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The Strength of Planar Fillet Weld Groups Subjected to a Shearing Force Applied Outside Their Planes

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SYNOPSIS

The method of analysis previously proposed for calculating the strength of fillet welded connections subjected to a shearing force only[1] *is now extended to analyse the strength of fillet welds which all lie in one plane and are subjected to a shearing force applied along a line of action which is parallel to, but outside, this plane and is so positioned as to give rise to a couple acting in a plane normal to the plane of the welds. The welds are assumed to have equal leg lengths. The effect of longitudinal residual stresses in the welds is allowed for. Analytical expressions, which depend upon the weld metal ultimate tensile strength and the weld geometry, are developed for predicting the strength of the welds; these expressions are simple enough to be used in design. The predictions of these formulae are compared with published experimental results, and agreement between the two is shown to be reasonable.*

NOTATION

d	Depth of column used in tests (Table 3).
e	Load eccentricity.
L	Distance between welds in a type A connection (Fig. 6a).
L_1	Length of tension fillet weld (Fig. 1a).
L_2	Length of shear fillet weld (Fig. 1b).
L'	Length of weld on beam web (Fig. 2a).

l, m, n	Direction cosines.
M	Bending moment.
M_0	Failure moment of connection subjected to pure bending.
N	Number of web welds carrying the applied load.
P	Shearing force transmitted by connection.
P_0	Value of P at failure when $M = 0$.
p	Principal stress (suffices 1, 2, 3 indicate three principal stresses).
$R_{1,2,3}$	Radii of Mohr's circles.
r_0	Co-ordinate of centre of rotation (Fig. 3).
w	Weld leg length.
x, y, z	Co-ordinate axes.
Y, Y_1, Y_2	Residual stresses in welds.
y_0	Co-ordinate of centre of rotation (Fig. 3).
η	Defines position of plane of discontinuity.
θ	Angle between load and direction of weld axis (Fig. 3).
ξ	e/L
ξ'	e/L'
$\Sigma, \Sigma', \Sigma''$	Defined in text.
σ	$P/(wL')$
σ (with suffices x, y, z)	Direct stress in specified directions.
σ_0	Material yield stress.
σ_u	Ultimate tensile strength of weld metal.
τ (with suffices)	Shearing stresses in specified directions.
ψ	Defines length of intermittent weld (Fig. 16).

1 INTRODUCTION

A new approach for the calculation of the strength of fillet welded connections was put forward in a recent paper.[1] It was proposed that the couples acting on the faces of the weld should be replaced by 'equivalent' direct and/or shearing forces and the strength of the welds when they are subjected to this revised set of forces should be determined. It was then assumed that this strength would provide a reasonable estimate of the

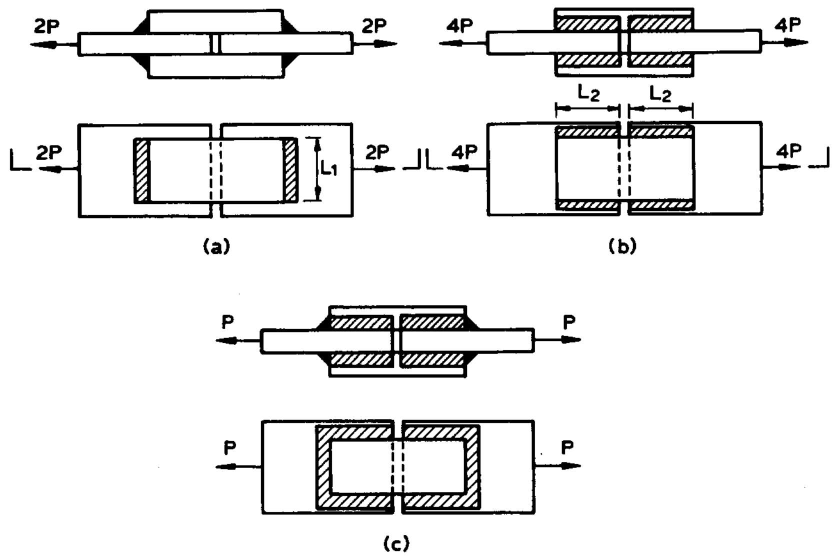

Fig. 1. Connections subjected to a shearing force only. (a) Tension fillet welds; (b) shear fillet welds; (c) all-round welds.

strength of the weld when it was subjected to the actual force system. The method was applied to some fillet welded connections which transmitted a shearing force only using tension and shear fillet welds (Fig. 1) and it was shown that very simple expressions, based only on the ultimate tensile strength (UTS) of the weld metal and the weld geometry, could be derived for calculating the strengths of the connections. The strengths predicted using these expressions were found to be in reasonably good agreement with test results.

The connections depicted in Fig. 1 are so loaded that, at failure, the connected members are displaced relative to each other without rotation; the welds then transmit a shearing force only. In many practical connections — such as bracket-to-column connections (Fig. 2) — the applied load is such that the connected members rotate relative to each other at failure; the welds are then subjected to a combination of a shearing force and couple. The welds in Fig. 2a are subjected to a shearing force acting along a line of action which is parallel to, but outside, the plane of the welds and it is assumed to be so placed as to give rise to a couple acting in a plane normal to the plane of the welds (a bending couple). The welds in Fig. 2b, on the other hand, are subjected to an eccentric shearing force in the plane of the welds which results in a couple acting in the same plane (a

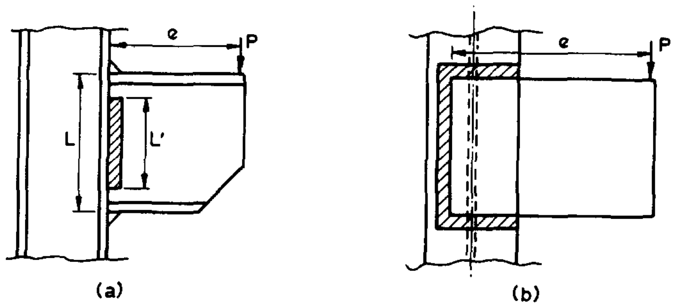

Fig. 2. Connections subjected to an eccentric shearing force. (a) Connection subjected to a shearing force and a bending couple; (b) connection subjected to a shearing force and a torque.

torque). Such connections are sometimes referred to as 'out-of-plane' and 'in-plane' connections respectively.[2] The method of analysis described above is now to be extended to assess the strength of welds loaded as shown in Figs 2a and 2b. The strengths of welds shown in Fig. 2a are assessed in this paper. The strengths of welds shown in Fig. 2b will be discussed in a separate paper.

2 PREVIOUS WORK

The conventional method of designing welds subjected to a shearing force and a bending couple (Fig. 2a) is to treat the weld metal as a 'beam' joining the abutting faces of the connected members. The cross-section of this 'beam' is assumed to consist of weld metal deposited along the lines of each of the welds. The material along each weld line is rectangular in shape, the length and the breadth of the rectangle being respectively equal to the length and the throat thickness of the weld. The applied shearing force is assumed to give rise to a shearing stress which is uniform over the entire cross-sectional area of the 'beam' and which acts in the plane of the 'beam' cross-section. The stresses caused by the bending couple are calculated by applying beam bending theory to the assumed 'beam' cross-section; these stresses act in a direction normal to the 'beam' cross-section. The resulting shearing forces per unit length are added vectorially and the weld is designed to restrict the shearing stress caused by the resultant force to a limiting value which is based on the yield stress of the connected members in many Codes of Practice.

That this approach leads to very conservative design was confirmed experimentally by Archer *et al.*[3] and later by Dawe and Kulak.[4] Archer *et al.* only investigated connections having the vertical welds shown in Fig. 2a and on the basis of their results, they proposed that the calculated bending stresses should be modified and combined with the shearing stress in such a way as to satisfy Tresca's yield criterion (see Mendelson[5]) in the weld. The weld strength was then based upon the shear strength of the *weld metal*. This proposal was limited to cases for which $e/L'(=\xi')<1{\cdot}3$ because it was found that the observed failure stresses tended to be lower than those predicted by theory when $\xi'>1$.

