Math 261-001
Quiz 6, October 10

Name	CSU ID #

Be sure to read each question carefully. You must choose and answer **exactly two** of the four problems. If you attempt more than two, only the first two will be graded. Write your final answers in the boxes provided. Each problem is worth the same amount of points. **Each problem is accompanied by a figure to help you visualize the region in question.**

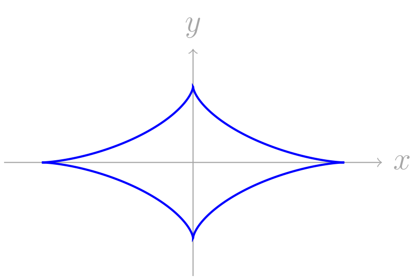
1. Let $\mathcal{C}$ be the curve which is the portion of the **astroid**

$$\left(\frac{x}{4}\right)^{\frac{2}{3}} + \left(\frac{y}{2}\right)^{\frac{2}{3}} = 1$$

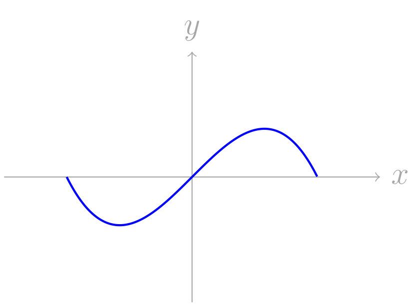
on the first quadrant of the plane. This curve admits the parametrization

$$r(t) = (4\cos^3(t), 2\sin^3(t)), \quad 0 \le t \le \pi/2.$$

Consider the vector field $F(x, y) = (y\cos(xy) - 1, 1 + x\cos(xy))$ and evaluate the line integral $\int_{\mathcal{C}} F \cdot \mathrm{d}\vec{x}$. [Suggestion: First verify if the vector field F is conservative.]



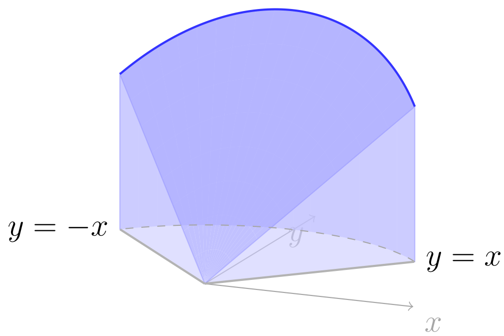
2. Ahhh, it's the refreshing sea breeze on your boat. Ahhh, and look! Out there is a shark eating fish, with your supreme math skills you picture the path the shark follows to be $y = x(1 - x^2)$ from the points $(-1, 0)$ to $(1, 0)$. Also, you realize there's more fish the further away you go, so that the fish density is $f(x) = 2^x$. Write down a path integral which describes the amount of fish eaten by the shark.



3. Some doorstops are triangular prisms, and some have the shape of a circular section with a wedge. The doorstop you're now thinking about can ideally be described as the volume enclosed by surfaces:

$$x^2 + y^2 = 4, \quad x = y, \quad -x = y, \quad z = 0, \quad \text{and} \quad z = x + y.$$

Using cylindrical coordinates, write an integral representing the mass of the doorstop assuming its density is $\delta(x, y) = e^{-x^2-y^2}$. (This is, it's more massive at the tip than outside).



4. You're in one of those fancy places which serve soup in a bread bowl. Before the soup eventually breaks through the bowl and makes a mess, you decide to calculate the mass of the soup inside. Assume the soup is densest around the center,

$$\delta(x, y, z) = \frac{1}{(x^2 + y^2 + z^2)^2}, \quad \text{(ignore the discontinuity at the origin).}$$

If the bowl is shaped like the sphere $x^2 + y^2 + z^2 = 1$ and has a circular opening at the plane $z = 1/\sqrt{2}$ so that the bowl is everything below the plane, how much (mass of) soup does the bread bowl hold? Calculate you answer using a triple integral with spherical coordinates.