Energy-Based Hysteresis modeling of Reinforced Concrete Members under Cyclic Loading

Tae-Sung EOM[1], Insung KIM[2], and Hong-Gun PARK[3]

SUMMARY

The seismic performance of reinforced concrete (RC) building structures can be significantly affected by the hysteresis shape and energy dissipation of the members under earthquake excitations. In this study, a hysteresis modeling approach for RC beams, columns, walls, and coupling beams is proposed with an emphasis on accurate estimation of energy dissipation. In the proposed method, the energy dissipation ratio, defined as the ratio of the hysteresis loop area to the idealized elastic-perfectly plastic parallelogram area, is theoretically evaluated, and then the hysteresis loop is reversely determined using the energy dissipation. Empirical coefficients are also used to account for reduction in energy dissipation due to shear deformation (i.e., cyclic pinching) and bond deterioration. Practical applications of the proposed method to RC columns and coupling beams are presented and the results are discussed.

Keywords: *coupling beam, nonlinear modeling, effective stiffness, modeling parameters, coupled wall*

ENERGY-BASED HYSTERESIS MODELING OF RC MEMBERS

The hysteresis shape of materials and members can be simulated using degrading stiffness models or energy-based models. NIST GCR 17-917-45 (ATC 2017), PEER/ATC 72-1 (ATC 2010). When a fiber modeling approach is used, the stress-strain hysteresis of concrete and steel materials needs to be simulated. Similarly, when a concentrated hinge is used, the force-deformation hysteresis of the member action needs to be simulated. The hysteresis shape should appropriately describe the phenomenological behavior (i.e., energy dissipation, unloading/reloading stiffness, and pinching) varying with governing actions.

Common hysteresis shapes under force or deformation reversals shown in **Fig. 1** illustrate stiffness degradations and energy dissipation under cyclic loading. Type A represents the behavior of the components with low pinching, such as columns subjected to low axial loads adequately detailed for toughness. Type B represents the behavior of the components with moderate pinching such as a column under moderate axial load. Type C represents the hysteretic behavior of the components with significant pinching such as a poorly detailed column with high axial load or the components sustaining sliding shear or splice failures.

The different hysteresis shapes depending on reinforcement details, load conditions, or failure modes, should be simulated in accordance with those observed in experiments. Where experimental data are limited, the energy-based hysteresis models described in **Fig. 2** can be used. In the energy-based hysteresis models, different shapes of hysteresis loops depending on reinforcement details and behavior modes can be simulated with the energy dissipation ratio κ_e.

The energy dissipation ratio κ_e is defined as the ratio of the hysteresis loop area to the idealized elastic-perfectly plastic parallelogram area. **Table 1** shows the energy dissipation ratio κ_e of RC beams, columns, walls, and coupling beams. In **Table 1**, the energy dissipation ratio κ_e is calculated by multiplying the theoretical energy dissipation ratio of the main reinforcement, which is longitudinal flexural reinforcement for beams, columns and

[1] *Professor, School of Architecture, Dankook University, Korea, e-mail: tseom@dankook.ac.kr*
[2] *Senior Associate, Degenkolb Engineers, San Francisco, CA, USA, e-mail: ikim@degenkolb.com*
[3] *Professor, Department of Architecture, Seoul National University, Korea, e-mail: parkhg@snu.ac.kr*

walls or diagonal reinforcement for diagonally reinforced coupling beams, by reduction factors of λ_s and λ_r. κ_e is directly calculated from design parameters such as material strength, reinforcement detail, and applied load. The descriptions of symbols used in calculating κ_e are shown in **Fig. 3**. λ_s and λ_r are empirical factors that account for the reduction in energy dissipation, due to the pinching caused by shear deformation and inappropriate reinforcement details, respectively. To be used along with modeling parameters in existing seismic evaluation guidelines such as ASCE 41-17, λ_r is determined based on splicing, anchoring, and conforming/nonconforming transverse reinforcement details. In case the controlling action is not associated with the main reinforcement, κe is taken as 0.15 as described in the footnotes of **Table 1**.

