Exercise 1. Consider the quadratic polynomial x^2-4x+8. Convert it to <u>vertex form</u>. *That is, convert it to the form $(x-h)^2+k$, where (h,k) is the vertex of this quadratic curve.*

$$x^2-4x+8=x^2-4x+4+4=(x-2)^2+4$$

Exercise 2. Match the following radicals of polynomials with their result after doing the correct trigonometric substitution.

1. $\sqrt{4-x^2}$, $x=2\cos(\theta)$	(2) $\tan(\theta)$
2. $\sqrt{9x^2-1}$, $x=\frac{1}{3}\sec(\theta)$	(3) $2\sec(\theta)$
3. $\sqrt{4+25x^2}$, $x=\frac{2}{5}\tan(\theta)$	(1) $2\sin(\theta)$

Exercise 3. Evaluate the integral $\int \frac{\sqrt{1-x^2}}{x}\mathrm{d}x$ using the correct trigonometric substitution. *Your answer must be a function depending on x.*

Take $x=\sin(\theta)$, then $\mathrm{d}x=\cos(\theta)\mathrm{d}\theta$. The integrand becomes

$$\frac{\sqrt{1-x^2}}{x}\mathrm{d}x=\frac{\sqrt{1-\sin^2(\theta)}}{\sin(\theta)}(\cos(\theta)\mathrm{d}\theta).$$

Then

$$\int\frac{\cos^2(\theta)}{\sin(\theta)}\mathrm{d}\theta=\int\frac{1-\sin^2(\theta)}{\sin(\theta)}\mathrm{d}\theta=\int(\csc(\theta)-\sin(\theta))\mathrm{d}\theta=-\log(\csc(\theta)+\cot(\theta))+\cos(\theta).$$

Since $\sin(\theta)=O/H$, then $A=\sqrt{1-x^2}$, and thus

$$I=-\log\left(\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)+\sqrt{1-x^2}.$$

Exercise 4. Evaluate the integral $\int\frac{\mathrm{d}x}{\sqrt{4x^2-4x+5}}$. *Your answer must be a function depending on x.*

We first convert the polynomial to vertex form $4x^2-4x+5=4x^2-4x+1+4=(2x-1)^2+4$. The first substitution to take $u=2x-1\Rightarrow \mathrm{d}u=2\mathrm{d}x\Rightarrow \mathrm{d}x=\frac{\mathrm{d}u}{2}$. The integral is now

$$\begin{aligned}\int\frac{1}{\sqrt{u^2+4}}\left(\frac{\mathrm{d}u}{2}\right)\xrightarrow[\mathrm{d}u=2\sec^2(\theta)]{u=2\tan(\theta)}\frac{1}{2}\int\frac{(2\sec^2(\theta)\mathrm{d}\theta)}{\sqrt{(2\tan(\theta))^2+4}}&=\int\frac{\sec^2(\theta)}{2\sec(\theta)}\mathrm{d}\theta\\&=\frac{1}{2}\int\sec(\theta)\mathrm{d}\theta.\end{aligned}$$

The integral of $\sec(\theta)$ is $\log(\sec(\theta)+\tan(\theta))$ and now we use the fact that $\tan(\theta)=O/A$, whence we obtain $O=u$ and $A=2$. Thus $H=\sqrt{u^2+4}$ and thus

$$I=\log\left(\frac{\sqrt{u^2+4}}{2}+\frac{u}{2}\right)=\log\left(\frac{\sqrt{(2x-1)^2+4}}{2}+\frac{2x-1}{2}\right).$$