Design for Eccentric and Inclined Loads on Bolt and Weld Groups

NESTOR R. IWANKIW

The 8th Edition AISC *Manual of Steel Construction* introduced the ultimate strength, instantaneous center of rotation method for computation of the tabulated bolt and weld group eccentricity coefficients C. As discussed in detail by Tide[1] and Brandt,[2,3] this marked a major departure from the traditional and conservative elastic analysis employed in the 7th Edition AISC Manual (and previous editions) and which many design textbook authors still emphasize. The reason for the change was the ultimate strength method better reflected the ductility and load redistribution capability within the group by explicit consideration of each connector's characteristic load-deformation curve.[4,5] The ultimate strength method's connector shape factor effect, somewhat analogous to the plastic section modulus to elastic section modulus ratio for steel beams, consequently accounts for this increase in connection strength. Unfortunately, this type of nonlinear analysis is certainly more complex, it involves trial iterations until satisfactory numerical convergence, thus, is probably only suitable for computerized solutions for those cases not covered by the Manual.

The AISC Manual eccentricity C tables have always been limited to only vertical loads acting on certain connector patterns. A frequent question arises on how to treat *inclined* eccentric loads resulting from a combination of vertical shear and horizontal axial force. As a partial answer to this possibility, the new 1st Edition AISC *Load and Resistance Factor Design* (LRFD) Manual has included C coefficients for 45° and 75° eccentric loads in addition to the usual vertical (0°) loads. More inclined load tables were calculated, but the new Manual space restrictions precluded their publication. However, in retrospect, these few additional tables may be of little practical value, since linear interpolation between load angles for both bolts and welds may be significantly unconservative and is not recommended in the LRFD Manual, particularly for angles larger than about 45°. Essentially, this leaves the engineer or detailer in a quandary when faced with a load angle other than 0°, 45° or 75° for LRFD design. For the Allowable Stress Design (ASD) 8th Edition Manual, any load angle besides 0° presents a design difficulty which is further compounded by this warning published in the AISC *Engineering for Steel Construction*:[6]

> Tables XIX to XXVI, inclusive, Manual Part 4, have been prepared to simplify design. These tables, based on the ultimate strength method, tabulate coefficients, the use of which will be explained later in the text. Since these coefficients were derived on the basis of physical tests with loading at the ultimate strength level, they should *only be used for the weld pattern indicated in the tables, and not in combination with any additional loading. For these cases, a special ultimate strength analysis is required.*

More recently, an AISC *Engineering Journal* article[7] has provided some ASD design aids consistent with the ultimate strength method for inclined loads on common C, L and line weld groups used in framing connections. Nevertheless, the general problem remains, i.e., how does one efficiently figure or design for an inclined eccentric load in a situation that is not covered by listed C coefficients? This subject is addressed in this paper, and a few possible design solutions, both old and new, will be reviewed and compared. The final goal is to provide some practical guidance and options on this topic for both ASD and LRFD applications.

Method 1—Ultimate Strength/Instantaneous Center of Rotation

As previously described, the nonlinear instantaneous center of rotation model is the basis for both the 8th Edition AISC Manual and the 1st Edition LRFD Manual connector eccentricity tables (C_u). It is definitely the preferred and most accurate method for this type of design problem.

Briefly, a maximum connector deformation establishes a linear strain distribution, which in turn, determines the corresponding connector forces from an empirical load-

Nestor R. Iwankiw is AISC Director, Research and Codes, Chicago, Illinois.

deformation relationship. Finally, moment and force equilibrium is checked relative to an assumed center of rotation (Fig. 1). If a balance is not achieved, the center of rotation r_o is adjusted and the connector forces correspondingly recalculated until equilibrium is satisfied within an acceptable numerical tolerance. Unless one has access to a computer and the resources to program this detailed solution procedure, it does not realistically present a viable option for a special analysis or for design by hand. However, the coefficients C_u calculated by this state-of-the-art method represent the best estimates of connection strength and will serve, for the purposes of this paper, as an upper-bound benchmark to the alternate and less accurate, but much quicker solutions.

Method 2—Elastic

Prior to the 8th Edition Manual, a pure elastic analysis was the standard way to calculate the eccentricity coefficients that herein will be designated C_e. This linear elastic solution always provides a conservative lower bound since additional loads can be redistributed to the connectors in a manner similar to additional simple beam flexural capacity beyond initial yield. However, the elastic method also results in an inconsistent factor of safety relative to failure. Hence, it has been superseded by the ultimate strength method in the AISC Manuals.

Since most elastic derivations have been restricted to only the applied vertical load case, the effects of the load horizontal component must be added to represent the general situation for a bolt group shown in Fig. 1. Note x_o is merely the horizontal distance from the connector centroid to the applied force line of action; the true load eccentricity

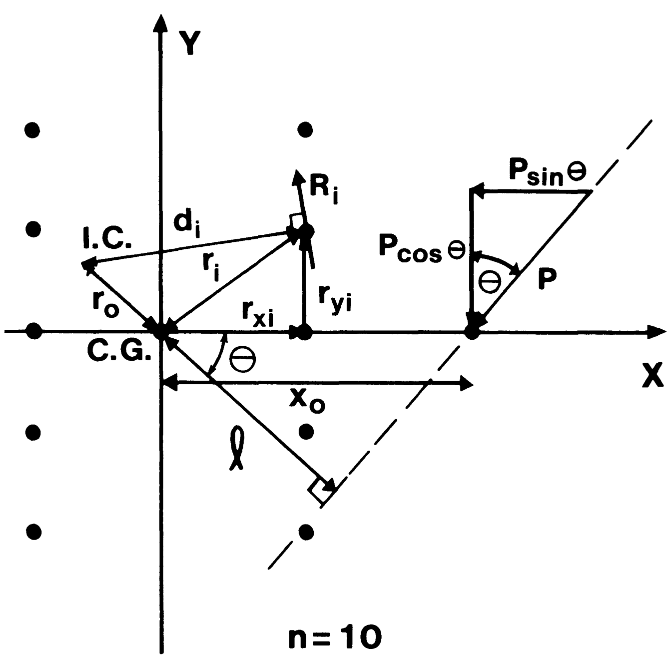


Fig. 1. Inclined and eccentric load on bolt group

ℓ perpendicular to the resultant force line of action can be calculated from geometry ($x_o = \ell$ only for vertical loads).

From Fig. 1, the elastic vectorial superposition of fastener stresses for a group of identical bolts results in this almost familiar equation:

$$R_{max} = \left[\left(\frac{P\cos\theta}{n} + \frac{Px_o(\cos\theta)r_{x,\,max}}{\Sigma r_i^2}\right)^2 + \left(\frac{P\sin\theta}{n} + \frac{Px_o(\cos\theta)r_{y,\,max}}{\Sigma r_i^2}\right)^2\right]^{1/2} \tag{1}$$

which after algebraic re-arrangement produces:

$$P = \frac{R_{max}}{\left\{\cos^2\theta\left[\frac{1}{n^2} + \frac{2x_o r_{x,\,max}}{n\Sigma r_i^2} + \frac{x_o^2(r_{x,\,max}^2 + r_{y,\,max}^2)}{(\Sigma r_i^2)^2}\right] + \sin^2\theta\frac{1}{n^2} + \sin\theta\cos\theta\left(\frac{2x_o r_{y,\,max}}{n\Sigma r_i^2}\right)\right\}^{1/2}} \tag{2}$$

The consideration of the applied force horizontal component has added the two $\sin\theta$ terms under the radical to the basic vertical load solution (first $\cos^2\theta$ term). By definition, the elastic C_e coefficient is simply one divided by the square root factor, so that

$$P = C_e r_v \tag{3}$$

r_v, the appropriate single fastener design strength (allowable load in ASD or ϕR_n in LRFD), replaces R_{max} consistent with the Manual's format. Note, this bolt coefficient C_e is only a function of the fastener pattern geometry and applied load angle and is independent of the selected design method or bolt design strength r_v. As a check, C_e by Eq. 2 will reduce to its expected maximum, i.e., the total number of fasteners n, when $\theta = 0°$ and $x_o = 0$ (concentric vertical load), or $x_o = 0$ (inclined concentric load), or $\theta = 90°$ (concentric horizontal load).

