Exercise 1. Draw the $\mathfrak{sl}_3$ crystal for weight $(3, 3, 0)$.

Exercise 2. Prove that the elements of the hyperoctahedral group, written in cycle notation as a permutation on $\{ \pm 1, \dots, \pm n \}$, has all of its cycles coming in either pairs of the form $(a_1 \dots a_k)(-a_1 \cdots - a_k)$, or of the form $(a_1 \dots a_k - a_1 \ - a_2 \cdots - a_k)$.

Exercise 3. Define the Lie algebra $\mathfrak{so}_{2n+1}$ as $\{ X : \ X^{\mathsf{T}} S + SX = 0 \}$ where

$$S = \begin{pmatrix} 1 & & \\ & 0_n & I_n \\ & I_n & 0_n \end{pmatrix}$$

and I_n is the $n \times n$ identity matrix and 1 is in the upper left corner. Write down what an arbitrary element X looks like, and using the fact that with respect to this setup the torus is simply the set of diagonal matrices X satisfying these conditions, explain how one obtains the type B root system.

Exercise 4. What is the dimension of the adjoint representation of $\mathfrak{so}_7$?

Exercise 5. Explain why the set of 5^{th} roots of unity in the plane don't form a root system. Which axioms of root systems does it satisfy?

Exercise 6. Compute the evacuation of the Young tableau below, and then evacuate again, and show you have returned to the starting tableau.

5			
2	7	8	
1	3	4	6

Exercise 7. Compute the Hall-Littlewood polynomial $\tilde{H}_{(2,1,1)}(x; q)$.

Exercise 8. Let $w = w_1 \dots w_n$ be a word of partition content, and suppose $w_1 \neq 1$. Let $w' = w_2 \dots w_n w_1$ be formed by cycling w_1 around to the end of the word. Show that $\boldsymbol{c}(w') = \boldsymbol{c}(w) = 1$ where $\boldsymbol{c}$ is cocharge. This operation is called *cyclage*.

Exercise 9. Give a counterexample showing that the formula in the above problem does not hold in general when $w_1 = 1$.