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# Nonlinear response of bolt groups under in-plane loading

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**Abstract**

The use of bolted joints to connect structural members together and to transfer in-plane forces between them has been extensively employed in civil, mechanical and aeronautic structures. Most of the available methods have been developed to estimate the loading capacity but not the deformation of bolt groups. In this paper, an iterative procedure is developed to calculate the non-linear deformation of bolt groups under in-plane eccentric loads based on the assumptions of elasto-plastic behaviour of bolts and rigid body movement of the bolt group. The force exerted on each bolt is proposed to be dependent on the instantaneous centre of rotation of the bolt group. A computer program based on the above theory has been implemented to simulate the full range non-linear response of bolt groups subjected to in-plane loads, in particular, the load–slip behaviour at the inelastic range. A numerical example is given, with comparison to the non-linear finite element analysis, to illustrate the effectiveness and accuracy of the method. The behaviour of a bolt group as well as individual bolts subjected to in-plane loads was studied in detail. It was found that the ultimate capacity of the bolt group could be about 30% higher than its elastic limit. A ductility factor of 3 to 4 might be required for the bolts in a bolt group to reach their ultimate loading capacity.


*Keywords:* Non-linear; Bolt group; In-plane loading; Elastic; Plastic; Slip; Centre of rotation

## 1. Introduction

The use of bolted joints to connect structural members together and to transfer in-plane forces between them has been extensively employed in civil, mechanical and aeronautic structures [1–3]. Many studies have been carried out to estimate the ultimate capacity of bolted joints in resisting eccentric in-plane loads for steel structures [4–12]. The classic approach [4] assumed all the bolts are fitted perfectly and deform only in an elastic manner. By such assumptions, the loads carried by the bolts in a bolt group at the elastic limit could be easily obtained. Alternatively, all the bolts could be assumed to be fully plastic and the ultimate load-carrying capacity of bolt groups was then evaluated [5]. Crawford and Kulak [8] proposed the ultimate strength method, which assumed that all bolts rotate about a certain instantaneous centre of rotation and the non-linear behaviour of individual bolts was taken into account. By considering equilibrium of the bolt group, the instantaneous centre of rotation and the ultimate strength of the connection could be found by an iterative approach. Surtees et al. [9] combined elastic and plastic analyses and proposed an elasto-plastic analysis, which postulated the behaviour of any bolt as being either elastic or plastic, and was governed by the distance of the bolts from the instantaneous centre of rotation of the connection. An alternative approach assuming the in-plane load was distributed in proportion to the available bearing area instead of the shear area was suggested for aluminium structures [13]. The aforementioned approaches are useful for estimating the strength but not the deformation of bolt groups.

While the major application of the bolt group analysis is on steelwork connections, the analysis can also be applied to anchor bolt groups subjected to in-plane eccentric loadings like reinforced concrete beams strengthened with steel plates at their sides [3,14–16]. It was pointed out [14–16] that not only the load-carrying capacity but also the load–slip relationship of bolts could significantly affect the load-carrying capacity of the beams strengthened with externally bolted steel plates. The importance in controlling the slip of anchor bolts was confirmed by an experimental study [3] of retrofitted concrete coupling beams by bolted side plate. Although load–slip relationships of individual bolts and bolt groups are so important in

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retrofitting, no theory has yet been proposed to calculate the non-linear deformation of bolted connections under external loads. Complicated non-linear finite element analysis appears to be the only means to simulate the complete load deformation relationship of bolt groups.

The objective of the present study is to develop an original iterative procedure to compute the non-linear load–deformation relationship of bolt groups, which consist of un-yielded and yielded bolts, under any in-plane external load. Apart from the normal assumptions used in elastic analysis of bolt groups, it is further postulated that the direction of force of the yielded bolts depends on the centre of rotation of the bolt group. By varying the centre of rotation, equilibrium between yielded bolt forces and applied loads could be sought in each loading step. For simplicity, the load–slip behaviour of individual bolts is idealized as a bi-linear relationship, which was commonly adopted in other similar studies [3,9,17]. An original computer program has been implemented to illustrate and validate the procedure.

Compared with the other available techniques, the new procedure offers several advantages. Firstly, the deformation of bolt groups in any geometry subjected to any in-plane load can be predicted. Secondly, the gradual shift of the instantaneous centre of rotation of bolt groups can be taken into account, improving its accuracy in the ultimate strength estimation. Finally, the limited slip capacity of bolts, which being difficult to be defined in finite element analyses, is considered in the calculation of the load–deformation response of bolt groups.

## 2. Non-linear theory on bolt groups

Detailed theory relating the applied loading and the corresponding deformation of a bolt group consisting of yielded and un-yielded bolts is established in this section. The theory can be applied to trace the complete load–deformation of bolt groups under eccentric in-plane loading.

### 2.1. Basic assumptions

The basic assumptions adopted in the development of the theory include:

(1) The bolts are perfectly fitted and connected by a plate, which acts as if rigid [4,13].
(2) The bolts deform in an elasto-plastic manner.
(3) The bolts are widely separated such that the interference between bolts can be neglected.

The first assumption idealizes the bolts as being perfectly fitted in the connection media. In practice, a clearance hole is often used to facilitate the installation of bolts, making the bolts not perfectly fitted and affecting the load deformation responses of bolts [6,7]. However, the influence is usually small and diminishes as more bolts are in close contact with the connection media through slips. So it is acceptable to ignore the above effect and the bolts may be assumed as perfectly fitted. However, when the bolt slips have to be limited, high strength friction grip bolts and dynamic set anchors (a proprietary

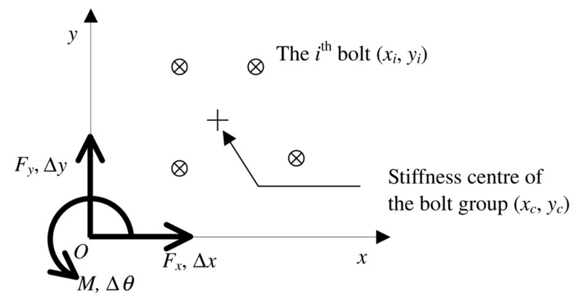


Fig. 1. General arrangement of bolt group.

technology of Hilti Company) may be used for steel and concrete structures, respectively, to control the undesirable slips. Moreover, based on the first assumption, only rigid body motion is admissible for the bolt group. As a result, the relative positions of all the bolts remain unchanged after deformation. It implies that bolts rotate about a point which is known as the instantaneous centre of rotation.

