PARAMETER IDENTIFICATION OF BOUC-WEN HYSTERESIS VIA CONSTRAINED UNSCENTED KALMAN FILTER

Sung-Yong KIM[1] and Cheol-Ho LEE[2],

SUMMARY

Civil structures often exhibit hysteretic behavior combined with strength and/or stiffness degradation when subjected to severe earthquakes. The Bouc-Wen hysteresis model and its extensions have been used extensively to model such hysteretic behavior. One of the challenging problems in the identification of Bouc-Wen parameters is that the model considers displacement and velocity in a system but acceleration is often the only available response measurement. The on-line identification of degrading and pinching hysteretic systems is a challenging problem because of its complexity. In recent years, the unscented Kalman filter (UKF) has been applied to the on-line parametric system identification of hysteretic differential models with degradation and pinching. In this paper, a constrained unscented Kalman filter (CUKF) is proposed for the simultaneous identification of nonlinear structural systems.

***Keywords**: System identification; Unscented Kalman filter; Hysteretic system; Seismic damage; Nonlinear response.*

INTRODUCTION

Over the past few decades, several structural identification techniques have been successfully implemented for high-rise and long-span structures based on the development of computing and measurement technology. In the earthquake engineering field, structural identification techniques aim to quantify damage levels and guide potentially necessary immediate retrofitting efforts. Structural assessment in seismic engineering focuses on real-time hazard assessment using model updating techniques to find optimal parameters for structural models that can describe various complex nonlinear phenomena including yielding, post-yielding, and strength and/or stiffness degradation. However, because it is difficult to describe complex inelastic behavior, the monitoring of seismically affected structures is considered to be a challenging problem.

Numerous models have been investigated for the description of nonlinear behaviors, and Bouc–Wen model is one of the most popular models, which was originally introduced in Bouc (1967) and extended in Wen (1976). Owing to its versatility, the Bouc-Wen model has been successfully extended to create several variations, for assessing strength degradation, stiffness degradation (Kottari et al., 2014), pinching effects (Baber and Wen, 1981; Baber and Noori, 1985; Foliente, 1995).

The Kalman filter (KF) algorithm is widely used to estimate system states in practical applications. It consists of two consecutive processes called prediction and estimation. In the prediction process, the KF calculates predicted states and covariances. In the following estimation process, the KF calculates Kalman gain based on the predicted covariances. Kalman gain is used to determine the weights between predicted states and measured values to calculate estimated states.

However, the KF is based on the assumption that a target system is linear, and it cannot be applied to the structural identification of structures subjected to severe earthquake forces, which exhibit nonlinear hysteretic behavior. To estimate the states of the nonlinear structures accurately, various numerical techniques have been investigated including the extended Kalman filter (EKF) (Ebrahimian et al., 2015; Zhou et al., 2018; Sen and Bhattacharya, 2016), unscented Kalman filter (UKF) (Gove and Hollinger, 2006; Wu and Smyth, 2008), and particle filter (Kwok et al., 2006). The EKF evaluates the Jacobian of a system at each time step, and requires state and observation functions for the differentiation of corresponding states. In contrast to the EKF, the UKF and particle filter do not require Jacobians because they transform a set of points using known nonlinear equations and combine the results to estimate the mean and covariance of a state. Therefore, they are more versatile in their application to highly nonlinear systems. However, the UKF and particle filter differ in that the particle filter chooses points randomly, whereas the UKF chooses points based on a specific algorithm. Therefore, the number of points used in a particle

[1] *Assistant professor, Changwon National University, Korea, e-mail: sungyong.kim@changwon.ac.kr*
[2] *Professor, Seoul National University, Korea, e-mail: ceholee@snu.ac.kr*

filter generally needs to be much greater than the number of points in a UKF.
This paper first reviews the original Bouc-Wen model and several variations that have been proposed to simulate strength and strength degradation. Next, we propose a constrained UKF (CUKF) that combines a constrained minimization technique with a conventional UKF to determine the appropriate constraints for state and parameter identification. The proposed method is then compared to conventional UKF-based parameter identification techniques to validate its robustness and efficacy in identifying Bouc-Wen parameters.

