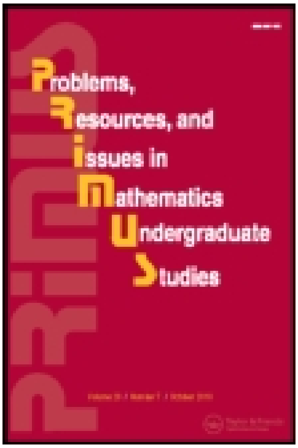

PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/upri20

The Coverage Issue

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Accepted author version posted online: 04 Jul 2011.
Version of record first published: 06 Jul 2012

PRIMUS, 22(4): 303–316, 2012

ISSN: 1051-1970 print / 1935-4053 online
DOI: 10.1080/10511970.2010.507622

The Coverage Issue

Stan Yoshinobu and Matthew G. Jones

Abstract: We address coverage versus depth as a false dichotomy, and reframe the issue in terms of critical questions for instructors to consider regarding student learning. We also review some key reasons to favor inquiry-based learning as an approach to undergraduate instruction.

Keywords: Coverage, inquiry-based learning, Moore method, traditional lecture method, mathematics education research.

1. WHAT IS COVERAGE?

A significant issue we face as mathematics instructors is how to cover all the material. Mathematics teachers of all levels have some external and internal pressures to "get through" all the required material. Courses, such as calculus, are jam packed with content, and books are literally several hundred or even thousands of pages long. When we teach these courses a serious concern is the amount of time we can spend on each topic we choose to teach.

We define *the coverage issue* to be the set of difficulties that arise in attempting to cover a lengthy list of topics. Principal among these is that material must be presented quickly, to ensure a course covers all the topics in the syllabus. As we will see, speeding through the material has a high cost.

The coverage issue and how we teach are intimately connected. To cover a long list of topics, the pace of a course must be relatively fast. Section 2.1 is covered on Monday. Section 2.2 will be covered on Wednesday, and so forth. The (perceived) need to cover all the topics encourages the use of the standard model.

The *standard model* (or traditional lecture method) for teaching is the instructional paradigm typical in mathematics courses at the secondary and college levels. It is a teacher-centered model based on the lecture method.

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The instructor provides clear explanations in class, and helps students solve exercises by modeling solutions to examples. The primary features of the standard model are the instructor's role as dispenser of knowledge and the students' role as passive consumers of knowledge. The instructor shows, and the students follow. Conversation, if it occurs at all, involves only the instructor posing a short query, a student responding, and the instructor evaluating the answer immediately as right or wrong [3].

The pressure to cover everything makes it difficult to use another teaching model. The following are some examples of how this sounds:

- "I need to cover all these topics, so I don't have enough time to do student-centered activities."
- "The next instructor expects that my students will know how to do . . . . So I have to cover these topics."
- "I have to cover these topics because they are on the state test."

On the surface, the coverage issue is about figuring out what content should be covered, and how that content can be effectively taught. However, there are fundamentally important issues about teaching and learning to consider.

2. WHAT IS THE PRICE OF COVERAGE?

To understand why coverage can be problematic, we draw on an example from mathematics education research. In Alan Schoenfeld's [16] article, "Good Teaching Bad Results: The Disasters of a 'Well Taught' Mathematics Course," a high school geometry teacher using the standard model is studied. This teacher does everything expected of him professionally, covering the standard syllabus for the course. His students succeed at the main tasks expected of them by society, such as scoring well on standardized tests. Schoenfeld finds that what looks like success is actually a mirage. One of the eye-opening results in Schoenfeld's article is the unintended effect on student beliefs. Students believed in statements such as, "Constructions in Geometry are unrelated to proofs," and, "Only geniuses can really understand mathematics."

