Thickness of shear flow path in RC beams at maximum torsional strength

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SUMMARY

The current maximum torsional design equations for predicting the torsional capacity of RC members underestimate the torsional strength of under-reinforced members and overestimate the torsional strength of over-reinforced members. This is because the design equations consider only the yield strength of torsional reinforcement and the cross-sectional properties of members in determining the torsional capacity. This paper presents an analytical model to predict the thickness of shear flow path in RC members subjected to pure torsion. The analytical model assumes that torsional reinforcement resists torsional moment with a sufficient deformation capacity until concrete fails by crushing. The ACI 318-19 code is modified by applying analytical results from the proposed model such as the average stress of torsional reinforcement and the effective gross area enclosed by the shear flow path. Comparison of the calculated and observed torsional strengths of existing 129 test members showed good agreement. Two design variables related to the compressive strength of concrete in the proposed model are approximated for design application. The accuracy of the ACI 318-19 code for the over-reinforced test members improved somewhat with the use of the approximations for the average stresses of reinforcements and the effective gross area enclosed by the shear flow path.

***Keywords**: RC beams; torsional reinforcement; torsional strength; combined loading; maximum torsional reinforcement amount.*

INTRODUCTION

Structural damage to columns, spandrel beams, or slabs in irregular reinforced concrete structures under static and dynamic excitations can be increased by shear forces and bending moments or by the combined effect of shear forces, bending moments, and torsional moments. To estimate the torsional strength of RC members, the current design codes (ACI 318-19, EC2-05, CSA-19, JSCE-07) have employed semi-empirical models in which empirical and theoretical approaches are combined and all models have almost identical forms.

Lee and Kim (2010) reported that the ACI 318-05 code underestimates the torsional strength of the beams tested by Fang and Shiau (2004) for amount of torsional reinforcement, $(p_l+f_{ly}+p_t+f_{ty})/f'_c$, less than or equal to 0.28, while the code overestimates the torsional strength for amount of torsional reinforcement $(p_l+f_{ly}+p_t+f_{ty})/f'_c$ greater than 0.28. Lee and Kim also observed that the EC2-02 and JSCE-02 codes show similar trends in estimating the torsional strength of the same beams at $(p_l+f_{ly}+p_t+f_{ty})/f'_c= 0.12$. The thresholds of the design codes, 0.28 and 0.12, may be attributed to differences in methods of calculating the gross area enclosed by the shear flow path, as presented in Table 1. Lee and Kim assumed that the discrepancy between the observed and the calculated torsional capacity could be attributed to the tension stiffening effect of concrete; they showed that the accuracy of the design codes can be improved simply by adopting the average stress of reinforcements embedded in concrete, instead of the local yield stress of the reinforcing bar. The experimental and analytical studies conducted by Lee et al. (2018) and Kim et al. (2019) also showed that, among the current design codes, the ACI 318-19 code provides the most reasonable predictions of torsional strength of RC members. While the current design codes are quite effective in estimating the torsional capacity of RC members, they require improvement to provide precise predictions in a broad range of amount of torsional reinforcement to concrete strength$(p_l+f_{ly}+p_t+f_{ty})/f'_c$. The test results of Peng and Wong (2011) showed that the ACI code provided quite conservative predictions for lightly reinforced walls with larger lengths. As shown in Table 1, the design codes limit the torsional reinforcement ratio to avoid unexpected structure failure by concrete crushing before the yielding of torsional reinforcements. Many results from previous experiments on RC beams designed according to design limits showed that torsional reinforcements reached their yield strains before the maximum torsional strength of the RC beams, and that the torsional reinforcements resisted torsional moment until concrete crushing. The experimental results from the study conducted by Kim et al. (2020) also showed that the maximum torsional strengths of under-reinforced and over-reinforced beams are mainly determined by concrete crushing. This means that cracked reinforced concrete should have a proper web thickness to resist the torsional moment, even after yielding of torsional reinforcements.

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Advanced sectional analysis techniques, in which iterative calculation is required, have been presented due to the development of computer analysis techniques (Choi and Lee 2019, Kuan et al. 2019, Rahal 2021). Some studies tried to analytically estimate the torsional strength of RC members under pure torsion with the help of FEA software (Jeong et al. 2021, Ibraheem and Mukhlif 2021). However, it is difficult for engineers or students with little experience in computer programming to design a torsional member using these analysis techniques. In this study, an analytical model is presented to predict the maximum torsional strength of RC beams subjected to pure torsion without iterative calculation. Based on the observed torsional behaviors of the twenty RC beams (Kim et al. 2020), this model assumes that yielded torsional reinforcements continue to resist a torsional moment until web concrete fails by crushing. In the model, therefore, the compressive strength of concrete and the deformation capacity of torsional reinforcements are considered the important variables that determine the maximum torsional strength of RC members. The proposed model utilizes the average stress-strain relationship of steel bars embedded in concrete (Belarbi and Hsu 1994), the rotation angle softened truss models (Hsu 1988), and the modified compression field theory, MCFT (Vecchio and Collins 1986), developed for reinforced concrete in torsion and shear. To verify the validity of the proposed model, the maximum torsional strengths calculated using the ACI 318-19 code modified by applying analytical results are compared with the observed maximum torsional strengths of the existing 129 test beams. Then, through comparison of the observed and analytical results from existing 129 test beams, two design variables related to the compressive strength of concrete in the ACI 318-19 code are approximated for design application.

LIMITATION OF EXISTING DESIGN CODES

The ACI 318-19, EC2-5, CSA-19, and JSCE-07 design codes propose torsional equations with almost identical forms; these are based on the space truss model and the thin-wall tube analogy as theoretical criteria. Table 1 compares the four design codes. Notably, the ACI 318-19 code specifies the equations for the torsional strength (ACI 318-19 Section R 22.7.6.1), as follows

$$T_n = \min \begin{cases} 2A_o \dfrac{A_t f_{ty}}{s} \cot\alpha \\ 2A_o \dfrac{A_l f_{ly}}{p_h} \tan\alpha \end{cases} \tag{1}$$

where T_n: torsional strength; A_o: gross area enclosed by shear flow path; A_t: cross-sectional area of one leg of a transverse reinforcement resisting torsion; A_l: total cross-sectional area of longitudinal torsional reinforcements; f_{ty} and f_{ly}: yield strength of transverse and longitudinal reinforcements, respectively; s: spacing of transverse reinforcement; p_h: perimeter of the centerline of outermost closed transverse reinforcement; α: angle of compression diagonals; and ρ_t and ρ_l: torsional reinforcement ratios in transverse and longitudinal directions, respectively.

