EXPLORING THE TRUE GEOMETRY OF THE INELASTIC INSTANTANEOUS CENTER METHOD FOR ECCENTRICALLY LOADED BOLT GROUPS

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ABSTRACT

Since 1971, presentations of the Instantaneous Center of Rotation Method for determining the capacity of eccentrically loaded bolt groups have included figures which indicate that the instantaneous center is located along a line perpendicular to the applied load and passing through the center of gravity of the bolt group. It is the purpose of the paper to show that this geometry is incorrect for all but a few very specific instances. The primary implication of this conclusion is that the search for the instantaneous center can not be limited to the one-dimensional line, but instead must include all points in a two-dimensional plane.

INTRODUCTION

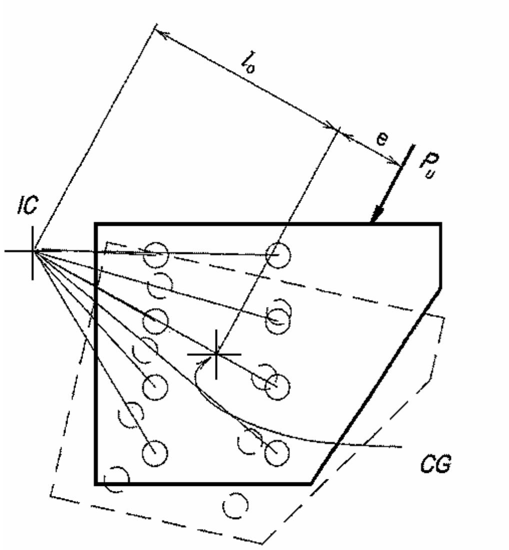

Figure 1.

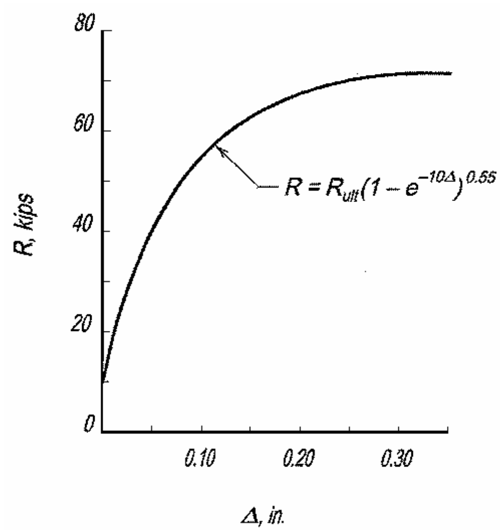

Figure 2.

The AISC LRFD, 3rd Edition Manual (1) presents Figure 7-2, reproduced here as Figure 1, to demonstrate the geometric relationship assumed when using the Instantaneous Center of Rotation Method to calculate the ultimate capacity of eccentrically loaded bolt groups. This figure indicates that the instantaneous center is located along a line perpendicular to the applied load and passing through the center of gravity of the bolt group. Crawford and Kulak (2) also indicate that this is the intended geometry in their paper. It will be shown however that it is impossible to satisfy equilibrium and the requisite nonlinear load-deformation relationship of the bolts with this geometry, except for the case where the load is parallel to one of the symmetry axes of a doubly symmetric bolt group, or a linear elastic constitutive equation is used. Therefore, the search for the instantaneous center of rotation cannot be limited to the one-dimensional line shown in Figure 1 but instead must include all points in a two-dimensional plane.

INVESTIGATION

In order to prove that equilibrium cannot be satisfied while maintaining both the geometric constraints of Figure 1 and the load-deformation constraints represented by Figure 2, we will look at the simple case of a two-bolt group, illustrated in Figure 3 (see section Notation).

From Figure 2 the required load-deformation relationship, based on empirical data, is:

$$R_i = R_{ult}\left(1 - e^{-10\Delta}\right)^{0.55}$$

Since the total deformation at each bolt is assumed to vary linearly with its distance from the instantaneous center, and the bolt furthest from the instantaneous center is assumed to reach its ultimate stress when Δ=0.34 inches, the force on each bolt can be calculated as follows:

$$R_1 = R_{ult}\left(1 - e^{-10(0.34)\left(\frac{r_1}{r_2}\right)}\right)^{0.55}$$

$$R_2 = R_{ult}\left(1 - e^{-10(0.34)}\right)^{0.55} = 0.9815R_{ult}$$

For simplicity assume that $R_2 = 1$. Note that theoretically R_2 should be unity but R_2=0.982 because of the empirical nature of the load-deformation equation.

