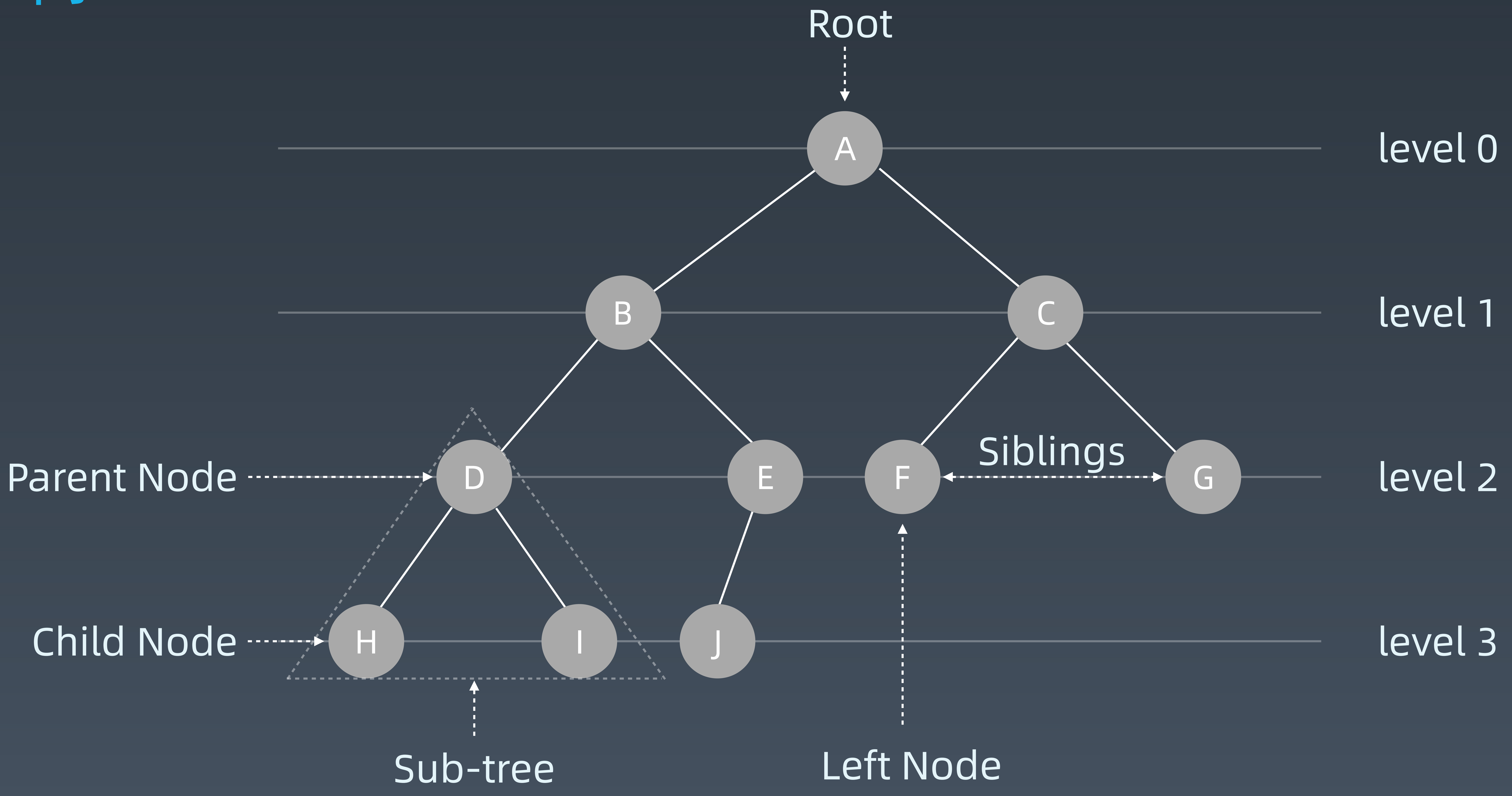
极客大学算法训练营
第十五课
高级树、AVL 树和红黑树

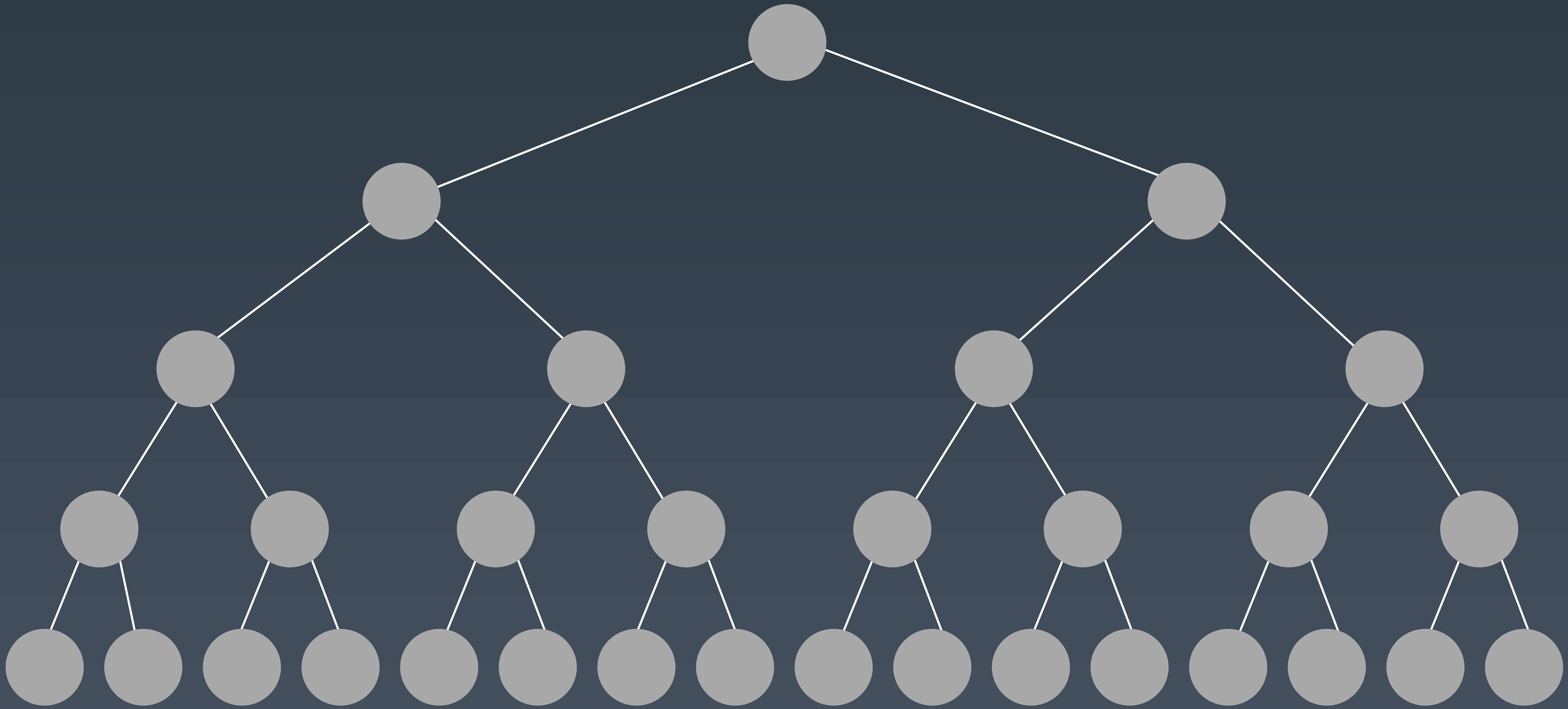
覃超

Sophon Tech 创始人，前 Facebook 工程师

树 Tree


Root
A
B
C
D
E
F
G
H
I
J
level 0
level 1
level 2
level 3
Parent Node
Child Node
Siblings
Sub-tree
Left Node

二叉树 Binary Tree



二叉树遍历

Pre-order/In-order/Post-order

1. 前序(Pre-order)：根-左-右

2. 中序(In-order)：左-根-右

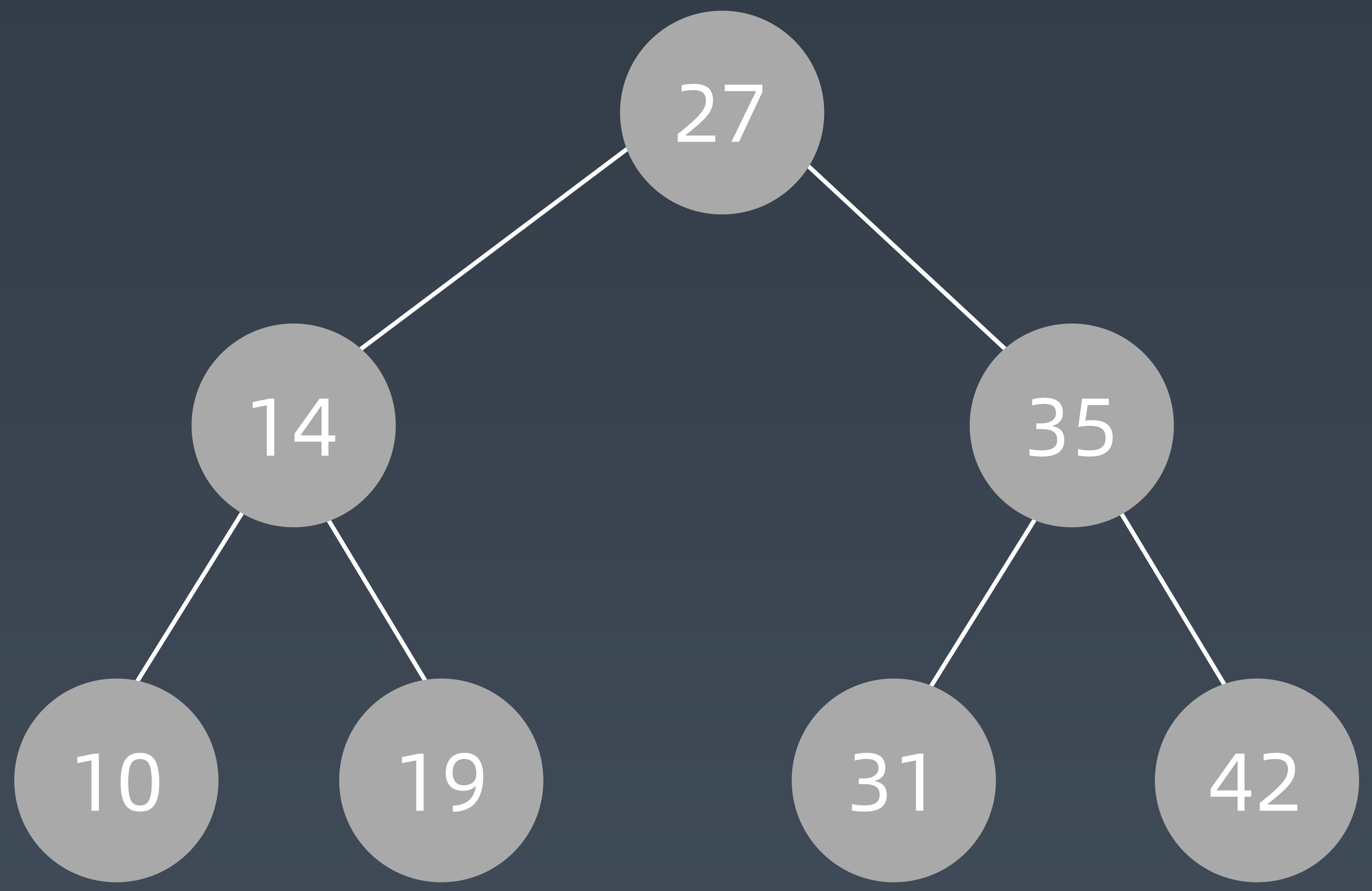
3. 后序(Post-order)：左-右-根

示例代码

```python
def preorder(self, root):

  if root:
    self.traverse_path.append(root.val)
    self.preorder(root.left)
    self.preorder(root.right)

def inorder(self, root):

  if root:
    self.inorder(root.left)
    self.traverse_path.append(root.val)
    self.inorder(root.right)

def postorder(self, root):

  if root:
    self.postorder(root.left)
    self.postorder(root.right)
    self.traverse_path.append(root.val)
```

Binary Search Tree
二叉搜索树

二叉搜索树 Binary Search Tree



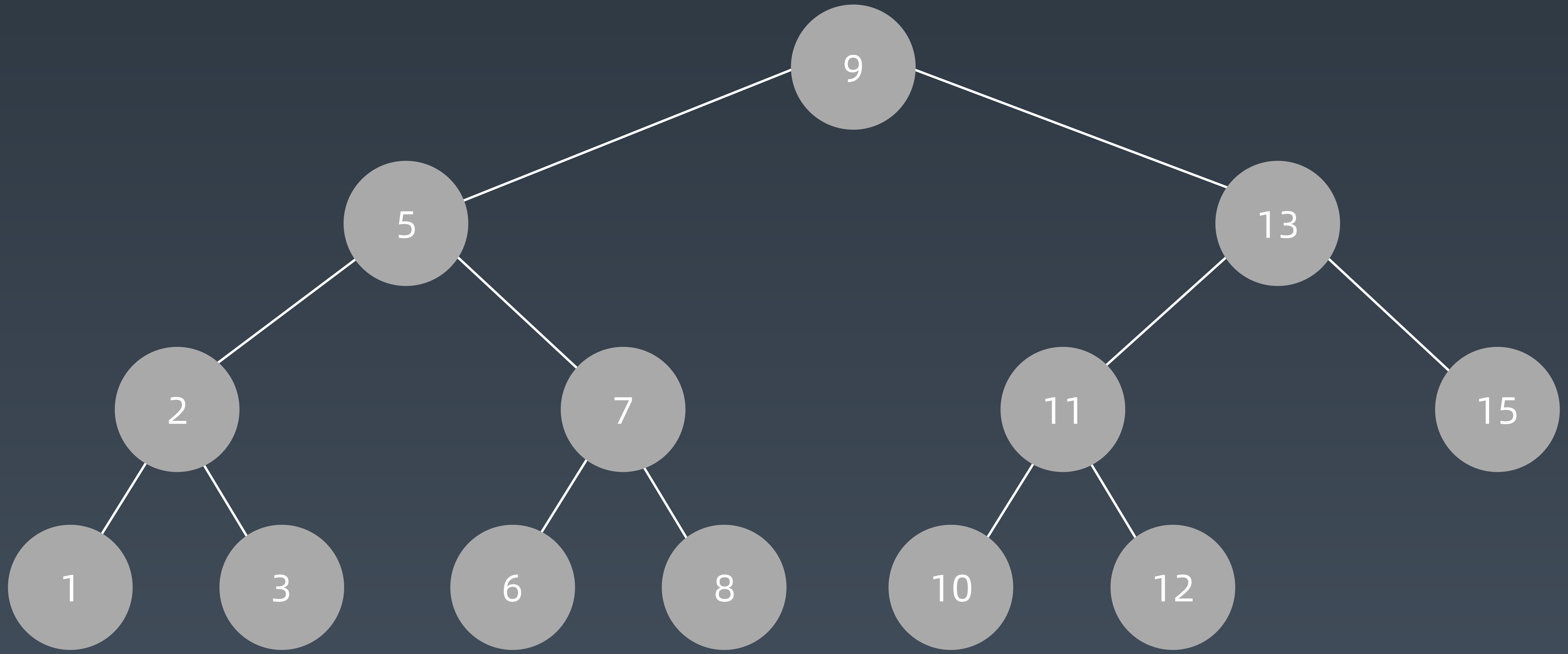
二叉搜索树 Binary Search Tree

二叉搜索树，也称二叉搜索树、有序二叉树（Ordered Binary Tree）、排序二叉树（Sorted Binary Tree），是指一棵空树或者具有下列性质的二叉树：