In general, the bolt eccentricity coefficients are uncoupled from the design strength regardless whether derived from an elastic analysis, as here, or from the previously described ultimate strength center of rotation method. Hence, the bolt coefficients from the 8th Edition Manual and the LRFD Manual are identical for identical situations.

For a similar elastic analysis of C-shaped or line weld groups, simply replace n by $(1 + 2k)$, Σr_i^2 by I_p (weld polar moment of inertia about centroid) and x_o by a (Fig. 2).

However, since the standard AISC Manual format assumes an E70 electrode strength per 1/16th in. of weld size per lineal inch as a basis, Eq. 3 is altered so that

$$P = C_e C_1 D\ell \tag{4}$$

C_1 = ratio of actual electrode strength to E70

D = number of sixteenths of weld size

ℓ = characteristic vertical length

$$C_e = \frac{f_D}{\left\{\cos^2\theta\left[\frac{1}{(1+2k)^2} + \frac{2ar_{x,\,max}}{(1+2k)I_p} + \frac{a^2(r^2_{x,\,max} + r^2_{y,\,max})}{I_p^2}\right] + \sin^2\theta\frac{1}{(1+2k)^2} + \sin\theta\cos\theta\left(\frac{2ar_{y,\,max}}{(1+2k)I_p}\right)\right\}^{1/2}} \tag{5}$$

where

f_D = design strength of E70 weld per sixteenth inch of weld size per lineal in., kips

= .3(70)(1/16)(.707) = 0.928 k/in./1/16 in. (ASD)

= .75(.6)(70)(1/16)(.707) = 1.392 k/in./1/16 in. (LRFD)

Again, a check of the default concentric load cases (a = 0, θ = 0°, or a = 0, or θ = 90°) simplifies the weld C_e by Eq. 5 to f_D (1 + 2k), its maximum value for use in Eq. 4. In contrast to bolts, because the f_D design strength is included in the eccentricity coefficient for Eq. 4, the comparable weld C values are numerically different between the 8th Edition and LRFD Manuals for identical cases.

In summary, the derived general elastic C_e coefficients for bolts and welds, as represented in Eqs. 2 and 5, respectively, can provide a rather quick, closed-form and lower bound answer for any inclined eccentric load applied to any connector pattern.

Method 3—Rotation of Inclined Load to Vertical

By this simple approach, any inclined load is just rotated to the vertical position and subsequently evaluated from the AISC Manual table (Fig. 3). It is a very quick and direct method and usually results in conservative answers. All angular orientations of the applied load thereby reduce to the same answer.

For high θ's and large x_o distance, this approximation will be unduly punishing as the exaggerated vertical load component and its increased eccentric moment will then have most effect. For an extreme example of an eccentric load that is almost horizontal (high θ), the actual eccentric moment ($P\ell$) would be negligible and the load could practically be considered concentric. However, blind adherence to this rule would produce a high calculated eccentric moment of Px_o. It is advisable to use another solution for such a situation.

In the case of smaller θ's and x_o's, this approximation is reasonable and is often within a few percent of the actual C_u, sometimes on the low side. The reason for this minor anomaly is the nature of the change in the trigonometric $\sin\theta$ and $\cos\theta$ functions for $\theta \leq 45°$, wherein the horizontal component related to $\sin\theta$ increases proportionately more than the vertical component related to $\cos\theta$ decreases. For design purposes, this small deviation may usually be ignored (see later examples) with the overall conclusion that such a rotation of the inclined load through small to moderate angles will always result in a safe approximate answer.

Method 4—Algebraic Addition of Direct Horizontal Shear and Eccentric Moment Resistances

This novel approximation developed by the author can be considered a refinement of Method 3. Instead of merely assuming that the full applied load is rotated to the vertical

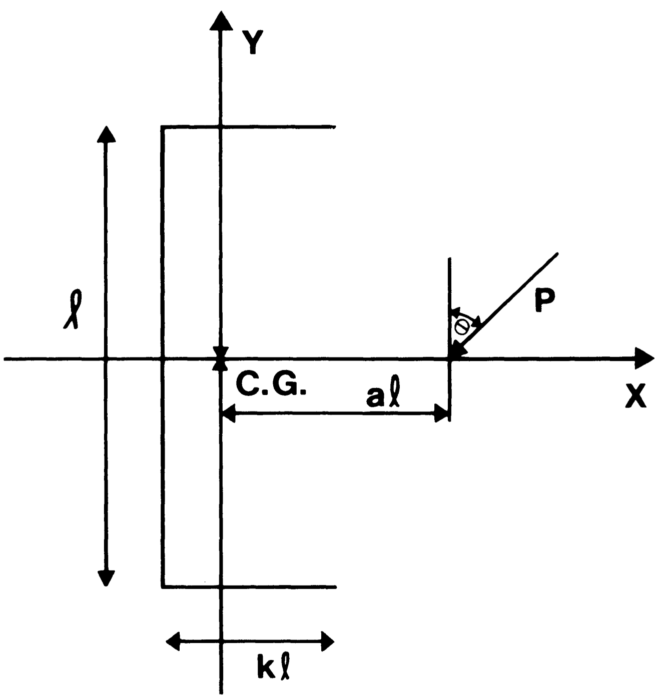
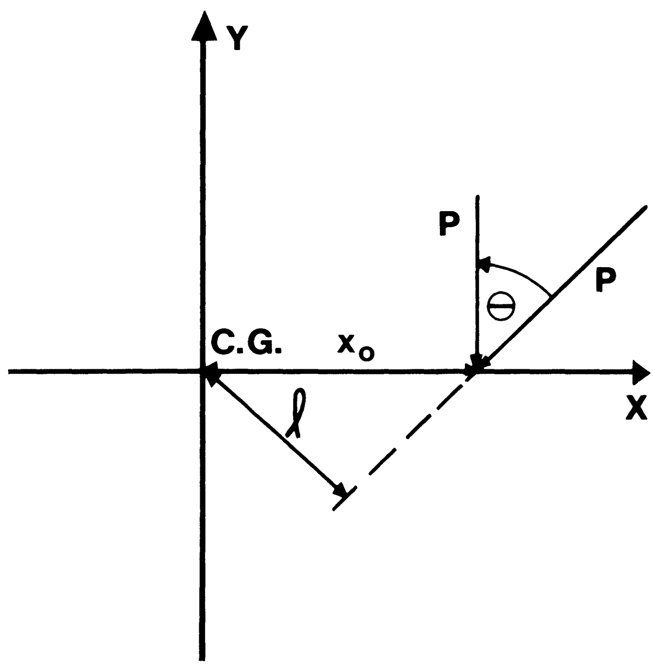


Fig. 2. C-shaped weld geometry



ℓ = actual moment arm = $x_o\cos\theta$

Fig. 3. Rotation of applied load to vertical

position, only its actual vertical component is used to identify an initial connector resistance to the applied eccentric moment by means of the conventional Manual C tables. This looked-up value is called C_o. The horizontal component of the applied load acts concentrically to the centroid of the bolt or weld group, by definition; thus, its effect can be independently and directly evaluated.

