Eccentric Connection Design: Geometric Approach

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Abstract: A noniterative and conservative method is developed to predict the ultimate capacity of eccentrically loaded steel connections. The method is based upon the inelastic behavior of the connectors and the geometry of the connection. This geometric approach can be used to analyze connections with skewed loads, arbitrary connection geometries, and varying bolt sizes. A simple interaction equation is developed so that connections with both high and low eccentricities may also be analyzed. Comparisons between these methods, experimental test results, and the ultimate strength method currently used in the American Institute of Steel Construction (AISC) design manuals are given. Results of the interaction equation and the geometric method are within a few percent of experimental test data and are closer than the ultimate strength method, without the need of an iterative approach. Several design examples reveal that the maximum results from the geometric approach and the interaction can be used in design.

Introduction

Many connections in steel structures must be designed for the condition in which the line of action of the applied load is offset from the centroid of the connection. This eccentricity of load creates an induced moment on the connection that must be resisted by additional shear forces in each connector. Examples of this type of connection include brackets, plate-girder-web splices, and beam-to-column connections.

Prior to the eighth edition of the AISC *Steel Construction Manual* (1980), connection analysis relied upon the assumption that the load-deformation response of the fasteners was elastic, which led to overly conservative results. Currently, the AISC *Allowable Stress Design* (ASD) (*Manual* 1989) and the *Load and Resistance Factor Design* (LRFD) (*Manual* 1986) use the ultimate strength method, which is an iterative approach that calculates a point of rotation about which the connectors pivot. This point, called the instantaneous center of rotation, assumes a nonlinear load-deformation response of the connectors (Crawford and Kulak 1971). Although the ultimate strength method is acceptable for standard connections, it cannot readily analyze connections with skewed loads, general connection geometries, and mixed connector sizes. There are examples in the ultimate strength method in which the normalized capacity of a connection (C_u) increases when a connector is removed. One example is a three-bolt row connection with bolt spacings of 15 cm (6 in.) and an eccentricity of 8 cm (3 in.), where C_u = 1.91 [Table XV, pages 4–66 (*Manual* 1989)]. By removing the bolt farthest from the load, a two-bolt row connection remains where C_u is increased to 2.0. The analysis of connections with skewed loads is treated with an elastic method (Iwankiw 1987) in the 1989 ASD manual. Skewed loadings based on the ultimate strength method are limited to angles of 0°, 45°, and 75° in the design tables in the 1986 LRFD manual.

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A simple approach for the analysis of eccentrically loaded connections that yields accurate results would be very useful for the connection designer. One such approach, based solely on the geometry of the connection, is presented.

FUNDAMENTALS

Fundamentals of the geometric approach are discussed here and shown in Fig. 1. The basic assumption in the geometric approach is that the applied load P, located at an eccentricity from the centroid of the connection e, is directly transferred to the individual connectors through the connection medium. This is based on a truss analogy in which each connector is attached directly to the load at point O (see Fig. 1) by truss members. Each of the connectors resists its share of load through shear forces. Other assumptions in the geometric approach include the following.

1. The direction of the load on an individual connector i, R_i, acts through point O and has a component, R_{yi}, that resists the load P. Point O is located at the intersection of the load and a line perpendicular to it through the centroid of the connection, as shown in Fig. 1. The component of force normal to the load, R_{xi}, and the component of force resisting the load, R_{yi}, are also shown in Fig. 1.
2. The deformation of each connector, Δ_i, is proportional to its length, l_i, from the point O: It is found from the equation $\Delta_i = \Delta_{\max} \cdot (l_i/l_{\max})$, where $\Delta_{\max}$ = maximum deformation of a connector, which occurs in the connector farthest from point O, at a distance $l_{\max}$.
3. The load in a connector is dependent on the deformation in that connector.
4. Equations of equilibrium of the forces and moments within the connection must be satisfied.

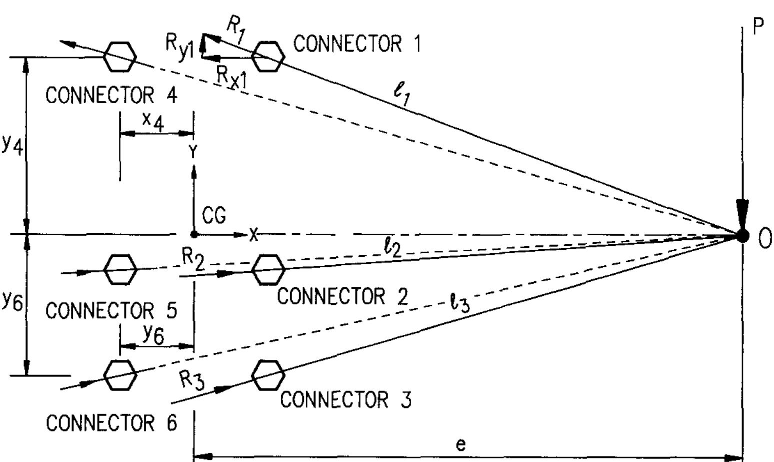

FIG. 1. Basics of Geometric Approach

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The components of the connector loads are given by the following equations:

