Name	CSU ID #

Be sure to read each question fully and carefully. Multiple choice answer bubbles must be fully filled in. There is space to the right of each multiple choice question to show work, if your work is correct you can get points even with an incorrect multiple choice answer.

1. For questions 1a through 1f consider the following information:

   The CSU Math Club is holding an election for President. Adam, Brenda, and Carlos are the candidates. The members vote, and the following preference schedule shows the results:

Number of votes	9	6	3	2	1
1st choice	Brenda	Adam	Carlos	Brenda	Adam
2nd choice	Carlos	Carlos	Adam	Adam	Brenda
3rd choice	Adam	Brenda	Brenda	Carlos	Carlos

(a) How many people voted in the Math Club presidential election? (2 points)
- ○ 1
- ○ 3
- ○ 9
- ○ 11
- ○ 21 (CORRECT)

(b) Who is the plurality winner in this election? (2 points)
- ○ Adam
- ○ Brenda (CORRECT)
- ○ Carlos
- ○ Nobody won, majority wasn't reached
- ○ There's a tie, so a tiebreaking process is necessary

(c) How many points does Adam score using the Borda count method? (2 points)
- ○ 21
- ○ 40 (CORRECT)
- ○ 44
- ○ 46
- ○ 63

(d) What happens in a pairwise comparison between Brenda and Carlos? (4 points)
- ○ Brenda wins against Carlos, 12 votes to 9 (CORRECT)
- ○ Brenda ties with Carlos, 10 votes each
- ○ Brenda wins against Carlos, 9 votes to 12
- ○ Carlos wins against Brenda, 12 votes to 9

(e) How many first-place votes are needed for a majority? (2 points)
- ○ 9
- ○ 10
- ○ 11 (CORRECT)
- ○ 15
- ○ 20

(f) What is the complete ranking of the candidates using the plurality with elimination method? (6 points)
- ○ Adam, Brenda, Carlos
- ○ Adam, Carlos, Brenda
- ○ Brenda, Adam, Carlos (CORRECT)
- ○ Brenda, Carlos, Adam
- ○ Carlos, Adam, Brenda
- ○ Carlos, Brenda, Adam

2. For questions 2a through 2i consider the following information:

Consider the group of friends Markus, Natalie, Oscar and Pauline who have invested money into a fund. They will take their decisions via a Weighted Voting Scheme based on how much money they each inverted into the fund. The following is the information on the deposited money:

Person	Markus	Natalie	Oscar	Pauline
Invested Money	??	$12000	$7000	$4000

The amount of money Markus deposited is unknown and will be the subject of our questions. Assume the quota for this setting is $q = 24$.

(a) If each person gets a vote per each $1000 they put into the fund, write down the weighted voting scheme for this setting assuming Markus gets M votes: (3 points)

$$[24 : M, \qquad , \qquad , \qquad] = [24 : M, 12, 7, 4]$$

(b) Find the minimum and maximum values for Markus' share of votes M, <u>pick 2 options</u>:
(4 points)

○ 7
○ 12 (CORRECT)
○ 17
○ 19
○ 23
○ 24 (CORRECT)

(c) Which value of M results in someone being a dictator? (2 points)

○ 7
○ 12
○ 17
○ 19
○ 23
○ 24 (CORRECT)

(d) Using the value of M you chose in the previous item, who is a dictator, why? (2 points)

○ (CORRECT) Markus is a dictator because they have enough votes to pass a motion single-handedly.

○ Markus and Natalie are dictators because any motion can pass without their votes.

○ Natalie is a dictator because the other people can pass motions without them.

○ Both Markus and Natalie are dictators because no motion can pass without both of their votes.

(e) Recall a player in a Weighted Voting Scheme has *no power* when they have no say in the outcome of the voting. For there to be exactly one player with no power, the value of M must be between... <u>Pick 2 options</u>: (4 points)

○ 7
○ 12
○ 17 (CORRECT)
○ 19 (CORRECT)
○ 23
○ 24

(f) For the previous values of M who is the player with no power? (2 points)

○ Oscar has no power because no motion can pass without their votes.

○ Oscar has no power because both Markus and Natalie or Markus and Pauline can pass resolutions on their own meaning Oscar's vote doesn't influence the decision.

○ Pauline has no power because they can pass motions single-handedly.

○ (CORRECT) Pauline has no power because both Markus and Natalie or Markus and Oscar can pass resolutions on their own meaning Pauline's vote doesn't influence the decision.

(g) Which values of M result in players (can be one, or more than one) with veto power? Pick 2 options: (4 points)

○ 7

○ 12 (CORRECT)

○ 17

○ 19

○ 23 (CORRECT)

○ 24

(h) Which value of M result in *exactly two* players with veto power? (2 points)

○ 7

○ 12 (CORRECT)

○ 17

○ 19

○ 23

○ 24

(i) For the previous value, which players have veto power, why? (2 points)

○ Both Natalie and Oscar have veto power because all motions can pass without their consideration.

○ (CORRECT) Both Markus and Natalie have veto power because no motion can pass without both of their votes.

○ Both Oscar and Pauline have veto power because no coalition can pass any motion at all.

○ Both Natalie and Pauline have veto power because they need the support of all the players to pass a motion.

3. Assume your family consists of your grandparents G, your parents P, your siblings S (you're part of this group), and your nephews N. When taking decisions as a family each group gets together and emits a vote.

The weight distribution is as follows:

- Your grandparents hold four times as many votes as your parents.
- Your parents hold twice as many votes as your siblings.
- Your siblings hold eight times as many votes as your nephews.
- And your nephews have only 3 votes.

The quota is the simple majority of the votes. We will show that your grandparents are dictators in this scheme by doing the following:

(a) Calculate the number of votes each party has and then find the quota by simple majority. (Hint: Remember you might need to round up). (8 points).

(b) Write down the Weighted Voting Scheme for this situation. (6 points).

(c) Verify that, indeed, your grandparents are dictators by comparing their number of votes with the quota. (4 points).

Sol:

(a) $N = 3, S = 24, P = 48, G = 192$ then $V = G + P + S + N = 267$. Simple majority is half of that rounded up: $267/2 = 133.5$ so $q = 134$.

(b) $[134 : 192, 48, 24, 3]$.

(c) As $192 > 134$, G is a dictator.