fig 1

fig 2

Let us begin with a simple question:

What are all the quadratic curves which pass through 4 points in general position in $\mathbb{R}^2$?

This question might be a bit tough to tackle right now, but let us consider a simplification. How about if the points are $(1, 1), (1, -1), (-1, -1)$ and $(-1, 1)$? At once the following idea should pop-in in our heads: *a circle*!

The circle which passes through these points is Ideally we would like to stretch and shrink thes circle in order to make it an ellipse. We know ellipses have equations of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

but to begin from our circle equation we will instead add coefficients to the equation $Ax^2+By^2 = 2$. These coefficients are bound by the points in the curve like $(x, y) = (1, 1)$. We may derive the relation

$$A(1)^2 + B(1)^2 = 2 \Rightarrow B = 2 - A$$

so we take $t = A$ to get the parametrized equation $tx^2 + (2 - t)y^2 = 2$. If we let t vary we start seeing the following behavior: If we let $t = 1$ we recover the original circle equation. When $t > 1$ we may choose for example $t = 3$ to get the equation $3x^2 - y^2 = 2$