What is more alarming is that these kinds of beliefs are widespread. In a meta-analysis of research on student attitudes and beliefs, Muis [14] finds that the beliefs described by Schoenfeld occur across grades and on a large scale. Muis defines the term *non-availing belief* to mean a belief that does not support or that inhibits learning. Muis collects from the research literature a lengthy list of non-availing beliefs commonly held by students. Some of these non-availing beliefs are:

1. Memorizing facts and formulas and practicing procedures is sufficient to learn mathematics.
2. Mathematics textbook problems can only be solved using the methods described in the textbook.
3. Teachers and textbooks are the mathematical authorities.
4. School mathematics is driven by rules and memorization, and is driven by procedures rather than concepts.
5. If a problem takes longer than 5–10 minutes, then there is something wrong with the student or the problem.
6. The goal of mathematics is to obtain one correct answer and do it quickly.
7. The teacher is the only source of determining whether an answer is correct or incorrect.
8. Students' role in the classroom is to receive knowledge by paying attention in class, and to demonstrate it has been received by producing correct answers.
9. The teacher's role is to transmit knowledge, and verify that it has been transmitted.
10. Only geniuses have what it takes to be good at mathematics.
11. Students prefer to have only one way of solving a problem, because it is less to memorize.
12. The processes of formal mathematics have little or nothing to do with discovery or invention.
13. Students who understand mathematics can solve assigned problems in 5 minutes or less.
14. One succeeds in school mathematics by performing the tasks, to the letter, as described by the teacher.
15. The various components of mathematics are unrelated.

Students who have several of these non-availing beliefs are at a disadvantage when learning mathematics. When they do mathematics, they interact with it in such a way that they take away a much narrower view of mathematics than intended.

We use the Schoenfeld article [16] in professional development workshops for college mathematics faculty. In the discussions pertaining to this article, several mathematicians recognized parallels to their own teaching. They have implemented similar strategies with good intentions. They thought they were being helpful, and students even appreciated their efforts. While they were helping students succeed on certain mathematical tasks, they did not realize that they were reinforcing some negative beliefs.

However, beliefs are not the only issue. Research consistently shows that students fail to learn important mathematical skills, attitudes, and ways of thinking. Students' learning issues are especially problematic when they transition to upper division mathematics, where proof becomes a more central part of the courses. Several researchers have investigated how students view and attempt to construct proofs [7, 8, 13, 17, 22]. What is clear from these studies is that students often do not rely on strategies that mathematicians frequently use, such as constructing and experimenting with examples, both to make sense of theorem statements and to provide insight into a method of proof. Students often do not look to make sense of the mathematics, but instead attempt to mimic solutions to superficially similar problem statements, or they manipulate symbols in hopes that the manipulations will result in a proof acceptable to an external authority (the professor).

Moore's findings summarize the situation well [13]. Moore finds that undergraduate mathematics majors have difficulty proving seemingly trivial statements. Students have difficulty with using notation, with conceptual understanding, and getting started on proofs. In the article, Moore reports that students relied on memorizing proofs, " . . . because they had not understood what a proof is or how to write one." [13, p. 264].

At the calculus level [18, 19], there is evidence that students in calculus cannot solve non-routine problems. Additionally, even after a year and a half of calculus, and currently being in a differential equations course, the students in the study [18] did not eventually learn to solve non-routine problems. Calculus students are not learning how to solve problems, despite having the requisite skills and content knowledge.

The price of coverage goes far beyond the skills and attitudes students acquire or fail to acquire. Sociologists Seymour and Hewitt [20] studied students in Science, Mathematics, and Engineering (SME) majors at a range of institutions. They found that many good, qualified students leave for other majors. About half of SME majors are lost to non-SME majors before graduation. The traditional theory for why students leave goes like this: students leaving SME majors are not cut out for the "hardness" of Science, Mathematics, and Engineering, and the weak students are "weeded out." The data presented by Seymour and Hewitt, however, say otherwise. Students did not necessarily switch because they were failing. On the basis of individual attributes, including GPA in the major, it is difficult to predict who will switch and who will not. The top four factors reported as contributing to students' decision for switching majors are:

1. lack of/loss of interest in SME;
2. non-SME major offers better education/more interest;
3. poor teaching by SME faculty;
4. curriculum overload, fast pace overwhelming.