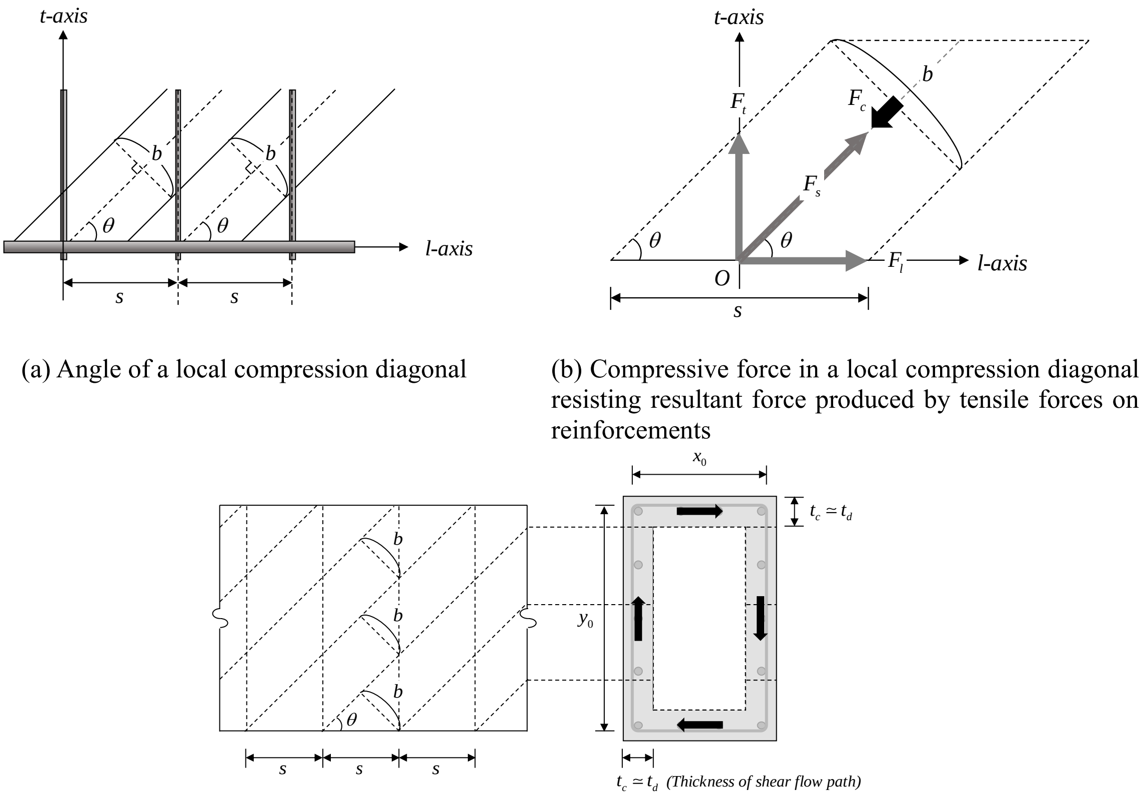
Table 1. Equations to compute torsional strength

ACI 318-19	EC2-05	CSA-19	JSCE-07
$T_n = \min \begin{cases} 2A_o \frac{A_t f_{ty}}{s} \cot\alpha \\ 2A_o \frac{A_l f_{ly}}{p_h} \tan\alpha \end{cases}$	$T_n = \min \begin{cases} 2A_k \frac{A_t f_{ty}}{s} \cot\alpha \\ 2A_k \frac{A_l f_{ly}}{u_k} \tan\alpha \end{cases}$	$T_n = 2A_o \frac{A_t f_{ty}}{s} \cot\alpha$	$T_n = \min \begin{cases} \frac{2a_1 A_0\, {}_w f_t}{x} \\ \frac{2a_s A_0\, {}_s f_t}{\varphi_o} \end{cases}$,
Brittle failure: $T_n = 0.67\sqrt{f_{ck}}\,1.7A_{oh}^2 / p_h$	Brittle failure: $T_n = 2\nu f_{ck} A_k t_{ef} \sin\alpha \cdot \cos\alpha$	Brittle failure: $T_n = 0.67\sqrt{f_{ck}}\,1.7A_{oh}^2 / p_h$	
$A_o = 0.85A_{oh}$ $A_{oh} = x_o \times y_o$, $p_h = 2(x_o + y_o)$ $30^\circ \le \alpha \le 60^\circ$ (suggested $\alpha = 45^\circ$) $\cot^2\alpha = \frac{\rho_l f_{ly}}{\rho_t f_{ty}}$ for predicting strength	$A_k = (B - t_{ef}) \times (H - t_{ef})$ $t_{ef} = (B \times H) / (2 \times (B + H))$ $2.5 \le \cot\alpha \le 1$ (suggested $\alpha = 45^\circ$) $\nu = 0.6(1 - f_{ck} / 250)$	$A_o = 0.85A_{oh}$ $A_{oh} = x_o \times y_o$, $p_h = 2(x_o + y_o)$ $\alpha = 29 + 7000\varepsilon_x$ $-0.0002 < \varepsilon_x \le 0.003$ (for design) $\cot^2\alpha = \frac{\rho_l f_{ly}}{\rho_t f_{ty}}$ (for prediction)	$A_0 = A_{oh} = x_o \times y_o$ in ACI 318-19 a_1 and a_s = A_t and A_l in other codes x = s in other codes ${}_s f_t$ = allowable stress of longitudinal reinforcement ${}_w f_t$ = allowable stress of transverse reinforcement φ_0 = p_h in other codes
Maximum torsional reinforcement ratio $\rho_{t\max} = \frac{5}{6} \frac{\sqrt{f_c'}}{f_{ty}} \frac{A_{oh}}{A_g} \tan\alpha$	Maximum torsional reinforcement ratio $\rho_{t\max} = \frac{f_c'}{f_{ty}} \frac{p_h}{p_c} \sin^2\alpha$	Maximum torsional reinforcement ratio $\rho_{t\max} = 0.25 \frac{f_c'}{f_{ty}} \frac{A_{oh}}{A_g} \tan\alpha$	Maximum torsional reinforcement ratio $\rho_{t\max} = \frac{1}{2} \frac{K_t f_{wcd}}{f_{ty}} \frac{p_h}{A_{oh} A_g} \tan\alpha$

