# Investigation on Instantaneous Center of Rotation Method For Design of Eccentrically Loaded Weld Group

**Sung−Yong Kim[1], Jong-Hyun Jung[2*], and Cheol-Ho Lee[3]**

[1] School of Architecture, Changwon National University, Changwon, Korea. *sungyong.kim@changwon.ac.kr*
[2*] School of Architecture, Kyungnam University, Changwon, Korea. *ironbell@kyungnam.ac.kr*
[3] Dept. of Architecture and Architectural Engineering, Seoul National University, Seoul, Korea. *ceholee@snu.ac.kr*

**Abstract**
The weld size of an eccentrically loaded weld group in plane can be determined using the instantaneous center of rotation method (ICRM) of the AISC design manual. ICRM assumes that the weld element farthest from the instantaneous center of rotation (IC), which can be determined upon the eccentricity and geometry of the weld group, controls the design strength of the eccentrically loaded weld group. However, in some cases, ICRM can unexpectedly provide two or more ICs. These multiple ICs may cause confusion among engineers on the selection of the mechanically correct IC. This paper illustrates the case of multiple ICs in designing eccentrically loaded weld group and discusses relevant problems.

**Keywords:** Instantaneous center of rotation method, Welded connections, Eccentric load

## 1. Introduction

The detailing of connections is a crucial part in designing of steel structures, and welded connections are generally considered as a direct and efficient means of connecting structural members. In many cases, the welded connections are subjected to an eccentric load. Analysis of such eccentric loading cases is complicated because the load-deformation behavior depends on the angle between the direction of the resistance and the axis of the fillet weld. The weld size of an eccentrically loaded weld group in plane can be determined using the instantaneous center of rotation method (ICRM) of the AISC design manual. However, in some cases, ICRM can unexpectedly provide two or more ICs. This study investigated the ICRM in details for design of eccentrically loaded weld group.

## 2. ICRM reformulated based on plastic theory

The method of instantaneous center of rotation (IC method) was earlier developed by Crawford and Kulak (1971) for bolted connections and adapted by Butler and Kulak (1972) for welded joints with in-plane eccentricity. For other inclined loads, Iwankiw (1987) proposed an equation to determine strength of connection as a function of load angle. The strength of an eccentrically loaded weld group can be determined by locating the instantaneous center of rotation (IC), using the load-deformation relationship of a weld segment. The procedure for the ICRM are described below.

### 2.1. ICRM

Suppose that a weld line of its width and length of $w$ and $l$, respectively, lies in the $x$-$y$ plane. Let $\mathbf{x}=(x, y, 0)$ be the position vector of the weld segments, of which coordinates are parameterized by arc length parameter $s$, and $\mathbf{x}_P=(x_P, y_P, 0)$ be the location vector to which the external load $\lambda\mathbf{P}=\lambda(P_x, P_y, 0)$ is applied, where $\lambda$ and $\mathbf{P}$ are the magnitude and direction vector of the applied load, respectively. The procedure for the strength calculation for the loaded weld group located is as follows (see

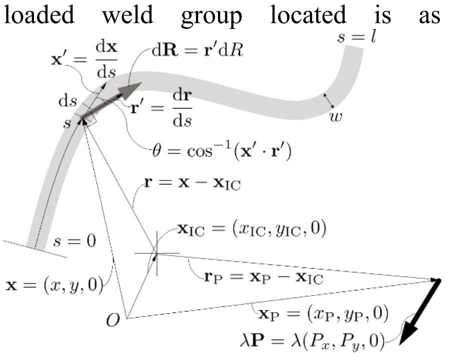



Figure *1*):

1) Select a trial location for the IC as $\mathbf{x}_{IC}=(x_{IC}, y_{IC}, 0)$. Note that the weld line, the IC and the external load were assumed to be located in the $x$-$y$ plane in this study.

