

# DEVELOPMENT OF ASYMMETRIC BOUC-WEN MODEL WITH LINEAR STRENGTH-DEGRADATION FUNCTIONS

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## ABSTRACT

In this study, an asymmetric Bouc-Wen model (BW model) is developed in which strength degradation is modeled with piecewise linear functions. This paper first reviews previous models, including the original BW model, the Wang-Wen model, and the generalized BW model and its modified version. In order to accurately describe the asymmetric strength degradation often encountered in many seismic engineering applications, this study extends the BW models with combining piecewise linear functions. A framework for parameter identification is then formulated, describing the objective function and constraint conditions required for the convex piecewise post-yielding functions. The proposed model is verified on the basis of the cyclic seismic test results of welded steel moment connections with composite floor slabs.

Keywords: Hysteresis model, Asymmetric hysteresis, Bouc-Wen model, Welded steel moment connections

## INTRODUCTION

Hysteresis is a nonlinear phenomenon in physical systems, and numerous models have been investigated for the description of the nonlinear behavior. The Bouc-Wen model is regarded as one of the most popular models which is originally introduced by Bouc (1967) and extended by Wen (1976). Due to its versatility, the Bouc-Wen model has been successfully extended to its variations to account for the strength degradation, stiffness degradation (Kottari et al., 2014) and pinching effect (Foliente, 1995). One of the major challenges in applying the Bouc-Wen model is how to describe the asymmetric hysteresis behavior. Many structural members and connections often exhibit asymmetric hysteresis behavior when subjected to cyclic loading. To account for the asymmetry, a number of studies were conducted to describe the asymmetric hysteresis (Colangelo et al., 1996; Kwok et al., 2007; and Zhu and Wang, 2012).

This study proposes the asymmetric Bouc-Wen model with linear strength-degradation functions, which combines the Bouc-Wen hysteresis and piecewise linear functions such that the strength degradation of the hysteretic behavior could be captured. The proposed model was validated using previous experimental results that exhibited severe asymmetric hysteresis.

## BOUC-WEN MODEL AND ITS EXTENSIONS

Hysteresis models describe two input and output variables as follows:

$$f = \alpha k_i x + (1 - \alpha) k_i z \quad (1)$$

where  $k_i$  is the elastic stiffness,  $\alpha$  is the ratio of the post-yield stiffness to the elastic stiffness; and  $z$  is an auxiliary variable introduced to simulate hysteretic behavior.

In the original Bouc-Wen model (BW model), the auxiliary variable  $z$  is controlled by the following nonlinear differential equation with a zero initial condition:

$$\dot{z} = \dot{x}(1 - |z|^n \psi) \quad (2)$$

where  $n$  is a parameter for the sharpness of the hysteresis loop and  $\psi$  is a nonlinear function that controls the other shape characteristics of the hysteresis loop expressed as follows:

$$\psi = \gamma + \beta \text{sgn}(\dot{x}z) \quad (3)$$

where  $\gamma$  and  $\beta$  control the slopes of hysteresis loop. The typical hysteresis loop generated by the BW model and its associated parameters are depicted in Figure 1.