A more detailed analytical procedure was proposed by Dawe and Kulak[4] to take into account earlier work by Butler and Kulak[6] which had

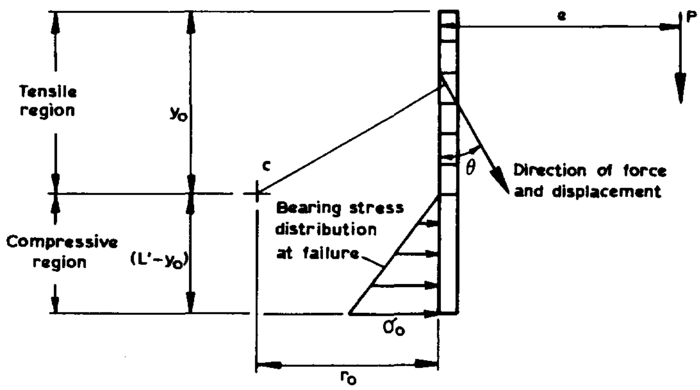

Fig. 3. Model of weld adopted by Dawe and Kulak.[4]

shown that both the strength of a weld and its deformation at failure depended upon the angle θ ($0\leq\theta\leq 90°$) between the direction of the resisting force in the weld and the direction of the weld length. They assumed that at failure one of the members connected by the weld would rotate relative to the other about some instantaneous centre, C (Fig. 3), the members themselves being assumed to remain rigid. The displacement at any point in the weld is then proportional to its distance from C and both the displacement and the resisting force act in a direction normal to the line joining the point to C. Clearly, the angle θ, and therefore the displacement required to cause failure at any point, varies along the weld length. Dawe and Kulak argued that it was necessary to determine the point in the weld which would reach its ultimate deformation first when they assumed that the entire weld length would fail. For any position of C, the angle θ is known for any point in the weld and the resisting force at that point can be obtained from the previously established load–displacement curve corresponding to the angle θ. It was further argued that the weld length in the compressive zone would carry a vertical shearing force only and the horizontal force would be provided by bearing between the connected parts. The bearing stress distribution was assumed to be linear with the maximum stress being set to the yield stress (σ_0) of the connected parts at failure (Fig. 3). To calculate the failure load, the weld length in the tensile region was divided into a number of small elements. The angle θ_i between the load resisted by a typical element i and the weld axis was determined and the force F_i in element i was obtained from the load–deformation curve corresponding to θ_i. The weld failed when the deformation of any element, say j, reached the ultimate deformation in the load–deformation curve corresponding to angle θ_j. The unknowns of the problem were the position of C (r_0 and y_0) and the magnitude of the collapse load. These could be obtained from the three equilibrium equations for the connection. However, since the equations are non-linear, a considerable computational effort was necessary to solve them and a computer was essential to perform the calculations. The method was used to determine the failure loads of three sets of tests which were described in the paper.

Another empirical method for the design of such connections has been proposed by the International Institute of Welding.[7]

3 SOME PRELIMINARY REMARKS

The technique to be used in the analysis proposed in this paper is the same as that detailed by Kamtekar.[1] The forces acting on the faces of the weld are determined and the couples necessary to keep the weld in equilibrium are replaced by 'equivalent' direct and/or shearing forces. This allows the weld to be divided into specified areas over which the forces can be assumed to be uniform. The stresses caused by these 'equivalent' force systems on three mutually perpendicular planes in each of these specified areas are calculated and the principal stresses are determined using the standard theory of stress analysis (see, for example, Mendelson[5] or Southwell[8]). When calculating the stresses it must be remembered that all welds loaded in the as-welded conditions have a self-equilibrating set of residual stresses locked into them as a result of the welding process; this can significantly affect their strength. A longitudinal tensile stress having a magnitude equal to the yield stress of the weld metal is usually present. The other residual stresses in the weld are small and are neglected in the analysis. As the weld is loaded the internal stresses redistribute themselves in such a way that the weld can carry the load but the weld metal remains at yield under the action of the internal stresses. In general, the principal stresses are functions of the applied load and the longitudinal residual stress in the weld (say Y). The value of Y is chosen so that the applied load at failure is a maximum.

Consider an element of the weld, bounded by planes parallel to the co-ordinate axes, which lies in an area of uniform stress. Let the stresses on

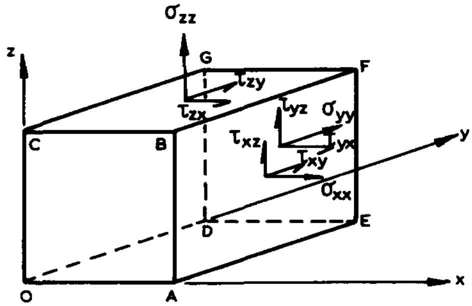

Fig. 4. Sign convention adopted for stresses. (Positive directions indicated for stresses on faces ABFE, DEFG and CBFG. Stresses on opposite faces are positive if acting in reverse directions.)

the planes be σ_{xx}, σ_{yy}, σ_{zz}, $\tau_{xy}\,(=\tau_{yx})$, $\tau_{yz}\,(=\tau_{zy})$ and $\tau_{zx}\,(=\tau_{xz})$, the positive directions being as indicated in Fig. 4. Suppose that one principal stress is of magnitude p and acts on a plane defined by the direction cosines (d.c.s.) (l, m, n) of its normal. Then, equilibrium requires that (see Mendelson,[5] Southwell[8]):

$$
\begin{aligned}
&l(\sigma_{xx}-p)+m\tau_{yx}+n\tau_{zx} = 0\\
&l\tau_{xy}+m(\sigma_{yy}-p)+n\tau_{zy} = 0\\
&l\tau_{xz}+m\tau_{yz}+n(\tau_{zz}-p) = 0
\end{aligned}
\qquad (1)
$$

where l, m, n are related by the identity

$$l^2+m^2+n^2 \equiv 1 \qquad (2)$$

For a known value of p, eqns (1) form a set of three homogeneous equations in l, m, n which have a non-trivial solution only if

$$\begin{vmatrix} (\sigma_{xx}-p) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_{yy}-p) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_{zz}-p) \end{vmatrix} = 0 \tag{3}$$

Expanding eqn (3) leads to a cubic equation in p, the roots of which can be shown to be real (see Southwell[8]); these roots are the values of the three principal stresses. In the case of welds, however, the sloping face is a free surface and so cannot be subjected to either a shearing stress or a direct stress. It must, therefore, be a principal plane on which the principal stress is zero. In other words, $p = 0$ must always be a root of eqn (3) and the problem reduces to finding the other two principal stresses which are the roots of a quadratic equation.

If the orientation of the principal planes is required, any of the calculated values of p can be substituted into eqns (1) and the resulting equations can be solved together with eqn (2) to give the d.c.s. of the normal to the plane on which the chosen value of p is the principal stress. The orientation of the plane on which the shearing stress is a maximum, which is the failure plane, can be deduced from the known orientations of the principal planes.

The principal stresses must always satisfy the yield criterion. It is assumed that yielding occurs according to von Mises' criterion and that, at failure, the weld metal is stressed to its UTS. Then the principal stresses must satisfy (see Mendelson[5])

$$2\sigma_u^2 = (p_1-p_2)^2+(p_2-p_3)^2+(p_3-p_1)^2 \tag{4}$$

When the technique described above is applied to a tension fillet weld, the 'equivalent' force system is as shown in Fig. 5 and the value of P at failure (Fig. 1a) is given by[1]

$$P = \frac{\sigma_u w L_1}{\sqrt{3}} \tag{5a}$$

By following the same procedure, the force carried by each shear weld at failure (Fig. 1b) can be shown to be[1]

$$P = \frac{\sigma_u w L_2}{\sqrt{6}} \tag{5b}$$

It can be shown that[1] the value of Y at failure is different for the two

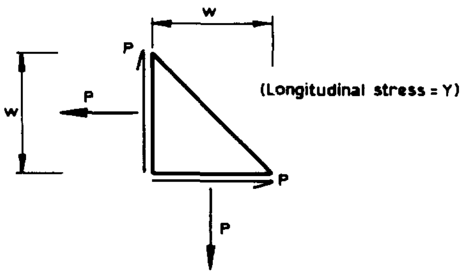

Fig. 5. 'Equivalent' force system for tension fillet weld.

cases, being equal to $\sigma_u/\sqrt{3}$ and zero for tension and shear fillet welds respectively.

It was proposed in the previous paper[1] that the strength of all-round welds (Fig. 1c) should be calculated as the sum of the strengths of the tension and shear fillet welds that participate in the failure mechanism. This implicitly assumes that the weld metal is ductile. This assumption is also used in the present paper.

Thus, three important assumptions are made in developing the solutions for the problems under consideration. Firstly, it is assumed that the strength of a weld subjected to the 'equivalent' force system is a reasonable estimate of its strength when subjected to the actual force system; there is no guarantee that this will be so. Secondly, it turns out that some of the 'equivalent' force systems require stresses to act on the triangular end sections of the welds. This is assumed to be possible even when the ends are free surfaces, in which case these stresses cannot, of course, be present in reality. Finally, the weld metal is assumed to be ductile so that each component weld will develop its full strength before failure occurs. It is essential to test the validity of these assumptions before the theory can be used with confidence; this is done by comparing the strengths of welds predicted by theory with experimental results.

4 ANALYSIS OF WELDS

4.1 Cases considered

Three types of out-of-plane fillet welded connections are analysed (Fig. 6):

(a) Type A connections which consist of a pair of fillet welds along the beam flanges only (Fig. 6a).