$$R_{yi} = R_i\left(\frac{|y_i|}{l_i}\right) \quad (1)$$

and

$$R_{xi} = -R_i\left(\frac{e - x_i}{l_i}\right)_{\text{if } y_i>0} \quad (2a)$$

or

$$R_i\left(\frac{e - x_i}{l_i}\right)_{\text{if } y_i<0} \quad (2b)$$

where, x_i and y_i = x (normal to the load) and y (parallel to the load) distances, respectively, of connector i from the centroid of the connection. The location of the centroid of a connection found by

$$x_c = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} = 0 \quad (3)$$

and

$$y_c = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i} = 0 \quad (4)$$

where A_i = cross-sectional area of a connector; and n = total number of connectors in the connection.

The behavior of the connectors is nonlinear (Fisher 1965) and is described by the following equation:

$$R_i = R_{\text{ult}}(1 - e^{-\mu\Delta_i})^{\lambda} \quad (5)$$

where R_i = shear capacity; R_{ult} = ultimate shear capacity; Δ_i = bolt deformation; and μ and λ = coefficients that describe the deformation characteristics of connector i. Eq. (5) may be rewritten as

$$R_i = A_i\sigma_{\text{ult}}\delta_i \quad (6)$$

where δ_i = percentage of the ultimate load in a connector; σ_{ult} = ultimate shear stress in the connector; and $R_{\text{ult}} = A_i\sigma_{\text{ult}}$. For 1.9 cm (3/4 in.) diameter A-325 bolts, μ = 10 and λ = 0.55 (Crawford and Kulak 1971). Substituting these into (5) yields

$$R_i = R_{\text{ult}}(1 - e^{-10\Delta_i})^{0.55} = R_{\text{ult}}\delta_i \quad (7)$$

Properties for these bolts in single shear are $\Delta_{\max}$ = 0.9 cm (0.34 in.); R_{ult} = 329 kN (74 kips); and σ_{ult} = 1,154 MPa (167 ksi).

Equilibrium of forces within a connection requires that the horizontal components (R_{xi}) sum to zero and that the sum of the vertical components (R_{yi}) resists the applied load P. All loads pass through the point O; therefore, the sum of moments is zero.

The requirement that the sum of the R_{xi} components equal 0 is shown by

$$\sum_{i=1}^{n} R_{xi} = \sigma_{\text{ult}} \sum_{i=1}^{n} \delta_i \left[\left(A_i \frac{e - x_i}{l_i} \right)_{\text{if } y<0} \text{ or } -\left(A_i \frac{e - x_i}{l_i} \right)_{\text{if } y>0} \right] \quad \ldots\ldots (8)$$

Typically, (8) is approximately zero, because the sum of the terms in the square brackets is nearly zero since the origin of the connection is located at the centroid [(3) and (4)]. If a connector i is located at $y_i = 0$, it has a component $R_{xi} = 0$, since in the limits as $y_i \to 0(+)$ and $y_i \to 0(-)$; R_{xi} has equal magnitudes but opposite signs.

The capacity of the connection R_g, which resists the load P, is given by

$$R_g = \sum_{i=1}^{n} R_{yi} = \sigma_{\text{ult}} \sum_{i=1}^{n} A_i \delta_i \left(\frac{|y_i|}{l_i} \right) \quad \ldots\ldots (9)$$

which is based solely on the geometry of the connection. When a connector i is located at $y_i = 0$, (9) yields that this connector does not resist any of the load P. In the rare case in which all of the connectors are located on the x-axis (or $y_i = 0$ for all $i = 1, n$), (9) yields a connection with zero capacity. This problem is addressed with an interaction equation later in this paper.

Eq. (9) can be normalized for connectors having the same cross-sectional area and ultimate stress by dividing it by $R_{\text{ult}} = A_i \sigma_{\text{ult}}$, which yields

$$C_g = \sum_{i=1}^{n} \delta_i \left(\frac{|y_i|}{l_i} \right) \quad \ldots\ldots (10)$$

where, C_g = normalized capacity of a connection based on the geometric approach.