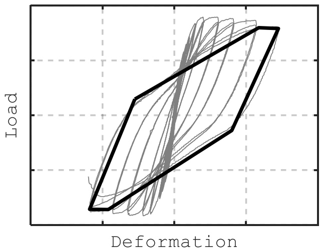
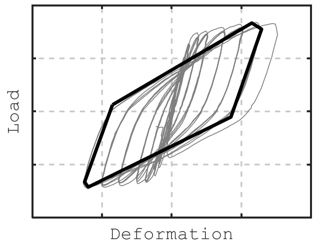
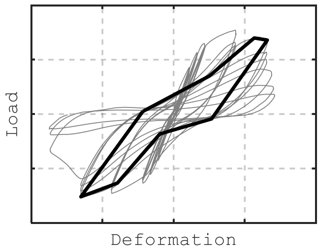
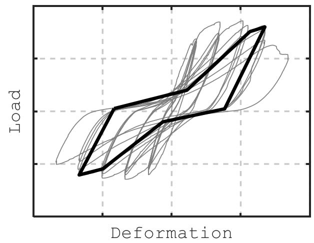
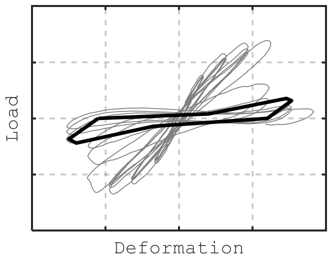
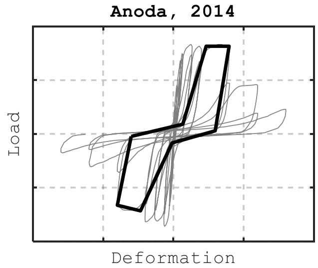
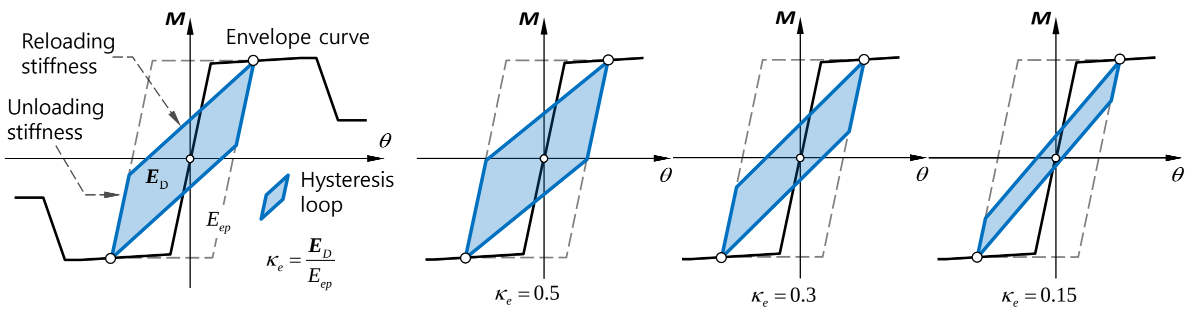
Pinching Level	Hysteresis Plots (Columns)	Hysteresis Plots (Walls)
Low: Type A	Sokoli and Ghannoum, 2016 	Dasti et al., 2017 
Moderate: Type B	Kim et al., 2011 	Christidis and Trezos, 2017 
Significant: Type C	Woods and Matamoros, 2010 	Anoda, 2014 
Experiment Numerical Model (one loop)		

Fig. 1 General hysteresis types of RC members under cyclic loading



(a) Energy dissipation ratio κ_e (b) Hysteresis shapes varying with energy dissipation ratios

Fig. 2 Hysteresis modeling approach using energy dissipation ratio

Table 1 Energy dissipation ratio for hysteresis modeling of deformation-controlled actions

