Exercise 1. Answer the following prompts, little to no explanation needed:

i) Consider the a geometric sequence (a_n) with initial condition $c=\frac{1}{3}$ and common ratio $r=\frac{-1}{2}$.

- Write an expression for the **general term** of a_n.

A geometric sequence is of the form cr^n so this one is $a_n=(1/3)(-1/2)^n$.

- What is the **value** of a_2?

$a_2=(1/3)(-1/2)^2=1/12$.

- Is this sequence **monotonic**?

No, $a_0=1/3$, $a_1=-1/6$ and $a_2=1/12$. $a_0>a_1$ and $a_1<a_2$ so neither increasing nor decreasing.

- What is the **limit** of this sequence? [**Hint**: If $\lim_{n\to\infty}|a_n|=0$ then $\lim_{n\to\infty}a_n=0$.]

The limit of the sequence $|a_n|=1/(3\cdot 2^n)$ is zero. By the hint, a_n must also converge to zero.

- What can you say about the behavior of the series $\sum_{n=0}^{\infty}a_n$? Is it **convergent or divergent**?

a_n is geometric with common ratio $r=(-1/2)$, then $|r|=\frac{1}{2}<1$ so this series must converge.

ii) Consider the sequence whose general term is $b_n=\frac{2n}{3n+1}$ for $n=0,1,2,...$

- Write the **values** of b_0, b_1, b_2 and b_3.

$b_0=0$, $b_1=1/2$, $b_2=4/7$ and $b_3=3/5$.

- Is this sequence monotonic? **Increasing or decreasing**?

Observing that $0<1/2<4/7<3/5$, it's possible to infer that the sequence is monotonically increasing.

- Is this sequence **bounded**? Above, below or neither?

It is bounded below by 0 and above by 1.

- What is the **limit** of this sequence as $n\to\infty$?

$\frac{2x}{3x+1}\xrightarrow[x\to\infty]{}\frac{\infty}{\infty}$, so by L'Hôpital, this function converges to $\frac{2}{3}$. Thus $b_n\to\frac{2}{3}$.

- What can you say about the behavior of the series $\sum_{n=0}^{\infty}b_n$? Is it **convergent or divergent**?

As $b_n\to\frac{2}{3}\neq 0$ it follows that the series is divergent.