MODULE FIVE PROBLEM SET

Brad Jackson

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

Indicate whether the two functions are equal. If the two functions are not equal, then give an element of the domain on which the two functions have different values.

(a)

$$f : \mathbb{Z} \to \mathbb{Z}, \text{ where } f(x) = x^2.$$
$$g : \mathbb{Z} \to \mathbb{Z}, \text{ where } g(x) = |x|^2.$$

Functions f and g have the same domain and target.
Moreover, for any integer x, $x^2 = |x|^2$
$f = g$

(b)

$$f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, \text{ where } f(x, y) = |x + y|.$$
$$g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, \text{ where } g(x, y) = |x| + |y|.$$

Functions f and g have the same domain and target.
Moreover, for each ordered pair of integers x and y, $|x + y| = |x| + |y|$

Problem 2

The domain and target set of functions f and g is $\mathbb{R}$. The functions are defined as:

- $f(x) = 2x + 3$
- $g(x) = 5x + 7$

(a) $f \circ g$?

$2(5x + 7) + 3$
$10x + 17$

(b) $g \circ f$?

$10x + 22$

(c) $(f \circ g)^{-1}$?

$(x - 17)/10$

(d) $f^{-1} \circ g^{-1}$?

$(x - 3)/2$

(e) $g^{-1} \circ f^{-1}$?

$(x - 22)/10$

Are any of the above equal?

None of the above are equal.

Problem 3

(a) Give the matrix representation for the relation depicted in the arrow diagram. Then, express the relation as a set of ordered pairs.

The arrow diagram below represents a relation.

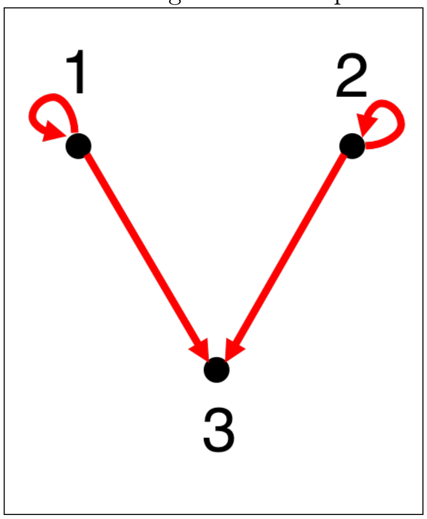
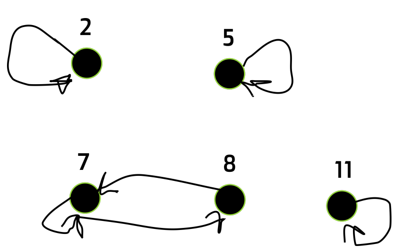


Figure 1: *An arrow diagram shows three vertices, 1, 2, and 3. An arrow from vertex 1 points to vertex 3, and another arrow from vertex 2 points to vertex 3. Two self loops are formed, one at vertex 1 and another at vertex 2.*

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{matrix}$$

R = (1,1)(1,3)(2,2)(2,3)

(b) Draw the arrow diagram for the relation.
The domain for the relation A is the set $\{2, 5, 7, 8, 11\}$. For x, y in the domain, xAy if $|x - y|$ is less than 2.



For each relation, indicate whether the relation is:

- Reflexive, anti-reflexive, or neither
- Symmetric, anti-symmetric, or neither
- Transitive or not transitive

Justify your answer.

(a) The domain of the relation L is the set of all real numbers. For x, $y \in \mathbb{R}$, xLy if $x < y$.

L is anti-reflexive and transitive:

(b) The domain of the relation A is the set of all real numbers. xAy if $|x-y| \leq 2$

reflexive, symmetric, transitive

(c) The domain of the relation Z is the set of all real numbers. xZy if $y = 2x$

Problem 5

The number of watermelons in a truck are all weighed on a scale. The scale rounds the weight of every watermelon to the nearest pound. The number of pounds read off the scale for each watermelon is called its measured weight. The domain for each of the following relations below is the set of watermelons on the truck. For each relation, indicate whether the relation is:

- Reflexive, anti-reflexive, or neither
- Symmetric, anti-symmetric, or neither
- Transitive or not transitive

Justify your answer.

(a) Watermelon x is related to watermelon y if the measured weight of watermelon x is at least the measured weight of watermelon y. No two watermelons have the same measured weight.

Reflexive because x is related to y.
Anti-symmetric because no two watermelons have the same weight.
Not transitive because z could be greater than x as all watermelons do not have the same weight.

