Homework 8
Due: Friday, March 31

Don't hand this in, but please work it out:

- Let $f(z)$ be a holomorphic function on Ω, and suppose that $\overline{N_R(z_0)} \subset \Omega$. Show that

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} f(z_0 + R\exp(i\theta))\, d\theta.$$

  (HINT: *Use the Cauchy integral formula.*)

1. Let Ω be any domain containing the unit circle $C_1(0)$. Show that there is no function $F(z)$, holomorphic on Ω, such that $e^{F(z)} = z$ on Ω. (HINT: *What would $F'(z)$ be? Can you prove $F'(z)$ admits no primitive on Ω?*)
2. Let Ω be a domain with $0 \notin \Omega$.
   (a) Suppose that $f(z)$ and $g(z)$ are continuous branches of the logarithm on Ω. Show that there is some integer n such that $g(z) = f(z) + 2\pi i n$. (HINT: *Ω is connected.*)
   (b) Suppose that $f(z)$ is a continuous branch of the logarithm. Show that $f(z)$ is holomorphic. (HINT: *Ω can be covered by simply connected subsets.*)
3. [SS]5.1. (HINT: *You will still need something like Step 3 in the proof from the book.*)

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