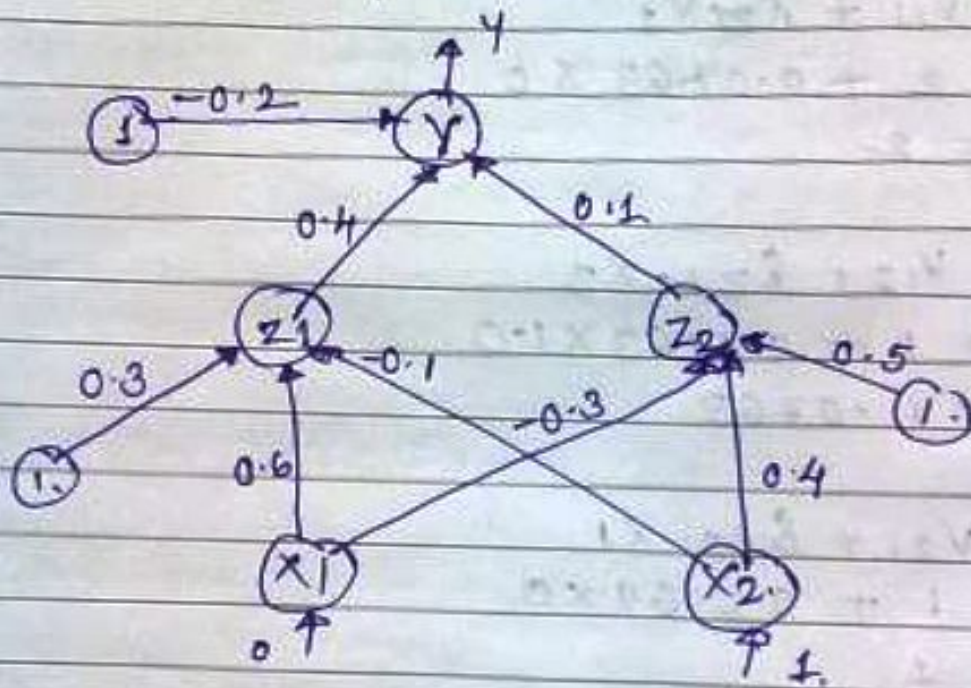


7) Using back propagation network, find the new weights for the net shown in figure. It is presented with the input pattern $[0, 1]$ and the target output is 1. Use learning rate $\eta = 0.25$ and binary sigmoidal activation function.



Solution: =

$$[v_{11}, v_{12}, v_{01}] = [0.6, -0.1, 0.3]$$

$$[v_{21}, v_{22}, v_{02}] = [-0.3, 0.4, 0.5]$$

$$[w_1, w_2, w_0] = [0.4, 0.1, 0.2]$$

$$\alpha = 0.25, t = 1, \lambda = 1$$

Activation function = binary sigmoidal.

$$\therefore f(x) = \frac{1}{1 + e^{-\lambda \text{net}}}$$

Calculate the input net. for z_1

$$\begin{aligned}Z_{in1} &= x_1 \times v_{11} + x_2 \times v_{12} + 0.3 \\&= 0 \times 0.6 + 1 \times -0.1 + 0.3 \\&= 0.2\end{aligned}$$

$$\begin{aligned}Z_{in2} &= x_1 \times v_{21} + x_2 \times v_{22} + 0.5 \\&= 0 \times -0.3 + 1 \times 0.4 + 0.5 \\&= 0.9\end{aligned}$$

$$z_1 = f(Z_{in1}) = \frac{1}{1 + e^{-0.2}} = 0.5498$$

$$z_2 = f(Z_{in2}) = \frac{1}{1 + e^{-0.9}} = 0.7109$$

$$\begin{aligned}y_{in} &= z_1 \times w_1 + z_2 \times w_2 + w_0 \\&= 0.5498 \times 0.4 + 0.7109 \times 0.1 + (-0.2) \\&= 0.09101\end{aligned}$$

$$y = f(y_{in}) = \frac{1}{1 + e^{-0.09101}} = 0.5227$$

Compute error term

$$\delta_k = (t_k - y_k) \cdot f'(y_{ink})$$

$$\begin{aligned}f'(y_{ink}) &= f(y_{in}) [1 - f(y_{in})] \\&= 0.5227 \cdot [1 - 0.5227] \\&= 0.2495\end{aligned}$$

$$\begin{aligned}\delta_0 &= (1 - 0.5227) \cdot 0.2495 \\&= 0.1191\end{aligned}$$