(b) Watermelon x is related to watermelon y if the measured weight of watermelon x is at least the measured weight of watermelon y. All watermelons have exactly the same measured weight.

Reflexive because $x \geq y$, therefore x is related to y.
Symmetric because all watermelons have the same weight.
Transitive because $x \geq y$ then $x \geq z$

Part 1. Give the adjacency matrix for the graph G as pictured below:

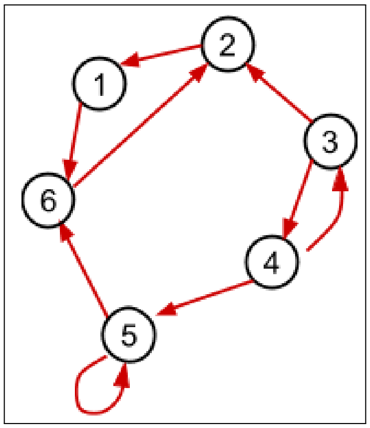


Figure 2: *A graph shows 6 vertices and 9 edges. The vertices are 1, 2, 3, 4, 5, and 6, represented by circles. The edges between the vertices are represented by arrows, as follows: 4 to 3; 3 to 2; 2 to 1; 1 to 6; 6 to 2; 3 to 4; 4 to 5; 5 to 6; and a self loop on vertex 5.*

0	0	0	0	0	1
1	0	0	0	0	0
0	1	0	1	0	0
0	0	1	0	1	0
0	0	0	0	1	1
0	1	0	0	0	0

Part 2. A directed graph G has 5 vertices, numbered 1 through 5. The 5×5 matrix A is the adjacency matrix for G. The matrices A^2 and A^3 are given below.

$$A^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Use the information given to answer the questions about the graph G.

(a) Which vertices can reach vertex 2 by a walk of length 3?

2, 4, and 5

(b) Is there a walk of length 4 from vertex 4 to vertex 5 in G? (Hint: $A^4 = A^2 \cdot A^2$.)

No, shown by the adjacency matrix below.

$$A^4 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Problem 7

Part 1. The drawing below shows a Hasse diagram for a partial order on the set $\{A, B, C, D, E, F, G, H, I, J\}$

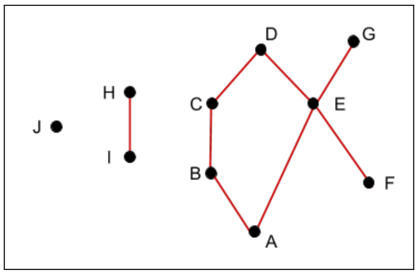


Figure 3: *A Hasse diagram shows 10 vertices and 8 edges. The vertices, represented by dots, are as follows: vertex J; vertices H and I are aligned vertically to the right of vertex J; vertices A, B, C, D, and E forms a closed loop, which is to the right of vertices H and I; vertex G is inclined upward to the right of vertex E; and vertex F is inclined downward to the right of vertex E. The edges, represented by line segments, between the vertices are as follows: Vertex J is connected to no vertex; a vertical edge connects vertices H and I; a vertical edge connects vertices B and C; and 6 inclined edges connect the following vertices, A and B, C and D, D and E, A and E, E and G, and E and F.*

(a) What are the minimal elements of the partial order?

I, A, and F

(b) What are the maximal elements of the partial order?

H, D, and G

(c) Which of the following pairs are comparable?

(A, D), (J, F), (B, E), (G, F), (D, B), (C, F), (H, I), (C, E)

(C,E)

Part 2. Each relation given below is a partial order. Draw the Hasse diagram for the partial order.

(a) The domain is $\{3, 5, 6, 7, 10, 14, 20, 30, 60\}$. $x \leq y$ if x evenly divides y.

(b) The domain is $\{a, b, c, d, e, f\}$. The relation is the set:

$\{(b, e), (b, d), (c, a), (c, f), (a, f), (a, a), (b, b), (c, c), (d, d), (e, e), (f, f)\}$

PROBLEM 8

Determine whether each relation is an equivalence relation. Justify your answer. If the relation is an equivalence relation, then describe the partition defined by the equivalence classes.

(a) The domain is a group of people. Person x is related to person y under relation M if x and y have the same favorite color. You can assume that there is at least one pair in the group, x and y, such that xMy.

(b) The domain is the set of all integers. xEy if $x + y$ is even. An integer z is even if $z = 2k$ for some integer k.