The second assumption is an extension of the elastic assumption in classic bolt group theory. In elastic analysis, it is assumed that bolts behave elastically until reaching the yield stress. This assumption is extended to the post-yield stage that the magnitude of the bolt force remains the same i.e. plastic stage. In reality, a clear yield point does not exist and the stiffness of the bolt drops gradually until failure [6,8]. However, the elasto-plastic assumption of bolts has been widely accepted for different engineering applications, including bolt group analysis [7,9] because of its simplicity and readiness to be analyzed in different circumstances. While the magnitude of bolt force is assumed to remain unchanged in the post-yield stage, it is further postulated that the direction of the yielded bolt force varies according to the incremental displacement. As the bolt group is assumed to displace in a rigid body manner, the bolt force of any yielded bolt would be perpendicular to the line joining the instantaneous centre of rotation ($x_o$, $y_o$) and the location of that bolt as shown in Fig. 3.

Furthermore, according to the first and third assumptions, the force exerted on each bolt can be readily obtained from the deformation of the bolt. After calculation of the polar moment of inertia and the stiffness of the bolt group, the relationship between the rigid body movement of the bolt group and the in-plane eccentric loads can be established.

### 2.2. Classic elastic theory on bolt groups

For completeness, the classic elastic theory [4] for analyzing bolt groups subjected to in-plane loads is reviewed. Based on that, the newly developed non-linear bolt group theory will be presented in the next section.

Fig. 1 shows a general arrangement of a group of bolts subjected to general external loads ($F_x$, $F_y$, $M$). The lateral stiffness of the $i$th bolt in the $x$ and $y$ directions are $k_{xi}$ and $k_{yi}$ respectively. The lateral movements of the bolt group at the origin of the coordinate system are denoted by ($\Delta x$, $\Delta y$, $\Delta\theta$). Based on the first assumption, the lateral movements of the $i$th

$$\left(-R_x^2 K_y - R_y^2 K_x + I_{zz} K_x K_y\right) \begin{Bmatrix} P_{xi} \\ P_{yi} \end{Bmatrix} = \begin{bmatrix} k_{xi}\left(I_{zz}K_y - R_y^2 - y_i R_x K_y\right) & k_{xi}\left(y_i R_y K_x - R_x R_y\right) & k_{xi}\left(R_x K_y - y_i K_x K_y\right) \\ k_{yi}\left(-R_x R_y + x_i R_x K_y\right) & k_{yi}\left(I_{zz}K_x - R_x^2 - x_i R_y K_x\right) & k_{yi}\left(-R_y K_x + x_i K_x K_y\right) \end{bmatrix} \begin{Bmatrix} F_x \\ F_y \\ M \end{Bmatrix}$$

Box I.

bolt in the $x$ and $y$ directions can be evaluated as,

$$\Delta u_i = \Delta x - y_i \Delta\theta \quad \text{and} \quad \Delta v_i = \Delta y + x_i \Delta\theta. \tag{1}$$

By multiplying the deformations of the bolts with its stiffness, the lateral forces ($P_{xi}$, $P_{yi}$) acting on the $i$th bolt in each direction are equal to,

$$\begin{Bmatrix} P_{xi} \\ P_{yi} \end{Bmatrix} = \begin{bmatrix} k_{xi} & 0 & -k_{xi} y_i \\ 0 & k_{yi} & k_{yi} x_i \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{Bmatrix}. \tag{2}$$

Considering the global force equilibrium of the applied loads and the bolt forces, one can have,

$$F_x = \sum_i P_{xi}, \quad F_y = \sum_i P_{yi} \quad \text{and} \quad M = \sum_i \left(P_{yi} x_i - P_{xi} y_i\right). \tag{3}$$

Substituting Eq. (2) into Eq. (3) and simplifying, the applied forces can be related to the bolt group displacement as

$$\begin{Bmatrix} F_x \\ F_y \\ M \end{Bmatrix} = \begin{bmatrix} K_x & 0 & -R_x \\ 0 & K_y & R_y \\ -R_x & R_y & I_{zz} \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{Bmatrix} \tag{4a}$$

where

$$K_x = \sum_i k_{xi}, \quad K_y = \sum_i k_{yi}, \quad I_{zz} = \sum_i \left(k_{xi} y_i^2 + k_{yi} x_i^2\right), \quad R_x = \sum_i k_{xi} y_i \quad \text{and} \quad R_y = \sum_i k_{yi} x_i. \tag{4b}$$

Solving Eq. (4a), the rigid body displacements of the bolt group can be determined as

$$\begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{Bmatrix} = \frac{1}{-R_x^2 K_y - R_y^2 K_x + I_{zz} K_x K_y} \times \begin{bmatrix} I_{zz}K_y - R_y^2 & -R_x R_y & R_x K_y \\ -R_x R_y & I_{zz}K_x - R_x^2 & -R_y K_x \\ R_x K_y & -R_y K_x & K_x K_y \end{bmatrix} \begin{Bmatrix} F_x \\ F_y \\ M \end{Bmatrix}. \tag{5}$$

The centre of rotation of the bolt group (as shown in Fig. 2) can be determined by the following equations,

$$x_o = -\frac{\Delta y}{\Delta\theta} \quad \text{and} \quad y_o = \frac{\Delta x}{\Delta\theta}. \tag{6}$$

It can be seen that the centre of rotation depends on the applied loads as well as the bolt group arrangement. By substituting Eq. (5) into Eq. (2), the forces at the $i$th bolt can be calculated from the applied forces. (See Box I.)