2) Compute the resisting force d**R** at any weld segment acting in a direction perpendicular to the radial line from the IC as follows:

$$\mathrm{d}\mathbf{R} = \mathbf{r}'\mathrm{d}R \tag{1}$$

where $\mathbf{r}'$ is the tangential vector of $\mathbf{r}=\mathbf{x}-\mathbf{x}_{IC}$, which can be computed as $\mathrm{d}\mathbf{r}/\mathrm{d}s$, and

$$\begin{aligned}\mathrm{d}R = 0.60F_{EXX}(1.0 + 0.5\sin^{1.5}\theta) \\ \times [p(1.9 - 0.9p)]^{0.3}w_e\mathrm{d}s\end{aligned} \tag{2}$$

where $w_e$ is the effective weld width, $\theta=\cos^{-1}(\mathbf{x}'\cdot\mathbf{r}')$ is the angle of loading ($\mathbf{r}'$) measured from the weld longitudinal axis ($\mathbf{x}'=\mathrm{d}\mathbf{x}/\mathrm{ds}$), degrees (see

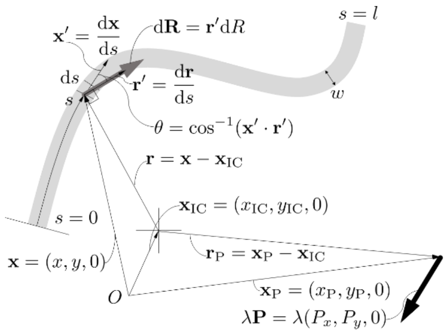



Figure *1*), $F_{EXX}$ is the weld electrode tensile strength, and $p$ is $\Delta/\Delta_{\mathrm{sup}}$ in which

$$\Delta_{\mathrm{sup}} = 1.087w(\theta + 6)^{-0.65} \leq 0.17w \tag{3}$$

and

$$\Delta = \frac{|\mathbf{r}|}{|\mathbf{r}|_{\mathrm{sup}}}\Delta_{\mathrm{sup}} \tag{4}$$

where $|\mathbf{r}|$ is the radius for the element (see

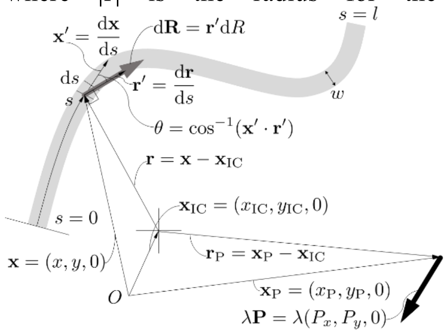


Figure *1*), and the subscript sup refers to the supremum. Instead of the maximum of the displacement function, the supremum was used in this study, because the infinitesimal element d$s$ rather than the finite element $\Delta s$ is used.

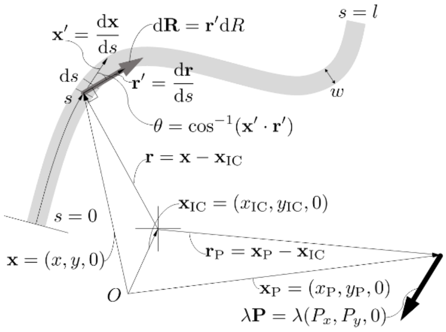


Figure 1 Schematic plot for ICRM

3) Check if the load factor of λ satisfies the equilibrium equations. If the equilibrium equations are satisfied within an acceptable tolerance, the analysis is complete. Otherwise, a new trial location of IC should be selected and the procedure repeated. The equilibrium equations are given as follows:

$$\lambda P_x - \int_0^l \left(\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s}\right)_x \mathrm{d}s = 0 \tag{5}$$

$$\lambda P_y - \int_0^l \left(\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s}\right)_y \mathrm{d}s = 0 \tag{6}$$

$$\left(\mathbf{r}_{\mathrm{P}} \times \lambda\mathbf{P} - \int_0^l \mathbf{r} \times \frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s}\mathrm{d}s\right) \cdot \mathbf{i}_z = 0 \tag{7}$$

where $\mathbf{r}_{\mathrm{P}} = \mathbf{x}_{\mathrm{P}}$-$\mathbf{x}_{\mathrm{IC}}$ is the distance vector of the applied load, and $\mathbf{i}_z$ is the unit vector for z direction. Note that, in order to extract the magnitude of the moment, the triple scalar product was used in Eq.(7).

A variety of numerical methods for the ICRM was proposed by various researchers, including Brandt (1982), Iwankiw (1987), and Lue et al. (2017). In the previous researches, Eqs.(5) to (7) were solved by following procedure: 1) calculate $\mathbf{P}_n$=$\lambda_n\mathbf{P}$ defined as the load value by which Eq.(7) is satisfied; 2) substitute $\mathbf{P}_n$ into the remaining equilibrium equations Eqs.(6) and (7) and compute their residuals; and 3) checks if the computed residuals are within acceptable tolerance. If they are not withing the tolerance, update the trial location of IC and repeat the procedure.