BOUC-WEN HYSTERETIC MODEL

The Bouc-Wen model describes two input and output variables as

$$f = \alpha k_i u + (1-\alpha) k_i z \tag{1}$$

where k_i is the elastic stiffness, α is the ratio of the post-yield stiffness to the elastic stiffness k_i; and z is an auxiliary variable incorporated to describe hysteretic behavior, which is controlled by the following nonlinear differential equation with a zero initial condition:

$$\dot{z} = \dot{u}(1 - |z|^n \psi) \tag{2}$$

where n is a shape parameter and ψ is a nonlinear function that controls the shape of the hysteresis loop defined as follows:

$$\psi = \gamma + \beta \operatorname{sgn}(\dot{u} z) \tag{3}$$

where γ and β are shape-control parameters that determine the slopes of the hysteresis in the u-z plane.

In the model proposed by Baber (1981), the auxilary variable z is modified in order to employ strength and stiffness degradation as follows:

$$\dot{z} = \frac{\dot{u}(1 - \nu |z|^n \psi)}{\eta} \tag{4}$$

where the parameters ν and η are expressed as linear functions of the dissipated energy and their slopes δ_ν and δ_η such as

$$\nu = 1 + \delta_\nu \varepsilon \tag{5}$$
$$\eta = 1 + \delta_\eta \varepsilon \tag{6}$$

with ε being the dissipated hysteretic energy as

$$\varepsilon = (1-\alpha) k_i \int_0^t z \dot{u} \, dt \tag{7}$$

It should be noted that, although the Bouc-Wen model and its variation can capture the various features of hysteresis such as hysteresis shape, strength degradation and stiffness degradation, its parameters does not have rigorous physical meaning. As such, a given set of parameters uniquely determines the hysteretic response, but a given hysteresis may not determine the parameters unambigously (Ma et al., 2004). In the perspective of structural identification, such an ambiguity of the model can yield a problem of searching for local fitting solutions. Hence, if some constraints on the parameters can also imposed based on a priori sense which is physically admissible, the complexity of the calculation and decreases the efficiency of the calibration can be improved.

In the following section, a conventional method for the system identification using the unscented kalman filter was introduced. And then projection was introduced.

UNSCENTED KALMAN FILTER WITH STATE LINEAR CONSTRAINTS

Unscented Kalman Filter

The Unscented Kalman Filter is a numerical technique used for the estimation of a nonlinear dynamic system. Consider a nonlinear dynamic system modeled as:

$$
\begin{aligned}
x_{k+1} &= F_k(x_k) + w_k \\
y_k &= H_k(x_k) + v_k
\end{aligned}
\tag{1}
$$

where x_k is the state variable, y_k is the measured value, w_k and v_k are process and sensor noise modeled as uncorrelated Gaussian, respectively, F_k is the nonlinear vector-valued transition dynamics of the system, H_k is a nonlinear vector-valued function which transforms a state vector into the appropriate measurement vector, and the subscript k stands for the k-th discrete time step.
Regarding n-dimension state variable matrix x following normal distribution with its mean $\bar{x}$ and covariance P_x, we calculate a normal distribution of y through a nonlinear function $\hat{x}_{k+1}$ by using (2N+1) sample sigma points (χ_i). The sigma points are declared as:

$$\chi_1 = \bar{x} \tag{5}$$

$$\chi_{i+1} = \bar{x} + \sqrt{(n+\lambda)P_x}, i = 1, \cdots, N \tag{6}$$

$$\chi_{i+N+1} = \bar{x} - \sqrt{(n+\lambda)P_x}, i = 1, \cdots, N \tag{6}$$

where $¥lambda = ¥sigma^2(n+¥kappa) - n$ is a scaling parameter, $¥sigma$ stands for spread of the sigma points around $¥bar{x}$, and $¥kappa$ is a secondary scaling factor. Here the weight values W_i is declared as:

$$W_1 = \frac{\lambda}{N+\lambda} \tag{5}$$

$$W_{i+1} = \frac{1}{2(N+\lambda)}, i = 1, \cdots, N \tag{6}$$

$$W_{i+N+1} = \frac{\lambda}{2(N+\lambda)}, i = 1, \cdots, N \tag{6}$$

The normal distribution of $\hat{x}_{k+1} = f(x)$ is calculated as:

$$\hat{x}_{k+1} = \sum_{i=1}^{2N+1} W_i f(\chi\_i) \tag{5}$$

$$P_{\hat{x}_{k+1}} = \sum_{i=1}^{2N+1} W_i \{f(\chi_i) - \bar{\hat{x}}_{k+1}\}\{f(\chi_i) - \bar{\hat{x}}_{k+1}\}^T \tag{6}$$

We start the UKF with initial value of estimated state variable $\hat{x}_0$ and covariance P_0. Each time step k, we calculate sigma points and their weight values as

$$(\chi_i, W_i) \leftarrow (\hat{x}_{k-1}, P_{k-1}, \kappa) \tag{3}$$

After that, we predict state variable, covariance, measured value as

$$
\begin{aligned}
(\hat{x}_{\bar{k}}, P_{\bar{k}}) &= \mathrm{UT}(f(\chi_i), W_i, Q) \\
(\hat{z}_k, P_z) &= \mathrm{UT}(h(\chi_i), W_i, R)
\end{aligned}
\tag{3}
$$

The Kalman gain is calculated as

$$
\begin{aligned}
P_{xz} = \sum_{i=1}^{2N+1} W_i \{f(\chi_i) - \hat{x}_{\bar{k}}\}\{f(\chi_i) - \hat{x}_{\bar{k}}\}^T \\
K_k = P_{xz} P_z^{-1}
\end{aligned}
\tag{3}
$$

The estimated state $\hat{x}_k$ and covariance P_k are finally calculated as

$$\hat{x}_k = \hat{x}_{\bar{k}} + K_k(z_k - \hat{z}_k) \tag{3}$$

Estimate projection

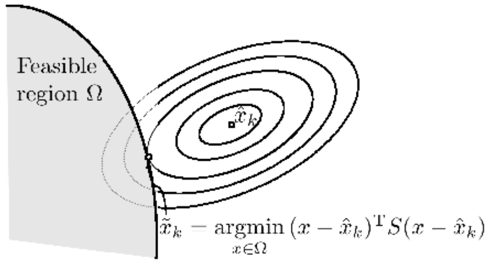

Figure 1 A geometric illustration of estimate projection

Having calculated the unconstrained estimate state $\hat{x}_k$, the constrained estimate can be obtained by projecting the unconstrained estimate onto the constraint surface. The constrained estimate can be obtained by solving the following minimization problem:

$$\tilde{x}_k = \underset{x\in\Omega}{\operatorname{argmin}}(x-\hat{x}_k)^T S(x-\hat{x}_k) \tag{3}$$

where $\hat{x}_\mathrm{k}$ and $\tilde{x}_k$ are the unconstrained estimate and the constrained estimate of the state, respectively, S is a positive-definite weight matrix, and Ω is the feasible region. A geometric illustration of the estimate projection is illustrated in Figure 1, where the feasible set Ω is shown shaded and the contour lines of the convex quadratic objective function are depicted.

