

Possibilistic c -Regression Models Clustering Algorithm

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Abstract—The purpose of this paper is to apply the possibilistic c -means (PCM) clustering algorithm to the fuzzy c -regression models (FCRM) clustering algorithm and propose a new clustering algorithm named possibilistic c -regression models (PCRM). The PCRM clustering algorithms relaxes the column sum constrain result in each cluster, it will alleviate the noisy data effectively. Finally, the simulation examples are provided to demonstrate the effectiveness of the PCRM clustering algorithm.

Index Terms—fuzzy clustering, possibilistic c -means (PCM), fuzzy c -regression models (FCRM).

I. INTRODUCTION

Fuzzy clustering methods have been widely and effectively used in various applications and research areas such as image recognition and pattern classification. The majority clustering algorithms are extended from fuzzy c -means (FCM) clustering algorithm [1-7], for example, Gustafson-Keseel (GK) clustering algorithm [8], Gath-Geva (GG) clustering algorithm [9], possibilistic fuzzy c -means (PFCM) [10], possibilistic c -means (PCM) clustering algorithm [7, 11] and enhanced possibilistic c -means (EPCM) clustering algorithm [12]. These clustering algorithms are characterized by hyper-spherical-shaped clusters.

However, the FCM clustering algorithm has the column sum of the fuzzy partition matrix equals to one result in each cluster is sensitive to the noise data [10].

In particular, Bezdek et al. proposed another fuzzy clustering algorithm called fuzzy c -regression models (FCRM) clustering algorithm [13] which assumes that the data are draw from c different hyper-plane-shaped clusters (regression models) rather than c hyper-spherical-shaped clusters. The difference between FCM and FCRM is briefly illustrated in Fig. 1.

In Fig. 1, there are six unlabeled data $\mathbf{z}_1, \dots, \mathbf{z}_6$ to be clustered, \mathbf{v}_1 and \mathbf{v}_2 are two hyper-spherical-shaped cluster centers from FCM, and the linear regression functions L_1 and L_2 are two hyper-plane-shaped clusters from FCRM. For the FCM, according to the distance from each datum to cluster representatives \mathbf{v}_1 and \mathbf{v}_2 , the data $\mathbf{z}_1, \mathbf{z}_2$ and \mathbf{z}_3 will have higher fuzzy membership degrees to the cluster one, and the data $\mathbf{z}_4, \mathbf{z}_5$ and \mathbf{z}_6 will have higher fuzzy membership degrees to the cluster two; while for the FCRM, the data $\mathbf{z}_1, \mathbf{z}_2$ and \mathbf{z}_4 will be classified to be the cluster one, the data $\mathbf{z}_3, \mathbf{z}_5$ and \mathbf{z}_6 will be classified to be the cluster two in that

they fit the linear regression functions L_1 and L_2 respectively. We see that the six unlabeled data are not suitable for FCM but suitable for FCRM.

Because the column sum constraint of the FCRM clustering algorithm are identical to the FCM clustering algorithm, the FCRM clustering algorithm also suffers this problem.

To overcome this problem, Krishnapuram and Keller [3] proposed the PCM clustering algorithm, which relaxes the column sum constraint and each cluster alleviates the noisy data effectively.

In this paper, we will relax the column sum constraint to the FCRM clustering algorithm and proposed a new clustering algorithm named possibilistic c -regression models (PCRM). It is seen that the proposed PCRM clustering algorithm not only conserves the hyper-plane-shaped characteristics but also alleviates the noisy data effectively.

The rest of the paper is organized as follows. Section II introduces the FCRM clustering algorithm. Section III proposes the PCRM clustering algorithm. In Section IV, we illustrate the effectiveness of the PCRM clustering algorithm with several examples. Section V gives the summary and conclusions.

II. FUZZY C -REGRESSION MODEL

Suppose that $\mathbf{Z} = \{\mathbf{z}_1, \dots, \mathbf{z}_L\}$ is an unlabeled data set, where $\mathbf{z}_j = (\mathbf{u}_j, y_j) \in \mathcal{R}^n \times \mathcal{R}$ for all $j \in \{1, \dots, L\}$, and let \mathbf{F} denote a $c \times L$ fuzzy c -partition matrix generated by the fuzzy c -regression models (FCRM) clustering algorithm as follows [1-9, 13-20]:

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1L} \\ f_{21} & f_{22} & \cdots & f_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ f_{c1} & f_{c2} & \cdots & f_{cL} \end{bmatrix} \quad (1)$$

where f_{ij} satisfy the following conditions [1-9, 13-20]:

$$f_{ij} \in [0, 1] \text{ for all } i \in \{1, \dots, c\} \text{ and } j \in \{1, \dots, L\} \quad (2)$$

$$0 < \sum_{j=1}^L f_{ij} < L \text{ for all } i \in \{1, \dots, c\}, \text{ and} \quad (3)$$

$$\sum_{i=1}^c f_{ij} = 1 \text{ for all } j \in \{1, \dots, L\}, \quad (4)$$

where c is the number of clusters and f_{ij} is the degree of membership of \mathbf{z}_j belonging to the i th cluster.

The regression model adopted in this paper is as follows [13-20]:

$$\begin{aligned} y_j &= \beta_{i0} + \beta_{i1}u_{j1} + \dots + \beta_{in}u_{jn} + e_j \\ &= [\mathbf{u}_j^T \mathbf{1}] \boldsymbol{\beta}_i + e_j, \end{aligned} \quad (5)$$

where $\mathbf{u}_j = [u_{j1}, \dots, u_{jn}]^T \in \mathcal{R}^n$, $\boldsymbol{\beta}_i \in \mathcal{R}^{n+1}$ is the parameter of the i th regression model (cluster) for all $i \in \{1, \dots, c\}$ and $\beta_{i0} \in \mathcal{R}$ denotes the offset or shift term. Then the estimation error is defined by [13-20]:

$$D_{ij}(\boldsymbol{\beta}_i) = |e_j|. \quad (6)$$

In such case, the regression model parameters $\boldsymbol{\beta}_i$ can be estimated by using the weighted least square (WLS) algorithm [19-23] as follows:

$$\boldsymbol{\beta}_i = [\boldsymbol{\Pi}^T \mathbf{G}_i \boldsymbol{\Pi}]^{-1} \boldsymbol{\Pi}^T \mathbf{G}_i \boldsymbol{\Gamma} \quad (7)$$

where

$$\boldsymbol{\Pi} = \begin{bmatrix} \mathbf{u}_1^T, 1 \\ \mathbf{u}_2^T, 1 \\ \vdots \\ \mathbf{u}_L^T, 1 \end{bmatrix} \in \mathcal{R}^{L \times (n+1)}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} \in \mathcal{R}^L \quad (8)$$

and

$$\mathbf{G}_i = \text{diag}[f_{i1}^m, \dots, f_{iL}^m] \in \mathcal{R}^{L \times L}. \quad (9)$$

Define [7, 13-20]

$$J_{FCRM}^m(\mathbf{F}, \mathbf{B}) = \sum_{j=1}^L \sum_{i=1}^c (f_{ij})^m (D_{ij}(\boldsymbol{\beta}_i))^2, \quad (10)$$

where $\mathbf{B} = (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \dots, \boldsymbol{\beta}_c)$ and $m \in (1, \infty)$ is the weighting exponent. For the FCRM clustering algorithm [13-20], the objective is to find (\mathbf{F}, \mathbf{B}) such that $J_{FCRM}^m(\mathbf{F}, \mathbf{B})$ is minimized.

FCRM Clustering Algorithm [13-20]:

Step 1: Given c ($1 < c < L$), $m \in (1, \infty)$, the termination threshold $\varepsilon > 0$, set an initial fuzzy c -partition matrix

$$\mathbf{F}^{(0)} = \begin{bmatrix} f_{11}^{(0)} & f_{12}^{(0)} & \dots & f_{1L}^{(0)} \\ f_{21}^{(0)} & f_{22}^{(0)} & \dots & f_{2L}^{(0)} \\ \vdots & \vdots & \ddots & \vdots \\ f_{c1}^{(0)} & f_{c2}^{(0)} & \dots & f_{cL}^{(0)} \end{bmatrix} \quad (11)$$

satisfying (2)-(4), and set iteration index $k = 0$.

Step 2: By (7), calculate the parameter vectors

$$\boldsymbol{\beta}_i^{(k)} \text{ for all } i \in \{1, \dots, c\}$$

Step 3: Update $\mathbf{F}^{(k)}$ to $\mathbf{F}^{(k+1)}$ by

$$f_{ij}^{(r+1)} = \left(\sum_{j=1}^c \left(\frac{D_{ij}(\boldsymbol{\beta}_i^{(k)})}{D_{ij}(\boldsymbol{\beta}_j^{(k)})} \right)^{\frac{2}{m-1}} \right)^{-1} \quad (12)$$

for all $i \in \{1, \dots, c\}$ and $j \in \{1, \dots, L\}$.