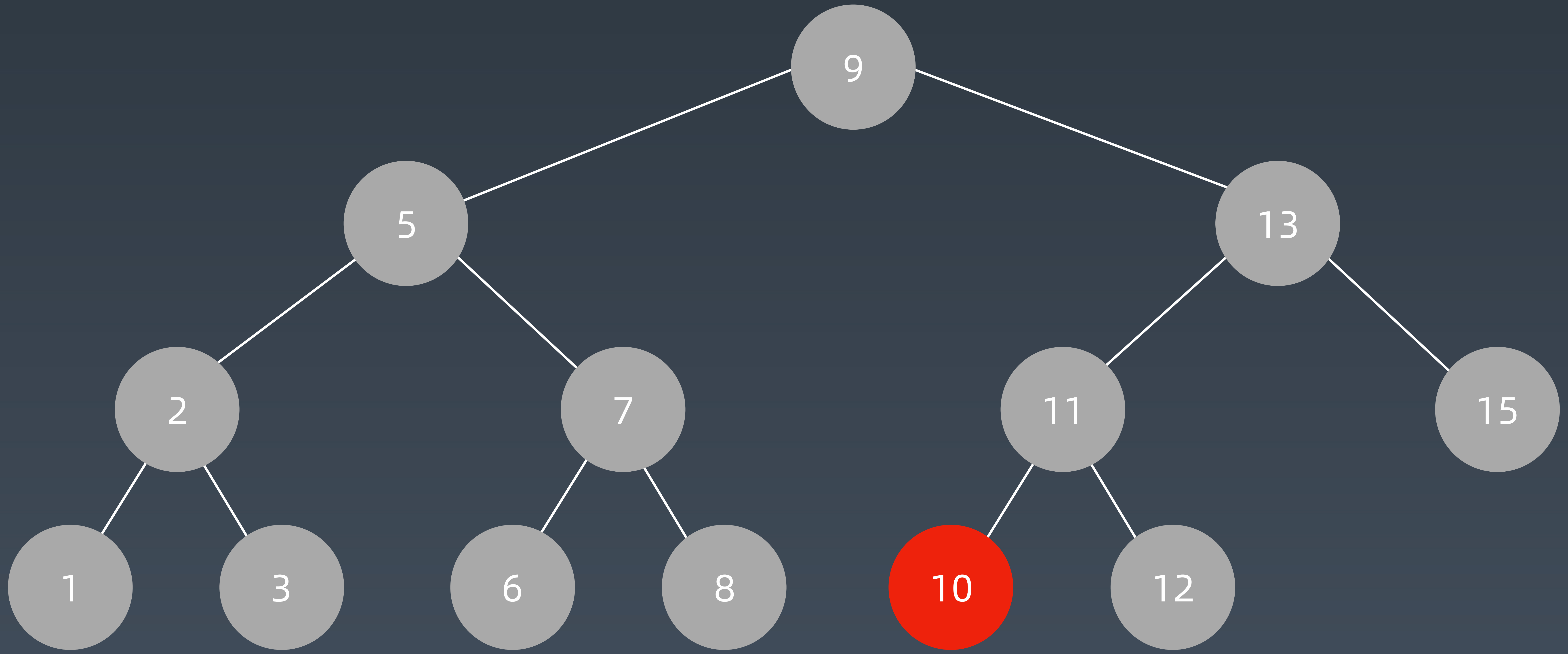
1. 左子树上所有结点的值均小于它的根结点的值；
2. 右子树上所有结点的值均大于它的根结点的值；
3. 以此类推：左、右子树也分别为二叉查找树。（这就是 重复性！）

中序遍历：升序排列

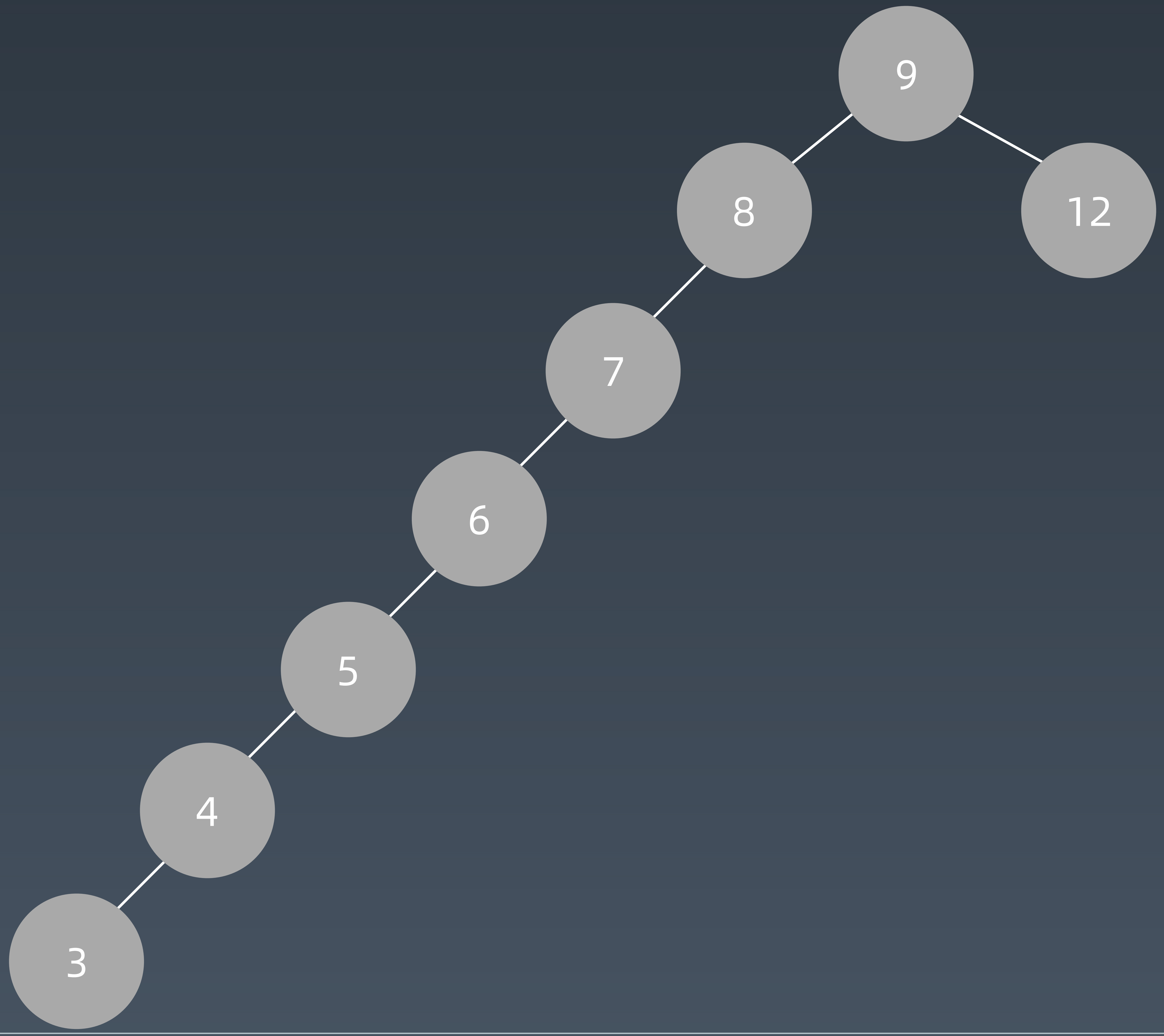
复习：二叉搜索树



如何查找结点



极端情况



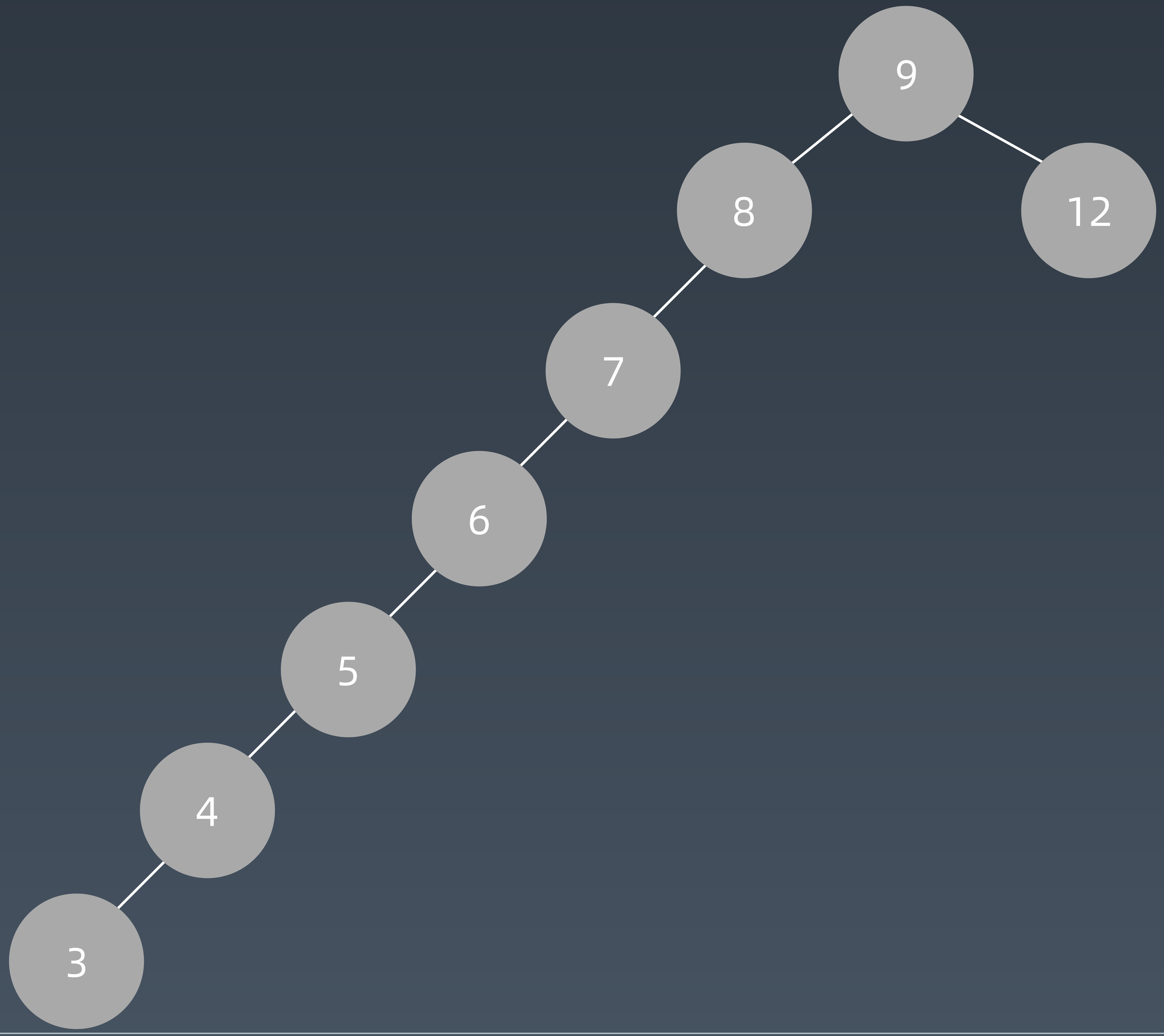
保证性能的关键

1. 保证二维维度！ —> 左右子树结点平衡（recursively）

2. Balanced

3. https://en.wikipedia.org/wiki/Self-balancing_binary_search_tree

思考：如何平衡？



AVL 树

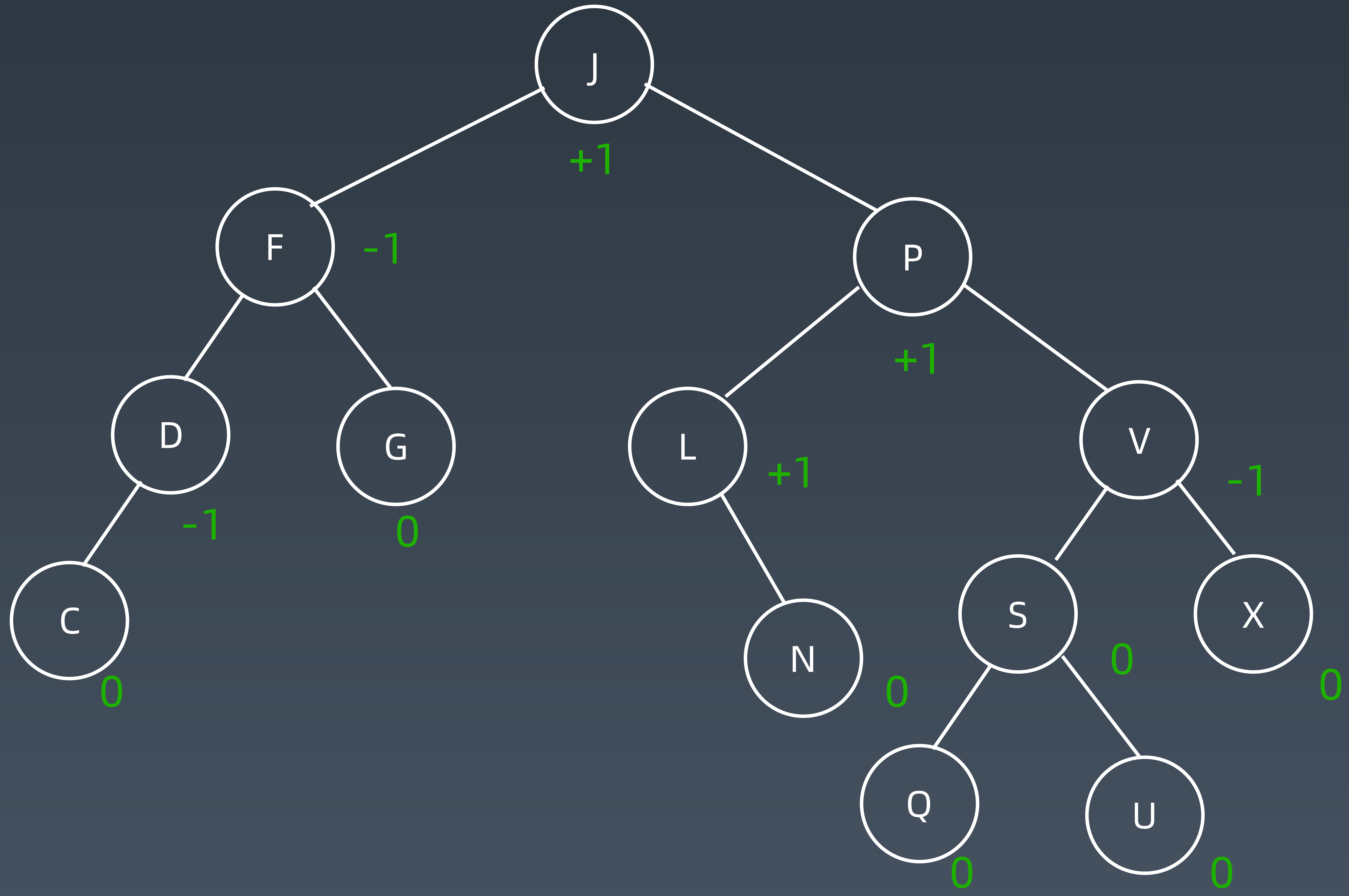
1. 发明者 G. M. Adelson-Velsky 和 Evgenii Landis

