Exercise 1. Consider a *wing of a plane* with density $\rho(x)=\frac{1}{x+1}$. The wing of the plane is bounded by the curves

$$y=0, \quad x=0, \quad x=3, \quad \text{and} \quad \text{a line } L \text{ connecting (0,1) and (3,1/3).}$$

I) Make a drawing which represents the wing in question.
II) Use the point slope formula to determine an equation for the line L.
III) If the wing is flat, in which order should we setup the integral to find its mass? dx or dy?
IV) Indicate in your diagram the bounds of integration. Write them as well as $a\leqslant x\leqslant b$ or $c\leqslant y\leqslant d$ depending on your choice for order of integration.
V) Find the `GREATER` and `LOWER` curves. Label them.
VI) Use the previous information to find the mass of the wing in question.

I) See diagram.
II) The points in question are (0,1) and (3,1/3) so the slope of the line is $m=\frac{1/3-1}{3-0}=\frac{-2}{9}$. From this we get

$$y=\frac{-2}{9}x+b\Rightarrow b=0(-2/9)+1=1\Rightarrow y=\frac{-2}{9}x+1.$$

III) Since the density is in x we should use a dx integral.
IV) See diagram and $0\leqslant x\leqslant 3$.
V) The greater one is $y=\frac{-2}{9}x+1$ and the lower one is $y=0$.
VI) The mass of the wing will be

$$\int_0^3 \frac{1}{x+1}\left(\frac{-2}{9}x+1-0\right)\mathrm{d}x.$$

Exercise 2. Consider the region in the 1st quadrant enclosed by the curves

$$y=x-2, \quad \text{and} \quad x=3.$$

Now suppose we rotate the region about the axis $x=2$. Do the following:

I) Draw the region in question.
II) Draw the solid of revolution obtained after rotation.
III) Which 2 methods can we use to find the volume of this shape? Recall the possibilities are rings/shells on x/y.
IV) Given your method of choice, find the bounds of the region. Label them either as $a\leqslant x\leqslant b$ or $c\leqslant y\leqslant d$.
V) Find the `GREATER` and `LOWER` curves. Label them.
VI) Use the previous information find the pair of parameters R,r or r,h given your choice of method. Label them.
VII) Construct the area function of your method. Label it.
VIII) With the previous information, find the volume of the shape in question.

I) See diagram.
II) See diagram.
III) As the region is being rotated about $x=2$ we can use shells in x or rings in y.
IV) The bounds are in x: $2\leqslant x\leqslant 3$ or in y: $0\leqslant y\leqslant 1$.
V) In dx order,

$$\text{Greater: } y=x-2, \quad \text{and} \quad \text{Lower: } y=0.$$

While in dy order

$$\text{Greater: } x=3, \quad \text{and} \quad \text{Lower: } x=y+2.$$

VI) In terms of rings in y we have

$$R=(3)-(2), \quad \text{and} \quad r=(3)-(y+2).$$

While using shells in x we have

$$h=(x-2)-(0), \quad \text{and} \quad r=(x)-(2).$$

VII) The area functions are

$$A(y)=\pi\left\{(3-2)^2-(3-(y+2))^2\right\}, \quad \text{and} \quad A(x)=2\pi(x-2)(x-2).$$

VIII) The volume of the shape is

$$V = \int_0^1 \pi\big[(3-2)^2-(3-(y+2))^2\big]\mathrm{d}y = \int_2^3 2\pi(x-2)(x-2)\mathrm{d}x.$$