A comparison of the direction and magnitudes of the forces in the connectors in the geometric approach, elastic method, and ultimate strength method is shown in Fig. 2. The results of experimental tests (Crawford and Kulak 1971) on eccentrically loaded bolted connections were used to develop the ultimate strength method, in which all connectors rotate about an instantaneous center of rotation. In the elastic centroid method all of the bolts are assumed to rotate about the centroid of the connection from the induced moment, $P \cdot e$. All of the bolts rotate about individual radii in the geometric approach. The connector forces in each of the three methods are consistent in direction and magnitude, as shown in Fig. 2.

Interaction Equation

An interaction equation, based upon the limiting cases of the eccentricities of a connection, is developed to generalize the geometric method. The equation is based upon (9), with a high eccentricity, and test results for long lap splices (low eccentricity). This interaction equation can be used to calculate the capacity of connections of either high or low eccentricities.

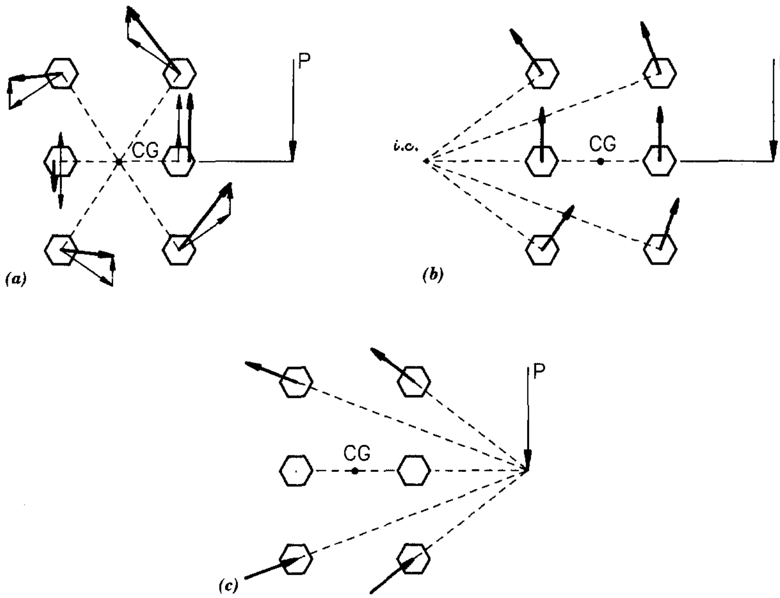

FIG. 2. Methods for Eccentrically Loaded Connections: (*a*) Elastic Center; (*b*) Ultimate Strength; (c) Geometric Approach

As the eccentricity of a connection becomes large, l_i approaches e for all connectors. Substituting this into (9) yields

$$R_{gh} = \left(\frac{1}{e}\right)\sigma_{\text{ult}}\sum_{i=1}^{n} A_i\delta_i|y_i| \quad (11)$$

where, R_g has been replaced by R_{gh} for connections with high eccentricities. In addition, since $l_i \rightarrow e$; $\Delta_i \rightarrow \Delta_{\max}$ and $\delta_i \rightarrow$ constant for all $i = 1, n$. For 1.9 cm (3/4 in.) diameter A-325 bolts, $\delta_i = 0.98$ from (7); and (11) can be approximated as

$$R_{gh} = \left(\frac{0.98}{e}\right)\sigma_{\text{ult}}\sum_{i=1}^{n} A_i|y_i| \quad (12)$$

The geometrical approach for eccentric connections becomes less valid as the eccentricity becomes small. As e approaches 0, or the load is located near the centroid of the connection, the connection acts more like a lap splice than an eccentrically loaded connection. An equation for the capacity of low-eccentricity connections is developed based on the test results of lap splices (Bendigo et al. 1963). The linear mean curve of the unbuttoning factor [shown in Fig. 8 in the paper by Bendigo et al. (1963)] is chosen to represent the variation of the capacity in connections of low eccentricity. This curve is a function of the size of the connection parallel to the load, and is shown in (14). The ultimate capacity, R_{gl}, of a long lap splice or a connection with low eccentricity is given by

$$R_{gl} = \gamma\sum_{i=1}^{n} A_i\sigma_{\text{ult}} \quad (13)$$

where

$$\gamma = 0.954 - 0.00765(y_{max} + |y_{min}|) \quad (14)$$

The term γ = strength-reduction factor for lap splices; and y_{max} and y_{min} = maximum and minimum values of y_i, respectively, in the entire connection. In the actual test results, the term $(y_{max} + |y_{min}|)$ represents the length of the splice parallel to the load.

