Literature Review for DA150X

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1 Background

We quickly outline the scope of the literature review through a brief background. *Please note that the following is more or less verbatim from what we wrote in our project specification, and not original work.*

The search problem may be formulated as finding some specific element within a large unordered set. Trivially, if we let $N = 2^n$ where n is the size of this set, then the probability of having found the element after examining k records in the set will be in $\mathcal{O}(\frac{k}{N})$. Thus it is easy to see why $\mathcal{O}(N)$ is the lower bound for a solution to this problem, at least if we are constrained to the realm of classical computation.

Quantum computing entails the use and harnessing of various properties of quantum mechanics such as superposition, interference, and quantum entanglement to preform calculations, in a manner not possible on systems relying solely on classical mechanics. The aforementioned phenomena have massive implications for how computing is performed.

Most notably, the quantum bit (qubit), which corresponds to the quantum version of the classical notion of a bit, can be 1, 0, or in both states simultaneously. This latter state is commonly referred to as a superposition or superstate. Whenever measured, it will however always be 1 or 0, the superstate collapses under measurement. The probability of the bit collapsing to 1 or 0 depends on the quantum state of the qubit prior to measurement. It is primarily this property that enables quantum computers to solve problems that classical computers cannot feasibly compute.

Qubits are prone to noise, and so qubit quality is of great importance for correct quantum computation. Quantum noise includes the various factors that can adversely affect the quantum state of qubits, to the detriment of computation results. This noise emanates from a variety of sources such as temperature, vibrations, cosmic radiation, magnetic fields and equipment contamination. The more tolerant the qubits are to this noise, the longer they are able to maintain their quantum states and thus longer and more complex calculations are possible.

Grover's algorithm is a quantum algorithm capable of finding the unique input to a black box function that produces a particular output value (note that this is merely a rephrasing of the search problem outlined above), in just $\mathcal{O}(\sqrt{N})$ steps. Notably, this has been shown to constitute the lower bound for a solution to the problem, in quantum mechanical systems. [1]

Grover's algorithm has been proven useful for quantum speedup of many classical algorithms, one such problem being that of boolean satisfiability [6], [2]. The problem of boolean satisfiability (SAT) is, in broad terms, the following problem: given a boolean expression B and of a set of variables $V = v_0, ..., v_n$, is there some assignment of truth-values to the variables such that $B(V*)$ evaluates to true. Problems of this nature are seen as the first NP-complete problems. [3]

2 Previous work

Dantsin, Kreinovich, and Wolpert [2] show that the application of Grover's for satisfiability problems cannot yield more than quadratic speed-up. This is a theoretical finding, not an experimental one.

What follows are a number of works pertaining to the simulation and experimentation of Grover's quantum algorithm.

Campbell, Khurana, and Montanaro [6] compares the performance of Grover's for boolean satisfiability against leading classical algorithms. More specifically, they simulate quantum computation, and analyse space and time complexity, of a quantum k-SAT algorithm for random instances of the problem for various k. Their experimental findings suggest quantum speedup is indeed possible, but that hurdles are yet manifest in lack of qubits and fault-tolerance.

Liu et al. [8] suggest the feasibility of Grover's algorithm (and others) on real quantum hardware (using IBM Q), although it is from their paper difficult to ascertain what problem instances were considered.

Mandviwalla, Ohshiro, and Ji [4] also explore the feasibility of Grover's algorithm by way of IBM Q, concluding that at the time of the study's publication, quantum computers could only accurately and reliably solve simple problems with small amounts of data. They also discovered large variations caused by "qubit choice", which we presume to mean the design of the quantum circuitry itself. Additionally, their experimental results suggest that quantum computers are slower than expected.

Nishio, Takahiko, and Rodney [5] elucidate above mentioned variations concerning qubit choice, by using SWAP-gates (combinations of CNOT gates) to improve accuracy of results. This is related to the fact that CNOT-gates place additional physical topological restrictions on circuitry design.

Matsuo, Hattori, and Yamashita [7] propose an approach to optimizing the number of SWAP-gates when building quantum circuitry on IBM Q. Interestingly, they utilize a SAT solver for finding optimal qubit placement.

References

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