HW 5
Math 672

Due Fri, Oct. 7 in class.

0. Read Chapter 4.

1. Consider the parallel lines $L = \mathbb{V}(x)$ and $L' = \mathbb{V}(x-1)$ in $\mathbb{A}^2$. Let $\overline{L}$ and $\overline{L'}$ denote their projective closures in $\mathbb{P}^2$ (embed $\mathbb{A}^2$ into $\mathbb{P}^2$ via the map $(x, y) \mapsto [x : y : 1]$). Recall that $\mathbb{P}^2$ is covered by the open charts $U_x = \{x \neq 0\}$, $U_y = \{y \neq 0\}$ and $U_z = \{z \neq 0\}$. Show that when restricted to the affine chart $U_y \cong \mathbb{A}^2$, the lines $\overline{L}$ and $\overline{L'}$ are no longer parallel.

2. Let $f_i(x_1, \ldots, x_n)$ and $g_i(x_1, \ldots, x_n)$ be polynomials for $1 \leq i \leq m$. Consider the open subset U of $\mathbb{A}^n$ defined by

$$U = \mathbb{A}^n \setminus \mathbb{V}(g(x_1, \ldots, x_n))$$

where $g(x_1, \ldots, x_n) = \prod_{i=1}^m g_i(x_1, \ldots, x_n)$. Define a function $F : U \to \mathbb{A}^m$ by

$$F(x_1, \ldots, x_n) = \left(\frac{f_1(x_1, \ldots, x_n)}{g_1(x_1, \ldots, x_n)}, \ldots, \frac{f_m(x_1, \ldots, x_n)}{g_m(x_1, \ldots, x_n)} \right).$$

Prove that F is a *morphism of quasiprojective varieties* (as defined in section 4.1).

3. 4.2.3

4. Let $U = \mathbb{A}^2 \setminus \{(0, 0), (1, 1)\}$. Find a basis of U consisting of affine varieties.

5. Let $V = \mathbb{A}^2$, let $U_1 = V \setminus \mathbb{V}(x)$, $U_2 = V \setminus (\mathbb{V}(x) \cup \mathbb{V}(x-1))$ and $U_3 = V \setminus \mathbb{V}(x, y)$. Calculate $\mathcal{O}_V(U_i)$ for $1 \leq i \leq 3$.