**Math 101-002**
**Exam 3, April 17**

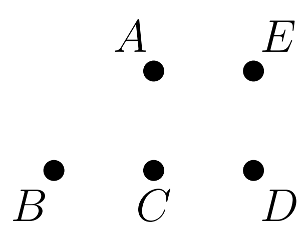
| Name | CSU ID # |
|---|---|
| | |

Be sure to read each question fully and carefully. Multiple choice answer bubbles must be fully filled in. There is space to the right of each multiple choice question to show work, if your work is correct you can get points even with an incorrect multiple choice answer.

1. For questions 1a through 1g consider the following information about a weighted graph $G$ with vertices $A$ through $E$:

| **Edge** | $AB$ | $AC$ | $AD$ | $AE$ | $BC$ | $CD$ | $DE$ |
|---|---|---|---|---|---|---|---|
| **Weight** | 1 | 7 | 10 | 5 | 3 | 4 | 2 |

(a) Based on the information presented in the tableau, fill in the edges of the graph $G$ with their corresponding weights: (4 points)




(b) What is the degree of separation between the vertices $B$ and $D$? (2 points)

◯ 1
◯ 2
◯ 3
◯ 4

(c) What is the degree of separation between the vertices $B$ and $E$? (2 points)

◯ 1
◯ 2
◯ 3
◯ 4

(d) Is there a pair of vertices in $G$ with degree of separation 3? (2 points)

◯ Yes.
◯ No.

(e) The redundancy of the graph $G$ is: (2 points)

$$\mathrm{red}(G) = \_\_\_\_.$$

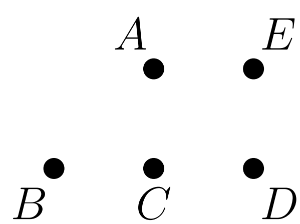
(f) Between the following, mark the option which is not a spanning tree of $G$: (2 points)

- ◯ $AB, BC, CD, AE$
- ◯ $AB, AD, AE, DE$
- ◯ $AC, AD, BC, DE$
- ◯ $AD, AE, BC, CD$

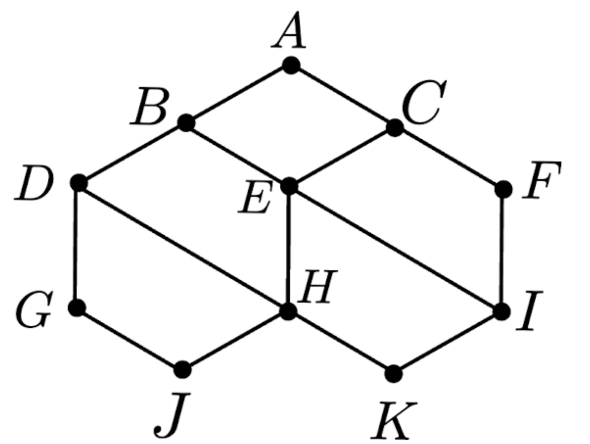
(g) Write down the weight of the minimal spanning tree produced by Kruskal's algorithm: (2 points)

$$\mathrm{weight}(MST) = \_\_\_\_.$$

Help yourself by drawing the tree in question in what follows:



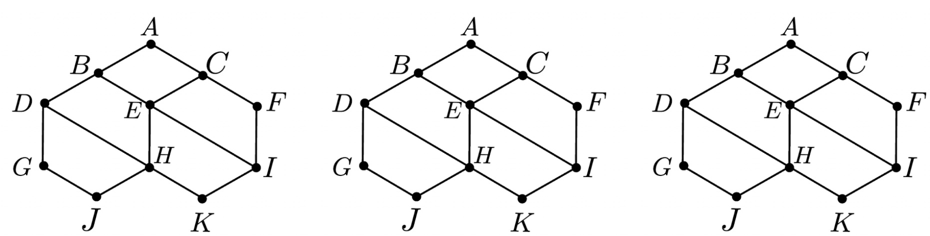
2. For questions 2a through 2d consider the following graph:




(a) Between the following options, which is a spanning tree that makes $D$ and $K$ have degree of separation $4$? (2 points)

- ○ $AB, AC, BD, BE, CF, DG, FI, GJ, HK, IK$
- ○ $AC, BD, BE, CE, CF, DG, EI, GJ, HJ, IK$
- ○ $AC, BE, CF, DH, EI, FI, GJ, HJ, HK, IK$
- ○ $AB, CE, CF, DG, EH, FI, GJ, HJ, HK, IK$

Help yourself with three other copies of the graph:



(b) Write down the degree of separation of the vertices $F$ and $G$: (2 points)

$$d(F, G) = \_\_\_$$

(c) Write down the redundancy of the graph $G$: (2 points)

$$\text{red}(G) = \_\_\_$$

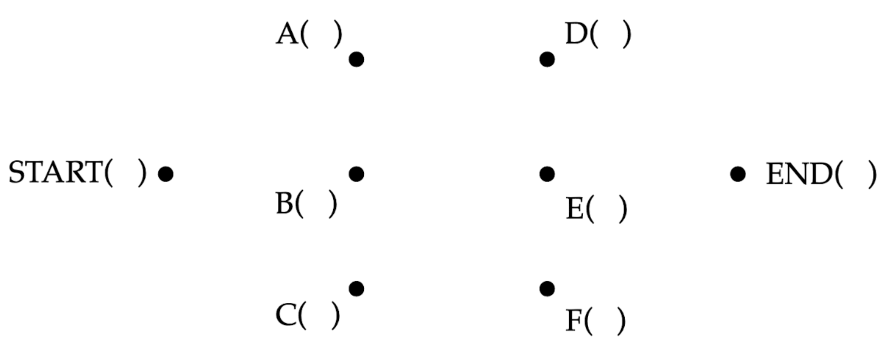
(d) Between the following pairs of vertices, only pair has a different degree of separation. Find it: (2 points)