The ACI 318-19 code recommends a value of α less than 30 degrees and not greater than 60 degrees; it suggests $\alpha = 45^\circ$ for non-prestressed members and $\alpha = 37.5^\circ$ for pre-stressed members. After torsional cracking develops, it is considered that torsional reinforcements and compression diagonals mainly resist torsion. The crack angle can also be calculated using an equation from the space truss model, as follows

$$\cot^2 \alpha = \frac{\rho_l f_{ly}}{\rho_t f_{ty}} \tag{2}$$

As presented in Table 1, the design codes limit the maximum torsional reinforcement ratios to avoid brittle failure of concrete sections due to concrete crushing. Existing studies on torsion (Hsu 1990, Rahal and Collins 1996, Fang and Shiau 2004, Lee et al. 2010, Peng and Wong 2011, Kim et al. 2019, Kim et al. 2020) showed that the design codes underestimate the torsional strength of under-reinforced RC members and overestimate that of over-reinforced RC members. Lee and Kim (2010) concluded that the accuracy of Eq. (1) can be improved simply by adopting the average yield stress of a steel bar, instead of the local yield stress of a bare steel bar. The existing experimental results (Fang and Shiau 2004, Peng and Wong 2011, Kim et al. 2020) also showed that concrete crushing mainly determines the maximum torsional strength of RC members subjected to pure torsion. The differences between the calculated and observed maximum torsional moments are derived from the following assumptions: i) the maximum torsional strength of RC members is determined only by the yield strength of torsional reinforcements, regardless of the contribution of concrete; ii) A_o in Eq. (1) is determined by multiplying the area enclosed by the centerline of the outermost closed transverse torsional reinforcement, A_{oh}, by a reduction factor. For instance, the ACI 318-19 code suggests $A_o = 0.85 A_{oh}$; however, this is a less accurate expression because the concrete strength affects the thickness of the shear flow path. The final assumption is that iii) the angle of the compression diagonals, α, is the same as the angle of the principal compressive stress of concrete. Furthermore, even though the strains of transverse or longitudinal transverse reinforcement do not reach the yield strain at maximum torsional strength, the design code uses the yield stress of the reinforcement. Therefore, the contribution of concrete to the torsional capacity should be considered to predict the maximum torsional strength of RC members more accurately.



(a) Angle of a local compression diagonal

(b) Compressive force in a local compression diagonal resisting resultant force produced by tensile forces on reinforcements

(c) Thickness of a local compression diagonal

Fig. 1. Assumption of force equilibrium and thickness of shear flow path in a RC beam.

ANALYTICAL APPROACH TO VARIABLES RELATED TO CONCRETE STRENGTH IN TORSIONAL DESIGN

Equilibrium equations in a local compression diagonal

In this study, three assumptions are made to establish an analytical model for estimating the thickness of the shear flow path of RC members subjected to pure torsion. i) Transverse and longitudinal reinforcements resist the compressive force acting on a local compression diagonal, as shown in Figs. 1(a) and (b).

$$\sqrt{F_l^2 + F_t^2} = F_s = F_c \tag{3}$$

$$\theta = \tan^{-1}\left(F_t / F_l\right) \tag{4}$$

where F_t and F_l: tensile forces acting on transverse and longitudinal torsional reinforcements in a local compressive diagonal; F_s: resultant force produced by transverse and longitudinal tensile forces; F_c: compressive force acting on a local compression diagonal; and θ: angle of a local compression diagonal. ii) The thickness of a local compression diagonal, t_c, required to resist F_s includes the diameter of the transverse torsional reinforcement, and t_c is distributed uniformly over the depth and width of a beam section, as shown in Fig. 1(b).

$$F_c = b \cdot t_c \cdot f_c' \tag{5}$$

$$b = s \cdot \sin\theta \tag{6}$$

where b: depth of a local compression diagonal; f_c: compressive strength of concrete; and s: spacing of transverse reinforcement. Substituting Eq. (6) into Eq. (5), tc can be expressed as

$$t_c = \frac{F_s}{s \cdot \sin\theta \cdot f_c'} \simeq t_d \tag{7}$$

In this study, t_c is assumed to be equal to the minimum thickness of the shear flow path, t_d. iii) The angle of principal compressive stress of concrete, α, derived from the equilibrium and compatibility equations in the transverse and longitudinal directions of cracked RC members, is the same as the average value of angles of local compression diagonals, θ, passing through transverse reinforcements, as shown in Fig. 1(c).

Average stress of torsional reinforcement at concrete crushing

Previous studies (Shima et al. 1987, Adebar 1989, Belarbi and Hsu 1994, Maekawa et al. 2003) reported that a stress-strain curve of a reinforcing bar embedded in concrete differs from that of a bare steel bar due to the bond action between concrete and reinforcements. Belarbi and Hsu (1994), Maekawa et al. (2003), and Chen and Kabeyasawa (2004) proposed an average stress-strain relationship considering the tension stiffening effect of concrete. The following equations present the average stress-strain relationship proposed by Belarbi and Hsu, in which the average stress of reinforcing bars depends on the concrete strength fc', and the ratio of reinforcement,ρ_s.

$$St)\ \bar{f}_i \le \left(0.93 - 2B_i\right) f_{iy}$$
$$\bar{f}_i = E_i \bar{\varepsilon}_i \tag{8a}$$

$$St)\ \bar{f}_i > \left(0.93 - 2B_i\right) f_{iy}$$
$$\bar{f}_i = \left(0.93 - 2B_i\right) f_{iy} + \left(0.02 + 0.25B_i\right) E_i \bar{\varepsilon}_i \tag{8b}$$

$$B_i = \left(f_{cr} / f_{iy}\right)^{1.5} / \rho_i \tag{9}$$

where $\bar{f_i}$ and $\bar{\varepsilon_i}$: average tensile stress and strain of reinforcements, respectively; E_i: elastic modulus of steel; f_{iy}:

yield strength of reinforcement; f_{cr}: cracking stress of concrete; ρ_i: ratio of reinforcing steel; and i: subscript indicating the direction of reinforcement. The existing typical space truss theories using strain compatibility conditions include the rotation angle softened truss models and the modified compression field theory. Using the existing space-truss models, the equilibrium and compatibility equations in the transverse and longitudinal directions of cracked RC members can be expressed as

$$\bar{\varepsilon}_t = \varepsilon_2 \sin^2 \alpha + \varepsilon_1 \cos^2 \alpha \tag{10}$$

$$\bar{\varepsilon}_l = \varepsilon_1 \sin^2 \alpha + \varepsilon_2 \cos^2 \alpha \tag{11}$$

$$0 = f_2 \sin^2 \alpha + f_1 \cos^2 \alpha + \rho_t \bar{f}_t \tag{12}$$

$$0 = f_1 \sin^2 \alpha + f_2 \cos^2 \alpha + \rho_l \bar{f}_l \tag{13}$$

where $\bar{\varepsilon}_t$ and $\bar{\varepsilon}_l$: average strains of torsional reinforcements in transverse and longitudinal directions, respectively; ε_1 and ε_2 : tensile and compressive strains of concrete, respectively; α : angle of principal compressive stress of concrete; f_1 and f_2 : principal tensile and compressive stresses of concrete, respectively; ρt and ρl : torsional reinforcement ratios in transverse and longitudinal directions, respectively; and $\overline{f_t}$ and $\overline{f_l}$: average stresses of reinforcements in transverse and longitudinal directions, respectively. First, because ε_2 in Eqs. (10)-(11) is very small compared with $\bar{\varepsilon}_i$ and ε_1, the relationship between $\bar{\varepsilon}_i$ and ε_1 at maximum torsional moment can be approximated as

