Exercise 1. Find an example of two curves in $\mathbb{P}^2$ that have the same degree but are not isomorphic.

Answer

Exercise 2. Do the following:

(a) Find the Hilbert polynomial P of a k-dimensional linear subvariety of $\mathbb{P}^n$.

(b) Describe the Hilbert scheme of varieties in $\mathbb{P}^n$ with Hilbert polynomial P.

Answer

Exercise 3. Assume that the variety $V \subseteq \mathbb{P}^n$ has the Hilbert polynomial $P(n)$. Calculate the Hilbert polynomial of the image variety $\nu_d(V) \subseteq \mathbb{P}^{\binom{n+d}{d}-1}$ of the Veronese map. ⟦ Hint: Do the case of $V = \mathbb{P}^1$ first. ⟧

Answer

Exercise 4. Using the theorem describing the defining equations for T_pV in terms of the equations for V, compute the tangent spaces of the curves in examples (1), (2), and (3) at the origin.

Answer

Exercise 5. Let $V \subseteq \mathbb{P}^n$ be a hypersurface defined by a homogeneous irreducible polynomial F. Find an explicit description of the tangent space to V at a point p. What conditions on p ensure that the tangent space to V at p has dimension $n-1$?

Answer