Math 101-002
Exam 2, March 13

Name	CSU ID #

Be sure to read each question fully and carefully. Multiple choice answer bubbles must be fully filled in. There is space to the right of each multiple choice question to show work, if your work is correct you can get points even with an incorrect multiple choice answer.

1. For questions 1a through 1d consider the following graph:

   ADD Drawing

   (a) What is the degree of the vertex A? (2 points)

   ◯ choices choices

   (b) Which edges are bridges (cut-edges)? (2 points)

   ◯ it's the skyfall (music cue)

   (c) How many vertices does this graph have? (2 points)

   ◯ meik et beter

   (d) Find the number of edges in this graph:

   ◯ es el skaifol

2. For questions 2a through 2i consider the following graph:

   ADD Drawing

   (a) If each person gets a vote per each \$1000 they put into the fund, write down the weighted voting scheme for this setting assuming Markus gets M votes: (3 points)

$$[24 : M, \quad , \quad , \quad] = [24 : M, 12, 7, 4]$$

(b) Find the minimum and maximum values for Markus' share of votes M, pick 2 options: (4 points)

- ◯ 7
- ◯ 12 (CORRECT)
- ◯ 17
- ◯ 19
- ◯ 23
- ◯ 24 (CORRECT)

(c) Which value of M results in someone being a dictator? (2 points)

- ◯ 7
- ◯ 12
- ◯ 17
- ◯ 19
- ◯ 23
- ◯ 24 (CORRECT)

(d) Using the value of M you chose in the previous item, who is a dictator, why? (2 points)

- ◯ (CORRECT) Markus is a dictator because they have enough votes to pass a motion single-handedly.
- ◯ Markus and Natalie are dictators because any motion can pass without their votes.
- ◯ Natalie is a dictator because the other people can pass motions without them.
- ◯ Both Markus and Natalie are dictators because no motion can pass without both of their votes.

(e) Recall a player in a Weighted Voting Scheme has *no power* when they have no say in the outcome of the voting. For there to be exactly one player with no power, the value of M must be between... Pick 2 options: (4 points)

- ◯ 7
- ◯ 12
- ◯ 17 (CORRECT)
- ◯ 19 (CORRECT)
- ◯ 23
- ◯ 24

(f) For the previous values of M who is the player with no power? (2 points)

- ◯ Oscar has no power because no motion can pass without their votes.
- ◯ Oscar has no power because both Markus and Natalie or Markus and Pauline can pass resolutions on their own meaning Oscar's vote doesn't influence the decision.
- ◯ Pauline has no power because they can pass motions single-handedly.
- ◯ (CORRECT) Pauline has no power because both Markus and Natalie or Markus and Oscar can pass resolutions on their own meaning Pauline's vote doesn't influence the decision.

(g) Which values of M result in players (can be one, or more than one) with veto power? Pick 2 options: (4 points)

- ◯ 7
- ◯ 12 (CORRECT)
- ◯ 17
- ◯ 19
- ◯ 23 (CORRECT)
- ◯ 24

(h) Which value of M result in *exactly two* players with veto power? (2 points)

- ◯ 7
- ◯ 12 (CORRECT)
- ◯ 17
- ◯ 19
- ◯ 23
- ◯ 24

(i) For the previous value, which players have veto power, why? (2 points)

- ◯ Both Natalie and Oscar have veto power because all motions can pass without their consideration.
- ◯ (CORRECT) Both Markus and Natalie have veto power because no motion can pass without both of their votes.
- ◯ Both Oscar and Pauline have veto power because no coalition can pass any motion at all.
- ◯ Both Natalie and Pauline have veto power because they need the support of all the players to pass a motion.

3. In this exercise we will explore graphs with Euler and Hamilton walks or circuits. Follow the instructions and complete each task as asked:

   - Explain the difference between a walk and a circuit of a graph. (4 points)
   - Explain the difference between Eulerian and Hamiltonian paths or circuits. (4 points)
   - Draw a graph (doesn't need to be very big) which contains an Euler tour but not an Euler circuit. (4 points)
   - Draw a graph which contains a Hamilton tour but doesn't have an Euler tour. (4 points)
   - Draw a graph which contains an Euler circuit but not a Hamilton tour. (4 points)
   - Extra: ¿Can you draw a graph with an Euler circuit and a Hamilton tour but not a Hamilton circuit? (4 extra points)
   - Extra: ¿Can you draw a graph with an Euler tour and a Hamilton circuit but not a Euler circuit? (4 extra points)

   Sol:

   (a) A walk starts and ends at different places whereas a circuit begins and ends at the same place.

   (b) Eulerian means that edges are the object of interest to traverse, while Hamiltonian means that the vertices are the traversed ones.

   (c) Any path graph.

   (d) A path graph with any pair of middle vertices connected.

   (e) Two triangles joined at a vertex, say a ribbon.

   (f) The ribbon graph again.

   (g) A cycle with more than four vertices with any two non-adjacent vertices connected.