Finally, these two effects are added algebraically, rather than vectorially, to provide a total estimate of connector strength (Fig. 4).

Theoretically, true superposition is only valid in the elastic response range, as the previously referenced *Engineering for Steel Construction* statement warns. However, this shortcut approach to the inelastic connector analysis simply adds the distributed horizontal load component effect to the pre-analyzed vertical load effect, regardless of the actual maximum connector force orientation due to the vertical component and its eccentricity. Since algebraic additions always result in a larger answer than by vectorial superposition, Method 4 should consistently be conservative.

To put this possible design approach into mathematical form compatible with the AISC Manual C tables, one starts with the simple trigonometric relationship between the horizontal P_H and vertical P_v applied load components as shown in Fig. 1.

$$\frac{P_H}{P_v} = \frac{P\sin\theta}{P\cos\theta} = \tan\theta \tag{6}$$

A part of the total connector capacity C_o will resist P_v and the remainder will resist P_H in direct shear according to the algebraic addition simplification. First, for bolt groups let

$$P_v = C'r_v \tag{7}$$

and

$$P_H = \left(\frac{C_o - C'}{C_o}\right) nr_v \tag{8}$$

where

n = *total* number of fasteners (different definition than given in AISC Manuals)

r_v = single fastener design strength

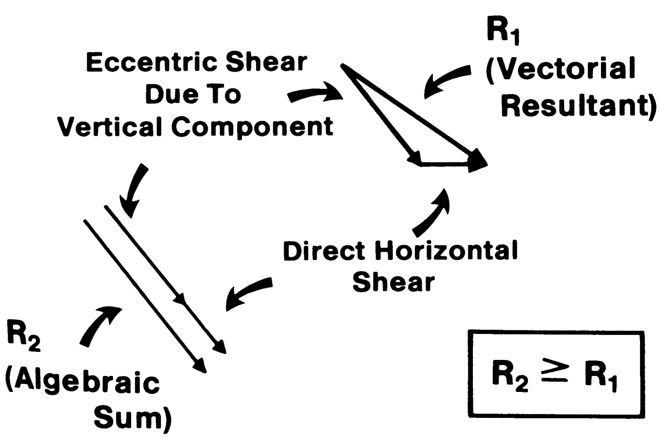


Fig. 4. Algebraic vs. vectorial addition

C_o = AISC Manual tabulated C value C_u for a given vertical load case

C' = derived eccentricity coefficient for only the vertical load component $\leq C$.

Eq. 8 represents the remainder of the bolt group capacity that can resist in direct shear the applied horizontal component.

From Eq. 6, $\left(\frac{C_o - C'}{C_o}\right) nr_v = C'r_v\tan\theta$

which may be solved for the intermediate unknown C':

$$C' = \frac{C_o n}{C_o\tan\theta + n} \tag{9}$$

If the applied load is truly vertical ($\tan\theta = 0$), Eq. 9 will correctly reduce to its maximum—$C' = C_o$ and $P_H = 0$; also, for a concentric horizontal load ($\tan 90 = \infty$), $C' = 0$, $P_v = 0$ and $P_H = nr_v$. Since the resistance components have now been identified by geometry, the resultant applied load P may be expressed as

$$P = C_a r_v = \sqrt{P_v^2 + P_H^2}$$

or, equivalently,

$$C_a = \sqrt{(C')^2 + \left(1 - \frac{C'}{C_o}\right)^2 n^2} \tag{10}$$

where

C_a = approximate eccentricity coefficient for total applied load by Method 4

Thus, C_a could be used to estimate the design eccentricity coefficient in the usual Manual format (for any load orientation that the Manual tabulates corresponding C_o values). Again, as a check, $C_a = C_o$ for $\theta = 0$ and $C_a = n$ for $\theta = 90°$. If one accepts the Method 3 premise that C_o is a true minimum value and subsequently combines this with the realization that n is the maximum value for concentric loading, the following practical working limits for bolt groups may be placed on C_a:

$$n \geq C_a \geq C_o \tag{11}$$

This corrects the unnecessarily low C_a coefficients that result from small x_o distances and small to moderate load angle configurations. For example, for a concentric load at $\theta = 45°$, C_a by Eq. 10 would be $0.707n$, whereas the proper answer is $1.0n$. This maximum error (on the safe side) is due exclusively to the algebraic addition simplification. Imposing $C_o = n$ for this case as the minimum C_a value rectifies this gross error.

For C-shaped or line weld groups, again a simple substitution of variables in the previous equations will provide a comparable C_a approximation: replace n with $f_D(1 + 2k)$; Σr_i^2 with I_p, and x_o by a (Fig. 2). Thus, the similar weld C_a algorithm will be as follows:

1. Compute C' for given weld configuration and applied load orientation

$$C' = \frac{C_o f_D(1 + 2k)}{C_o \tan\theta + f_D(1 + 2k)} \tag{12}$$

2. Determine C_a

$$C_a = \sqrt{(C')^2 + \left(1 - \frac{C'}{C_o}\right)^2 f_D^2(1 + 2k)^2} \tag{13}$$

3. Finally, check C_a limits

$$f_D(1 + 2k) \geq C_a \geq C_o \tag{14}$$

With such assumptions and derivation, only Eqs. 9, 10 and 11 for bolts and 12, 13 and 14 for welds are needed to compute the approximate C_a. Again, note bolt C' and C_a equations are independent of r_v, while their weld counterparts are a function of f_D. Similar to the elastic Method 2 solution, the calculations involved are very straightforward and direct.

C_a usually results in higher design coefficients than C_e because one of its components, C_o, is based on the ultimate strength Method 1 of the AISC Manual tables. Also, because of the conservatism of arithmetic addition of connector resistances, C_a is also assured of being lower than the maximum C_u values by Method 1. Thus, C_a captures both the computational ease of C_e and some of the redistribution benefits of C_u without their respective previously discussed shortcomings. The C_a method will generally be most effective for larger angles and larger eccentricities. Otherwise, the minimum C_o limit will prevent any major inaccuracies.

Method 4 clearly disputes the previously quoted statement in the Introduction from the AISC *Engineering for Steel Construction*.[6] It demonstrates AISC Manual ultimate strength coefficients ($C_o = C_u$) may be used in combination with an additional imposed horizontal load component, as explained here, to provide a satisfactory and quick solution to any inclined eccentric load problem.

Method 5—Full Plastic Analysis

Another alternate procedure possible for non-tabulated cases is the fully plastic approach.[8,9]

For a load P_u, acting at an eccentricity ℓ, the elastic center of rotation at a distance r_o from the centroid of the bolt group (Fig. 1) is given by:

in which
$$r_o = k_o^2 / \ell \tag{15}$$

k_o = polar radius of gyration of the bolt group relative to the centroid $= \sqrt{\Sigma r_i^2 / n}$

r_i = distance from centroid to the ith bolt

n = total number of bolts

Using this center of rotation, the applied ultimate moment is $P_u(\ell + r_o)$ and the resisting moment, if all bolts are assumed to have reached their ultimate capacity R_{ult} at complete redistribution is:

$$M_u = R_{ult} \sum_{i=1}^{n} d_i \tag{16}$$

in which d_i = distance from the center of rotation to the ith bolt. This approximation predicts failure when:

$$C_p = \frac{P_u}{R_u} = \frac{\sum_{i=1}^{n} d_i}{\ell + r_o} \tag{17}$$

where C_p = plastic eccentricity coefficient

A check for transverse force equilibrium is not necessary.

