Exercise 1. Define a *line* in $\mathbb{P}^2$ to be a closed subset of the form $L = \{ [x : y : z] : ax + by + cz = 0 \}$ for some constants $a, b, c \in \mathbb{C}$, not all zero.

i) If $(a, b, c) = (1, 0, 0)$, we saw in class that $\mathbb{P}^2 \backslash L = \{ [x : y : z] : x \neq 0 \} = U_x$ could be identified with $\mathbb{C}^2$.

   Similarly, show that for any line L there is a bijection $\mathbb{P}^2 \backslash L \simeq \mathbb{C}^2$.

ii) Prove that any two distinct lines L_1 and L_2 intersect in a single point.

iii) Prove that there is a unique line L through any two distinct points in $\mathbb{P}^2$.

Answer