

Exercise 1. Describe a “ballot-type” condition for a word of 1’s and 2’s to be lowest weight for \mathfrak{sl}_2 , that is, that F sends the word to 0. Prove that your condition is correct. Do the same for \mathfrak{sl}_3 and the two lowering operators.

Answer

Recall a word is Yamanouchi when every suffix has partition content. This occurs when reading right-to-left, $\#i's \geq \#i + 1's$. The reverse condition of this is precisely being a ballot word, when every prefix left-to-right has more $i + 1$'s than i 's.