Because of the simplified assumption all bolts are at their ultimate load R_{ult} this analysis provides unconservative results, compared to the most accurate Method 1-C_u values, on the order of about 15%. Thus, it is recommended, if this fully plastic analysis method is used, the computed capacity C_p coefficient must be reduced appropriately, since this plastic method provides an unreachable upper bound. For this reason and because of complications introduced by weld load-deformation dependence on the resistance force angle relative to the weld axis, this bolt plastic analysis will not be extended any further.

Comparisons and Evaluation

All but two (Methods 3 and 4) of the previously discussed eccentricity design procedures are well documented and do not require any additional justification. Method 5 is a mere extension of the usual steel plastic design theory. Methods 1 and 2 have been developed rigorously, they have served as standard procedures for years and are widely accepted. However, the extra complications caused by the horizontal component of an inclined load warrant the additional elaborations of this article and the introduction of approximate Methods 3 and 4 solutions which, in actuality, can be condensed through Eqs. 11 or 14 into basically one new procedure.

To give a better illustration of the similarities, differences and parameter sensitivity of the new approximate answers relative to the other more established methods, a few sample problems have been analyzed and the results tabulated.

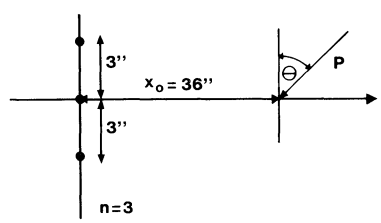


Table 1

Load Angle, θ	C_u^* (Method 1)	C_e (Method 2)	C_o (Method 3)	C_a (Method 4)	C_p (Method 5)
0	$\boxed{0.17}$	0.17	$\boxed{0.17}$	0.17	0.17
15	0.17	0.17		0.17	0.18
30	0.19	0.186		0.186	0.20
45	$\boxed{0.23}$	0.222		0.224	0.24
60	0.33	0.303		0.305	0.35
75	$\boxed{0.62}$	0.532		0.534	0.67
90	3.0	3.0		3.0	3.0

* Preferred and most accurate.
$\boxed{}$ Values from LRFD Manual Table X

For this large eccentricity bolt problem, the baseline C_o value of 0.17 can be read directly from either the 8th Edition or the LRFD Manual Table X. The bracketing of the approximate C_a by $C_o = 0.17$ and $n = 3.0$ is evident throughout the entire load angle range. Furthermore, complete reliance on only the Method 3 rule would yield terribly conservative answers for the larger angles and is not recommended. As expected, the elastic Method 2 C_e's are consistently below the Method 1 C_u's, which, in turn, are below the Method 5 C_p's. The C_a's closely track the corresponding C_e's for this problem and, thus, are definitely reasonable. At the extremes, $\theta = 0$ and 90, all the methods give essentially identical answers.

The LRFD Manual caution against linear interpolation between angles will now be demonstrated. Using only the preferred Method 1, interpolation between 60° and 90° for a desired C_u value at 75° will produce 1.67, whereas the actual C_u is a much lower 0.62 (only 37% of interpolated value). The largest possible interpolation errors ordinarily will occur in this higher angle range where the eccentricity coefficient increases very rapidly as the horizontal concentric load position ($\theta = 90°$) is approached. This nature of the C_u versus θ function is clearly illustrated in Fig. 5. Nevertheless, even straight-line interpolations between 45° and 75° C_u's (now given in the LRFD Manual) for 60° will yield 0.425, compared to the actual C_u of 0.33 (about 78% of interpolated value). Even though the unconservative interpolation error will decrease for load angles less than 45° from the vertical, this type of linear interpolation is strongly discouraged in favor of one of the rational methods presented in this paper. Alternately, as suggested in the LRFD Manual, it is safe to use the C_u values tabulated for the next lower angle, i.e., for the latter example of a load angle of 60°, simply use the 45° coefficient of 0.23. Such an approximation, much like employed for Method 3, can be rather conservative in many cases.

Another configuration with 9 total bolts in a single row was analyzed and the results presented in Table 2 for a small (Case A) and large (Case B) load offset distance (x_o):

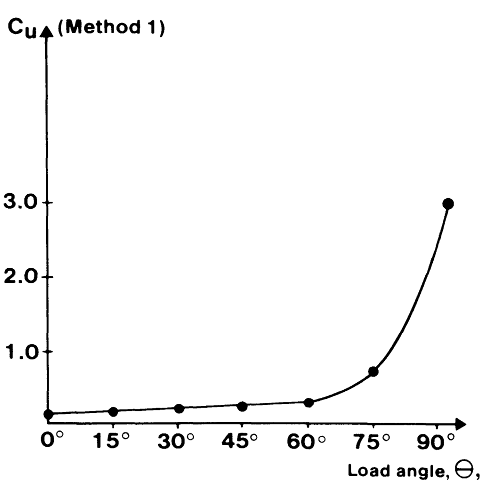


Fig. 5. C_u's in Table 1

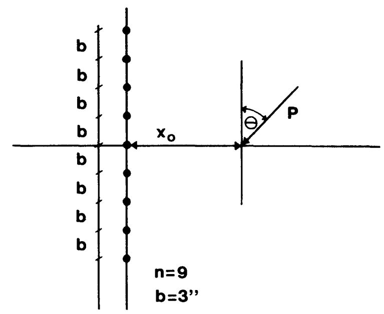


Table 2
Case A—x_o = 2 in.

Load Angle, θ	C_u^* (Method 1)	C_e (Method 2)	C_o (Method 3)	C_a by Eq. 10 (Method 4)	C_p (Method 5)
0	$\boxed{8.52}$	8.356	$\boxed{8.52}$	8.52	8.71
15	8.47	7.748	8.52	~~7.04~~	8.71
30	8.44	7.432	8.52	~~6.36~~	8.72
45	$\boxed{8.48}$	7.397	8.52	~~6.19~~	8.78
60	8.59	7.644	8.52	~~6.16~~	8.87
75	$\boxed{8.73}$	8.179	8.52	~~7.26~~	8.96
90	9.0	9.0		9.0	9.0
Case B—x_o = 36 in.					
Load Angle, θ	C_u (Method 1)	C_e (Method 2)	C_o (Method 3)	C_a by Eq. 10 (Method 4)	C_p (Method 5)
0	$\boxed{1.54}$	1.238	$\boxed{1.54}$	1.54	1.69
15	1.59	1.236	1.54	1.52	1.74
30	1.74	1.325		1.62	1.93
45	$\boxed{2.07}$	1.54		1.86	2.32
60	2.75	2.0		2.38	3.11
75	$\boxed{4.30}$	3.17		3.63	4.84
90	9.0	9.0		9.0	9.0

* Preferred and most accurate.
$\boxed{}$ Values from LRFD Manual Table X

More observations can be made and trends noted from the Table 2 data. It is obvious that the case B, $x_o = 36$ large load distance from the bolt group centroid severely reduces all the eccentricity coefficients until the load position approaches a horizontal orientation with little real eccentricity. In contrast, the almost concentric load position of case A, $x_o = 2$ results in rather large C coefficients for all inclinations. This demonstrates the powerful effect of eccentric moment on connector group capacity.