NUMERICAL SIMULATION

A single degree of freedom (SDOF) system with Bouc-Wen hysteresis is considered in this example. The equation of motion of the system subjected to a ground acceleration is given by

$$\ddot{u} + 2\zeta\dot{u} + \alpha\omega_n^2 u + (1-\alpha)\omega_n^2 z = -\ddot{u}_g \tag{3}$$

where u is the displacement, ζ is the damping ratio, ω_n is the natural frequency, u_g is the ground displacement, α is the post-yielding stiffness ratio, an overdot stands for the time derivative, and z is an auxiliary variable for describing the Bouc-Wen hysteresis governed by following differential equation:

$$\dot{z} = \dot{u}[1 - |z|^n\{\gamma + \beta\,\mathrm{sgn}(\dot{u}z)\}] \tag{3}$$

To utilize the parameter identification via the UKF, an augmented state vector is introduced as follows:

$$x = [u, v, z, \alpha, \gamma, \beta, n]^T \tag{3}$$

where $v = \dot{u}$ is the velocity of the structure. The output equation for the measurement can be written as follows:

$$y = \ddot{u} + \ddot{u}_g + w \tag{3}$$

where w is the measurement noise. In this study, a Gaussian white noise process with 1 m/s^2 RMS value is imposed to its measurement. To utilize the system identification via the Kalman filter methods, formulated differential equation was reformulated into the discretized form using the fourth-order Runge-Kutta integration method.
In this example, the 1940 El Centro ground motion was selected as the input excitation. The initial values for the original state vector x are chosen as zero. The post-yielding parameter α was chosen as 0.09, which is acceptable in general for steel material. The shape parameter n, β and γ were chosen as 1.2, 20 and 40, respectively. The initial guesses were determined to have the same or similar order of magnitude of the corresponding true values.

To utilize the CUKF, the following constraint conditions were determined based on physical intuition. First, the parameters α, γ, β and n were set to be positive. Second, the upper bound of the stiffness ratio α was set to 0.2. Finally, $\beta + \gamma$ is bounded in a range of 35 and 70. The constraint conditions, to which the estimated state would be projected, constitute the feasible region:
Then, the procedure to find the constrained estimate $\hat{x}_k$ falls into a linearly constrained quadratic optimization problem.

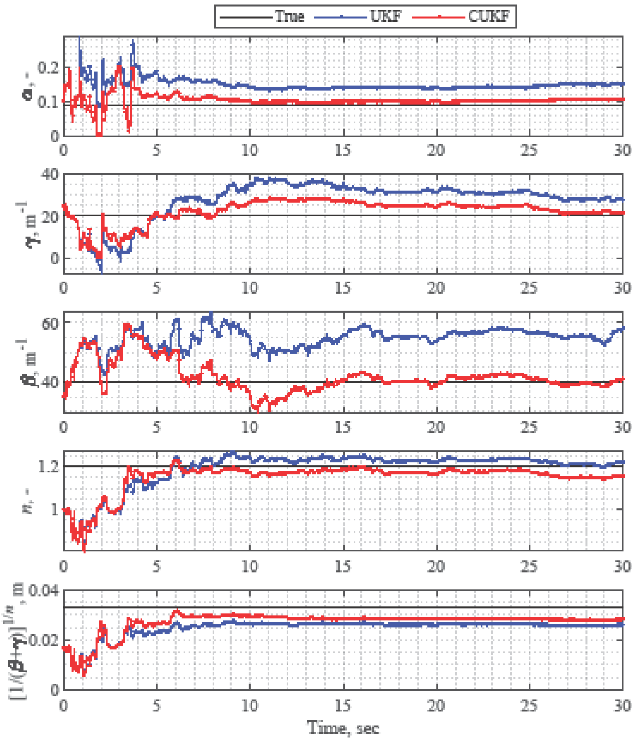

Figure 2 Results of parameter identification for non-degrading hysteresis

Figure 2 presents the time histories of the parameters estimated by the UKF and CUKF. By carefully inspecting the time histories for identifying parameter α, one can see that the UKF attempted to seek a value of α larger than 0.3 at approximately 1 s and 4 s, which is an infeasible region from a physical perspective. In contrast, the CUKF estimated parameters closer to the true values compared to the UKF by searching for the parameters within admissible physical constraints. The inaccuracy of parameters combined with good agreement with predicted measurements indicates overfitting for the UKF. The parameters of the Bouc-Wen model are empirical, meaning they are not derived from fundamental mechanics, and a given response may not determine parameters unambiguously (Ma et al., 2004). Therefore, it is helpful for applying the CUKF to system parameter identification if certain parameters can be constrained in a physical sense.

