

Exercise 1. Consider the curve $r(t) = (t, (4-t^2)^2)$ for $1 \leq t \leq 3$.

1. Find a parametrization of the line segment between the endpoints of this curve.
2. Find the velocity vector for this curve.
3. Verify the following given solution and correct it if there are any mistakes:

$$\text{Parametrization: } t(3) + (1-t)(1) = 2t - 1$$

$$\text{Velocity vector: } v(t) = r'(t) = (1, -4t(4-t^2))$$

1. **Find the endpoints:**

$$r(1) = (1, (4-1^2)^2) = (1, 9)$$

$$r(3) = (3, (4-3^2)^2) = (3, 25)$$

2. **Parametrize the line segment between the endpoints:**

$$\begin{aligned} \text{The line segment is: } & t(3, 25) + (1-t)(1, 9) \\ & = (3t + 1 - t, 25t + 9 - 9t) \\ & = (2t + 1, 16t + 9), \quad 0 \leq t \leq 1 \end{aligned}$$

3. **Find the velocity vector:**

$$\begin{aligned} v(t) = r'(t) &= \left(\frac{d}{dt}t, \frac{d}{dt}(4-t^2)^2 \right) \\ &= (1, -4t(4-t^2)) \end{aligned}$$

Mistakes:

1. The endpoints are misinterpreted as they should be vectors. The points $t=1$ and $t=3$ represent the initial and end times. There's also no time interval for the new parametrization.
2. The endpoints should be determined by evaluating the curve at the initial and ending times $t=1$ and $t=3$.