## 2.2. Reformulation of ICRM from virtual work principle

The virtual work principle relates a system of forces in equilibrium to a system of compatible displacements. Stated simply, if a body in equilibrium is given a set of admissible displacement, then the work done by the external loads on these external displacements is equal to the work done by the internal forces on the internal deformation, i.e.,

$$\delta W_E = \delta W_I \tag{8}$$

where $\delta W_E$ and $\delta W_I$ are the work done by the external loads on the virtual displacement set and that done the internal forces on the virtual displacement, respectively.

For the weld line considered, the virtual work equation can be rewritten as follows:

$$\delta\mathbf{u}_{\mathrm{P}} \cdot \lambda\mathbf{P} = \int_0^l \delta\mathbf{u} \cdot \frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}s}\mathrm{d}s \tag{9}$$

where $\delta\mathbf{u}$ and $\delta\mathbf{u}_{\mathrm{P}}$ are the virtual displacements for the weld line and external force, respectively.

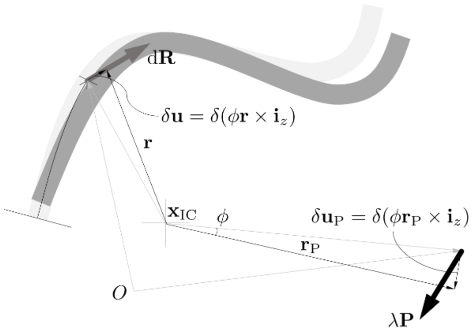



Figure 2 Virtual displacement of given weld line

Using geometrical relationship, the virtual displacement sets $\delta\mathbf{u}$ and $\delta\mathbf{u}_{\mathrm{P}}$ is determined as follows:

$$\delta\mathbf{u} = \delta(\phi\mathbf{r} \times \mathbf{i}_z) \tag{10}$$

$$\delta\mathbf{u}_{\mathrm{P}} = \delta(\phi\mathbf{r}_{\mathrm{P}} \times \mathbf{i}_z) \tag{11}$$

It should be note that the virtual displacement sets $\delta\mathbf{u}$ and $\delta\mathbf{u}_{\mathrm{P}}$ depend on not only the angle $\phi$ but also the location of weld element and load point $\mathbf{r}$ and $\mathbf{r}_{\mathrm{P}}$, which in turn depends on the assumed location of the IC, $\mathbf{x}_{\mathrm{IC}}$.

Using calculus of variation, the virtual displacements $\delta\mathbf{u}$ and $\delta\mathbf{u}_{\mathrm{P}}$ can be expressed as follows:

$$\delta\mathbf{u} = \delta\phi(\mathbf{r} \times \mathbf{i}_z) - \phi(\delta\mathbf{x}_{\mathrm{IC}} \times \mathbf{i}_z) \tag{12}$$

$$\delta\mathbf{u}_{\mathrm{P}} = \delta\phi(\mathbf{r}_{\mathrm{P}} \times \mathbf{i}_z) - \phi(\delta\mathbf{x}_{\mathrm{IC}} \times \mathbf{i}_z) \tag{13}$$

where $\delta\phi$ is the virtual rotation angle and $\delta\mathbf{x}_{\mathrm{IC}}=(\delta x_{\mathrm{IC}}, \delta y_{\mathrm{IC}}, 0)$ is the virtual location of the IC. Finally, Eq.(9) can be written into the following form:

$$\delta y_{\mathrm{IC}}\left[\lambda P_x - \int_0^l \left(\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s}\right)_x \mathrm{d}s\right] = 0 \tag{14}$$

$$\delta x_{\mathrm{IC}}\left[\lambda P_y - \int_0^l \left(\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s}\right)_y \mathrm{d}s\right] = 0 \tag{15}$$

$$\delta\phi\left(\mathbf{r}_{\mathrm{P}} \times \lambda\mathbf{P} - \int_0^l \mathbf{r} \times \frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s}\mathrm{d}s\right) \cdot \mathbf{i}_z = 0 \tag{16}$$

The virtual work principle is applicable when the exact displacement and internal forces are not available. Any equilibrium set of forces in Eqs.(14) to (16) may be used to obtain the approximate load factor $\lambda$ by enforcing the virtual work equation. Thus, different solutions can be obtained for different choice of virtual displacements for given equilibrium equation.

