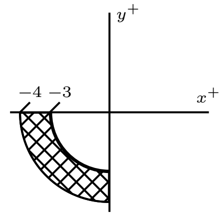
**Exercise 1.** Consider the figure which describes a region in the plane. The marked points are $x = -4$ and




$x = -3$. Describe the region in polar coordinates.

The region is described by a circular section with radii $3 \leqslant r \leqslant 4$. Since we are on the whole 3rd quadrant, this gives us values of $\theta$ from $\pi$ to $3\pi/2$.

**Exercise 2.** Consider the complex number $z = e^{2\pi i/3}$. Show that the argument of $\bar{z}$ coincides with the argument $z^2$.

The number $\bar{z}$ is $e^{-2\pi i/3}$. To find the argument between 0 and $2\pi$ we add $2\pi$ to the multiple of $i$ in the exponent.

$$2\pi - \frac{2\pi}{3} = \frac{6\pi - 2\pi}{3} = \frac{4\pi}{3}.$$

On the other hand, $z^2 = e^{4\pi i/3}$, so the argument is $\frac{4\pi}{3}$. It follows that the arguments coincide.

**Exercise 3.** Consider the complex number $w = 2i$. What is the real part of $\frac{w}{1+w^2}$?

The number in question is

$$\frac{2i}{1+(2i)^2} = \frac{2i}{1+4i^2} = \frac{-2i}{3}.$$

Therefore the real part of the number in question is zero.

**Exercise 4.** Solve the equation $z + 3\bar{z} = 8 - 5i$ where $z$ is a complex number.

If $z = x + iy$, then $\bar{z} = x - iy$. The equation in question becomes

$$x + iy + 3(x - iy) = 8 - 5i \Rightarrow 4x - 2iy = 8 - 5i.$$

The real and imaginary parts must coincide so we get

$$4x = 8,\ -2y = -5 \Rightarrow\ x = 2,\ y = \frac{5}{2}.$$

Therefore our complex number is $z = 2 + \frac{5i}{2}$.