Also for simplicity introduce $\eta = \left(1 - e^{-10(0.34)\left(\frac{r_1}{r_2}\right)}\right)^{0.55}$.

The tangent of the applied load from Figure 3 is $\tan\theta = \dfrac{R_x}{R_y}$

From the geometry the component forces on the bolts can be calculated as:

$$R_{2x} = R_{ult}\frac{r_{2y}}{r_2} \qquad R_{2y} = R_{ult}\frac{r_{2x}}{r_2}$$

$$R_{1x} = R_{ult}\eta\frac{r_{1y}}{r_1} \qquad R_{1y} = R_{ult}\eta\frac{r_{1x}}{r_1}$$

The tangent of the resultant resisting force on the bolts can then be found to be:

$$\tan\theta_R = \frac{\sum R_x}{\sum R_y} = \frac{\eta \frac{r_{1y}}{r_1} + \frac{r_{2y}}{r_2}}{r_{0x}\left[\frac{\eta}{r_1} + \frac{1}{r_2}\right]}$$

Since $\tan\theta_R$ must equal $\tan\theta$ for equilibrium to be satisfied it can be shown that:

$$\eta \frac{r_{1y}}{r_1} + \frac{r_{2y}}{r_2} = r_{0y}\left[\frac{\eta}{r_1} + \frac{1}{r_2}\right] \qquad (1)$$

It can also be shown that this equation can only be satisfied when $r_{1y} = r_{2y} = r_{0y}$ which only occurs when θ = 0 with a doubly symmetric bolt group or when $\eta = r_1 / r_2$ which is the linearly elastic case.

Having proven that the prescribed geometry cannot be satisfied for the two bolt case, it can also be demonstrated for other bolt groups by moving the instantaneous center along the line perpendicular to the applied load and passing through the centroid of the bolt group and then plotting the resultant theta angle, θ_R , as shown for a three bolt group in Figure 4. As can be seen from the graph θ_R is asymptotic to the value of θ, but will never equal θ for a finite r_0. Therefore, equilibrium cannot be achieved.

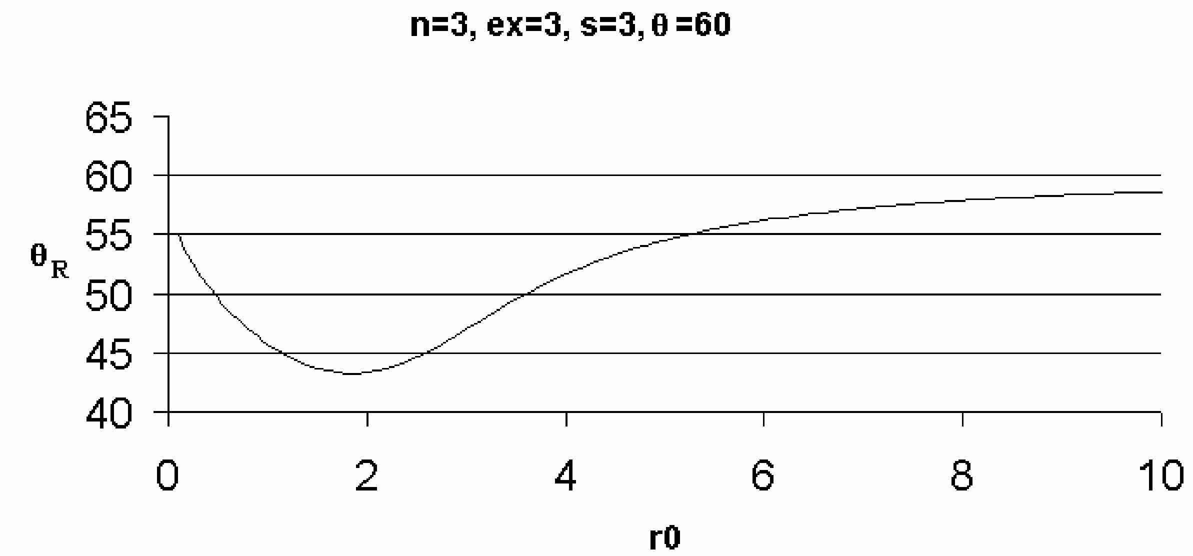

Figure 4.