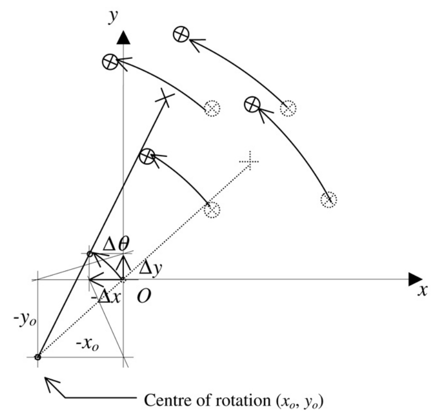


Fig. 2. Rigid body movement of bolt group.

### 2.3. Non-linear theory on bolt groups

In this section, a non-linear theory for calculating the distributions of loads and displacements in a bolt group with yielded and un-yielded bolts is presented. It is also employed to find the load–slip relationship of general bolt groups under in-plane loads. Since a non-linear solution of the centre of rotation is to be sought, incremental loading and an iterative procedure have been used in the solution. For simplicity, the proportional applied load is considered in the analysis.

Assuming all the incremental applied loads ($\Delta F_x$, $\Delta F_y$, $\Delta M$) are equal and considering the $j-1$th loading step, the total external load ($F_x$, $F_y$, $M$) applied to the bolt group is simply equal to the sum of all the incremental loads from the 1st to the $j-1$th loading steps. Further assuming some of the bolts have yielded and by considering the global force equilibrium of the bolt group, the incremental load sharing between the yielded and un-yielded bolts at the $j$th loading step is expressed as,

$$\begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta M \end{Bmatrix} = \begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta M \end{Bmatrix}_{\text{elastic}}^j + \begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta M \end{Bmatrix}_{\text{inelastic}}^j. \tag{7}$$

As mentioned in the classic bolt group theory, the force vector of the elastic bolts can be found according to Eq. (4a),

$$\begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta M \end{Bmatrix}_{\text{elastic}}^j = \begin{bmatrix} K_x & 0 & -R_x \\ 0 & K_y & R_y \\ -R_x & R_y & I_{zz} \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{Bmatrix}^j. \tag{8}$$

Here only the un-yielded bolts are considered in formulating the above equation.

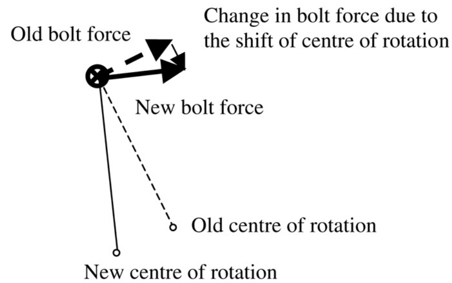


Fig. 3. Post-yielded response of bolt.

The force components produced by the $i$th yielded bolt about the origin of the coordinate system are related to the instantaneous centre of rotation $(x_o^j, y_o^j)$ and the incremental rotation $\Delta\theta^j$, and is expressed as,

$$P_{xi}^j = P_{yfi}\left[\frac{(y_i^j - y_o^j)}{\sqrt{\left(x_i^j - x_o^j\right)^2 + \left(y_i^j - y_o^j\right)^2}}\right] \times \operatorname{sgn}(\Delta\theta^j) \quad (9a)$$

$$P_{yi}^j = P_{yfi}\left[\frac{(x_o^j - x_i^j)}{\sqrt{\left(x_i^j - x_o^j\right)^2 + \left(y_i^j - y_o^j\right)^2}}\right] \times \operatorname{sgn}(\Delta\theta^j) \quad (9b)$$

$$P_{\theta i}^j = x_i^j P_{yi}^j - y_i^j P_{xi}^j \quad (9c)$$

where $P_{yfi}$ is the yielded force of the $i$th bolt. Hence the change in bolt forces of the yielded bolt from the $j-1$th to the $j$th load step (as illustrated in Fig. 3) and the sum of the bolt forces contributing to balance the incremental external load $(\Delta F_x, \Delta F_y, \Delta M)$ are,

$$\Delta P_{xi}^j = P_{xi}^j - P_{xi}^{j-1}, \quad \Delta F_{x\text{inelastic}}^j = \sum_i \Delta P_{xi}^j \quad (10a)$$

$$\Delta P_{yi}^j = P_{yi}^j - P_{yi}^{j-1}, \quad \Delta F_{y\text{inelastic}}^j = \sum_i \Delta P_{xi}^j \quad (10b)$$

$$\Delta P_{\theta i}^j = P_{\theta i}^j - P_{\theta i}^{j-1}, \quad \Delta M_{\text{inelastic}}^j = \sum_i \Delta P_{\theta i}^j. \quad (10c)$$

For the un-yielded bolts, making use of Eqs. (6) and (10),

$$\begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta M \end{Bmatrix}_{\text{elastic}}^j = \begin{bmatrix} K_x & 0 & -R_x \\ 0 & K_y & R_y \\ -R_x & R_y & I_{zz} \end{bmatrix} \begin{Bmatrix} y_o^j \\ -x_o^j \\ 1 \end{Bmatrix} \times \Delta\theta^j. \quad (11)$$

Eqs. (9)–(11) show that the bolt forces of both yielded and un-yielded bolts are functions of the instantaneous centre of rotation $(x_o^j, y_o^j)$ and the incremental rotation $\Delta\theta^j$ of the bolt group. Hence, by determining the instantaneous centre of rotation and the incremental rotation at each loading step, the non-linear response of the bolt group could be solved readily.