Step 4: If $\|\mathbf{F}^{(k)} - \mathbf{F}^{(k+1)}\| < \varepsilon$, stop; otherwise, set $k = k + 1$ and return to **Step 2**.

The following simple example illustrates the problem associated with the FCRM clustering algorithm.

Given two linear equations

$$\begin{aligned} y &= 5u - 5 + \varepsilon, \\ y &= -9u + 1 + \varepsilon, \end{aligned}$$

where $\boldsymbol{\beta}_1 = [5 \ -5]^T$ and $\boldsymbol{\beta}_2 = [-9 \ 1]^T$ and ε is white Gaussian random noise with zero mean and standard deviation σ .

We randomly generate 10 training input points u whose magnitude is uniformly distributed over range $[-1, 1]$ to the two linear equations with noise free ($\varepsilon = 0$). Additionally, randomly generate 2 training input points u whose magnitude is uniformly distributed over range $[-1, 1]$ to the three linear equations, and set the Gaussian random noise ε with standard deviation $\sigma = 1.5$. The total number of generated data of the noise free data set $\{\mathbf{z}_1, \dots, \mathbf{z}_{20}\}$ and noisy data set $\{\mathbf{z}_{21}, \mathbf{z}_{22}, \mathbf{z}_{23}, \mathbf{z}_{24}\}$ are 20 and 4, respectively.

We choose $c = 2$ and $m = 2$. Fig. 2 shows two-dimensional data sets \mathbf{Z}_{20} and \mathbf{Z}_{24} with 20 and 24 points whose coordinates are given in Table I.

We denote the noise free data set by

$$\mathbf{Z}_{20} = \{\mathbf{z}_1, \dots, \mathbf{z}_{20}\},$$

the noisy data set by

$$\mathbf{N}_4 = \{\mathbf{z}_{21}, \mathbf{z}_{22}, \mathbf{z}_{23}, \mathbf{z}_{24}\}$$

and

$$\mathbf{Z}_{24} = \mathbf{Z}_{20} \cup \mathbf{N}_4.$$

We apply the FCRM clustering algorithm to the data set \mathbf{Z}_{20} and \mathbf{Z}_{24} and initialized the elements of the partition matrix \mathbf{F} using random values drawn from $[0, 1]$.

The results of the fuzzy membership values are shown in Table I and the parameter estimates of the FCRM $\mathbf{B}_{FCRM} = [\beta_1, \beta_2]$ are shown in (13) and (14).

$$\mathbf{B}_{FCRM}^{\mathbf{Z}_{20}} = \begin{bmatrix} 5 & -9 \\ -5 & 1 \end{bmatrix}, \quad (13)$$

$$\mathbf{B}_{FCRM}^{\mathbf{Z}_{24}} = \begin{bmatrix} 3.998674 & -9.20997 \\ -5.11839 & 0.92358 \end{bmatrix}. \quad (14)$$

Fig. 2 shows the clustering result of the data set \mathbf{Z}_{24} obtained from the FCRM clustering algorithm.

According to the parameter estimates (14), we see that the noisy data (\mathbf{z}_{21} , \mathbf{z}_{22} , \mathbf{z}_{23} , and \mathbf{z}_{24}) degrade the performance of the FCRM clustering algorithm result in inaccuracy of the parameter estimates \mathbf{B} . Intuitively, the addition of the noisy data should not have high membership degree in either cluster in Table I (red numbers).

III. POSSIBILISTIC C-REGRESSION MODEL

To overcome the aforementioned problem of FCRM, we applied the characteristics of the possibilistic c -means (PCM) [7, 11] clustering algorithm to the FCRM clustering algorithm and proposed a new clustering algorithm named possibilistic c -regression models (PCRM).