As previously mentioned in the five methods of analysis, the Method 4 algebraic combination will produce unnecessarily low C_a values by Eq. 10 for small x_o distances and small to moderate load angles. Case A demonstrates this problem and its suggested quick resolution by simple repeated use of the Method 3-C_o as the minimum limit given in Eq. 11. One may argue against this approach by pointing to the slight decrease in the baseline Method 1-C_u, $\theta = 0$ value for Case A through about 45°. However, this maximum reduction is only on the order of 1-2%, which is within the numerical tolerance for this solution method, and is hardly noteworthy for design purposes. This calculated

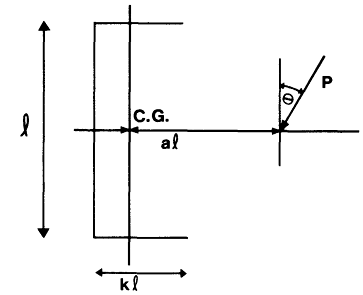


Table 3
Case A—a = 0.2, k = 0.5

Load Angle, θ	C_u^* (Method 1)	C_e (Method 2)	C_o (Method 3)	C_a (Method 4)
0	$\boxed{2.494}$	1.88	$\boxed{2.494}$	2.494
15	2.50	1.81	2.494	~~2.08~~--
30	2.53	1.82	2.494	~~1.90~~--
45	$\boxed{2.59}$	1.91	2.494	~~1.95~~--
60	2.67	2.11	2.494	~~1.95~~--
75	$\boxed{2.75}$	2.41	2.494	~~2.22~~--
90	2.784	2.784		2.784
Case B—a = 2.0, k = 0.5				
0	$\boxed{0.538}$	0.39	$\boxed{0.538}$	0.538
15	0.556	0.393	0.538	0.530
30	0.614	0.425		0.559
45	$\boxed{0.735}$	0.499		0.638
60	0.977	0.656		0.806
75	$\boxed{1.504}$	1.057		1.208
90	2.784	2.784		2.784

* Preferred and most accurate.
$\boxed{}$ Values from LRFD Manual Table XXII.

reduction admittedly is more pronounced for the elastic C_e's (maximum drop from θ = 0° of about 15%), but this does not represent a real strength decrease according to the Method 1 ultimate strength theory based on tests. Hence, there should be little qualms about accepting C_o as the minimum design coefficient for any load angle.

Case B further shows the design advantages of C_a over the lower C_e values while remaining on the safe side of the upper bound C_u's. As predicted before, the Method 5-C_p's are always above the C_u's and can be used only with due care to maintain the intended factor of safety for ASD or the reliability index for LRFD.

The bolt C coefficients shown in Tables 1 and 2 may be used for either LRFD or ASD, since they are uncoupled from any design parameters.

Analysis of weld groups will follow the same trends as noted for bolts. For example, a general C-shaped weld with an inside eccentric load (Table XXIII in 8th Edition Manual; Table XXII in LRFD Manual) produce the results tabulated in Table 3 for LRFD design.

First of all, the maximum capacity for a concentric loading, such as θ = 90°, is simply $f_D\ (1 + 2k) = 1.392(2) = 2.784$ k/in./1/16 in. Again, for the small load offset Case A, all the eccentricity coefficients are much larger than for the large load offset Case B. For those load angles wherein C_a is less than C_o, use C_o. The C_a values will then be bounded by the C_u and C_e numbers, as expected. Similar to bolt usage, do not use linear interpolation between load angles for tabulated weld C values.

A reminder that one major difference between bolts and welds lies in the inclusion of the E70 weld design strength f_D in the weld eccentricity coefficient itself. Thus, since the ratio of 1.392/0.928 = 1.5, the LRFD weld eccentricity coefficients will be about this much greater than comparable ASD values. Consequently, Table 3 values are intended exclusively for LRFD design.

Two simple design problems in the Appendix illustrate the easy applicability of the algebraic C_a method for inclined loads.

Summary and Conclusions

This article has reviewed and critically evaluated five rational methods for the general analysis/design of eccentric and inclined loads on bolt and weld groups. The strengths and weaknesses of each have been discussed and demonstrated. Relative comparisons were made for a few sample problems:

1. Method 1. Ultimate strength/instantaneous center of rotation is recognized as the best predictor of connector group strength. Because of its demanding computational requirements, however, it may be difficult to implement for non-routine special applications.
2. Method 2. The elastic approach C_e has been extended in this paper to include horizontal load components. This still presents a rather conservative but expedient option for analysis/design.
3. Method 3. It simply uses the tabulated C coefficients for vertical loads in the AISC Manuals as the default safe value for any load angle. While very easy, this will be extremely punishing for certain situations.
4. Method 4. A new approximate Method 4 has been derived from algebraic addition of connector strengths and proposed as a convenient alternative for cases not directly covered in the AISC Manual. The algorithm involves the solution of only two short equations, which, in combination with the previous Method 3 minimum C_o value limit, produces answers bounded by the elastic and ultimate strength values, a clearly acceptable design range.

This latter Method 4 algebraic addition has a valid conservative basis and is, thus, proposed as an attractive design option wherein the favored instantaneous center of rotation solution is not feasible. Readers are encouraged to try the C_a approach when applicable and execute additional comparisons. A more thorough parameter study, possibly by an engineering intern or a university student, would be useful to explore some different connector configurations or other aspects of the C_a method, such as new refinements or unforeseen limitations.

ACKNOWLEDGMENT

The author thanks W. A. Thornton, Cives Steel Co., and R. H. R. Tide, Wiss, Janney, Elstner & Associates, for sharing their insights on this general design topic and for providing the stimulation for further contemplation and improvements.

APPENDIX—DESIGN EXAMPLES

Ex. 1. Two vertical rows of 6 bolts, 5½ in. apart, standard 3-in. spacing, eccentric load P at 16 in. from group centroid inclined at an angle of 60° (similar to example on pg. 4-61 8th Edition Manual and pg. 5-62 LRFD Manual). Using ⅞-in. A325-N bolts, what is maximum design capacity P_{max}?

ASD

1. From Table XII, 8th Edition Manual: $C_o = 3.55$
2. Eq. 9 $C' = \dfrac{3.55(12)}{3.55 \tan 60 + 12} = 2.35$
3. Eq. 10 $C_a = \sqrt{2.35^2 + \left(1 - \dfrac{2.35}{3.55}\right)^2 (12^2)} = 4.69$

   $3.55 < 4.69 < 12$ **o.k.**
4. Using $r_v = 12.6$ kips, $P_{max} = 4.69 \times 12.6 = 59.1$ kips

LRFD

1. From Table XII, angle = 0°, LRFD Manual: $C_o = 3.55$
2. and 3. Identical to ASD, $C_a = 4.69$
4. Using $\phi r_v = 21.1$ kips, $P_{max} = 4.69 \times 21.1 = 99$ kips

Ex. 2. C-shape weld, E70xx electrodes, with $\ell = 10$ in., $kl = 5$ in., eccentric service load $P = 87$ kips at 75° with $al = 8.75$ (Fig. 2). Determine minimum required weld size.