The top four reasons why students switch out of SME majors reflect the teaching and learning environment. Curriculum overload and poor teaching highlight two related facets of the coverage issue that encourage students to leave SME majors. Students, including some who are succeeding in the courses, are reacting to the coverage issue and the standard model by leaving the major.

Studies such as these give reason to believe that the standard model may fall short in important ways, and that the coverage issue is widespread and casts a shadow over our entire system. Students have non-availing beliefs, cannot prove even simple statements, and cannot apply their knowledge to non-routine situations. Confronted with difficulty and a pace that many cope with by memorizing rather than understanding, a significant number leave for other majors, despite living in an era when technical fields are becoming ever more important and lucrative. The high cost of racing through material is quite clear.

We introduce another definition that will help codify our intent to look at mathematics instruction on broader terms. Consider the idea of *mathematical proficiency*, defined as consisting of five components: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition [9, p. 116]. Viewed through this lens, teaching is much more than a list of topics in a syllabus. Implied messages are sent to students through classroom experiences. These experiences, as a whole, shape attitudes and influence *how* students learn mathematics. The negative impact of the standard model on the components other than procedural fluency suggests that there is a need to examine alternatives. In the next section, we offer such an alternative.

3. WHAT IS INQUIRY-BASED LEARNING AND HOW IS IT DIFFERENT?

Inquiry-based learning (IBL) is an instructional paradigm in which the students interact, under the guidance of the instructor, with a curriculum typically consisting of a set of axioms, definitions, and problems. The core idea is that students are engaged in an apprenticeship into the practice of mathematics. Students actively participate in contributing their mathematical ideas to solve problems, rather than applying teacher-demonstrated techniques to similar exercises. The instructor facilitates student progress, ensuring that the class can move toward increasingly sophisticated ways of thinking. A key feature is that students present their ideas to small groups or to the entire class, and classmates peer review their work. Students construct their own ideas and evaluate the mathematics of their classmates for themselves, rather than rely on an external authority. In short, students do mathematics like research mathematicians do mathematics.

One historical source for this approach is the Moore Method, named for mathematician R. L. Moore [5, 11]. Although the Moore Method is a type of IBL, IBL allows for classroom activities that Moore prohibited, such as student collaboration. These differences are minor, however, and, in the broader view, the Moore Method and IBL are essentially equivalent.

To illustrate what IBL looks like in a proof-based classroom, consider a course in Real Analysis that one of the authors has taught for several years. In this course, the syllabus reflects many differences, compared to the standard model. In addition to the usual homework assignments, midterms, and a final, a portfolio and in-class presentations are used to evaluate student performance. Typically, the portfolio is worth about 10–20% of the final course grade, and the presentations are worth about 25–35%. Thus, emphasis is placed on the ability to demonstrate, through writing and proving, that a student can *do* mathematics.

Students in this Real Analysis course are not allowed to use a traditional textbook, look up answers to problems, or ask people not in the class for help. The students, instead, work with assistance from the instructor on a carefully crafted sequence of problems. Such a course cannot progress through topics linearly in time. Students may be stuck on problems for what could seem like an eternity to the instructor. Indeed, learning is nonlinear, and students will be stuck for a while and then make leaps forward. When students are stuck, the instructor helps them by posing questions, using group activities, and helping the students find and solve smaller problems or special cases that could help them deal with the larger problems. Coverage as measured by the number of topics is de-emphasized, but time spent *doing* mathematics is increased.

The use of IBL techniques extends far beyond proof-based courses. IBL methods can be implemented in any mathematics course. Indeed, we have used IBL methods in courses such as Calculus, Precalculus, Introductory Statistics, Mathematics for Elementary Teachers, and graduate-level courses in a Master's in Mathematics Teaching. The point emphasized is that IBL methods are universal, and not restricted to proof-based courses. They can be implemented in any course, with some adaptations. In lower-division courses, inquiry is usually implemented as problem-solving experiences, some of which may be open-ended.

