Math 261-001

Quiz 11, **December 5**

Name	CSU ID #

Be sure to read each question carefully. You must choose and answer **exactly two** of the four problems. If you attempt more than two, only the first two will be graded. Write your final answers in the boxes provided. Each problem is worth the same number of points.

1. Consider an open-top cylindrical can with radius r and height h. If it has surface area 48π, maximize its volume given that constraint. Use the method of **Lagrange multipliers**.

2. Let's say you have a space station with $x > 0$ alien chefs, and each chef can produce $y > 0$ space sandwiches per hour. Assume the cost of operating the station is given by:
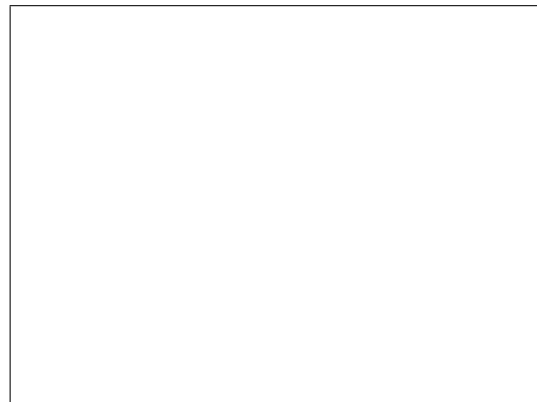
$$C(x, y) = 2x^2 + 3y,$$

where having more chefs increases the cost due to larger living quarters and additional supplies. For a fixed production of 3888 space sandwiches per hour, find the minimum value of the cost function. Use the method of **Lagrange multipliers**.

3. Consider the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $2x + 2y + z = 2$.

   Without solving, set up the system needed to find the points of the intersection closest to the origin.

   Hint: Use the square of the distance function $f(x, y, z) = x^2 + y^2 + z^2$ and realize that the Lagrange equation is:

   $$\nabla f = \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2.$$

4. The **Cobb-Douglas model of production** states that the amount of product P produced depends on the amount of labor x and materials y via:

   $$P = kx^a y^b,$$

   where a, b are constants such that $a + b = 1$.

   Assume that labor costs A dollars per unit, and materials cost B dollars per unit. You have a fixed budget of C dollars to spend on labor and materials, such that the cost is distributed as:

   $$Ax + By = C.$$

   Use **Lagrange multipliers** to maximize production $P(x, y)$. Your answer will depend on a, b, A and B.