ASD (Problem similar to example on pg. 4-75, 8th Edition Manual)

1. $k = 0.5$, $a = 0.875$, use Table XXIII, 8th Edition Manual, $C_o = 0.704$
2. Eq. 12 $C' = \dfrac{.704\ (.928)\ [1 + 2(.5)]}{.704 \tan 75 + .928[1 + 2(.5)]} = 0.27$
3. Eq. 13 $C_a = \sqrt{.27^2 + \left(1 - \dfrac{.27}{.704}\right)^2 (.928)^2[1 + 2(.5)]^2}$

   $C_a = 1.176$

   $0.704 < 1.176 < .928\ [1 + 2(.5)] = 1.856$ **o.k.**
4. $C_1 = 1.0$

   $$D = \frac{87}{(1.176)(10)} = 7.4, \text{ say } 8$$

   $$\text{min. weld size} = \frac{8}{16} = ½ \text{ in.}$$

LRFD Same as for ASD, but eccentric factored load $P_u = 130$ kips (problem similar to example on pg. 5-90, LRFD Manual).

1. $k = 0.5$, $a = 0.875$, use Table XXII (angle = 0°), LRFD Manual, $C_o = 1.136$
2. Eq. 12 $C' = \dfrac{1.136\ (1.392)[1 + 2(.5)]}{1.136 \tan 75 + 1.392[1 + 2(.5)]} = 0.45$
3. Eq. 13 $C_a = \sqrt{.45^2 + \left(1 - \dfrac{.45}{1.136}\right)^2 (1.392)^2[1 + 2(.5}$

   $C_a = 1.74$

   $1.136 < 1.74 < 1.392\ [1 + 2(.5)] = 2.784$ **o.k.**

(Note—exact C_u value can be obtained for this case from LRFD Table XXII, angle = 75°, equal to 2.17)

4. Using computed approximate $C_a = 1.74$, $C_1 = 1.0$

   $$D = \frac{130}{1.74(10)} = 7.47, \text{ say } 8$$

   $$\text{min. weld size} = \frac{8}{16} = ½ \text{ in.}$$

REFERENCES

1. *Tide, R. H. R.* Eccentrically Loaded Weld Groups—AISI Design Tables *AISC Engineering Journal, 4th Qtr., 1980, Chicago, Ill. (p. 90).*
2. *Brandt, G. D.* Rapid Determination of Ultimate Strength of Eccentrically Loaded Bolt Groups *AISC Engineering Journal, 2nd Qtr., 1982, Chicago, Ill. (p. 94).*
3. *Brandt, G. D.* A General Solution for Eccentric Loads on Weld Groups *AISC Engineering Journal, 3rd Qtr., 1982, Chicago, Ill. (p. 150).*
4. *Butler, L. J., S. Pal and G. L. Kulak* Eccentrically Loaded Weld Connections *ASCE Journal of the Structural Division, Vol. 98, No. ST5 (pp. 989–1,005).*
5. *Crawford, S. F. and G. L. Kulak* Eccentrically Loaded Bolted Connections *ASCE Journal of the Structural Division, Vol. 97, No. ST3 (pp. 765–783).*
6. *American Institute of Steel Construction, Inc.* Engineering for Steel Construction *1st Ed., 1984, Chicago, Ill.*
7. *Thornton, W. A., K. Loomis and T. Kane* A Design Aid for Connection Angle Welds Subjected to Combined Shear and Axial Loads *AISC Engineering Journal, 4th Qtr., 1985, Chicago, Ill. (p. 178).*
8. *Shermer, C. L.* Plastic Behavior of Eccentrically Loaded Connections *AISC Engineering Journal, 2nd Qtr., 1971, New York, N.Y. (p. 48).*
9. *Marsh, C.* Discussion on "Rapid Determination of Ultimate Strength of Eccentrically Loaded Bolt Groups" *AISC Engineering Journal, 4th Qtr., 1982, Chicago, Ill. (p. 214).*

U.S. Postal Service

STATEMENT OF OWNERSHIP, MANAGEMENT AND CIRCULATION

Required by 39 U.S.C. 3685)

1A. TITLE OF PUBLICATION	1B. PUBLICATION NO.	2. DATE OF FILING
Engineering Journal, American Institute of Steel Construction	0 0 1 3 8 0 2 9	Dec. 2, 1987

3. FREQUENCY OF ISSUE	3A. NO. OF ISSUES PUBLISHED ANNUALLY	3B. ANNUAL SUBSCRIPTION PRICE
Quarterly	Four	$11.00

4. COMPLETE MAILING ADDRESS OF KNOWN OFFICE OF PUBLICATION *(Street, City, County, State and ZIP+4 Code) (Not printers)*

400 North Michigan Avenue, Chicago, Illinois 60611

5. COMPLETE MAILING ADDRESS OF THE HEADQUARTERS OF GENERAL BUSINESS OFFICES OF THE PUBLISHER *(Not printer)*

American Inst. of Steel Construction, Inc.
400 North Michigan Avenue, Chicago, Illinois 60611

6. FULL NAMES AND COMPLETE MAILING ADDRESS OF PUBLISHER, EDITOR, AND MANAGING EDITOR *(This item MUST NOT be blank)*

PUBLISHER *(Name and Complete Mailing Address)*

American Inst. of steel Construction, Inc.
400 North Michigan Avenue, Chicago, Illinois 60611

EDITOR *(Name and Complete Mailing Address)*

George E. Harper, Director of Publications American Institute of Steel Const., Inc.
400 North Michigan Avenue, Chicago, Illinois 60611

MANAGING EDITOR *(Name and Complete Mailing Address)*

NONE

7. OWNER *(If owned by a corporation, its name and address must be stated and also immediately thereunder the names and addresses of stockholders owning or holding 1 percent or more of total amount of stock. If not owned by a corporation, the names and addresses of the individual owners must be given. If owned by a partnership or other unincorporated firm, its name and address, as well as that of each individual must be given. If the publication is published by a nonprofit organization, its name and address must be stated.) (Item must be completed.)*

FULL NAME	COMPLETE MAILING ADDRESS
American Institute of Steel Construction, Inc.	400 North Michigan Avenue Chicago, IL 60611

8. KNOWN BONDHOLDERS, MORTGAGEES, AND OTHER SECURITY HOLDERS OWNING OR HOLDING 1 PERCENT OR MORE OF TOTAL AMOUNT OF BONDS, MORTGAGES OR OTHER SECURITIES *(If there are none, so state)*

FULL NAME	COMPLETE MAILING ADDRESS
NONE	

9. FOR COMPLETION BY NONPROFIT ORGANIZATIONS AUTHORIZED TO MAIL AT SPECIAL RATES *(Section 423.12 DMM only)*
The purpose, function, and nonprofit status of this organization and the exempt status for Federal income tax purposes *(Check one)*

(1) ☐ HAS NOT CHANGED DURING PRECEDING 12 MONTHS

(2) ☐ HAS CHANGED DURING PRECEDING 12 MONTHS

(If changed, publisher must submit explanation of change with this statement.)