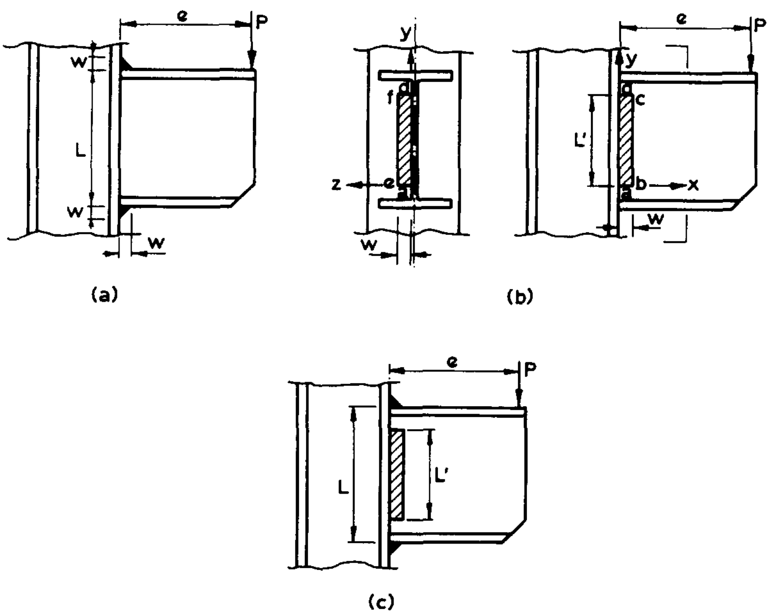

Fig. 6. Types of connections considered. (a) Type A connection; (b) type B connection; (c) type C connection.

(b) Type B connections which consist of welds deposited along the beam web only (Fig. 6b).

(c) Type C connections which consist of a combination of a pair of welds along the beam flanges and welds along the beam web (Fig. 6c).

The welds are assumed to have equal leg lengths.

In each case the weld is subjected to an eccentric shearing force P so that in addition to this force, the connection must also transmit a moment of magnitude Pe. In the following analysis it is assumed that failure occurs in the weld only and that the connected members are strong enough to resist the applied force. It is also assumed that the applied force is transmitted by the welds only, the effect of bearing between the connected members being ignored. This is equivalent to assuming that there will be a gap between the connected members as will often prove to be the case in practice due to unavoidable tolerances and erection requirements.

4.2 The analysis of type A connections

4.2.1 The force system on the weld

The shearing force P and the bending couple Pe applied to the beam at the position of the welds will be transferred to the welds along faces *ab* and *de* (Fig. 7). The shearing force will induce a tensile force P_1 on face *ab* and a compressive force P_2 on face *de*. The couple will apply equal and opposite shearing forces of unknown magnitude P_3 together with moments M_1 and

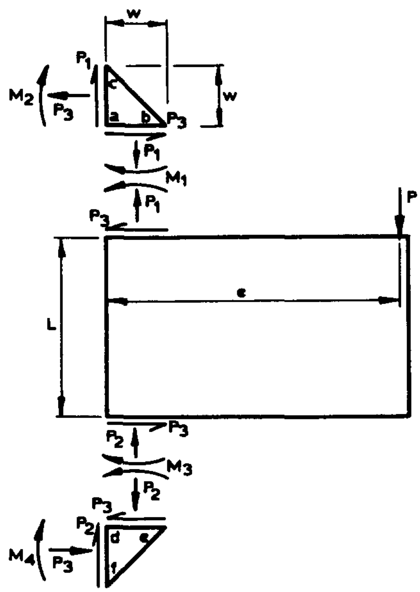

Fig. 7. Force system on a type A connection.

M_3, also of unknown magnitude. These forces are transferred to the column by faces *ac* and *df* by developing tensile and shearing forces and moments. Since it is intended that moments should be replaced by a suitable combination of direct and/or shearing forces, it is possible at this stage to postulate that the couple Pe should lead to shearing forces along faces *ab* and *de* only (i.e. set $M_1 = M_3 = 0$). Consider now the forces acting on the beam (Fig. 7). Assuming that e is large compared with the leg length of the weld, the equilibrium equations for the connection become

$$P = P_1 + P_2 \tag{6}$$

and

$$Pe = P_3 L$$

or

$$P_3 = P\xi \tag{7}$$

For the force system on weld *abc* to be in equilibrium, it is necessary that

$$M_2 = \frac{w}{2}(P\xi - P_1)$$

The moment M_2 can be replaced either by a tensile force of magnitude $(P\xi - P_1)$ on face *ab* together with a shearing force of the same magnitude along *ac* to give the 'equivalent' force distribution of Fig. 8a or by a compressive force of magnitude $(P\xi - P_1)$ on face *ac* together with a shearing force of the same magnitude along face *ba* to give the 'equivalent' force distribution of Fig. 8b.

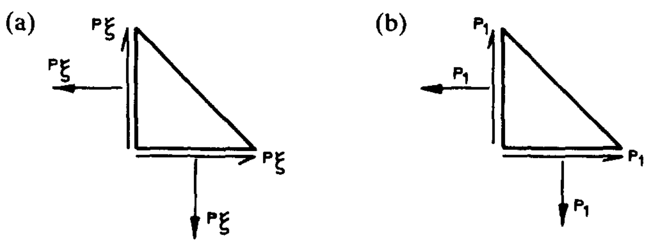

Fig. 8. 'Equivalent' force distributions for tension weld in type A connections. (a) In terms of $P\xi$; (b) in terms of P_1.

4.2.2 The strength of the connection

The force distributions of both Figs 8a and 8b are similar to those for a tension fillet weld (Fig. 5) so that the failure load corresponding to each can be obtained without further analysis. Comparing Figs 8a and 5, eqn (5) gives

$$P\xi = \frac{\sigma_u w L_1}{\sqrt{3}} \tag{8a}$$

and comparing Figs 8b and 5, eqn (5) gives

$$P_1 = \frac{\sigma_u w L_1}{\sqrt{3}} \tag{8b}$$

A similar argument can be applied to the weld *def* to either give eqn (8a) again or to give

$$P_2 = \frac{\sigma_u w L_1}{\sqrt{3}} \tag{9}$$

If it is assumed that the connection fails only when both the welds *abc* and *def* have failed, it follows that either

$$P\xi = \frac{\sigma_u w L_1}{\sqrt{3}}$$

giving

$$P = \frac{\sigma_u w L_1}{\xi\sqrt{3}} \tag{10a}$$

or

$$P_1 = P_2 = \sigma_u w L_1/\sqrt{3}$$

whence eqn (6) gives

$$P = \frac{2\sigma_u w L_1}{\sqrt{3}} \tag{10b}$$

Thus, *two* failure loads have been determined for the connection and it is necessary to decide which of them is correct. The two failure loads, in fact, correspond to two different failure modes. The welds are subjected to a shearing force (P) and a couple ($M = Pe$) and one of the failure loads (eqn (10a)) corresponds to the case where M is large compared to P and the other (eqn (10b)) corresponds to the case where P is large compared to M. It would be expected that eqn (10a) would define the failure load for the connection when e (and, therefore, ξ) was large and eqn (10b) would define the failure load when ξ was small. When $\xi = 0$, the connection represents a pair of tension fillet welds for which eqn (10b) is the correct solution. The problem is similar to the one that arises when considering the failure of an I-section beam at a particular cross-section which is subjected to a combination of a bending moment and a shearing force. The load-carrying capacity of the cross-section can be reached either as a result of the whole cross-section yielding due to the applied loads or as a result of the web of the beam yielding due to the applied shearing force.[9] For this problem, an empirical relationship obtained by satisfying the

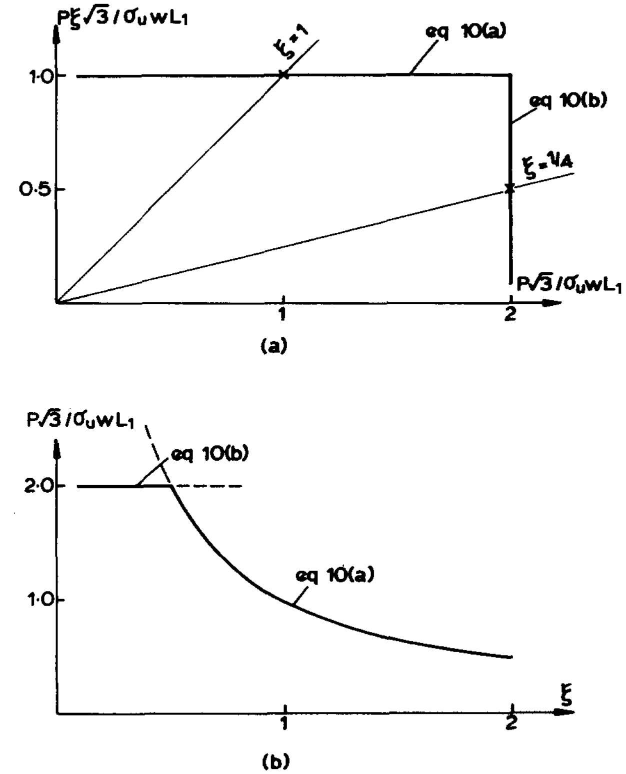

Fig. 9. Failure load for type A connection. (a) Interaction curve; (b) relationship between P and ξ.

yield criterion *locally* was proposed for determining the load-carrying capacity of the cross-section under consideration. Due to the approach adopted for the present problem, P and M have effectively been uncoupled, leading to two expressions for the failure load. For any given connection, the *lower* of the two loads obtained from eqns (10a) and (10b) must be the maximum load that the connection can transmit.

