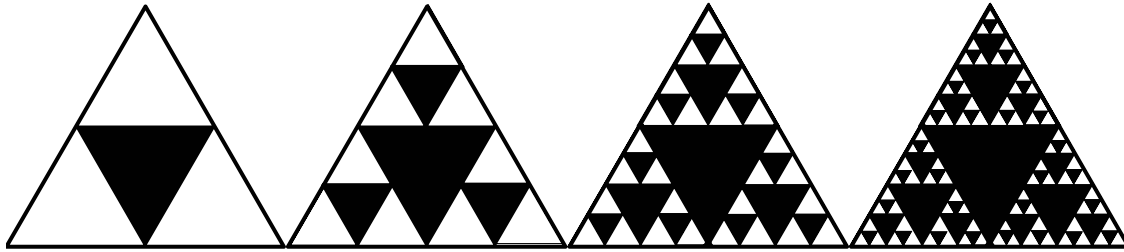


Exercise 1. Consider the series $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^{5r}}$ where r is a real number and do the following:

- i) Identify the general term of the series and simplify it.
- ii) Is our series a p -series? How can you justify that?
- iii) State the convergence result for p series. This means, given ANY p -series $\sum \frac{1}{n^p}$, for what values of p will that converge?
- iv) Determine for which values of r will our series converge. [Hint: The answer to this item is NOT the same as the last item.]

Exercise 2 (Filling a triangle). Consider an empty triangle of area A which we start filling with smaller triangles. The objective of this question is to determine if we can fill completely the triangle in question.



We start adding a triangle with area $a_1 = \frac{A}{4}$ in the middle, then the second step adds 3 triangles of area $\left(\frac{A/4}{4}\right) = \frac{A}{16}$. So in total we are adding an area of $a_2 = \frac{3A}{16}$.

- i) In the third step, how many triangles do we add? What is the area of each of the new smaller triangles? In total how much area a_3 are we adding in the third step?
- ii) Derive a formula for the area added a_n at the n^{th} step by considering how many triangles are we adding and the area of each of those new triangles.
- iii) As a sequence, is a_n geometric? If it is, what's is initial term and common ratio?
- iv) Consider the series $\sum_{n=1}^{\infty} a_n$, in terms of area, what does this series represent? What do the partial sums represent?
- v) Write an expression for the partial sums of this series.
- vi) Using the information above, determine if we fill up the triangle.