Exercise 1. Consider the curve $r(t)=(t,(4-t^2)^2)$ for $1\leqslant t\leqslant 3$. Answer the following tasks:

1. What are the value $t=1,t=3$ and $r(1)=(1,9),r(3)=(3,1)$ called? What is the difference between them?
2. Is the point (2,4) on the curve? If so, what is it's local coordinate/parameter?
3. Is the point with local coordinate $t=0$ on the curve? There's more than one way to show this, can you mention 2 ways to do it?

1. The values $t=1$ and $t=3$ are the parameter values corresponding to the initial and final positions of the curve, respectively. The points $r(1)=(1,9)$ and $r(3)=(3,1)$ represent the actual starting and ending points in space. The key distinction is that parameters describe positions along the curve, while the points $r(1)$ and $r(3)$ are the spatial coordinates corresponding to these parameters.
2. To determine if the point (2,4) lies on the curve, we solve for t in the equations $t=2$ and $(4-t^2)^2=4$. Substituting $t=2$ gives:
$$(4-2^2)^2=0^2=0\neq 4.$$
Since the equations are inconsistent, the point (2,4) is not on the curve.
3. The point corresponding to $t=0$ is not on the curve because $t=0$ is outside the given parameter range $1\leqslant t\leqslant 3$. One way to verify this is by checking if $t=0$ falls within the interval. Another method is to substitute $t=0$ into the parametric equations and verify if the resulting point lies on the curve.

Exercise 2. Consider the plane which satisfies the following:

- Passes through the origin.
- Is orthogonal to the line between the points (4,–5,0) and (2,–3,1).

Verify if the point (1,1,0) is on the plane.

Given that the plane passes through the origin (0,0,0) and is orthogonal to the line ℓ passing through (4,–5,0) and (2,–3,1), the normal vector $\vec{n}$ to the plane is parallel to the direction vector of ℓ, denoted by $\vec{v}_\ell$. We compute:
$$\vec{v}_\ell=(2,-3,1)-(4,-5,0)=(-2,2,1).$$
The equation of the plane can then be written as:
$$\vec{n}\cdot\vec{x}=0 \iff (-2,2,1)\cdot(x,y,z)=0 \iff -2x+2y+z=0.$$
To verify if the point (1,1,0) lies on the plane, substitute into the plane equation:
$$-2(1)+2(1)+0=0.$$
Since the equation holds true, the point (1,1,0) is indeed on the plane.