HW 3
Math 672

Due Wed, Sep. 14 in class.

0. Read Sections 2.4, 2.5, 2.6.

1. Let $q = (a_1, \ldots, a_n)$ be a point in $\mathbb{A}^n$. Using the fact that $\mathbb{I}(q)$ is a maximal ideal in $\mathbb{C}[x_1, \ldots, x_n]$, prove that the coordinate ring of q is isomorphic to $\mathbb{C}$. If $i : q \to \mathbb{A}^n$ is the inclusion map, show that the pullback homomorphism

$$i^{\#} : \mathbb{C}[x_1, \ldots, x_n] \to \mathbb{C}[q] = \mathbb{C}$$

   sends a function $f(x_1, \ldots, x_n)$ to the complex number $f(a_1, \ldots, a_n)$ obtained by evaluating at that point.

2. Prove that if $F : V \to W$ is an isomorphism of affine algebraic varieties, then the pullback homomorphism is a ring isomorphism.

3. Let $V \subset \mathbb{A}^n$ and $W \subset \mathbb{A}^m$ be affine algebraic varieties. Let $\widetilde{F} : \mathbb{A}^n \to \mathbb{A}^m$ be a morphism. Show that $\widetilde{F}$ maps V to W (i.e. $\widetilde{F}(V) \subset W$) if and only if the pullback $\widetilde{F}^{\#} : \mathbb{C}[y_1, \ldots, y_m] \to \mathbb{C}[x_1, \ldots, x_n]$ sends $\mathbb{I}(W)$ to $\mathbb{I}(V)$. (hint: $W = \mathbb{V}(\mathbb{I}(W))$)

4. 2.6.1

5. (won't be graded) 2.5.1 and 2.5.2

6. (For 2 points extra credit) Come up with something you are confused about and ask me about it in office hours. If you can't make office hours then talk to me and we can find a different time to meet.