Suppose that $\mathbf{Z} = \{\mathbf{z}_1, \dots, \mathbf{z}_L\}$ is an unlabeled data set, where $\mathbf{z}_j = (\mathbf{u}_j, y_j) \in \mathfrak{R}^n \times \mathfrak{R}$ for all $j \in \{1, \dots, L\}$, and let \mathbf{P} denote a $c \times L$ possibilistic c -partition matrix generated by PCRM clustering algorithm as follows [7, 11, 12]:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1L} \\ p_{21} & p_{22} & \dots & p_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ p_{c1} & p_{c2} & \dots & p_{cL} \end{bmatrix} \quad (15)$$

where p_{ij} of \mathbf{P} satisfy the following conditions [7, 11, 12]:

$$p_{ij} \in [0, 1] \text{ for all } i \in \{1, \dots, c\} \text{ and } j \in \{1, \dots, L\}, \quad (16)$$

$$0 < \sum_{j=1}^L p_{ij} < L \text{ for all } j \in \{1, \dots, L\}, \text{ and} \quad (17)$$

$$\max_i p_{ij} > 0 \text{ for all } j \in \{1, \dots, L\}, \quad (18)$$

where c is the number of clusters and p_{ij} is the degree of possibility (typicality) of \mathbf{z}_j belonging to the i th cluster.

The regression model and the estimation error of PCRM are represented as (5) and (6), respectively. The regression model parameters β_i can be calculated by (7).

The objective function for the PCRM clustering algorithm is defined by [7, 11]

$$J_{PCRM}^m(\mathbf{P}, \mathbf{B}) = \sum_{j=1}^L \sum_{i=1}^c (p_{ij})^m (D_{ij}(\beta_i))^2 + \sum_{i=1}^c \gamma_i \sum_{j=1}^L (1 - p_{ij})^m, \quad (19)$$

$$\gamma_i = \rho \frac{\sum_{j=1}^L \sum_{i=1}^c (p_{ij})^m (D_{ij}(\beta_i))^2}{\sum_{j=1}^L (p_{ij})^m} \text{ for all } i \in \{1, \dots, c\},$$

where $\mathbf{B} = (\beta_1, \beta_2, \dots, \beta_c)$, ρ is a constant and $m \in (1, \infty)$ is the weighting exponent. For the PCRM clustering algorithm, the objective is to find (\mathbf{P}, \mathbf{B}) such that $J_{PCRM}^m(\mathbf{P}, \mathbf{B})$ is minimized.

PCRM Clustering Algorithm:

Step 1: Given c ($1 < c < L$), $m \in (1, \infty)$, the termination threshold $\varepsilon > 0$, set an initial possibilistic c -partition matrix

$$\mathbf{P}^{(0)} = \begin{bmatrix} p_{11}^{(0)} & p_{12}^{(0)} & \dots & p_{1L}^{(0)} \\ p_{21}^{(0)} & p_{22}^{(0)} & \dots & p_{2L}^{(0)} \\ \vdots & \vdots & \ddots & \vdots \\ p_{c1}^{(0)} & p_{c2}^{(0)} & \dots & p_{cL}^{(0)} \end{bmatrix} \quad (20)$$

satisfying (16)-(18), and set iteration index $k = 0$.

Step 2: By (7), calculate the parameter vectors

$$\beta_i^{(k)} \text{ for all } i \in \{1, \dots, c\}.$$

Step 3: Update $\mathbf{P}^{(k)}$ to $\mathbf{P}^{(k+1)}$ by [7, 11]

$$p_{ij} = \left(1 + \left[\frac{(D_{ij}(\beta_i^{(k)}))^2}{\gamma_i} \right]^{\frac{1}{m-1}} \right)^{-1} \quad (21)$$

for all $i \in \{1, \dots, c\}$ and $j \in \{1, \dots, L\}$.

Step 4: If $\|\mathbf{P}^{(k)} - \mathbf{P}^{(k+1)}\| < \varepsilon$, stop; otherwise, set $k = k + 1$ and return to **Step 2**.

IV. NUMERICAL EXAMPLES

To illustrate the PCRM clustering algorithm, we consider the following examples and compare the clustering results with that of obtained by FCRM clustering algorithm.

Example 1: Consider the data set \mathbf{Z}_{24} in Section II, choose $c = 2$ and $m = 2$.

We apply the PCRM clustering algorithm to the data set \mathbf{Z}_{24} , the parameter estimates are identical to (13), i.e., the PCRM clustering algorithm alleviates the noisy data effectively. The clustering results are shown in Fig. 3 and Table II. From the simulation results shown in Fig. 3 and Table II, it is seen that the clustering results of the PCRM clustering algorithm are better than the FCRM clustering algorithm.

Example 2: Consider the three linear equations

$$\begin{aligned} y &= 2u + 5 + \varepsilon, \\ y &= -3u + 4 + \varepsilon, \\ y &= 5u - 2 + \varepsilon, \end{aligned}$$

where $\beta_1 = [2 \ 5]^T$, $\beta_2 = [-3 \ 4]^T$, $\beta_3 = [5 \ -2]^T$ and ε is white Gaussian random noise with zero mean and standard deviation σ .

