Development of Geometrically Nonlinear Mass Damper with Mass Moment of Inertia for Seismically-Excited Buildings

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SUMMARY

Mass dampers are one of the widely accepted control techniques for seismically-excited tall buildings. When these mass dampers are optimally tuned to the primary natural frequencies of buildings, the structural responses (i.e., floor displacements and accelerations) can be effectively mitigated. However, the tuned mass dampers may introduce a large displacement when buildings are subjected to intensive earthquake loadings. To address this shortcoming, some researchers suggested adding nonlinear restoring forces to mass dampers, such as forming a track nonlinear energy sink. Still, a sufficiently large mass in this nonlinear mass damper is a critical issue. Therefore, this research develops a track nonlinear energy sink with a mass moment of inertia. The proposed mass damper has not only the feature of nonlinear restoring forces but also increased effective mass by rotational components. In this study, the equation of motion for a building with the proposed mass damper is derived. A design method based on the frequency-domain input-output relationship is established. To further verify the damper performance, a prototype track nonlinear energy sink with a mass moment of inertia is fabricated and experimentally evaluated by real-time hybrid simulation. As seen in the experimental results, the proposed mass damper outperforms the conventional track nonlinear energy sink. Moreover, only adequately effective mass, i.e., sufficient momentum to maintain static friction, is feasible to generate control performance against input ground motion.

Keywords*: Seismic Control; Track Nonlinear Energy Sink with Rotational Mass; Frequency-Domain Input-Output Relationship; Real-Time Hybrid Simulation; Mass Damper*

INTRODUCTION

A tuned mass damper is one of the effective control strategies to mitigate structural responses against strong winds and perhaps earthquakes. However, when subjected to significant excitation, excessive displacements would be found in tuned mass dampers. Meanwhile, a narrow band of effective frequencies may lower the control performance of tuned mass dampers when the detuning effect exists. Instead of tuned mass dampers, researchers developed a track nonlinear energy sink (track-NES), which provides nonlinear restoring forces and avoids the detuning effect by an increased band of effective frequencies (Wang et al. 2014; Wang et al. 2016; Lu et al. 2017; Wang et al. 2020b). Also, the excessive mass damper displacements can be improved by track-NES. Still, the condition for the reduced displacements is valid only when the effective translational mass in track-NES is sufficiently large. Therefore, increasing the translational inertial force of track-NES without introducing too much dead load to a structure becomes a challenging issue.

The track nonlinear energy sink was developed with a nonlinear track, and the control performance was experimentally verified on a two-story laboratory-scale building (Wang et al. 2014). In this development, track-NES was designed to slide on a fourth-order polynomial track, effectively reducing mass movement. To better lower the mass movement, Wang *et al.* (2016) added a one-sided stopper on the track and formed a new type of

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track-NES, namely a single-sided vibro-impact track nonlinear energy sink. This stopper not only limited the movement of the mass but also transferred energy from lower modes to higher modes of structures more effectively. Lu *et al.* (2017) developed an optimal design method for track-NES and experimentally investigated the materials of wheels on the mass. The results showed that increasing the friction between the wheels and track can better reduce the floor accelerations of a building structure against earthquakes. Wang *et al.* (2020a) numerically evaluated the control effectiveness of track-NES for a 32-story tall building. Track-NES only required a smaller stroke and damping to achieve similar control performance compared to the tuned mass damper. In the design of track-NES, researchers employed time-domain approaches to determine the optimal parameters. For instance, given an input time history, the parameters can be optimally determined when the best control objective is met. Moreover, the linearization of track-NES yields an opportunity to decide associated parameters by the optimal design approach of tuned mass dampers. Still, the mass ratio between the track-NES and primary structure is too small in these developments to be sufficient for seismic control applications.

In this study, a track nonlinear energy sink with a mass moment of inertia is developed and experimentally verified. The proposed mass damper features nonlinear restoring forces and has increased effective mass by rotational components. First, the equation of motion for a building with the proposed mass damper is derived. A design method based on the frequency-domain input-output relationship is established. In addition, a prototype track nonlinear energy sink with a mass moment of inertia is fabricated and experimentally evaluated by real-time hybrid simulation. The experimental results demonstrate that the proposed mass damper provides better control performance than the conventional track nonlinear energy sink. Moreover, the kinetic friction between the wheels and track should be continuously maintained to generate desired control effectiveness against earthquakes.