Equations (10a) and (10b) define an interaction curve between P and M. The relationships can be plotted non-dimensionally in two ways. They can be shown either as a plot of P against $P\xi$ (Fig. 9a) which gives an interaction curve consisting of two straight lines parallel to the axes, or as a plot of P against ξ (Fig. 9b). Since the lower of the values given by eqns (10a) and (10b) is the required failure load for any connection, it can be seen that eqn (10a) governs when $\xi \geq 1/2$ and eqn (10b) governs when

$\xi \leq 1/2$. It is perhaps worth noting that Fig. 9a is of precisely the same form as the interaction curve predicted for biaxially loaded welds in ref. 1.

The failure load for any type A connection can be obtained from Fig. 9a by noting that a straight line passing through the origin has a slope ξ which is known for a specified connection. For a particular connection, therefore, it is only necessary to draw the appropriate straight line and read off the failure load as the load corresponding to the point where this line intersects the interaction curve. Lines corresponding to connections for which $\xi = 1/4$ and $\xi = 1$ are indicated in Fig. 9a.

4.3 The strength of type B connections

The applied eccentric shearing force is transferred to the weld as a vertical shearing force of magnitude P and a bending couple of magnitude Pe. Both these are resisted by the weld by producing shearing stresses on the plane *abcd* in Fig. 6b. They are transferred to the column flange as shearing and direct forces on the plane *adfe* (Fig. 6b). In addition to these forces there are couples present at the weld/beam and weld/column face interfaces but these are not considered in detail because they are to be replaced by 'equivalent' direct and/or shearing forces.

For the analysis to proceed along the proposed lines, the stresses must be uniform in specified regions of the weld. The applied couple is resisted by shearing stresses which act on face *abcd* in a direction normal to the line of action of the applied force P. Since there must be no out-of-balance force in this direction and since the shearing stresses are assumed to be uniform over parts of the weld, they must change sign somewhere along the length L' making the stress distribution discontinuous. This discontinuity is similar to the one that is present in the stress distribution down the depth of a beam at a section at which a plastic hinge has formed. To be statically admissible, the stress distributions in the regions on either side of the plane of discontinuity must satisfy certain conditions. Consider an element of material through which the plane of discontinuity (which is perpendicular to the y-axis, say) passes (Fig. 10). The stresses in regions 1 and 2 are uniform so that the forces on the faces of the element in the two regions are as indicated in Fig. 11. Remembering that in each region $\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}$ and $\tau_{zx} = \tau_{xz}$, it can be seen that the forces in the x and z directions in the two regions will be in equilibrium whatever the values of σ_{xx}, σ_{zz} and τ_{xz} may be, i.e. these stresses may have different values in the regions on either side of the plane of discontinuity without affecting

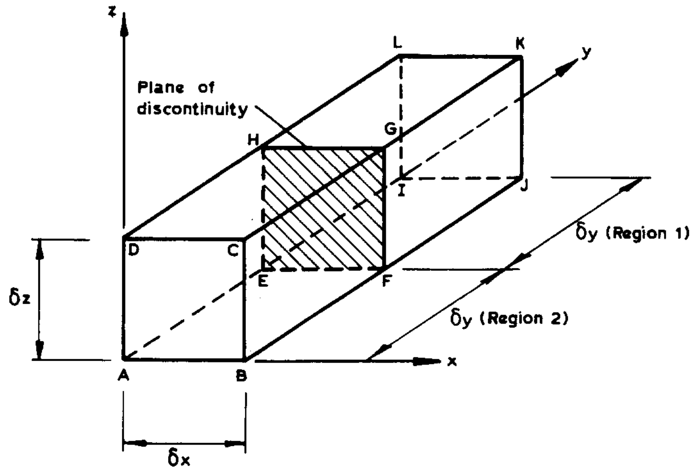

Fig. 10. Element of material showing plane of discontinuity.

equilibrium. The stresses in the y-direction must be such that the forces on the plane of discontinuity ($EFGH$ in Fig. 11) must be in equilibrium. This requires

$$F^1_{yx} = F^2_{yx},$$

$$F^1_{yy} = F^2_{yy}$$

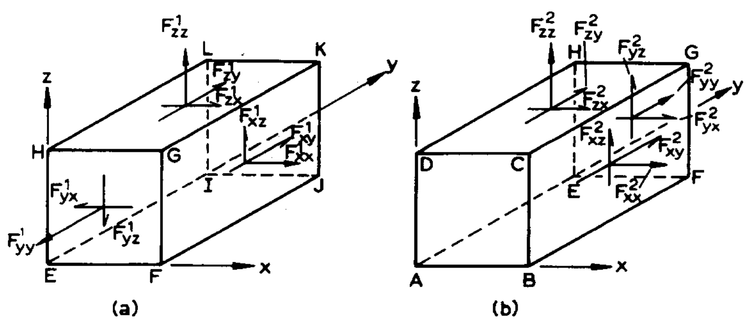

Fig. 11. Forces on faces of element on either side of plane of discontinuity. (Forces shown on planes $BFGC$, $EFGH$, $DCGH$, $FJKG$ and $HKGL$ only. Forces on opposite planes are equal in magnitude but act in reverse directions — sign convention as in Fig. 4.)

$F^i_{xx} = (\sigma_{xx})_i A_x,\ F^i_{xy} = (\tau_{xy})_i A_x,\ F^i_{xz} = (\tau_{xz})_i A_x$

$F^i_{yy} = (\sigma_{yy})_i A_y,\ F^i_{yx} = (\tau_{yx})_i A_y,\ F^i_{yz} = (\tau_{yx})_i A_y$

$F^i_{zz} = (\sigma_{zz})_i A_z,\ F^i_{zx} = (\tau_{zx})_i A_z,\ F^i_{zy} = (\tau_{zy})_i A_z$

$i = 1, 2$ (region); $A_x = (\delta y)(\delta z)$; $A_y = (\delta z)(\delta x)$; $A_z = (\delta x)(\delta y)$

and

$$F^1_{yz} = F^2_{yz}$$

which requires

$$(\tau_{yx})_1 = (\tau_{yx})_2, (\sigma_{yy})_1 = (\sigma_{yy})_2$$

and

$$(\tau_{yz})_1 = (\tau_{yz})_2$$

i.e. equilibrium requires that the stresses parallel to the y-axis *must* have the same values on either side of the plane of discontinuity.

The bending couple Pe is resisted by shearing stresses on plane *abcd* (Fig. 6b) acting parallel to the x-axis. Suppose that this stress changes sign once only — at a distance $\eta L'$ from the top of the weld. For the loading of Fig. 6b, the shearing force in the length $\eta L'$ of the weld acts in the positive direction of the x-axis and the shearing force in the rest of the weld acts in the opposite direction. Denote the length $\eta L'$ of the weld as region 1, the rest of the weld as region 2 and the shearing stresses resisting the bending couple in the two regions as $(\tau_{zx})_1$ and $(\tau_{zx})_2$ respectively. The shearing forces in the two regions are then $(\tau_{zx})_1 \eta w L'$ and $(\tau_{zx})_2(1-\eta)\,wL'$ respectively. Forces of equal magnitude but of opposite sign act on the beam web, as indicated in Fig. 12.

The shearing force P gives rise to a shearing stress τ_{zy} on plane *abcd*. Since this stress must have the same value in both regions, it follows that

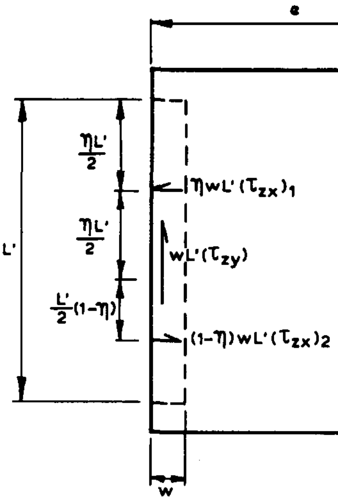

Fig. 12. Forces on the beam web (type B connection).

the shearing force in the weld on plane *abcd* is $\tau_{zy} wL'$, acting vertically downwards. An equal and opposite force acts on the beam web (Fig. 12).

