PS2 Putnam II-2023

We will need to use the following toolkit to solve the problems:

- ⋄ Quadratic forms
- ⋄ Relationship between quadratic forms and conics
- ⋄ Minkowski's theorem for the geometry of numbers.

Exercise 1 (Polish Olympiad)**.** Let a, b, c be positive integers with

$$ac = b^2 + b + 1$$

Prove that the equation

$$ax^2 - (2b+1)xy + cy^2 = 1$$

has integer solutions (x, y).

Answer

First observe

Exercise 2 (Hungarian Olympiad)**.** Suppose n is a positive integer such that

$$x^2 + xy + y^2 = n$$

has rational solutions (x, y). Show that the equation also has integer solutions.