MODELING OF Track Nonlinear Energy Sink with Mass Moment of Inertia

In the modeling, a track-NES with rotational mass attached to the top of a building is considered, as shown in Figure 1. First, the rotational mass is formed by a flywheel coupled with a multi-gear system. Thus, the equation of motion for the proposed track-NES with rotational mass alone is written by

$$\mathbf{M}_{\rm s}\ddot{\mathbf{x}} + \mathbf{C}_{\rm s}\dot{\mathbf{x}} + \mathbf{K}_{\rm s}\mathbf{x} = -\mathbf{M}_{\rm s}\mathbf{1}\ddot{x}_{\rm g} - \mathbf{b}f_{\rm N} \tag{1}$$

$$(m_{\rm N} + m_{\rm rot})(1 + h'^2)\ddot{u}_{\rm N} + c_{\rm N}\dot{u}_{\rm N} + m_{\rm N}gh' + (m_{\rm N} + m_{\rm rot})\dot{u}_{\rm N}^2 h'h'' = -m_{\rm N}(\ddot{x}_2 + \ddot{x}_{\rm g}) \tag{2}$$

where

$$u_{\rm N} = x_{\rm roof} - x_{\rm N},\ f_{\rm N} = m_{\rm N}(\ddot{u}_{\rm N} + \ddot{x}_2 + \ddot{x}_{\rm g})$$

$\mathbf{M}_{\rm s}$, $\mathbf{C}_{\rm s}$, and $\mathbf{K}_{\rm s}$ are the mass, damping, and stiffness matrices of the building; $\mathbf{1}$ indicates a vector consisting of 1 in all entries; $\ddot{x}_{\rm g}$ is the ground acceleration; $\mathbf{b}$ is the influence matrix due to the translational inertial force provided by the track-NES; $m_{\rm N}$ is the total mass including the flywheel, gears, and wheels; $m_{\rm rot}$ indicates the effective translational mass introduced by all rotatable components (i.e., the flywheel, gears, and wheels); h is the track shape as a function of horizontal displacement, $u_{\rm N}$, relative to the roof of the building, $x_{\rm roof}$, and $x_{\rm N}$ is the track-NES displacement relative to the ground; $c_{\rm N}$ denotes the inherent damping coefficient existing in the track-NES and is typically relatively small; g is the gravitational acceleration. This derivation implies the rolling friction between the wheels and track in the rotational mass; thus, an adequate normal force should always exist and result in static friction to maintain the wheel rolling. If the wheels stop rolling, $m_{\rm rot}$ will become zero and the overall equation is changed into the conventional track-NES as the one in Wang *et al.* (2014).

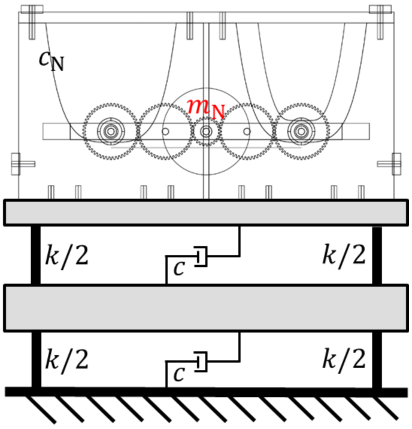

Figure 1 Illustration of a two-story building with a track-NES having a flywheel triggered by a multi-gear system.

Frequency-Domain Input-Output Relationship

The frequency-domain input-output relationship from base excitation to the proposed track-NES with rotational mass is investigated. This frequency-domain input-output relationship allows an understanding of the response magnification and effective frequency range of the track-NES. Moreover, this frequency-domain relationship also offers opportunities to explore the interaction between the track-NES and primary structure and even design this track-NES. Therefore, the nonlinear effects of this track-NES can be directly included in the design without model linearization.

