HW 8
Math 672

Due Fri, Dec. 2 in class.

0. Read Chapter 8.

1. Recall that $\mathbb{P}^n$ is defined (as a set) to be the set of equivalence classes of points in $\mathbb{C}^{n+1} \setminus \mathbf{0}$ where
$$(x_0, \dots, x_n) \equiv (x'_0, \dots, x'_n)$$
if there exists a complex number $\lambda \in \mathbb{C} \setminus \mathbf{0}$ such that $x_i = \lambda \cdot x'_i$ for all i.
For $d \in \mathbb{Z}$ an integer, define $\mathcal{O}_{\mathbb{P}^n}(d)$ to be the set of equivalence classes of points in $(\mathbb{C}^{n+1} \setminus \mathbf{0}) \times \mathbb{C}$ where
$$(x_0, \dots, x_n, t) \equiv (x'_0, \dots, x'_n, t)$$
if there exists a complex number $\lambda \in \mathbb{C} \setminus \mathbf{0}$ such that $x_i = \lambda \cdot x'_i$ for all i *and* $t = \lambda^d t'$.
There is a map $\pi : \mathcal{O}_{\mathbb{P}^n}(d) \to \mathbb{P}^n$ which forgets the last coordinate.
What are the fibers of the map π? What is another way of writing this space if $d = 0$?

2. Show that the map $\pi : \mathcal{O}_{\mathbb{P}^n}(d) \to \mathbb{P}^n$ by finding a local trivialization $\{U_i, \phi_i\}$. Using this trivialization, what are the maps
$$\psi_{ij} : U_i \cap U_j \times \mathbb{C} \to U_i \cap U_j \times \mathbb{C}$$
where recall that ψ_{ij} is defined to be $\phi_j \circ \phi_i^{-1}|_{\pi^{-1}(U_i \cap U_j)}$.

3. A *section* of $\mathcal{O}_{\mathbb{P}^n}(d)$ is a morphism $s : \mathbb{P}^n \to \mathcal{O}_{\mathbb{P}^n}(d)$ such that $\pi \circ s$ is the identity map on $\mathbb{P}^n$. The space of sections of $\mathcal{O}_{\mathbb{P}^n}(d)$ is a finite dimensional vector space for each d. Find a basis for the space of sections of $\mathcal{O}_{\mathbb{P}^n}(d)$.

4. (Read 8.5 for help) Fix some $d > 1$. Consider the map from $\mathbb{P}^n$ to a (larger) projective space defined by the complete linear series
$$|\mathcal{O}_{\mathbb{P}^n}(d)|.$$
Prove that this map is the same as the Veronese embedding.

5. 8.4.1