Consider now the equilibrium of the beam web. Remembering that the forces due to the weld act over a width equal to the leg length (w) and assuming that $w << e$, equilibrium gives (Fig. 12):

$$P = \tau_{zy} wL'$$

so that

$$\tau_{zy} = \frac{P}{wL'} = \sigma \tag{11}$$

$$\eta wL' (\tau_{zx})_1 = (1-\eta) wL' (\tau_{zx})_2$$

so that

$$(\tau_{zx})_2 = \frac{\eta}{1-\eta}(\tau_{zx})_1 \tag{12}$$

and

$$Pe = \frac{1}{2}\eta wL'^2 (\tau_{zx})_1$$

so that

$$(\tau_{zx})_1 = \frac{2Pe}{\eta wL'^2} = \frac{2\sigma\xi'}{\eta} \tag{13}$$

The shearing stresses $(\tau_{zx})_1$ and $(\tau_{zx})_2$ are transmitted to the column face by direct stresses of the same magnitude on face *adfe* and the shearing force P is transmitted to the column face as a shearing force of magnitude P on face *adfe* in the positive direction of the y-axis giving rise to a shearing stress τ_{xy} which must have the same value in both regions 1 and 2 as discussed earlier (Fig. 13). Therefore, in both regions

$$|\tau_{xy}| = P/wL' = \sigma \tag{14}$$

These forces by themselves are not in equilibrium because the couples acting on the faces *abcd* and *adfe* have, so far, been neglected. First consider the equilibrium of region 1 of the weld. The $(\tau_{zx})_1$ stress requires a complementary shearing stress $(\tau_{xz})_1$ on face *adfe* which in turn requires a

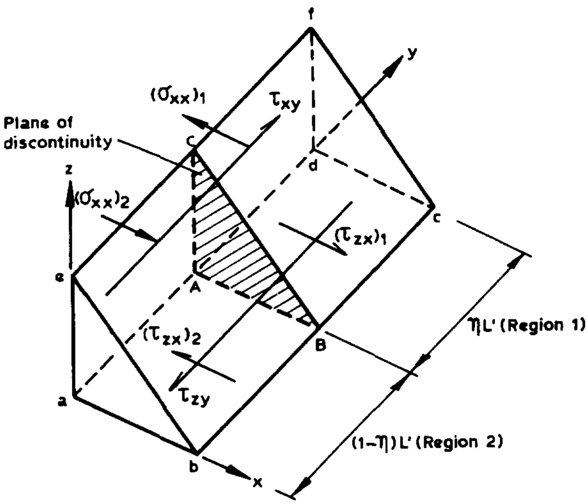

Fig. 13. Stresses in the weld due to applied shearing force and couple (not in equilibrium).

tensile stress $\sigma_{zz}(= (\tau_{xz})_1)$ on face *abcd* to maintain equilibrium. Similarly, the shearing stresses τ_{zy} and τ_{xy} require complementary shearing stresses τ_{yz} and τ_{yx} on the end faces of region 1. Thus, the stresses in region 1 are as indicated in Fig. 14a. The complementary shearing stresses together with the associated direct stresses ensure that the force system is in equilibrium — they form the force system which is 'equivalent' to the

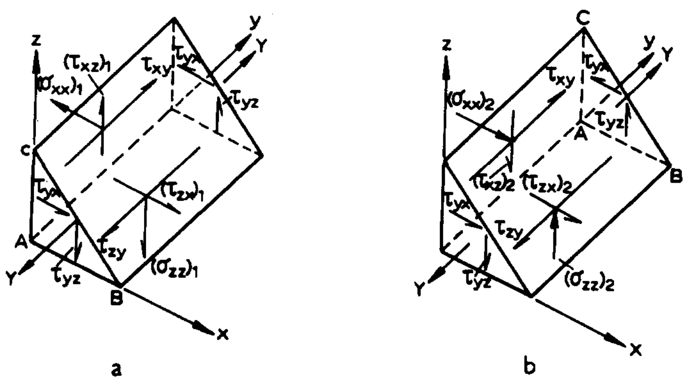

Fig. 14. 'Equivalent' stresses in type B welds. (a) Stresses in region 1; (b) stresses in region 2.

couples on faces *abcd* and *adfe*. Using the sign convention of Fig. 4, the stresses in region 1 are

$$\sigma_{xx} = \frac{2\sigma\xi'}{\eta}, \; \tau_{xy} = \tau_{yx} = -\sigma$$

$$\sigma_{yy} = Y, \; \tau_{yz} = \tau_{zy} = \sigma$$

$$\sigma_{zz} = \frac{2\sigma\xi'}{\eta}, \; \tau_{zx} = \tau_{xz} = -\frac{2\sigma\xi'}{\eta}$$

Substituting these stresses into eqn (3) and expanding the determinant gives:

$$p\{2\sigma^2 - (Y-p)(\Sigma-p)\} = 0 \tag{15}$$

where

$$\Sigma = 4\sigma\xi'/\eta$$

The principal stresses in region 1 are then determined as

$$p_1 = 0; p_{2,3} = \tfrac{1}{2}(\Sigma + Y \pm R_1); \; R_1^2 = (\Sigma - Y)^2 + 8\sigma^2$$

Since the material in region 1 satisfies the yield criterion at failure, these values of the principal stresses can be substituted into eqn (4) to give

$$\sigma_u^2 = Y^2 - Y\Sigma + \Sigma^2 + 6\sigma^2 \tag{16}$$

Now consider the equilibrium of region 2. Arguing in exactly the same way as for region 1, the stress system which satisfies equilibrium is the one shown in Fig. 14b. (σ_{yy} must be the same in the two regions.) Using the sign convention of Fig. 4 for the stresses and using eqn (12) gives

$$\sigma_{xx} = -\frac{2\sigma\xi'}{1-\eta}; \; \tau_{xy} = \tau_{yx} = -\sigma$$

$$\sigma_{yy} = Y; \; \tau_{yz} = \tau_{zy} = \sigma$$

$$\sigma_{zz} = -\frac{2\sigma\xi'}{1-\eta}; \; \tau_{zx} = \tau_{xz} = \frac{2\sigma\xi'}{1-\eta}$$

The equation from which the principal stresses are determined then becomes

$$p\{2\sigma^2-(Y-p)(\Sigma'+p)\} = 0 \tag{17}$$

where

$$\Sigma' = 4\sigma\xi'/(1-\eta)$$

The principal stresses can now be calculated as

$$p_1 = 0; p_{2,3} = \tfrac{1}{2}(Y-\Sigma') \pm R_2$$

where

$$R_2^2 = (Y+\Sigma')^2+8\sigma^2$$

At failure these stresses satisfy eqn (4) so that

$$\sigma_u^2 = Y^2+\Sigma'+(\Sigma')^2+6\sigma^2 \tag{18}$$

There are three unknowns (σ, Y and η) left in the problem and only two equations (eqns (16) and (18)) to determine them. A third equation can be obtained by applying the condition that Y will assume that value at failure which will maximise the failure load.

An intuitive argument can be used to determine the value of Y at collapse. It can be argued that the shearing stresses $(\tau_{zx})_1$ and $(\tau_{zx})_2$ caused by the bending couple should be equal in magnitude. Equation (12) then gives $\eta = \tfrac{1}{2}$. It follows that $\Sigma = \Sigma'$ and eqns (16) and (18) then require $Y = 0$. Alternatively, an expression for Y can be obtained by equating the right-hand sides of eqns (16) and (18), whence,

$$Y\left\{\frac{1}{\eta(1-\eta)}\right\} = \frac{4\sigma\xi'}{\eta(1-\eta)}\left\{\frac{1}{\eta}-\frac{1}{1-\eta}\right\} \tag{19}$$

Thus either $\eta = 0$, or $\eta = 1$, or

$$Y = \frac{4\sigma\xi'(1-2\eta)}{\eta(1-\eta)} \tag{20}$$

$\eta = 0$ and 1 correspond to the special case where $e = 0$. In general, therefore, eqn (20) gives the value of Y. This can be substituted into either of eqns (16) and (18) to give:

$$\sigma_u^2 = \left\{ \frac{4\sigma\xi'}{\eta(1-\eta)} \right\}^2 (1 - 3\eta + 3\eta^2) + 6\sigma^2 \tag{21}$$

Differentiating eqn (21) with respect to η and setting $d\sigma/d\eta = 0$ gives three possible solutions; $\eta = 0$, 1 or ½. The first two have already been considered so that $\eta = ½$ is the solution in general. Equation (20) then gives $Y = 0$, as before. Using these values of Y and η in any of eqns (16), (18) and (21) finally gives the failure load as:

$$P = \sigma w L' = \frac{\sigma_u w L'}{\sqrt{(6 + 64\xi'^2)}} \tag{22}$$

Equation (22) can be plotted in non-dimensional form (Fig. 15) as a relationship between P and ξ'. Alternatively, it can be plotted as an

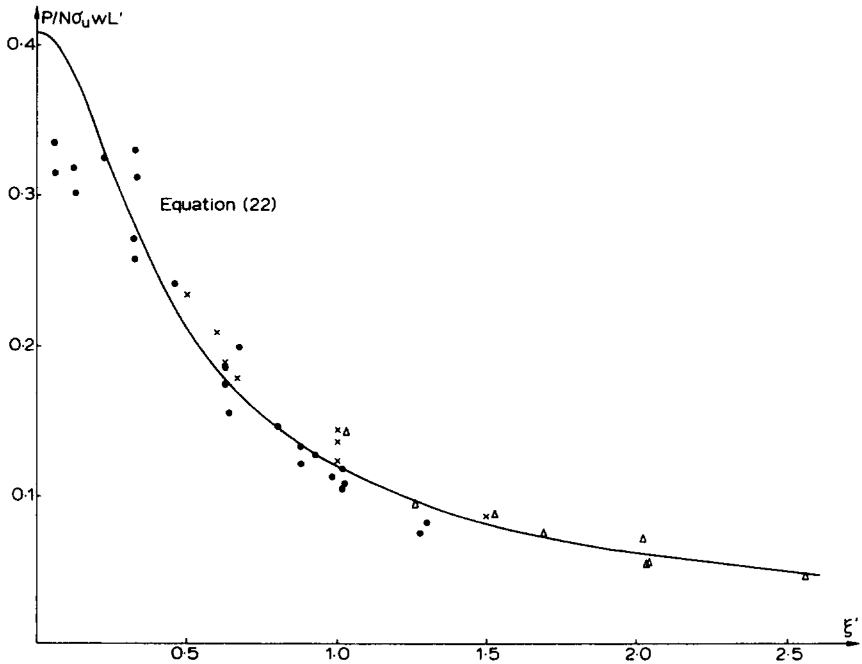

Fig. 15. Failure load of type B connections. (●) Archer *et al.*[3] ($N = 2$); (×) Butler *et al.*[11] ($N = 4$); (Δ) Dawe and Kulak[4] ($N = 2$).

