Exercise 1. Compute the chromatic symmetric function of the triangle graph, that is, the complete graph K_3, and express it in terms of elementary symmetric functions and in terms of Schur functions.

Answer

Observe that $\chi(K_3) = 3$ which means that there's no proper colorings with 1 or 2 colors. Thus we must color vertices $1, 2, 3$ with colors $i, j, k \in \mathbb{N}$. However there's $3!$ ways of doing this, so that each monomial $x_i x_j x_k$ is accounted $3!$ times. We thus have that

$$X_{K_3} = 3! m_{(1,1,1)} = 3! s_{(1,1,1)} = 3! e_3.$$