Member	Energy dissipation ratio κ_e
Beam[1]	$\kappa_e = \frac{3}{2}\frac{f_{yE}A_{se2}h_e}{\left(M_{PE}+M_{NE}\right)}\lambda_s\lambda_r\ \left(\geq 0.15\right)$ $\lambda_s = \frac{l_{ss}}{5h}\ \left(\leq 1.0\right)$ λ_r = 0.5 for actions controlled by inadequate development, splice, or embedment of longitudinal reinforcement; 1.0 for all other actions
Column[2]	$\kappa_e = \begin{cases} \frac{3}{2}\frac{f_{yE}A_{se}h_e}{\left(M_{PE}+M_{NE}\right)}\lambda_s\lambda_r \quad \left(\geq 0.15\right) \text{ for rectangular section} \\ \frac{3}{2\pi}\frac{f_{yE}A_{st}d_r}{\left(M_{PE}+M_{NE}\right)}\lambda_s\lambda_r \quad \left(\geq 0.15\right) \text{ for circular section} \end{cases}$ $\lambda_s = \frac{l_{ss}}{3h}\ \left(\leq 1.0\right)$ λ_r = 1.0 for actions controlled by adequate development or splicing along the clear height; 0.5 for all other actions
Wall[3]	$\kappa_e = \frac{3}{2}\frac{f_{yE}A_{se2}h_e + 0.25\rho_{ww}f_{yE}l_{ww}^2t_w}{\left(M_{PE}+M_{NE}\right)}\lambda_s\lambda_r\ \left(\geq 0.15\right)$ $\lambda_s = \frac{l_{ss}}{3h}\ \left(\leq 1.0\right)$ λ_r = 1.0 for conforming and nonconforming reinforced concrete structural walls and associated components controlled by flexure; 0.5 for nonconforming reinforced concrete structural walls and associated components controlled by flexure
Coupling beam[4]	$\kappa_e = \begin{cases} \frac{3}{2}\frac{f_{yE}A_{se2}h_e}{\left(M_{PE}+M_{NE}\right)}\lambda_s\lambda_r \quad \left(\geq 0.15\right) \text{ for conventionally reinforced} \\ \frac{3}{2}\frac{f_{yE}A_{sd}}{V_{cbE}}\sin\alpha \quad \left(\geq 0.15\right) \text{ for diagonally reinforced} \end{cases}$ $\lambda_s = \frac{l_{ss}}{4h}\ \left(\leq 1.0\right)$ λ_r = 1.0 for conforming transverse reinforcement; 0.5 for nonconforming transverse reinforcement

[1] For beams controlled by shear, κ_e is taken as 0.15.
[2] For rectangular columns reinforced unsymmetrically, A_{se} is replaced by A_{se2}, the smaller of areas of concentrated reinforcements on both sides. Where biaxial bending is applied, κ_e is taken as the smaller of the energy dissipation ratios calculated in each orthogonal direction. For columns controlled by shear, κ_e is taken as 0.15.
[3] For T-shaped walls or walls with unsymmetrical boundary elements, A_{se} is replaced by A_{se2}, the smaller of the areas of concentrated reinforcement on either side. For reinforced concrete structural walls and associated components controlled by shear and shear-friction, κ_e is taken as 0.15.
[4] Conventionally reinforced: coupling beam with longitudinal and transverse reinforcements,
diagonally reinforced: coupling beam reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan. For conventionally reinforced coupling beams controlled by shear, κ_e is taken as 0.15.

The symbols used in Table 1 are as follows (see **Fig. 3**).

κ_e = energy dissipation ratio
λ_s = energy dissipation ratio reduction factor accounting for shear deformation
λ_r = energy dissipation ratio reduction factor accounting for reinforcement details
α_{cb} = angle between diagonal bars and longitudinal axis of coupling beam
ρ_{ww} = ratio of area of distributed longitudinal reinforcement to gross concrete area perpendicular to that reinforcement in the web of wall (other than the boundary region)
f_{yE} = expected yield strength of steel reinforcement

A_{se} = area of longitudinal tension or compression reinforcement near extreme tension or compression fiber, respectively, in rectangular column or the boundary element of wall (area of reinforcement on one side)

A_{se2} = area of longitudinal tension or compression reinforcement confined by hoops in beam or coupling beam, whichever is smaller

A_{sd} = area of diagonal reinforcement in coupling beam

A_{st} = total area of longitudinal reinforcement in circular column

b = section width

h = overall thickness or depth of member

h_e = center distance between concentrated reinforcements at both ends of beam or column, or at both boundary regions of wall

ℓ_w = length of entire wall or a segment of wall considered in the direction of shear force

ℓ_{ww} = length of wall web excluding length of boundary elements

ℓ_{ss} = shear span length, M_{UD}/V_{UD}, one-half clear span for beams or coupling beams and one-half clear height for columns unless more detailed analysis is performed

t_w = thickness of wall web

d_r = center-to-center diameter of longitudinal reinforcement circle placed along the circumference of circular column

M_{PE} = expected positive moment strength, taken as M_{Cult} in beam and coupling beam or M_{CultGE} in column and wall