In Precalculus, students may focus extensively on building a cohesive understanding of basic classes of functions (linear, quadratic, higher-degree polynomial, rational functions, exponential functions, logarithmic functions, and periodic or trigonometric functions). This can be achieved through tasks such as having students interpret a data table or a graph in context, determining a symbolic description of a function from a given data table, graph, or written context, and asking students to select the most appropriate class of function to model a given situation. Students may also be asked to read key definitions or theorems, and give examples and non-examples or restatements, a task that helps students learn to make sense of mathematical text. Students will often be asked to solve problems without similar solved problems being given as models. The central theme of these activities is that students are asked to build an understanding through connecting representations of a function, and through interpreting the mathematics for themselves.

A sample inquiry-based activity in a Mathematics for Elementary Teachers course is to have students build their own algorithms for addition, subtraction, multiplication, and division in base 5. Changing bases requires students to develop a deeper understanding of place value and the positional system. Asking students to build their own algorithms prepares them to analyze the standard algorithms (in the decimal system), as well as be more sensitive to the learning issues their future students may face. Combined with appropriate readings from Mathematics Education, these inquiry-based activities can be tied to school curriculum and children's mathematical thinking.

Another example of IBL in a non-proof course is the Inquiry-Oriented Differential Equations Project. Rasmussen and Kwon described the course goals as follows:

> We wanted students to essentially reinvent many of the key mathematical ideas and methods. . . . We wanted challenging tasks, often situated in realistic situations, to serve as the starting point for students' mathematical inquiry. . . . We wanted a balanced treatment of analytic, numerical, and graphical approaches, but we wanted these various approaches to emerge more or less simultaneously for learners. [15, p. 190].

Students in the course therefore spent extensive time using, developing, and comparing different approaches, rather than applying clearly articulated processes to well-posed problems that provide the opportunity to practice the solution method of the day. Thus, IBL can take many forms, but has as a basic feature the focus on student thinking, sense-making, and student-driven development of key mathematics.

IBL is a paradigm shift in mathematics instruction. Mason summarized the change in orientation toward instruction as, "A revised didactic contract, seeking to balance developing competency with enculturation into mathematical thinking, rather than succumbing to student desire to minimize effort and simply be trained in requisite behavior." [12, p. 72]. This shift involves spending extended time on key concepts, and, thus, coverage of the topics will look different from in the standard treatment, as some topics are shifted into the background for reduced emphasis.

4. WHAT HAPPENS TO COVERAGE IN AN INQUIRY-BASED COURSE?

Use of IBL will mean that critical topics will be treated in greater depth to allow for students to explore and develop their own understandings. Once an instructor has determined which topics are most critical, the question arises as to what to do with the less critical material. The authors typically rely on one of three strategies for such topics: outside reading assignments, mini-lectures, or elimination. If a topic has been chosen for de-emphasis, but some of the results are important for later in the course or subsequent courses, then students may be asked to read additional mathematics outside of class time, and turn in an assignment based on the reading, with no corresponding class time spent on the topic. The results are then assumed for future class sessions.

An alternative to this approach is to provide a mini-lecture on a topic. A mini-lecture might last up to one half of a course meeting, and may be a way to either open or close a unit of study. For instance, students studying techniques of integration in calculus may be given a brief lecture on the method of partial fractions as a relatively less essential method, before moving on to applications of integration.

Finally, when considered in light of the learning goals of the course, some topics may be eliminated. In the aforementioned Mathematics for Elementary Teachers course, in order to provide time to explore non-decimal bases, ancient numeral systems were eliminated as a topic. Although of some value for historical perspective, the authors have found that the main point of understanding the value and meaning of our positional system of numeration can be made using base 5.

Given students' inadequate mastery of topics in the standard model, and inability to use their knowledge to solve problems, reducing the number of topics seems justified. Moreover, the reduction of the number of topics, and going beyond primarily focusing on procedural skills may be less costly than one might expect, as we will see next.

