Exercise 1. Explain what does the mnemonic **LIATE** or **LIPTE** means and how should one use it when integrating by parts.

LI(A/P)TE is an acronym which means Logarithm, Inverse (trigonometric), Algebraic/Polynomial (function), Trigonometric, Exponential. It determines the order of precedence for which function to differentiate when using integration by parts.

Exercise 2. When using integration by parts, give an example of an integral where $\mathrm{d}v = 1\mathrm{d}x$ and u is the function we are looking to integrate. It is **not required** to solve the integral.

Any integral of the form $\int \log(x)\mathrm{d}x$, $\int \arcsin(x)\mathrm{d}x$, $\int \arctan(x)\mathrm{d}x$ is a valid answer. Also the integral $\int \log^2(x)\mathrm{d}x$, the third question of this quiz, is a valid answer.
Answers such as $\int xe^x\mathrm{d}x$, $\int x\sin(x)\mathrm{d}x$ are not, because you have to take $\mathrm{d}v = x\mathrm{d}x$, for example.
There are some edge cases where another integral will be accepted as an answer.

Exercise 3. Find an antiderivative for the function $\log^2(x)$ using integration by parts.

By integration by parts:

$$\begin{cases} u = \log^2(x) \Rightarrow \mathrm{d}u = \dfrac{2\log(x)}{x}\mathrm{d}x \\ \mathrm{d}v = \mathrm{d}x \Rightarrow v = x \end{cases} \Rightarrow I = x\log^2(x) - \int 2\log(x)\mathrm{d}x = \underline{x\log^2(x) - 2x\log(x) + 2x}$$

Exercise 4. Use integration by parts to evaluate the integral $\displaystyle\int \left(\frac{\log(x)}{x}\right)^3 \mathrm{d}x$.

By integration by parts:

$$\begin{cases} u = \log^3(x) \Rightarrow \mathrm{d}u = \dfrac{3\log^2(x)}{x}\mathrm{d}x \\ \mathrm{d}v = \dfrac{1}{x^3}\mathrm{d}x \Rightarrow v = \dfrac{-1}{2x^2} \end{cases} \Rightarrow I = \frac{-\log^3(x)}{2x^2} - \int \frac{-3\log^2(x)}{2x^3}\mathrm{d}x = \frac{-\log^3(x)}{2x^2} + \frac{3}{2}I_2$$

Applying parts to I_2 we get

$$\begin{cases} u_2 = \log^2(x) \Rightarrow \mathrm{d}u_2 = \dfrac{2\log(x)}{x}\mathrm{d}x \\ \mathrm{d}v_2 = \dfrac{1}{x^3}\mathrm{d}x \Rightarrow v_2 = \dfrac{-1}{2x^2} \end{cases} \Rightarrow I_2 = \frac{-\log^2(x)}{2x^2} - \int \frac{-\log(x)}{x^3}\mathrm{d}x = -\frac{\log^2(x)}{2x^2} + I_3$$

Once again we take

$$\begin{cases} u_3 = \log(x) \Rightarrow \mathrm{d}u_3 = \dfrac{1}{x}\mathrm{d}x \\ \mathrm{d}v_3 = \dfrac{1}{x^3}\mathrm{d}x \Rightarrow v_3 = \dfrac{-1}{2x^2} \end{cases} \Rightarrow I_3 = \frac{-\log(x)}{2x^2} - \int \frac{-1}{2x^3}\mathrm{d}x = -\frac{\log(x)}{2x^2} + \frac{1}{2}\left(\frac{-1}{2x^2}\right)$$

And collecting terms we get

$$I = \frac{-\log^3(x)}{2x^2} + \frac{3}{2}\left[-\frac{\log^2(x)}{2x^2} + \left(-\frac{\log(x)}{2x^2} + \frac{1}{2}\left(\frac{-1}{2x^2}\right)\right)\right] = \underline{\frac{-\log^3(x)}{2x^2} - \frac{3\log^2(x)}{4x^2} - \frac{3\log(x)}{4x^2} - \frac{3}{8x^2}}$$