To generalize the geometrical method, an equation is derived based on a circular interaction for σ_{ult} between connections of low eccentricity (13) and large eccentricity (12), and results in

$$\sigma_{ult}^2 = \left(\frac{R_{gl}}{\gamma \sum_{i=1}^{n} A_i}\right)^2 + \left(\frac{R_{gh}e}{0.98 \sum_{i=1}^{n} A_i|y_i|}\right)^2 \quad (15)$$

which can be rearranged as

$$R_{int} = \frac{\sigma_{ult}}{\sqrt{\left(\frac{1}{\gamma \sum_{i=1}^{n} A_i}\right)^2 + \left(\frac{e}{0.98 \sum_{i=1}^{n} A_i|y_i|}\right)^2}} \quad (16)$$

where R_{int} = capacity based on the interaction.

If all of the connectors have the same cross-sectional area, (16) can be normalized by dividing it by $R_{ult} = A_i\sigma_{ult}$, which yields

$$C_{int} = \frac{1}{\sqrt{\left(\frac{1}{n\gamma}\right)^2 + \left(\frac{e}{0.98 \sum_{i=1}^{n} |y_i|}\right)^2}} \quad (17)$$

where C_{int} = normalized capacity of a connection based on the interaction and is only a function of the geometry of the connection. Note that if all of the connectors are located on a line perpendicular to the load, (17) and (14) yield that $C_{int} = 0.954 \cdot n$, which is independent of e.

Results

To validate the geometric approach, normalized connection capacities calculated from (10), and the interaction equation, (17), are compared with experimental eccentric-connection test results (C_{test}) for 1.9 cm (3/4 in.) diameter A-325 bolts (Crawford and Kulak 1971)] and results obtained from the ultimate strength method (C_u). The normalized results from the test data are obtained by dividing the measured capacity of the connection by the ultimate capacity of a single bolt in double shear; $R_{ult} = 2 \times 329$ kN (74 k) = 658 kN (148 k). These results are shown in Table 1.

The geometric approach is the closest of all the methods, with errors from +1% to +9% and a mean of 5.8% and standard deviation of 2.5%. Results from the interaction equation have errors from +5% to +11%, with a mean of 8.0% and standard deviation of 2.6%. Errors in the ultimate strength method ranged from +7% to +27%, with a mean of 14.8% and

TABLE 1. Comparison of Methods and Experimental Results

Mark (1)	e in. (2)	e cm (3)	$n_x \times n_y$ (4)	S_y in. (5)	S_y cm (6)	S_x in. (7)	S_x cm (8)	Test kips (9)	Test (kN) (10)	C_{test} (11)	C_g (12)	C_{int} (13)	C_u (14)
(*a*) Normalized Bolt Capacities													
B1	8	20	1 × 5	2.5	6	—[a]	—[a]	225	1,001	1.52	1.62	1.69	1.70
B2	10	25	1 × 5	3	8	—[a]	—[a]	240	1,023	1.55	1.57	1.63	1.66
B3	12	30	1 × 5	3	8	—[a]	—[a]	190	485	1.28	1.35	1.39	1.39
B4	13	33	1 × 6	3	8	—[a]	—[a]	251	1,116	1.70	1.84	1.89	1.88
B5	15	38	1 × 6	3	8	—[a]	—[a]	221	983	1.49	1.63	1.66	1.65
B6	12	30	2 × 4	3	8	2.5	6	264	1,174	1.78	1.86	1.89	2.02
B7	15	38	2 × 4	3	8	2.5	6	212	943	1.43	1.52	1.53	1.61
B8	15	38	2 × 5	2	6	2.5	6	266	1,183	1.80	1.88	1.91	2.09
(*b*) Normalized Rivet Capacities													
TP-1	2.5	6	1 × 3	3	8	—[a]	—[a]	216	961	2.02	1.51	1.78	2.00
TP-2	3.5	9	1 × 3	3	8	—[a]	—[a]	161	716	1.51	1.28	1.43	1.64
TP-3	6.5	17	1 × 3	3	8	—[a]	—[a]	100	445	0.94	0.82	0.86	0.92
TP-4	2.5	6	1 × 6	3	8	—[a]	—[a]	550	2,446	5.15	4.36	4.55	5.22
TP-5	4.5	11	1 × 6	3	8	—[a]	—[a]	440	1,957	4.12	3.61	3.82	4.25
TP-6	6.5	17	1 × 6	3	8	—[a]	—[a]	362	1,610	3.39	3.00	3.17	3.38
TP-7	3.5	9	2 × 2	3	8	2.5	6	222	987	2.08	1.60	1.53	1.85
TP-8	6.5	17	2 × 2	3	8	2.5	6	120	534	1.12	0.89	0.88	1.15
TP-9	3.5	9	2 × 4	3	8	2.5	6	568	2,526	5.32	4.59	4.87	5.28
TP-10	6.5	17	2 × 4	3	8	2.5	6	354	1,575	3.31	3.11	3.22	3.48

