Exercise 1. Consider a region R bounded by the curves

$$y=x, \quad y=-x, \quad \text{and} \quad x=1,$$

additionally the region has density $\rho(y)=e^y$. Now suppose we rotate the region about the axis $y=-4$. Do the following:

I) Draw the region in question.
II) Draw the solid of revolution obtained after rotation.
III) Which method should we use to find the volume of this shape?
IV) Find the bounds of the region. Label them either as $a \leqslant x \leqslant b$ or $c \leqslant y \leqslant d$.
V) Find the `GREATER` and `LOWER` curves by writing their equations.
VI) Find the parameters (*either* R,r *or* r,h) used to build your area function.
VII) With the previous information, write out the integral which represents the mass of the solid obtained.

I) See diagram.
II) See diagram.

III) As we have a rotation about a $y=b$ line, we can either use shells in y or rings in x. The density is in terms of y so we should use shells in y.
IV) The regions starts at $y=-1$ and ends at $y=1$. So $-1 \leqslant y \leqslant 1$.
V) We must divide the region into two pieces:

$$\begin{cases} \text{GREATER: } x=1 \\ \text{LOWER1: } x=-y \quad \text{for} \quad -1 \leqslant y \leqslant 0 \end{cases} \quad \text{and} \quad \begin{cases} \text{GREATER: } x=1 \\ \text{LOWER2: } x=y \quad \text{for} \quad 0 \leqslant y \leqslant 1 \end{cases}$$

VI) As we are using shells we must find r,h. These are:

$$\begin{cases} h_1 = \text{GREATER} - \text{LOWER1} = [1-(-y)] \quad \text{for} \quad -1 \leqslant y \leqslant 0 \\ h_2 = \text{GREATER} - \text{LOWER2} = [1-(y)] \quad \text{for} \quad 0 \leqslant y \leqslant 1 \\ r = \text{dist(axis,coordinate)} = y-(-4) \end{cases}$$

VII) Adding this facts together we get

$$m = \int_{-1}^{0} 2\pi(y+4)(1+y)e^y \mathrm{d}y + \int_0^1 2\pi(y+4)(1-y)e^y \mathrm{d}y.$$

Exercise 2. Consider the tank formed after rotating the curve $y = x^3$ with $0 \leqslant x \leqslant 1$ about the axis $x = 0$. Suppose tank is filled with *radioactive waste* with density $\rho(y) = 100 + 25y^2$. Do the following:

I) Draw the curve and the tank formed by rotating.
II) Make a diagram of an infinitesimal slice of fluid and label the height and the radius accordingly. With this write an expression for its volume.
III) Suppose there's a tube at the top with length 1m. Whats's the distance from the slice to the top.
IV) What do the bounds of integration in the work integral represent? Find them and write them as $a \leqslant y \leqslant b$.
V) With the previous information, write an integral expression for the work required to pump out water from the tank.

I) See diagram.
II) See diagram.

The volume is $V = \pi r^2 \mathrm{d}y$ with $r = x$ but we can solve x from $y = x^3$ to get $V = \pi(\sqrt[3]{y})^2 \mathrm{d}y$.

III) The total distance is

$$D_{TOT} = (1 - y) + 1 = 2 - y.$$

IV) The bounds represent where the fluid begins and where it ends. In this case we have $0 \leqslant y \leqslant 1$.
V) The work will be

$$\int_0^1 (\rho g V) D_{TOT} = \int_0^1 (100 + 25y^2)(\pi y^{2/3})(2 - y)\mathrm{d}y.$$