interaction curve as follows. The weld is subjected to a shearing force P and a moment $M(=Pe)$. Let P_0 be the value of P at failure when the eccentricity is zero and let M_0 be the value of the moment at failure when P is zero. P_0 can be obtained directly from eqn (22) by setting $\xi' = 0$ giving

$$P_0 = \frac{\sigma_u w L'}{\sqrt{6}} \tag{23}$$

as expected, since the weld is now a shear fillet weld. M_0 can be obtained from eqn (22) by noting that it is the value of M as $P \rightarrow 0$ and $e \rightarrow \infty$ with Pe having a constant value M_0. This gives

$$M_0 = \frac{\sigma_u w L'^2}{8} \tag{24}$$

The (P, M) interaction curve can be obtained using eqns (22)–(24) as the circle

$$\left(\frac{P}{P_0}\right)^2 + \left(\frac{M}{M_0}\right)^2 = 1 \tag{25}$$

The failure load given by eqn (22) is the load transmitted by *one* beam-web-to-column-flange weld in Fig. 6b. If the connection consists of N such welds, each carrying the same load, the failure load given by eqn (22) must be multiplied by N. The non-dimensionalised curve of Fig. 15 remains unaltered, however, provided that the load term is plotted as $P/(N\sigma_u w L')$.

It turns out that the 'equivalent' force system derived for this type of connection is exactly the same as that which can be derived for the vertical weld in Fig. 2b. The strength of the latter weld is also, therefore, given by eqn (22).

The analysis developed above is for a single continuous weld only. The connection may sometimes be made using the weld arrangement of Fig. 16. The analysis of this type of weld is simpler than that of a type B connection because the continuity conditions required to satisfy the equilibrium equations at a plane of discontinuity are no longer necessary. In particular, the longitudinal residual stress in the welds need not be the same. The analysis gives rise to an interesting anomaly. It is given here for that reason and also because it enables the predictions of the proposed theory to be compared with test results quoted by Biggs *et al.*[10]

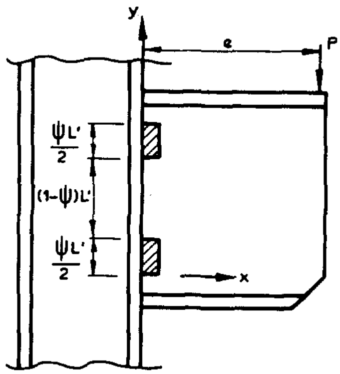

Fig. 16. Intermittent type B welds (see Biggs *et al.* [10]).

The bending couple Pe is again replaced by a pair of shearing stresses τ_{zx} in the welds. The stress in the upper weld acts in the positive direction of the x axis and that in the lower weld acts in the opposite direction. Assuming them to be uniform in each of the welds, their value is given by

$$|\tau_{zx}| = \frac{4\sigma\xi'}{\psi(2-\psi)}$$

The shearing force P gives rise to the shearing stress τ_{zy} at the weld/beam interface and assuming it to be uniform over the weld area,

$$|\tau_{zy}| = \frac{\sigma}{\psi}$$

As for the type B connection, the τ_{zx} stresses give rise to a direct stress and complementary shearing stress on the weld/column interface which, in turn, requires a direct stress on the weld/beam interface for equilibrium. The τ_{zy} stresses give rise to shearing stresses τ_{xy} on the weld/column interface and both these stresses require complementary shearing stresses on the end faces of the weld to maintain equilibrium. The stress system in the upper weld (using the sign convention of Fig. 4) is

$$\sigma_{xx} = \sigma_{zz} = \frac{4\sigma\xi'}{\psi(2-\psi)},\ \tau_{xy} = \tau_{yx} = -\tau_{yz} = -\tau_{zy} = -\frac{\sigma}{\psi}$$

$$\sigma_{yy} = Y_1,\ \tau_{zx} = \tau_{xz} = -\frac{\sigma}{\psi}$$

All the stresses in the lower weld, except σ_{yy}, have the same absolute values. The signs of σ_{xx}, σ_{zz}, τ_{zx} and τ_{xz} are, however, reversed. The longitudinal stress can be different and is denoted by Y_2.

The principal stresses in the upper weld can be obtained using eqn (3) as

$$p_1 = 0,\ p_{2,3} = \frac{1}{2}\{(Y+\Sigma'') \pm R_3\}$$

where

$$\Sigma'' = \frac{8\sigma\xi'}{\psi(2-\psi)}$$

and

$$R_3^2 = (Y-\Sigma'')^2 + 8\left(\frac{\sigma}{\psi}\right)^2$$

The stresses will satisfy the yield criterion if

$$\sigma_u^2 = Y_1^2 - \Sigma''\ Y_1 + (\Sigma'')^2 + 6\left(\frac{\sigma}{\psi}\right)^2 \qquad (26)$$

Similarly it can be shown that the stresses in the lower weld will satisfy the yield criterion if

$$\sigma_u^2 = Y_2^2 + \Sigma''\ Y_2 + (\Sigma'')^2 + 6\left(\frac{\sigma}{\psi}\right)^2 \qquad (27)$$

To satisfy both eqns (26) and (27) and to give a maximum value of σ, $Y_1 = -Y_2 = \frac{1}{2}\Sigma''$, whence

$$\sigma = \frac{\sigma_u\psi(2-\psi)}{\sqrt{6\{(2-\psi)^2 + 8\xi'^2\}}} \qquad (28)$$

and

$$P = \sigma w L' \qquad (29)$$

P_0 and M_0 are obtained as before from eqn (29) and are given by

$$P_0 = \frac{\sigma_u\psi w L'}{\sqrt{6}} \qquad (30)$$

and

$$M_0 = \frac{\sigma_u \psi(2-\psi) w L'^2}{4\sqrt{3}} \tag{31}$$

Equations (28)–(31) lead to the same interaction formula given by eqn (25).

It may appear at first sight that a value for the failure load of a *continuous* weld can be deduced from eqn (29) by putting $\psi = 1$. This would give

$$P = \frac{\sigma_u w L'}{\sqrt{(6+48\xi'^2)}} \tag{32}$$

The failure load predicted by eqn (32) is greater than the one predicted by eqn (22). This is merely due to the fact that the condition concerning the equilibrium of longitudinal forces on either side of a plane of discontinuity has been violated in arriving at eqn (32). This cannot, therefore, be the correct solution for *continuous* welds.

4.4 The strength of type C connections

No special analysis is carried out for this type of connection it being assumed that its strength will simply be given by the sum of the strengths of the flange welds (type A connection, eqns (10a) and (10b)) and of the web welds (type B connections, eqn (22)). In using this approach it is implied that the welds have adequate ductility to enable each weld to develop its full strength before the connection fails. The same assumption was made for obtaining the strength of all-round welds (Fig. 1c) in a previous paper[1] where the assumption was found to be reasonable (this conclusion being reached by comparing the failure loads predicted by theory with experimental results) and enabled the failure load to be easily calculated. The assumption will again be judged by comparing theoretical predictions with experimental results. It should be noted that this approach ensures that the forces in the welds are in equilibrium with the applied external load and that each weld satisfies the failure criterion given by eqn (4).

The collapse load of a type C connection (Fig. 6c) is given by

$$P = \frac{2\sigma_u w L_1}{\sqrt{3}} + \frac{N\sigma_u w L'}{\sqrt{(6+64\xi'^2)}} \text{ if } \xi \leq \frac{1}{2} \tag{33a}$$

$$P = \frac{\sigma_u w L_1}{\xi\sqrt{3}} + \frac{N\sigma_u w L'}{\sqrt{(6+64\xi'^2)}} \text{ if } \xi \geq \frac{1}{2} \tag{33b}$$

where N is the number of web welds which help to carry the applied load. Although it is obviously possible to combine the two terms it is preferable to leave them separate because it is then easier to allow for the case where the flange and web welds are of different sizes.

4.5 Commentary

The strength of a type B connection is a maximum when the longitudinal residual stress in the weld is zero. Thus, the strength of such a connection will not be affected by stress relieving. The strength of a type A connection, on the other hand, does depend upon the longitudinal residual stress in the weld and will be affected by stress relieving. It was previously shown[1] that initially stress-relieved tension fillet welds would fail at a load obtained from eqn (5) by replacing $\sqrt{3}$ by 2. The same change is necessary in eqns (10) and (33) if the strength of stress-relieved type A and type C connections is required. The effect of stress-relieving on the strength of the intermittent weld of Fig. 16 may be investigated by putting $Y_1 = Y_2 = 0$. Equations (26) and (27) become identical and the factor which multiplies ξ'^2 in eqn (28) becomes 64 instead of 48. The anomaly mentioned previously then disappears.

