DISCUSSION

Design for Eccentric and Inclined Loads on Bolts and Weld Groups

Paper by NESTOR IWANKIW
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Discussion by **Miguel Angel Dodes Traian**

I have read the article and think there is an expedient way to get a value for analysis and design of bolt groups, which is implicit in the explanation.

Method 2, which defines the elastic approach C_e, is a closed form and a lower bound value. On the other side, Method 5 defines the fully plastic approach C_p, which is an unreachable upper bound.

The real solution is between these two limits, thus a very close and direct approximation is the arithmetic mean of both values, that is (using the article's nomenclature):

$$C_e = \Sigma d^2/[(r_o + \ell).\ d_{\max}]$$

$$C_p = \Sigma d_i/(r_o + \ell)$$

$$C_{\text{mean}} = (C_e + C_p)/2$$

In the examples of this article and some others of the AISC Manual, the difference with C_u (considered as the most accurate) is between -7% and $+2\%$, which I consider an excellent approximation for a non-iterative procedure.

Addendum/Closure by **Nestor Iwankiw**

Traian's interest in the article and helpful design suggestion of an approximate coefficient C_{mean} for the analysis of bolt group cases not tabulated in the AISC Manual is appreciated. This average of the lower C_e and upper C_p bound limits results in a reasonable approximation, as indicated by the reported accuracy of C_{mean} relative to C_u. The discussion also gives a useful reminder of an equivalent alternate formula for bolt C_e in terms of elastic center of rotation geometry.

Even though Traian, in the original article, did not address full plastic analysis of welds, C_e and Method 5 C_p can also be similarly defined for weld groups relative to the elastic center of rotation:

$$C_e = \frac{f_D I_{\text{pic}}}{(r_o + \ell)d_{\max}} \qquad \text{(D-1)}$$

$$C_p = \frac{f_D \Sigma(d_i L_i)}{(r_o + \ell)} \qquad \text{(D-2)}$$

where

I_{pcg} = polar moment of inertia of weld group about centroid
k_o = polar radius of gyration of weld group relative to centroid = $\sqrt{\dfrac{I_{pcg}}{\Sigma L_i}}$
I_{pic} = polar moment of inertia of weld group about instantaneous center = $I_{pcg} + r_o^2(\Sigma L_i)$
L_i = length of ith weld line element
d_i = distance from center of rotation to centroid of ith weld element
(f_D, ℓ, r_o defined in paper)

Unfortunately, a few sample problems have demonstrated that C_p is sensitive to the number and location of weld subdivisions (elements), especially at corners and extremities. Probably, at least 10-20 weld elements are necessary to obtain a reasonable answer, even for fairly regular patterns. In addition, actual weld ductility could limit significantly the assumed complete redistribution and strength, represented by C_p, particularly for larger connections. Because of these uncertainties and complications, general use of C_p for weld design cautiously is not recommended pending further studies and comparisons.

Consequently, for routine design applications Method 4 will be shown computationally more convenient and reliable in view of its simple overall representation of both bolt and weld group strength under all load angles. If necessary for special cases, optimization of this trial design capacity could be attempted in combination with another rational method, such as C_{mean}. The remainder of this discussion focuses mainly on further development and practical application of Method 4.

In the original paper, the proposed Method 4 Eqs. 9, 10 and 11 for bolts and 12, 13 and 14 for welds were presented in somewhat "raw" form. Their derivation was based on the conservative assumption of simple arithmetic, rather than vectorial, addition of maximum connector strength as an exaggerated load effect. Some additional simplifications can be made to this formulation which are helpful both computationally and conceptually. Through substitution and algebraic rearrangement, the referenced equations will be reduced to a more convenient and normalized form for application. A plot of the functional relationship also illustrates better the behavioral trends relative to load angle.

First, define $C_{\max}$ as the total number of bolts n or the maximum concentric weld capacity, (f_D (1 + $2k$) for

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C-shapes), as applicable. Next, let

$$A = \frac{C_{\max}}{C_o} \geq 1.0 \tag{D-3}$$

where C_o is the AISC Manual-tabulated C_u for a given vertical load case. For a particular connector pattern and load eccentricity distance, A is a constant relative to the load angle θ; it serves as the single characteristic input property of the connector geometry.