2. Balance Factor（平衡因子）：
   是它的左子树的高度减去它的右子树的高度（有时相反）。
   balance factor = {-1, 0, 1}

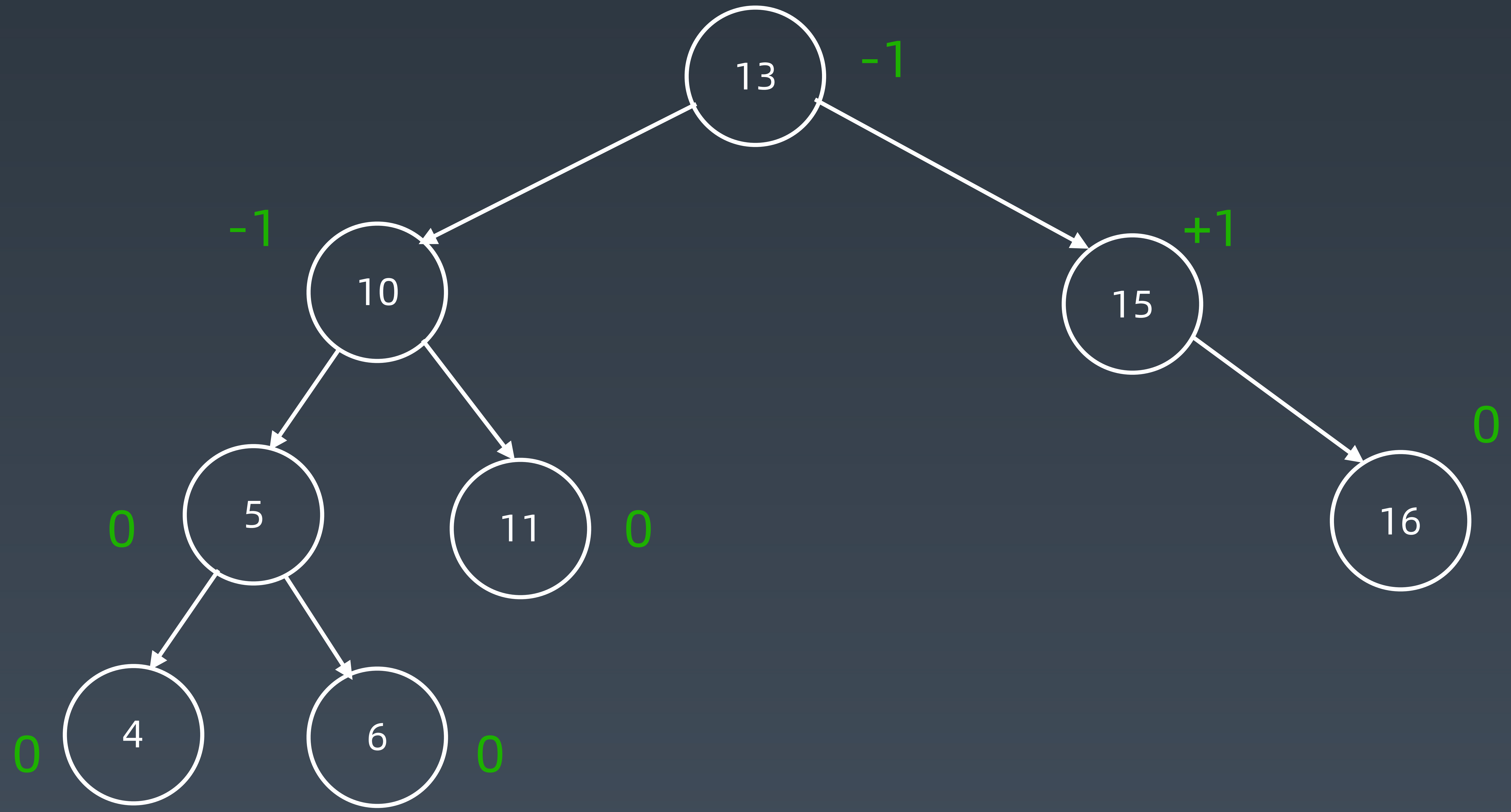
3. 通过旋转操作来进行平衡（四种）

4. https://en.wikipedia.org/wiki/Self-balancing_binary_search_tree

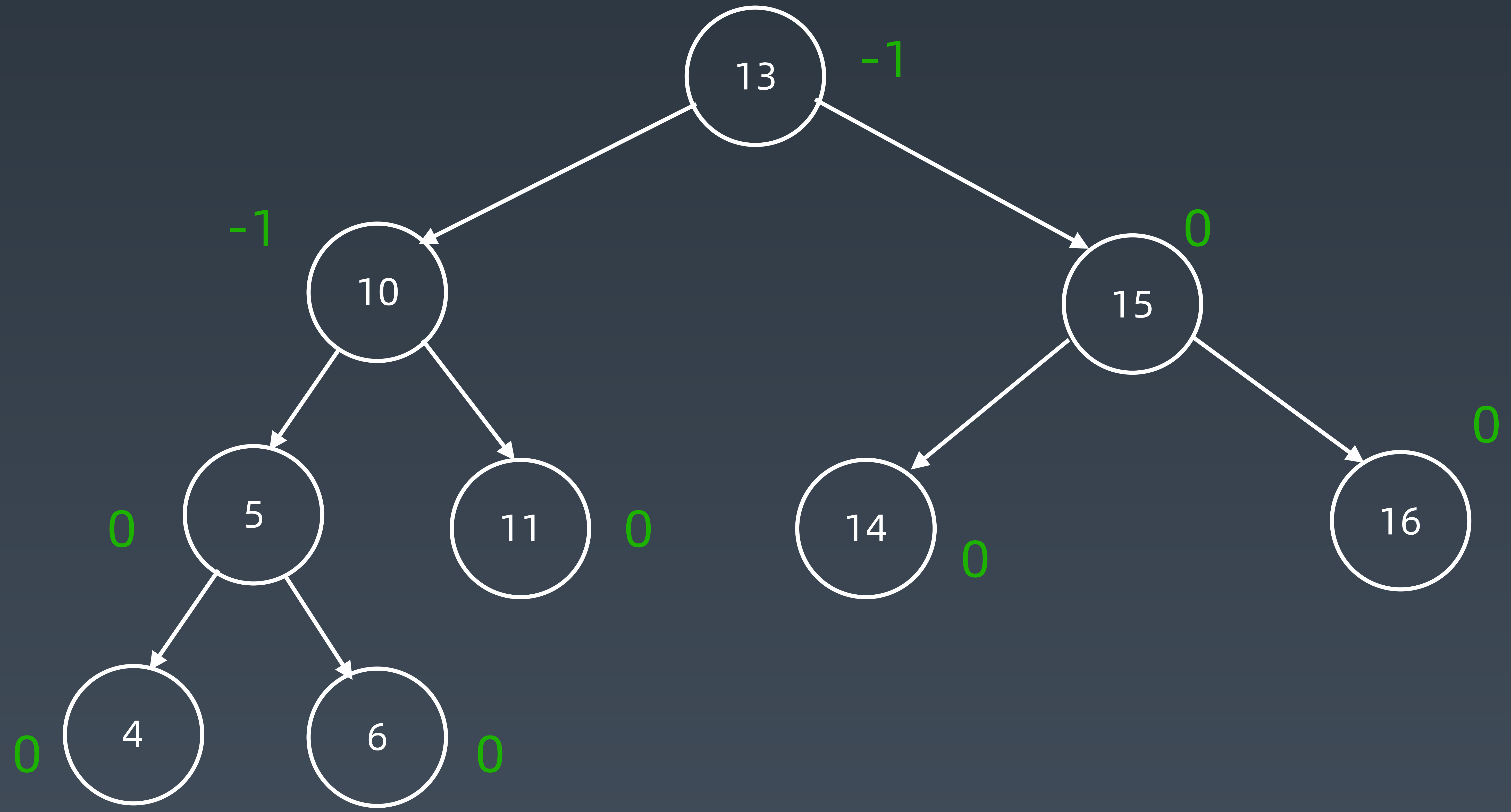
记录左右子树高度



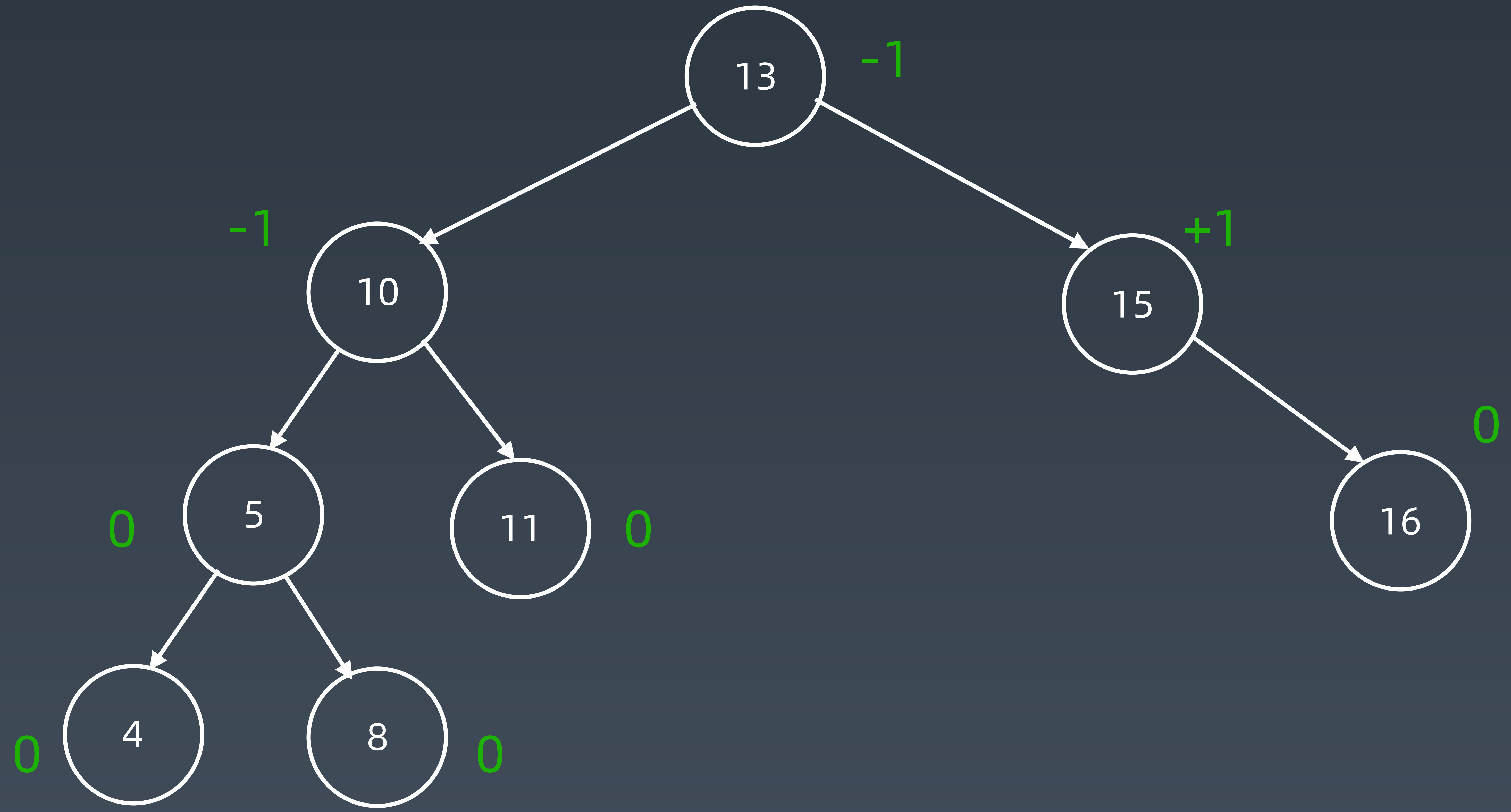
增加 14



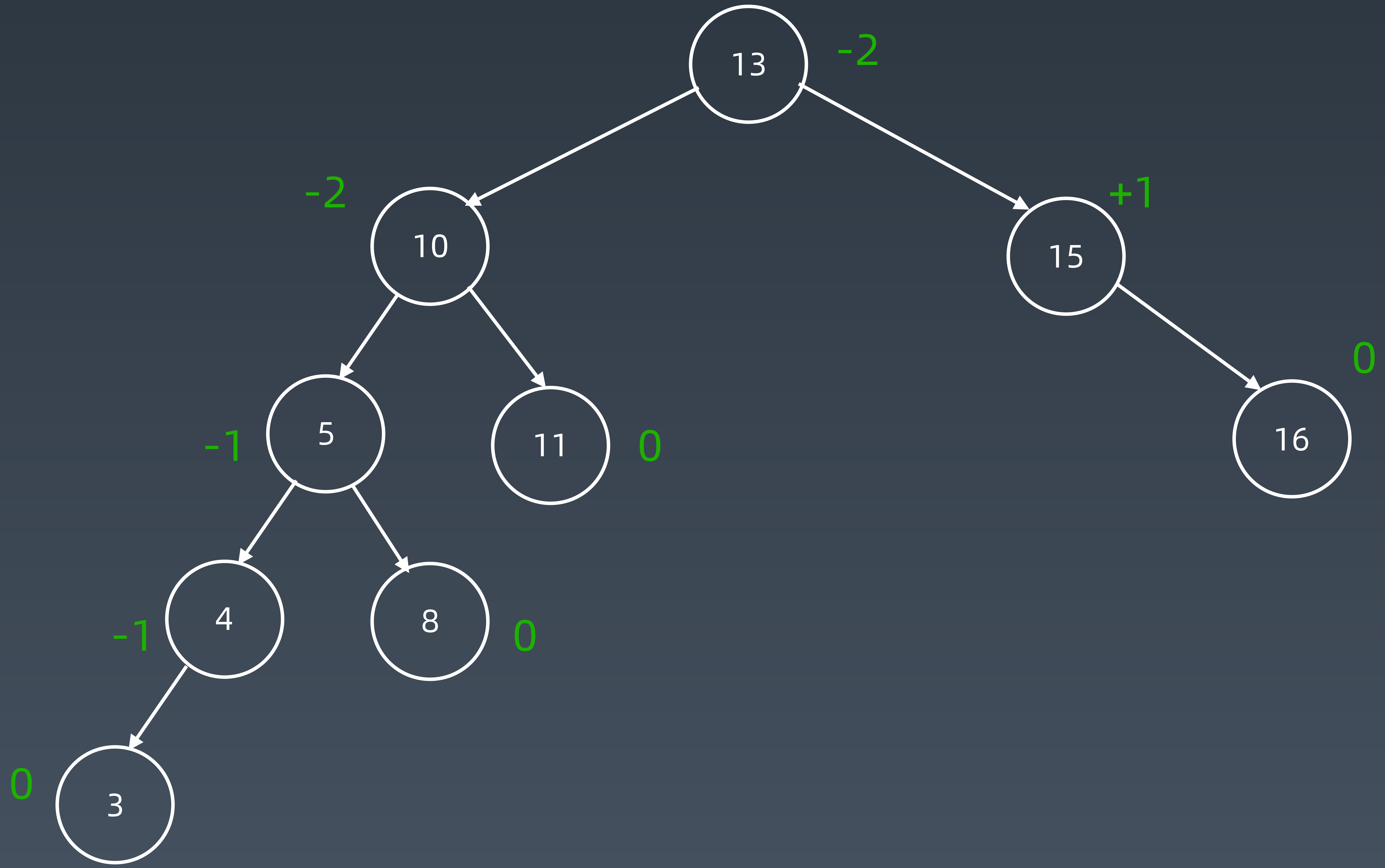
增加 14



