

**HW 2**  
**Math 672**

Due Friday, in class.

1. Read 2.1-2.4 of *The Rising Sea: Foundations of Algebraic Geometry* (Ravi Vakil), 2022 version.
2. Let  $(\mathbb{R}, \leq)$  denote the category whose objects are real numbers and there exists a morphism  $f : x \rightarrow y$  if and only if  $x \leq y$ . Let  $(\mathbb{Z}, \leq)$  denote the category whose objects are integers and again there exists a morphism  $f : x \rightarrow y$  if and only if  $x \leq y$ . Note that the inclusion  $\mathbb{Z} \rightarrow \mathbb{R}$  induces a (fully faithful!) functor  $i : (\mathbb{Z}, \leq) \rightarrow (\mathbb{R}, \leq)$ . Show that  $i$  is left adjoint to the “floor functor”  $\lfloor - \rfloor : (\mathbb{R}, \leq) \rightarrow (\mathbb{Z}, \leq)$  which, on objects, sends the real number  $x$  to the largest integer which is less than or equal to  $x$ . Check also that  $i$  is right adjoint to the “ceiling functor” which, on objects, sends the real number  $x$  to the smallest integer which is greater than or equal to  $x$ .
3. 1.6.D
4. Let  $\mathcal{C}$  be an abelian category and let  $C$  be an object of  $\mathcal{C}$ . Show that  $\text{Hom}_{\mathcal{C}}(C, -) : \mathcal{C} \rightarrow \text{Ab}$  is a left-exact covariant functor.
5. 2.2.F The following will not be graded, but are good practice. They are listed in decreasing order of importance.
6. 1.3.H
7. 1.6.F (a)
8. 1.6.I