M_{NE} = expected negative moment strength, taken as M_{Cult} in beam and coupling beam or M_{CultGE} in column and wall

V_{cbE} = expected shear strength of diagonally reinforced coupling beam; evaluated in accordance with ACI 318 18.10.7.4 using expected material properties

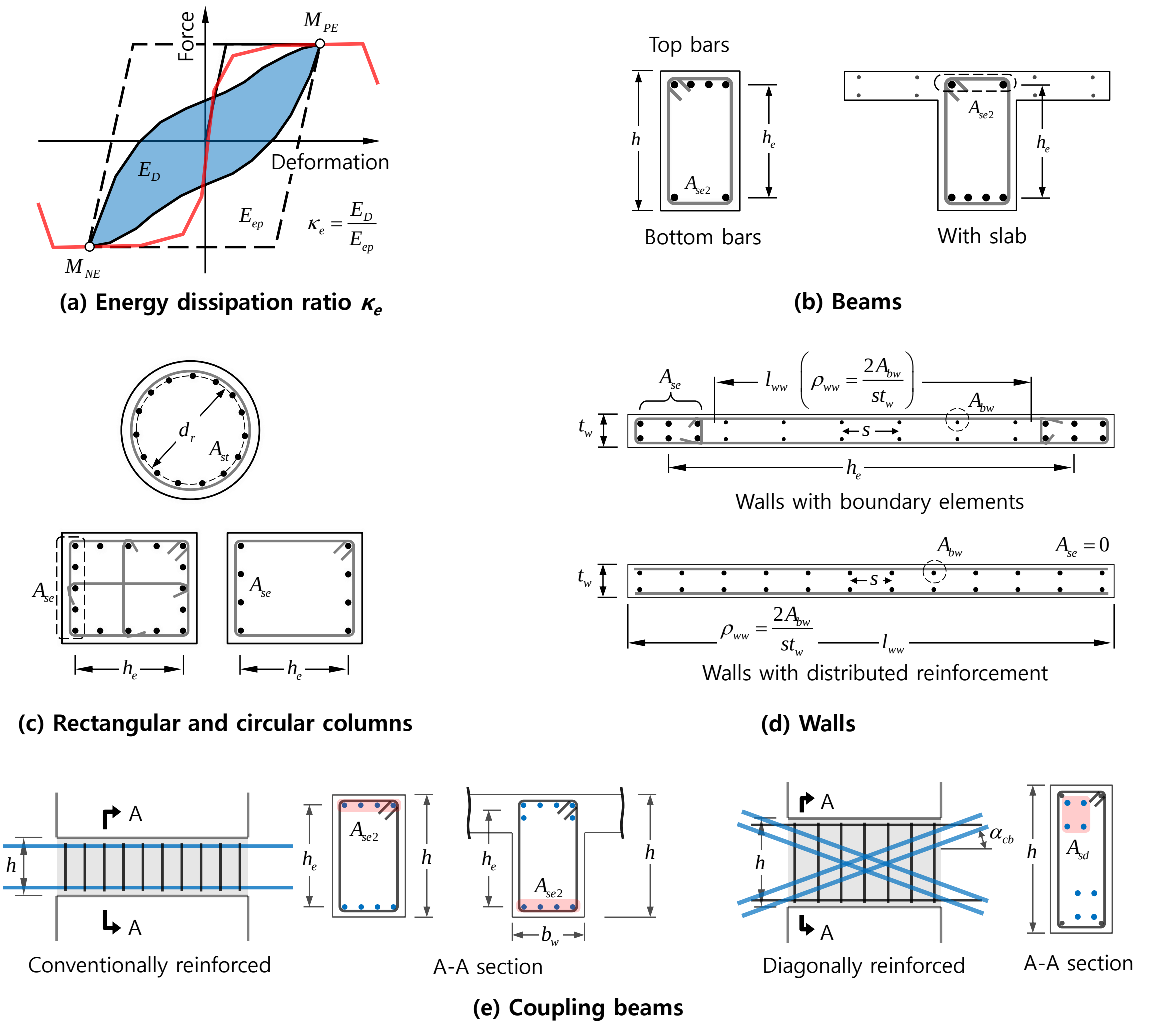


Fig. 3 Calculation of energy dissipation ratio

VERIFICATION AND PRACTICAL APPLICATION

Columns

Cantilever columns can be idealized with the elastic column element with a moment hinge at the bottom, as shown in **Fig. 4 (a)**. For the elastic column element accounting for elastic deformation, the effective flexural and shear rigidities EI_e and GA_e are assigned. For the moment hinge element depicting only plastic rotation, the load-deformation relation shown in **Fig. 4 (b)** is used: the envelope relation is plotted as the YULRX relation, and the hysteresis loop is plotted as a solid parallelogram. Here, the hysteresis loop area representing the energy dissipation per load cycle (or E_D) is determined by the energy dissipation ratio κ_e.