First, the right-hand side in Eq. (2) is changed into harmonic excitation. Assume that the track shape is a fourth-order polynomial function as

$$h(u_{\mathrm{N}}) = au_{\mathrm{N}}^4 \tag{3}$$

where a is the coefficient of the track shape controlling the nonlinearity of the restoring force provided by the proposed track-NES. By the harmonic integration method (Wang et al. 2020b), the input-output relationship can be obtained by solving

$$(-A\Omega^2(1+H) + -5a^2A^7\Omega^2(1+H) + 4a^2A^7\Omega^2(1+H) + 2agA^3)^2 + \left(\frac{c_{\mathrm{N}}A\Omega}{m_{\mathrm{N}}}\right)^2 = B^2 \tag{4}$$

where H represents the m_{rot}-to-m_{N} ratio. Given an excitation frequency, Ω, the harmonic displacement amplitude, A, can be solved by a known input harmonic amplitude, B, of which is an acceleration. For example, the mass, damping coefficient, and track shape coefficient are 2.425 kg, 1.6 N-sec/m, and 2000 1/m^3. Then, Figure 2 displays the input-output relationship with and without rotational mass. As seen, the displacement amplitude highly depends on excitation magnitudes, and larger excitation magnitudes would increase the effective frequency range. Adding rotational mass shifts the effective range to low frequencies. Also, the displacement amplitudes consist of two phases, i.e., stable and unstable phases. By looking into the stable phase, an approximate resonant frequency exists. For instance, the resonant frequency is between 0.8-1.3 Hz in Figure 2(a) and 1.2-1.8 Hz in Figure 2(b). Increasing the input magnitude results in a lower resonant frequency. The frequency-domain input-output relationship provides rich information for the frequency content of the proposed track-NES with rotational mass.

(a) (b)

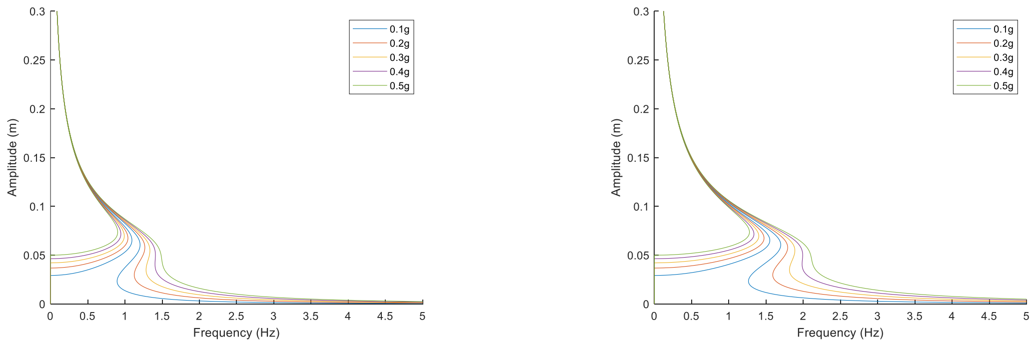

Figure 2 Frequency-domain displacement amplitudes of track-NES under harmonic excitation: (a) with and (b) without rotational mass.

The frequency-domain relationship derived by Eq. (3) can be applied to obtain responses of the primary structure when the track-NES with rotational mass is attached. Because the output amplitude of the track-NES is a function of excitation frequency and amplitude, the calculation of structural frequency-domain responses needs an iterative process by

$$Y^{j+1}_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}}}(\Omega) = H_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}},\ddot{x}_{\mathrm{g}}}(\Omega)B_{\ddot{x}_{\mathrm{g}}} + H_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}},f_{\mathrm{N}}}(\Omega)H_{f_{\mathrm{N}},\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}}}(\Omega,B)Y^{j}_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}}}(\Omega) \tag{5}$$

where

$$Y^{1}_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}}}(\Omega) = H_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}},\ddot{x}_{\mathrm{g}}}(\Omega)B_{\ddot{x}_{\mathrm{g}}}$$

$H_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}},\ddot{x}_{\mathrm{g}}}(\Omega)$ and $H_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}},f_{\mathrm{N}}}(\Omega)$ are constant frequency response functions from the primary structure; $B_{\ddot{x}_{\mathrm{g}}}$ is the ground excitation magnitude and is assumed to be constant over frequencies; $H_{f_{\mathrm{N}},\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}}}(\Omega,B)$ can be derived by a similar approach as in Eq. (3); $Y^{j}_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}}}(\Omega)$ is the calculated absolute acceleration at the roof at the j-th step. The iterative process ends when the error satisfies

$$\left\| Y^{j+1}_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}}}(\Omega) - Y^{j}_{\ddot{x}^{\mathrm{abs}}_{\mathrm{roof}}}(\Omega) \right\|_2 < \varepsilon \tag{6}$$

where ε is a sufficiently small number. A primary structure is considered with a single degree of freedom and has a mass and damping ratio of 350 kg and 2 %, while the natural frequency of this structure varies from 0.6 to 1.8 Hz. The parameters of the track-NES used to generate Figure 2 are employed in this example. By Eq. (4), Figure 3 exhibits the absolute accelerations of the primary structure. The results demonstrate the statistically best control performance when the structure has a natural frequency at 1.0 and 1.4 Hz for the track-NES with and without rotational mass. Indeed, this approach in Eq. (4) can yield a possible design method to be developed.

