HW 4
Math 672

Due Wed, Sept 28 in class.

0. Read Chapter 3 and Section 4.1.

1. Define a *line* in $\mathbb{P}^2$ to be a closed subset of the form $L = \{[x : y : z] | ax + by + cz = 0\}$ for some constants $a, b, c \in \mathbb{C}$, not all zero.

   (a) If $(a, b, c) = (1, 0, 0)$, we saw in class that $\mathbb{P}^2 \setminus L = \{[x : y : z] | x \neq 0\} = U_x$ could be identified with $\mathbb{C}^2$. Similarly, show that for any line L there is a bijection $\mathbb{P}^2 \setminus L \cong \mathbb{C}^2$.

   (b) Prove that any two distinct lines L_1 and L_2 intersect in a single point.

   (c) Prove that there is a unique line L through any two distinct points in $\mathbb{P}^2$.

2. Consider the sequence $\{p_n = (n^3, 2n^2, 3n^3) | n \in \mathbb{N}\} \subset \mathbb{C}^3$. Identifying $\mathbb{C}^3$ with $U_0 = \{[x_0 : x_1 : x_2 : x_3] | x_0 \neq 0\} \subset \mathbb{P}^3$ as in section 3.1, what is the limit of this sequence as $n \mapsto \infty$.

3. In $\mathbb{A}^2$, let V be the line $\mathbb{V}(x)$, let W be the line $\mathbb{V}(x - 1)$, and let Z be the curve $\mathbb{V}(y - x^2)$. Let $\overline{V}$, $\overline{W}$, and $\overline{Z}$ denote the respective *projective closures* in $\mathbb{P}^2$. Find all of the points in the following intersections: $\overline{V} \cap \overline{W}$, $\overline{V} \cap \overline{Z}$, $\overline{W} \cap \overline{Z}$.

4. 3.2.2

5. (a) Find an affine variety whose coordinate ring is isomorphic to $\mathbb{C}[x, 1/x, y]$, i.e. rational functions whose denominator is a polynomial in x. (hint: see exercise 4.1.1 and the preceding paragraphs)

   (b) Find an affine variety whose coordinate ring is $\mathbb{C}[x, y, 1/(x^2 + y^2)]$, i.e. rational functions whose denominator is a polynomial in $x^2 + y^2$.