Math 101-002
Exam 2, **March 13**

Name	CSU ID #

Be sure to read each question fully and carefully. Multiple choice answer bubbles must be fully filled in. There is space to the right of each multiple choice question to show work, if your work is correct you can get points even with an incorrect multiple choice answer.

1. For questions 1a through 1k consider the following graph G:
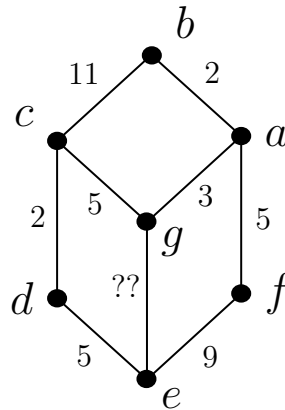
(a) Write down the list of vertices of G: (4 points)

$$V = \{ \qquad\qquad\qquad\qquad \}.$$

(b) Write down the list of edges of G: (4 points)

$$E = \{ \qquad\qquad\qquad\qquad \}.$$

(c) Out of the following, which is **NOT** an edge in the graph? (2 points)
   - ○ ag
   - ○ dg
   - ○ ef
   - ○ bc

(d) How many vertices does this graph have? (2 points)
   - ○ 3
   - ○ 6
   - ○ 7
   - ○ 8

(e) The degrees of the vertices d, e, f and g respectively are: (2 points)

○ $5, 6, 7, 8$

○ $1, 2, 3, 4$

○ $2, 3, 2, 3$

○ $6, 7, 6, 7$

(f) A path of length 3 from a to d passing through f is: (2 points)

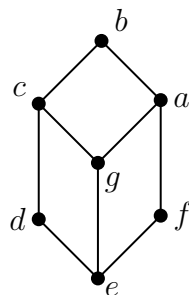○ ab, bc, cd

○ ag, gc, cd

○ ag, ge, ed

○ af, fe, ed

(g) Does this graph have cut-edges? (2 points)

○ Yes.

○ No.

(h) State whether this graph has an Euler tour. If it does, write it down, if not state why it doesn't. (2 points)

(i) State whether this graph has an Hamilton tour. If it does, write it down, if not state why it doesn't. (2 points)

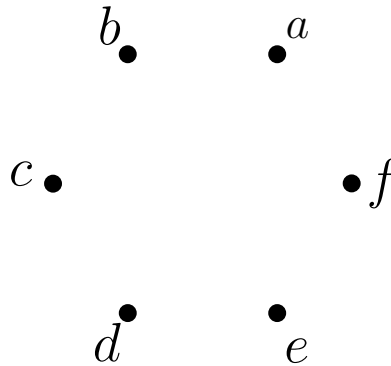(j) The graph G doesn't have an Euler circuit, Eulerize it by adding edges: (2 points)

(k) State whether this graph has an Hamilton tour. If it does, write it down, if not state
why it doesn't. (2 points)

2. For questions 2a through 2g consider the information about the graph G given by the fol-
lowing lists:

$$V = \{a, b, c, d, e, f\}$$
$$E = \{ad, ae, bc, bd, cd, cf, de, df, ef\}$$

(a) Fill in the edges of the graph G: (4 points)

b a

c f

d e

(b) List all the vertices adjacent to d: (2 points)
$$N(d) = \{ \qquad\qquad\qquad\qquad \}$$

(c) What is the degree of the vertex d? (2 points)
$$\deg(d) =$$

(d) Find the sum of the degrees of all vertices: (2 points)
   ○ 6
   ○ 9
   ○ 12
   ○ 18

(e) State whether this graph has an Euler tour. If it does, write it down, if not state why it doesn't. (2 points)

(f) State whether this graph has an Euler circuit. If it does, write it down, if not Eulerize it. (2 points)

(g) For the previous value, which players have veto power, why? (2 points)

○ Both Natalie and Oscar have veto power because all motions can pass without their consideration.

○ (CORRECT) Both Markus and Natalie have veto power because no motion can pass without both of their votes.

○ Both Oscar and Pauline have veto power because no coalition can pass any motion at all.

○ Both Natalie and Pauline have veto power because they need the support of all the players to pass a motion.

3. In this exercise we will explore graphs with Euler and Hamilton walks or circuits. Follow the instructions and complete each task as asked:

- Explain the difference between a walk and a circuit of a graph. (4 points)
- Explain the difference between Eulerian and Hamiltonian paths or circuits. (4 points)
- Draw a graph (doesn't need to be very big) which contains an Euler tour but not an Euler circuit. (4 points)
- Draw a graph which contains a Hamilton tour but doesn't have an Euler tour. (4 points)
- Draw a graph which contains an Euler circuit but not a Hamilton tour. (4 points)
- Extra: ¿Can you draw a graph with an Euler circuit and a Hamilton tour but not a Hamilton circuit? (4 extra points)
- Extra: ¿Can you draw a graph with an Euler tour and a Hamilton circuit but not a Euler circuit? (4 extra points)

Sol:

(a) A walk starts and ends at different places whereas a circuit begins and ends at the same place.

(b) Eulerian means that edges are the object of interest to traverse, while Hamiltonian means that the vertices are the traversed ones.

(c) Any path graph.

(d) A path graph with any pair of middle vertices connected.

(e) Two triangles joined at a vertex, say a ribbon.

(f) The ribbon graph again.

(g) A cycle with more than four vertices with any two non-adjacent vertices connected.