**Exercise 1.** Consider a region $R$ bounded by the curves

$$y=x, \quad y=-x, \quad \text{and} \quad x=1,$$

additionally the region has density $\rho(y)=e^y$. Now suppose we rotate the region about the axis $y=-4$. Do the following:

I) Draw the region in question.
II) Draw the solid of revolution obtained after rotation.
III) Which method should we use to find the volume of this shape?
IV) Find the bounds of the region. Label them either as $a \leqslant x \leqslant b$ or $c \leqslant y \leqslant d$.
V) Find the `GREATER` and `LOWER` curves by writing their equations.
VI) Find the parameters (*either* $R$,$r$ *or* $r$,$h$) used to build your area function.
VII) With the previous information, write out the integral which represents the mass of the solid obtained.

I) See diagram.
II) See diagram.
III) As we have a rotation about a $y=b$ line, we can either use shells in $y$ or rings in $x$. The density is in terms of $y$ so we should use shells in $y$.
IV) The regions starts at $y=-1$ and ends at $y=1$. So $-1 \leqslant y \leqslant 1$.
V) We must divide the region into two pieces:

$$\begin{cases} \text{GREATER: } x=1 \\ \text{LOWER1: } x=-y \quad \text{for} \quad -1 \leqslant y \leqslant 0 \end{cases} \quad \text{and} \quad \begin{cases} \text{GREATER: } x=1 \\ \text{LOWER2: } x=y \quad \text{for} \quad 0 \leqslant y \leqslant 1 \end{cases}$$

VI) As we are using shells we must find $r$,$h$. These are:

$$\begin{cases} h_1=\text{GREATER}-\text{LOWER1}=[1-(-y)] \quad \text{for} \quad -1 \leqslant y \leqslant 0 \\ h_2=\text{GREATER}-\text{LOWER2}=[1-(y)] \quad \text{for} \quad 0 \leqslant y \leqslant 1 \\ r=\text{dist(axis,coordinate)}=y-(-4) \end{cases}$$

VII) Adding this facts together we get

$$m=\int_{-1}^{0} 2\pi(y+4)(1+y)e^y \mathrm{d}y+\int_{0}^{1} 2\pi(y+4)(1-y)e^y \mathrm{d}y.$$

**Exercise 2.** Consider the tank formed after rotating the curve $y=x^3$ with $0 \leqslant x \leqslant 1$ about the axis $x=0$. Suppose tank is filled with *radioactive waste* with density $\rho(y)=100+25y^2$. Do the following:

I) Draw a cross-section of the tank.
II) Make a diagram of an infinitesimal slice of fluid and label the height and the radius accordingly.
III) Write a expression that describes the radius of the infinitesimal cylinder in question.
IV) Write an expression for weight of the water at any particular height.
V) Suppose a hose of length 1 sits at the top of the tank. Write the bounds of the work integral given this.
VI) With the previous information, write an integral expression for the work required to pump out water from the tank.