Normalizing Eqs. 9 and 12 through division by C_o yields the intermediate solution:

$$\frac{C'}{C_o} = \frac{A}{(\tan\theta + A)} \tag{D-4}$$

Furthermore, a similar division by C_o of Eqs. 10 and 13 and substitution of D-4 for $\frac{C'}{C_o}$ results in the final short expression for $\frac{C_a}{C_o}$ only as a function only of θ:

$$\frac{C_a}{C_o} = \frac{A}{(\tan\theta + A)\cos\theta}$$

or equivalently, (D-5)

$$\frac{C_a}{C_o} = \frac{A}{(\sin\theta + A\cos\theta)}$$

This solution could also have been obtained directly from D-4 and the geometry of Fig. 1 by recognizing C' as the eccentricity coefficient for only the vertical load component. The working limits imposed by 11 and 14 convert readily for the normalized $\frac{C_a}{C_o}$ variable to:

$$A \geq \frac{C_a}{C_o} \geq 1 \tag{D-6}$$

Upon this effective elimination of D-4, the simplified Eqs. D-3, D-5 and D-6 are all that is necessary to easily apply Method 4 for bolt or weld design/analysis. With this normalized dependent variable $\frac{C_a}{C_o}$ and D-5, graphs can be prepared for specific A values and for load angles from 0-90°, as in Figs. D1 and D2. The $A = 1$ case in Fig. D1 is presented only to show the minimum $\frac{C_a}{C_o}$ for a concentric loading for which the lower boundary of D-6 obviously would govern i.e. $C_a = C_o = C_{\max}$. The figures give a clear overall picture of the C_a sensitivity to both A and θ. The $\frac{C_a}{C_o}$ ordinates and curve slopes increase with higher A values and the end values at $\theta = 0°$ and $90°$ are 1.0 and A, as expected. The D-5 function curvature is concave upward (consistent with the C_u plot in Fig. 5) meaning that linear interpolation can be greatly misleading, especially for larger θ's and larger A's. Fig. D2 is Fig. D1 restricted to a narrower practical $\frac{C_a}{C_o}$ range with the lower imposed limit of 1.0.

These two figures can help to identify immediately the importance of load angle for a given problem as well as to provide for a quick graphical solution estimate for the eccentricity coefficient C of the total resultant load P. Given an A value, the range of so-called small-to-moderate-angles for which the eccentricity coefficient is essentially invariant from its vertical minimum value C_o can be approximated from the dip in the curve below 1.0. For example, for the more common smaller load eccentricity distances, A will usually be between 1 and 2; consequently any load angle less than about 55° will not increase $\frac{C_a}{C_o}$ (see original paper Table 2, case A, $A = \frac{9}{8.52} = 1.06$). On the other hand, larger load eccentricities (Table 1, $A = \frac{3}{0.17} = 17.6$) produce a rapid rise in

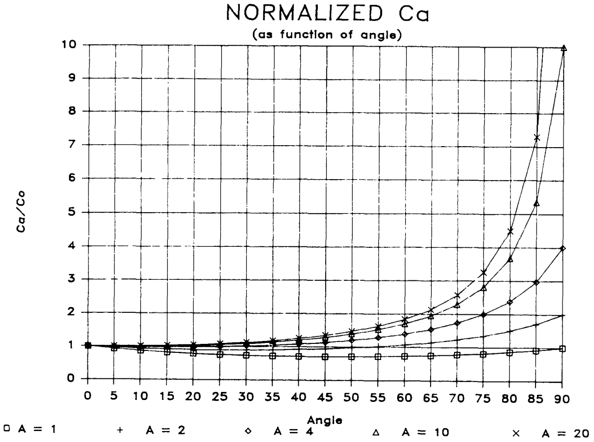

Figure D-1

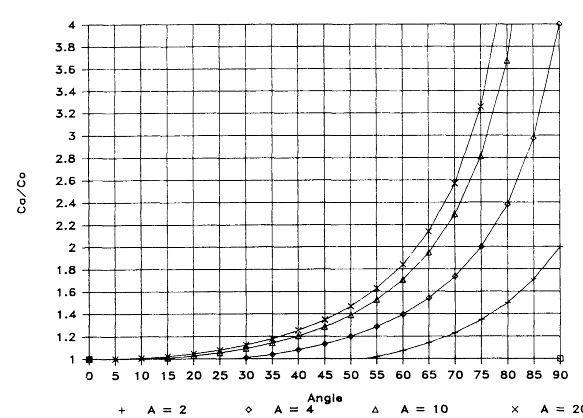

Figure D-2

$\frac{C_a}{C_o}$ for θ greater than 15°. These observations confirm the numerical comparisons made beforehand in the paper.

