**Exercise 1.** If $V = \mathbb{V}(F_1, \ldots, F_r)$ is an affine variety in $\mathbb{A}^n$, then the tangent bundle $TV$ is a subvariety of $\mathbb{A}^n \times \mathbb{A}^n$. Find the equations defining $TV$ in $\mathbb{A}^n \times \mathbb{A}^n$. You should label your coordinates of $\mathbb{A}^n \times \mathbb{A}^n$ as $(x_1, \ldots, x_n, y_1, \ldots, y_n)$. Do the case $r = 1$ first.

Answer

**Exercise 2.** Let $\Gamma$ be the graph of a rational map $X \dashrightarrow Y$. Prove that the projection $\Gamma \to X$ is a birational equivalence.

Answer

**Exercise 3.** Recall that the Cremona transform is the rational map $\phi : \mathbb{P}^n \dashrightarrow \mathbb{P}^n$ defined as

$$[x_0 : \cdots : x_n] \mapsto [1/x_0 : \cdots : 1/x_n].$$

Find equations defining the graph of $\phi$ as a subvariety of $\mathbb{P}^n \times \mathbb{P}^n$.

Answer

**Exercise 4.** Let $B$ be the blowup of $\mathbb{P}^2$ at $[0 : 0 : 1]$. Find equations defining $B$ as a subvariety of $\mathbb{P}^2 \times \mathbb{P}^1$. Show that there is a morphism defined everywhere from $B$ to $\mathbb{P}^1 \times \mathbb{P}^1$.

Answer

**Exercise 5.** An algebraic variety is *rational* if it's birationally equivalent to projective space (of some dimension). Show that the nodal plane curve defined by the equation $y^2 - x^2 - x^3 = 0$ is rational. [[ Hint: Project from the node. ]]

Answer