$$\bar{\varepsilon}_t = \varepsilon_1 \cdot \cos^2 \alpha \tag{14}$$

$$\bar{\varepsilon}_l = \varepsilon_1 \cdot \sin^2 \alpha \tag{15}$$

The current design codes limit the amount of torsional reinforcement used to prevent RC structure failure by concrete crushing before the yielding of torsional reinforcements. Based on the assumptions that the torsional strength of RC members is determined by concrete crushing after the yielding of torsional reinforcements, and that f_1 is very small compared with f_2 and $\overline{f_l}$ in Eqs. (12)-(13), $\overline{f_t}$ and $\overline{f_l}$ at maximum torsional moment can be approximately determined as

$$\bar{f}_t = -\frac{f_2 \sin^2 \alpha}{\rho_t} = -\frac{\upsilon f_c \sin^2 \alpha}{\rho_t} \tag{16}$$

$$\bar{f}_l = -\frac{f_2 \cos^2 \alpha}{\rho_l} = -\frac{\upsilon f_c \cos^2 \alpha}{\rho_l} \tag{17}$$

$$v = 1/(0.8 + 170 \cdot \varepsilon_1) \tag{18}$$

υ is the softening parameter proposed by Vecchio and Collins (1986). It is well known that cracked concrete subjected to tensile strain is weaker than concrete in a standard cylinder test. This softening parameter was developed to predict the deterioration in compression resistance of cracked reinforced concrete under uniaxial loading. The successful application of υ was found in existing studies (Rahal and Collins 1995, Lee and Kim 2008, Lee and Kim 2010). $\overline{f_t}$ and $\overline{f_l}$ can be rewritten as functions of ε_1 by substituting Eqs. (13)-(14), respectively, into $\bar{\varepsilon}_i$ in Eq. (8b). Then, by substituting $\overline{f_t}$ (ε_1) and $\overline{f_l}$ (ε_1) into Eqs. (16)-(17), respectively, the principal tensile strain of concrete, ε_1, at maximum torsional moment can be calculated as

$$\varepsilon_1 = \frac{-b_t + \sqrt{b_t^2 - 4a_t c_t}}{2 \cdot a_t} = \frac{-b_l + \sqrt{b_l^2 - 4a_l c_l}}{2 \cdot a_l} \tag{19}$$

where

$a_t = 170(0.02 + 0.25B_t)E_t \rho_t \cos^2 \alpha$

$b_t = 170(0.91 - 2B_t) f_{ty} \rho_t + 0.8(0.02 + 0.25B_t)E_t \rho_t \cos^2 \alpha$

$c_t = 0.8(0.91 - 2B_t) f_{ty} \rho_t - f_c' \sin^2 \alpha$

$a_l = 170(0.02 + 0.25B_l)E_l \rho_l \sin^2 \alpha$

$b_l = 170(0.91 - 2B_l) f_{ly} \rho_l + 0.8(0.02 + 0.25B_l)E_l \rho_l \sin^2 \alpha$

$c_l = 0.8(0.91 - 2B_l) f_{ly} \rho_l - f_c' \cos^2 \alpha$

α is determined from the relation between the second and third equations in Eq. (19). Thus, $\bar{\varepsilon}t$ and $\bar{\varepsilon}$ at a maximum torsional moment can be calculated by substituting $\varepsilon 1$ and α into Eqs. (14)-(15). Finally, by substituting the average strain of torsional reinforcements $\bar{\varepsilon_t}$ and $\bar{\varepsilon_l}$ into Eq. (8b), the average tensile stresses of torsional reinforcements at maximum torsional moments $\overline{f_t}$ and $\overline{f_l}$ can be obtained.

Prediction of maximum torsional strength of RC beams at concrete crushing

F_t and F_l resisting a compressive force of a local compression diagonal are calculated by multiplying $\overline{f_t}$ and $\overline{f_l}$ by a cross-sectional area of single legs of transverse and longitudinal reinforcements A_t and A_{li}, respectively, as follows

$$F_t = A_t \cdot \overline{f_t} \tag{20}$$

$$F_l = A_{li} \cdot \overline{f_l} \tag{21}$$

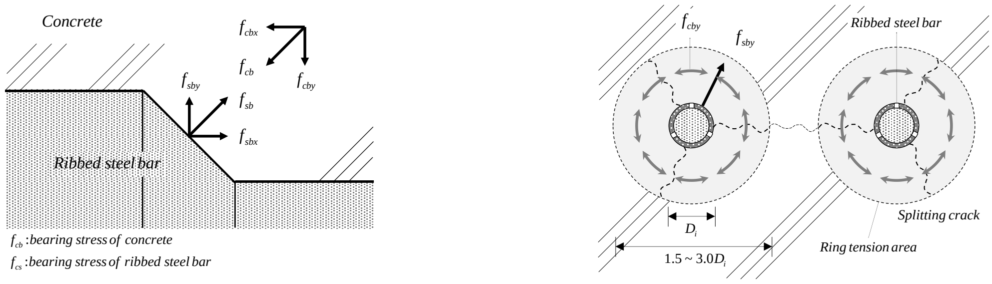
where A_{li}: cross-sectional area of longitudinal reinforcement. θ is obtained by substituting Eqs. (20) and (21) into Eq. (4). Finally, the minimum thickness of the shear flow path t_d of RC beams at a maximum torsional moment is calculated by substituting F_t, F_l, and θ into Eq. (7). If the value of t_d calculated using this model is more than twice the distance from the center of the transverse reinforcement to the concrete cover, it is recommended to use the smallest value in Eq. (22) as the thickness of the shear flow path t_d.