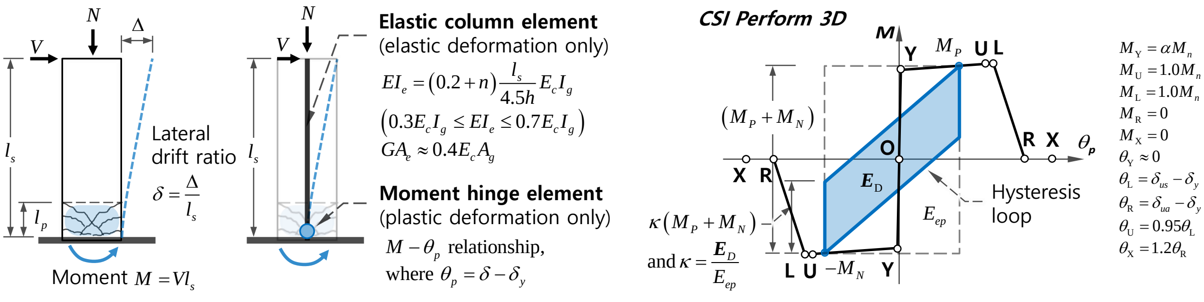


Fig. 4 Moment hinge model for cantilever columns

The proposed energy dissipation ratio κ_e and hysteresis model were applied to existing column specimens in the literature. The predicted load-deformation relations including the hysteresis loops are compared with the test results in **Fig. 5**. The modeling results of three rectangular column specimens with $l_s/h \ge 3.0$ are plotted in **Fig. 5 (a)**; four rectangular column specimens with $l_s/h < 3.0$ in **Fig. 5 (b)**; and three circular column specimens ($l_s/h \ge 3.0$) in **Fig. 5 (c)**. For all column specimens, the predicted hysteresis shapes agree with the test results. Particularly, the hysteresis loops defined by the energy dissipation ratios κ_e (= 0.15 ~ 0.596) capture the overall cyclic responses with reasonable precision.