## 3. Computer implementation of nonlinear bolt group theory

To study the inelastic behaviour of bolt groups, a computer program, namely BOGA, written in FORTRAN was developed, based on the theory described in the previous section. In this program, incremental proportional loads are applied to the bolt group and equilibrium is maintained in each loading step by iteration. The bolt group responses in terms of bolt forces and displacements are calculated by summing up all the incremental responses from each loading step. Detailed description of the program is given below.

### 3.1. Description of the methodology

The algorithm of the computer program for the determination of the instantaneous centre of rotation and the incremental rotation at each loading step has been presented in Fig. 4 and the details are described herein.

Step 1: The required parameters of the bolt problem are defined. These include,

(1) The geometry of the bolt group (i.e. the number and the locations of bolts).
(2) The load–slip relationship of bolts (i.e. the bolt stiffness, the yield and ultimate displacements of the bolts).
(3) The incremental load applied to the bolt group and the target load steps.
(4) A preset tolerance to stop the iteration process at each loading step.

Step 2: To start with, the incremental load $(\Delta F_x \Delta F_y, \Delta M)$ is applied to the bolt group.

Step 3: Compare the calculated displacement with the yield displacement of the bolts to determine the yielded bolts. If none of the bolts are yielded, the bolt group displacements $(\Delta x^j, \Delta y^j, \Delta\theta^j)$ in the $j$th loading step are calculated. Then the instantaneous centre of rotation $(x_o^j, y_o^j)$ is found; the incremental displacements $(\Delta u_i^j, \Delta v_i^j)$, and the incremental bolt forces $(\Delta P_{xi}^j, \Delta P_{yi}^j)$ can be evaluated accordingly. The process can then go back to Step 2. However, if any of the bolts are yielded during the $j$th loading step, the process has to go on to Step 4.

Step 4: Iteration has to start to determine the instantaneous centre of rotation and the incremental rotation so that force equilibrium can be satisfied in this loading step. A trial instantaneous centre of rotation, denoted as $(x_{ot}^j, y_{ot}^j)$, at the $j$th loading step is determined based on the previous loading or iteration step. The subscript '$t$' stands for a trial or temporary value for which force equilibrium has not been satisfied completely.

Step 5: With the trial centre of rotation, together with the sign of incremental rotation $\operatorname{sgn}(\Delta\theta_t^j)$ (based on the previous loading or iteration step), the force vector of any yielded bolt is found by Eq. (11). Then $\Delta M_{\text{inelastic}}$ is computed by summing up all the moments contributed from individual yielded bolts. The incremental rotation $\Delta\theta_t^j$ is solved by considering moment equilibrium according to Eqs. (9) and (13). The translational

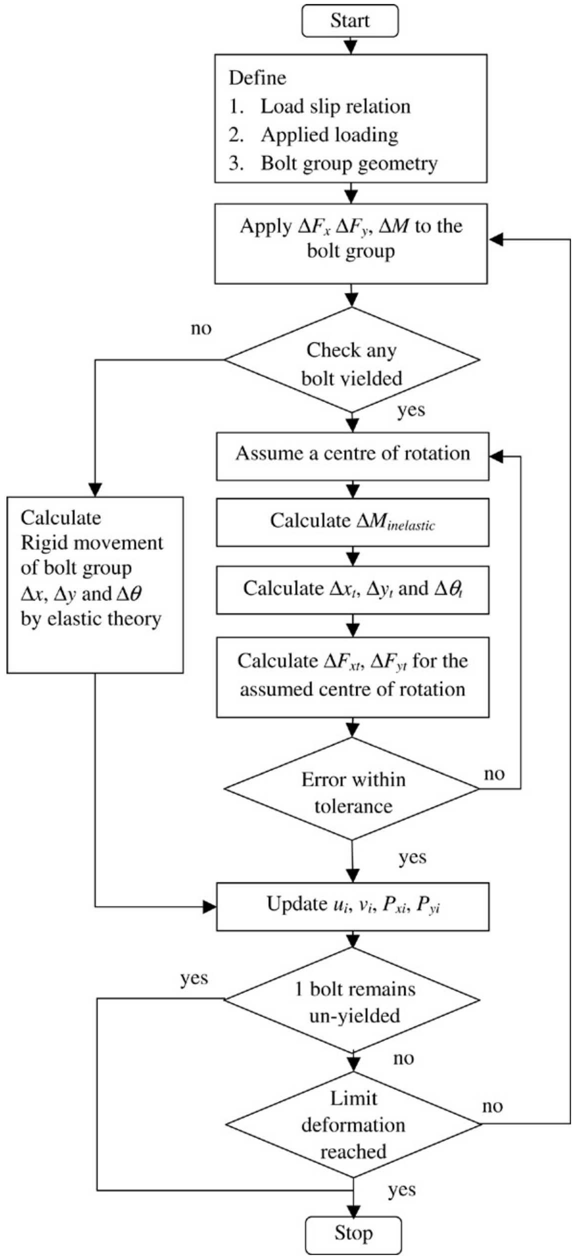


Fig. 4. Schematic diagram of the iteration process.

movement ($\Delta x^j$, $\Delta y^j$) of the bolt group is then determined by Eq. (6). Hence the incremental displacements ($\Delta u_{it}^j$, $\Delta v_{it}^j$) of each bolt can be calculated by Eq. (1).

Step 6: The trial values of $\Delta P_{xit}^j$, $\Delta P_{yit}^j$, $\Delta F_{xt}^j$ and $\Delta F_{yt}^j$, where equilibrium has not been fully satisfied, are calculated.

Step 7: $\Delta F_{xt}^j$ and $\Delta F_{yt}^j$ is checked against the values of $\Delta F_x$ and $\Delta F_y$. If the error is less than the preset tolerance, the trial instantaneous centre of rotation represents the true value. The iteration can be stopped and the process can go to Step 8. Otherwise, a new trial instantaneous centre of rotation is determined by the steepest descent method [18], which will be described in detail in Section 3.2. Then the iteration goes back to Step 4.