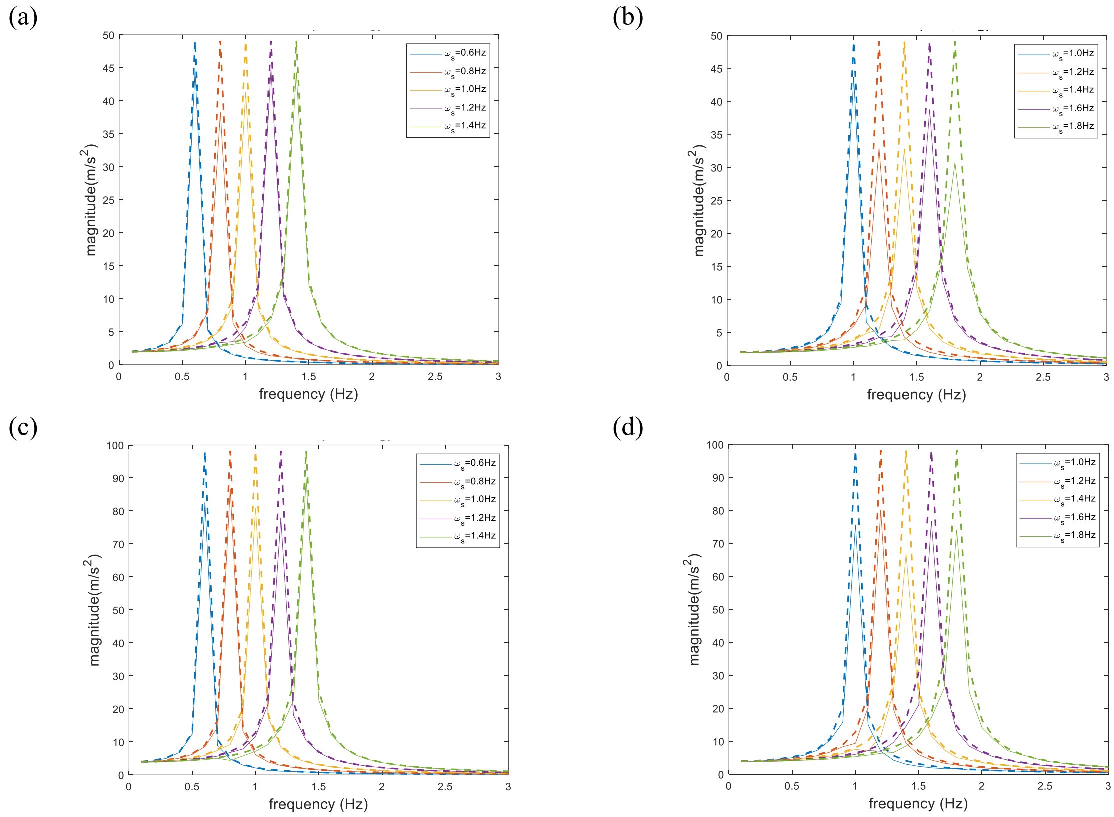

Figure 3 Frequency-domain responses of absolute structural accelerations: (a) with and (b) without rotational mass when $B_{\ddot{x}_g} = 0.2$ g and (c) with and (d) without rotational mass when $B_{\ddot{x}_g} = 0.4$ g.

EXPERIMENTAL VERIFICATION OF PROPOSED TRACK NES WITH ROTATIONAL MASS

A prototype track-NES with rotational mass is fabricated in-house and experimentally evaluated by performance testing and real-time hybrid simulation using a shake table. The total mass of moveable components in this track-NES is 19.41 kg, while the rotatable components have a total mass of 11.29 kg. Because some gears in this track-NES are detachable, the rotational mass can be composed into different combinations, i.e., the resulting m_{rot}-to-m_{N} ratio can be 0, 1, and 3. The track shape is formed by a fourth-order polynomial function with a shape coefficient of 2,000. In addition, the wheels considered in this experiment are made of steel with and without a rubber coating. This prototype track-NES is directly placed on the uniaxial shake table, and a laser displacement transducer and accelerometers are instrumented to measure relative displacements between the track-NES and shake table and absolute accelerations at the track-NES and shake table. All measurements are sampled at 200 Hz.