[a]Not applicable.

standard deviation of 7.2%. In each case the ultimate strength method was the least accurate and most unconservative.

Normalized capacities of rivet tests on 1.9 cm (3/4 in.) diameter rivets in double shear, where $R_{ult} = 2 \times 238$ kN (53.4 k) = 475 kN (106.8 k) (Yarimci and Slutter 1963), are also compared to the geometric approach, the interaction equation, and the ultimate strength method. These capacities are shown in Table 1. Note that the results from these three methods cannot accurately be compared with test results for a riveted connection, since these methods are based on the behavior of 1.9 cm (3/4 in.) diameter A-325 bolts [(7)]. Results from the interaction equation are within −3% to −26%, with a mean value of −9.9% and standard deviation of 8.1%. The geometric approach has errors from −6% to −25%, with a mean of −15.7%. Errors in the ultimate strength method ranged from −9% to +11%, with a mean of 0.9% and standard deviation of 5.6%. Results from the geometric approach and the interaction equation are consistently conservative when compared to the experimental results and the ultimate strength method.

No comparison is made for the results from lap splices, where $e = 0$, since the interaction equation is based on the experimental data [Fig. 8 in Bendigo et al. (1963)] and is in close agreement. The geometric approach and the ultimate strength method are not intended for the analysis of lap splices.

Design Examples

Several examples are given to compare the ultimate strength method currently used in the AISC manuals (*Steel* 1980; *Manual* 1986, 1989), the geometric approach (10), and the interaction equation (17) for eccentric connections. One large (48 connectors) and one small (four connectors) connection is analyzed, along with a skewed, eccentrically loaded connection (nine connectors), with various angles of loading.

The largest design example in the AISC *Manuals* (1980, 1986, 1989) based on the ultimate strength method is a four-column by 12-row (4 × 12) connection. Spacings of the columns that are parallel to the load (S_x) and of the rows that are normal to the load (S_y) are 7.6 cm (3 in.). Results for eccentricities ranging from 0 to 91 cm (0 to 36 in.) are shown in Table 2.

TABLE 2. Results for Two Design Examples

e		4 × 12			2 × 2		
in. (1)	cm (2)	C_g (3)	C_{int} (4)	C_u (5)	C_g (6)	C_{int} (7)	C_u (8)
0	0	36.6	33.7	48.0	2.78	3.77	4.00
2	5	36.0	33.3	46.0	2.40	2.32	2.54
4	10	34.4	32.1	43.2	1.44	1.37	1.67
6	15	32.0	30.4	39.6	0.97	0.95	1.22
8	20	29.4	28.4	35.8	0.73	0.72	0.96
10	25	26.9	26.4	32.0	0.59	0.58	0.79
12	30	24.6	24.4	28.7	0.49	0.49	0.67
16	41	20.9	20.8	23.4	0.37	0.37	0.51
20	51	17.9	17.9	19.6	0.29	0.29	0.41
24	61	15.6	15.6	16.8	0.24	0.24	0.34
30	71	13.0	13.0	13.8	0.20	0.21	0.28
36	91	11.1	11.1	11.6	0.16	0.16	0.23

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For connections of eccentricities of greater than (25 cm) (10 in.) the results of all of the methods are within 18%. As the eccentricity becomes small, or the connection approaches a lap splice (where $e = 0$), the results from the different methods vary greatly. The ultimate strength method predicts the full capacity of all 48 connectors; the interaction equation predicts 33.7 (70% of the full capacity), and the geometric approach yields 36.6 (76%). Actual test results from lap splices (Bendigo et al. 1963) show a 70% reduction in capacity for a connection with a length of 84 cm (33 in.) [(14)]. The interaction equation and the geometric approach are conservative in all cases when compared with the ultimate strength method; with the results from the interaction equation closer to the ultimate strength method than the geometric approach.