增加 3



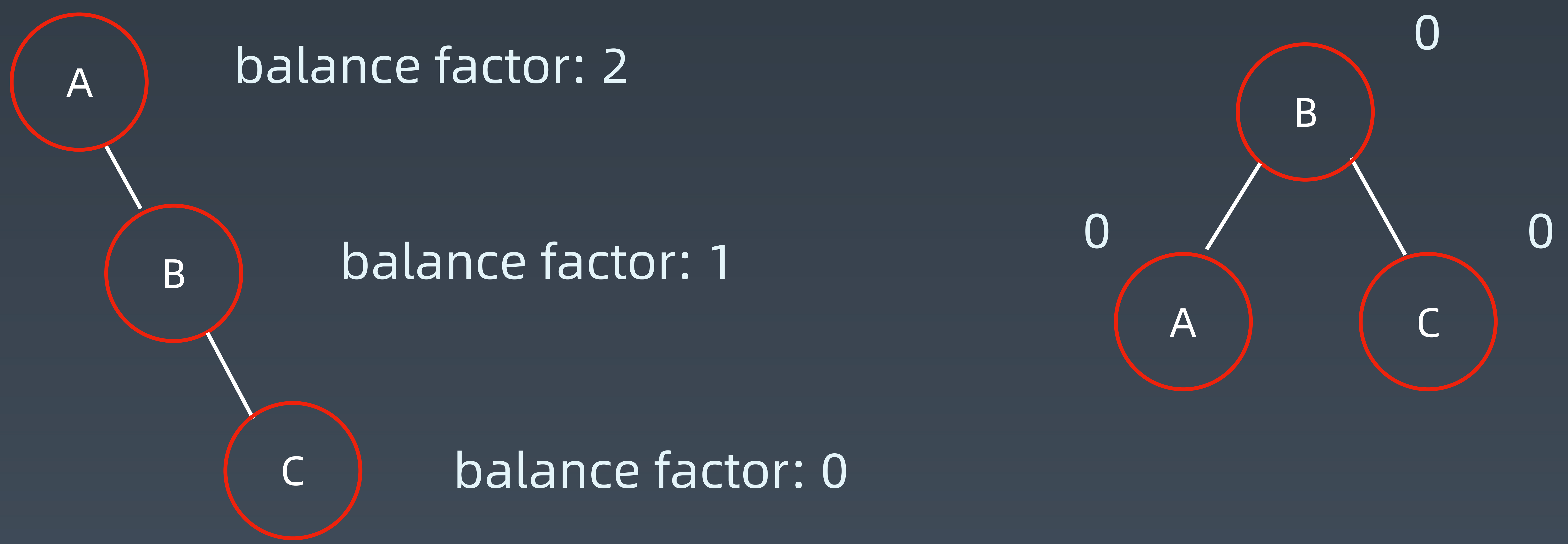
增加 3



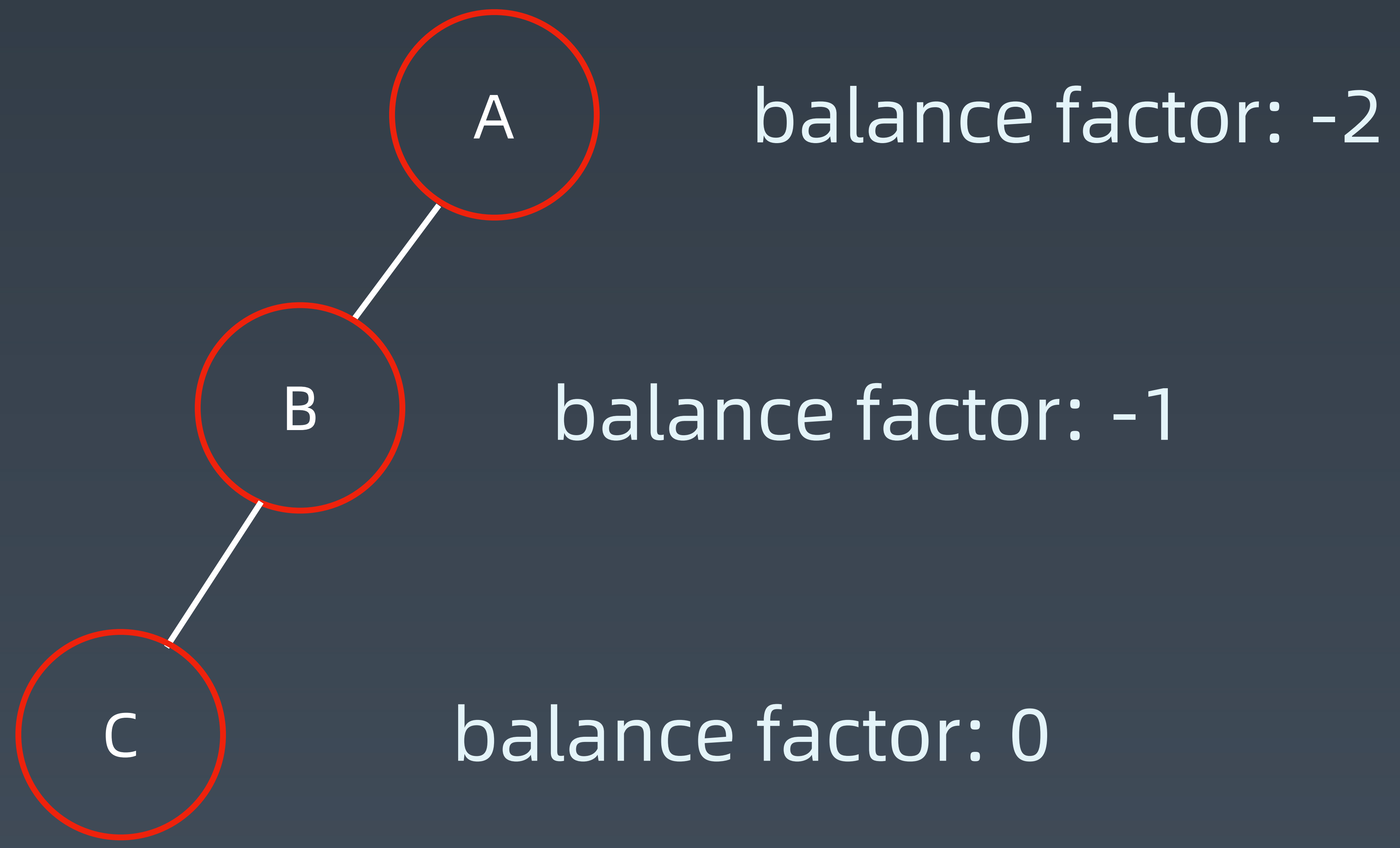
旋转操作

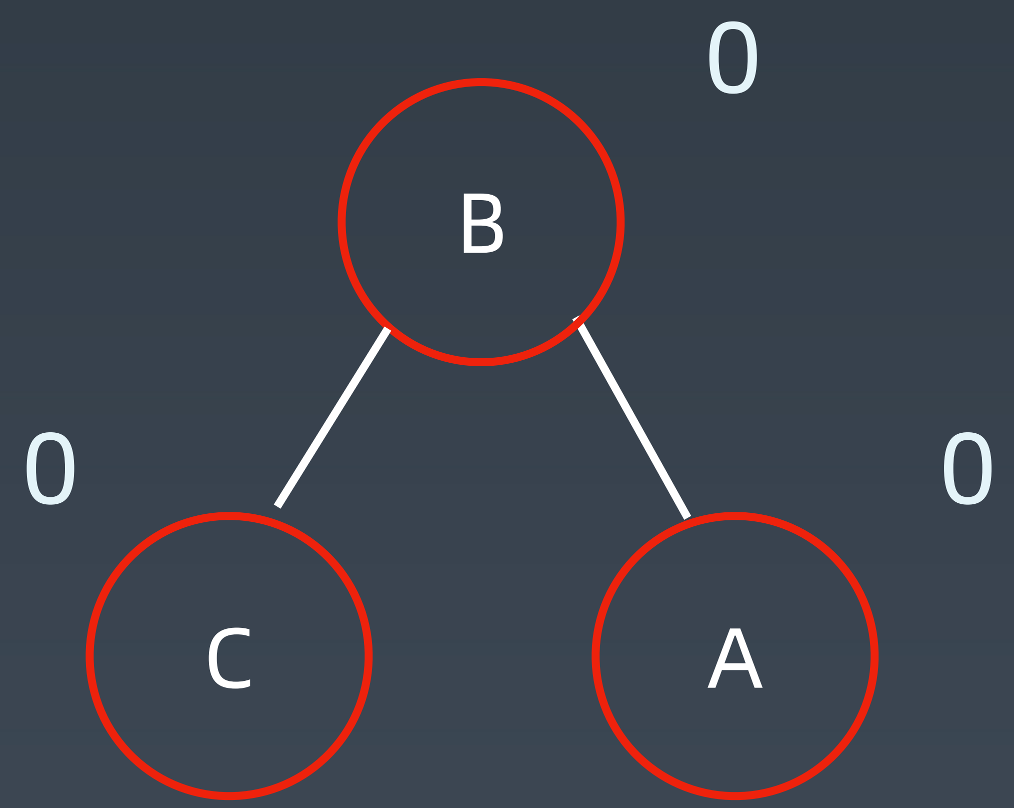
1. 左旋
2. 右旋
3. 左右旋
4. 右左旋

子树形态：右右子树 —> 左旋

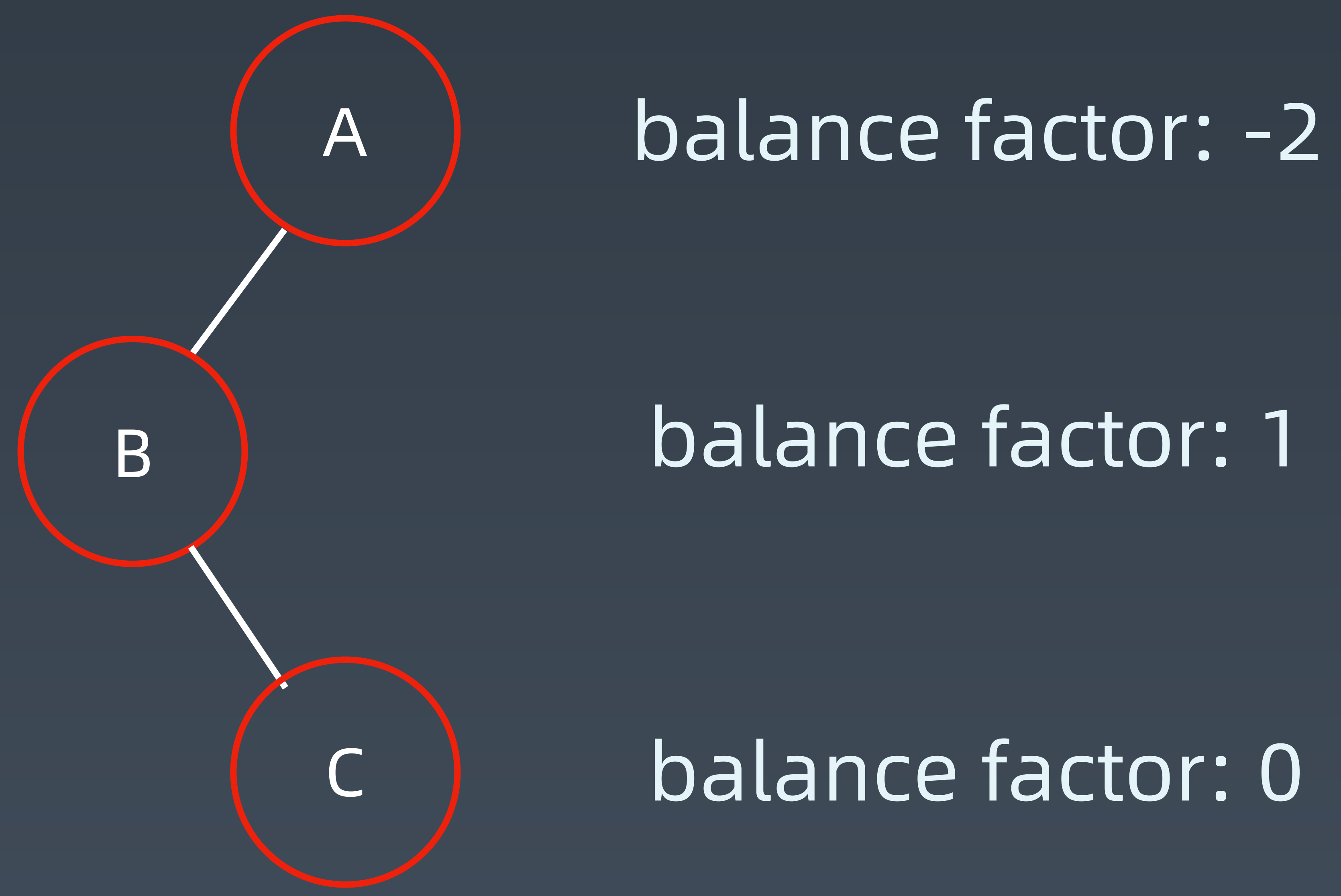

A
balance factor: 2
B
balance factor: 1
C
balance factor: 0
0
B
0
0
A
C

子树形态：左左子树 —> 右旋

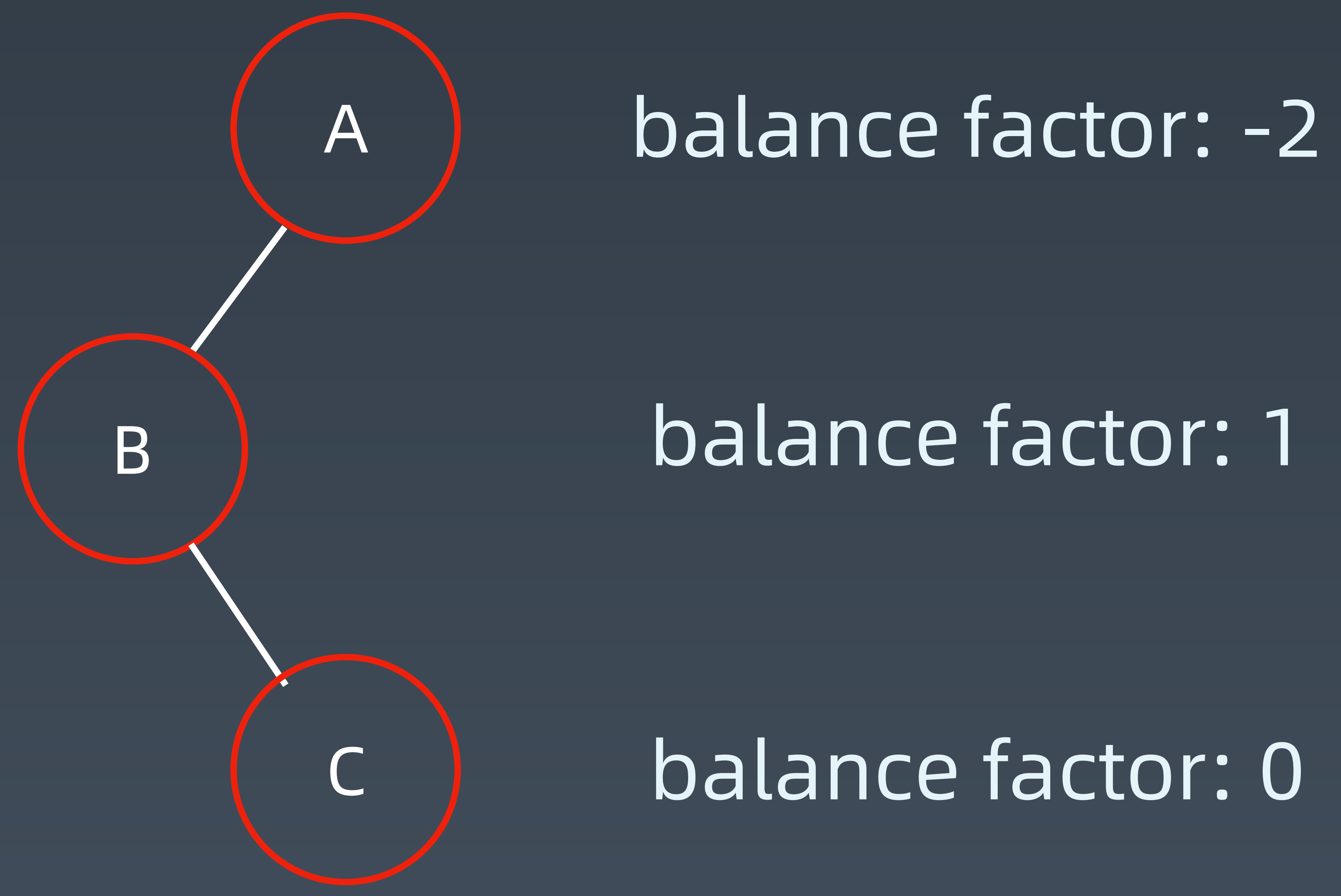

A