We randomly generate 15 training input points u whose magnitude is uniformly distributed over range $[-1, 1]$ to the three linear equations with noise free ($\varepsilon = 0$). Additionally, randomly generate 5 training input points u whose magnitude is uniformly distributed over range $[-1, 1]$ to the three linear equations, and set the Gaussian random noise ε with standard deviation $\sigma = 1$. The total number of generated data of the noise free data set and noisy data set are 45 and 15, respectively. We choose $c = 3$ and $m = 2$. The data set and the clustering results are shown in Fig. 4 and Table III. From the simulation results shown in Fig. 4 and Table III, it is seen that the FCRM clustering algorithm is sensitive for the noisy data; however, the PCRM clustering algorithm can alleviate the noisy data effectively.

Example 3: Consider the three linear equations

$$\begin{aligned} y &= 10u_1 - u_2 + 3 + \varepsilon, \\ y &= 4u_1 + u_2 - 5 + \varepsilon, \\ y &= -u_1 - 7u_2 + 3 + \varepsilon, \end{aligned}$$

where

$$\begin{aligned} \mathbf{u} &= [u_1 \ u_2]^T, \\ \beta_1 &= [10 \ -1 \ 3]^T, \\ \beta_2 &= [4 \ 1 \ -5]^T, \\ \beta_3 &= [-1 \ -7 \ 3]^T. \end{aligned}$$

We randomly generate 200 training input vectors \mathbf{u} whose magnitude is uniformly distributed over range $[-1, 1]$ to the three linear equations with noise free ($\varepsilon = 0$). Additionally, we randomly generate 100 training input vectors \mathbf{u} whose magnitude is uniformly distributed over range $[-1, 1]$ to the three linear equations, and set the Gaussian random noise ε with standard deviation $\sigma = 0.8$. The total number of generated data of the noise free data set and noisy data set are 600 and 300, respectively. We choose $c = 3$ and $m = 2$. The clustering results are presented in Table IV. From Table IV, it is seen that the PCRM clustering algorithm performs better than the FCRM clustering algorithm for the noisy data clustering.

V. CONCLUSION

In this paper, we have discussed the FCRM clustering algorithm and pointed out the problem of FCRM. We combined the PCM with the FCRM clustering algorithm and proposed the PCRM clustering algorithm. The simulation results shows that the proposed PCRM clustering algorithm can alleviate the noisy data effectively and make the clustering results suitable.

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Table I. The fuzzy membership values resulting from FCRM for Z_{20} and Z_{24} (Noisy data with zero mean and standard deviation 1.5)

$\begin{smallmatrix} z_j \\ j \end{smallmatrix}$	Data		FCRM on Z_{20}		FCRM on Z_{24}	
	u	y	F_1^T	F_2^T	F_1^T	F_2^T
1	-0.84531	-9.22657	1.0000	0.0000	0.003296	0.996704
2	-0.32293	-6.61465	1.0000	0.0000	0.000762	0.999238
3	0.161235	-4.19382	1.0000	0.0000	0.01177	0.98823
4	-0.04953	-5.24765	1.0000	0.0000	0.000216	0.999784
5	0.610641	-1.9468	1.0000	0.0000	0.123565	0.876435
6	0.061556	-4.69222	1.0000	0.0000	0.002545	0.997455
7	-0.54538	-7.7269	1.0000	0.0000	0.00196	0.99804
8	0.41897	-2.90515	1.0000	0.0000	0.998456	0.001544
9	-0.70273	-8.51367	1.0000	0.0000	0.002709	0.997291
10	0.316232	-3.41884	1.0000	0.0000	0.156658	0.843342
11	0.267965	-1.41169	0.0000	1.0000	0.998738	0.001262
12	-0.54139	5.872487	0.0000	1.0000	0.999996	4.00E-06
13	-0.63554	6.719896	0.0000	1.0000	0.999992	7.84E-06
14	-0.66729	7.005655	0.0000	1.0000	0.999991	9.24E-06
15	-0.70079	7.307072	0.0000	1.0000	0.999989	1.07E-05
16	-0.59451	6.350547	0.0000	1.0000	0.999994	6.09E-06
17	0.909918	-7.18926	0.0000	1.0000	0.998908	0.001092
18	-0.96818	9.71365	0.0000	1.0000	0.999977	2.29E-05
19	0.915029	-7.23526	0.0000	1.0000	0.998924	0.001076
20	-0.94862	9.537539	0.0000	1.0000	0.999978	2.21E-05
21	0.942221	-2.35876			0.065467	0.934533
22	-0.40481	-5.17444			0.048304	0.951696
23	0.050146	3.161799			0.947289	0.052711
24	0.724678	-8.52435			0.912022	0.087978