And the changes in weights between hidden and output layers.

$$\begin{aligned} \Delta w_{1new} &= w_{1old} + \eta \delta o z_1 \\ &= 0.4 + 0.25 \times 0.1191 \times 0.5498 \\ &= 0.4 + 0.0164 \\ &= 0.4164 \end{aligned}$$

$$\begin{aligned} w_{2new} &= w_{2old} + \eta \delta o z_2 \\ &= 0.1 + 0.25 \times 0.1191 \times 0.7109 \\ &= 0.12117 \end{aligned}$$

Compute error signal terms of hidden layer

$$\delta y_j = y_j(1 - y_j) \cdot \sum_{k=1}^K \delta o_k \cdot w_{kj}$$

Two hidden nodes in hidden layer, z_1, z_2 .

$$\begin{aligned} \delta z_1 &= [0.5498 \times (1 - 0.5498)] \times 0.1191 \times 0.4 \\ &= 0.2475 \times 0.1191 \times 0.4 \\ &= 0.2475 \times 0.04764 \\ &= 0.0118 \end{aligned}$$

$$\begin{aligned} \delta z_2 &= [0.7109 \times (1 - 0.7109)] \times 0.1191 \times 0.1 \\ &= 0.2055 \times 0.1191 \times 0.1 \\ &= 0.00245 \end{aligned}$$

Adjust weights of hidden layer

$$v \leftarrow v + \eta \delta y \cdot z$$

$$\begin{aligned}
 V_{11} &= V_{11} + \eta \cdot \delta Z_1 \cdot X_1 \\
 &= 0.6 + 0.25 \times 0.0118 \times 0 \\
 &= 0.6
 \end{aligned}$$

$$\begin{aligned}
 V_{12} &= V_{12} + \eta \times \delta Z_1 \times X_2 \\
 &= -0.1 + 0.25 \times 0.0118 \times 1 \\
 &= -0.1 + 0.00295 \\
 &= -0.09705
 \end{aligned}$$

$$\begin{aligned}
 V_{21} &= V_{21} + \eta \times \delta Z_2 \times X_1 \\
 &= -0.8 + 0.25 \times 0.00245 \times 0 \\
 &= -0.8
 \end{aligned}$$

$$\begin{aligned}
 V_{22} &= V_{22} + \eta \times \delta Z_2 \times X_2 \\
 &= 0.4 + 0.25 \times 0.00245 \times 1 \\
 &= 0.4006125
 \end{aligned}$$

Update ~~weights~~ bias

$$\begin{aligned}
 V_{01} &= V_{01} + \eta \cdot \delta Z_1 \\
 &= 0.3 + 0.25 \times 0.0118 \\
 &= 0.3 + 0.00295 \\
 &= 0.30295
 \end{aligned}$$

$$\begin{aligned}
 V_{02} &= V_{02} + \eta \cdot \delta Z_2 \\
 &= 0.5 + 0.25 \times 0.00245 \\
 &= 0.5006125
 \end{aligned}$$

$$\begin{aligned}
 W_0 &= W_0 + \eta \cdot \delta O \\
 &= -0.2 + 0.25 \times 0.1191 \\
 &= -0.170225
 \end{aligned}$$

Unipolar continuous

$$f(\text{net}) = \frac{1}{1 + \exp(-\text{net})}$$

$$\delta o_k = (d_k - o_k) (1 - o_k) o_k \quad \text{for } k=1 \dots K$$

$$\delta y_j = y_j (1 - y_j) \cdot \sum_{k=1}^K \delta o_k \cdot w_{kj} \quad \text{for } j=1 \dots J$$

Bipolar continuous

$$f(\text{net}) = \frac{2}{1 + \exp(-\text{net})} - 1$$

$$\delta o_k = \frac{1}{2} (d_k - o_k) (1 - o_k^2) \quad \text{for } k=1 \dots K$$

$$\delta y_j = \frac{1}{2} (1 - y_j^2) \sum_{k=1}^K \delta o_k \cdot w_{kj} \quad \text{for } j=1 \dots J$$