$$t_d \text{ from Eq. (7)}, \qquad t_d \leq 2c_c + d_t \tag{22a}$$

$$t_d = min[2c_c + d_t, 2.5d_t, 2.5d_l]\, , t_d > 2c_c + d_t \tag{22b}$$

where c_c: thickness of cover concrete; t_d: diameter of transverse torsional reinforcement; and d_l: diameter of longitudinal torsional reinforcement. When torsional reinforcement is subjected to tensile force due to the extension of diagonal cracks, bearing stress occurs at the reinforcement ribs. The bearing stress due to mechanical contact between reinforcement ribs and concrete induces compressive stress in the concrete surrounding the reinforcement. Splitting cracks resulting from the tensile stress in the concrete area are formed in a circular pattern around the reinforcement, as shown in Fig. 2. This behavior is known as the ring tension effect. According to previous studies (Tepfers 1982, Kanakubo et al. 1997, Sato et al. 2003, Kim et al. 2014, Kanakubo and Hosoya 2015), the area in which ring tension occurs is known to be in the range of 1.5 to 3.0 times the diameter of the reinforcing bars. The expansion of splitting cracks formed around the reinforcement may cause spalling of the concrete cover, reducing A_o and the torsional capacity of the section. Rahal and Collins (1996) showed that although spalling is one factor causing the reduction in the torsional strength, account must be taken of the fact that, in many tests, no spalling, was observed before the maximum torsional capacity was reached (Hsu 1968, Mitchell and Collins 1974, McMullen and Rangan 1978, El-Degwy and McMullen 1985). Peng and Wong (2011) also showed that concrete cover spalling was only occurred at the corners of specimens at the maximum torsional moment. They concluded that the effect of concrete spalling on the torsional strength of walls was negligible. Kim et al. (2020) compared the cross-sectional cuts of RC beams that experienced torsional failure to evaluate the effective area of concrete resistance and internal cracks. They found that cracks were formed along with the transverse reinforcement and concentrated around the longitudinal reinforcement.

Based on the existing test results, it is assumed in this study that the thickness of the shear flow path including the concrete cover resists torsional moment by the ring tension effect until the maximum torsional strength is reached. Furthermore, it is considered that, when the bond strength between the reinforcement and concrete is lost, only the cracked concrete area resists the torsional moment. The ACI 318-19 code assumes that the shear stress flows along the centers of the outmost closed stirrups. In Eq. (22), $2c_c + d_t$, $2.5d_t$ and $2.5d_l$ are the values representing the assumptions of the code, the reinforcement details, and the ring tension effect at maximum torsional strength.



(a) Bearing stresses of reinforcement (b) Ring tension

Fig. 2. Bond splitting behavior of reinforced concrete

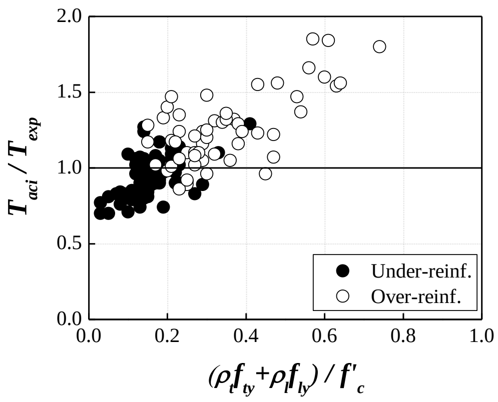
In the calculation of the torsional strength of RC members, the ACI 318-19 code suggests that the gross area enclosed by the shear flow path, A_o, is 0.85 times the area enclosed by the outmost closed stirrups, A_{oh}. However, this is an empirical expression that does not consider the properties of any materials or the reinforcement ratio. It is considered here that the concrete around the stirrups is relatively ineffective after maximum torsional strength is exceeded, and the gross area enclosed by the shear flow path at peak torsional moment can be defined in terms of A_{oh} and t_d, as follows

$$A_{oe} = (x_0 - t_d)(y_0 - t_d) \tag{23}$$

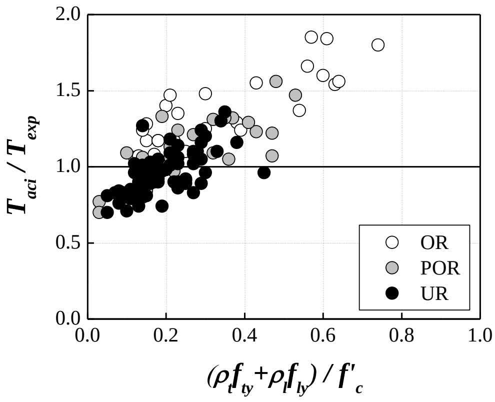
where A_{oe}: effective gross area enclosed by shear flow path at a peak torsional moment; x_0: shorter center-to-center dimension of transverse reinforcement; and y_0: longer center-to-center dimension of transverse reinforcement. The ACI 318-19 code can be modified by substituting the analytical results for α, $\overline{f_t}$, $\overline{f_l}$, and A_{oe} into Eq. (1), as follows

$$T_n = min\left(2\frac{A_{oe}A_t\bar{f}_t}{s}\cot\alpha, 2\frac{A_{oe}A_l\bar{f}_l}{p_h}\tan\alpha\right) \tag{24}$$

The analytical results were applied to certain design variables related to the thickness of the shear flow path and the stress of torsional reinforcements in the ACI 318-19 code, these can also be applied to the same design variables in EC2-05, CSA-19, and JSCE-07 because the current design codes are almost identical.