5. WHAT DOES RESEARCH SAY ABOUT STUDENT LEARNING IN AN IBL SETTING?

IBL may be effective in helping students to become mathematically proficient. We present some key evidence from mathematics education, in which IBL students develop conceptual understanding and appropriate mathematical habits and beliefs.

Two recent research projects in mathematics education studied courses in which the instruction centers on the students' efforts to create mathematics. The first of these projects, the Inquiry-Oriented Differential Equations project [10, 15], focused on having students develop and grapple with multiple representations for differential equations, under the guidance of the instructor, as described in the previous section. Their work showed that students in the inquiry-oriented class, and in a comparison traditional lecture course, performed similarly on procedural items, but that the students in the inquiry-oriented course outperformed the students from the traditional course on modeling and conceptual items.

On a retention test given one year after the course, students in the inquiry course performed similarly, except on modeling tasks, on which students from the inquiry course continued to outperform students from the traditional course, even though 75% of the students in the traditional section had taken a course in numerical differential equations in the interim year, which the inquiry students had not taken. On the retention test, modeling questions consisted of selecting an appropriate model of a scenario, among given choices of differential equations, and developing a differential equation and initial condition from a given scenario. Thus, it might be argued that the cost of coverage using the standard model was that students were only able to solve a narrow range of well-posed problems presented in textbook-like format, while they performed relatively poorly on items requiring application of the mathematics to real scenarios.

In a separate project, Smith compares a subset of students in an inquiry-based number theory course, taught by an experienced user of IBL, to students enrolled in a different section of the same course taught using the standard model [21]. Students in this course develop number theory from its foundation, with the major focus of activity being students presenting their proof attempts, while their peers critique the attempts and decide whether the attempt establishes the theorem.

The perceptions of proof and strategies used in constructing proof were different between the two groups, with those in the inquiry-based course using more strategies to make sense of the mathematics as a way to gain insight into a proof, while those in the traditional course manifested problematic perceptions, such as viewing proof as an algorithm rather than a means to understand a result. These perceptions also corresponded with the students using different strategies to write proofs. Those in the inquiry-based course were able to use examples to understand and to construct formal proofs, while those who experienced traditional instruction avoided using examples, since they felt examples were not directly part of producing a proof, and, instead, they tended to search their memories for problems with similar surface features. Thus, the evidence in both the differential equations and the number theory cases suggests that there are significant benefits to designing instruction so that students' thinking is central to the activities of the course.

Much more work has been done at the calculus level in comparing reform calculus to traditional lecture. Reform calculus has taken many forms. However, it shares with IBL a focus on student thinking and an emphasis on problems, particularly contextual problems and the use of multiple representations. We review two reports of interest in this area.

Chappell compared the students of four accomplished instructors, two with reform approaches and two with traditional approaches to calculus, on both conceptual and procedural items on the midterm and final exams [4]. Students in the reform sections significantly outperformed students in the traditional sections on the conceptual items on both the midterm and final, and on the procedural items on the midterm, while no significant differences were found on procedural items on the final exam. Based on her examination of student responses, Chappell concluded that students in the reform courses were "better able to explain in words or graphically why a particular procedure made sense." [4, p. 54]. Moreover, "Item analysis supported the conjecture that knowledge acquired with understanding can be extended to solve unfamiliar problems more readily than strictly procedural knowledge," [4, p. 55]. Next, we turn to a second calculus reform study.

Darken, Wynegar, and Kuhn [6] found that, of eight studies comparing calculus students on common final exams, four found significant differences in favor of the reform sections on problem solving, graphs, or conceptual questions; two found differences in favor of reform sections but did not report the significance; one did not report on such questions, and only one found significant differences in favor of the traditional lecture section. Among the six of these studies that reported a comparison on traditional test items, all found that the two types of instruction yielded either no significant differences or similar scores. Thus, this review supports the findings of Chappell [4]. We will move next to a different setting, that of high school mathematics.

