Ex. 6.9.3 : Design a fuzzy controller to control the feed amount of purifier for the water purification plant. Raw water is purified by injecting chemicals. Assume input as water temperature and grade of water, output as amount of purifier. Use three descriptors for input and output variables. Design rules to control action and defuzzification. Design should be supported by figures whenever necessary. Clearly indicate that when temperature is low, grade is low then chemical used is in large amount. **MU - May 13, 20 Marks**

Soln. :

Step 1 : *Identify input and output variables and decide descriptors for the same.*

- Here input variables are water temperature and grade of water.
- Water temperature is measured in °C. grade of water is measured in percentage.
- Descriptors for water temperature are

{C, M, H} C - Cold
M - Medium
H - High

- Descriptors for grade are

{L, M, H} L - Low
M - Medium
H - High

- Amount of purifier is measured in grams.
- Descriptors for amount of purifier are

{S, M, L} S - Small
M - Medium
L - Large

Step 2 : Fuzzification

- Define membership functions for each of the input and output variables.
- We use triangular MFs because of its simplicity.

(1) Membership functions for water temperature

$$\mu_C(x) = \frac{50-x}{50}, \quad 0 \le x \le 50$$

$$\mu_M(x) = \begin{cases} \frac{x}{50}, & 0 \le x \le 50 \\ \frac{100-x}{50}, & 50 < x \le 100 \end{cases}$$

$$\mu_H(x) = \frac{x-50}{50}, \quad 50 \le x \le 100$$

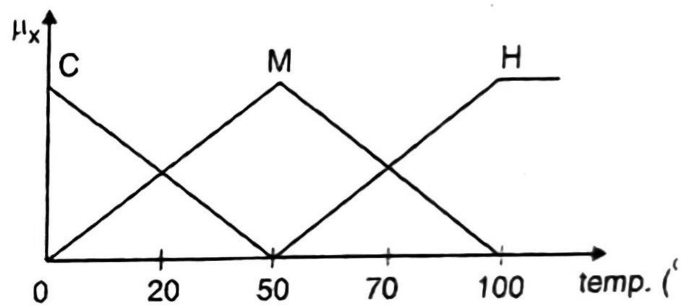

Fig. P. 6.9.3 : Membership functions for water temp

(2) Membership functions for grade of water

$$\mu_L(y) = \frac{50-y}{50}, \quad 0 \le y \le 50$$

$$\mu_M(y) = \begin{cases} \frac{y}{50}, & 0 \le y \le 50 \\ \frac{100-y}{50}, & 50 < y \le 100 \end{cases}$$

$$\mu_H(y) = \frac{y-50}{50}, \quad 50 \le y \le 100$$

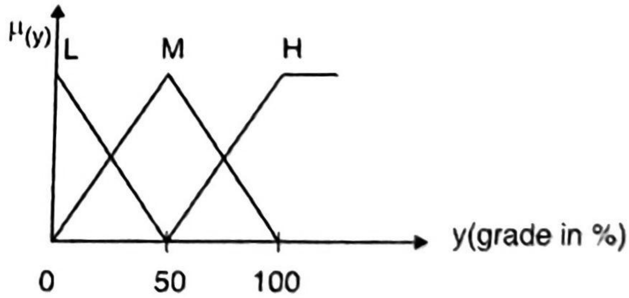

Fig. P. 6.9.3(a) : Membership functions for grade of water

(3) Membership functions for amount of purifier

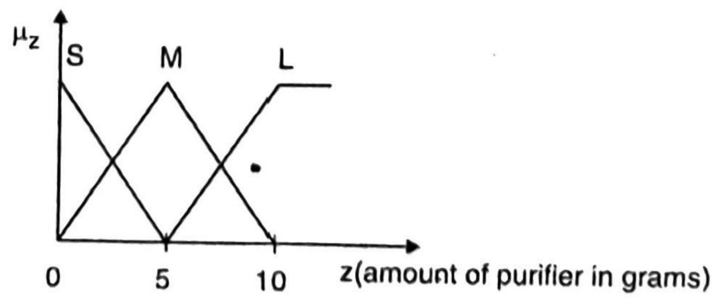

Fig. P. 6.9.3(b) : Membership functions for purifier

$$\mu_S(z) = \frac{5-z}{5}, \quad 0 \le z \le 5$$

$$\mu_M(z) = \begin{cases} \frac{z}{5}, & 0 \le z \le 5, \\ \frac{10-z}{5}, & 5 < z \le 10 \end{cases}$$

$$\mu_L(z) = \frac{z-5}{5}, \quad 5 \le z \le 10$$

tep 3 : Form a Rule base

Temp \ grade	L	M	H
C	L	M	S
M	L	M	M
H	M	S	S

- The above matrix represents in all nine rules. For example,
- First rule can be, "If temperature is cold and grade is low then amount of purifier required is large."
- Similarly all nine rules can be defined using if-then rules.

