# MODULE ONE PROBLEM SET



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Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

## Problem 1

In the following question, the domain of **discourse** is a set of male patients in a clinical study. Define the following predicates:

- $P(x):$ $x$ was given the placebo

- $D(x):$ $x$ was given the medication

- $M(x):$ $x$ had migraines

Translate each of the following statements into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

*Sample question: Some patient was given the placebo and the medication.*

- $\exists x\ (P(x)\ \wedge\ D(x))$
- *Negation:* $\neg\exists x\ (P(x)\ \wedge\ D(x))$
- *Applying De Morgan's law:* $\forall x\ (\neg P(x)\ \vee\ \neg D(x))$
- *English: Every patient was either not given the placebo or not given the medication (or both).*

(a) Every patient was given the medication or the placebo or both.

$\forall x\ (D(x)\ \vee\ P(x))$

Negation:
$\neg\forall x\ (D(x)\ \vee\ P(x))$

Applying De Morgan's law:
$\exists x\ (\neg D(x)\ \wedge\ \neg P(x))$

English:
There is a patient who was not given the medication and not given the placebo.

(b) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \to q \equiv \neg p \vee q$.)

$\forall x\ (P(x)\ \to M(x))$

Negation:
$\neg\forall x\ (P(x)\ \to M(x))$

Applying De Morgan's law:
$\exists x\ (P(x)\ \wedge\ \neg M(x))$

English:
Some patient took the placebo and did not have migraines.

(c) There is a patient who had migraines and was given the placebo.

$\exists x\ (M(x)\ \wedge\ P(x))$

Negation:
$\neg\exists x\ (M(x)\ \wedge\ P(x))$

Applying De Morgan's law:
$\forall x\ (\neg M(x)\ \vee\ \neg P(x))$

English:
Every patient did not have migraines or did not take the placebo.

## Problem 2

Use De Morgan's law for quantified statements and the laws of propositional logic to show the following equivalences:

(a) $\neg\forall x\ (P(x) \wedge \neg Q(x)) \ \equiv\ \exists x\ (\neg P(x) \vee Q(x))$

Applying De Morgan's law:
$\exists x \neg (P(x) \wedge \neg Q(x))$

Applying De Morgan's law:
$\exists x\ (\neg P(x) \vee \neg\neg Q(x))$

Double negation law:
$\exists x\ (\neg P(x) \vee Q(x))$

(b) $\neg\forall x\ (\neg P(x) \rightarrow Q(x)) \ \equiv\ \exists x\ (\neg P(x) \wedge \neg Q(x))$

Applying De Morgan's law:
$\exists x \neg (\neg P(x) \rightarrow Q(x))$

Conditional identity:
$\exists x \neg (\neg(\neg P(x)) \vee Q(x))$

Double Negation:
$\exists x \neg (P(x) \vee Q(x))$

Applying De Morgan's law:
$\exists x\ (\neg P(x) \wedge \neg Q(x))$

(c) $\neg\exists x\ \big(\neg P(x) \vee (Q(x) \wedge \neg R(x))\big) \ \equiv\ \forall x\ \big(P(x) \wedge (\neg Q(x) \vee R(x))\big)$

## Problem 3

The domain of **discourse** for this problem is a group of three people who are working on a project. To make notation easier, the people are numbered 1, 2, 3. The predicate $M(x,\ y)$ indicates whether x has sent an email to $y$, so $M(2,\ 3)$ is read "Person 2 has sent an email to person 3." The table below shows the value of the predicate $M(x,\ y)$ for each $(x,\ y)$ pair. The truth value in row $x$ and column $y$ gives the truth value for $M(x,\ y)$.

| $M$ | 1 | 2 | 3 |
|---|---|---|---|
| 1 | $T$ | $T$ | $T$ |
| 2 | $T$ | $F$ | $T$ |
| 3 | $T$ | $T$ | $F$ |

**Determine if the quantified statement is true or false. Justify your answer.**

(a) $\forall x\, \forall y\, (x \neq y)\ \rightarrow\ M(x,\ y))$

For all senders and recievers where the sender and reciver is not the same, then the sender has sent an email to the reciever. This is true.

(b) $\forall x\, \exists y\ \ \neg M(x,\ y)$

For all senders there exists a reciever such that the sender is not the reciever. This is true because the domain of discourse is not 1 dimensional.

(c) $\exists x\, \forall y\ \ M(x,\ y)$

There exists a sender for every reciever such that the sender is the reciever. This is false because $M(2,\ 2)$ and $M(3,\ 3)$ are False.

## Problem 4

Translate each of the following English statements into logical expressions. The domain of **discourse** is the set of all real numbers.

(a) The reciprocal of every positive number less than one is greater than one.

$\forall y\,((y < 1) \land (1/y > 1))$

(b) There is no smallest number.

$\exists x\,\forall y\,(x < y)$

(c) Every number other than 0 has a multiplicative inverse.

$\forall x\,\exists y\ ((x \neq 0)\ \land (xy = 1))$

The sets $A$, $B$, and $C$ are defined as follows:

$$A = tall, grande, venti$$
$$B = foam, no - foam$$
$$C = non - fat, whole$$

Use the definitions for $A$, $B$, and $C$ to answer the questions. Express the elements using $n$-tuple notation, not string notation.

(a) Write an element from the set $A \times B \times C$.

(grande, no-foam, whole)

(b) Write an element from the set $B \times A \times C$.

(foam, tall, non-fat)

(c) Write the set $B \times C$ using roster notation.

$B \times C = \{(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)\}$