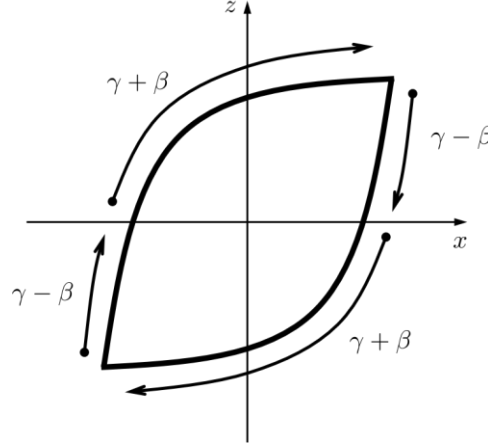


Figure 1 Typical loops of the Bouc-Wen hysteresis model

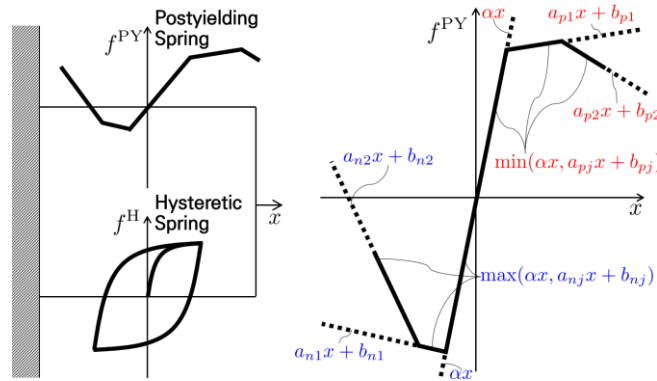


Figure 2 Graphical illustration for the asymmetric Bouc-Wen model proposed

### ASYMMETRIC BOUC-WEN MODEL PROPOSED

A new hysteresis model is proposed by combining the Bouc-Wen hysteresis model with a post-yielding function composed of piecewise linear functions. The proposed model is expressed as follows (see Figure 2 for the case of  $k_i=1$ ):

$$f = f^{\text{PY}} + f^{\text{H}} = \begin{cases} \min(\alpha k_i x, a_{pj} k_i x + b_{pj}) + (1 - \alpha) k_i z & x \geq 0 \\ \max(\alpha k_i x, a_{nj} k_i x + b_{nj}) + (1 - \alpha) k_i z & x < 0 \end{cases} \quad (4)$$

where  $f^{\text{PY}}, f^{\text{H}}$ , are the post-yielding and hysteresis forces, respectively,  $a_{pj}$  and  $b_{pj}$  are the slope and the intercept of the  $j$ -th linear function for the positive displacement, respectively, and  $a_{nj}$  and  $b_{nj}$  are those for the negative displacement, respectively.

One of the key features of the proposed model is its versatility; the proposed model is not restricted to a particular hysteresis function, and can be combined with any kind of hysteretic models. One can, for example, combine the proposed post-yielding function with a hysteresis model with deterioration that depends on the peak deformation, which is a common approach to describe strength deterioration. Also, the proposed model is capable of predicting the behavior of a cycle test where the deformation amplitude is not steadily increasing from cycle to cycle but varying randomly.

The rheological property in the proposed model should be noted. The proposed model is derived from rheology. Thus it can fit a highly asymmetric strength degradation of hysteresis, but may loosen physical meaning. For example, the non-hysteretic force  $f^{PY}$  is shown to be positive where displacement  $x$  is negative.

## MODEL VALIDATION

A series of experimental tests previously conducted by Kim and Lee (2017) were used for the identification and validation. Two specimens were selected for the validation. PN500 and PN500C were the benchmark specimens for the bare steel and composite connections (see Figures 3). The column had a length of 3.5 m, which was pinned at the top and bottom, and the distance between the column centerline and the actuator loading point was 3.5 m. Lateral bracing was provided at a distance of 2.5 m from the column centerline. A cyclic test was performed following the AISC loading.

