Exercise 1. Suppose we are lifting an empty open box with a *weightless rope* up a building and after some time it starts raining. We will calculate the work necessary to pull the box up given the following conditions:

$$\begin{cases} \text{Weight of box} = w_B = 16 \text{ N}, \\ \text{Capacity of box} = \text{cap}_B = 10 \text{ N}, \\ \text{Length of rope} = \ell_R = 15 \text{ m}. \end{cases} \qquad \begin{cases} \text{Pulling velocity} = v_{\text{pull}} = 1 \text{ ms}^{-1}, \\ \text{Rain starts at } t_{\text{rain}} = 5 \text{ s}, \\ \text{Rain speed} = v_{\text{rain}} = 5 \text{ Ns}^{-1}. \end{cases}$$

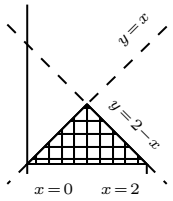
i) Will the box be filled with water **before** reaching the top? If your answer is yes, at what height after beginning is the box full?
ii) Express the work required to pull as a sum of integrals. It is not necessary to solve them.

i) The box will be filled before reaching the top. Rain starts at 5 s, at that moment the box is at height 5 m. Since the rain falls at 5 Ns^{-1}, it takes $t_{\text{fill}} = (\text{cap}_B)/(v_{\text{rain}}) = 2$ s. Then the box will be filled at 7 m.
ii) We need to pull the box and the water, so the integral would be

$$W = \underbrace{\int_0^{15} 16 \mathrm{d}y}_{\text{box}} + \underbrace{\int_5^7 5y \mathrm{d}y + \int_7^{15} 10 \mathrm{d}y}_{\text{water}}$$

Exercise 2. Consider the region enclosed by the x-axis, and the lines $y = x$, $y = 2 - x$.

i) Sketch the region in question and highlight the enclosed area.
ii) Suppose that the region defines a metal plate with density $\rho(x) = \sin(\pi x)$. Express the mass of the plate as an integral.



The mass of the plate is given by the integrals

$$\int_0^1 x \sin(\pi x) \mathrm{d}x + \int_1^2 (2-x) \sin(\pi x) \mathrm{d}x.$$

Or with y (right-minus-left) orientation

$$\int_0^1 ((2-y) - y) \left(\frac{1}{\pi} \arcsin(y) \right) \mathrm{d}y.$$