Ising 2D

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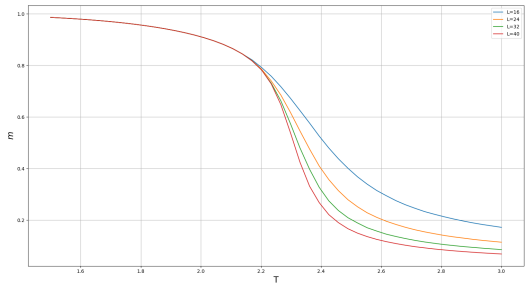
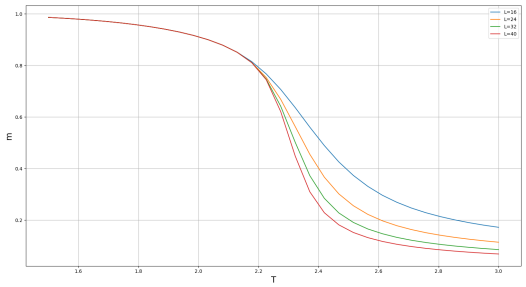
1 Introduction

Here we have used Metropolis Monte Carlo Simulation for a 2 dimensional Ising Model in this report observed that there is a phase transition, order parameter being the magnetization.

We have calculated the observables(magnetization and susceptibility) after the initial state of all up spins equilibrate. We have assumed that this equilibration occurs within 10^4 MC steps for the lengths $L = 16, 24, 32, 40$ considered here. For the calculation of the observables, we have considered two different ensemble sizes generated from 10^7 and 10^8 MC steps. The evolution of the magnetization is plotted below:

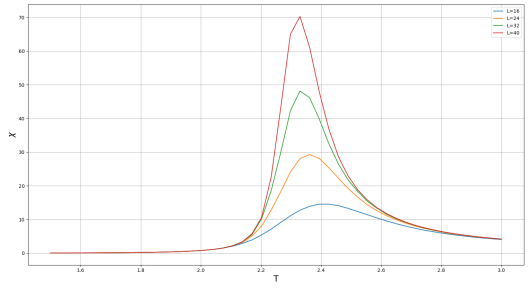


(a) Ensemble size = 10^7

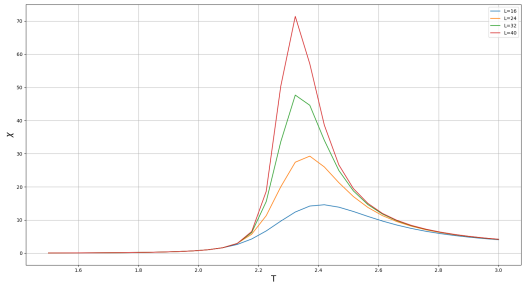


(b) Ensemble size = 10^8

The evolution of the susceptibility is plotted below:



(a) Ensemble size = 10^7

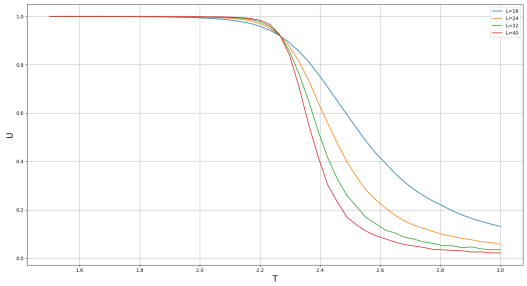


(b) Ensemble size = 10^8

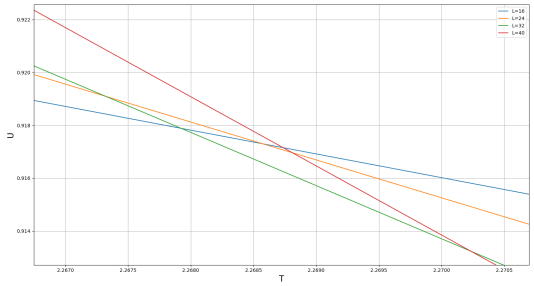
Next we attempt to estimate the value of the critical temperature for this model. For this purpose we first define the Binder's Cumulant:

$$U = \frac{1}{2}\left(3 - \frac{\langle m^4 \rangle}{\langle m^2 \rangle^2}\right) \tag{1}$$

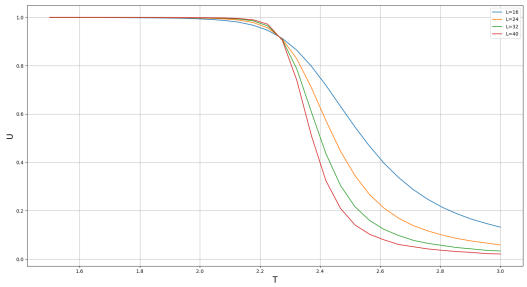
Then we make use of the fact that $U(T = T_c)$ is independent of the size($= L$) of the system. We plot the Binder Cumulant curve vs temperature for a set of lengths $L = 16, 24, 32, 40$ and identify the crossing point. This too we do for two ensemble sizes :