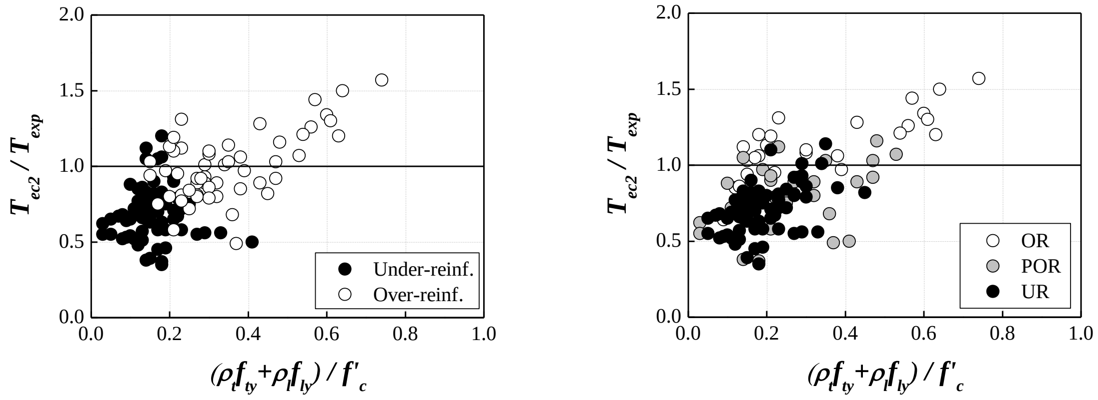


Under-reinforced VS Over-reinforced



Types of failure modes

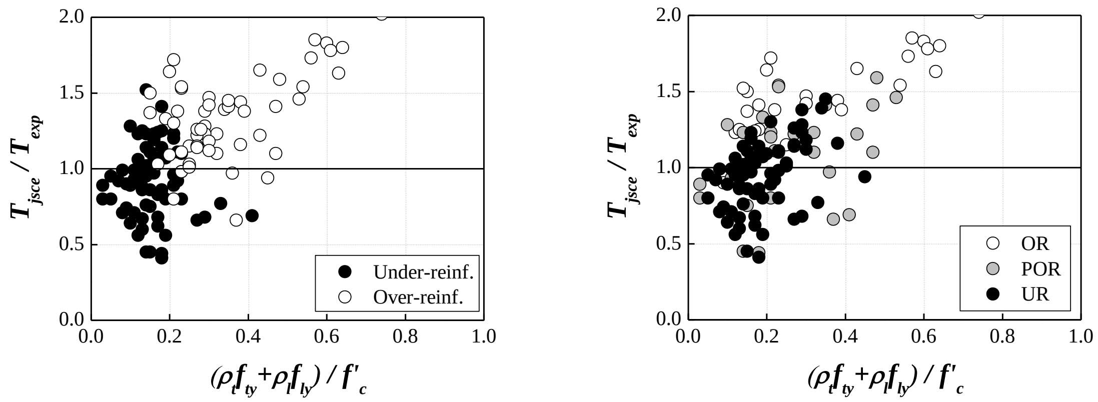
(a) ACI 318-19



Under-reinforced VS Over-reinforced

Types of failure modes

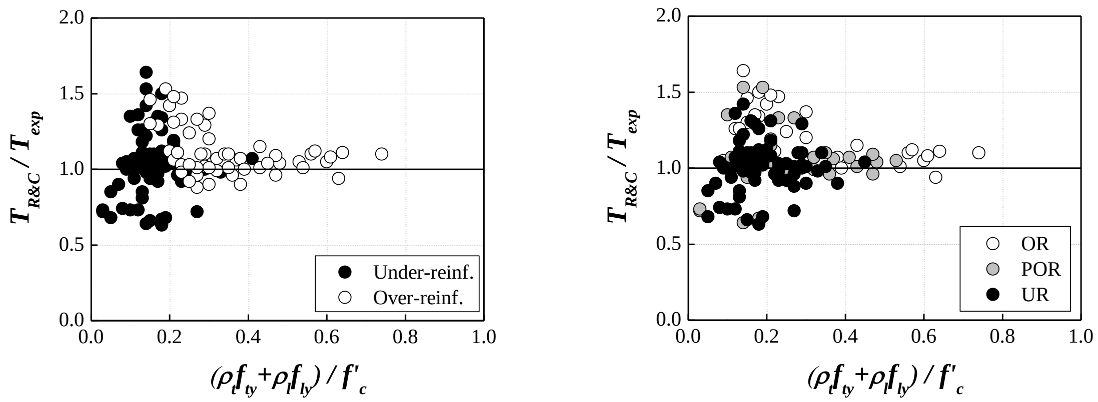
(b) EC2-05



Under-reinforced VS Over-reinforced

Types of failure modes

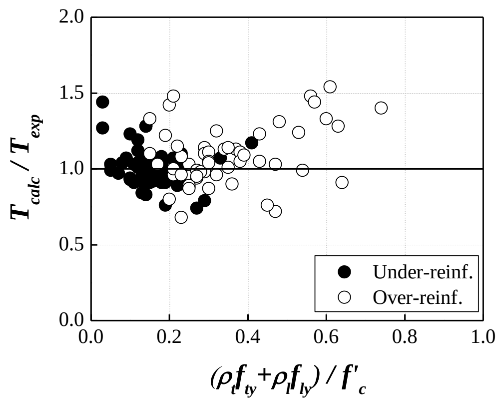
(c) JSCE-07



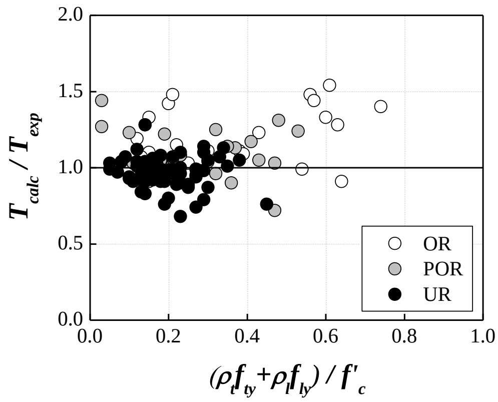
Under-reinforced VS Over-reinforced

Types of failure modes

(d) Rahal and Collins [12]



Under-reinforced VS Over-reinforced



Types of failure modes

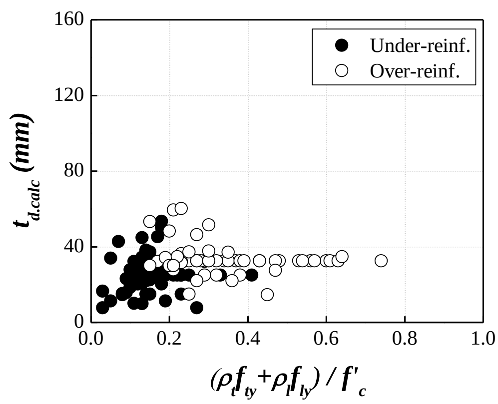
(e) Proposed model

Fig. 3. Comparisons of calculated and observed torsional strengths.

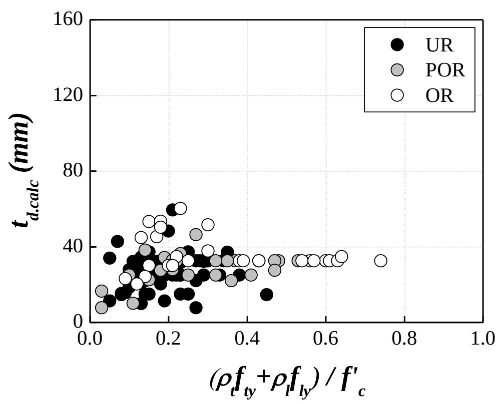
COMPARISON WITH EXISTING BEAM TEST RESULTS

To verify the validity of the proposed model, the torsional strengths calculated using Eq. (24), T_{calc}, are compared with the torsional strengths observed in existing beam tests, T_{exp}, available in the literature. The aspect ratios of the test beams, B/H, are in the range of 0.13 to 1.00. To evaluate the effect of reinforcement ratio on the analytical results, the test beams are classified into over-reinforced beams and under-reinforced beams. T_{exp} were compared with T_{calc} values and those calculated using EC2-05, JSCE-07, and the model proposed by Rahal and Collins (1996).

For the 129 test beams, the average values of the ratios of calculated to observed torsional strength predicted by ACI 318-19, EC2-05, JSCE-07, Rahal and Collins' model, and the proposed model were 1.06, 0.82, 1.09, 1.07, and 1.04, respectively. The corresponding coefficients of variation were 24.1%, 30.0%, 29.5%, 18.2%, and 15.4%, respectively. The results of the comparative analysis showed that, compared to the current design codes, the proposed model and Rahal and Collins' model provide more reasonable predictions for under-reinforced, partially over-reinforced, and over-reinforced beams. However, for the test beams with larger heights such as No.123 to No.129, the proposed model made conservative predictions. Peng and Wong (2011) pointed out that the ACI 31-19 code makes quite conservative predictions for lightly reinforced members with larger heights. These results are attributed to the proposed model's using the ACI 318-19 code format to verify its validity.