First, performance tests on the prototype track-NES with rotational mass are conducted by harmonic excitation on the shake table. Figure 4 presents the experimental frequency-domain displacement responses of the track-NES against the theoretical ones. As found in the results, the low friction coefficient in the steel wheels introduces difficulties in maintaining the required static frictional force. Thus, switches between rolling and sliding wheels are observed in Figure 4(a). Because the rubber coating can continuously provide a sufficient static frictional force, the frequency-domain displacement responses demonstrate a good agreement between the experiment and theory. Moreover, these test results also validate the derivation in Eq. (3) for the frequency-domain displacements.

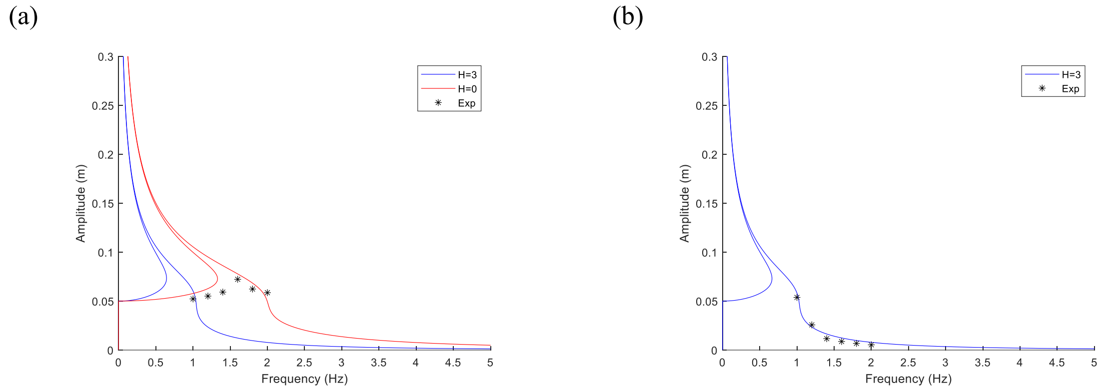

Figure 4 Experimental results of frequency-domain displacements obtained from track-NES: wheels (a) without and (b) with a rubber coating.

Real-time hybrid simulation (RTHS) is carried out to evaluate the control performance when the proposed track-NES with rotational mass is placed on top of a two-story scaled building. In this RTHS, this scaled building is numerically modeled and simulated in the dSPACE controller. This building has a 500-kg mass per floor and a modal damping ratio of 2% for all modes. The stiffness parameters of these two stories are identical and result in the 1st modal frequency of 1.2 Hz. Five earthquake records are considered in this performance evaluation, and the peak ground accelerations (PGA) are scaled accordingly from 0.1 g to 0.3 g. Note that each earthquake record employed in the experiment contains horizontal ground accelerations, and PGA levels consider the stroke limit of the shake table. Meanwhile, the track-NES is experimentally evaluated by shake table testing. This shake table is executed in the displacement control, and the displacement input per time step derives from the absolute displacement on the 2nd floor in real time. Then, the restoring force provided by the track-NES is the movable mass multiplied by the measured absolute acceleration and sent back to the numerical model. Because the total weight of the prototype track-NES with rotational mass is much less than the force capacity of the shake table, this RTHS directly employs the default proportional-integral-derivative (PID) control in the actuator controller. This PID control only utilizes a hard-tuning proportional gain and a slight derivative gain. This RTHS is carried out with a sampling rate of 1,024 Hz while the dSPACE controller collects all physical and numerical responses of this structure-control system.

Figure 5 presents the evaluation results of the track-NES attached to the top of the two-story building. In this figure, the locked NES indicates a mass added on the roof with the same amount of the prototype track-NES, particularly for a fair comparison. The seismic excitation considered in this example is the 0.3-g Kobe earthquake record. When the friction coefficient is low, the steel wheels may not always roll and would slide on the track. On the contrary, the wheels with a rubber coating can maintain the rolling behavior and further reduce structural responses. In addition, Figure 6 shows the maximum displacements of the track-NES against PGA levels in all earthquake events. Although the sliding behavior may reduce the track-NES movements, the wheels with a rubber coating can still yield better control performance for the building while slightly increasing the travel distances. In summary, the rotational mass added in the track-NES benefits the control performance of response mitigation for buildings.