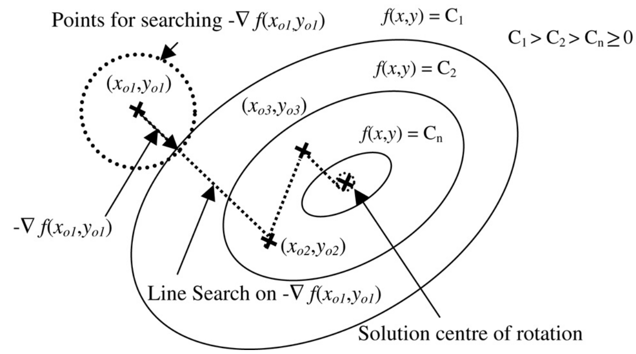


Fig. 5. Iterative scheme using steepest descent method.

Step 8: The incremental displacements are added to the cumulative displacements ($u_i^{j-1}$, $v_i^{j-1}$) and the locations of bolts are updated accordingly. Then the yield conditions of bolts are updated by considering the cumulative displacements of individual bolts.

Step 9: The total number of bolts remaining un-yielded is checked by comparing the cumulative slip with the yield slip of the bolts. And the slip is also checked against the ultimate slip of the corresponding bolt. Once only one bolt remains elastic, or any of the bolts reaches its ultimate slip, the bolt group is identified as reaching its ultimate capacity and the solution procedure is terminated. Otherwise, the program proceeds to Step 2 of the $j+1$th loading step and the next incremental load is applied to the bolt group.

### *3.2. Steepest descent method for determination of the instantaneous centre of rotation*

To evaluate the instantaneous centre of rotation at each loading step, an iterative procedure using the steepest descent method [18] is developed as illustrated in Fig. 5. As mentioned previously, the solution has to satisfy global force equilibrium in the $x$- and $y$-directions, and hence the goal of the scheme is to find a centre of rotation satisfying the above equilibrium conditions. To drive the iteration loop, the centre of rotation of the previous loading step ($x_o^{j-1}$, $y_o^{j-1}$) is set as the first trial ($x_{ok}$, $y_{ok}$) of which $k=1$.

In general, the steepest descent method is represented by the following equation,

$$\begin{pmatrix} x_{o(k+1)} \\ y_{o(k+1)} \end{pmatrix} = \begin{pmatrix} x_{ok} \\ y_{ok} \end{pmatrix} - \lambda_k \begin{pmatrix} \partial f\,(x_{ok}, y_{ok})/\partial x \\ \partial f\,(x_{ok}, y_{ok})/\partial y \end{pmatrix}. \tag{12}$$

In the above equation, $x_{o(k+1)}$ and $y_{o(k+1)}$ refers to the instantaneous centre of rotation to be determined in the $k$th iteration step. The error terms of bolt group forces in the $x$- and $y$-directions are defined as,

$$\begin{aligned} & h(x,y) = \Delta F_{xt}\,(x,y) - \Delta F_x \\ \text{and} \quad & g(x,y) = \Delta F_{yt}\,(x,y) - \Delta F_y. \end{aligned} \tag{13}$$

The function defining the solution criteria is represented as $f(x,y)$,

$$f(x,y) = h(x,y)^2 + g(x,y)^2. \tag{14}$$

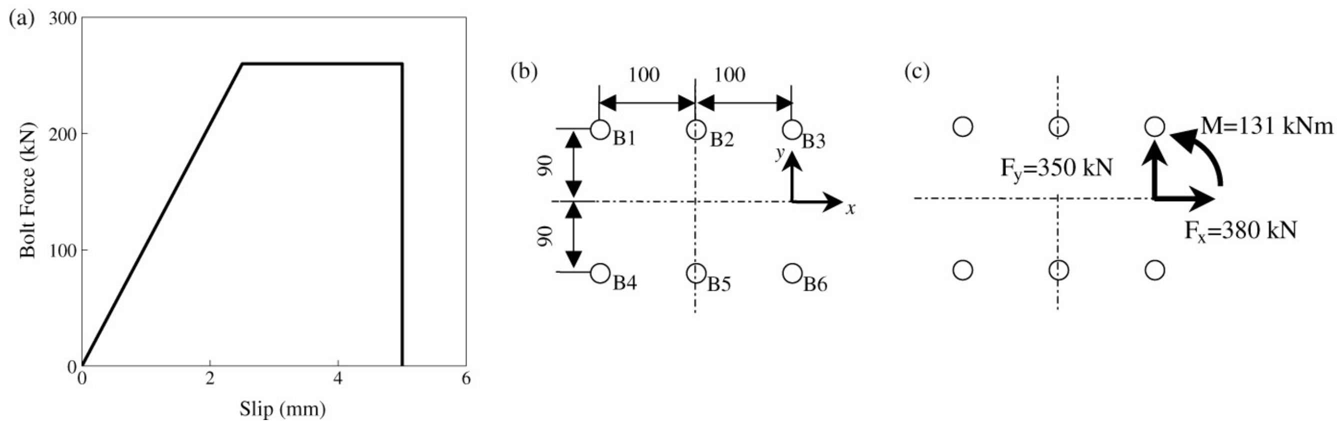


Fig. 6. Numerical example (a) load–slip relationship of bolt, (b) geometry of bolt group and (c) applied load.