balance factor: -2
B
balance factor: -1
C
balance factor: 0

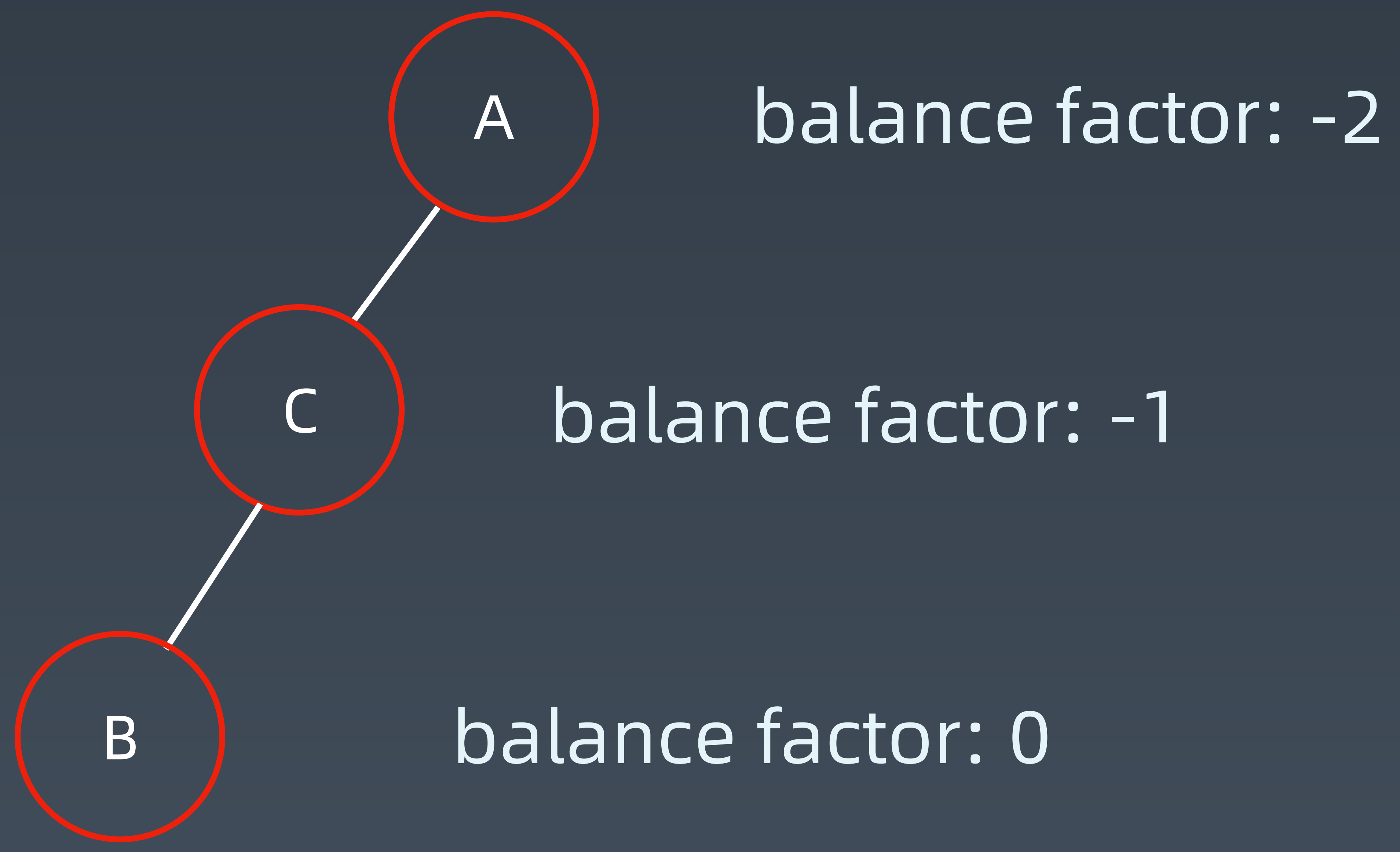

0
B
0
0
C
A

子树形态：左右子树 —> 左右旋

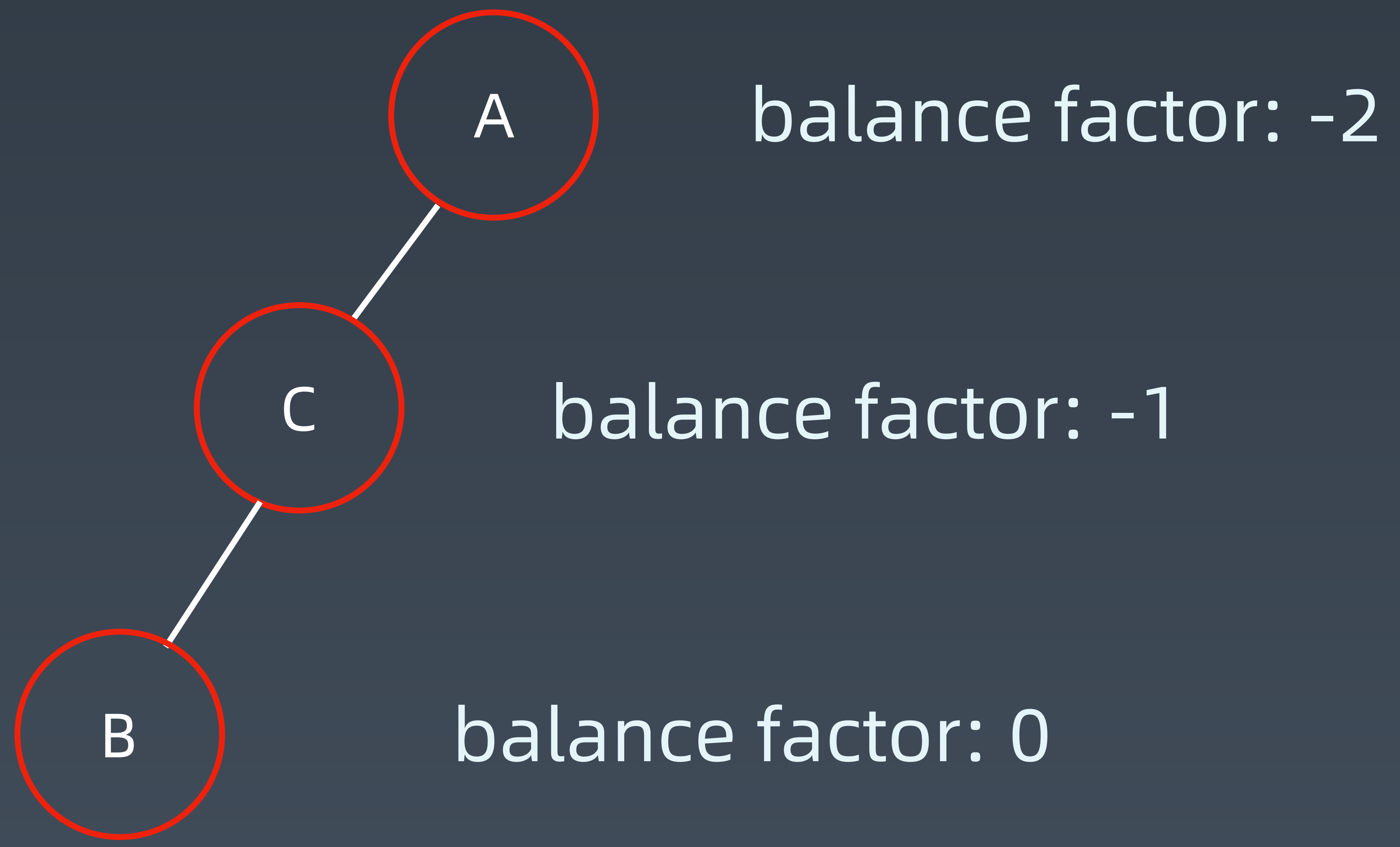


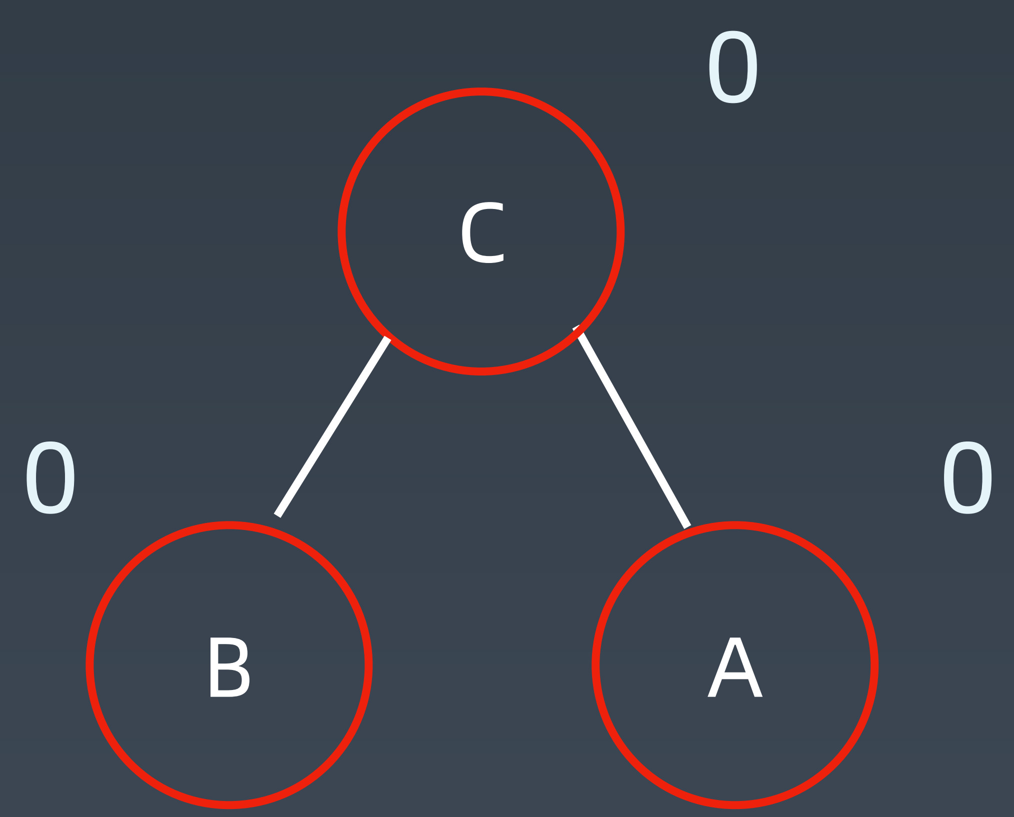
子树形态：左右子树 —> 左右旋



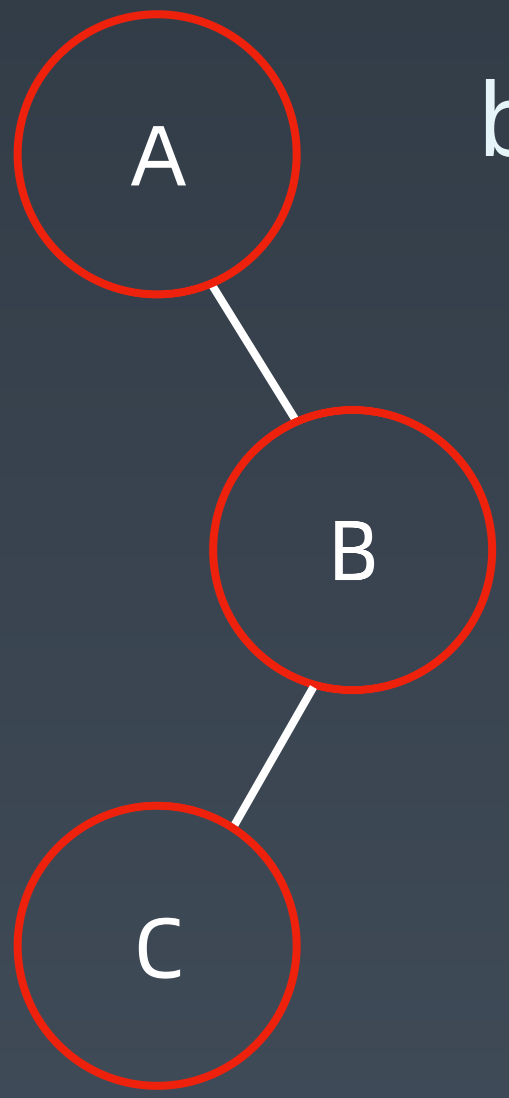


子树形态：左右子树 —> 左右旋





子树形态：右左子树 —> 右左旋

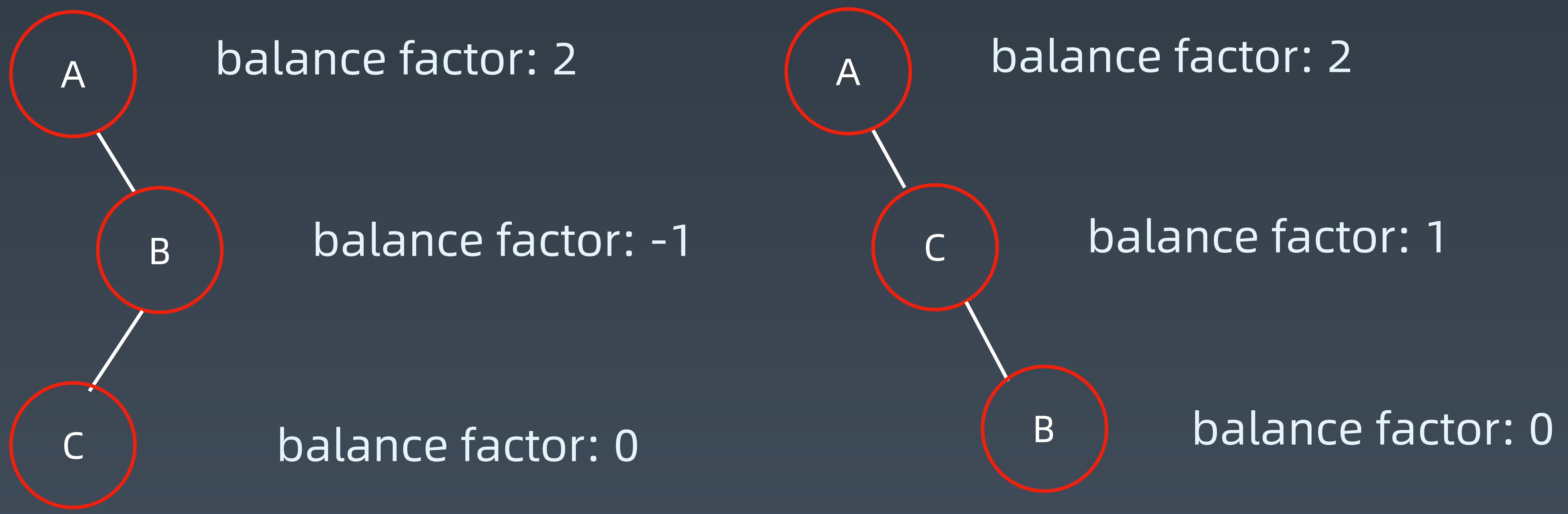


balance factor: 2

