

Exercise 1. Let $H \subseteq \mathbb{P}^n$ be a hyperplane. H is defined as the zero locus of a linear equation

$$H = \mathbb{V}(a_0x_0 + \cdots + a_nx_n), \quad a_0, \dots, a_n \in \mathbb{C}.$$

Prove that $H \simeq \mathbb{P}^{n-1}$. (Also, be comfortable with the fact that $\mathbb{P}^n \setminus L \simeq \mathbb{A}^n$.)

Answer

Exercise 2. Let $\nu_2 : \mathbb{P}^2 \rightarrow \mathbb{P}^5$ be the Veronese embedding of degree 2. Write out the equations defining the image of ν_2 .

Answer

Recall that this particular Veronese map takes a point in \mathbb{P}^2 to all possible monomials of degree 2 in \mathbb{P}^5 . This means that

$$\nu_2([u : v : w]) = [u^2 : v^2 : w^2 : uv : vw : wu].$$

Recall that the image of the Veronese map is defined using a multi-indexed array, so let us relabel in that sense:

$$[u^2 : v^2 : w^2 : uv : vw : wu] = [z_{2,0,0} : z_{0,2,0} : z_{0,0,2} : z_{1,1,0} : z_{0,1,1} : z_{1,0,1}].$$

The defining equations for the image are given by

$$z_I z_J = z_K z_L, \quad I + J = K + L,$$

where I, \dots, L are our multi-indices. The only ways to sum our multi-indices in a non-trivial manner are:

$$\begin{cases} (2, 0, 0) + (0, 1, 1) = (1, 1, 0) + (1, 0, 1) \\ (0, 2, 0) + (1, 0, 1) = \end{cases}$$