Permutation Statistics.

Ignacio Rojas

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Abstract

When being taught permutations, it's common to introduce the cycle notation. There are other ways to write permutations, for example, in *list notation*. For example in S_4:

$$(123) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix} = 2314$$

In list notation it's possible count certain characteristics of a permutation, for example, *the number of inversions*. This particular permutation has 2 inversions because 2 is to the left of 1 and 3 is to the left of 1.

This is an initial example of a permutation *statistic*. We will talk about different statistics, see how they are related, find generating functions for them and express them in a different way as a q-analogue.

The idea for this colloquium is to introduce this topic with interactive examples among the participants, so come prepared with pen and paper and with an open mind!

1 Introduction