Math 261-001
Quiz 7, October 24

Name	CSU ID #

Be sure to read each question carefully. You must choose and answer **exactly two** of the four problems. If you attempt more than two, only the first two will be graded. Write your final answers in the boxes provided. Each problem is worth the same number of points.

1. Ahhh, look at that, a Green's Theorem problem (so relatable, right?). You are asked to integrate over the triangle with vertices $(0, 0)$, $(1, 2)$, and $(0, 2)$, traversed counterclockwise. Please do the following:

   - Give a parametrization of the boundary of the triangle.
   - In terms of x and y, describe the interior of the triangle.

Boundary:
Interior:

2. In this problem, $\boldsymbol{\nabla}(f)$ is the gradient of a scalar function, $\boldsymbol{\nabla} \times (F)$ is the curl of a vector field F, and $\boldsymbol{\nabla} \cdot (F)$ is its divergence. Do the following:

   - Find $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times F)$ for $F(x, y, z) = (e^y, e^z, e^x)$.
   - Compute $\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot G))$ for $G(x, y, z) = (x^3 y, y^3 z, z^3 x)$.

$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times F) =$
$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot G)) =$

3. Consider the conservative vector field

$$F(x,y,z) = \left(\frac{1}{(x-y)^2} - \frac{1}{(x-z)^2}, \frac{1}{(y-z)^2} - \frac{1}{(y-x)^2}, \frac{1}{(z-x)^2} - \frac{1}{(z-y)^2} \right).$$

Compute its potential.

4. Consider the vector field $F(x, y) = (0, x)$.

- Compute its two-dimensional curl.
- Write down another vector field G with the same curl. (There may be more than one answer.)
- Compute the integral $\oint_C F \cdot d\mathbf{x}$, where C is the circle centered at the origin with radius R. (Hint: use Green's Theorem!)

$\nabla \times (F) =$

$G =$

$\oint_C F \cdot d\mathbf{x} =$