The procedure adopted has been one of postulating a statically admissible stress field for the weld which exactly satisfies the yield criterion everywhere. Only one such stress field has been considered for each connection and it is possible that there may be other stress fields which will give higher estimates of the failure load.

5 COMPARISON BETWEEN THEORY AND EXPERIMENTS

5.1 General comments

The importance of comparing the predictions of the theory developed in this paper with experimental results has been made clear already. For such a comparison to be meaningful, however, it is necessary to have accurate information about two parameters for the welds used in the tests — the weld leg lengths and the UTS of the weld metal. Very few tests were found which gave accurate information on these parameters.

For the welds to be uniform it is necessary to fabricate the specimens by depositing oversize welds which can be machined to the correct size. This method of fabrication is obviously expensive with the result that very few specimens used in published experimental studies were prepared in this way. In the majority of cases the welds under test were carefully deposited by a skilled welder and the leg lengths were measured. However skilful the operator may be, this process is bound to lead to some non-uniformity in the welds.

To determine the UTS of the weld metal, tension specimens must be obtained from deposited weld metal. Such tests were not generally performed and it is therefore difficult to relate the actual UTS of the weld metal to the weld strength determined experimentally. Instead of performing tension tests, investigators usually perform control tests on tension and/or shear fillet welds; the specimens used are similar to those

TABLE 1
Values of σ_u Deduced From Control Tests

Investigators	*Value of σ_u (ksi) from*	
	tension fillet weld	*shear fillet weld*
Archer *et al.*[3]	109	97
Butler *et al.*[11]	109	104
Dawe and Kulak[4]		
Series 1 (type B welds)	—	91
Series 3 (type C welds)	—	96

Value of σ_u for tests of Biggs *et al.*[10] is 550 N mm^{-2} (deduced from pure bending tests).

shown in Figs 1a and 1b. If eqns (5a) and (5b) are accepted as correct, a value for the UTS of the weld metal can be deduced by applying them to the results of the control tests. This procedure was used to estimate the UTS of the weld metal used in the tests described by Archer *et al.*,[3] Butler *et al.*[11] and Dawe and Kulak;[4,12] the estimated values are given in Table 1. The UTS values deduced from the tension fillet welds and shear fillet welds do not vary appreciably but the former values were used in the comparisons because earlier work[1] had suggested that eqn (5a) gave closer estimates of the failure loads than eqn (5b). Tests reported by two other investigators are also used in making comparisons between theory and experiment. Biggs *et al.*[10] performed no control tests but they did perform pure bending tests on welds of the type shown in Fig. 16. A value of 550 Nmm^{-2} was deduced for the UTS of their weld metal from these tests by applying eqn (31). This is close to the value of 520 Nmm^{-2} that was deduced for σ_u from their tests on biaxially loaded welds. Johnson[13] did not give any results from which the UTS of the weld metal could be deduced and a different procedure, described later, was used to compare his results with the proposed theory.

The tests used for making the following comparisons were conducted between 1959 and 1981 and different units were used by the investigators. The comparisons are generally made in non-dimensional form but, where this is not convenient, the units used in the original investigation have been retained.

Several test results can be found in the literature for type B connections but only a limited number of results are available for type C and type A connections. Type B connections are therefore discussed first followed by type C and type A connections.

5.2 Type B connections

These were investigated by Archer *et al.*,[3] Butler *et al.*,[11] Dawe and Kulak[4,12] and Biggs *et al.*[10]

Each specimen tested by Archer *et al.* was made up by welding a beam to a column using a pair of vertical welds, one on each side of the beam web. The welds in the various specimens were deposited at different positions down the depth of the beam, the intention being to see whether bearing between the connected members played a significant part in the failure of the weld. The leg lengths of the deposited welds were measured. The ratio ξ' was varied in the tests between 0·06 and 1·3, a total of 24 tests being performed. The beam was loaded as a cantilever and the failure load P was measured. The non-dimensional failure loads (with $N = 2$) are compared with the prediction of eqn (22) in Fig. 15.

Butler *et al.*[11] performed tests which, in fact, modelled the vertical weld in the connection shown in Fig. 2b but, as noted earlier, the strength of this weld is also given by eqn (22). The specimens consisted of a central beam with an angle welded to each side of its web at each end using a vertical fillet weld. The outstanding legs of the angles were bolted to end support fixtures which allowed free axial movement and rotation. The distance between the weld and the end support fixtures was varied to provide values of ξ' between 0·5 and 1·5. The beam was loaded at its mid-point, effectively providing four identically loaded welds (i.e. $N = 4$). The load applied to the beam at failure was recorded but the weld sizes were not measured. The non-dimensionalised failure loads, calculated by assuming that the weld leg lengths were ¼″ as specified, are plotted in Fig. 15.

Dawe and Kulak[4,12] tested connections made up by welding a wide flange section beam to a column. The beam was loaded as a cantilever and orientated so that it was bent about its minor axis, the connection being made using a weld between the outside of each flange of the beam and the column ($N = 2$). ξ' was varied between 1 and 2·5. Average values of measured leg lengths were quoted, together with the failure loads. These results are also plotted in Fig. 15.

The specimens used by Biggs *et al.*[10] consisted of three parts — two re-usable end pieces and a central test section which was bolted to them. The test section was of H shape in plan, the welds under test being those

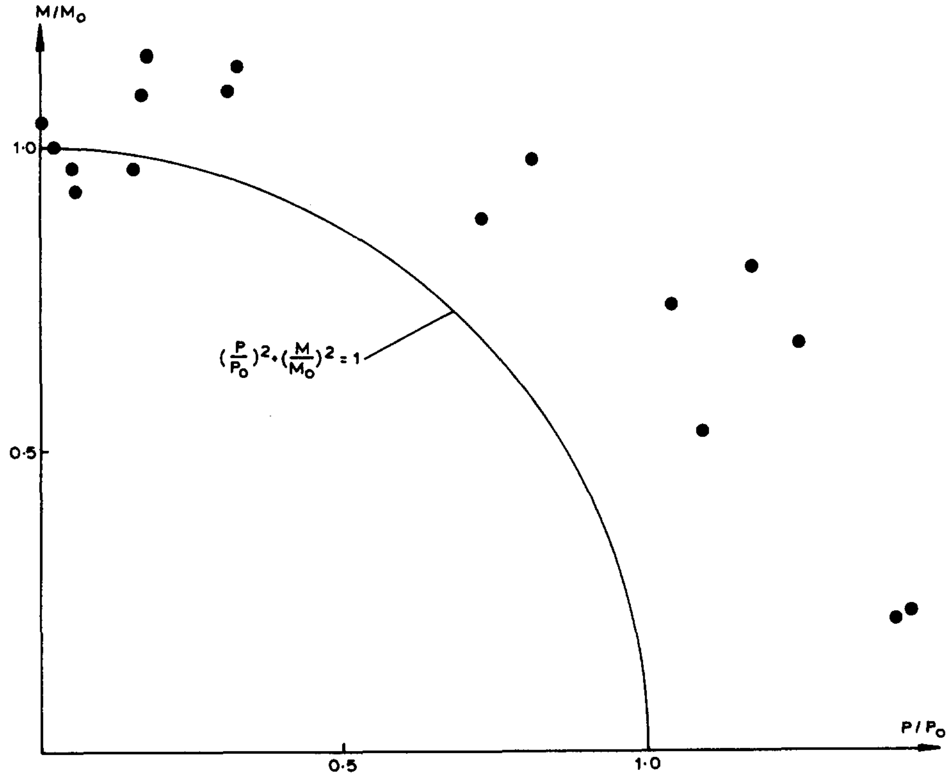

Fig. 17. Interaction curve for intermittent type B welds (together with test results by Biggs *et al.*[10]).

joining the web and the flanges of the H shape. For testing, the H shape was orientated such that the welds were vertical. The weld detail was as shown in Fig. 16 with $L = 150$ mm and $\psi = 0{\cdot}2$. The welds were machined to give a leg length of 4 mm. The specimen was supported and loaded in such a way that the welds were subjected to various combinations of bending moment (M) and shearing force (P), varying from pure bending ($\xi' \rightarrow \infty$) at one end to low bending and high shear ($\xi' \rightarrow 0$) at the other. The observed results are plotted in Fig. 17 together with the interaction curve given by eqn (25).

5.3 Type C and type A connections

Only a few results were found with which the proposed theory could be compared. Four test results for type C connections were given by Dawe and Kulak[4,12] and two by Johnson[13] who also gave the only two results that seem to be available for type A connections.

