Exercise 1. Find the value of the sum $\frac{4!}{0!(4-0)!}+\frac{4!}{1!(4-1)!}+\frac{4!}{2!(4-2)!}+\frac{4!}{3!(4-3)!}+\frac{4!}{4!(4-4)!}$. Show your work by calculating the factorials in question.

First we find the values of the factorials in question:

$$0!=1,\ 1!=1,\ 2!=2,\ 3!=6,\ 4!=24.$$

The sum in question is then

$$\frac{24}{1\cdot 24}+\frac{24}{1\cdot 6}+\frac{24}{2\cdot 2}+\frac{24}{6\cdot 1}+\frac{24}{24\cdot 1}=1+4+6+4+1=16.$$

Exercise 2. Suppose we wish to form a committee (president, secretary, treasurer, attorney and communicator) out of five people, U,V,X,Y and Z. In how many ways can this be done? (You may leave your answer in terms of factorials.)

With this in hand, explain what does the factorial mean? (*It's the number of ways such that...*)

There are 5 positions to be held in the committee, the first slot can be held by any of the five people, so 5 options. The second slot can be held by any of the four remaining people. Proceeding in that way we see that there are

$$5\cdot 4\cdot 3\cdot 2\cdot 1=5! \text{ ways to form the committee.}$$

In this sense, the factorial is the number of ways to arrange n distinct objects into n distinct slots.

Exercise 3. Compute the limit $\lim_{n\to\infty}\frac{e^{\frac{\log(n)}{2}}}{n}$.

We may first notice that $e^{\frac{\log(n)}{2}}=\left(e^{\log(n)}\right)^{\frac{1}{2}}=\sqrt{n}$ so we may simplify the expression in the limit to

$$\frac{\sqrt{n}}{n}=\frac{1}{\sqrt{n}}\xrightarrow[n\to\infty]{}0.$$

Exercise 4. Compute the limit $\lim_{n\to\infty}\cos\left(\frac{n!}{n^n}\right)$.

As cosine is a continuous function it suffices to find the limit of the sequence inside and then plug it into the cosine. The limit in question is

$$\lim_{n\to\infty}\frac{n!}{n^n}=0, \text{ because } n!\ll n^n.$$

So the value of our expression is $\cos(0)=1$.