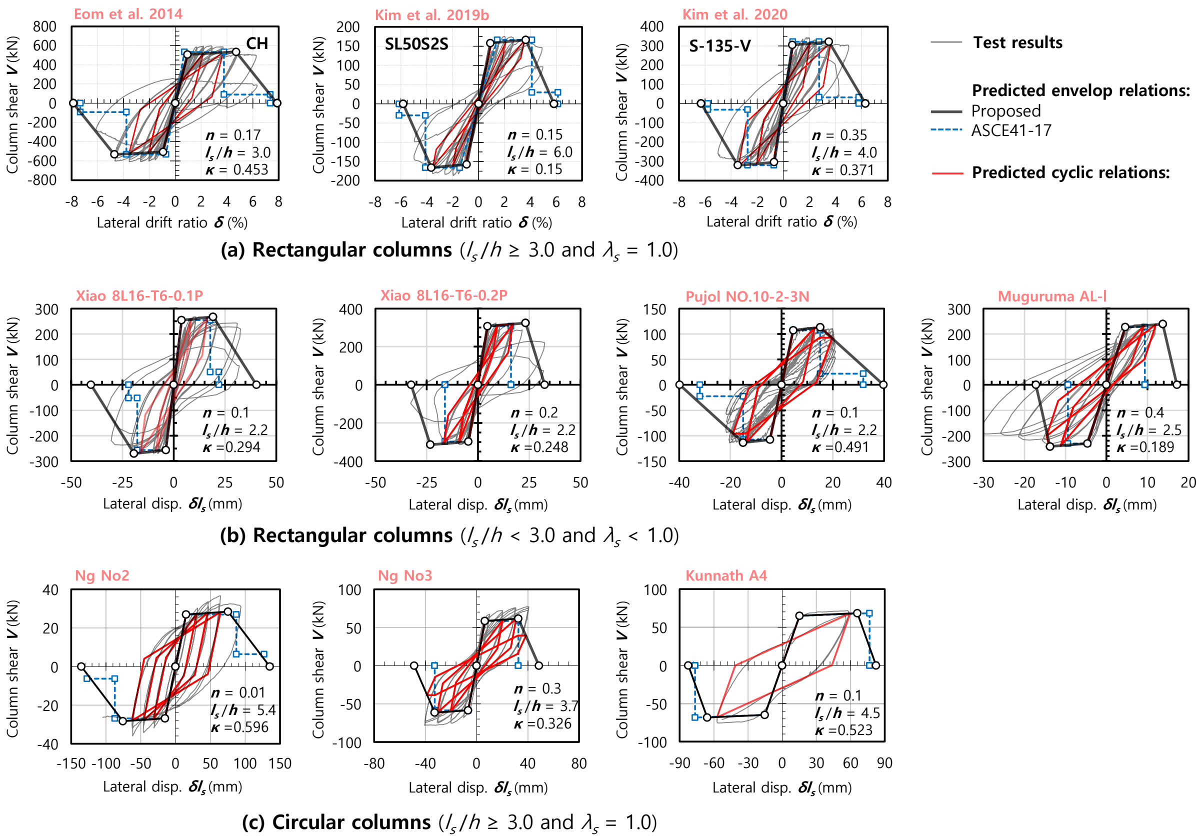


Fig. 5 Comparison of predicted cyclic relations with test results

Coupling beams

Two modeling methods for coupling beams are shown in **Fig. 6** (Naish et al. 2013). Both modeling methods use an elastic beam element to account for linear elastic behavior before flexural yielding; the effective flexural and shear rigidities of the beam element are taken as $0.3E_cI_g$ and $0.04[l/h]GA_w$, respectively. After flexural yielding, plastic deformations are considered with different hinge models; moment hinge elements at both ends shown in **Fig. 6 (a)** and single shear hinge element at the midspan shown in **Fig. 6 (b)**. For the moment hinge elements, a rigid plastic M-θ_p relation shown in **Fig. 7** is used; for the shear hinge element, a rigid plastic V-Δ_p relation compatible to the M-θ_p relation is used (that is, $V = 2M/l$ and $\Delta_p = \theta_p l$). The moment hinges at beam ends in **Fig. 6 (a)** are more natural for slender coupling beams with conventional reinforcement layouts and controlled by flexure. The shear hinge at the midspan in **Fig. 6 (b)** may be more appropriate for short coupling beams with diagonal reinforcement layouts and controlled by shear.

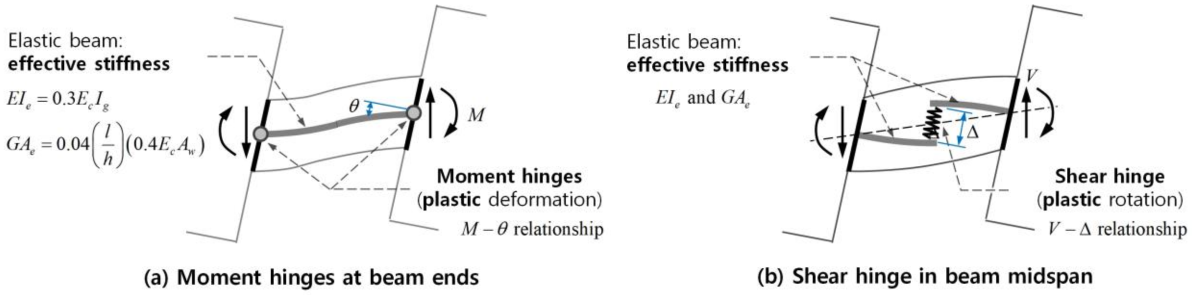


Fig. 6 Modeling methods for coupling beams

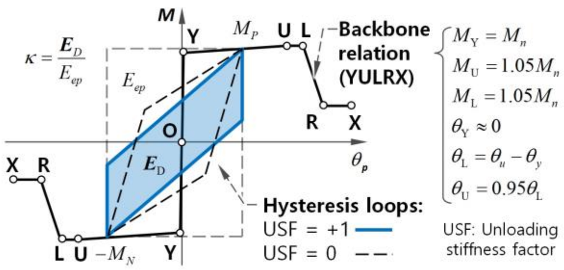


Fig. 7 Rigid plastic force-deformation relation for moment hinge model

For validation, the hysteresis shapes of 17 existing coupling beam specimens were plotted in **Figs. 8** and **9**. The calculated energy dissipation ratios κ_e based on the actual material strengths and reinforcement details reported in the literature are also described in each plot. The modeling of load-deformation relations including hysteresis loops were implemented by CSI Perform 3D (2018), as follows.

