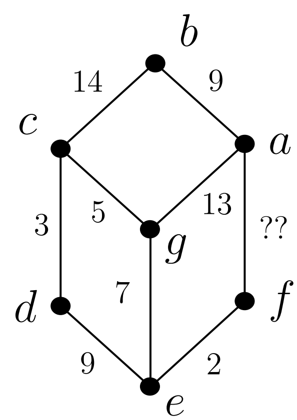
Math 101-002
Exam 2 Makeup, **March 26**

Name	CSU ID #

Be sure to read each question fully and carefully. Multiple choice answer bubbles must be fully filled in. There is space to the right of each multiple choice question to show work, if your work is correct you can get points even with an incorrect multiple choice answer.

1. For questions 1a through 1p consider the following weighted graph G:



(a) Write down the list of vertices of the graph G: (2 points)

$$V = \{\_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_\}\,.$$

(b) Write down the list of edges of the graph G: (2 points)

$$E = \{\_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_, \_\_\_\}\,.$$

(c) Out of the following, which is **NOT** an edge in the graph? (2 points)

- ○ bg
- ○ cd
- ○ eg
- ○ af

(d) How many vertices does this graph have? (2 points)

- ○ 3
- ○ 6
- ○ 7
- ○ 8

(e) The degree of the vertex a is: (2 points)

$$\deg(a) = \_\_\_.$$

(f) The degree of the vertex e is: (2 points)

$$\deg(e) = \_\_\_.$$

(g) A path of length 5 from c to g passing through a is: (2 points)

- ◯ cg
- ◯ cd, de, eg
- ◯ cd, de, ef, fa, ag
- ◯ ga, ab, ba, ag, gc

(h) Write down the weight of the previous path: (2 points)

$$\text{weight} = \_\_\_.$$

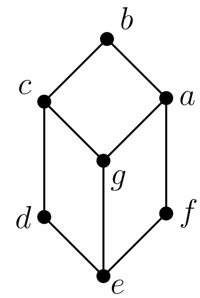
(i) Does this graph G have cut-edges (bridges)? (2 points)

- ◯ Yes.
- ◯ No.

(j) State whether this graph has an Euler walk. If it does, write it down, if not state why it doesn't. (2 points)

(k) State whether this graph has an Hamilton walk. If it does, write it down, if not state why it doesn't. (2 points)

(l) The graph G doesn't have an Euler circuit, Eulerize it by adding edges: (2 points)



(m) Assume you start traveling the graph G at the vertex f. If you are applying the Nearest-Neighbor algorithm and you traveled to the vertex a instead of e, then the weight of af was: (2 points)

- ◯ 1
- ◯ 2
- ◯ 3
- ◯ 4

(n) When traveling the graph G, you arrived at the vertex e from f. Following the Nearest-Neighbor algorithm, which vertex should you visit next? (2 points)

- ◯ a
- ◯ b
- ◯ d
- ◯ g

(o) When applying the Cheapest-Link algorithm, suppose you have already picked the edges ef, cd, cg and eg. Which edge should be picked next? (2 points)

- ◯ ab
- ◯ bc
- ◯ de
- ◯ ef

(p) Suppose you're applying Cheapest-Link algorithm to G. For af to be 3rd edge you pick, its weight should be: (2 points)

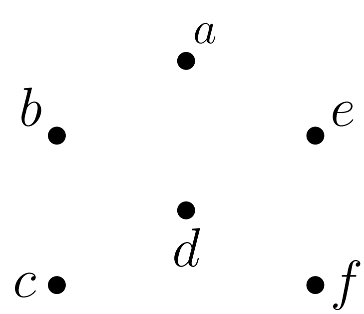
$$\text{weight}(eg) = \_\_\_\_.$$

2. For questions 2a through 2k consider the information about the graph G given by the following lists:

$$V = \{a, b, c, d, e, f\},$$
$$E = \{ad, ae, bc, bd, cd, de, df, ea, ef\},$$
$$W = \{7, 6, 8, 9, 5, 6, 4, 10, 7\}.$$

The weights are ordered respecting the order of the edges.

(a) Fill in the weighted edges of the graph G (Hint: Pay attention to the repeated edge): (4 points)



(b) List all the vertices adjacent to d: (2 points)

$$N(d) = \{\underline{\quad\;}, \underline{\quad\;}, \underline{\quad\;}, \underline{\quad\;}, \underline{\quad\;}\}$$

(c) What are the degrees of the vertices a and f? (4 points)

$$\deg(a) = \underline{\quad\;} \quad \text{and} \quad \deg(f) = \underline{\quad\;}.$$

(d) Find the sum of the degrees of all vertices: (2 points)

- ○ 6
- ○ 9
- ○ 12
- ○ 18

(e) State whether this graph has an Euler walk. If it does, write it down, if not state why it doesn't. (2 points)

(f) The graph G doesn't have an Euler circuit. Only one edge is needed to make it Eulerian. Which is that edge? (2 points)

$$\text{missing edge} = \_\_\_\_.$$

(g) Does this graph G have cut-edges (bridges)? (2 points)

- ◯ Yes.
- ◯ No.

(h) What is the weight of the path $\{bc, cd, df, fe\}$? (2 points)

- ◯ 5
- ◯ 8
- ◯ 20
- ◯ 24

(i) When traveling the graph G starting at e, your Nearest-Neighbor algorithm took you to a and then d. The next vertex you should visit is: (2 points)

- ◯ b
- ◯ c
- ◯ e
- ◯ f

(j) The Nearest-Neighbor algorithm starting at the vertex b produces a walk that ends at which vertex? (2 points)

- ◯ a
- ◯ b
- ◯ d
- ◯ f

(k) Which is the edge that you pick first in G when applying the Cheapest-Link algorithm? (2 points)

$$1^{\text{st}}\text{edge} = \_\_\_\_.$$

3. Short answer:

   (a) Explain the difference between Eulerian and Hamiltonian walks or circuits. (4 points)

   (b) Recall that a graph is called a *complete graph* when it contains all possible edges, is a graph that contains a *Hamilton circuit* complete? Briefly explain.(4 points)

4. In the following space draw the requested graphs and answer briefly if requested:

   (a) A complete graph on 3 vertices. Does this graph have an Euler circuit? (4 points)

   (b) A complete graph on 4 vertices. Does this graph have an Euler circuit?. (4 points)

   (c) A graph with an Euler circuit and a Hamilton walk, but not a Hamilton circuit. (4 points)

5. (25 points) When modeling a walk through a graph it is usual to forget the edges which are not being traveled. An example of this would be a trip from your house to pick up your friends and then go to a concert. You would only consider the streets you're traveling, not the whole load of streets in the city.

   But consider the scenario at the planning moment, before you begin your trip and you're deciding where to go. Realistically, you can grab a map of your city (or your phone) and see every place there is to go up to a certain distance. At this planning stage, you can decide which route is best to take and you can optimize it globally.

   - In this situation, do think you should model this using a walk or a circuit? Are you trying to go for Hamiltonian or Eulerian? Is there another way that you could approach this problem?
   - In such complete graphs, is it always possible to find a Hamilton circuit? How about an Euler circuit? Can you explain why and reinforce your reasoning by using the previous part's questions as examples?

   Sol:

   - 8 pts clearly explains whether the situation is more Hamilton or Euler and adds evidence to support reasonings.
   - 6 pts discusses how friends locations affects choice of approach
   - 6 pts thoroughly considers whether a hybrid strategy would be beneficial.
   - 5 pts similarly discusses how a time constraint could affect the choice.