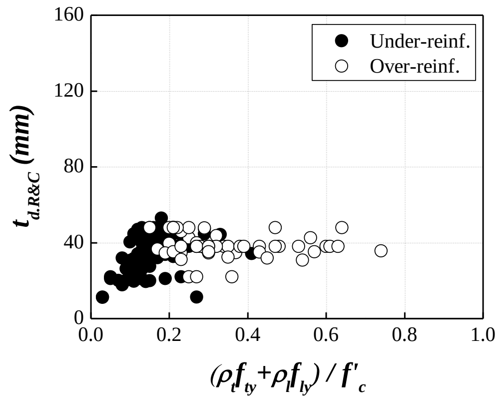


Under-reinforced VS Over-reinforced

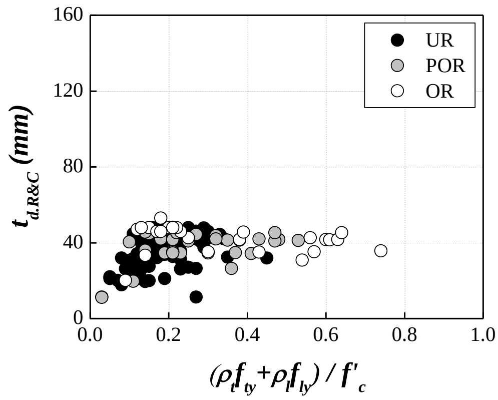


Types of failure modes

(a) Eq. (21)

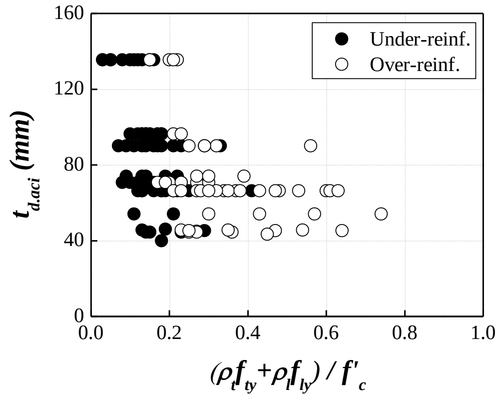


Under-reinforced VS Over-reinforced

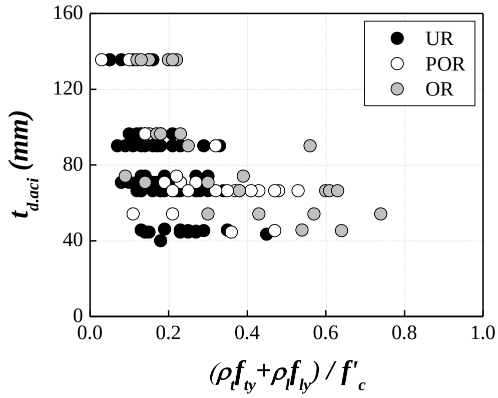


Types of failure modes

(b) Eq. (24)



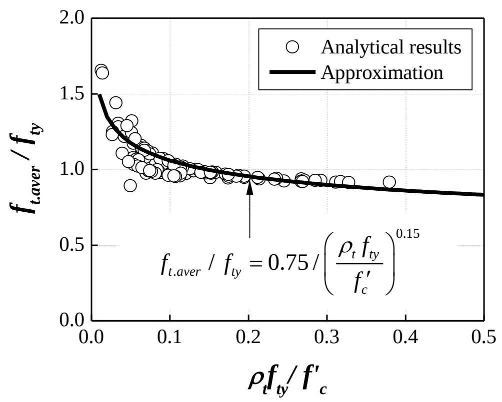
Under-reinforced VS Over-reinforced



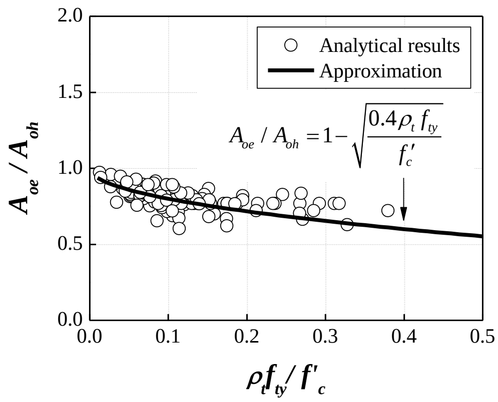
Types of failure modes

(c) Eq. (25)

Fig. 4. Thickness of shear flow path for $\left(\rho_t f_{ty} + \rho_l f_{ly}\right) / f_c'$



(a) $\overline{f}_t / f_{ty}$ for $\rho_t f_{ty} / f_c'$



(b) A_{oe} / A_{oh} for $\rho_t f_{ty} / f_c'$

Fig. 5. Approximation of analytical results.

Fig. 3 compares the calculated and observed torsional strengths for the ratio of the amount of torsional reinforcement to the concrete strength, $(\rho_t f_{ty}+\rho_l f_{ly})/f`_c$. The figures on the left and right, respectively, show the analytical results classified into reinforcement ratios (over-reinforced and under-reinforced) and failure modes (over-reinforced, partially over-reinforced, and under-reinforced torsional failures) of the test beams. For the reinforcement ratios, the current design codes underestimate the torsional strength of the under-reinforced beams and overestimate the torsional strength of the over-reinforced beams. Furthermore, for the failure modes, the design codes overestimate the torsional strength of the test beams that have undergone over-reinforced torsional failure. However, the proposed model and Rahal and Collins' model overestimate the torsional strength of the test beams with the values of $(\rho_t f_{ty}+\rho_l f_{ly})/f`_c$ >0.4 and$(\rho_t f_{ty}+\rho_l f_{ly})/ f`_c$ <0.3 for the over-reinforced test beams, respectively. This is because while Rahal and Collins' model does not consider the deformation capacity of torsional reinforcement in the under-reinforced members, the proposed model does not consider the torsional failure by concrete crushing before the yielding of torsional reinforcement in the over-reinforced members. Rahal and Collins (1996) proposed a simplified equation to compute the thickness of the compressive field of stress, as follows

$$t_{d.R\&C} = \left[\frac{(\rho_{bl}/\rho_l)(A_l f_{ly}+A_p f_{py})}{0.5\alpha_1 f_c' p_c} + \frac{(\rho_{bt}/\rho_t)(A_t f_{ty})}{0.55\alpha_1 f_c' s}\right]/\beta_1 \tag{25}$$

