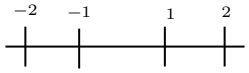
Exercise 1 (10 points)**.** Consider the function $f(x)=\frac{2}{(x-2)^2(x^2-1)}$.

i) What are the discontinuities of this function? Label all of them as $x=(...)$.
ii) Make a diagram of the interval $[-2,2]$ including the discontinuities of $f(x)$ and the endpoints of the interval.
iii) Explain why the integral $\int_{-5}^{5} f(x)\mathrm{d}x$ is improper.
iv) Write the previous integral in limit notation. (It is not necessary to solve the integral.)

The discontinuities of this function are the zeroes of the denominator:

$$x=2, \quad x=1, \quad x=-1.$$

The diagram in question is



The integral is improper because the interval contains discontinuities of the function we are integrating.

$$\int_{-2}^{2} f(x)\mathrm{d}x = \lim_{a\to -1}\int_{-2}^{a} f(x)\mathrm{d}x + \lim_{b\to -1}\int_{b}^{0} f(x)\mathrm{d}x + \lim_{c\to 1}\int_{0}^{c} f(x)\mathrm{d}x + \lim_{d\to 1}\int_{d}^{1/2} f(x)\mathrm{d}x + \lim_{e\to 2}\int_{1/2}^{e} f(x)\mathrm{d}x$$

Exercise 2. Consider an empty box with capacity $\mathrm{cap}_B=18\mathrm{N}$. Assume it's raining at $v_{\mathrm{RAIN}}=3\mathrm{Ns}^{-1}$.

i) How long does it take for the box to fill up?
ii) If the box dangles from a 10m long rope and we are pulling at $1\mathrm{ms}^{-1}$, will the box fill up before reaching the top? If so, at what point?
iii) If the answer to the last question was positive, what will be the weight of the water from that point to the top? If the box doesn't fill, write *the weight is variable*.
iv) Write an expression (not necessary to solve) for the work required to pull the box with water up. (The box and rope are weightless.)

i) $\mathrm{cap}_B/v_{\mathrm{RAIN}}=t_{\mathrm{fill}}=6\mathrm{s}$.
ii) Yes, it fills up at 6m.
iii) The water weighs as much as the box can hold, so 18N.
iv) $W_W=\int_0^6 3y\mathrm{d}y+\int_6^{10}18\mathrm{d}y$.

Exercise 3. The same box with capacity $\text{cap}_B = 18\text{N}$ is now full of water. Assume it leaks water at $v_{\text{LEAK}} = 9\text{Ns}^{-1}$.

i) How long does it take for the water to leak completely?
ii) If the box dangles from a 3m long rope and we are pulling at 1ms^{-1}, will the water leak out before reaching the top? If so, at what point?
iii) If the answer to the last question was positive, is there work being done after the water leaks? (Assume the box and rope are weightless) If the box doesn't stop leaking, write *the box is still leaking.*
iv) Write an expression (not necessary to solve) for the work required to pull the box with water up. (The box and rope are weightless.)

i) $\text{cap}_B / v_{\text{LEAK}} = t_{\text{LEAK}} = 2\text{s}$.
ii) Yes, it leaks out at 2s.
iii) No, there's no weight.
iv) $W_W = \int_0^2 (18 - 9y)\,\mathrm{d}y$.