Jo Boaler conducted two separate in-depth studies of IBL approaches to mathematics, compared to traditional approaches [1], and [2], with Megan Staples. In the first case [1], she conducted a three-year study of two high schools, one dubbed Amber Hill, using a traditional approach, and the other, Phoenix Park, using an IBL curriculum emphasizing a project-based curriculum.

Among Boalar's findings, students at Amber Hill exhibited what she termed cue-based behavior: "If a question seemed inappropriately easy or difficult, if it required some non-mathematical thought, or if it required an operation other than the one they had just learned about, many students would stop working." [1, p. 48]. She includes examples of these students asking for help with contextual problems in a lesson. In each case, when asked, the student could explain what to do; however, they had asked for help or skipped the question because it did not fit their expectation of applying a similar procedure, and at a similar level of complexity, as the other problems assigned. Further, at Amber Hill, 64% of the students prioritized remembering over thinking as central to mathematics on a questionnaire, while, at Phoenix Park, only 35% prioritized remembering.

In addition, Phoenix Park students outperformed Amber Hill students on contextual problems, but did about as well on typical textbook-like problems, which is surprising given the de-emphasis on skills at Phoenix Park. Boaler characterized the Amber Hill students as having developed "an inert procedural knowledge that was of limited use to them in anything other than textbook situations." [1, p. 59], a characterization supported by proclamations by the students themselves as well as their performance. A second study by Boaler and Staples [2] with a different set of high schools, either using an IBL-type curriculum or not, generated similar kinds of results and distinctions between the groups.

In sum, there is ample evidence to suggest that IBL courses, as compared to courses using the standard model, enable students to develop greater strategic competence and a more productive disposition without much, if any, sacrifice in their level of procedural fluency. The clear theme among this diverse body of studies is that there is much to be gained by refocusing courses around students' thinking and, where appropriate, looking at problems in real contexts. This body of research demonstrates that IBL produces gains on a broad array of measures, while the reduction in coverage has minimal cost.

6. WHAT IS THE POINT OF (MATHEMATICS) EDUCATION?

Considering mathematical proficiency as the goal of mathematics education, IBL has demonstrated some advantages, particularly in terms of adaptive reasoning and productive disposition, while sacrificing little if any procedural fluency, as we have seen. In practice, then, IBL courses will spend more time on critical topics, as students develop their own reasoning skills and prove theorems for themselves. This means that some topics may receive lighter treatment than in the standard model. An overlooked aspect of education is that leaving students to fill in gaps on their own is a fact of life. No degree program can fully prepare students to master enough content for all possibilities. For a long time, mathematics has been too large for anyone to master all of it. There is always more to learn. But we can prepare our students to be effective, thoughtful, and curious learners who have the ability and tools to learn and discover more on their own. In the Google age of information, knowledge, as an end in itself, has become less important, while the value of processing and evaluating information and solving problems has increased. If students are mathematically proficient, the evidence reviewed suggests we can then be confident of their ability to learn additional material as needed for future mathematical endeavors.

7. CONCLUSION

Coverage isn't just about more or less material and the relative depth on particular topics. Moreover, we are not trivializing the importance of content. Content matters. Mathematical content is the vehicle for developing mathematical thinkers. But for the content learning to achieve greatest effect, it must not come at the expense of achieving our main goals for mathematics education, and education more broadly. Instead, it must be achieved in the process of training students as mathematical thinkers—people apprenticed in doing Mathematics like mathematicians.

The coverage issue can bind us. If we are beholden to the notion that we must lecture daily to make it through all the material, we must accept that many students are left with underdeveloped learning abilities and nonavailing beliefs. The data speak ever more loudly that our standard model is failing many students, and pushing away students who are capable of succeeding. Therefore, we have suggested extensive use of inquiry-based learning to develop students' mathematical abilities, with additional content developed through occasional, well-timed presentations and other out-of-class assignments. With this balanced approach, we can produce students who are truly mathematical thinkers.