The above function combines the errors in bolt group forces in both $x$- and $y$-directions. By minimizing $f(x, y)$, both functions $h(x, y)$ and $g(x, y)$ converge to zero. The direction $-\nabla f(x_{ok}, y_{ok})$ represents the greatest decrement direction of the function $f$ and is found by substituting the points at a fixed distance which is preset by the program around the trial centre of rotation $(x_{ok}, y_{ok})$. The value $\lambda_k$ is defined as the distance between the trial point $(x_{ok}, y_{ok})$ and the point $(x, y)$ on the direction $-\nabla f(x_{ok}, y_{ok})$ with a minimum value of $f$. It is determined by performing a line search along the direction $-\nabla f(x_{ok}, y_{ok})$. Mathematically, it can be represented by a simple formula,

$$\lambda_k = \text{argmin}_{s>0}\left(f\left(x_{ok} - s\frac{\partial f\,(x_{ok}, y_{ok})}{\partial x},\right.\right.$$
$$\left.\left. y_{ok} - s\frac{\partial f_y\,(x_{ok}, y_{ok})}{\partial y}\right)\right). \quad (15)$$

The iterative procedure is completed when the errors of the forces in the $x$- and $y$-directions of the bolt group are within the preset tolerance. That is,

$$h(x_{ok}, y_{ok}) \le \varepsilon, \qquad g(x_{ok}, y_{ok}) \le \varepsilon. \quad (16)$$

Once the iterative procedure has been completed, the solution point is returned to the main program where the subsequent rigid body movements and bolt slips are then computed as previously described.

## 4. Numerical study of nonlinear response of a bolt group

An example is given to simulate a bolt group subjected to in-plane loads. The results calculated by BOGA are compared to those from the non-linear finite element analysis to validate the developed program. The bolt group considered in the numerical example consists of 6 numbers of identical M20 bolts with the idealized load–slip relationship as shown in Fig. 6(a). The physical properties of the bolts are as follows: the stiffness is 104 kN/mm, the capacity is 260 kN, the yield displacement is 2.5 mm and the ultimate displacement is 5 mm. The geometric arrangement of the bolts is depicted in Fig. 6(b). The origin of the coordinates, where the load is being applied, is located at the middle between Bolt Nos. 3 and 6 as shown in Fig. 6(b). The total load applied to the bolt group includes lateral forces ($F_x$ = 380 kN, $F_y$ = 350 kN) and a twisting moment ($M$ = 131 kN m), as shown in Fig. 6(c).

### 4.1. *Non-linear finite element analysis*

The non-linear finite element package ATENA [19] is employed to analyze the problem and the results are used to validate BOGA. In the finite element analysis, the mesh configuration of a single bolt is shown in Fig. 7. The load–slip behaviour of the bolt is simulated by a ring of bilinear material which would deform when subjected to in-plane loads, and the load–deformation relationship of the material was adjusted to match the load–slip behaviour of the bolt. The inner ring of the bilinear material represents the shank of the bolt, and in the analysis, the inner ring is set as fixed to simulate the bolt anchored in a rigid medium, while the outer ring of the bilinear material is joined to the steel plate in the analysis. The finite element mesh of the bolt group is shown in Fig. 8. The bolts are inter-connected through the steel plate to build up the group action. The external in-plane loads are applied onto the steel plate and distributed to the bolts. The deformation of the bolt group is taken to be the displacement of the point on the steel plate where the centroid of the bolt group is located. The calculated load–displacement relationship in the $x$- and $y$-directions and the moment–rotation relationship obtained from the analysis are plotted in Fig. 9.

### 4.2. *Simulation by BOGA*

The program BOGA, as mentioned in Section 3, is also adopted to simulate the non-linear response of the same bolt group. The total applied load is divided into 500 equal incremental loads. The load–deformation relationship obtained by BOGA is super-imposed on Fig. 9 to compare with the results simulated by non-linear finite element analysis. It is observed that the results from the non-linear finite element analysis terminate at a load level which is slightly lower than the predicted ultimate capacity by BOGA. However, it can be seen in Fig. 9 that the results from the two methods closely

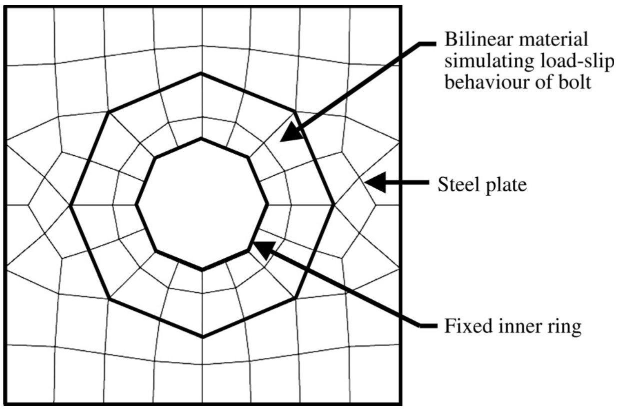


Fig. 7. Mesh detail for a single bolt.

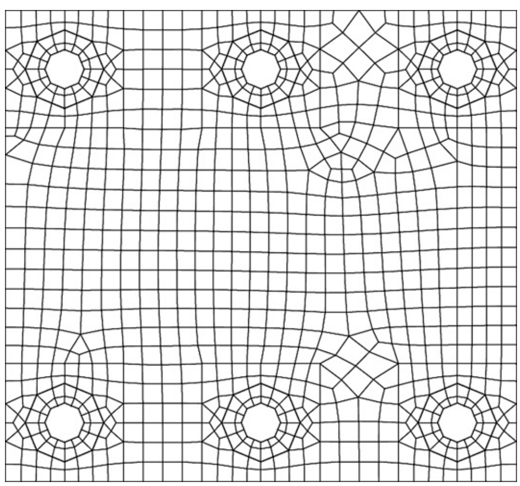

Fig. 8. Mesh for non-linear finite element analysis.

agree with each other, with a maximum error less than 5%. Both analyses show an initial elastic stage, followed by a gradual decrease in stiffness until reaching the load. From the consistency of the results obtained from the two analyses, it can be concluded that the proposed numerical technique is applicable to model the load–deformation relationship of bolt groups in a simple and accurate way.

### 4.3. Result interpretations

In this section, the behaviour of the bolt group subjected to eccentric in-plane loading based on the result obtained from BOGA is reported.

