

**Exercise 1** (Bessel Function). Consider the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2}$ , this is  $J_0(1)$  where  $J$  is called the Bessel J function. Do the following:

- i) From the convergence tests we've seen in class name a test that proves that the series converges.
- ii) State the conditions which guarantee convergence based on the test you named.
- iii) Apply the test to prove that  $J_0(1)$  converges.

**Exercise 2.** Suppose a function  $f(x)$  can be represented by a polynomial  $p(x)$  about  $a=3$ . We have

$$p(x) = 5 + 3(x-3)^2 - (x-3)^3 + \frac{2(x-3)^4}{5!} + 2(x-3)^5 + \frac{1}{2}(x-3)^6.$$

Identify the following:

I)  $f(3)$

II)  $f'(3)$

III)  $f''(3)$

IV)  $f^{(3)}(3)$

V)  $f^{(5)}(3)$

**Exercise 3.** We will approximate  $e^{1/10}$  using Taylor series.

- I) Write out the first four terms of  $e^x$ 's Taylor series.
- II) Evaluate  $x = \frac{1}{10}$  to obtain an approximation of  $e^{1/10}$ .