REFERENCES

1. Boaler, J. 1998. Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education.* 29: 41–62.
2. Boaler, J., and M. Staples. 2008. Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record.* 110: 608–645.
3. Cazden, C., and H. Mehan. 1990. Principles from sociology and anthropology: Context, code, classroom, and culture. In M. C. Reynolds (Ed.), *The Knowledge Base for Beginning Teachers*, pp. 47–57. New York: Pergamon.
4. Chappell, K. K. 2006. Effects of concept-based instruction on calculus students' acquisition of conceptual understanding and procedural skill. In F. Hitt, G. Harel, and A. Selden (Eds.), *Research in Collegiate Mathematics Education VI* (pp. 27–60). Providence, RI: American Mathematical Society.
5. Coppin, C. A., W. T. Mahavier, E. L. May, et al. 2009. *The Moore Method—A Pathway to Learner-Centered Instruction*. Washington, DC: Mathematical Association of America.
6. Darken, B., R. Wynegar, and S. Kuhn. 2000. Evaluating calculus reform: A review and a longitudinal study. In E. Dubinsky, A. H. Schoenfeld, and J. Kaput (Eds.), *Research in Collegiate Mathematics Education IV* (pp. 16–41). Providence, RI: American Mathematical Society.
7. Harel, G., and L. Sowder. 1998. Students' proof schemes: Results from exploratory studies. In A. H. Schoenfeld, J. Kaput, and E. Dubinsky (Eds.), *Research in Collegiate Mathematics Education III* (pp. 234–283). Providence, RI: American Mathematical Society.
8. Housman, D., and M. Porter. 2003. Proof schemes and learning strategies of above-average mathematics students. *Educational Studies in Mathematics.* 53: 139–158.
9. Kilpatrick, J., J. Swafford, and B. Findell. 2001. *Adding It up: Helping Children Learn Mathematics*. Washington, DC: National Research Council.
10. Kwon, O.N., C. Rasmussen, and K. Allen. 2005. Students' retention of mathematical knowledge and skills in differential equations. *School Science and Mathematics.* 105: 227–239.

11. Mahavier, W.S. 1999. What is the Moore Method? *PRIMUS*. 9: 339–254.
12. Mason, J. 2001. Mathematical teaching practices at tertiary level: Working group report. In D.A. Holton (Ed.),*The Teaching and Learning of Mathematics at University Level: An ICMI Study* (pp. 71–86). Hingham, MA: Kluwer.
13. Moore, R. C. 1994. Making the transition to formal proof. *Educational Studies in Mathematics*. 27: 249–266.
14. Muis, K. R. 2004. Personal epistemology and mathematics: A critical review and synthesis of research. *Review of Educational Research*. 74: 317–377.
15. Rasmussen, C., and O. N. Kwon. 2007. An inquiry-oriented approach to undergraduate mathematics. *Journal of Mathematical Behavior*. 26: 189–194.
16. Schoenfeld, A. H. 1988. When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist*. 23: 145–166.
17. Selden, A., and J. Selden. 2003. Validation of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem? *Journal for Research in Mathematics Education*. 34: 4–36.
18. Selden, A., J. Selden, S. Hauk, and A. Mason. 2000. Why can't calculus students access their knowledge to solve non-routine problems? In E. Dubinsky, A. H. Schoenfeld, and J. Kaput (Eds.), *Research in Collegiate Mathematics Education IV* (pp. 128–153). Providence, RI: American Mathematical Society.
19. Selden, J., A. Selden, and A. Mason. 1994. Even good calculus students can't solve nonroutine problems. In J. J. Kaput and E. Dubinsky (Eds.), *Research Issues in Undergraduate Mathematics Learning, MAA Notes No. 33* (pp. 19–26). Washington, DC: Mathematical Association of America.
20. Seymour, E., and N. M. Hewitt. 1997. *Talking about Leaving*. Boulder, CO: Westview Press.
21. Smith, J. C. 2005. A sense-making approach to proof: Strategies of students in traditional and problem-based number theory courses. *Journal of Mathematical Behavior*. 25: 73–90.
22. Weber, K. 2001. Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics*. 48: 101–119.

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