Exercise 1 (6 points)**.** Consider the series $\sum_{n=1}^{\infty}\frac{1}{n^{3p}}$. For what values of $p\in\mathbb{R}$ does the series **converge**?

By the p-series test, it must hold that $3p>1$. Therefore $p>1/3$.

Exercise 2 (6 points)**.** Suppose $q\in\mathbb{R}$ and consider the geometric series $\sum_{n=1}^{\infty}\frac{2^{qn}}{3^n}$.

Find the common ratio and determine **for what values of q does the series converge**. [Your answer should be an inequality depending of q.]

The geometric series in question has as a general term

$$\frac{2^{qn}}{3^n}=\frac{(2^q)^n}{3^n}=\left(\frac{2^q}{3}\right)^n.$$

So the common ratio is $\frac{2^q}{3}$. By the geometric series test, the common ratio should be smaller than one in absolute value, so

$$\left|\frac{2^q}{3}\right|<1 \iff 2^q<3 \iff q<\frac{\log(3)}{\log(2)}.$$

Exercise 3 (8 points)**.** The series $\sum_{n=1}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}$ is divergent. **Explain why** it diverges using one of the convergence tests we've seen in class. [*Hint:* $\sqrt[n]{n}\to 1$ *as* $n\to\infty$.]

Rewriting the general term as $\frac{1}{n^{1+\frac{1}{n}}}=\frac{1}{n\sqrt[n]{n}}$ we can limit-compare to $b_n=\frac{1}{n}$. It holds that

$$\frac{a_n}{b_n}=\frac{n}{n\sqrt[n]{n}}=\frac{1}{\sqrt[n]{n}}\to 1>0.$$

By the Limit Comparison Test, our series behaves as the harmonic series which is divergent. It holds that our series diverges.