THE AISC COEFFICIENTS C FOR ECCENTRICALLY LOADED BOLT GROUPS

Finding errors in the presentation of the theory underlying the calculation of the instantaneous center calls into question the values in Tables 7-17 through 7-24 presented in the AISC Manual (1). Fortunately these values were produced by a program based on Brandt's work (Brandt (3, 4)). Brandt's procedure never restricts its search for the instantaneous center to the line perpendicular to the applied load and passing through the centroid of the connection, though from a programming standpoint this would seem to be the most efficient approach. Instead both the location of the instantaneous center and the angle of the resultant are allowed to drift off of the values implied by Crawford and Kulak's theory until equilibrium is satisfied. Being based solely on the load-deformation relationship and equilibrium, the C-values presented by AISC are correct. The authors have checked numerous cases and are satisfied that both the load-deformation relationship and equilibrium are satisfied using Brandt's approach.

NOTATION (Also see Figure 3)

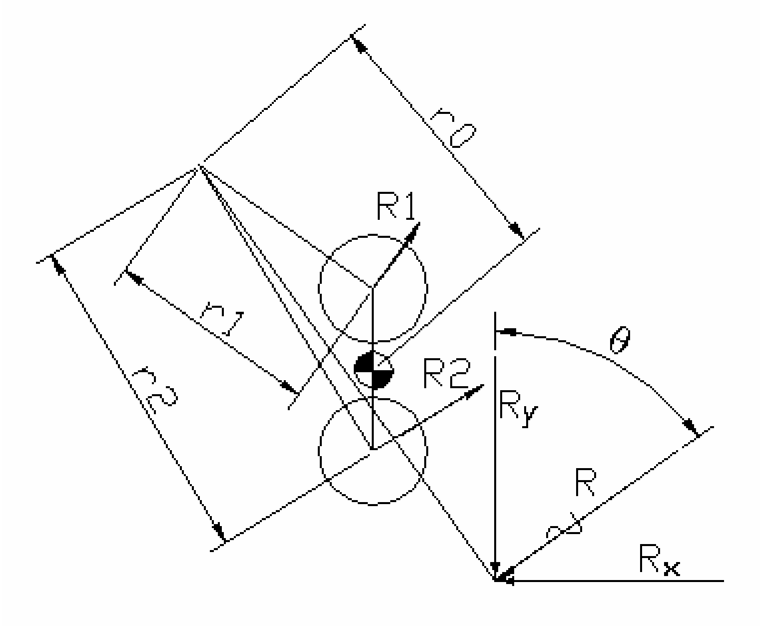

Figure 3.

R_{ult} = ultimate shear strength of one bolt, kips

R_i = nominal shear strength of one bolt at a deformation Δ_i, kips

Δ_i = total deformation in the bolt and the connection elements, in.

Δ = maximum deformation in the bolt and the connection elements = 0.34 in.

θ = angle of the applied load measured from the vertical

θ_R = angle of the calculated resisting force measured from the vertical

r_i = distance from the center of the bolt to the instantaneous center

r_{ix} = horizontal distance from the center of the bolt to the instantaneous center

r_{iy} = vertical distance from the center of the bolt to the instantaneous center

r_0 = distance from the instantaneous center to the applied load

r_{0x} = horizontal distance from the instantaneous center to the centroid of the bolt group

r_{0y} = vertical distance from the instantaneous center to the centroid of the bolt group

$$\eta = \left(1 - e^{-10(0.34)\left(\frac{r_1}{r_2}\right)}\right)^{0.55}$$

REFERENCES

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2. Crawford, S. F. and G. L. Kulak Eccentrically Loaded Bolted Connections Journal of the Structural Division, ASCE, Vol. 97, ST3, March 1971, pages 765-783.
3. Brandt, G. Donald: Rapid Determination of Ultimate Strength of Eccentrically Loaded Bolt Groups. Engineering Journal, American Institute of Steel Construction, Vol. 19, No. 2, 2nd Quarter 1982.
4. Brandt, G. Donald: A General Solution for Eccentric Loads on Weld Groups. Engineering Journal, American Institute of Steel Construction, Vol. 19, No. 3, 3rd Quarter 1982.