10. EXTENT AND NATURE OF CIRCULATION *(See instructions on reverse side)*	AVERAGE NO. COPIES EACH ISSUE DURING PRECEDING 12 MONTHS	ACTUAL NO. COPIES OF SINGLE ISSUE PUBLISHED NEAREST TO FILING DATE
A. TOTAL NO. COPIES *(Net Press Run)*	10,452	9,868
B. PAID AND/OR REQUESTED CIRCULATION 1. Sales through dealers and carriers, street vendors and counter sales	NONE	NONE
2. Mail Subscription *(Paid and/or requested)*	8,337	8,107
C. TOTAL PAID AND/OR REQUESTED CIRCULATION *(Sum of 10B1 and 10B2)*	8,337	8,107
D. FREE DISTRIBUTION BY MAIL, CARRIER OR OTHER MEANS SAMPLES, COMPLIMENTARY, AND OTHER FREE COPIES	40	32
E. TOTAL DISTRIBUTION *(Sum of C and D)*	8,377	8,139
F. COPIES NOT DISTRIBUTED 1. Office use, left over, unaccounted, spoiled after printing	2,115	1,729
2. Return from News Agents	NONE	NONE
G. TOTAL *(Sum of E, F1 and 2—should equal net press run shown in A)*	10,452	9,868

11. **I certify that the statements made by me above are correct and complete**

SIGNATURE AND TITLE OF EDITOR, PUBLISHER, BUSINESS MANAGER, OR OWNER

George E Harper, Editor

PS Form 3526, Dec. 1985 *(See instruction on reverse)*

DISCUSSION

Design for Eccentric and Inclined Loads on Bolts and Weld Groups

Paper by NESTOR IWANKIW
(4th Quarter, 1987)

Discussion by **Miguel Angel Dodes Traian**

I have read the article and think there is an expedient way to get a value for analysis and design of bolt groups, which is implicit in the explanation.

Method 2, which defines the elastic approach C_e, is a closed form and a lower bound value. On the other side, Method 5 defines the fully plastic approach C_p, which is an unreachable upper bound.

The real solution is between these two limits, thus a very close and direct approximation is the arithmetic mean of both values, that is (using the article's nomenclature):

$$C_e = \Sigma d^2/[(r_o + \ell).\, d_{\max}]$$

$$C_p = \Sigma d_i/(r_o + \ell)$$

$$C_{\text{mean}} = (C_e + C_p)/2$$

In the examples of this article and some others of the AISC Manual, the difference with C_u (considered as the most accurate) is between -7% and $+2\%$, which I consider an excellent approximation for a non-iterative procedure.

Addendum/Closure by **Nestor Iwankiw**

Traian's interest in the article and helpful design suggestion of an approximate coefficient C_{mean} for the analysis of bolt group cases not tabulated in the AISC Manual is appreciated. This average of the lower C_e and upper C_p bound limits results in a reasonable approximation, as indicated by the reported accuracy of C_{mean} relative to C_u. The discussion also gives a useful reminder of an equivalent alternate formula for bolt C_e in terms of elastic center of rotation geometry.

Even though Traian, in the original article, did not address full plastic analysis of welds, C_e and Method 5 C_p can also be similarly defined for weld groups relative to the elastic center of rotation:

$$C_e = \frac{f_D I_{\text{pic}}}{(r_o + \ell)d_{\max}} \qquad \text{(D-1)}$$

$$C_p = \frac{f_D \Sigma(d_i L_i)}{(r_o + \ell)} \qquad \text{(D-2)}$$

where

I_{pcg} = polar moment of inertia of weld group about centroid
k_o = polar radius of gyration of weld group relative to centroid = $\sqrt{\dfrac{I_{pcg}}{\Sigma L_i}}$
I_{pic} = polar moment of inertia of weld group about instantaneous center = $I_{pcg} + r_o^2(\Sigma L_i)$
L_i = length of ith weld line element
d_i = distance from center of rotation to centroid of ith weld element
(f_D, ℓ, r_o defined in paper)

Unfortunately, a few sample problems have demonstrated that C_p is sensitive to the number and location of weld subdivisions (elements), especially at corners and extremities. Probably, at least 10-20 weld elements are necessary to obtain a reasonable answer, even for fairly regular patterns. In addition, actual weld ductility could limit significantly the assumed complete redistribution and strength, represented by C_p, particularly for larger connections. Because of these uncertainties and complications, general use of C_p for weld design cautiously is not recommended pending further studies and comparisons.

Consequently, for routine design applications Method 4 will be shown computationally more convenient and reliable in view of its simple overall representation of both bolt and weld group strength under all load angles. If necessary for special cases, optimization of this trial design capacity could be attempted in combination with another rational method, such as C_{mean}. The remainder of this discussion focuses mainly on further development and practical application of Method 4.

In the original paper, the proposed Method 4 Eqs. 9, 10 and 11 for bolts and 12, 13 and 14 for welds were presented in somewhat "raw" form. Their derivation was based on the conservative assumption of simple arithmetic, rather than vectorial, addition of maximum connector strength as an exaggerated load effect. Some additional simplifications can be made to this formulation which are helpful both computationally and conceptually. Through substitution and algebraic rearrangement, the referenced equations will be reduced to a more convenient and normalized form for application. A plot of the functional relationship also illustrates better the behavioral trends relative to load angle.

First, define $C_{\max}$ as the total number of bolts n or the maximum concentric weld capacity, (f_D (1 + $2k$) for

---

Miguel Traian is Engineering Manager for Acrow Argentina, South America.

C-shapes), as applicable. Next, let

$$A = \frac{C_{\max}}{C_o} \geq 1.0 \tag{D-3}$$

where C_o is the AISC Manual-tabulated C_u for a given vertical load case. For a particular connector pattern and load eccentricity distance, A is a constant relative to the load angle θ; it serves as the single characteristic input property of the connector geometry.

Normalizing Eqs. 9 and 12 through division by C_o yields the intermediate solution:

$$\frac{C'}{C_o} = \frac{A}{(\tan\theta + A)} \tag{D-4}$$

Furthermore, a similar division by C_o of Eqs. 10 and 13 and substitution of D-4 for $\frac{C'}{C_o}$ results in the final short expression for $\frac{C_a}{C_o}$ only as a function only of θ:

$$\frac{C_a}{C_o} = \frac{A}{(\tan\theta + A)\cos\theta}$$

or equivalently, (D-5)

$$\frac{C_a}{C_o} = \frac{A}{(\sin\theta + A\cos\theta)}$$

This solution could also have been obtained directly from D-4 and the geometry of Fig. 1 by recognizing C' as the eccentricity coefficient for only the vertical load component. The working limits imposed by 11 and 14 convert readily for the normalized $\frac{C_a}{C_o}$ variable to:

$$A \geq \frac{C_a}{C_o} \geq 1 \tag{D-6}$$

Upon this effective elimination of D-4, the simplified Eqs. D-3, D-5 and D-6 are all that is necessary to easily apply Method 4 for bolt or weld design/analysis. With this normalized dependent variable $\frac{C_a}{C_o}$ and D-5, graphs can be prepared for specific A values and for load angles from 0-90°, as in Figs. D1 and D2. The $A = 1$ case in Fig. D1 is presented only to show the minimum $\frac{C_a}{C_o}$ for a concentric loading for which the lower boundary of D-6 obviously would govern i.e. $C_a = C_o = C_{\max}$. The figures give a clear overall picture of the C_a sensitivity to both A and θ. The $\frac{C_a}{C_o}$ ordinates and curve slopes increase with higher A values and the end values at $\theta = 0°$ and 90° are 1.0 and A, as expected. The D-5 function curvature is concave upward (consistent with the C_u plot in Fig. 5) meaning that linear interpolation can be greatly misleading, especially for larger θ's and larger A's. Fig. D2 is Fig. D1 restricted to a narrower practical $\frac{C_a}{C_o}$ range with the lower imposed limit of 1.0.