- ○ $A$ and $H$
- ○ $B$ and $F$
- ○ $I$ and $G$
- ○ $C$ and $J$

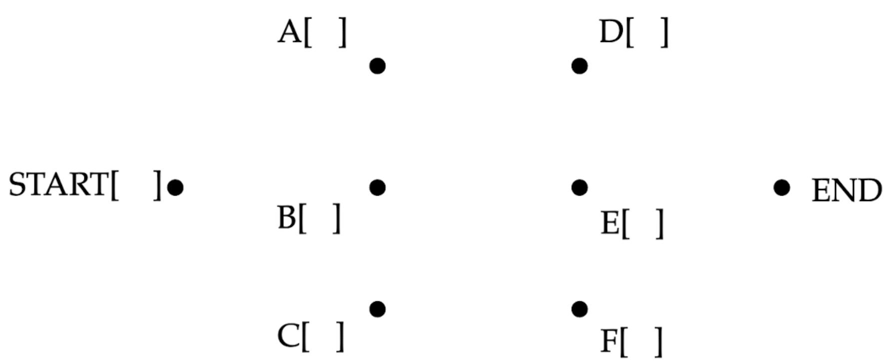
3. For questions 3a through 3e consider the following tableau with information on a project with six tasks $A$ through $E$:

| Task | Time | Precedent tasks |
|---|---|---|
| $A$ | 2 | |
| $B$ | 4 | |
| $C$ | 3 | |
| $D$ | 4 | $A, B$ |
| $E$ | 2 | $A, C$ |
| $F$ | 1 | $B, C$ |

(a) Fill in the project digraph with the processing times and precedence relations: (4 points)




(b) Apply the backflow algorithm to find the critical time of each task: (4 points)



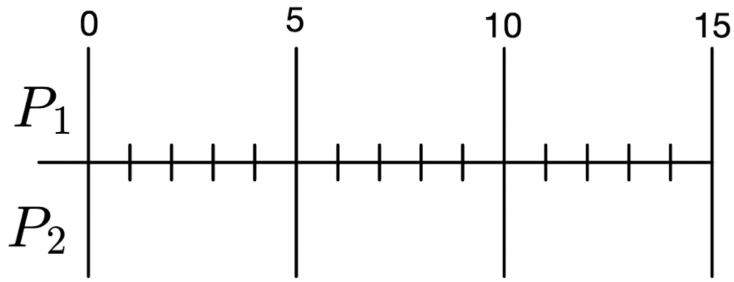
(c) What is the critical time for this project? (2 points)

$$\text{Project Critical Time} = \_\_\_.$$

(d) Write down a priority list of tasks based on the critical path algorithm: (2 points)

$$\text{Project List} = \{\_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_\}.$$

(e) Using the critical path algorithm create a schedule for two workers: (4 points)



4. Short answer:

   (a) Suppose a connected graph has $16$ vertices and $64$ edges. How many edges will any of its spanning trees have? (4 points)

   (b) We defined a *tree* as a connected graph with no cycles. Mention any of the three properties equivalent to being a tree. (4 points)

   (c) Consider your family tree as a graph where the relation is $A$ is connected to $B$ if $A$ is $B$'s kid. Assume you have a nephew, what is the degree of separation between yourself and your nephew? (4 points)

5. In the following space draw the requested graphs:

   (a) A directed graph with a directed cycle. (4 points)

   (b) A directed graph without a directed cycle. (4 points)

6. (25 points) **Discussion:** In class, we've studied idealized scheduling with three key assumptions:

   - **Versatility:** any processor can do any task,
   - **Uniformity:** all processors take the same time per task, and
   - **Persistence:** once a processor starts a task, they finish it without interruption.

   Let's now imagine a **restaurant kitchen**, where a team of chefs is preparing a coordinated multi course meal. Tasks (like dishes or sauces) must follow a strict order. Some can't start until others are partially or fully completed.

   However, real kitchens don't behave like that. For instance:

   - Chef 1 is a pastry expert, but Chef 2 always burns desserts.
   - Chef 3 grills much faster than the others.
   - Chef 2 gets pulled away mid-prep to speak with a health inspector.

   Based on this scenario, address the following points:

   - Which assumptions break down in this context, and how?
   - How would these breakdowns affect the overall scheduling of tasks?
   - How would scheduling be affected with 2 chefs vs. 3 chefs?
   - What kinds of strategies or considerations would help adapt the critical path algorithm (or other scheduling methods) to a real kitchen?

   Use examples from the context and course concepts to support your ideas. There's no single "correct" answer. Focus on thoughtful discussion. Please answer on the next page.

   Rubric:

| Category | Criteria | Points |
|---|---|---|
| Identifying Broken Assumptions | Clearly identifies which of versatility, uniformity, and persistence break down, with examples from the scenario | 0-6 |
| Effects on Scheduling | Discusses how the breakdowns would affect scheduling in practice (e.g., bottlenecks, inefficiencies, delays) | 0-6 |
| Comparison of 2 vs. 3 Chefs | Offers a meaningful comparison of how scheduling changes with fewer or more chefs | 0-5 |
| Adaptation of Strategies | Proposes thoughtful ways to adapt scheduling approaches (e.g., modifying the critical path algorithm, assigning by skill, etc.) | 0-6 |
| Clarity & Organization | Response is clear, coherent, and logically structured | 0-2 |

**Discussion Answer**: