Exercise 1 (Exercise 1.a). Suppose b > a and one computes $T \leftarrow b \leftarrow a$, in other words, inserting b into T with the RSK insertion algorithm and then inserting a into the result. Show that the insertion path of a lies weakly left of the insertion path of b.

Answer

We will divide the proof into two cases and in both of them we will show that $IP(a) \leq IP(b)$ where IP is the element's insertion path. The cases are:

Either b was inserted at the end of the first row, or it wasn't.

 \diamond Assume b is the last element of the first row. Then, as a < b, a will bump c < b in the first row to the second row.

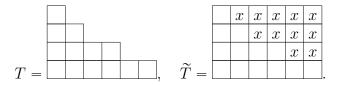
Exercise 2 (Exercise 3). Let π be a permutation in S_n , and let ℓ be the length of the longest increasing subsequence of π and d the length of the longest decreasing subsequence of π . Show that $\ell \cdot d \ge n$.

Answer

Via the RSK bijection we can identify π with a SYT T of shape $\lambda \vdash n$. Also, we have that $\ell = \lambda_1$ and $d = \ell(\lambda)$. Now note that $\lambda_1 \cdot \ell(\lambda) \ge n$.

By considering T we can see that it is a subset of the tableau $(\lambda_1, \ldots, \lambda_1)$ ($\ell(\lambda)$ times) which represents a partition of $\lambda_1 \cdot \ell(\lambda)$. This implies the desired inequality as the size of the table T is n and the next one has size $\lambda_1 \cdot \ell(\lambda)$. Finally the RSK bijection gives us the result we are searching for.

As an example, consider the partitions (6, 4, 2, 1) and (6, 6, 6, 6):



In this case n = 13 and by doing the process we get a partition of 24 which is larger than 13.