#### 4.3.1. General performance of bolt group

The calculated load–displacement relationships in the $x$- and $y$-directions, as well as the moment–rotation relationship of the bolt group are shown in Fig. 9. A general multi-linear behaviour is observed for both rotational and translational movements. It is found that the capacity of the bolt group is significantly higher than the elastic limit. The initial yielding of the bolt group occurs at loading step 386 (76.2% of applied load). Immediately beyond the elastic limit, the stiffness in the $x$, $y$ and rotational directions of the bolt group dropped by 50%, 50% and 30%, respectively. Such stiffness is retained as the load approaches the bolt group's ultimate load capacity. And at loading step 482, Bolt No. 6 reaches its ultimate slip displacement and the program terminates.

The full range behaviour of the bolt group, from the initial loading to the ultimate capacity, can be divided into several stages, according to the yield conditions of individual bolts within the bolt group. The bolt force vectors at the end of each stage are presented in Fig. 10. The numerical values of the magnitude and direction of bolt forces at various key steps are listed in Table 1. It is observed that the bolt forces are unevenly distributed in the bolt group and the variation depends on the location of the bolts. Furthermore, the yielding sequence of the bolts follows the descending order of the bolt forces in the bolt group in the elastic analysis.

It is also discovered that rotation of bolt force vectors may not be unidirectional in either the clockwise or anticlockwise direction. For instance, the bolt force vector of Bolt No. 6 was rotating in a clockwise direction in the first 456 loading step, but then swings back gradually afterwards (Table 1). A similar situation is observed for other bolts. But the case of unidirectional rotation of the bolt force vector also exists for some of the bolts. Hence the actual rotations of bolt force vectors beyond the elastic limit are hard to determine by classic bolt group theory.

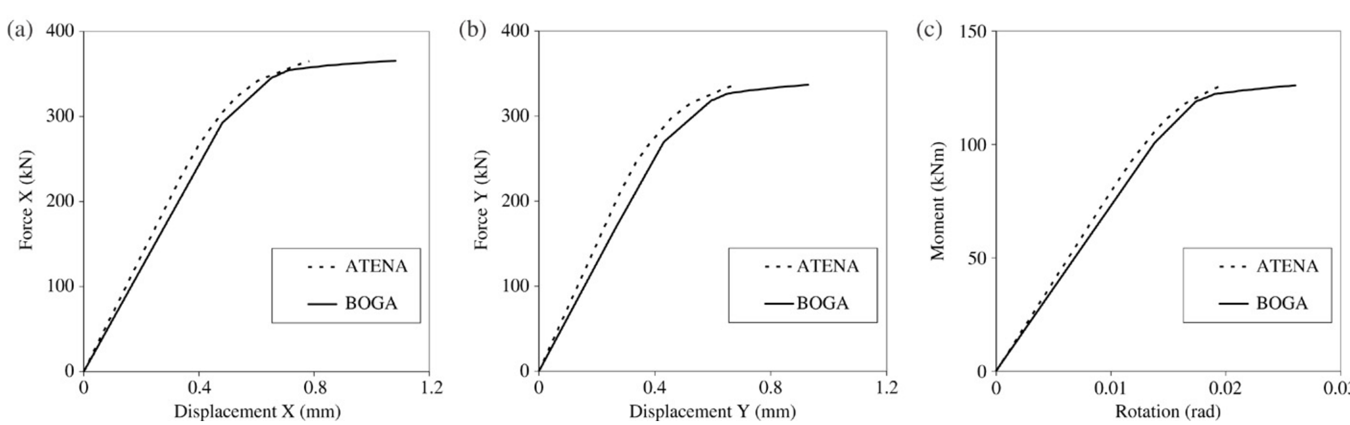


Fig. 9. Comparison between finite element analysis with the proposed analysis in (a) moment–rotation, (b) force–displacement in $x$-direction and (c) force–displacement in $y$-direction.

Table 1
Bolt force values and directions

| | Bolt force $P$ (kN) | | | | | | Bolt force direction $\theta$ (Clockwise from $+y$ direction) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | No. 1 | No. 2 | No. 3 | No. 4 | No. 5 | No. 6 | No. 1 | No. 2 | No. 3 | No. 4 | No. 5 | No. 6 |
| Step 386 | 128.29 | 92.25 | 205.39 | 204.81 | 184.38 | 260.00 | 218.62 | 298.50 | 336.45 | 118.73 | 75.61 | 43.02 |
| Step 454 | 153.76 | 113.91 | 260.00 | 258.73 | 237.23 | 260.00 | 218.25 | 301.84 | 337.92 | 117.11 | 74.62 | 42.47 |
| Step 456 | 155.09 | 114.96 | 260.00 | 260.00 | 239.39 | 260.00 | 218.26 | 301.86 | 338.07 | 117.22 | 74.60 | 41.83 |
| Step 467 | 167.97 | 124.70 | 260.00 | 260.00 | 260.00 | 260.00 | 218.23 | 301.95 | 338.22 | 115.29 | 74.52 | 42.24 |
| Step 482 | 220.19 | 162.75 | 260.00 | 260.00 | 260.00 | 260.00 | 216.09 | 305.06 | 348.14 | 109.19 | 73.65 | 46.09 |