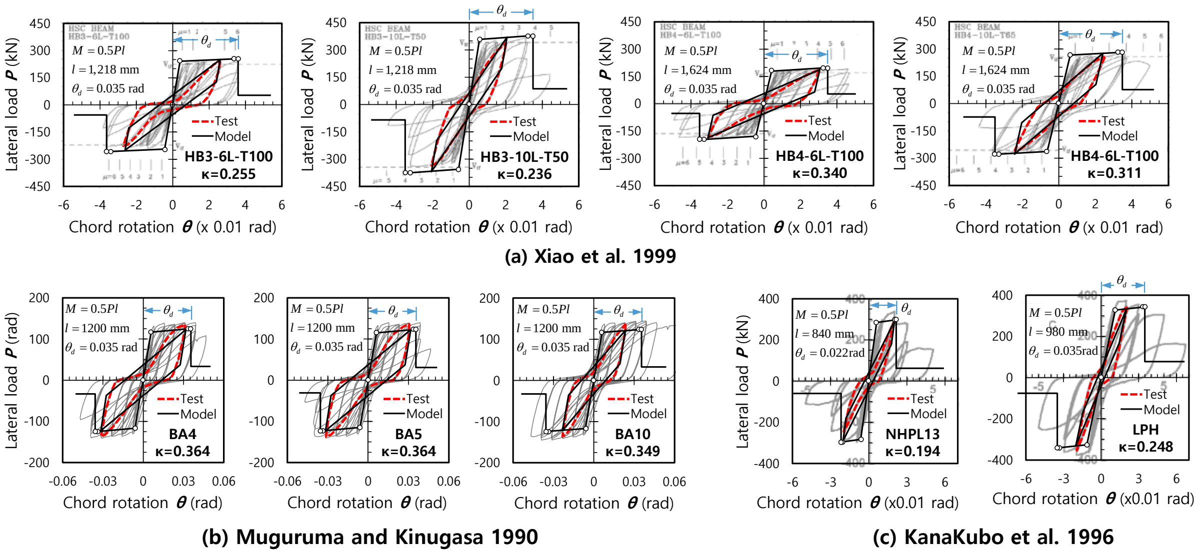


Fig. 8 Cyclic load-deformation relations of conventionally reinforced coupling beams: predictions versus tests

- For the current version of CSI Perform 3D (2018), the shear hinge model does not use the energy dissipation ratio κ_e for determining hysteresis loops. Thus, the moment hinge model shown in **Fig. 6 (a)** was used for all specimens, whether controlled by flexure or shear. The rigid-plastic behavior of the moment hinge elements is defined by five-point backbone curves (YULRX curves) and parallelogram hysteresis loops (see **Fig. 7**).
- Two types of hysteresis loops were considered using the calculated energy dissipation ratios κ_e (see the solid and dashed parallelograms in **Fig. 7**). The areas of two hysteresis loops were the same, but the unloading/reloading stiffnesses were differently assigned.

