Exercises
Math 673

1. Let $\mathbb{P}^1_{\mathbb{C}}$ denote the scheme constructed from

$$U_0 = \operatorname{Spec} \mathbb{C}[y] \cong \mathbb{A}^1_{\mathbb{C}}$$

and

$$U_1 = \operatorname{Spec} \mathbb{C}[x] \cong \mathbb{A}^1_{\mathbb{C}}$$

by identifying $U_0 \setminus \{0\} = \operatorname{Spec} \mathbb{C}[y, y^{-1}]$ with $U_1 \setminus \{0\} = \operatorname{Spec} \mathbb{C}[x, x^{-1}]$ via the ring isomorphism $\mathbb{C}[y, y^{-1}] \cong \mathbb{C}[x, x^{-1}]$ given by $x = y^{-1}$. Denote by $\mathcal{O}(1)$ the invertible sheaf on $\mathbb{P}^1_{\mathbb{C}}$ defined by the transition functions $T_{01} = (x) = (y^{-1})$ and $T_{10} = (y) = (x^{-1})$.

   (a) Calculate the global sections of $\mathcal{O}(1)$.

   (b) Consider the irreducible Weil divisors $V(x) \subset U_1$, $V(y) \subset U_0$, and $V(x-2) \subset U_0 \cap U_1$. Let $D = [V(x)] + [V(y)] - [V(x-2)]$. What is $D|_{U_0}$?

   (c) Find a *rational* section s of $\mathcal{O}(1)$ such that $\operatorname{div}(s) = D$.

   (d) Calculate the global sections of $\mathcal{O}(D)$.

   (e) Prove that $\mathcal{O}(1) \cong \mathcal{O}(D)$.

2. Let $X = \operatorname{Spec} k[x, y]$, let $D = [V(y)]$. Prove that $\mathcal{O}_X(D) \cong \mathcal{O}_X$.