An interesting fact about this $\frac{C_a}{C_o}$ solution is that it can be reduced to the following simple linear interaction between the eccentric shear $\frac{P_v}{C_o r_v}$ and direct shear $\frac{P_H}{C_{max} r_v}$ capacities, merely by substituting $P = C_a r_v$, $\sin\theta = \frac{P_H}{P}$, $\cos\theta = \frac{P_v}{P}$, and $A = \frac{C_{max}}{C_o}$ in D-3:

$$\frac{P_H}{C_{max}\, r_v} + \frac{P_v}{C_o\, r_v} = 1.0 \qquad \text{(D-7)}$$

or

$$\frac{C_H}{C_{max}} + \frac{C_v}{C_o} = 1.0$$

where

C_H = coefficient for only the horizontal load component (direct shear)

C_v = eccentricity coefficient for only the vertical load component (eccentric shear)
= C' in (7)

This equivalency to a typical conservative interaction diagram (see Fig. D–3) further legitimizes Method 4 from another perspective.

To digress briefly, another different approach to the inclined eccentric load problem would have been to bypass any simplifying theoretical considerations and only develop a curve-fit approximation, C_c, to the actual ultimate strength C_u. This type of equation could have the mathematical form

$$\frac{C_c}{C_o} = \left(\frac{\theta}{90}\right)^m (A\text{-}1) + 1 \qquad \text{(D-8)}$$

where m is an appropriate exponent. The benefits of such a general formula are that the lower limit of 1.0 in D-6 automatically is satisfied and that the approximation of the larger C_u coefficients could probably be further improved. Unfortunately, a basic linear (m = 1) or quadratic (m = 2) function in θ definitely is inadequate, thus the proper higher order exponent needs to be established. Since this development requires a more exhaustive study of the Method 1 ultimate strength solutions to identify the best predictor equation(s), it is left for possible future work. For now, the Method 4 formula D-5 appears to be quite satisfactory.

In summary, the advantages of the modified $\frac{C_a}{C_o}$ format in D-5 or D-7 are its simplicity, easy derivability, applicability to either bolts or welds and broad representation of capacity as merely a trigonometric function of load angle. The behavioral nonlinearity and computational complexity of the preferred Method 1 — ultimate strength solution can be safely and quickly approximated by this new C_a coefficient for any eccentric load angle. The only prerequisite is the baseline C_o coefficient for vertical load (using Method 1, such as the eccentrically loaded connector geometries tabulated in the AISC Manuals) must be available to define the A parameter. The Method 4 procedure will cover concisely all the other possible applied load orientations, thereby reducing a very difficult and tedious ultimate strength problem to a most manageable one.

The Appendix-Design examples are briefly re-done, using the modified $\frac{C_a}{C_o}$ equation:

Ex. 1 — ASD & LRFD

$$C_o = 3.55;\ \theta = 60°$$
$$C_{max} = 2 \times 6 = 12$$
$$A = \frac{12}{3.55} = 3.38$$
$$\text{(D-3)}\ \frac{C_a}{C_o} = \frac{3.38}{(.866 + 3.38(.5))} = 1.322$$
$$\therefore C_a = 1.322(3.55) = 4.69,\ \textbf{o.k.}$$

Ex. 2 — ASD

$$C_o = 0.704;\ \theta = 75°$$
$$C_{max} = 1.856$$
$$A = \frac{1.856}{0.704} = 2.64$$
$$\text{(D-3)}\ \frac{C_a}{C_o} = \frac{2.64}{(.966 + 2.64(.259)} = 1.6$$
$$\therefore C_a = 1.6\,(0.704) = 1.13\ \ \textbf{o.k.}$$

Ex. 2 — LRFD

$$C_o = 1.136;\ \theta = 75°$$
$$C_{max} = 2.784$$
$$A = \frac{2.784}{1.136} = 2.45$$
$$\text{(D-3)}\ \frac{C_a}{C_o} = \frac{2.45}{(.966 + 2.45(.259))} = 1.53$$
$$\therefore C_a = 1.53(1.136) = 1.74\ \ \textbf{o.k.}$$

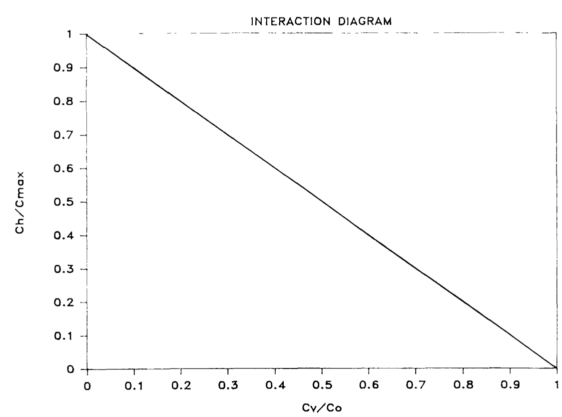

Figure D-3