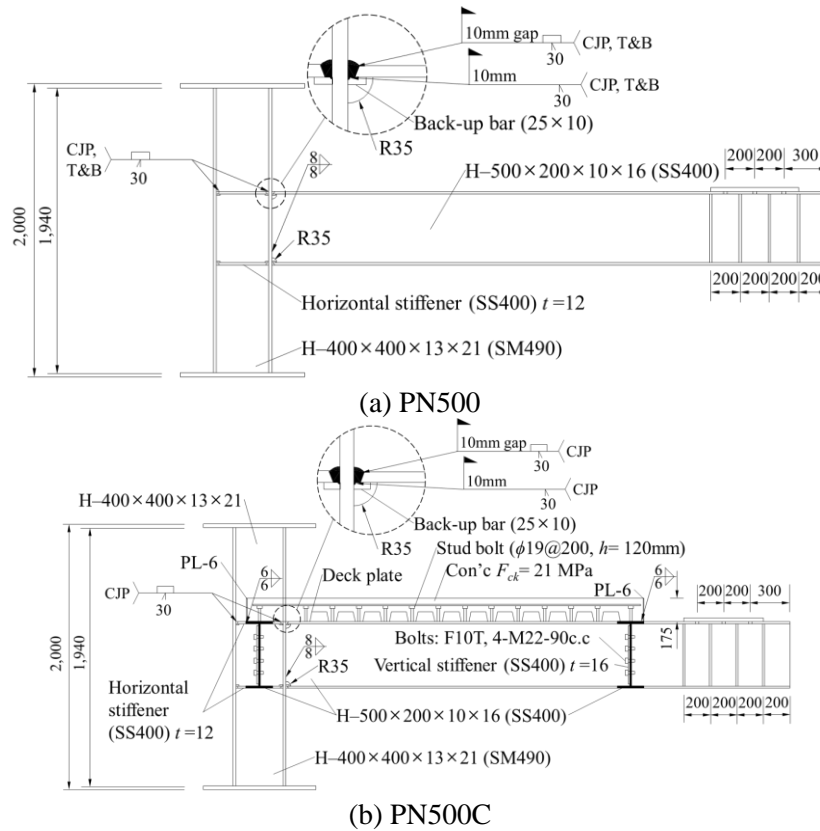


Figure 3 Side view of the test specimens (Kim and Lee, 2017)

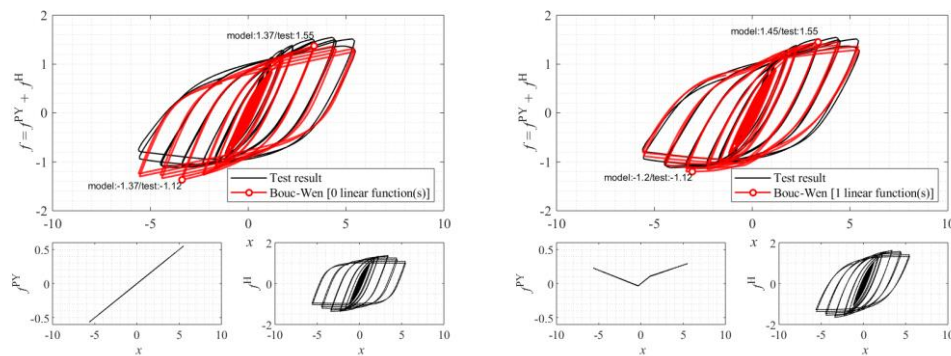


Figure 4 Hysteresis behavior of PN500C as tested and as predicted by the proposed model

Figure 4 shows a comparison of the experimental and predicted results obtained from the original BW model and proposed model. It can be seen that the original BW model could not accurately simulate asymmetric hysteresis. The model yielded the same peak value of  $1.2M_p$  in both the positive and the negative moment, whereas the maximum values in the experimental results were, respectively,  $1.55M_p$  at the positive moment and  $1.12M_p$  at the negative moment, with a difference of 38%. On the other hand, introducing a linear function enabled the model to predict the asymmetric hysteresis more accurately, proving the positive and negative maximum moments as  $1.45M_p$  and  $1.2M_p$ .

## CONCLUSIONS

In this study, a phenomenological hysteresis model is proposed, which can describe severe asymmetric hysteretic behavior. The proposed model was validated on the basis of the hysteretic behavior of welded steel moment connections with highly composite slabs in which the connections exhibited severe asymmetric hysteretic behavior. It was shown that the proposed model and its parameter identification procedure could simulate the asymmetric hysteresis well compared to the existing BW class models.

## ACKNOWLEDGMENTS

This work was supported by the National Research Foundation of Korea (NRF) grant (no. NRF-2015R1A2A1A10054506) funded by the Korean government (MSIP).

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