Table II. The parameter estimates of the FCRM and PCRM

$\begin{smallmatrix} B \\ j \end{smallmatrix}$	FCRM		PCRM	
	β_1	β_2	β_1	β_2
	3.998674	-9.20997	5	-9
	-5.11839	0.92358	-5	1

Table III. The parameter estimates of the FCRM and PCRM

$\begin{smallmatrix} B \\ j \end{smallmatrix}$	FCRM			PCRM		
	β_1	β_2	β_3	β_1	β_2	β_3
	2.416417	-2.83823	6.673489	2	-3	5
	5.104936	3.806156	-0.81155	5	4	-2

Table IV. The parameter estimates of the FCRM and PCRM

$\begin{smallmatrix} B \\ j \end{smallmatrix}$	FCRM			PCRM		
	β_1	β_2	β_3	β_1	β_2	β_3
	9.950224	3.992198	-1.02904	10	4	-1
	-0.99794	0.981925	-6.98756	-1	1	-7
	2.970877	-4.97258	2.951883	3	-5	3

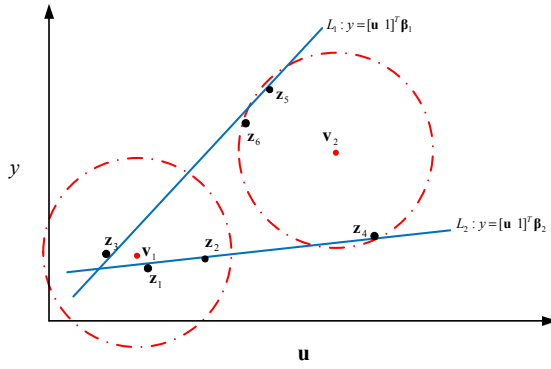


Figure 1. Illustrative example of FCM and FCRM

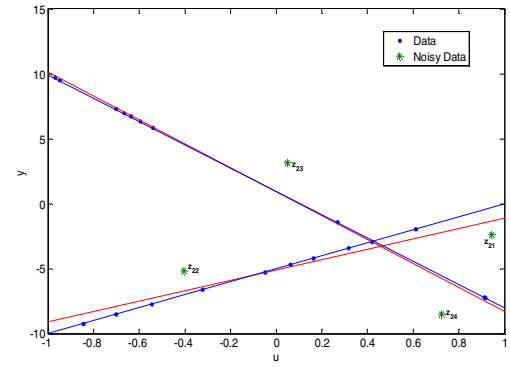


Figure 3. The clustering result of the data set Z_{24} from the FCRM (red line) and PCRM (blue line).

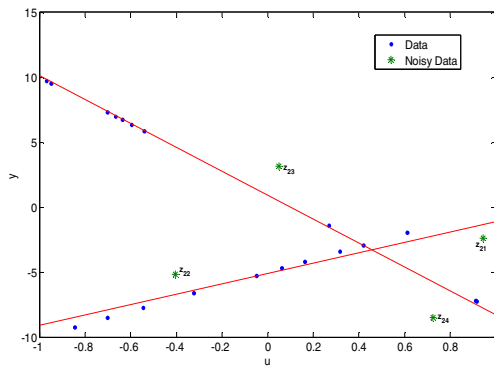


Figure 2. The clustering result of the data set Z_{24} from the FCRM (red line).

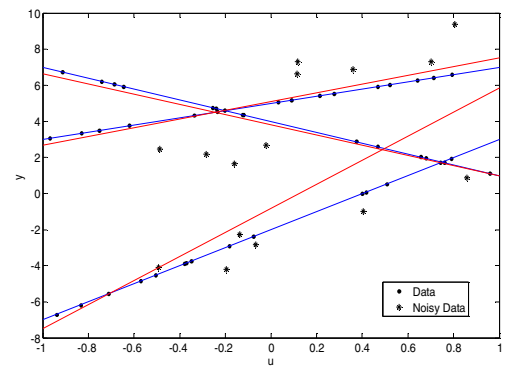


Figure 4. The clustering result of the data set from the FCRM (red line) and PCRM (blue line).