

**Math 101-002**

**Exam 1, February 13**

Name	CSU ID #
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Be sure to read each question fully and carefully. Multiple choice answer bubbles must be fully filled in. There is space to the right of each multiple choice question to show work, if your work is correct you can get points even with an incorrect multiple choice answer.

1. For questions 1a through 1f consider the following information:

The CSU Math Club is holding an election for President. Adam, Brenda, and Carlos are the candidates. The members vote, and the following preference schedule shows the results:

ADD TABLEUX

- (a) How many people voted in the Math Club presidential election? (2 points)

☐ 3  
☐ 5  
☐ 9  
☐ 10  
☐ 20

- (b) Who is the plurality winner in this election? (2 points)

☐ Adam  
☐ Brenda  
☐ Carlos  
☐ Nobody won, majority wasn't reached  
☐ There's a tie, so a tiebreaking process is necessary

- (c) How many points does Adam score using the Borda count method? (2 points)

☐ aaa

- (d) What happens in a pairwise comparison between Brenda and Carlos? (2 points)

☐ aaa

- (e) How many first-place votes are needed for a majority? (2 points)

☐ aaa

- (f) What is the complete ranking of the candidates using the plurality with elimination method? (6 points)

☐ Adam, Brenda, Carlos  
☐ Adam, Carlos, Brenda  
☐ Brenda, Adam, Carlos

- ☐ Brenda, Carlos, Adam
- ☐ Carlos, Adam, Brenda
- ☐ Carlos, Brenda, Adam

2. Consider the Weighted Voting Scheme  $[q : 9, 7, 1]$  formed by players  $P_1, P_2$  and  $P_3$ .

(a) What are the minimum and maximum possible values of  $q$ ? Pick 2 options. (4 points)  
[Hint: Recall that the quota lies between half the total votes and the total votes.]

- ☐ 3
- ☐ 5
- ☐ 11
- ☐ 17
- ☐ 23

(b) Which value of  $q$  results in a dictator for this scheme? (2 points)

- ☐ 7
- ☐ 8
- ☐ 9
- ☐ 10
- ☐ 11

(c) Using the value of  $q$  you chose in the previous item, who is a dictator, why? (2 points)

- ☐  $P_1$  is a dictator because  $P_1$  has enough votes to pass a motion single-handedly.
- ☐  $P_1$  is a dictator because any motion can pass without their votes.
- ☐ Both  $P_1$  and  $P_2$  are dictators because they have enough votes to pass motions together.
- ☐ Both  $P_1$  and  $P_2$  are dictators because no motion can pass without both of their votes.

(d) For the previous value, who has no power, why? (2 points)

- ☐ Both  $P_2$  and  $P_3$  have no power because they can pass a motion together.
- ☐ Both  $P_2$  and  $P_3$  have no power because  $P_1$  is a dictator.
- ☐ All players have no power because  $P_1$  is a dictator.
- ☐ All players have no power because no one can pass a motion single-handedly.

- (e) For which value of  $q$  is there exactly one player with veto power? (2 points)
- ☐ 8
  - ☐ 9
  - ☐ 10
  - ☐ 11
  - ☐ 12
- (f) For the previous value, which player has veto power, why? (2 points)
- ☐  $P_1$  has veto power because  $P_1$  has enough votes to pass a motion single-handedly.
  - ☐  $P_1$  has veto power because  $P_2$  and  $P_3$  together have only 8 votes.
  - ☐  $P_2$  has veto power because  $P_2$  has enough votes to pass a motion single-handedly.
  - ☐  $P_2$  has veto power because  $P_1$  and  $P_3$  together have only 10 votes.
- (g) Which values  $q$  guarantee that *two players* will have veto power? Pick 2 options.(4 points)
- ☐ 7
  - ☐ 9
  - ☐ 11
  - ☐ 15
  - ☐ 17
- (h) For the previous values, which players have veto power, why? (2 points)
- ☐ Both  $P_1$  and  $P_3$  have veto power because no motion can pass without at least one of their votes.
  - ☐ Both  $P_1$  and  $P_2$  have veto power because no motion can pass without at least one of their votes.
  - ☐ Both  $P_1$  and  $P_2$  have veto power because no motion can pass without both of their votes.
  - ☐ Both  $P_1$  and  $P_3$  have veto power because no motion can pass without both of their votes.
- (i) Which value of  $q$  guarantees that all players have veto power? (2 points)
- ☐ 7
  - ☐ 9
  - ☐ 11
  - ☐ 15
  - ☐ 17

3. Assume you're on a family trip with your two uncles ( $U$ ) and two cousins ( $C$ )—you are also represented by the letter  $C$ . Family trip decisions are made by a majority vote (at least three people must vote "Yes"), but at least one uncle must vote "Yes" for the decision to pass (i.e., the three cousins alone cannot carry the motion).

Each uncle has the same number of votes, and each cousin has the same number of votes.

Using a voting scheme  $[q : U, U, C, C, C]$ , find the smallest possible values for  $q$ ,  $U$ , and  $C$  that satisfy these conditions.