Step 4 : Rule Evaluation

Assume water temperature = 5° and grade = 30

Water temperature = 5° maps to the following two MFs of "temperature" variable.

$$\mu_c(x) = \frac{50-x}{50} \text{ and } \mu_M(x) = \frac{x}{50}$$

Similarly, grade = 30

maps to the following two MFs of "grade" variable.

$$\mu_L(y) = \frac{50-y}{50} \text{ and } \mu_M(y) = \frac{y}{50}$$

Evaluate $\mu_c(x)$ and $\mu_M(x)$ for x = 5° we get,

$$\mu c(5) = \frac{50-5}{50} = \frac{9}{10} = 0.9 \quad \text{...(1)}$$

$$\mu_M(5) = \frac{5}{50} = \frac{1}{5} = 0.1 \quad \text{...(2)}$$

Evaluate $\mu L(y)$ and $\mu M(y)$ for y = 30

$$\mu L(30) = \frac{50-30}{50} = \frac{2}{5} = 0.4 \quad \text{...(3)}$$

$$\mu M\,(30) = \frac{30}{50} = \frac{3}{5} = 0.6 \qquad \ldots(4)$$

- The above four equation represents following for rules that we need to evaluate.
  1. If temperature is **cold** and grade is **low**.
  2. If temperature is **cold** and grade is **medium**.
  3. If temperature is **medium** and grade is **low**.
  4. If temperature is **medium** and grade is **medium**.
- Since the antecedent part of each rule is connected by **and** operator we use **min** o
- perator to evaluate strength of each rule

$$\begin{aligned}
\text{Strength of rule 1} : S_1 &= \min\,(\mu_c\,(5), \mu_L\,(30)) \\
&= \min\,(0.9, 0.4) = 0.4 \\
\text{Strength of rule 2} : S_2 &= \min\,(\mu_c(5), \mu_M(30)) \\
&= \min\,(0.9, 0.6) = 0.6 \\
\text{Strength of rule 3} : S_3 &= \min\,(\mu_M(5), \mu_L(30)) \\
&= \min\,(0.1, 0.4) = 0.1 \\
\text{Strength of rule 4} : S_4 &= \min\,(\mu_M(5), \mu_M(30)) = \min\,(0.1, 0.6) = 0.1
\end{aligned}$$

Temp \ Grade	$\mu_L(30)$	$\mu_M(30)$	
$\mu_C(5)$	0.4	0.6	X
$\mu_M(5)$	0.1	0.1	X
	X	X	X

(a) Rule strength table

Max MF

Temp \ Grade		
L	M	S
L	M	M
M	S	S

(b) Rule base table

Fig. P. 6.9.3(c) : Rule strength and its mapping to corresponding output MF

Step 5 : Defuzzification

- Since, we use "mean of max" defuzzification technique, we first find the rule with maximum strength.

$$\begin{aligned}
&= \max\,(S_1, S_2, S_3, S_4) \\
&= \max\,(0.4, 0.6, 0.1, 0.1) = 0.6
\end{aligned}$$

- This corresponds to rule 2.
- Thus rule 2 : "Temperature is cold and grade is medium" has maximum strength 0.6.
- The above rule corresponds to the output MF $\mu_M(z)$. This is show is shown in Fig. P. 6.9.3(c).