The capacity for a 2 × 2 connection with $S_y = S_x = 7.6$ cm (3.0 in.) is compared for all three methods. This case is run to check the validity of the interaction equation and the geometric approach for shallow connections. These results, shown in Table 2 for eccentricities from 0 to 91 cm (0 to 36 in.), have a maximum deviation of 30% between all three methods; with the capacities in the ultimate strength method being consistently higher than the two methods presented in this paper. Detailed calculations for a 2 × 2 connection with an eccentricity of 25 cm (10 in.) are given in Appendix I.

One example is made to show the ease of the proposed methods in analyzing connections with skewed loads. This connection is a 3 × 3 with connector spacings of 6 cm (3 in.). A normalized eccentricity x_0, used in the AISC *Manual* (1986), is shown in Fig. 3, along with the relationship between e and x_0. Results for x_0 from 0 to 91 cm (0 to 36 in.) from the geometric approach, interaction equations, and the ultimate strength method [Table XIV, AISC LRFD (*Manual* 1986)] for skewed loads at 0°, 45°, and 75° are given in Table 3. Deviations in the results range from 9% to 40%, with the higher discrepancies occurring with the higher eccentricities. Results from both of the proposed methods are consistently lower than from the ultimate strength method. For all cases, the interaction equation results are closer to the ultimate strength method than the geometric approach.

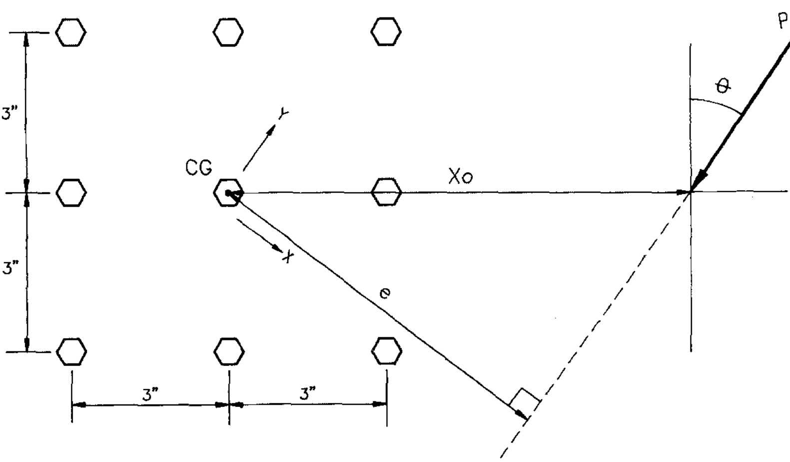

FIG. 3. 3 × 3 Connection with Skewed Load

TABLE 3. Results for 3 × 3 Connection with Skewed Loading

X_0		$\theta = 0°$			$\theta = 45°$			$\theta = 75°$		
in. (1)	cm (2)	C_g (3)	C_{int} (4)	C_u (5)	C_g (6)	C_{int} (7)	C_u (8)	C_g (9)	C_{int} (10)	C_u (11)
0	0	4.67	8.17	9.00	4.65	8.00	9.00	5.01	8.08	9.00
2	5	4.28	5.99	6.89	4.41	6.62	7.54	4.95	7.88	8.59
4	10	3.53	3.88	4.97	3.84	4.73	5.84	4.87	7.36	8.07
6	15	2.70	2.77	3.85	3.11	3.52	4.78	4.79	6.70	7.41
8	20	2.11	2.13	3.11	2.52	2.76	4.00	4.70	6.00	6.75
10	25	1.71	1.72	2.59	2.11	2.26	3.40	4.59	5.36	6.17
12	30	1.44	1.45	2.21	1.81	1.90	2.95	4.40	4.80	5.67
16	41	1.09	1.09	1.70	1.39	1.45	2.81	3.81	3.92	4.87
20	64	0.87	0.88	1.37	1.13	1.16	1.89	3.25	3.28	4.24
28	71	0.63	0.63	0.99	0.82	0.83	1.38	2.43	2.44	3.35
36	91	0.49	0.49	0.77	0.64	0.65	1.08	1.93	1.93	2.74

CONCLUSIONS

A noniterative and seemingly conservative approach for eccentrically loaded connections that is based solely on the geometry of the connection was presented. Based on this approach, an interaction equation for high eccentricities and experimental data for lap splices (low eccentricity) is derived. Both the interaction equation and the geometric approach are shown to yield results consistently better than the ultimate strength method when compared to test data from lap splices and eccentrically loaded bolted connections. The maximum value of the results from the interaction equation and the geometric approach for eccentric connections is shown to be reasonable for use in design. Both proposed methods can be generalized for any combination of connectors, geometries, and loadings.

