

Exercise 1 (Stein&Shakarchi 3.15(c)). Let w_1, \dots, w_n be points on the unit circle in the complex plane. Prove that there exists a point z on the unit circle such that the product of the distances from z to the points w_j , $1 \leq j \leq n$, is at least 1.

Conclude that there exists a point w on the unit circle such that the product of the distances from w to the points w_j , $1 \leq j \leq n$, is exactly equal to 1.

Answer

Consider the function

$$g(z) = \prod_{k=1}^n (z - w_k)$$

which is holomorphic on $B(0, 1)$ and continuous on $\overline{B}(0, 1)$. questions