Exercise 1. Analyze the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ and determine if it converges or diverges.

Seeing factorials leads us to believe that we can use the ratio test. The consecutive ratios are of the form:

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!(n+1)!}{(2n+2)!}}{\frac{(n!)(n!)}{(2n)!}} = \frac{(2n)!(n+1)!(n+1)!}{(2n+2)!(n!)(n!)} = \frac{(n+1)(n+1)}{(2n+2)(2n+1)} = \frac{n^2+o(n^2)}{4n^2+o(n^2)} \xrightarrow[n\to\infty]{} \frac{1}{4}.$$

By the ratio test, as the consecutive ratios tend to $\frac{1}{4} < 1$, it follows that our series converges.