The next example considers an SDOF structure inherent from the above, but the degrading effect is included. Similarly an augmented state vector is introduced to formulate the state space equation as follows:

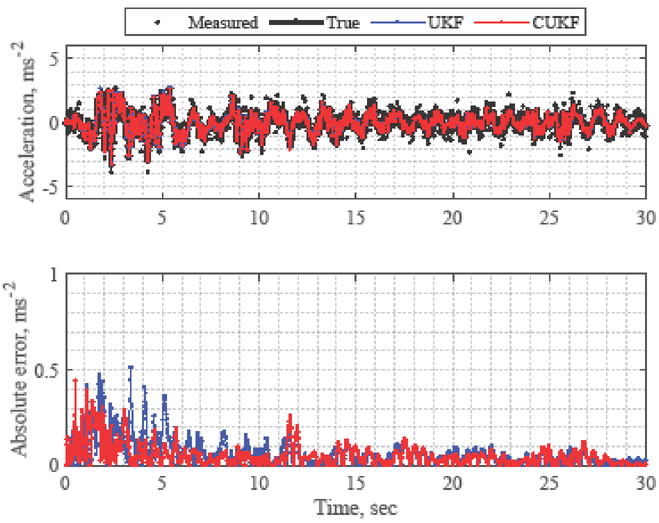

Figure 3 Comparison of acceleration time history results for degrading hysteresis identified by the UKF and CUKF (top), and the corresponding absolute errors (bottom)

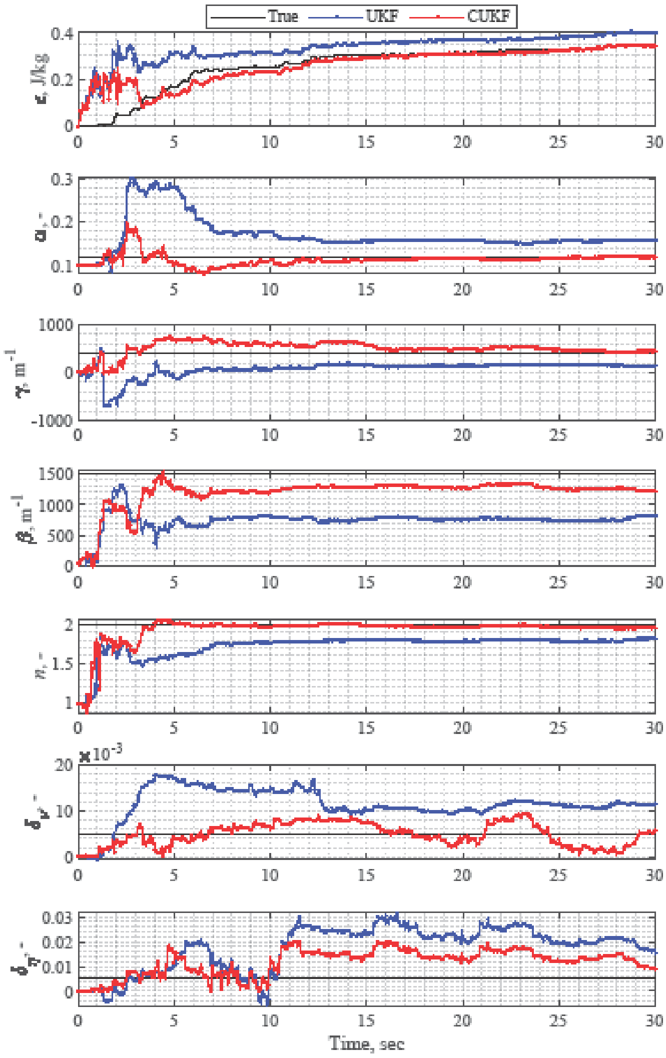

Figure 4 Results of parameter identification for degrading hysteresis

Figure 3 shows the comparison of the accelerations estimated by the UKF and CUKF. It was observed that the acceleration histories estimated by UKF and CUKF correlates well with the true simulated response such that they could not be distinguished from the true response, and their absolute errors were only up to 0.5 m/s^2.