The setup for the tests conducted by Dawe and Kulak was similar to the one described above for their tests on type B connections except that the wide flange section beam was bent about its major axis when loaded. The connection was made using a weld on the outside of each of the beam flanges (giving a type A connection) and a pair of welds down the depth of the beam web between the roots of the fillets (giving a type B connection with $N = 2$). A 6WF25 section was used as the beam for specimens C1 and C2 and an 8WF40 section was used for the others.[12] The mean values of the measured leg lengths were recorded, together with the observed failure loads. The latter are compared in Table 2 with the theoretical failure loads calculated using eqn (33b). In general, the size of the type A weld in tension was different from the weld in compression. The size of the tension weld was used in the theoretical calculations.

Johnson did not perform any control tests from which the UTS of the weld metal could be deduced. The technique adopted for using his test results was to compute the failure loads for each of his specimens in terms of σ_u using whichever of eqns (33) and (10) was relevant and to use the observed failure load to calculate a value for σ_u for each specimen. If the theory is correct, all the σ_u values would be the same and would lie in the range specified by the British Standard applicable to the electrodes used.

Johnson's specimens were made up by welding an RSJ to either one or both flanges of a stub column of a suitable rolled section. The RSJ's were loaded in such a way that the eccentricity of the applied shearing force was

TABLE 2
Comparison Between Theory and Test Results of Dawe and Kulak[4]—Type C Welds

		Tension flange welds				*Web welds*			*Failure loads (kips)*	
Specimen no.	*e (in)*	*w (in)*	L_1 *(in)*	L^a *(in)*	ξ	*w (in)*	L' *(in)*	ξ'	*Theory*[b]	*Experiment*
C1	15	0·30	4·29	6·38	2·351	0·31	5·50	2·727	45·3	46·6
C2	20	0·29	4·43	6·38	3·135	0·29	5·52	3·623	33·3	38·4
C3	15	0·32	4·21	8·25	1·818	0·27	7·15	2·098	62·9	66·0
C4	20	0·29	4·19	8·25	2·424	0·32	7·12	2·809	47·1	46·1

[a]Overall depth of WF section used.
[b]Obtained from eqn (33b) with $N = 2$ and $\sigma_u = 96$ ksi (see Table 1).

54 in from the column face. Several beam/column connection details were investigated but only four of these are of interest in the present work. For two of these (specimens 2 and 7) the connection was made by depositing welds on both sides of each of the beam flanges and also on each side of the beam web. The sizes of the welds on the outsides of the beam flanges were the same as were those of the welds on the insides of the beam flanges and of the welds on either side of the web. The other two connection details (specimens 11 and 14) were similar except that only the flange welds were used. Thus, specimens 2 and 7 were type C connections and specimens 11 and 14 were type A connections. However, the situation was somewhat complicated by the fact that the flange connections represented two pairs of type A welds. The strength of each pair of welds was determined separately and the contribution of the type A connection to the total failure load was assumed to be the sum of the strengths of each pair of welds. For the welds on the outsides of the flanges L was assumed to be the overall depth of the RSJ used and for the welds on the insides of the flanges L was assumed to be the depth between the roots of the fillets. This latter length was also assumed to be the value of L' for the web welds. A further complication was that the values of the failure 'loads' were quoted as the 'bending moment at failure'. It was not clear whether this was the maximum bending moment at failure (i.e. at the column centre-line) or the bending moment transmitted by the connection (i.e. at the column face). The quoted values were assumed to be bending moments at the column centre-line. The values calculated for σ_u using this procedure are given in Table 3.

TABLE 3
Comparison Between Theory and Test Results of Johnson[13]

Specimen no.	Weld type	Column depth	Flange welds						Web welds,	Failure moment, M (tonf-in)		σ_u (tonf in^{-2})
			Outside			Inside						
		d (in)	w (in)	L_1 (in)	L (in)	w (in)	L_1 (in)	$L = L'$ (in)	w (in)	Experiment	Theory	
2	C	5	3/16	4·0	7	3/16	3·75	5·18	3/16	155	$6{\cdot}684\sigma_u$	23·2
7	C	7	3/8	4·0	9	3/8	3·70	7·67	1/4	315	$17{\cdot}600\sigma_u$	17·9
11	A	10	1/2	4·5	10	1/4	4·20	7·84	—	510	$19{\cdot}387\sigma_u$	26·3
14	A	8	1/2	4·5	10	3/8	4·20	7·84	—	530	$21{\cdot}611\sigma_u$	24·5

e = 54 in for all specimens.
Theoretical bending moment at failure = $(54 + d/2)$. (Failure load calculated using eqn (33b).)

5.4 Discussion

Before commenting on the results given in Figs 15 and 17 it is perhaps worth commenting on the values of the weld metal UTS used in the calculations. Some of the values given in Table 1 are considerably higher than the minimum values specified in Codes of Practice current at the time the experiments were performed. For example, the specimens of Butler *et al.*[11] and Dawe and Kulak[4] were all fabricated using E60 electrodes for which the specified minimum UTS is 60 ksi. This is much lower than the values given in Table 1. However, the values in Table 1 are remarkably consistent and it may well be that they are good estimates of the *actual* UTS of the weld metal used in the tests, particularly in view of the fact that the UTS value deduced for the specimens of Biggs *et al.*[10] is known to be reasonable and those deduced for three of Johnson's specimens (Table 3) are in reasonable agreement with the minimum UTS specified by the then current version of BS639.* In any case, this discrepancy does not affect the conclusions drawn from Figs 15 and 17 because it is always possible to specify that σ_u should be *defined* as the failure stress deduced from control tests on tension and/or shear fillet welds.

Figure 15 shows that the predictions of the proposed theory for type B connections are in reasonable agreement with experimental results over a wide range of values of ξ' ($0{\cdot}06 \leq \xi' \leq 2{\cdot}56$). This is heartening in view of the fact that a single, simple equation can be used to estimate the strength of all type B connections. The information required to perform the calculations is the value of σ_u, which is usually supplied by manufacturers of electrodes, and the weld geometry. The only doubt about the proposed theory would seem to be at the lower end of the range for ξ' where it appears to overestimate the failure load. There seems to be no obvious explanation for this since the theory should agree with experimental results when $e = 0$ and the connection becomes a shear fillet weld.

Figure 17 shows that for the type of connection shown in Fig. 16, eqn (25) tends to underestimate the strength in general. Because of the method used for estimating σ_u, theory would be expected to agree with experiment when $P/P_0 = 0$. The theoretical predictions become increasingly more conservative as P/P_0 increases. This may well be

* BS639: Specification for covered electrodes for the metal-arc welding of mild steel.

because the value of σ_u has been underestimated. A test result involving a simple shear fillet welded connection would have been very illuminating.

The results in Table 2 show that the strength of type C connections can be predicted satisfactorily using the proposed technique. These results when allied with the results for all-round welds (Fig. 1c) studied previously[1] suggest that the assumption made about the ductility of the weld metal is reasonable, but this remains to be verified for the general case. The results also suggest that bearing between connected members does not significantly affect the weld strength. Thus the complication introduced into the calculations by allowing for bearing could well be ignored in practice.

The results in Table 3 show that the deduced UTS values for three of Johnson's four specimens are in agreement when the scatter to be expected in tests on welds is taken into account. Further, these values are in reasonable agreement with the value of 26 tonf in^{-2} specified by BS639 to be the minimum UTS for the electrodes used by Johnson (E.319 and E.217).

Thus, the available results suggest that the proposed theory is satisfactory for estimating the strength of fillet welds subjected to a shearing force and a bending couple. The formulae which define the strength of the connection (eqns (10a) and (10b) for type A connections, eqn (22) for type B connections and eqns (33a) and (33b) for type C connections) are attractive because of their simplicity and the proposed method is thought to be more useful to designers than the computation technique suggested by Dawe and Kulak.[4]

It is noted that eqn (10b) has not been verified using published experimental results. This needs to be done because it also affects the strength of type C connections when $\xi \leq 1/2$.

The possibility was raised earlier that other stress fields could be postulated for the welds which might lead to higher estimates of the failure load. Comparisons carried out between experiment and theory do suggest, however, that the stress fields actually used give acceptable results.

6 CONCLUSIONS

The method proposed previously for predicting the strength of welds subjected to a shearing force only has been successfully extended to cover the case of welds subjected to a shearing force and a bending couple. As before, the actual set of forces acting on the weld was replaced by an 'equivalent' set and the resulting stresses were made to satisfy a yield criterion. This enabled very simple expressions to be derived for estimating the strength of the connections depicted in Figs 6 and 16. The theory was developed assuming that

(i) bearing between the connected members could be ignored; and
(ii) the deposited weld metal would have sufficient ductility to enable each weld to develop its full strength before failure.

The effect of longitudinal residual stresses locked into the welds was allowed for and it turned out that the strength of type B connections made using continuous welds would not be affected by stress relieving, although the strengths of the other connection details would be. The strength of type A, type B and type C connections loaded in the as-welded condition can be estimated using eqns (10), (22) and (33), respectively. The validity of these expressions was confirmed by comparing their predictions with experimental results obtained by others. The agreement between theory and experiment was satisfactory, suggesting that the assumptions made in the analysis are reasonable and that the expressions derived for predicting the strengths of the weld configurations considered can be used with confidence.

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