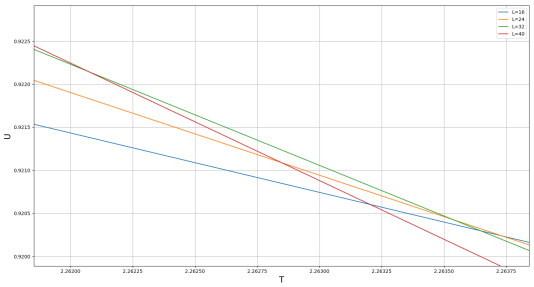
(a) Ensemble size = 10^7



(b) Ensemble size = 10^7 (Zoomed near T_c)



(a) Ensemble size = 10^8



(b) Ensemble size = 10^8 (Zoomed near T_c)

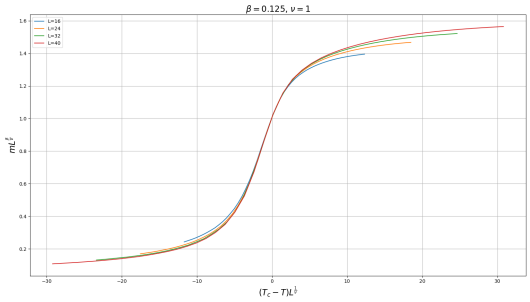
From the intersections we find that the calculated T_c is around 2.263 (Analytical calculations give $T_c \approx 2.269$)

Finally we try to estimate the critical exponents γ, β and ν for this model by making use of the Finite Size scaling relations:

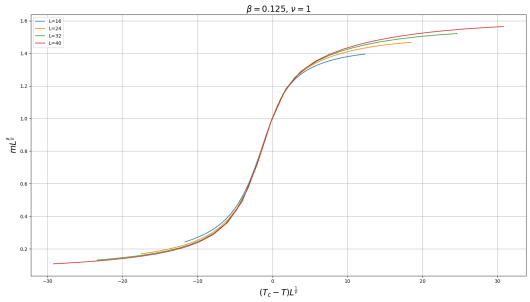
$$m = L^{-\frac{\beta}{\nu}} \phi_m \left((T_c - T) L^{\frac{1}{\nu}} \right) \tag{2}$$

$$\chi = L^{\frac{\gamma}{\nu}} \phi_\chi \left((T_c - T) L^{\frac{1}{\nu}} \right) \tag{3}$$

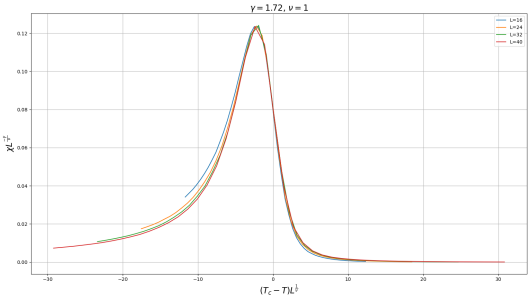
Where ϕ_m and ϕ_χ are scaling functions. At exact values of the exponents, the curves of $mL^{\frac{\beta}{\nu}}$ vs $(T_c - T)L^{\frac{1}{\nu}}$ and $\chi L^{-\frac{\gamma}{\nu}}$ vs $(T_c - T)L^{\frac{1}{\nu}}$ will be independent of the system size and the curves would collapse on each other. The corresponding figures for best collapse observed here are:



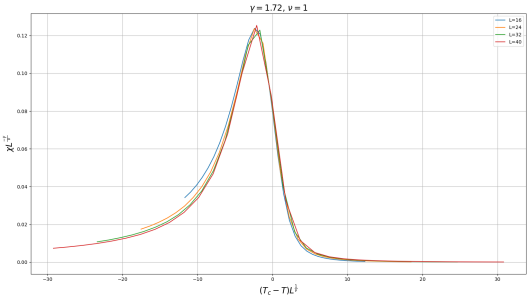
(a) Ensemble size = 10^8



(b) Ensemble size = 10^8



(a) Ensemble size = 10^8



(b) Ensemble size = 10^8

Analytical calculations for the 2D Ising Model yield the values for the critical exponents : $\nu = 1$, $\beta = \frac{1}{8} = 0.125$ and $\gamma = \frac{7}{4} = 1.75$.