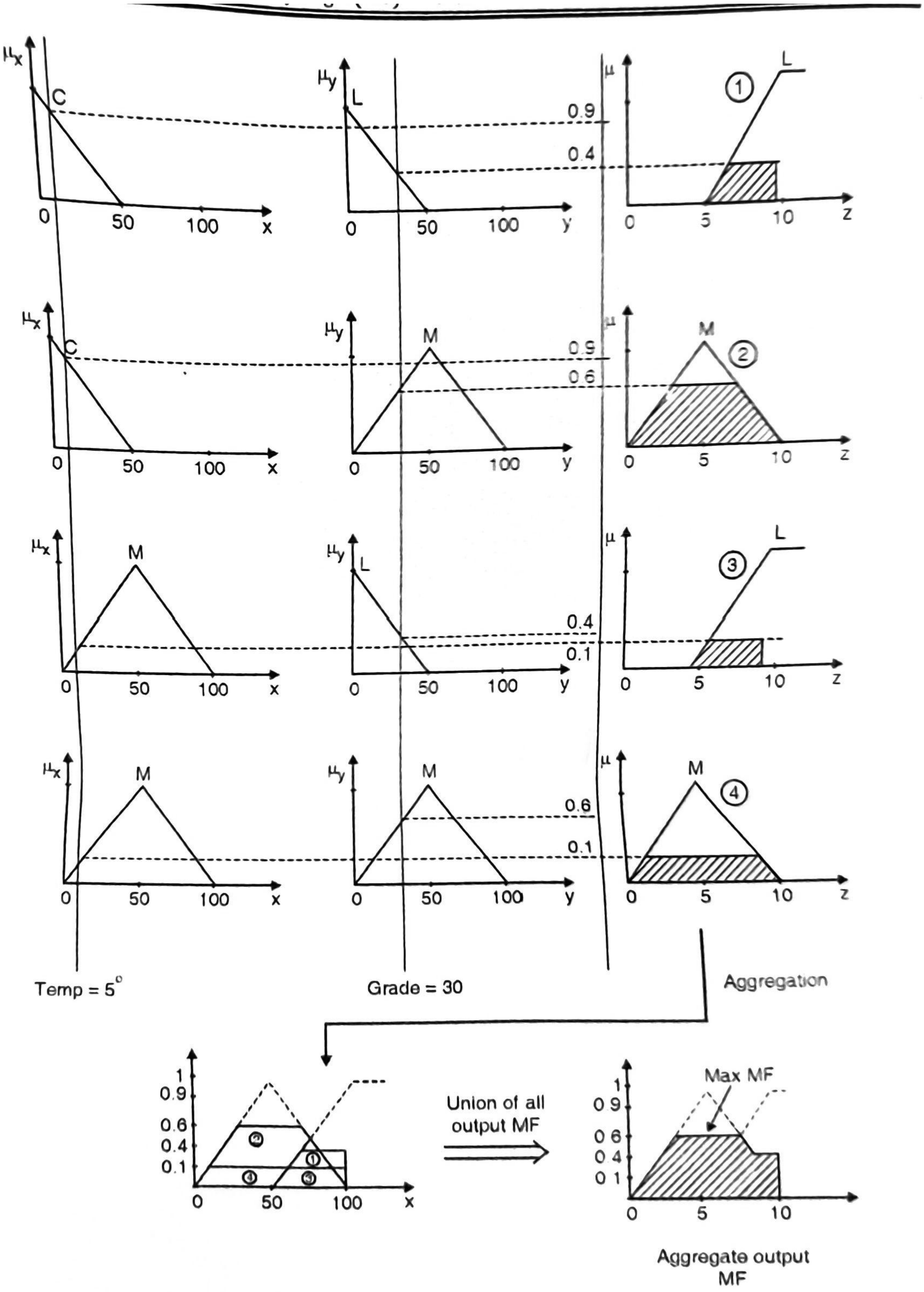

Fig. P. 6.9.3(d) : Process of rule evaluation and defuzzification

- To find out final defuzzified value, we now take average (i.e. mean) of $\mu_M(z)$.

$$\mu_M(z) = \frac{10-z}{5} \qquad \mu_M(z) = \frac{z}{5}$$

$$0.6 = \frac{10-z}{5} \qquad 0.6 = \frac{z}{5}$$

$$\therefore z = 13 \qquad \therefore z = 3$$

$$\therefore z^* = \frac{13+3}{2} = \mathbf{8\ grams}$$

Ex. 6.9.4 : Design a fuzzy controller for a train approaching or leaving a station, the inputs are distance from a station and speed of the train. The output is brake power used. Use,

(i) Triangular membership functions

(ii) Four descriptors for each variables

(iii) Five to six rules.

(iv) Appropriate defuzzification method. **MU - Dec. 11, 20 Marks**

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

Here inputs are

- *Distance of a train from the station, measured in meters and*
- *Speed of train measured in km/hr.*
- Output variable is brake power measured in %.
- As mentioned, we take four descriptors for each of the input and output variables.
- For distance ⇒ {VSD, SD, LD, VLD}

VSD : Very Short Distance

SD : Short Distance

LD : Large Distance

VLD : Very Large Distance

- For speed ⇒ {VLS, LS, HS, VHS}

VLS : Very Low Speed

LS : Low Speed

HS : High Speed

VHS : Very High Speed