Project Specification

Application of Grover's Algorithm for Boolean Satisfiability

Authors: Edvard All, Dilvan Sabir
Supervisor: Stefano Markidis

KTH
DA150x
February 25, 2022

1 Introduction

The search problem may be formulated as finding some specific element within a large unordered set. Trivially, if we let $N = 2^n$ where n is the size of this set, then the probability of having found the element after examining k records in the set will be in $\mathcal{O}(\frac{k}{N})$. Thus it is easy to see why $\mathcal{O}(N)$ is the lower bound for a solution to this problem, at least if we are constrained to the realm of classical computation.

Quantum computing entails the use and harnessing of various properties of quantum mechanics such as superposition, interference, and quantum entanglement to preform calculations, in a manner not possible on systems relying solely on classical mechanics. The aforementioned phenomena have massive implications for how computing is performed.

Most notably, the quantum bit (qubit), which corresponds to the quantum version of the classical notion of a bit, can be 1, 0, or in both states simultaneously. This latter state is commonly referred to as a superposition or superstate. Whenever measured, it will however always be 1 or 0, the superstate collapses under measurement. The probability of the bit collapsing to 1 or 0 depends on the quantum state of the qubit prior to measurement. It is primarily this property that enables quantum computers to solve problems that classical computers cannot feasibly compute.

Qubits are prone to noise, and so qubit quality is of great importance for correct quantum computation. Quantum noise includes the various factors that can adversely affect the quantum state of qubits, to the detriment of computation results. This noise emanates from a variety of sources such as temperature, vibrations, cosmic radiation, magnetic fields and equipment contamination. The more tolerant the qubits are to this noise, the longer they are able to maintain their quantum states and thus longer and more complex calculations are possible.

Grover's algorithm is a quantum algorithm capable of finding the unique input to a black box function that produces a particular output value (note that this is merely a rephrasing of the search problem outlined above), in just $\mathcal{O}(\sqrt{N})$ steps. Notably, this has been shown to outline the lower bound for a solution to the problem, in quantum mechanical systems. [1]

Grover's algorithm has been proven useful for quantum speedup of many classical algorithms, one such problem being that of boolean satisfiability [2], [3]. The problem of boolean satisfiability (SAT) is, in broad terms, the following problem: given a boolean expression B and of a set of variables $V = v_0, ..., v_n$, is there some assignment of truth-values to the variables such that $B(V*)$ evaluates to true. Problems of this nature are seen as the first NP-complete problems. [4]

Boolean satisfiability problems have far-ranging applications: equivalence checking in combinational circuits; automatic test-pattern generation for identification of fabrication defects in integrated circuits; planning in artificial agents; haplotyping in bio-informatics; and even quantum compilation, i.e. translating algorithms expressed in high-level abstraction quantum programming languages into architecturally dependant instructions for quantum hardware [5], [6].

2 Problem statement

Although the theoretical foundations underlying Grover's algorithm are sound, real applications of the algorithm on quantum hardware for non-trivial problem sizes remains to be properly explored. This holds especially true with respect to quantum fault-tolerance, as available hardware remains inadequate (with regards to number of qubits) to satisfactorily implement quantum error correction necessary to account for these faults.

The goal of this project is to investigate the performance of k-SAT solving algorithms incorporating Grover's algorithm, with respect to the inclusion of Grover's algorithm. To elaborate, we wish to probe what real speedup is acquired by making use of Grover.

Furthermore, we wish to investigate the conditions of quantum fault-tolerance in parallel with above process.

3 Approach

Our plan is to implement and compare three classes of algorithms:

1. k-SAT solver using Grover's under noise
2. k-SAT solver using Grover's with no noise
3. A classical implementation of a k-SAT solver

We aim to achieve this through the use of quantum machines for our experiments, made available by the IBM quantum cloudware platform [7]. However, given the limited availability of such machines, we will primarily rely on simulating quantum computation.

As simulation will be the bedrock of this experimentation, above noise will naturally too have to be simulated. Qiskit [8] is an open source software development kit for quantum computation. As the framework enables both simulation and quantum computation actual, it seems highly suited for the project.

Furthermore, because noise and error may manifest in a number of different ways, it seems reasonable to investigate results under different simulated conditions (with respect to noise), given time is not a limiting constraint to this end.

References

[1] Lov K Grover. "A fast quantum mechanical algorithm for database search". In: *Proceedings of the twenty-eighth annual ACM symposium on Theory of computing*. 1996, pp. 212–219.

[2] Earl Campbell, Ankur Khurana, and Ashley Montanaro. "Applying quantum algorithms to constraint satisfaction problems". In: *Quantum* 3 (2019), p. 167.

[3] Evgeny Dantsin, Vladik Kreinovich, and Alexander Wolpert. "On quantum versions of record-breaking algorithms for sat". In: *ACM SIGACT News* 36.4 (2005), pp. 103–108.

[4] Yakir Vizel, Georg Weissenbacher, and Sharad Malik. "Boolean Satisfiability Solvers and Their Applications in Model Checking". In: *Proceedings of the IEEE* 103.11 (2015), pp. 2021–2035. DOI: `10.1109/JPROC.2015.2455034`.

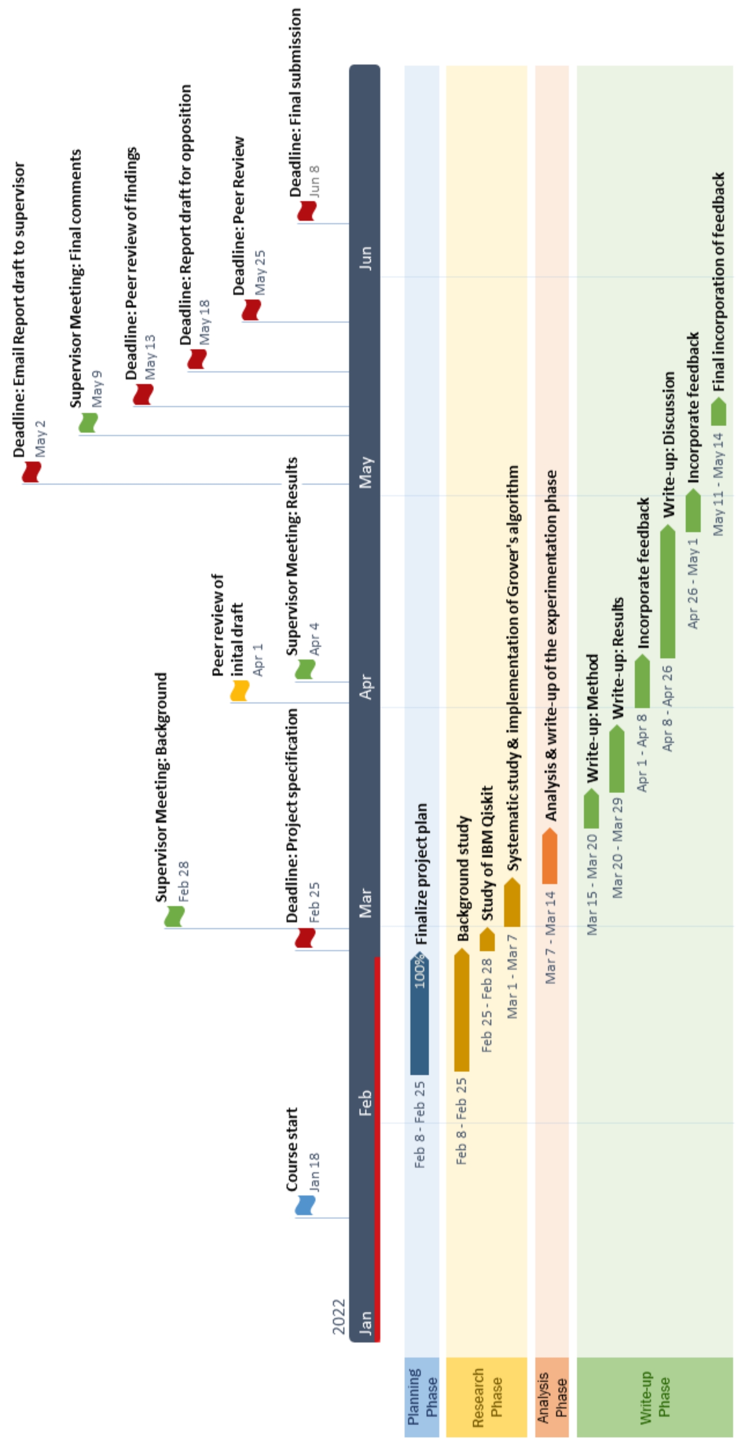
[5] Joao Marques-Silva. "Practical applications of Boolean Satisfiability". In: *2008 9th International Workshop on Discrete Event Systems*. 2008, pp. 74–80. DOI: `10.1109/WODES.2008.4605925`.

[6] Mathias Soeken, Giulia Meuli, Bruno Schmitt, et al. "Boolean satisfiability in quantum compilation". In: *Philosophical Transactions of the Royal Society A* 378.2164 (2020), p. 20190161.

[7] Chris Fisher. *IBM: Quantum Computing*. Apr. 2009. URL: `https://www.ibm.com/quantum-computing/`.

[8] *Open-source quantum development*. URL: `https://qiskit.org/`.

4 Timeplan



2022
Jan
Feb
Mar
Apr
May
Jun
Course start
Jan 18
Deadline: Project specification
Feb 25
Supervisor Meeting: Background
Feb 28
Peer review of inital draft
Apr 1
Supervisor Meeting: Results
Apr 4
Deadline: Email Report draft to supervisor
May 2
Supervisor Meeting: Final comments
May 9
Deadline: Peer review of findings
May 13
Deadline: Report draft for opposition
May 18
Deadline: Peer Review
May 25
Deadline: Final submission
Jun 8
Planning Phase
Feb 8 - Feb 25
100%
Finalize project plan
Research Phase
Feb 8 - Feb 25
Background study
Feb 25 - Feb 28
Study of IBM Qiskit
Mar 1 - Mar 7
Systematic study & implementation of Grover's algorithm
Analysis Phase
Mar 7 - Mar 14
Analysis & write-up of the experimentation phase
Write-up Phase
Mar 15 - Mar 20
Write-up: Method
Mar 20 - Mar 29
Write-up: Results
Apr 1 - Apr 8
Incorporate feedback
Apr 8 - Apr 26
Write-up: Discussion
Apr 26 - May 1
Incorporate feedback
May 11 - May 14
Final incorporation of feedback