The interaction equation is shown to be a simple design tool for the analysis of eccentric connections, while yielding results consistent with actual test data; and it is conservative when compared with the ultimate strength method for several test cases. This interaction equation can easily analyze connections with arbitrary geometries, loads with high and low eccentricities, and loads skewed with respect to the axes of the connection. Results from the interaction equation can easily be programmed on a hand calculator by entering only: (1) The depth of the connection in the direction of the load; (2) the number of connectors; (3) the eccentricity of the load; and (4) the sum of the distances of the connectors in the direction of the load.

Both the geometric approach and the interaction equation have the capability to analyze connections with varying connector sizes. No attempt is made to use these methods for this type of analysis since no test data or methods have yet been developed.

Actual verification of the geometric approach, the interaction equation and the ultimate strength method, should be conducted through more experimental testing. The most pressing needs seem to be connections with skewed loadings and quantifying the response of connectors other than 1.9 cm (3/4 in.) diameter A-325 bolts.

APPENDIX I. EXAMPLE CALCULATION

Calculations for the normalized connection capacities C_g [(10)] and C_{int} [(17)] of the 2 × 2 connection shown in Fig. 4 are detailed in the following.

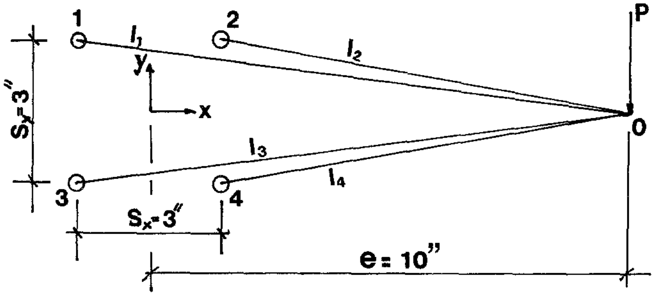

FIG. 4. Example Connection Detailed in Appendix I

Coordinates x_i and y_i for the four bolts are

$$x_1 = 3.8 \text{ cm } (-1.5 \text{ in.}); \; y_1 = 3.8 \text{ cm } (1.5 \text{ in.}) \quad (18a)$$

$$x_2 = 3.8 \text{ cm } (1.5 \text{ in.}); \; y_2 = 3.8 \text{ cm } (1.5 \text{ in.}) \quad (18b)$$

$$x_3 = 3.8 \text{ cm } (-1.5 \text{ in.}); \; y_3 = 3.8 \text{ cm } (-1.5 \text{ in.}) \quad (18c)$$

$$x_4 = 3.8 \text{ cm } (1.5 \text{ in.}); \; y_4 = 3.8 \text{ cm } (-1.5 \text{ in.}) \quad (18d)$$

Distances of each bolt from the point O l_i are given by

$$l_1 = [(10.0 + 1.5)^2 + (1.5)^2]^{0.5} = 29.5 \text{ cm } (11.6 \text{ in.}) \quad (19a)$$

$$l_2 = [(10.0 - 1.5)^2 + (1.5)^2]^{0.5} = 21.9 \text{ cm } (8.6 \text{ in.}) \quad (19b)$$

$$l_3 = [(10.0 + 1.5)^2 + (1.5)^2]^{0.5} = 29.5 \text{ cm } (11.6 \text{ in.}) \quad (19c)$$

$$l_4 = [(10.0 - 1.5)^2 + (1.5)^2]^{0.5} = 21.9 \text{ cm } (8.6 \text{ in.}) \quad (19d)$$

The maximum length $l_{max} = l_1 = l_3 = 29.5$ cm (11.6 in.) is found from these values. Connection deformations $\Delta_i = \Delta_{max}(l_i/l_{max})$, where $\Delta_{max} = 0.9$ cm (0.34 in.), are

$$\Delta_1 = 0.34 \left(\frac{11.6}{11.6}\right) = 0.9 \text{ cm } (0.34 \text{ in.}) \quad (20a)$$

$$\Delta_2 = 0.34 \left(\frac{8.63}{11.6}\right) = 0.6 \text{ cm } (0.25 \text{ in.}) \quad (20b)$$

$$\Delta_3 = \Delta_1 = 0.9 \text{ cm } (0.34 \text{ in.}); \text{ and } \Delta_4 = \Delta_2 = 0.6 \text{ cm } (0.25 \text{ in.}) \quad (20c)$$

The percentage reductions in the connections δ_i as calculated from (7) are

$$\delta_1 = [1.0 - e^{(-10\times 0.34)}]^{0.55} = 0.98 \quad (21a)$$

$$\delta_2 = [1.0 - e^{(-10\times 0.25)}]^{0.55} = 0.96 \quad (21b)$$

$$\delta_3 = \delta_1 = 0.98; \text{ and } \delta_4 = \delta_2 = 0.96 \quad (21c)$$

Now the normalized connection capacity C_g can be calculated from (10) as

$$C_g = 2\left(0.98\frac{1.5}{11.6} + 0.96\frac{1.5}{8.63}\right) = 0.59 \quad (22)$$

which agrees with the value in Table 2.

