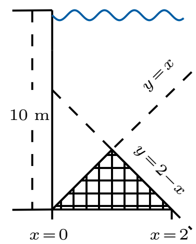
Exercise 1. Suppose we have a triangular plate at the bottom of a pool. It is enclosed by the following curves:

$$\{y=x,\ y=2-x,\ y=0\}.$$

Suppose that the pool contains a fluid of density $\rho=1$ and the full depth of the pool is 10 m. Do the following:

I) Make a diagram which illustrates the situation described above. Highlight the enclosed region formed by the curves to show that that is the plate.
II) Express the pressure as an integral. [`Remember: Pressure is the integral of depth times width times the density.`]



Notice that the curve $y=2-x$ is to the right of $y=x$. We need to express these functions in terms of y, so the curves are

$$y=2-x \Rightarrow x=2-y, \text{ and } y=x \Rightarrow x=y$$

In this case $R-L=(2-y)-(y)=2-2y$. At height y the depth D is found by the equation

$$y+D=\text{total depth} \Rightarrow y+D=10 \Rightarrow D=10-y.$$

The plate's max height occurs at the point where $2-x=x$ which is $x=1$. Plugging this in either curve's equation we get $y=1$. We conclude that the pressure is

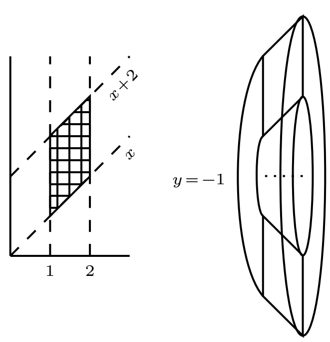
$$\int_0^1 (10-y)(2-2y)(1)\mathrm{d}y = 29/3.$$

Exercise 2. Consider the region in the 1st quadrant enclosed by the curves

$$\{x=1,\ x=2,\ y=x,\ y=x+2\}.$$

If we rotate the region about the axis $y=-1$ we obtain a solid of revolution.

I) Sketch the region in question and make a rough sketch of how the solid of revolution looks like.
II) Use the method of rings to find the volume of the solid.



In this case, we don't need to find the intersections of the curves. The bounds of integration are given to us to be $\underline{x=1 \text{ and } x=2}$. We construct the radii:

$$R=U-\text{axis}=(x+2)-(-1)=x+3,\ r=D-\text{axis}=x-(-1)=x+1.$$

The area function is $A(x)=\pi[(x+3)^2+(x+1)^2]=\pi(2x^2+8x+10)$ and so the volume is

$$V=\int_1^2 \pi(2x^2+8x+10)\mathrm{d}x = 80\pi/3.$$