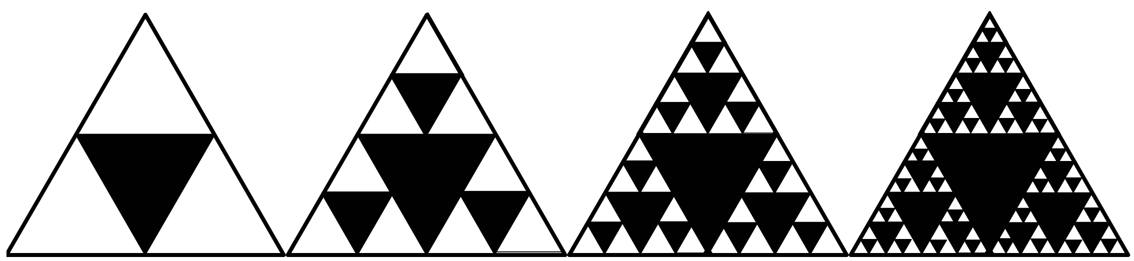
Exercise 1. Consider the series $\sum_{n=1}^{\infty} \frac{3\sqrt{n^3}}{7n^{5r}}$ where r is a real number and do the following:

I) Identify the general term of the series and simplify it.
II) Is our series a p-series? How can you justify that?
III) State the convergence result for p series. This means, given ANY p-series $\sum \frac{1}{n^p}$, for what values of p will that converge?
IV) Determine for which values of r will our series converge. [Hint: The answer to this item is NOT the same as the last item.]

1. The general term is $\frac{3\sqrt{n^3}}{7n^{5p}}$. We may simplify it to $\frac{3}{7}\frac{1}{n^{5r-3/2}}$.
2. Indeed it's a p-series, as the general term looks like 1 over n to some power, in this case $5r - \frac{3}{2}$.
3. A general p series converges when $p > 1$.
4. We require that $5r - \frac{3}{2} > 1$ so solving for r we get $5r > \frac{5}{2}$ which means that $r > \frac{1}{2}$. So for values of r larger than $\frac{1}{2}$, our series converges.

Exercise 2 (Filling a triangle)**.** Consider an empty triangle of area A which we start filling with smaller triangles. The objective of this question is to determine if we can fill completely the triangle in question.



We start adding a triangle with area $a_1 = \frac{A}{4}$ in the middle, then the second step adds 3 triangles of area $\left(\frac{A/4}{4}\right) = \frac{A}{16}$. So in total we are adding an area of $a_2 = \frac{3A}{16}$.

I) In the third step, how many triangles do we add? What is the area of each of the new smaller triangles? In total how much area a_3 are we adding in the third step?
II) Derive a formula for the area added a_n at the n^{th} step by considering how many triangles are we adding and the area of each of those new triangles.
III) As a sequence, is a_n geometric? If it is, what's is initial term and common ratio?
IV) Consider the series $\sum_{n=1}^{\infty} a_n$, in terms of area, what does this series represent? What do the partial sums represent?
V) Using the information above, determine if we fill up the triangle.

1. TO DO