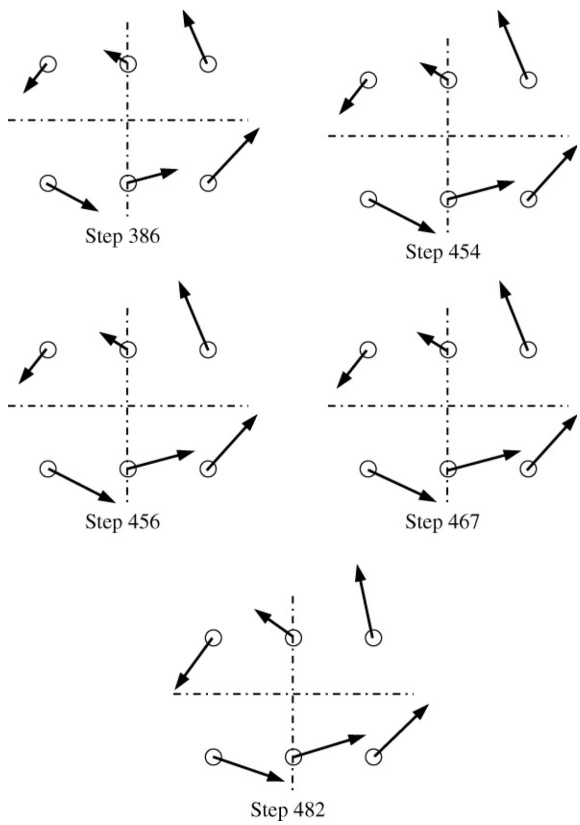


Fig. 10. Bolt force vectors at various loading steps.

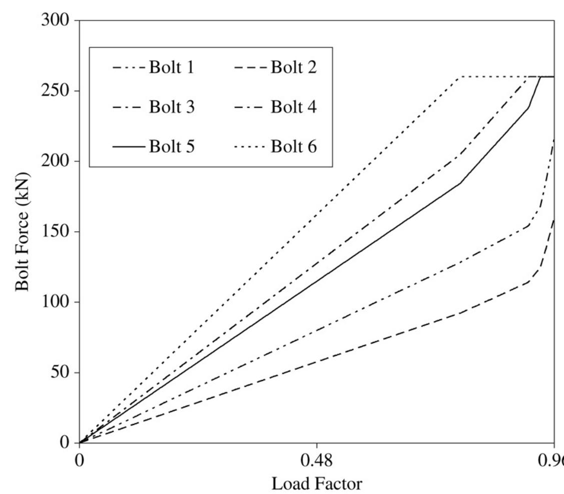


Fig. 11. Relationship between bolt forces and load factor.

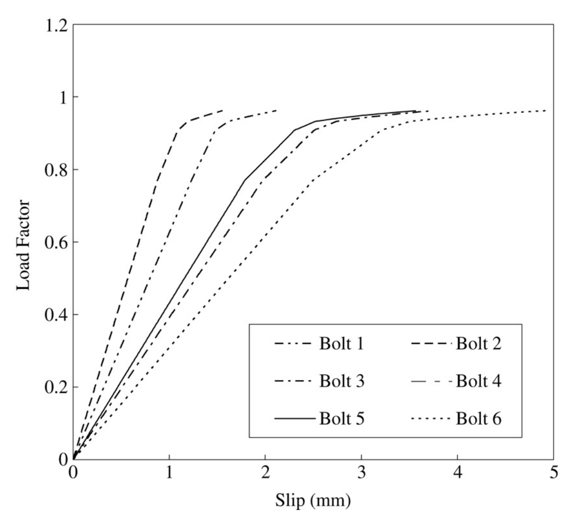


Fig. 12. Relationship between load factor and bolt slip.

It is also found that as the applied loading in the bolt group approaches the ultimate capacity, the rate of rotation of the bolt force vectors generally increases when compared with the early loading stages. For example, the total rotation of the force vector of Bolt No. 1 from loading step 456 to loading step 482 is 5 times more than the cumulative rotation of the previous loading steps.

### *4.3.2. Behaviour of individual bolts*

Fig. 11 shows the bolt force variation against the load factor of the bolt group. It is observed that the load sharing on individual bolts is uneven, and the maximum bolt force may be several times larger than the minimum force. And in general, the bolts with higher load sharing are located at the corner of the bolt group. Besides that, as the bolts with maximum load sharing become yielded, the other bolts would take up more loading in the subsequent loading steps. But the increase in load taken by each bolt, again, is uneven. As demonstrated in Fig. 9, after Bolt No. 6 has yielded, Bolt Nos. 3, 4 and 5 significantly increase in the load sharing in the subsequent loading steps, but the corresponding increments of Bolt Nos. 1 and 2 are insignificant.

The relationship between the slips of individual bolts and the load factor of the bolt group is plotted in Fig. 12. For different bolts, the ultimate slips at the moment capacity of the bolt group is reached are different, varying from the minimum slip of Bolt No. 2, which has just become yielded (1.6 mm) to the maximum slip of 5 mm (Bolt No. 6, reaching deformation capacity). A numerical trial has been performed assuming that the bolts have unlimited deformation capacity. It is observed that at the instant when all the bolts become yielded, the maximum slip (7–9 mm) of some of the bolts is much higher than the average slip of the bolt group (2–3 mm). This implies that in order to let the bolts in a group to attain its maximum capacity simultaneously, the ductility demand for individual bolts could be quite high, recalling the fact that the yield displacement of bolts are set at 2.5 mm, implying a ductility factor of 3 to 4 may be required for the bolts to reach the ultimate loading capacity in a bolt group.

## 5. Conclusions

This paper presents an original procedure to calculate the non-linear load–deformation responses of bolt groups subjected to in-plane translational and rotational loadings. All the bolts are considered to behave elasto-plastically. By assuming the direction of yielded bolt force is perpendicular to the line joining the instantaneous centre of rotation to the location of the corresponding bolt, the relationship between the forces of the yielded bolts and the instantaneous centre of rotation of the bolt groups is derived. After solving the centre of rotation in each load increment by the steepest descent method, the complete response of the bolt group and the behaviour of individual bolts can be evaluated accordingly.

Computer program BOGA has been implemented to evaluate the non-linear behaviour of bolt groups. Its reliability and accuracy have been verified by comparing the numerical results with those from non-linear finite element analysis. The numerical example studied by the newly developed program revealed that the ultimate capacity of the bolt group could be about 30% higher than the elastic limit of the same bolt group. A ductility factor of 3 to 4 may be required for the bolts in a bolt group to fully utilize their ultimate loading capacity.

## Acknowledgements

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