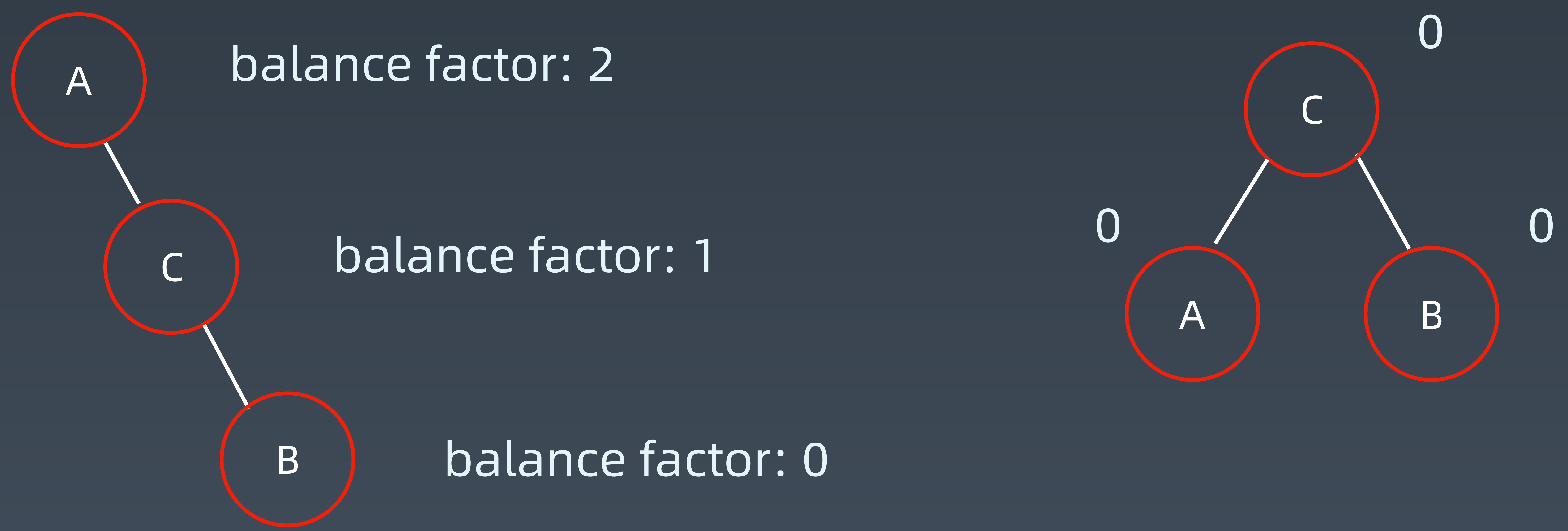
balance factor: -1

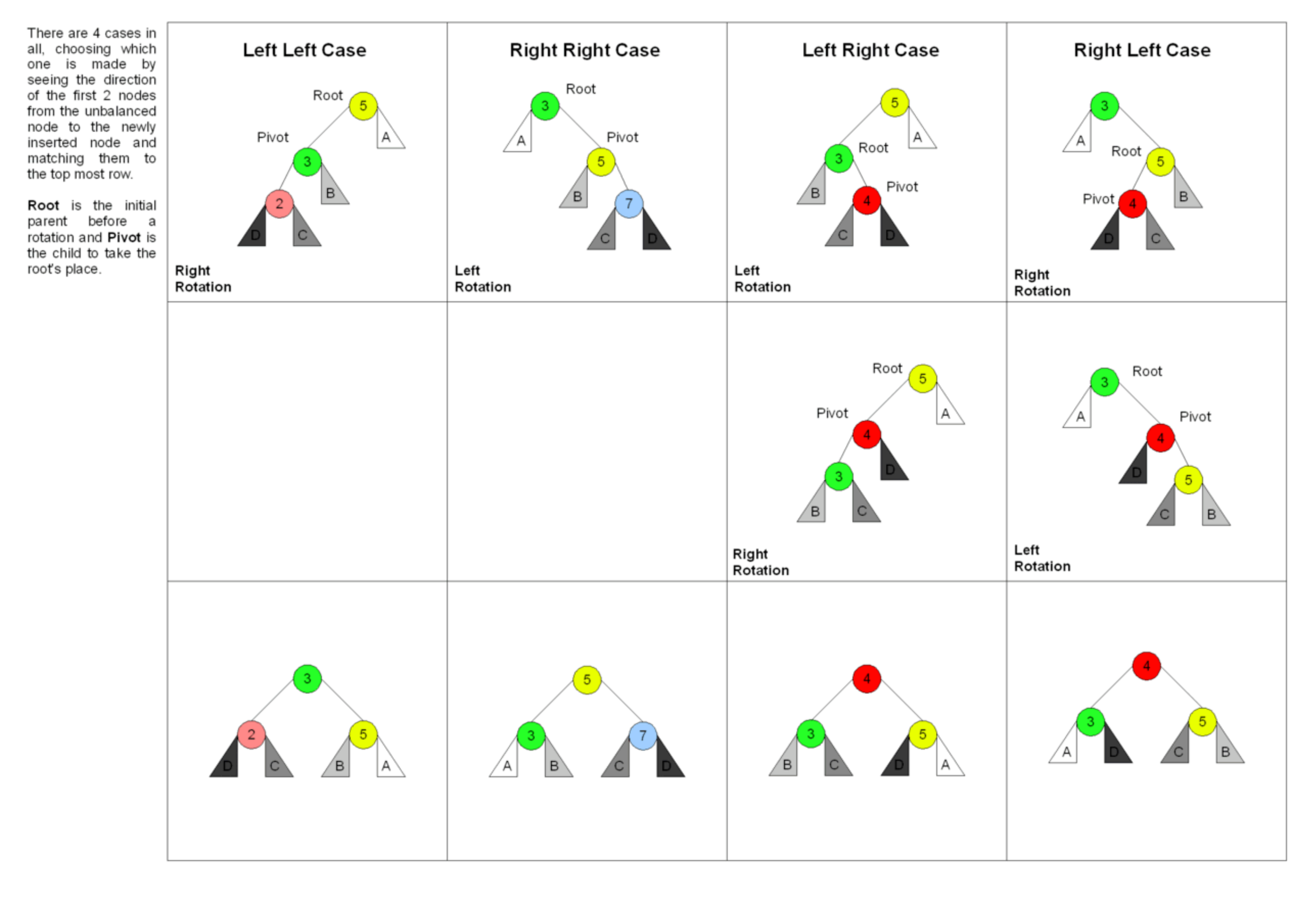
balance factor: 0

子树形态：左右子树 —> 左右旋



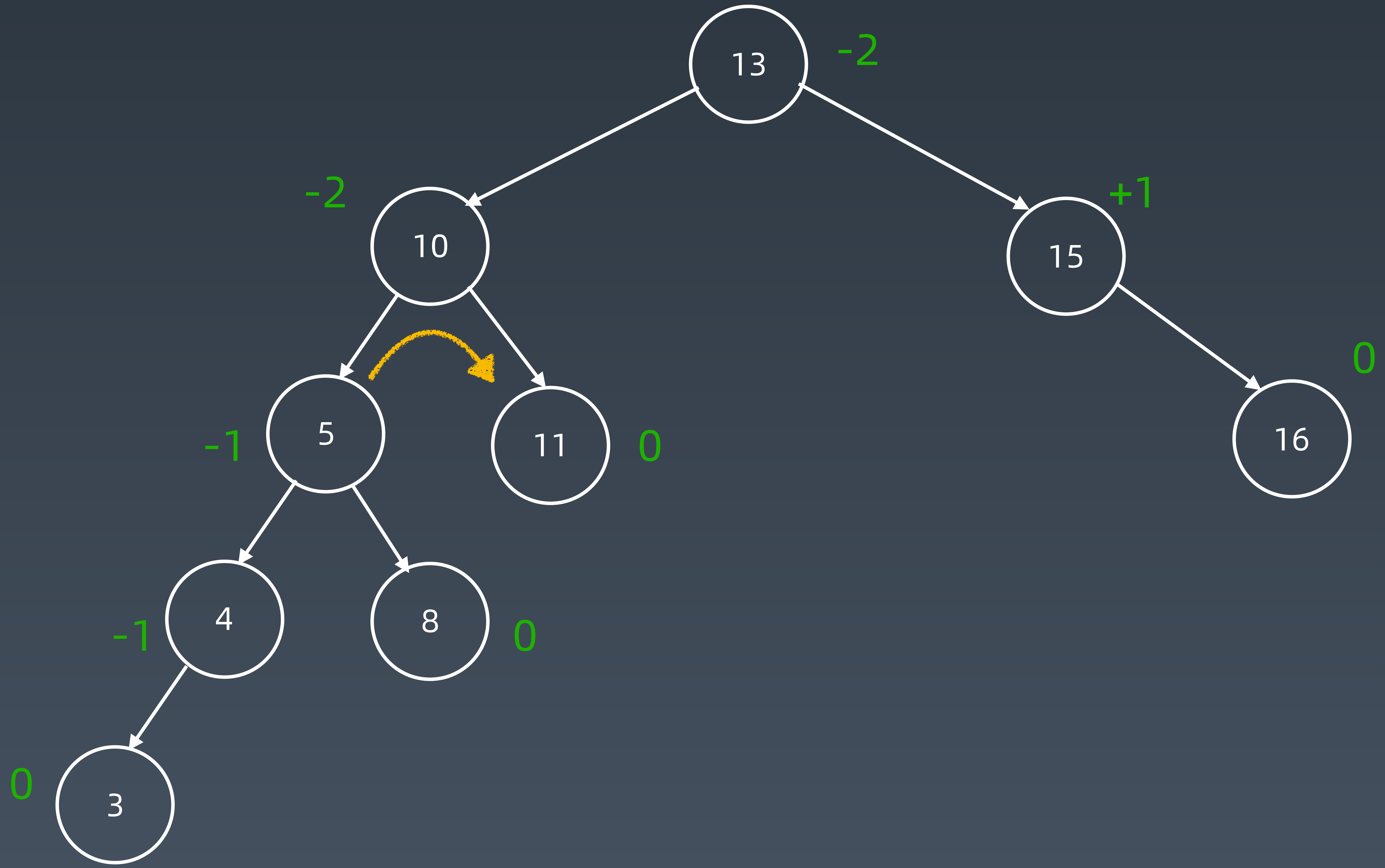
子树形态：左右子树 —> 左右旋



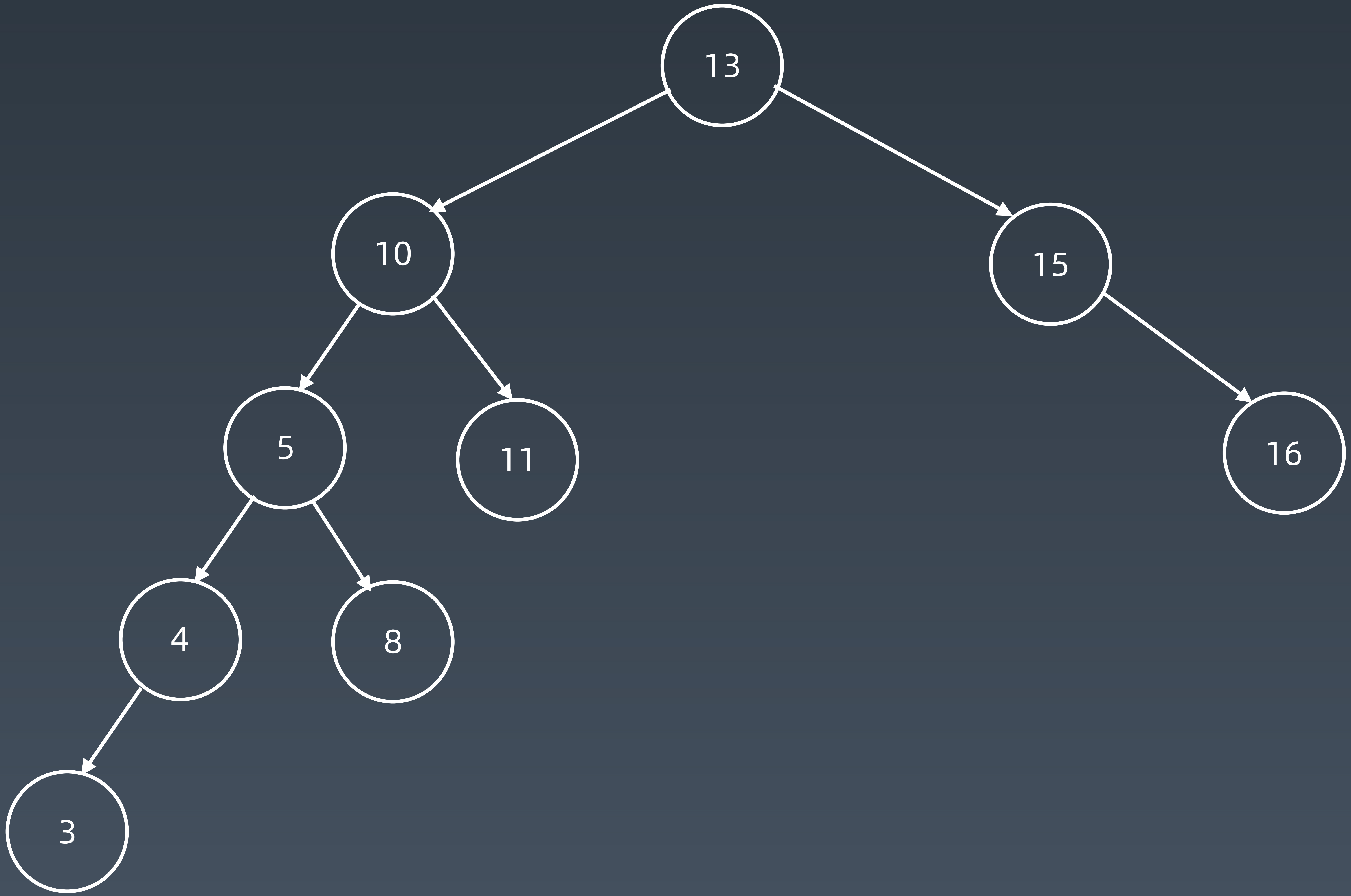


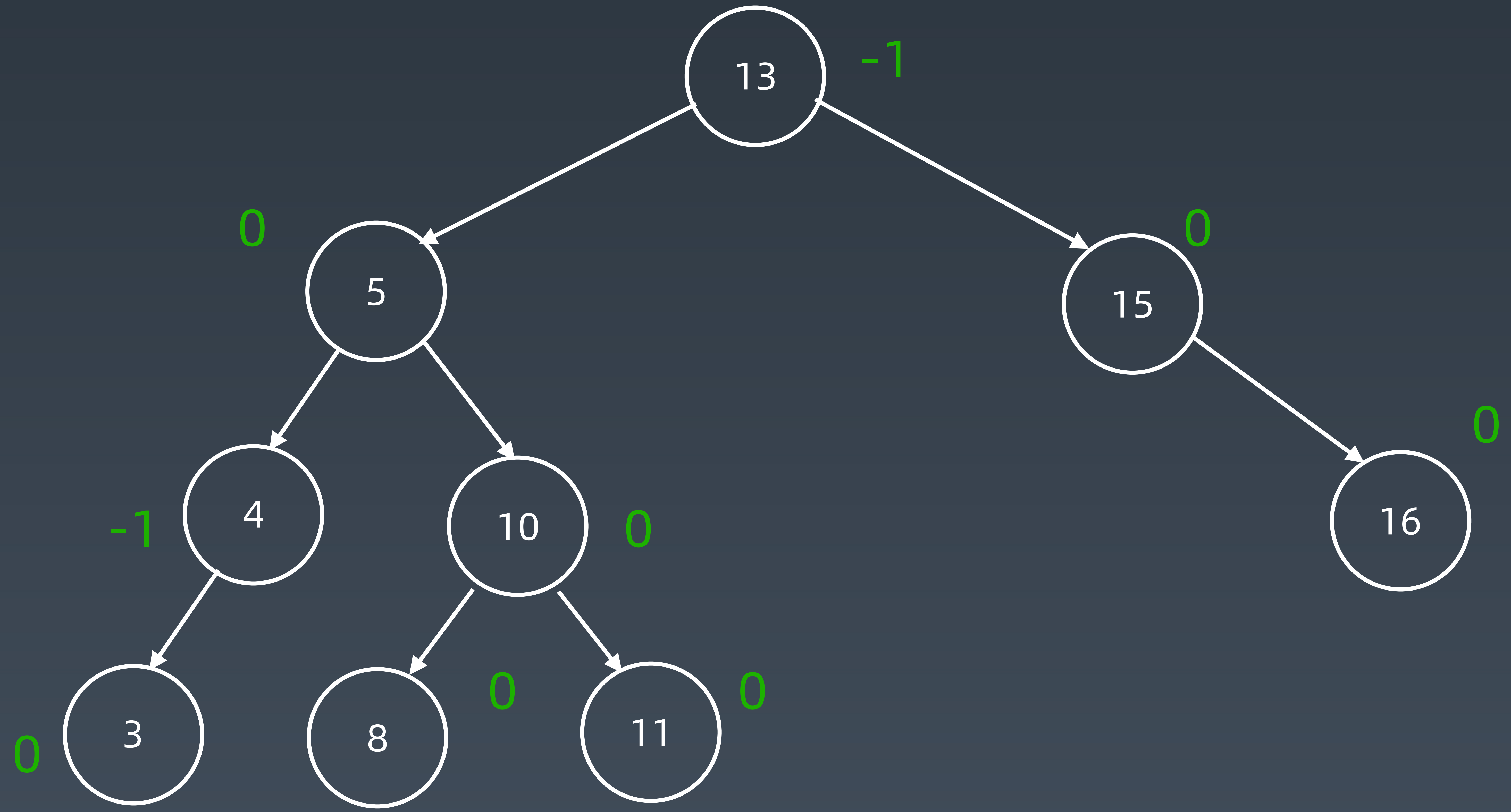
https://en.wikipedia.org/wiki/Tree_rotation#/media/File:Tree_Rebalancing.gif