## 3. Design example

Figure 3 shows the C-shaped loaded weld group examined, and only the upper half part of the weld configuration is considered in order to reduce computational burden. The horizontal lengths are 60 mm each and the vertical length is 160 mm (i.e. the vertical length of the upper half part is 80 mm). The eccentric load is applied at 120 mm from the far left of the weld group. The weld size is 10 mm, and E70 electrodes are used. In the analysis, the weld configuration is divided into segments 10 mm long.

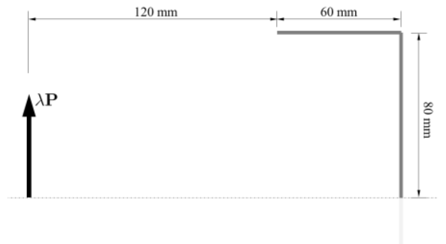



Figure 3 Upper half part of the examined weld group

Figure 4 shows the results obtained by ICRM. It can be seen that three different ICs were observed at 14.88 mm, 36.33 mm and 68.85 mm from the far left of the weld group, respectively, all of which satisfy the internal force equation described in Eq.(2). Figure 5 represents the functions of $P_n$ and $R_y$ and equilibrium points obtained by ICRM. It should be noted that three different values of the load factor were calculated for each displacement sets.

## 3. Conclusions

This paper investigated the ICRM for design of eccentrically loaded weld group. It was shown that, in some cases, multiple ICs can be determined, and the reason was explained by reformulating the ICRM from the virtual work principle. Further research is ongoing to propose the recommendations to select correct IC.

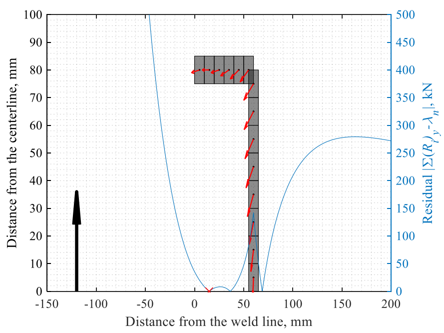
## 4. References


Butler, L. J., and Kulak G. L. (1972). "Eccentrically loaded welded connections" *Journal of the Structural Division* 98(5), pp. 989-1005.

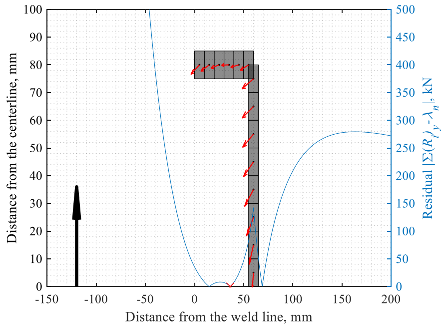
Chen, W. F., and Sohal, I. (2013). *Plastic Design and Second-Order Analysis of Steel Frames*, Springer.

Crawford, S. H., and Kulak, G. L. (1972). "Eccentrically loaded bolted connections" *Journal of the Structural Division* 97(3), pp. 765-783.

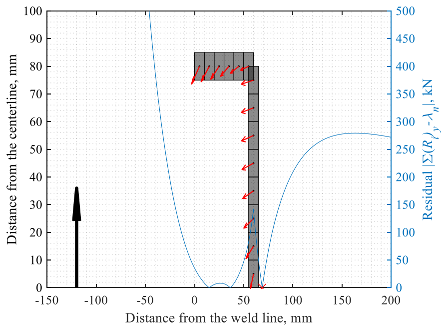
Iwankiw, N. R. (1987). "Design for Eccentric and Inclined Loads on Bold and Weld Groups" *Engineering Journal*, 24(4), pp.164-171.


(a) $x_{\mathrm{IC}}$=14.88 mm




(b) $x_{\mathrm{IC}}$= 36.33 mm




(c) $x_{\mathrm{IC}}$ = 68.85 mm

Figure 4 Three different locations of IC

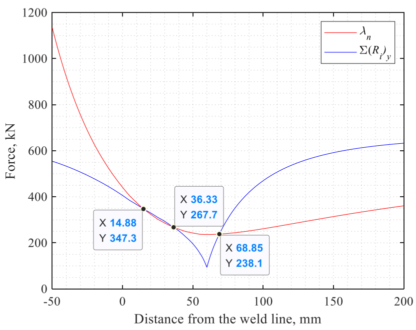



Figure 5 Calculated weld strengths and locations of corresponding ICs