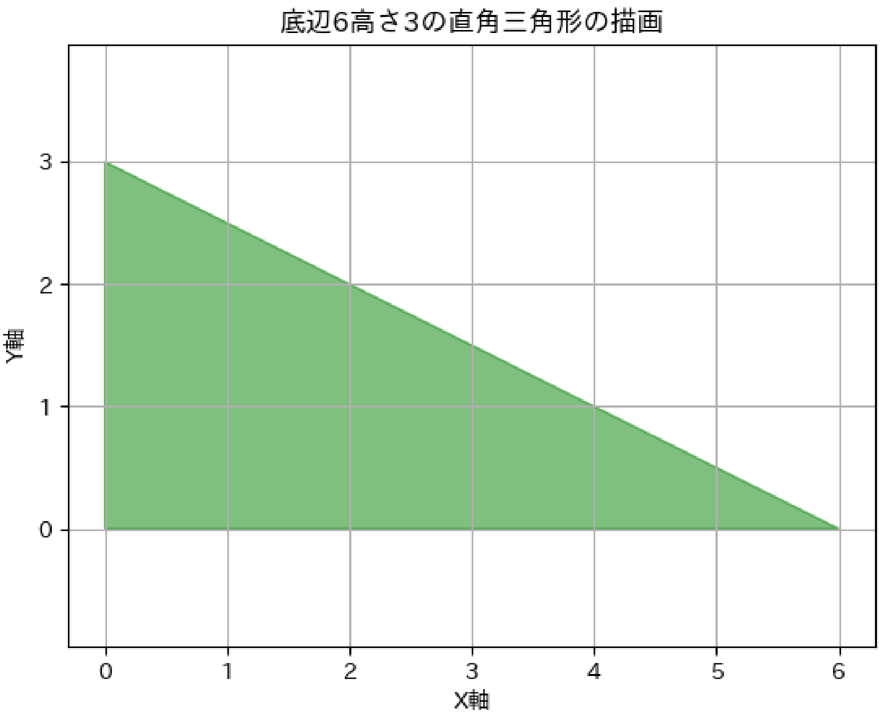

図 2　底辺 6 高さ 3 の直角三角形

図 2 のように $(0,0), (6,0), (0,3)$ の頂点で構成される直角三角形の重心を求める.

$$
\begin{aligned}
C\colon 0 \leqq x \leqq 6, 0 \leqq y \leqq 3 - \frac{x}{2} \\
dV = dxdy \\
V_c = \frac{1}{2} \times 6 \times 3 = 9
\end{aligned}
$$

とする.

$$
\begin{aligned}
x_g &= \frac{1}{V_c}\int_C x dV \\
&= \frac{1}{V_c}\int_0^6 \int_0^{3-\frac{x}{2}} x dy dx \\
&= \frac{1}{V_c}\int_0^6 x(3-\frac{x}{2}) dx \\
&= \frac{1}{V_c}(\frac{9}{2}x - \frac{1}{6}x^3)\Big|_0^6 \\
&= \frac{1}{V_c}(\frac{9}{2}\times 6 - \frac{1}{6}\times 6^3) \\
&= \frac{1}{V_c}\times 18 \\
&= 2
\end{aligned}
$$

$$
\begin{aligned}
y_g &= \frac{1}{V_c}\int_C y dV \\
&= \frac{1}{V_c}\int_0^6 \int_0^{3-\frac{x}{2}} y dy dx \\
&= \frac{1}{V_c}\int_0^6 \frac{1}{2}y^2\Big|_6^{3-\frac{x}{2}} dx \\
&= \frac{1}{V_c}\int_0^6 \frac{1}{2}((3-\frac{x}{2})^2 - 6^2) dx \\
&= \frac{1}{V_c}\int_0^6 \frac{x^2-12x+36}{8} dx \\
&= \frac{1}{V_c}(\frac{1}{24}x^3 - 3x^2 + 18x)\Big|_0^6 \\
&= \frac{1}{V_c}(\frac{1}{24}\times 6^3 - 3\times 6^2 + 18\times 6) \\
&= \frac{1}{V_c}\times 9 \\
&= 1
\end{aligned}
$$

2.3 扇形の重心

図 3 のように半径 6, 角度 30° の扇形の重心を求める.

$$
\begin{aligned}
C\colon 0 \leqq r \leqq 6, 0 \leqq \theta \leqq \frac{\pi}{6} \\
dV = r dr d\theta \\
V_c = 6^2\pi\frac{1}{12} = 3\pi \\
\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}
\end{aligned}
$$