向右旋转

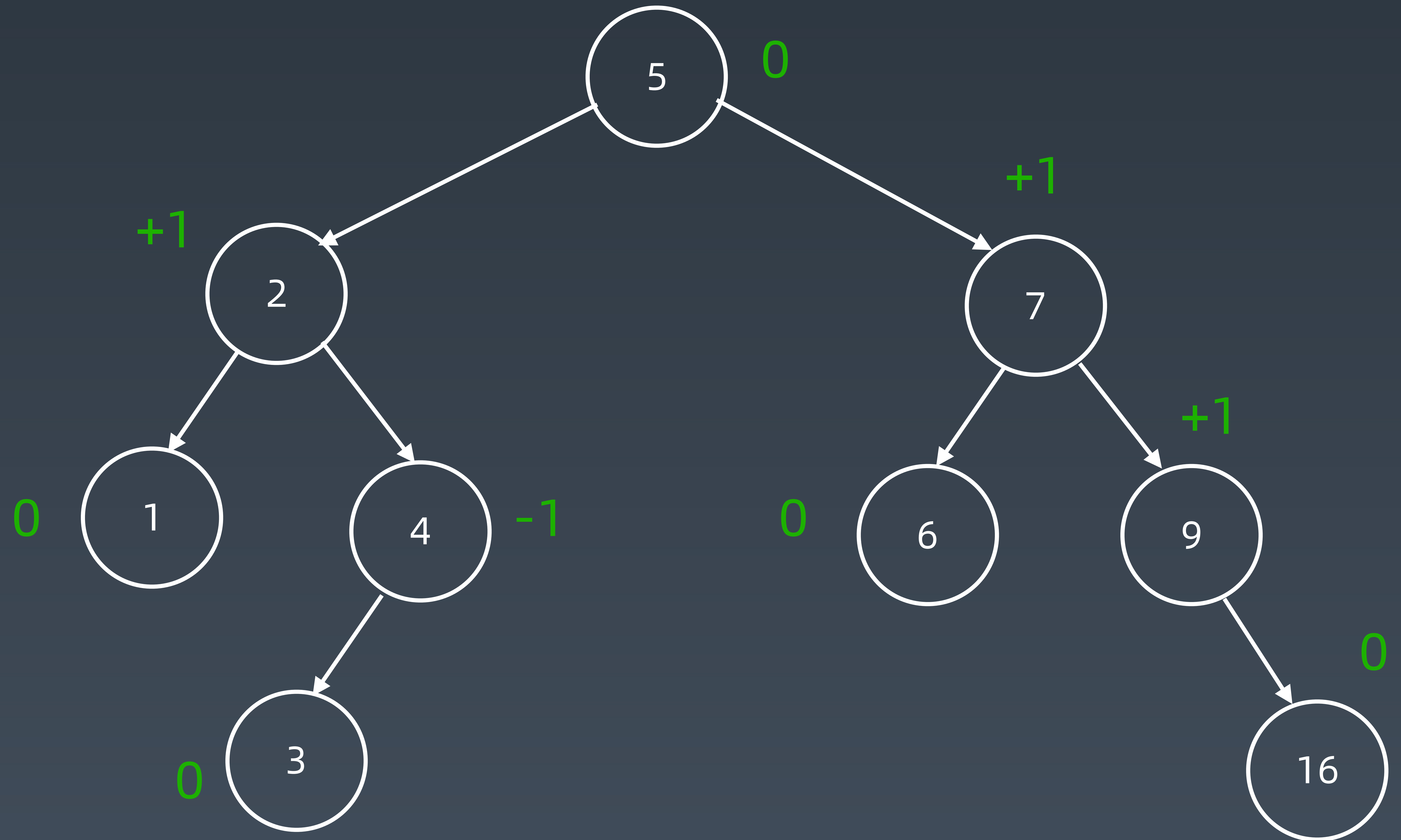


向右旋转

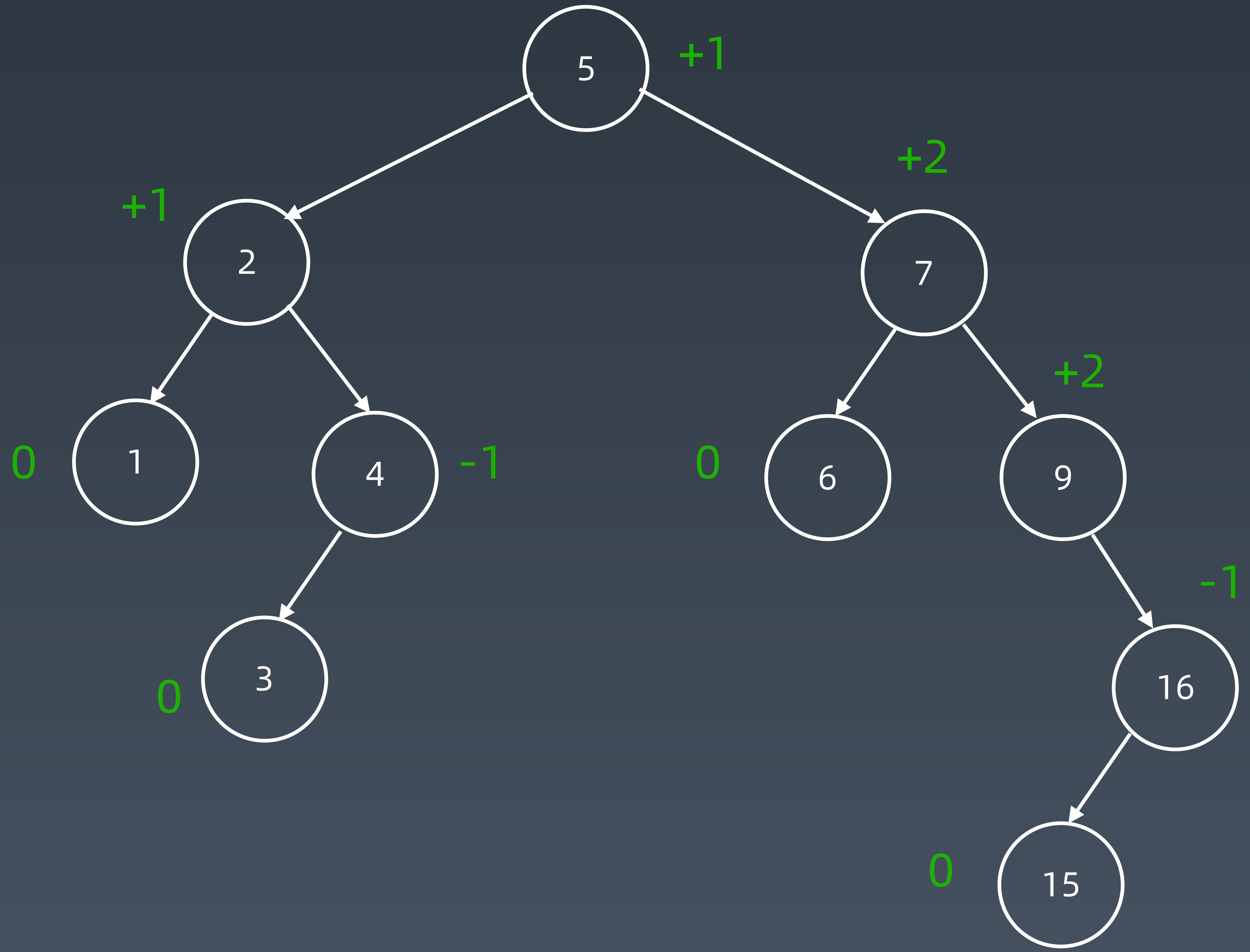



13
-1
5
0
15
0
16
0
4
-1
10
0
3
0
8
0
11
0

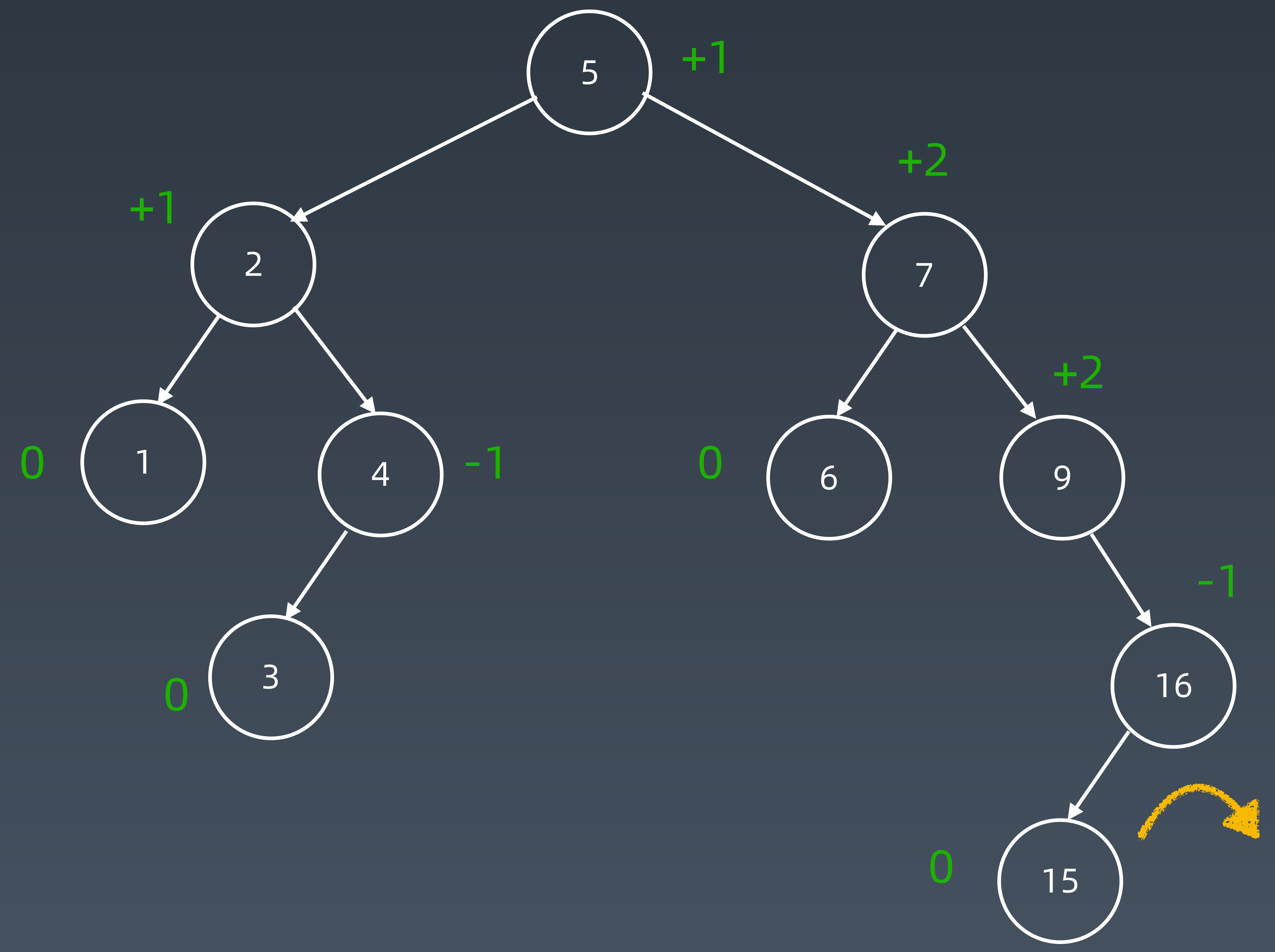
增加 15



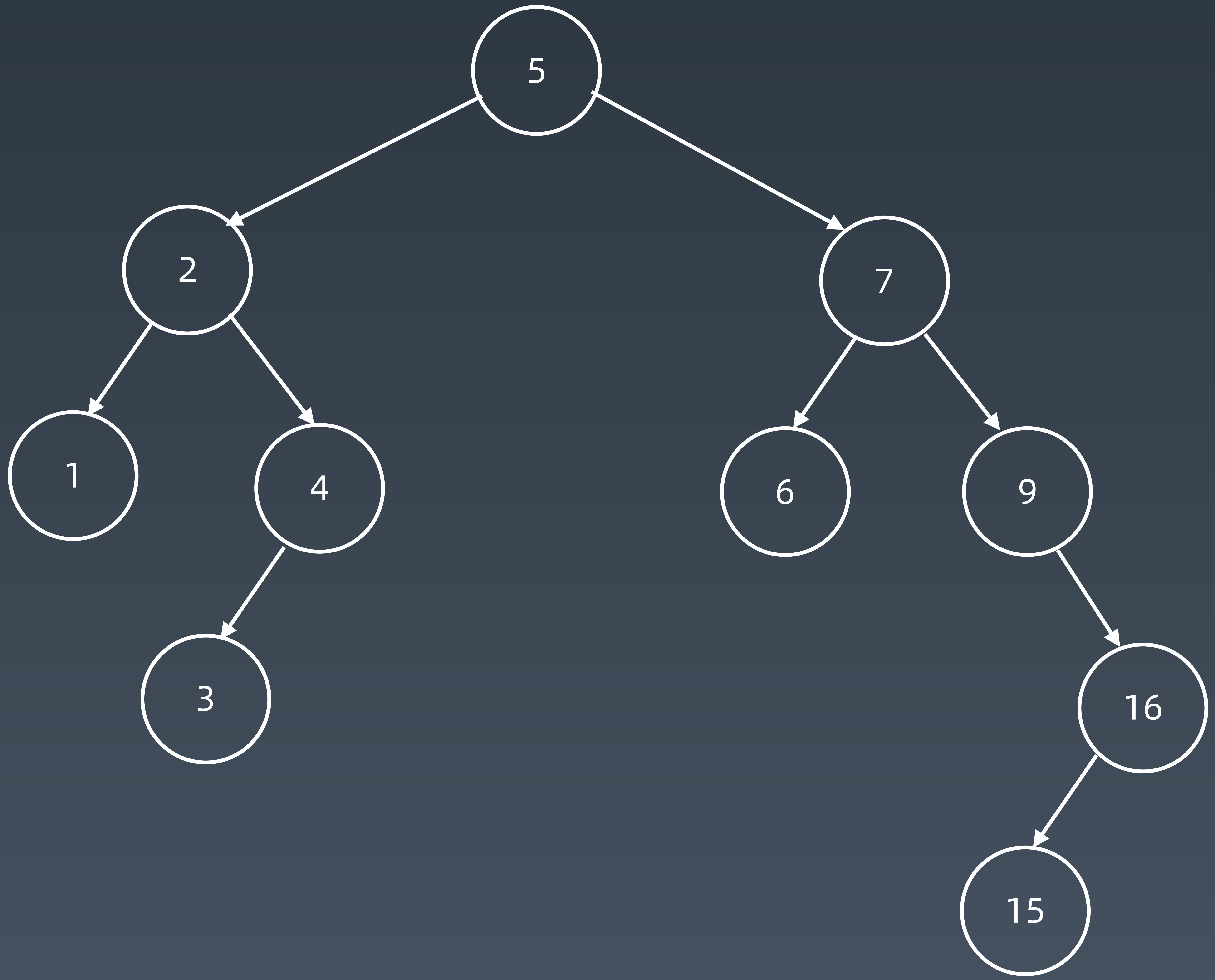
增加 15

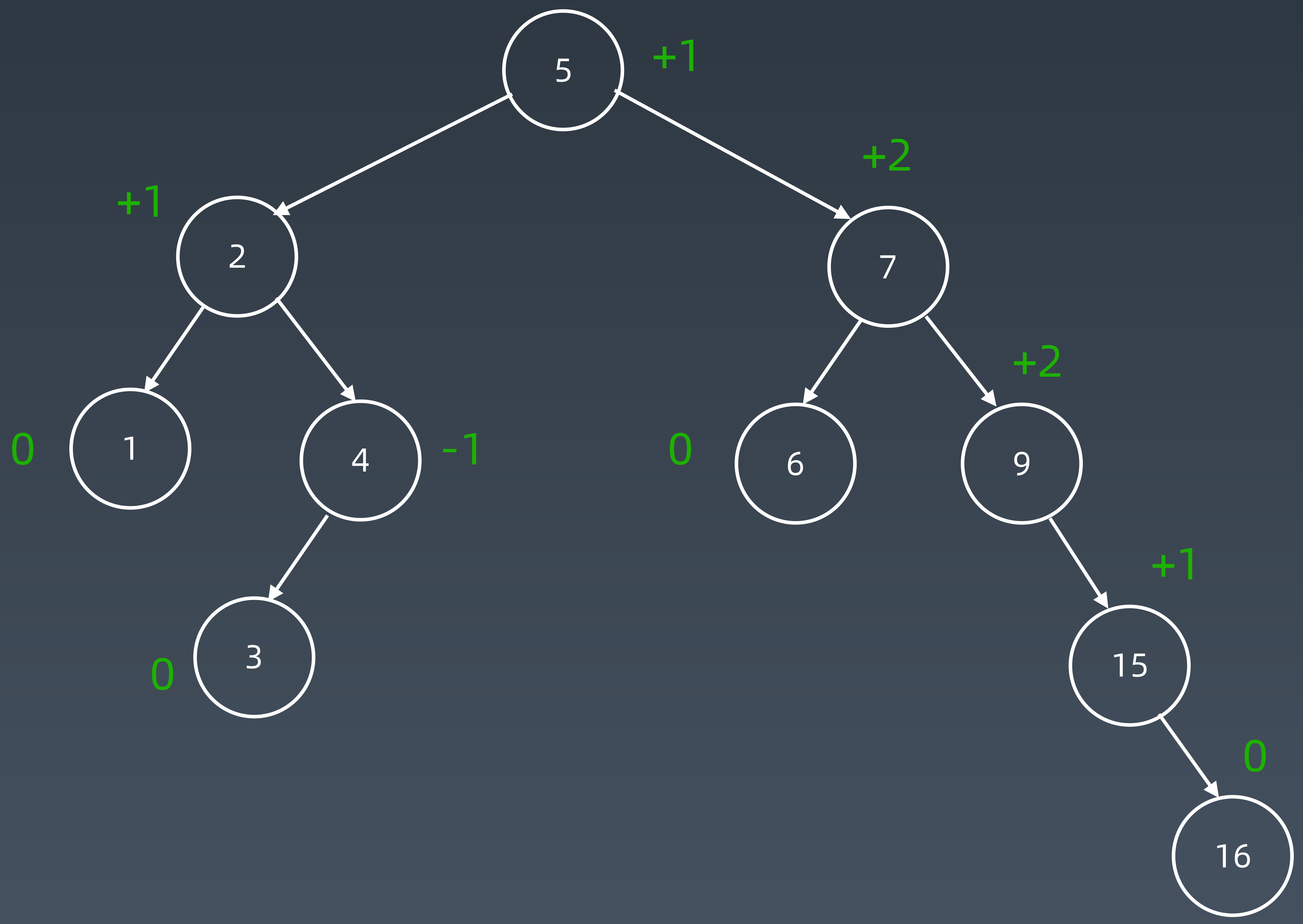


向右旋转



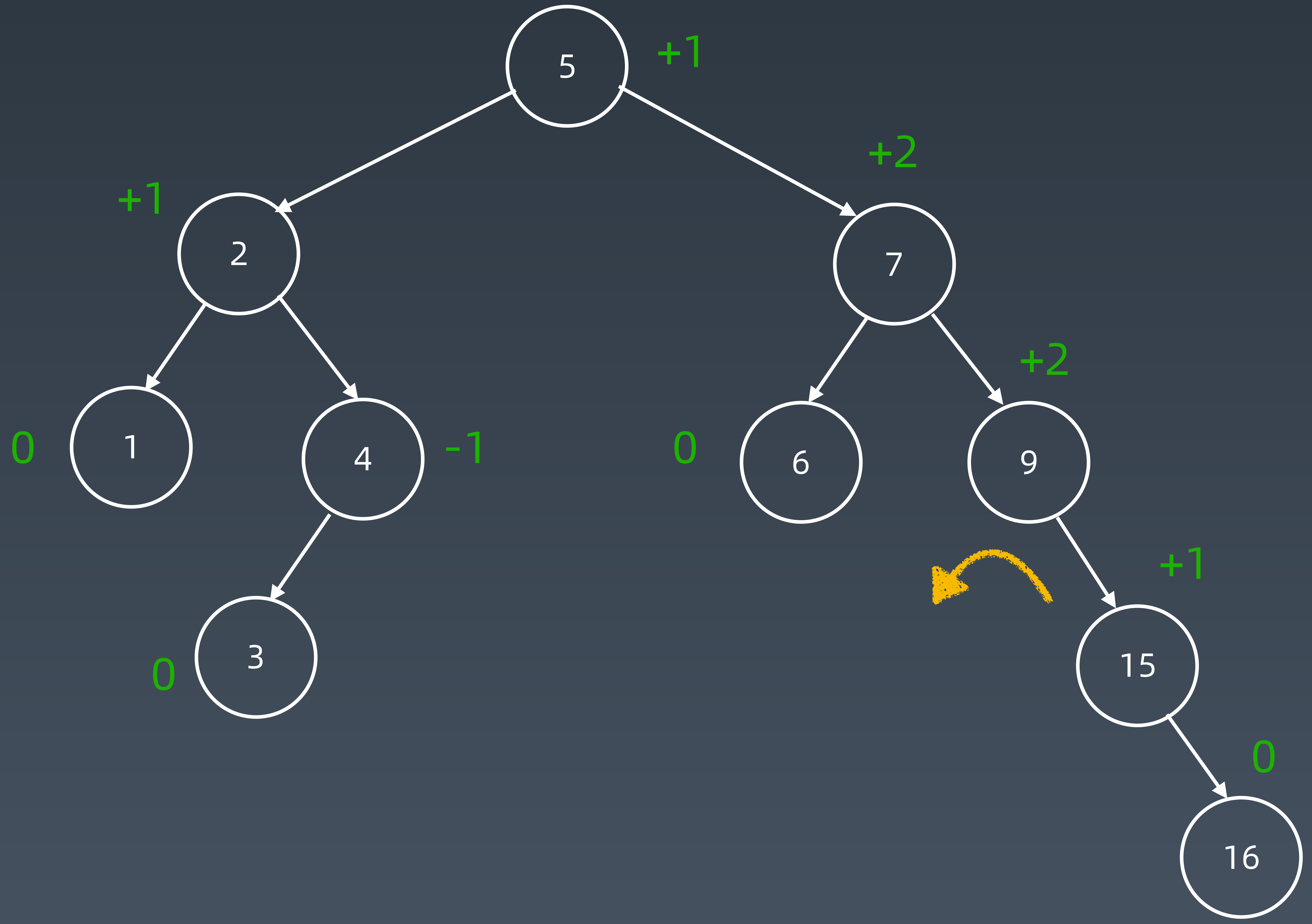
向右旋转



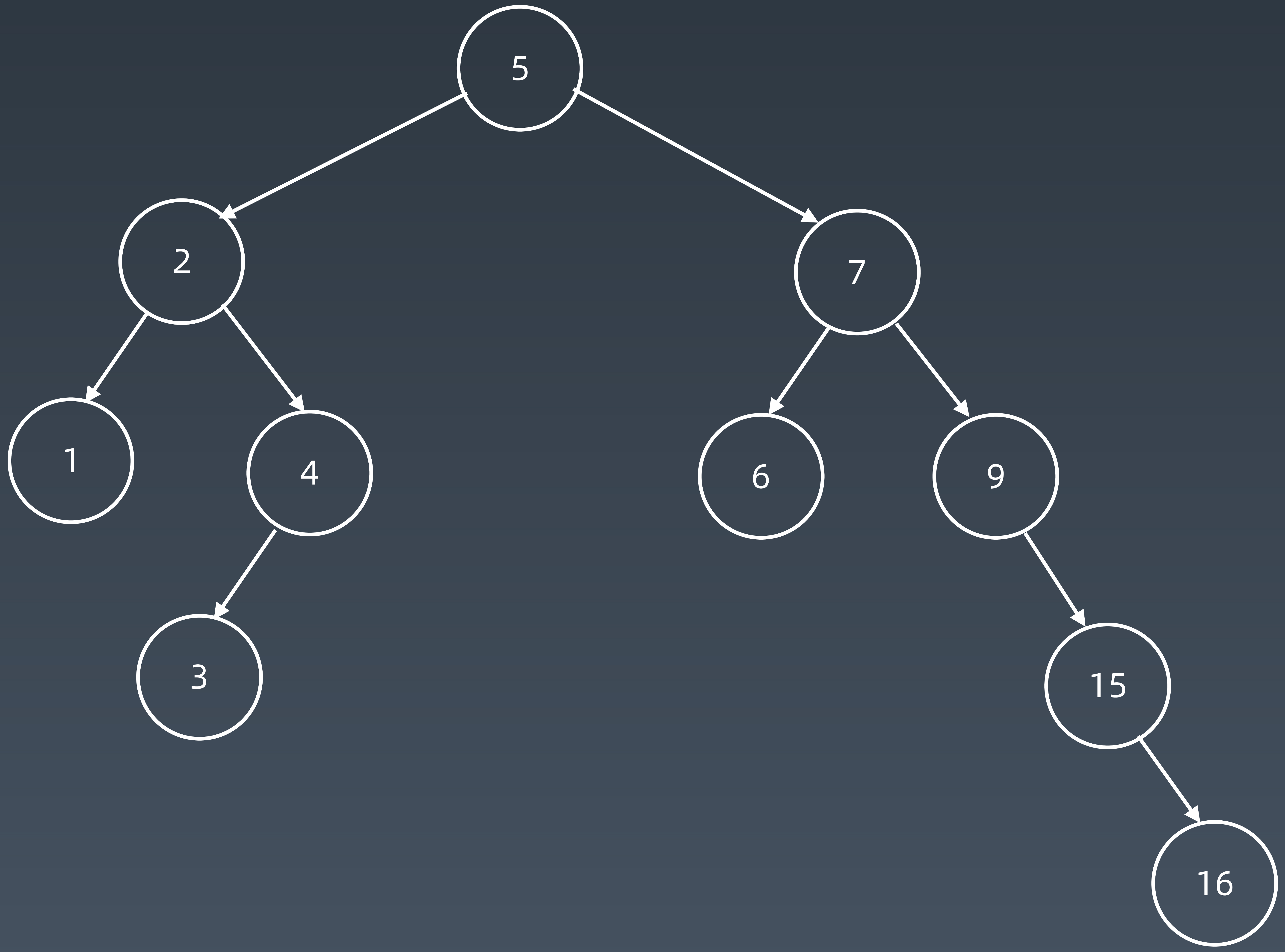


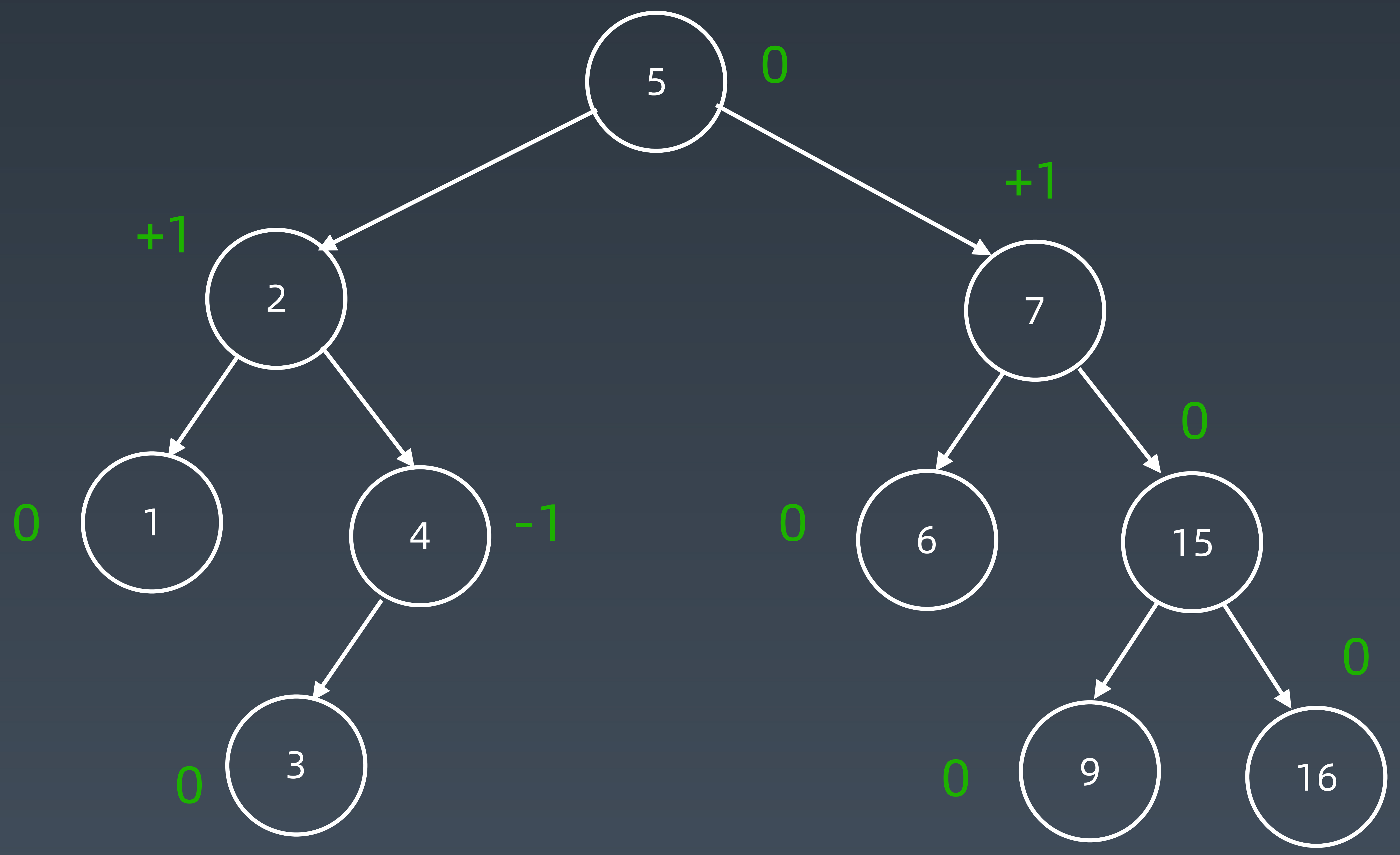
5
+1
2
+1
7
+2
1
0
4
-1
6
0
9
+2
3
0
15
+1
16
0

向左旋转



向左旋转




5
0
2
+1
7
+1
1
0
4
-1
6
0
15
0
3
0
9
0
16
0

AVL 总结

1. 平衡二叉搜索树
2. 每个结点存 balance factor = {-1, 0, 1}
3. 四种旋转操作

不足：结点需要存储额外信息、且调整次数频繁

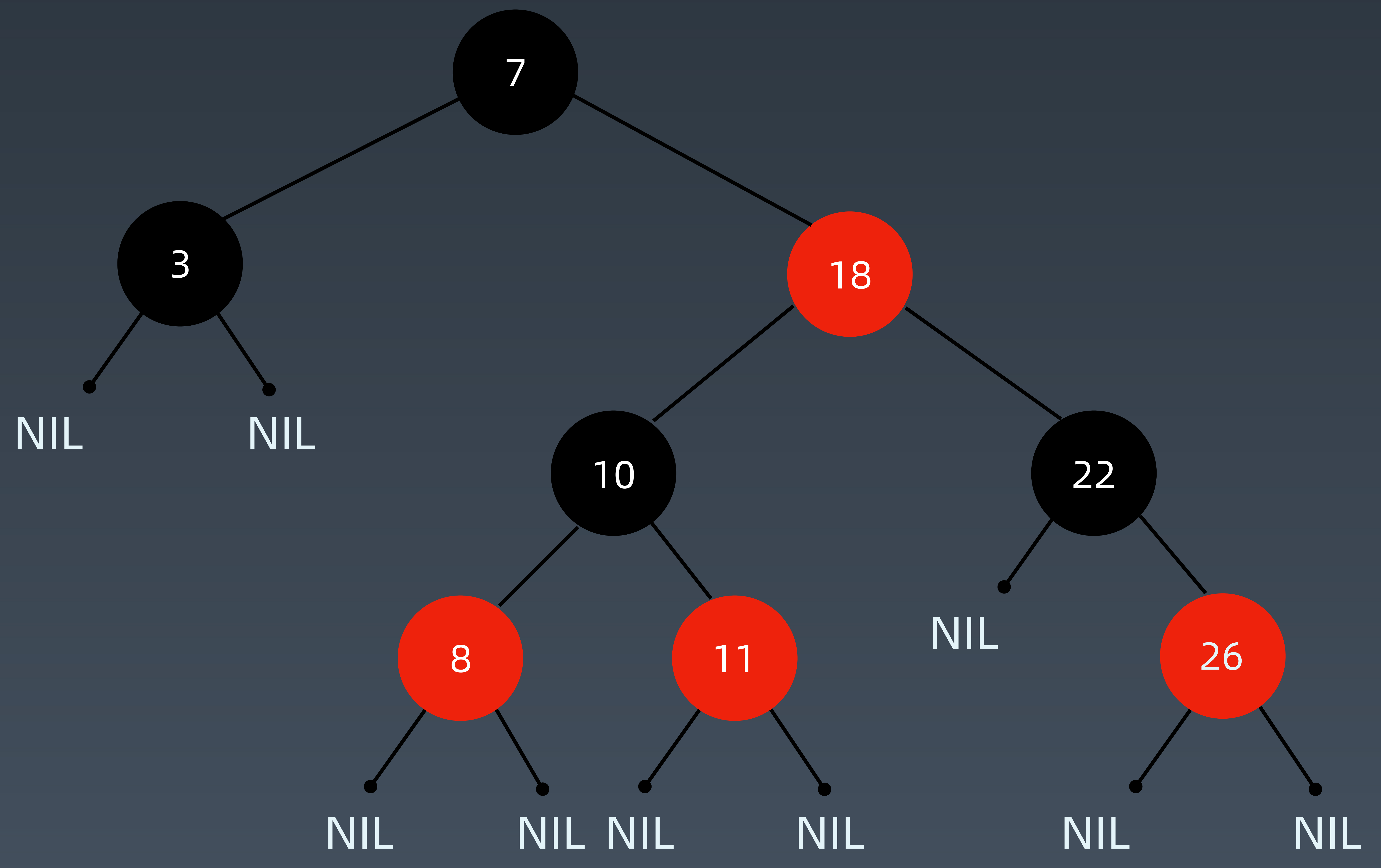
红黑树

Red-black Tree

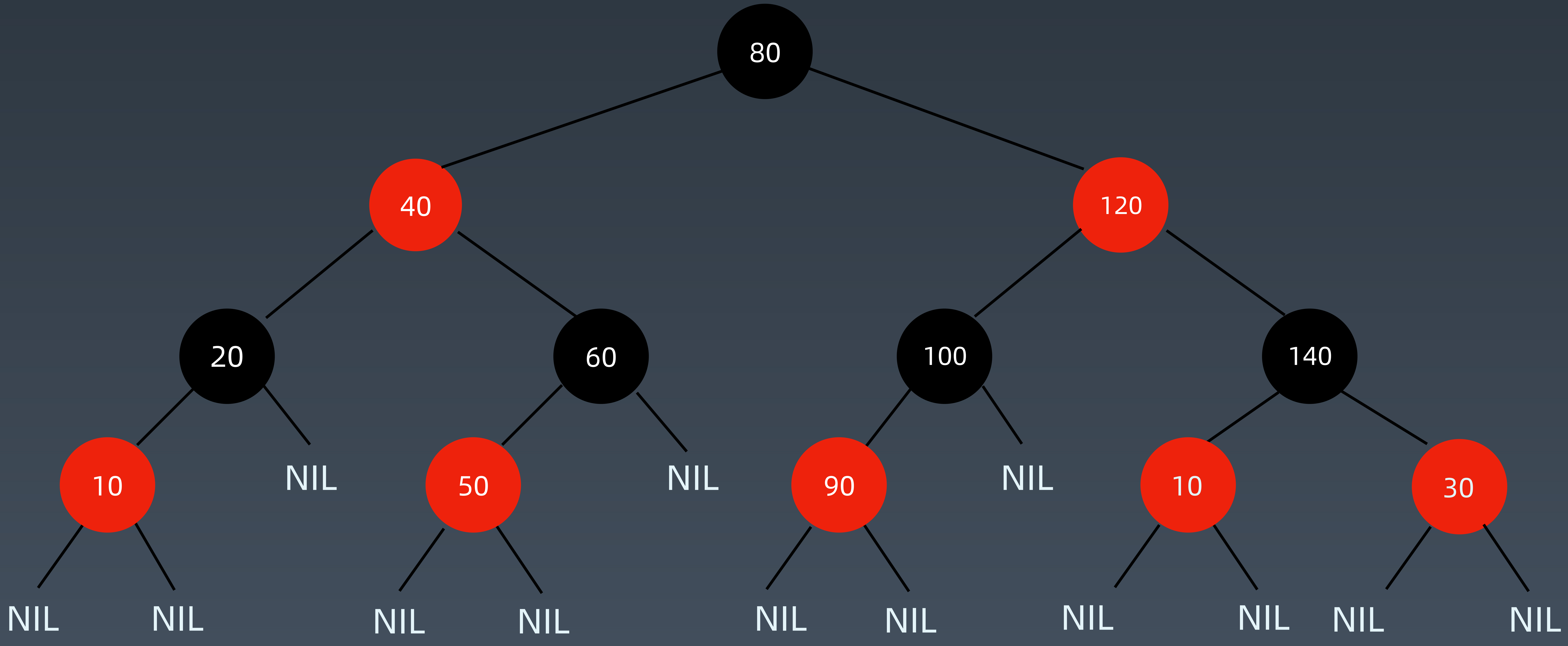
红黑树是一种近似平衡的二叉搜索树（Binary Search Tree），它能够确保任何一个结点的左右子树的高度差小于两倍。具体来说，红黑树是满足如下条件的二叉搜索树：

- 每个结点要么是红色，要么是黑色
- 根节点是黑色
- 每个叶节点（NIL节点，空节点）是黑色的。
- 不能有相邻接的两个红色节点
- 从任一节点到其每个叶子的所有路径都包含相同数目的黑色节点。

Red-black Tree



Red-black Tree



关键性质

从根到叶子的最长的可能路径不多于最短的可能路径的两倍长。

对比

- AVL trees provide faster lookups than Red Black Trees because they are more strictly balanced.
- Red Black Trees provide faster insertion and removal operations than AVL trees as fewer rotations are done due to relatively relaxed balancing.
- AVL trees store balance factors or heights with each node, thus requires storage for an integer per node whereas Red Black Tree requires only 1 bit of information per node.
- Red Black Trees are used in most of the language libraries like map, multimap, multisetin C++ whereas AVL trees are used in databases where faster retrievals are required.

THANKS!

极客大学