Exercise 1 (Exercise 1 and 2)**.** Let P be a finite poset and $L = \mathcal{J}(P)$ be the corresponding distributive lattice. If $X \subseteq P$ is a lower-order ideal, then use the corresponding lowercase letter x to denote the associated element of L.

i) Show that x covers y in L if and only if $Y = X \backslash \{ m \}$, where m is a maximal element of X.

ii) Show that x is join-irreducible in L if and only if X is a principal ideal of P.

Answer

i) Let us begin by assuming that x covers y in L this means that $x \supseteq y$ and $\nexists z(x > z > y)$. In terms of elements, this means that $|x \backslash y| = 1$. Take n to be the element we removed from x, if n is not maximal, then y is no longer an order ideal as $n \leqslant s$ where s is a maximal element in y.

   On the other hand $X \backslash \{ m \} \subseteq X$, so we only have to prove that $X \backslash \{ m \}$ is an ideal. FINISH

ii)