

Exercise 1. Consider the vectors $\vec{u} = \langle 1, 3, 4 \rangle$ and $\vec{v} = -2\hat{j} + 3\hat{k}$. Examine the following attempt to evaluate $3\vec{v} + 2\vec{u}$:

$$3\vec{v} + 2\vec{u} = 3\langle 1, 3, 4 \rangle + 2(-2\hat{j} + 3\hat{k}) \quad (1)$$

$$= (3+9+4) + 2(-2\hat{j} + 3\hat{k}) \quad (2)$$

$$= 16 + 2(-2\hat{j} + 3\hat{k}) \quad (3)$$

$$= 18(-2\hat{j} + 3\hat{k}) \quad (4)$$

$$= (-16\hat{j} + 21\hat{k}) \quad (5)$$

$$= \langle -16, 0, 21 \rangle \quad (6)$$

In the following space complete the tasks below:

- Answer: should the result of this operation be a vector or a scalar?
- Identify at least 3 mistakes in the process. Refer to the equation numbers where errors occur.

Expected Result: The result should be a vector since we are performing vector addition.

Errors Identified:

- **Line 1:** The vectors \vec{u} and \vec{v} are incorrectly placed.
- **Line 2:** The components of \vec{u} are incorrectly added together instead of remaining as a vector.
- **Line 4:** An incorrect scalar addition is performed, it's impossible to add a scalar to a scalar multiplying a vector.
- **Line 5:** The components are added instead of multiplied by the scalar.
- **Line 6:** The y component is misplaced as the x component.

Exercise 2. Consider the following pairs of vectors:

$$\hat{i} + \sqrt{3}\hat{j} \quad \text{and} \quad \sqrt{3}\hat{i} + 3\hat{j} \quad (1)$$

$$2\hat{i} + 4\hat{j} + 6\hat{k} \quad \text{and} \quad 4\hat{i} + 6\hat{j} + 8\hat{k} \quad (2)$$

$$\hat{i} - \sqrt{3}\hat{j} + \hat{k} \quad \text{and} \quad -3\hat{i} + 3\hat{k} \quad (3)$$

In the following space complete the tasks below:

- Define parallel and orthogonal vectors.
- Identify which pair is parallel, which is orthogonal, and which is neither.

Definitions:

- Two vectors are **parallel** if one is a scalar multiple of the other.
- Two vectors are **orthogonal** if their dot product equals zero.

Solutions:

- **First pair:** Parallel, because $\sqrt{3}\langle 1, \sqrt{3} \rangle = \langle \sqrt{3}, 3 \rangle$.
- **Second pair:** Neither, as they are not multiples of each other and their dot product is not zero.

- **Third pair:** Orthogonal, because

$$\langle 1, -3, 1 \rangle \cdot \langle -3, 0, 3 \rangle = (1)(-3) + (-3)(0) + (1)(3) = 0.$$