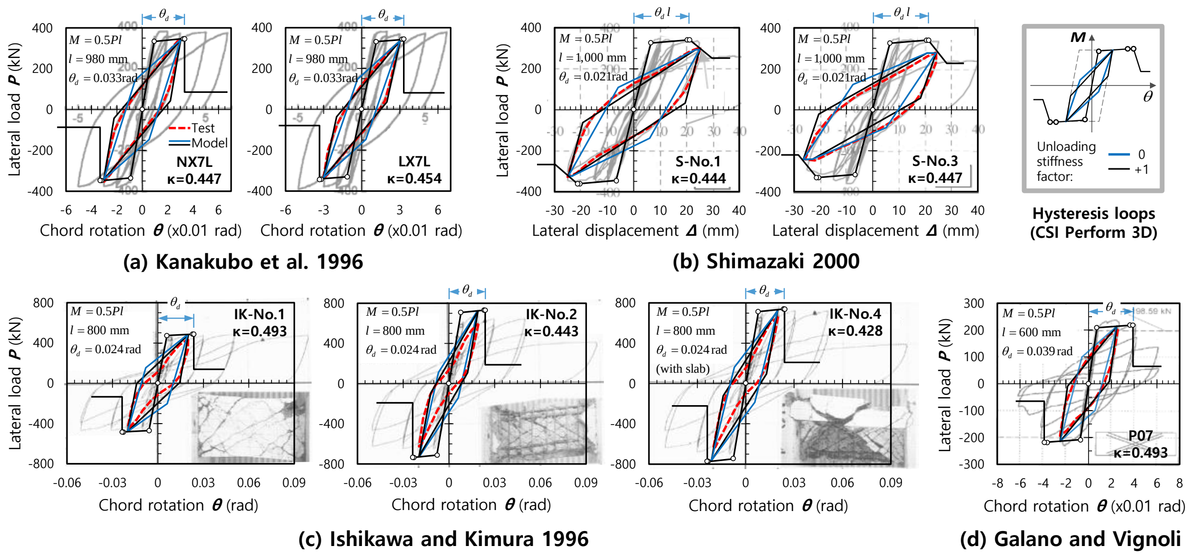


Fig. 9 Cyclic load-deformation relations of diagonally reinforced coupling beams: predictions versus tests

The hysteresis shapes of conventionally reinforced coupling beams are shown in **Fig. 8**. Overall, the hysteresis loops determined by the energy dissipation ratios κ_e are in good agreements with the tests. However, the unloading and reloading stiffnesses do not accurately capture the pinched cyclic curves in some specimens. The hysteresis shapes of diagonally reinforced coupling beams are shown in **Fig. 9**. The predicted hysteresis loops agree well with the tests. Note that the energy dissipation ratios κ_e vary with reinforcement details in each specimen. For conventionally reinforced coupling beams, the κ_e values are relatively small ranging from 0.194 to 0.364, and thus the hysteresis shapes in **Fig. 8** are relatively thin and narrow. For diagonally reinforced coupling beams, the κ_e values (= 0.428 ~ 0.493) are relatively large, and the hysteresis shapes in **Fig. 9** dissipate more energies. The proposed method well captures such hysteresis shape variations.

REFERENCES

ACI 318 (2019), *Building Code Requirements and for Structural Concrete (ACI 318-19) and Commentary*, American Concrete Institute, Farmington Hills, MI. USA.

ASCE 41 (2017). *Seismic Rehabilitation of Existing Buildings* (ASCE/SEI 41-17), American Society of Civil Engineers, Reston, VA, 416 pgs.

CSI (2018), "Perform 3D, Nonlinear Analysis and Performance Assessment for 3D Structures User Guide," Version 7. Computers and Structures, Inc.: Berkeley, CA.

Eom, T.-S., Park, H.-G., and Kang, S.-M. (2009). "Energy-Based Cyclic Force-Displacement Relationship for Reinforced Concrete Short Coupling Beams," *Engineering Structures*, 31, 2020-2031.

Eom, T.-S., and Park, H.-G. (2010a). "Evaluation of Energy Dissipation of Slender Reinforced Concrete Members and Its Applications," *Engineering Structures*, 32(9), 2884-2893.

Eom, T.-S. and Park, H.-G. (2013). "Evaluation of Shear Deformation and Energy Dissipation of Reinforced Concrete Members Subjected to Cyclic Loading," *ACI Structural Journal*, 110(5), 845-84.

Eom, T.-S., Hwang, H.-J., and Park, H.-G. (2015). "Energy-Based Hysteresis Model for Reinforced Concrete Beam-Column Connections," *ACI Structural Journal*, 112(2), 157-166.

Naish, D., Fry, A., Klemencic, R., Wallace J. W. (2013). "Reinforced concrete coupling beams—Part II: Modeling," *ACI Structural Journal*, 110(6): 1067–1075.

Park, H.-G., and Eom, T.-S. (2006), "A Simplified Method for Estimating the Amount of Energy Dissipated by Flexure-Dominated Reinforced Concrete Members for Moderate Cyclic Deformations," *Earthquake Spectra*, 22(2), 459-490.