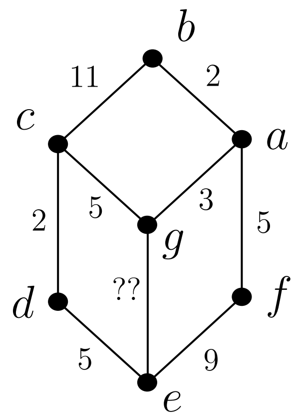
Math 101-002
Exam 2, March 13

Name	CSU ID #

Be sure to read each question fully and carefully. Multiple choice answer bubbles must be fully filled in. There is space to the right of each multiple choice question to show work, if your work is correct you can get points even with an incorrect multiple choice answer.

1. For questions 1a through 1p consider the following graph G:



(a) Write down the list of vertices of the graph G: (2 points)

$$V = \{\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad}\}\,.$$

(b) Write down the list of edges of the graph G: (2 points)

$$E = \{\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad},\underline{\quad\quad}\}\,.$$

(c) Out of the following, which is **NOT** an edge in the graph? (2 points)

- ◯ ag
- ◯ dg
- ◯ ef
- ◯ bc

(d) How many vertices does this graph have? (2 points)

- ◯ 3
- ◯ 6
- ◯ 7
- ◯ 8

(e) The degree of the vertex d is: (2 points)

$$\deg(d) = \_\_\_.$$

(f) The degree of the vertex g is: (2 points)

$$\deg(g) = \_\_\_.$$

(g) A path of length 3 from a to d passing through f is: (2 points)

- ◯ ab, bc, cd
- ◯ ag, gc, cd
- ◯ ag, ge, ed
- ◯ af, fe, ed

(h) Write down the weight of the previous path: (2 points)

$$\text{weight} = \_\_\_.$$

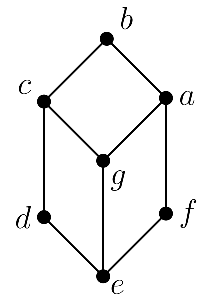
(i) Does this graph G have cut-edges (bridges)? (2 points)

- ◯ Yes.
- ◯ No.

(j) State whether this graph has an Euler walk. If it does, write it down, if not state why it doesn't. (2 points)

(k) State whether this graph has an Hamilton walk. If it does, write it down, if not state why it doesn't. (2 points)

(l) The graph G doesn't have an Euler circuit, Eulerize it by adding edges: (2 points)



(m) Assume you're traveling the graph G and just came in to the e from d. For g to be your nearest neighbor, the weight of the edge eg should be: (2 points)

- ◯ 6
- ◯ 12
- ◯ 18
- ◯ 24

(n) When traveling the graph G, you arrived at the vertex c from b. Following the Nearest-Neighbor algorithm, which vertex should you visit next? (2 points)

- ◯ a
- ◯ b
- ◯ d
- ◯ g

(o) When applying the Cheapest-Link algorithm, suppose you have already picked the edges ab, cd, ag and cg. Which edge should be picked next? (2 points)

- ◯ af
- ◯ de
- ◯ ef
- ◯ bc

(p) Suppose you're applying Cheapest-Link algorithm to G. For eg to be 4^{th} edge you pick, its weight should be: (2 points)

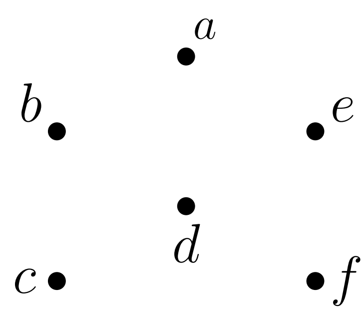
$$\text{weight}(eg) = \_\_\_.$$

2. For questions 2a through 2k consider the information about the graph G given by the following lists:

$$V = \{a, b, c, d, e, f\},$$
$$E = \{ad, ae, bc, bd, cd, de, df, ea, ef\},$$
$$W = \{7, 10, 4, 6, 5, 9, 8, 6, 7\}.$$

The weights are ordered respecting the order of the edges.

(a) Fill in the weighted edges of the graph G (Hint: Pay attention to the repeated edge): (4 points)



(b) List all the vertices adjacent to d: (2 points)

$$N(d) = \{\_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_\}$$

(c) What are the degrees of the vertices d and e? (4 points)

$$\deg(d) = \_\_\_ \quad \text{and} \quad \deg(e) = \_\_\_.$$

(d) Find the sum of the degrees of all vertices: (2 points)

- ○ 6
- ○ 9
- ○ 12
- ○ 18

(e) State whether this graph has an Euler walk. If it does, write it down, if not state why it doesn't. (2 points)

(f) The graph G doesn't have an Euler circuit. Only one edge is needed to make it Eulerian. Which is that edge? (2 points)

$$\text{missing edge} = \underline{\quad\quad}.$$

(g) Does this graph G have cut-edges (bridges)? (2 points)

- ◯ Yes.
- ◯ No.

(h) What is the weight of the path $\{bc, cd, df, fe\}$? (2 points)

- ◯ 8
- ◯ 16
- ◯ 24
- ◯ 28

(i) When traveling the graph G starting at e, your Nearest-Neighbor algorithm took you to a and then d. The next vertex you should visit is: (2 points)

- ◯ b
- ◯ c
- ◯ e
- ◯ f

(j) The Nearest-Neighbor algorithm starting at the vertex b produces a walk that ends at which vertex? (2 points)

- ◯ a
- ◯ b
- ◯ d
- ◯ f

(k) Which is the edge that you pick first in G when applying the Cheapest-Link algorithm? (2 points)

$$1^{\text{st}}\text{edge} = \underline{\quad\quad}.$$

3. Short answer:

   (a) Explain the difference between a walk and a circuit of a graph. (4 points)

   (b) Explain the difference between Eulerian and Hamiltonian paths or circuits. (4 points)

4. In the following space draw the requested graphs:

   (a) A graph with an Euler walk but without and Euler circuit. (4 points)

   (b) A graph with a Hamilton walk but without an Euler walk. (4 points)

   (c) A graph with an Euler circuit and a Hamilton walk, but not a Hamilton circuit. (4 points)

5. (25 points) Realistically, when you're traveling in your local city, you're very likely to know your neighborhood and your route to your worksite. It is also likely that you know the places you frequent often such as the supermarket and houses of friends you visit often.

   In this hypothetical scenario, you're driving to a concert this weekend and need to pick up several friends along the way. Some friends live close to you, while others are farther away in areas you're not familiar with. You also don't know the concert venue very well.

   For this scenario, assume you have a physical map of the city and the addresses of your friends, but no access to GPS or navigation apps.

   In the following space address the following prompts and discuss thoroughly and explain your reasonings:

   - Do you think that this situation is better modeled through an Eulerian or Hamiltonian approach? Are you trying to make a walk or a circuit?
   - Since some friends live nearby and others farther away, where you're not familiar, would that influence your choice of algorithm to pick up? Why?
   - Could a mix of algorithms make for a better route?
   - If you're running late or some friends aren't ready yet, how does that change your approach? Would it affect your choice of algorithm or the order in which you pick people up? Why or why not?