These two figures can help to identify immediately the importance of load angle for a given problem as well as to provide for a quick graphical solution estimate for the eccentricity coefficient C of the total resultant load P. Given an A value, the range of so-called small-to-moderate-angles for which the eccentricity coefficient is essentially invariant from its vertical minimum value C_o can be approximated from the dip in the curve below 1.0. For example, for the more common smaller load eccentricity distances, A will usually be between 1 and 2; consequently any load angle less than about 55° will not increase $\frac{C_a}{C_o}$ (see original paper Table 2, case A, $A = \frac{9}{8.52} = 1.06$). On the other hand, larger load eccentricities (Table 1, $A = \frac{3}{0.17} = 17.6$) produce a rapid rise in

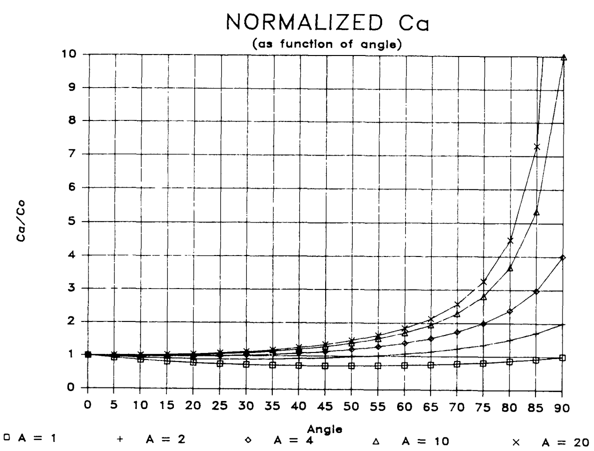


Figure D-1

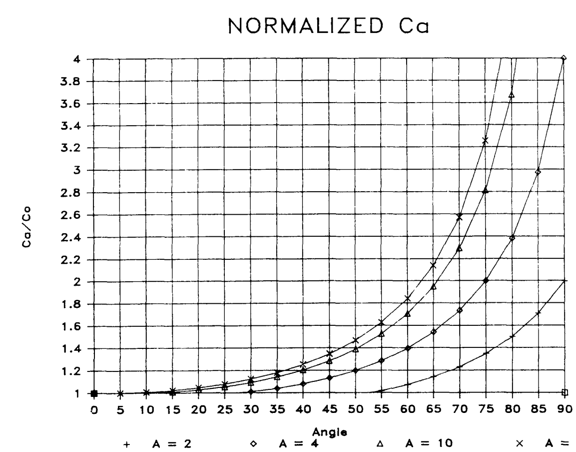


Figure D-2

$\frac{C_a}{C_o}$ for θ greater than 15°. These observations confirm the numerical comparisons made beforehand in the paper.

An interesting fact about this $\frac{C_a}{C_o}$ solution is that it can be reduced to the following simple linear interaction between the eccentric shear $\frac{P_v}{C_o r_v}$ and direct shear $\frac{P_H}{C_{max} r_v}$ capacities, merely by substituting $P = C_a r_v$, $\sin\theta = \frac{P_H}{P}$, $\cos\theta = \frac{P_v}{P}$, and $A = \frac{C_{max}}{C_o}$ in D-3:

$$\frac{P_H}{C_{max}\, r_v} + \frac{P_v}{C_o\, r_v} = 1.0 \qquad \text{(D-7)}$$

or

$$\frac{C_H}{C_{max}} + \frac{C_v}{C_o} = 1.0$$

where

C_H = coefficient for only the horizontal load component (direct shear)

C_v = eccentricity coefficient for only the vertical load component (eccentric shear)
= C' in (7)

This equivalency to a typical conservative interaction diagram (see Fig. D–3) further legitimizes Method 4 from another perspective.

To digress briefly, another different approach to the inclined eccentric load problem would have been to bypass any simplifying theoretical considerations and only develop a curve-fit approximation, C_c, to the actual ultimate strength C_u. This type of equation could have the mathematical form

$$\frac{C_c}{C_o} = \left(\frac{\theta}{90}\right)^m (A\text{-}1) + 1 \qquad \text{(D-8)}$$

where m is an appropriate exponent. The benefits of such a general formula are that the lower limit of 1.0 in D-6 automatically is satisfied and that the approximation of the larger C_u coefficients could probably be further improved. Unfortunately, a basic linear (m = 1) or quadratic (m = 2) function in θ definitely is inadequate, thus the proper higher order exponent needs to be established. Since this development requires a more exhaustive study of the Method 1 ultimate strength solutions to identify the best predictor equation(s), it is left for possible future work. For now, the Method 4 formula D-5 appears to be quite satisfactory.

In summary, the advantages of the modified $\frac{C_a}{C_o}$ format in D-5 or D-7 are its simplicity, easy derivability, applicability to either bolts or welds and broad representation of capacity as merely a trigonometric function of load angle. The behavioral nonlinearity and computational complexity of the preferred Method 1 — ultimate strength solution can be safely and quickly approximated by this new C_a coefficient for any eccentric load angle. The only prerequisite is the baseline C_o coefficient for vertical load (using Method 1, such as the eccentrically loaded connector geometries tabulated in the AISC Manuals) must be available to define the A parameter. The Method 4 procedure will cover concisely all the other possible applied load orientations, thereby reducing a very difficult and tedious ultimate strength problem to a most manageable one.

The Appendix-Design examples are briefly re-done, using the modified $\frac{C_a}{C_o}$ equation:

Ex. 1 — ASD & LRFD

C_o = 3.55; θ = 60°

C_{max} = 2 × 6 = 12

$A = \frac{12}{3.55} = 3.38$

(D-3) $\frac{C_a}{C_o} = \frac{3.38}{(.866 + 3.38(.5))} = 1.322$

$\therefore C_a = 1.322(3.55) = 4.69$, **o.k.**

Ex. 2 — ASD

C_o = 0.704; θ = 75°

C_{max} = 1.856

$A = \frac{1.856}{0.704} = 2.64$

(D-3) $\frac{C_a}{C_o} = \frac{2.64}{(.966 + 2.64(.259))} = 1.6$

$\therefore C_a = 1.6\,(0.704) = 1.13$ **o.k.**

Ex. 2 — LRFD

C_o = 1.136; θ = 75°

C_{max} = 2.784

$A = \frac{2.784}{1.136} = 2.45$

(D-3) $\frac{C_a}{C_o} = \frac{2.45}{(.966 + 2.45(.259))} = 1.53$

$\therefore C_a = 1.53(1.136) = 1.74$ **o.k.**

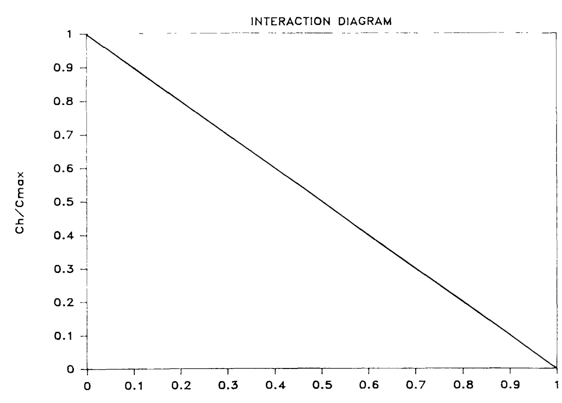


Figure D-3