The second normalized connection capacity C_{int} can be calculated from (17), where the eccentricity $e = 25.4$ cm (10 in.) and the total number of connectors $n = 4$. Summation of the absolute values of $y_i = 4 \times 1.5 =$ (15.2 cm) (6.0 in.). The strength-reduction factor calculated from (14) is

$$\gamma = 0.954 - 0.00765(1.5 + |-1.5|) = 0.93 \quad (23)$$

Now the connection capacity is calculated from (17) as

$$C_{int} = \left[\left(\frac{1}{4 \times 0.93}\right)^2 + \left(\frac{10.0}{0.98 \times 6.0}\right)^2\right]^{-0.5} = 0.58 \quad (24)$$

which also agrees with that in Table 2.

Appendix II. References

Bendigo, R. A., Hansen, R. M., and Rumpf, J. L. (1963). "Long bolted joints." *J. Struct. Div.*, 89(6), ASCE, 187–213.

Crawford, S. F., and Kulak, G. L. (1971). "Eccentrically loaded bolted connections." *J. Struct. Div.*, ASCE, 97(3), 765–783.

Fisher, J. W. (1965). "Behavior of fasteners and plates with holes." *J. Struct. Div.*, ASCE, 91(6), 265–286.

Iwankiw, N. R. (1987). "Design for eccentric and inclined loads on bolt and weld groups." *Engrg. J.*, 24(4), 164–171.

Manual of steel construction; Allowable stress design. (1989). 9th Ed., American Institute of Steel Construction, (AISC), Chicago, Ill.

Manual of steel construction; Load and resistance factor design. (1986). 1st Ed., American Institute of Steel Construction, (AISC), Chicago, Ill.

Steel constuction manual. (1980). 8th Ed., American Institute of Steel Construction, (ASIC), Chicago, Ill.

Yarimci, E., and Slutter, R. G. (1963). "Results of tests on riveted connections." *Fritz Engrg. Lab. Rep. No. 200.63.403.1*, Lehigh University, Bethlehem, Pa.

Appendix III. Notation

The following symbols are used in this paper:

A_i = cross-sectional area of connector i;
C_g = normalized connection capacity from geometric approach;
C_{int} = normalized connection capacity from interaction equation;
C_{test} = normalized connection capacity from test results;
C_u = normalized connection capacity from ultimate strength method;
d_i = diameter of connector;
e = eccentricity of load in connection;
l_i = length of connector i from point O;
l_{max} = maximum value of l_i;
n = total number of connectors in connection;
n_x = number of columns (normal to x) in rectangular connection;
n_y = number of rows (normal to y) in rectangular connecton;
P = applied load;
R_g = connection capacity from geometric approach;

R_{gh} = connection capacity with high eccentricity;
R_{gl} = connection capacity with low eccentricity;
R_i = shear load in connector i;
R_{int} = connection capacity from interaction equation;
R_{ult} = ultimate capacity of connector;
R_{xi} = the component of R_i normal to P;
R_{yi} = component of R_i parallel to P;
S_x = spacing of columns in rectangular connection;
S_y = spacing of rows in rectangular connection;
x_c = x-coordinate of centroid of connection = 0;
x_i = distance of connector i from the centroid of connection normal to P;
x_0 = normalized eccentricity used in AISC manuals;
y_c = y-coordinate of centroid of connection = 0;
y_i = distance of connector i from centroid of connection parallel to P;
y_{max} = maximum value of y_i;
y_{min} = minimum value of y_i;
Δ_i = deformation of connector i;
Δ_{max} = maximum deformation of connector, corresponding to σ_{ult};
δ_i = percent reduction of capacity of connection with eccentric load;
γ = reduction in capacity of lap splice connection;
θ = skew angle of load on rectangular connection;
λ = parameter in capacity of loaded connector;
μ = parameter in capacity of loaded connector; and
σ_{ult} = ultimate shear stress in connector.