

Exercise 1. Consider the vectors $\vec{u} = (1, 3, 4)$ and $\vec{v} = -2\hat{j} + 3\hat{k}$. Evaluate $3\vec{v} + 2\vec{u}$.

1. Identify and explain the mistakes in the following procedures:

$$3(1, 3, 4) + 2(-2\hat{j} + 3\hat{k}) = (3 + 9 + 4) + (-4 + 6) = 18$$

$$\begin{aligned} 3(0\hat{i} - 2\hat{j} + 3\hat{k}) + 2(1, 3, 4) \\ = (0\hat{i} - 2\hat{j} + 3\hat{k}) + (2, 3, 4) \\ = (2, -1, 7) \stackrel{?}{=} 2\hat{i} - \hat{j} + 7\hat{k} \end{aligned}$$

2. Write out the correct procedure and answer.

Mistakes:

Firstly both answers are disorganized which reduces the possibility of understanding the information given.

1. In the first procedure, the substitution is incorrect, and the terms are not added correctly as the result should be in a consistent vector form.
2. In the second procedure, the scaling of the vectors is incorrect.

The last term in the answer, while a repetition of the answer, is not a mistake.

Correct Procedure:

$$\begin{aligned} 3\vec{v} + 2\vec{u} &= 3(0\hat{i} - 2\hat{j} + 3\hat{k}) + 2(1\hat{i} + 3\hat{j} + 4\hat{k}) \\ &= (0, -6, 9) + (2, 6, 8) \\ &= (2, 0, 17) \end{aligned}$$

The correct answer is $(2, 0, 17)$.

Exercise 2. Consider the curve $r(t) = (t, (4 - t^2)^2)$ for $1 \leq t \leq 3$.

1. Find a parametrization of the line segment between the endpoints of this curve.
2. Find the velocity vector for this curve.
3. Verify the following given solution and correct it if there are any mistakes:

$$\begin{aligned} \text{Parametrization: } t(3) + (1-t)(1) &= 2t - 1 \\ \text{Velocity vector: } v(t) = r'(t) &= (1, -4t(4 - t^2)) \end{aligned}$$

1. **Find the endpoints:**

$$\begin{aligned} r(1) &= (1, (4 - 1^2)^2) = (1, 9) \\ r(3) &= (3, (4 - 3^2)^2) = (3, 25) \end{aligned}$$

2. Parametrize the line segment between the endpoints:

$$\begin{aligned}\text{The line segment is: } & t(3,25) + (1-t)(1,9) \\ & = (3t+1-t, 25t+9-9t) \\ & = (2t+1, 16t+9), \quad 0 \leq t \leq 1\end{aligned}$$

3. Find the velocity vector:

$$\begin{aligned}v(t) = r'(t) &= \left(\frac{d}{dt}t, \frac{d}{dt}(4-t^2)^2 \right) \\ &= (1, -4t(4-t^2))\end{aligned}$$

Mistakes:

1. The endpoints are misinterpreted as they should be vectors. The points $t=1$ and $t=3$ represent the initial and end times. There's also no time interval for the new parametrization.
2. The endpoints should be determined by evaluating the curve at the initial and ending times $t=1$ and $t=3$.