CONCLUSIONS

In this study, a track-NES with rotational mass was successfully developed and experimental verified using real-time hybrid simulation. The rotatable components in this track-NES introduced the additional effective mass, allowing more restoring forces to transfer to the primary structure. Using the harmonic integration method, the frequency-domain input-output relationship from the base excitation to the responses of track-NES with rotational mass was established. Meanwhile, an iterative method was constructed to calculate the frequency-domain response of structures for a system with a track-NES with rotational mass, and this method offered an opportunity to design this track-NES for a structure. Then, a prototype track-NES with rotational mass was facbricated in-house and tested using shake table testing. The established frequency-domain input-output relationship was experimentally verified through harmonic excitation. The overall performance of a scaled building with a prototype track-NES was evaluated by real-time hybrid simulation. As seen in the results, the rotational components in this track-NES better mitigated structural responses and provided higher energy against earthquakes. Moreover, maintaining sufficient static friction enabled rotatable components to spin continuously and increased the effective kinetic energy in this track-NES.

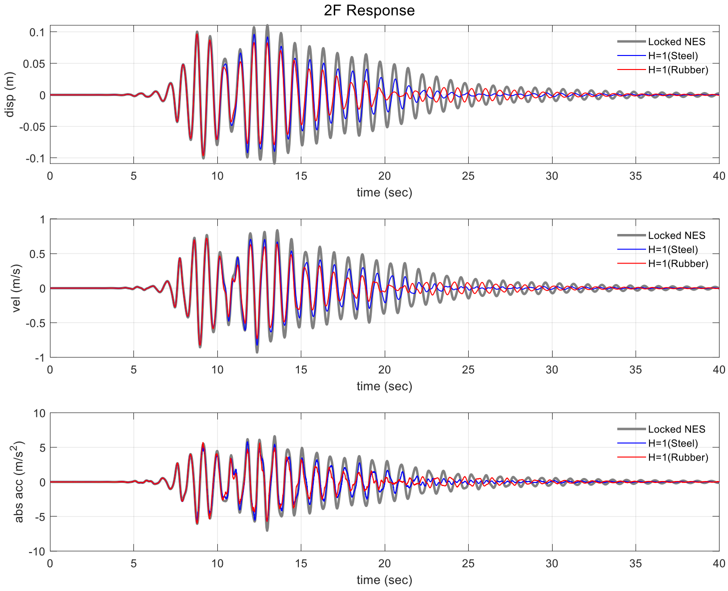

Figure 5 Seismic performance of the scaled building when subjected to the E-W ground motion of the 0.3-g Kobe earthquake record.

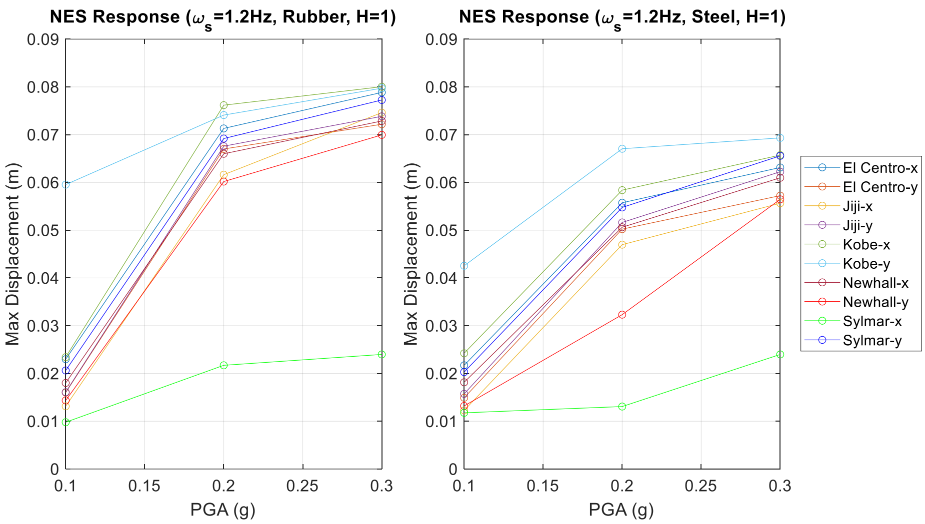

Figure 6 Maximum displacements of track-NES with rotational mass against PGAs in all earthquake events.

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