Homework 3
Due: Friday, February 10

1. For $\alpha \in \mathbb{C}$ and $r > 0$, let $\gamma_r(\alpha)$ be the arc given by

$$[0, 2\pi] \xrightarrow{\quad z \quad} \mathbb{C}$$

$$t \longmapsto r\exp(it) + \alpha.$$

   Let n be an integer. Directly calculate the integral

$$\int_{\gamma_1(0)} z^n \, dz.$$

2. Evaluate the integral

$$\int_{\gamma_1(0)} \mathrm{Re}(z) \, dz$$

   in two different ways:

   (a) Use the definition directly. (HINT: *You can model your calculation on the work we did in class to compute* $\int_{\gamma_1(0)} \overline{z} \, dz$.)

   (b) Use the fact that $\mathrm{Re}(z) = \frac{z+\overline{z}}{2}$, and some known integrals.

3. Let f be a function on a domain Ω. Let $\gamma \subset \Omega$ be a closed contour.

   Suppose that, for each $\epsilon > 0$, there exists a polynomial $P_\epsilon(z)$ such that, for each point z on the contour γ, one has

$$|f(z) - P_\epsilon(z)| < \epsilon.$$

   Show that $\int_\gamma |f(z)| \, dz = 0$, and thus conclude that

$$\int_\gamma f(z) \, dz = 0.$$

4. [SS] 2.1. *This is* exercise *1, not* problem *1. To be done after class on Monday, February 6.*