- To compute the final defuzzified value, we take average of $\mu_{HP}(z)$.

$$\mu_{HP}(z) = \frac{z-20}{60} \qquad \mu_{HP}(z) = \frac{100-z}{20}$$

$$\therefore 0.8 = \frac{z-20}{60} \qquad 0.8 = \frac{100-z}{20}$$

$$\therefore z = 68 \qquad \therefore z = 84$$

$$\therefore z^* = \frac{68+84}{2} = 76\%$$

Ex. 6.9.5 : Design a fuzzy controller for a train approaching station. Inputs are speed and Distance and output is break power. Use triangular membership function. Consider two descriptor for input and three descriptors for output. Derive a set of rules for control action and defuzzification. The design should be supported by figures wherever possible. Design a fuzzy controller for a train with high speed and small distance. **MU - Dec. 15, 20 Marks**

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same :

- Here inputs are
- Distance of a drain from the station measured in meters.
- Speed of a train measured in km/hr.
- Output variable is brake power measured in %
- As mentioned, we take two descriptors for inputs.
- Descriptors for distance = > {SD,LD}
  - SD : Short Distance
  - LD : Long Distance
- Descriptors for speed => {LS , HS}
  - LS : Low Speed
  - HS : High Speed
- We take three descriptors for output variable.
- Descriptors for brake power => {LP, MP, HP}
  - LP : Low Power
  - MP : Medium Power
  - HP : High Power

Step 2 : Define membership functions for each of the input and output variables

- As mentioned, we use triangular membership functions.

- Membership functions for distance.

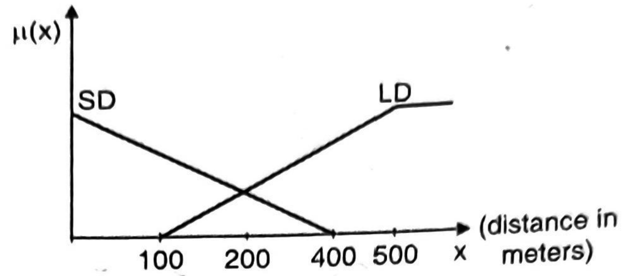

Fig. P. 6.9.5(a) : Membership functions for distance

$$\mu_{SD}(x) = \frac{400 - x}{400}; \qquad 0 \leq x \leq 400$$

$$\mu_{LD}(x) = \frac{x - 100}{400}; \qquad 100 \leq x \leq 500$$

- Membership functions for speed

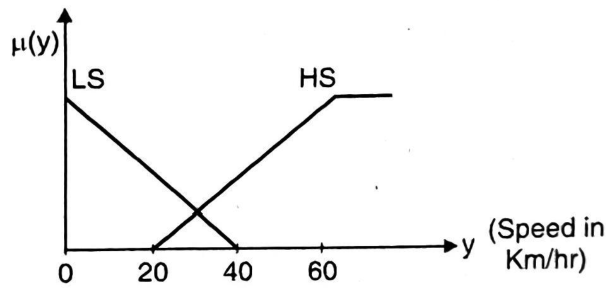

Fig. P. 6.9.5(b) : Membership functions for speed

$$\mu LS(y) = \frac{40 - y}{40}; \qquad 0 \leq y \leq 40$$

$$\mu HS(y) = \frac{y - 20}{40}; \qquad 20 \leq y \leq 60$$

- Membership functions for brake power

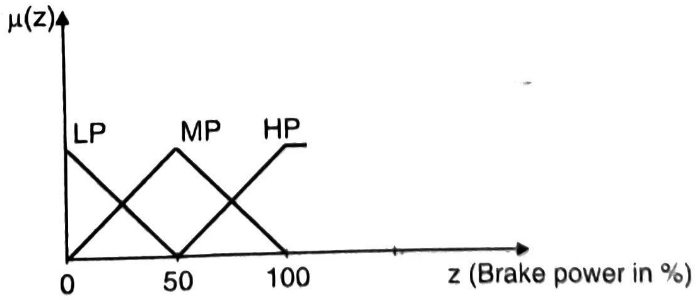

Fig. P. 6.9.5(c) : Membership functions for brake power

$$\mu LP(z) = \frac{50 - z}{50}; \qquad 0 \leq z \leq 50$$

$$\mu MP(z) = \begin{cases} \dfrac{z}{50} & ; \quad 0 \le z \le 50 \\ \dfrac{100 - z}{50} & ; \quad 50 \le z \le 100 \end{cases}$$

