

Exercise 1. Consider a region R bounded by the curves

$$y = x, \quad y = -x, \quad \text{and} \quad x = 1,$$

additionally the region has density $\rho(y) = e^y$. Now suppose we rotate the region about the axis $y = -4$. Do the following:

- i) Draw the region in question.
- ii) Draw the solid of revolution obtained after rotation.
- iii) Which method should we use to find the volume of this shape?
- iv) Find the bounds of the region. Label them either as $a \leq x \leq b$ or $c \leq y \leq d$.
- v) Find the **GREATER** and **LOWER** curves by writing their equations.
- vi) Find the parameters (*either R, r or r, h*) used to build your area function.
- vii) With the previous information, write out the integral which represents the mass of the solid obtained.

i) See diagram.

ii) See diagram.

iii) As we have a rotation about a $y = b$ line, we can either use shells in y or rings in x . The density is in terms of y so we should use shells in y .

iv) The regions starts at $y = -1$ and ends at $y = 1$. So $-1 \leq y \leq 1$.

v) We must divide the region into two pieces:

$$\begin{cases} \text{GREATER: } x = 1 \\ \text{LOWER1: } x = -y \end{cases} \text{ for } -1 \leq y \leq 0 \quad \text{and} \quad \begin{cases} \text{GREATER: } x = 1 \\ \text{LOWER2: } x = y \end{cases} \text{ for } 0 \leq y \leq 1$$

vi) As we are using shells we must find r, h . These are:

$$\begin{cases} h_1 = \text{GREATER} - \text{LOWER1} = [1 - (-y)] & \text{for } -1 \leq y \leq 0 \\ h_2 = \text{GREATER} - \text{LOWER2} = [1 - (y)] & \text{for } 0 \leq y \leq 1 \\ r = \text{dist}(\text{axis}, \text{coordinate}) = y - (-4) \end{cases}$$

vii) Adding this facts together we get

$$m = \int_{-1}^0 2\pi(y+4)(1+y)e^y dy + \int_0^1 2\pi(y+4)(1-y)e^y dy.$$

Exercise 2. Consider the tank formed after rotating the curve $y = x^3$ with $0 \leq x \leq 1$ about the axis $x = 0$. Suppose tank is filled with *radioactive waste* with density $\rho(y) = 100 + 25y^2$. Do the following:

- i) Draw the curve and the tank formed by rotating.
- ii) Make a diagram of an infinitesimal slice of fluid and label the height and the radius accordingly. With this write an expression for its volume.
- iii) Suppose there's a tube at the top with length 1m. What's the distance from the slice to the top.
- iv) What do the bounds of integration in the work integral represent? Find them and write them as $a \leq y \leq b$.
- v) With the previous information, write an integral expression for the work required to pump out water from the tank.