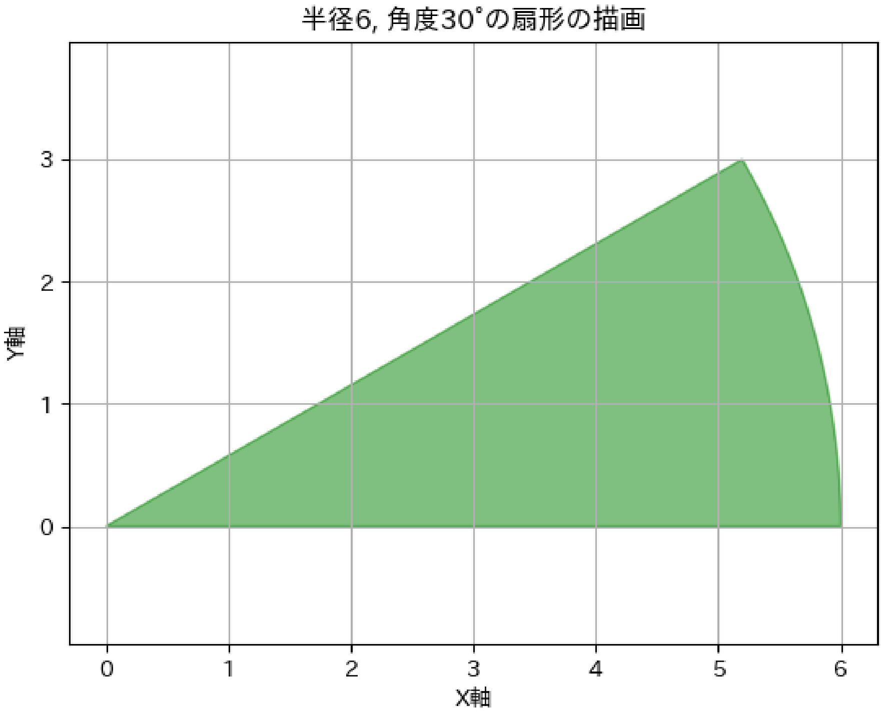

図 3　半径 6, 角度 30° の扇形

$$
\begin{aligned}
x_g &= \frac{1}{V_c}\int_C x dV \\
&= \frac{1}{V_c}\int_0^{\frac{\pi}{6}}\int_0^6 r\cos\theta r dr d\theta \\
&= \frac{1}{V_c}\int_0^{\frac{\pi}{6}}\cos\theta\int_0^6 r^2 dr d\theta \\
&= \frac{1}{V_c}\int_0^{\frac{\pi}{6}}\cos\theta\frac{1}{3}r^3\Big|_0^6 d\theta \\
&= \frac{1}{V_c}\int_0^{\frac{\pi}{6}}\cos\theta\times 72 d\theta \\
&= \frac{1}{V_c}72\sin\theta\Big|_0^{\frac{\pi}{6}} \\
&= \frac{1}{V_c}72\sin\frac{\pi}{6} \\
&= \frac{1}{V_c}72\times\frac{1}{2} \\
&= \frac{12}{\pi} \\
&=\simeq 3.82
\end{aligned}
$$

$$
\begin{aligned}
y_g &= \frac{1}{V_c}\int_C y dV \\
&= \frac{1}{V_c}\int_0^{\frac{\pi}{6}}\int_0^6 r\sin\theta r dr d\theta \\
&= \frac{1}{V_c}\int_0^{\frac{\pi}{6}}\sin\theta\int_0^6 r^2 dr d\theta \\
&= \frac{1}{V_c}\int_0^{\frac{\pi}{6}}\sin\theta\frac{1}{3}r^3\Big|_0^6 d\theta \\
&= \frac{1}{V_c}\int_0^{\frac{\pi}{6}}\sin\theta\times 72 d\theta \\
&= \frac{1}{V_c}72(-\cos\theta)\Big|_0^{\frac{\pi}{6}} \\
&= \frac{1}{V_c}72(-\cos\frac{\pi}{6}+1) \\
&= \frac{1}{V_c}72\times\frac{\sqrt{3}}{2} \\
&= \frac{1}{3\pi}(72-36\sqrt{3}) \\
&= \frac{24-12\sqrt{3}}{\pi} \\
&=\simeq 1.02
\end{aligned}
$$