$$\mu MP(z) = \frac{z - 50}{50} \; ; \qquad 50 \le z \le 100$$

Step 3 : Form a rule base

dist \ speed	LS	HS
SD	MP	HP
LD	LP	MP

The above matrix represents four rules; for example, first rule can be, "If distance is short (SD) and speed is low (LS) then required brake power is medium (MP)".

Step 4 : Rule evaluation

- We design a controller for **high speed** and **small distance.**
- Consider

Distance = 110 meters and

Speed = 50 km/hr

Distance = 110

maps to the following two MFs of "distance" variable

$$\mu_{SD}(x) = \frac{400 - x}{400} \text{ and}$$

$$\mu_{LD}(x) = \frac{x - 100}{400}$$

And Speed = 50 km/hr

maps to the following MF of a "Speed" variable.

$$\mu_{HS}(y) = \frac{y - 20}{40}$$

- Evaluate $\mu_{SD}(x)$ and $\mu_{LD}(x)$ for $x = 110$, we get,

$$\mu_{SD}(110) = \frac{400 - 110}{400} = \frac{290}{400} = 0.725 \qquad \text{...(1)}$$

$$\mu_{SD}(110) = \frac{110 - 100}{400} = \frac{10}{400} = 0.025 \qquad \text{...(2)}$$

- Similarly, evaluate $\mu_{HS}(y)$ for $y = 50$, we get,

$$\mu_{HS}(50) = \frac{50 - 20}{40} = \frac{30}{40} = 0.75 \qquad \text{...(3)}$$

- Above three equations lead to the following two rules that we need to evaluate
  1. Distance is short and speed is high.
  2. Distance is long and speed is high.
- Since the antecedent part of each rule is connected by 'and' operator, we use "min" operator to evaluate strength of each rule.
- Strength of rule 1

$$\begin{aligned} S1 &= \min(\mu_{SD}(110), \mu_{SD}(50)) \\ &= \min(0.725, 0.75) = 0.725 \end{aligned}$$

- Strength of rule 2

$$\begin{aligned} S2 &= \min(\mu LD(110), \mu SD(50)) \\ &= \min(0.025, 0.75) = 0.025 \end{aligned}$$

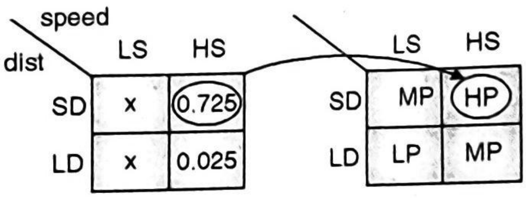

Fig. P. 6.9.5(d) : Rule strength table and its mapping to corresponding output HF

Step 5 : Defuzzification

- We use "mean of max" defuzzification method.
- We first find the rule with maximum strength.

$$\begin{aligned} &= \max(S1, S2) \\ &= \max(0.725, 0.025) = 0.725 \end{aligned}$$

- Which corresponds to rule 1.
- Thus, rule 1 "If distance is short and speed is high" has maximum strength 0.725.
- The above rule corresponds to the output MF μ_{HP} (z). This mapping is shown in Fig. P. 5.10.5(d).
- To compute the final defuzzified value, take MF of μ_{HP} (z).

$$\begin{aligned} \mu HP(z) &= 0.725 \\ \frac{z-50}{50} &= 0.725 \\ z - 50 &= 50 \times 0.725 \\ z - 50 &= 36.25 \\ z &= 50 + 36.25 = 86.25\ \% \end{aligned}$$

So, for distance = 110 meters and speed = 50 km/hr

equired brake power is 86.25 %.