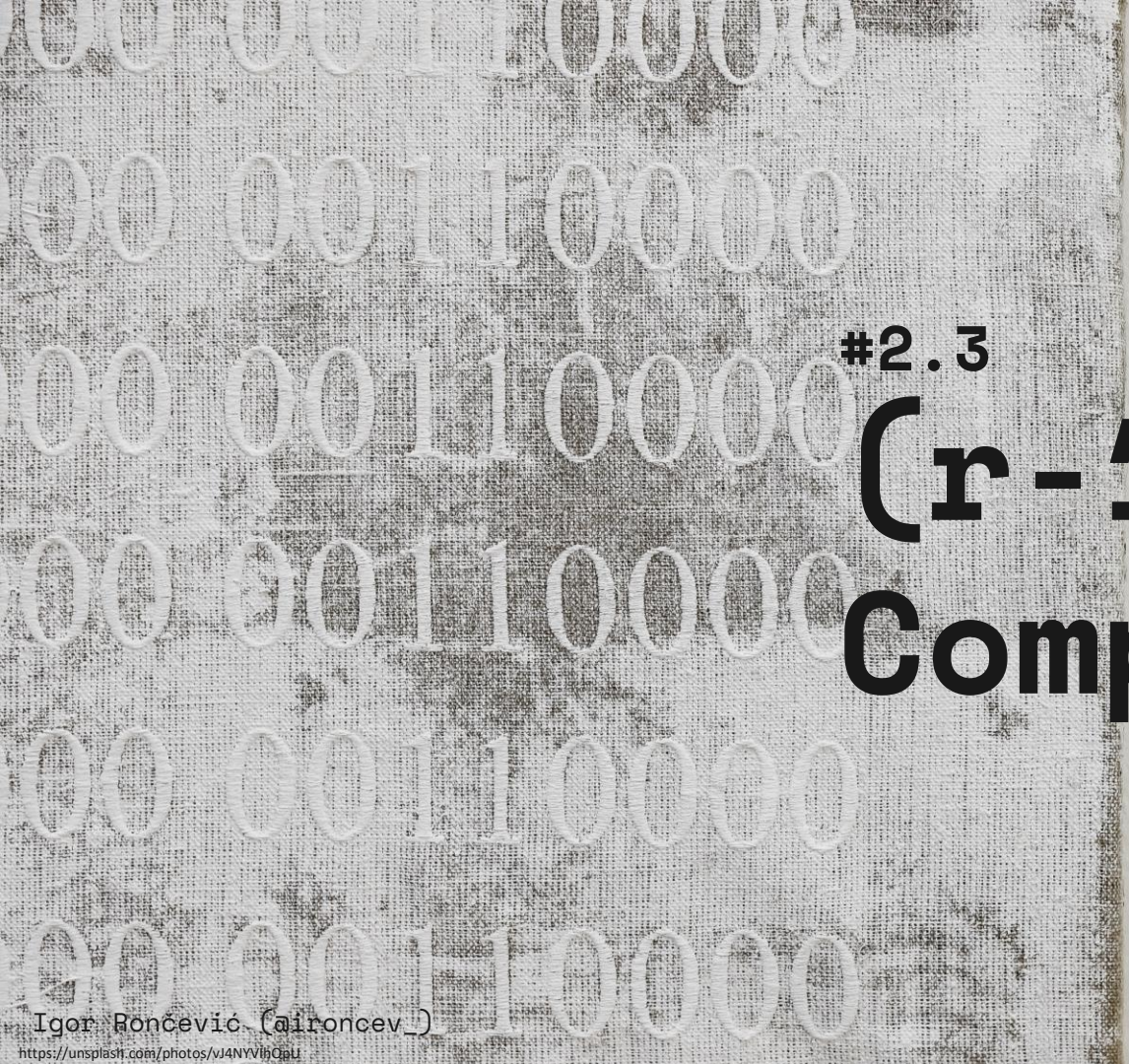

#2.3

(r-1)'s Complement

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https://unsplash.com/photos/vJ4NYVIhQpU

🔄 Motivation: “Subtraction by Addition”

$$214 - 78 =$$

$$214 - 078 =$$

$$214 + 1000 - 1000 - 078 =$$

$$214 + 999 + 1 - 078 - 1000 =$$

$$214 + 999 - 078 + 1 - 1000 =$$

$$214 + 921 + 1 - 1000 =$$

$$1136 - 1000 =$$

$$136$$

This is indeed only addition ;-)

Extremely easy to calculate ;-)
Just the “distance” between each digit and 9.

9’s complement of 078.

In the end, we just need to drop the leading 1. No heavy subtraction.

🔄 Motivation: “Subtraction by Addition”

$$214 - 78$$

$$214 - 078$$

$$214 + \mathbf{921} + 1$$

921 is **9’s complement** of 078.

$$1136$$

x

$$136$$

🔄 E.g., 1's Complement in Binary System

$$101 - 11 =$$

$$101 - 011 =$$

$$101 + 1000 - 1000 - 011 =$$

$$101 + 111 + 1 - 011 - 1000 =$$

$$101 + 111 - 011 + 1 - 1000 =$$

Extremely easy to calculate ;-)
Just swap 0s and 1s.

$$101 + 100 + 1 - 1000 =$$

1's complement of 011.

$$1010 - 1000 =$$

$$010$$

🔄 E.g., 1's Complement in Binary System

$$101 - 11$$

$$101 - 011$$

100 is **1's complement** of 011.

$$101 + \mathbf{100} + 1$$

$$1010$$

x

$$10$$

🔄 (r-1)'s or Diminished Radix Complement

$$N = (d_n \dots d_1 d_0)_r$$

Number N represented as a numeral of n+1 digits in the positional system with base r. E.g., $(123)_{10}$.

$$(r^{n+1} - 1) - N$$

(r-1)'s complement or **diminished radix complement** of the number N represented as a numeral in the positional system with base r. E.g., $(10^3\text{-}1)_{10}\text{-}(123)_{10}=(876)_{10}$.

$$d_{max} = v^{-1}(r-1)$$

Digit with the maximum value in the positional system with base r. E.g., ***9***.

$$(r^{n+1} - 1) = \left(d_{max_n} \dots d_{max_1} d_{max_0}\right)_r$$

$(r^{n+1}\text{-}1)$ in the positional system with base r will always contain exactly n+1 digits of the maximum value. E.g., $(999)_{10}$.

🔄 E.g., (r-1)'s Complement of Numerals

$N = (d_n \dots d_1 d_0)_r$	d_{max}	$(r^{n+1}-1)$	$(r^{n+1}-1) - N$
$(901)_{10}$	**9**	$(999)_{10}$	$(098)_{10}$
$(023)_{10}$	**9**	$(999)_{10}$	$(976)_{10}$
$(F2A)_{16}$	***F***	$(FFF)_{16}$	$(0D5)_{16}$
$(0F3)_{10}$	***F***	$(FFF)_{16}$	$(F0C)_{16}$
$(101)_2$	**1**	$(111)_2$	$(010)_2$
$(010)_2$	**1**	$(111)_2$	$(101)_2$