Exercise 1. Evaluate the definite integral $\int_1^e \frac{1}{t}(3\log(t)+1)^2$.

$$\int_1^e \frac{1}{t}(3\log(t)+1)^2 = \int \frac{1}{t}u^2 \frac{\mathrm{d}u}{\frac{3}{t}} \left(\substack{u=3\log(t)+1 \\ \mathrm{d}u=(3/t)\mathrm{d}t}\right) = \frac{1}{3}\int u^2 \mathrm{d}u = \frac{u^3}{9}$$

$$= \frac{1}{9}(3\log(t)+1)^3 |_1^e = \frac{1}{9}(3+1)^3 - \frac{1}{9}(0+1)^3 = \underline{7}.$$

Exercise 2. Use integration by parts to find an antiderivative of $4x^7\sin(x^4)$.

$$\int 4x^7 \sin\left(x^4\right)\mathrm{d}x = \int 4x^7 \sin(t)\frac{\mathrm{d}t}{4x^3}\left(\substack{t=x^4 \\ \mathrm{d}t=4x^3\mathrm{d}x}\right) = \int x^4 \sin(t)\mathrm{d}t \text{ (but } x^4 = t)$$

$$= \int t\sin(t)\mathrm{d}t = t(-\cos(t)) - \int(-\cos(t))\mathrm{d}t$$

$$= \underline{-t\cos(t)+\sin(t)+C = -x^4\cos\left(x^4\right)+\sin\left(x^4\right)+C}$$

Exercise 3. Calculate the following indefinite integral: $\int x^2 e^{-x}\mathrm{d}x$. [Hint: Watch out for the minus sign.]

Call I the integral in question, by integration by parts:

$$\begin{cases} u = x^2 \Rightarrow \mathrm{d}u = 2x\mathrm{d}x \\ \mathrm{d}v = e^{-x}\mathrm{d}x \Rightarrow v = -e^{-x} \end{cases} \Rightarrow I = -x^2e^{-x} - \int(-e^{-x})2x\mathrm{d}x = -x^2e^{-x} + 2\int xe^{-x}\mathrm{d}x.$$

Applying IBP once more we obtain

$$\begin{cases} u = x \Rightarrow \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^{-x}\mathrm{d}x \Rightarrow v = -e^{-x} \end{cases} \Rightarrow I = -x^2e^{-x} + 2\left[-xe^{-x} - \int(-e^{-x})\mathrm{d}x\right] = -x^2e^{-x} - 2xe^{-x} + 2(-e^{-x}) + C.$$

The final result is thus $\underline{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$

Exercise 4. Find an antiderivative for the function $\log^2(x)$ using integration by parts.

By integration by parts:

$$\begin{cases} u = \log^2(x) \Rightarrow \mathrm{d}u = \frac{2\log(x)}{x}\mathrm{d}x \\ \mathrm{d}v = \mathrm{d}x \Rightarrow v = x \end{cases} \Rightarrow I = x\log^2(x) - \int 2\log(x)\mathrm{d}x = \underline{x\log^2(x) - 2x\log(x) + 2x}$$