Figure 4 shows the time histories of the parameters estimated by the UKF and CUKF. It can be observed that the UKF tried to seek a value which would be regarded as being unacceptable, while the CUKF could estimate the parameter closer to the true value than the UKF by searching the parameters within the constraint.

SUMMARY AND CONCLUSIONS

In this study, a phenomenological hysteresis model is proposed, which can describe severe asymmetric hysteretic behavior. First, the Bouc-Wen (BW) class models, including the original BW model, the Wang-Wen model, and the generalized BW model and its modified version, were briefly reviewed. A new hysteresis model was then proposed by combining the BW class hysteresis models with a post-yielding convex function composed of piece-wise linear functions. A framework for parameter identification was formulated, describing the objective function and constraint conditions required for the convex piecewise post-yielding functions.

The proposed model was validated on the basis of the hysteretic behavior of welded steel moment connections with highly composite slabs in which the connections exhibited severe asymmetric hysteretic behavior. It was shown that the proposed model and its parameter identification procedure could simulate the asymmetric hysteresis well compared to the existing BW class models. The convergence of the proposed model was also examined.

Inclusion of one or two linear functions in the original BW or Wang-Wen model is recommended for better accuracy and more optimal numerical performance.

REFERENCES

Bouc, R. (1967), “Forced vibrations of mechanical systems with hysteresis”, *Proc. of the Fourth Conference on Nonlinear Oscillations*, Prague.

Wen, Y. K. (1976), “Method for random vibration of hysteresis systems”, *Journal of the Engineering Mechanics Division*, 102(2): 249-263.

Kottari, A. K., Charalampakis, A. E., Koumousis, V. K. (2014), “A consistent degrading Bouc-Wen model”, *Engineering Structures* 60: 235-240.

Baber, T. T., Wen, Y. –K. (1981), “Random vibration of hysteretic degrading systems”, *Journal of the Engineering Mechanics Division* 107 (6): 1069-1087.

Baber, T. T., Noori, M. N. (1985), “Random vibration of degrading, pinching systems”, *Journal of Engineering Mechanics* 111 (8): 1010-1026.

Ebrahimian, H., Astroza, R., Conte, J. P. (2015), “Extended Kalman filter for material parameter estimation in nonlinear structural finite element models using direct differentiation method”, *Earthquake Engineering & Structural Dynamics* 44: 1495-1522.

Zhou, Z., Zhao, L., Xie, L., Xue, S., Tang, H., Wan, C. (2018), “Parameter Identification for Structural Health Monitoring with Extended Kalman Filter Considering Integration and Noise Effect”, *Applied Sciences* 8 (12): 2480.

Sen, S., Bhattacharya, B. (2016), “Online structural damage identification technique using constrained dual extended Kalman filter”, *Structural Control and Health Monitoring*: 1-12.

Gove, J. H., Hollinger, D. Y. (2006), “Application of a dual unscented Kalman filter for simultaneous state and parameter estimation in problems of surface-atmosphere exchange”, *Journal of Geophysical Research* 111: 1-21.

Wu, M., Smyth, A. W. (2008), “Real-time parameter estimation for degrading and pinching hysteretic models”, *International Journal of Non-Linear Mechanics* 43 (9): 822-833.

Kwok, N. Ha, Q., Nguyen, T., Li, J., Samali, B. (2006), “A novel hysteretic model for magnetorheological fluid dampers and parameter identification using particle swarm optimization”, *Sensors and Actuators A: Physical* 132 (2): 441-451.

Ma, F., Zhang, H., Bockstedte, A., Foliente, G. C., Paevere, P. (2004), “Parameter analysis of the differential model of hysteresis”, *Journal of Applied Mechanics* 71 (3) 342.