where A_c: outer area of concrete cross-section; p_c: perimeter enclosed by concrete outer dimensions; A_p: total area of prestressed reinforcement; f_{py}: yield strength of prestressed reinforcement; ρ_{bl} and ρ_{bt}: balanced longitudinal and transverse reinforcement ratios, respectively; and α_1 and β_1: stress factor (=0.9) and depth factor (=0.833) in equivalent stress block, respectively. $t_{d.R\&C}$ is limited by a simple check of concrete spalling, which suggested that spalling took place if the thickness of the concrete cover exceeded 30 percent of A_c/p_c. Rahal and Collins suggested an average value of thickness of concrete ($t_{d.aver}$= $0.5A_c/p_c$) based on the analytical results for 86 beams. Instead, the ACI 318-19 code suggests a minimum required thickness of hollow sections for which the presence of the core does not reduce the torsional capacity, as follows

$$t_{d.aci} = A_{oh}/p_h \tag{26}$$

Fig. 4 compares $t_{d.calc}$, $t_{d.R\&C}$, and $t_{d.aci}$. Test results were classified into reinforcement ratios and failure modes. The proposed model and Rahal and Collins' simplified equation provide different values of t_d for $(\rho_t f_{ty}+\rho_l f_{ly})/f`_c$. However, the trends of $t_{d.calc}$ and $t_{d.R\&C}$ are similar to each other and they converge to constant values as $(\rho_t f_{ty}+\rho_l f_{ly})/f`_c$ increases. For the over-reinforced test beams, the values of t_d predicted by the proposed model are slightly smaller than those predicted by Rahal and Collins' model. This is because the two models limit t_d according to criteria established in each study.
However, the values of $t_{d.aci}$ from ACI 318-19 depend only on the cross-sectional properties of the test beams. For instance, the value of $t_{d.aci}$ of test beams No. 1 to No. 16 is 90.0 mm, and the value of $t_{d.aci}$ of test beams No. 17 to No. 32 is 70.7 mm.

APPROXIMATION OF ANALYTICAL RESULTS FOR DESIGNAPPLLICATION

From Fig. 4, it can be inferred that $\overline{f_l}$, $\overline{f_t}$, and A_{oe} may also converge to constant values because they are correlated to t_d, which is affected by $f_{c'}$ and ρ_s. If the proposed model is revised so that $\overline{f_l}$, $\overline{f_t}$, and A_{oe} converge to constant values with the increase of $(\rho_t f_{ty} + \rho_l f_{ly})/f_{c'}$, the torsional strength of over-reinforced members can be predicted more accurately. The comparative analysis showed that the proposed model provides more reasonable predictions than other models. However, calculations of f_{ty}, f_{ly} , α, and A_{oe} are not convenient for the design of torsional members. An approach to approximate these variables via regression analysis of analytical results is introduced. The ratio of $\overline{f_l}$ to f_{ty}, and the ratio of A_{oe} to A_{oh} for $\rho_t f_{ty}/f_{c'}$ are plotted in Fig. 5. The mean value of A_{oe}/A_{oh} for the 129 test beams is about 0.82, which is slightly lower than the reduction factor recommended in the ACI 318-19 code. However, it can be seen that $\overline{f_l}/f_{ty}$ and A_{oe}/A_{oh} decrease with the increase of $\rho_t f_{ty}/f_{c'}$. The correlation between the two ratios and $\rho_t f_{ty}/f_{c'}$ was approximated as presented in Fig. 5; then, $\overline{f_t}$ and A_{oe} can be

calculated using the following approximations. The approximation for $\overline{f_l}$ is the same as Eq. (27). The angle of principal compressive stress of concrete, α, can be calculated by substituting $\overline{f_l}$ and $\overline{f_t}$ from Eq. (27) into Eq. (2) instead of f_{ly} and f_{ty}. In addition, substituting A_{oe} as calculated by Eq. (28) into Eq. (23), t_d can be approximately obtained. However, it should be noted that there may be a deviation of 40 percent or more between the values of t_d as determined by the approximations and as determined by Eq. (22). For the 129 test beams, the average of T_{app}/T_{exp} is 1.00, and the coefficient of variation is 13.8 percent. Fig. 6 shows the ratios of T_{app} to T_{exp} for $(\rho_t f_{ty} + \rho_l f_{ly})/f_{c'}$. Comparing Fig. 4 (a) and Fig. 6, it can be seen that the accuracy of the ACI 318-19 code for the over-reinforced test beams was improved somewhat by using the approximations of $\overline{f_l}$, $\overline{f_t}$, and A_{oe}.
From the results of the comparative analysis presented in this study, it was found that when $f_{c'}$ remains constant, an increase in ρ_l or f_{ly} leads to a decrease in A_{oe}, $\overline{f_l}$, and $\overline{f_t}$. This trend shows that, instead of reducing the deformation of reinforcement, the increase of the amount of torsional reinforcement ratio $(\rho_t f_{ty} + \rho_l f_{ly})/f_{c'}$ increases the value of t_d required to resist torsional moment. Furthermore, even if $(\rho_t f_{ty} + \rho_l f_{ly})/f_{c'}$ is increased, t_d, $\overline{f_l}$, and $\overline{f_t}$ converge to values constrained by reinforced concrete sections. This trend shows that over-reinforced torsional members may fail by concrete crushing before the yielding of torsional reinforcement. Therefore, considering the deformation capacity of reinforcement for under-reinforced torsional members and the limit in the thickness of the shear flow path for over-reinforced torsional members, the accuracy of the current design codes is expected to be further improved.

CONCLUSIONS

This paper presented an analytical model to predict the maximum torsional strength of RC beams considering material and cross-sectional properties. The proposed model assumes that torsional reinforcements with sufficient deformation capacity resist compressive forces acting on local compression diagonals until concrete fails by crushing. This model is based on the average stress-strain relationship of steel bars embedded in concrete, the rotation angle softened truss model, and the modified compression field theory. The maximum torsional strengths calculated by the proposed model were compared with those of existing test beams.

· Calculated and observed torsional strengths of existing 129 test beams subjected to pure torsion showed good agreement. The accuracy of the ACI 318-19 code after applying the analytical results from the proposed model improved 36% in terms of coefficient of variation. The proposed model provides more reasonable predictions than those of the current design codes; however, the proposed model slightly overestimates the torsional strength of over-reinforced members.

· Even if the torsional reinforcement ratios in both directions are the same, the angle of the local compression diagonal, θ, decreases when the cross-sectional area of longitudinal reinforcement is larger than that of transverse reinforcement. This leads to an increase in the thickness of the shear flow path. In this case, the calculated t_d is larger than $2c_c$+d_t and the centerline of the shear flow path no longer coincides with that of the outermost closed stirrups. The minimum thickness of the shear flow path, considering the reinforcement details and the ring tension, serves to limit the theoretical value of t_d of torsional members in which the cross-sectional area of longitudinal reinforcement differs from that of transverse reinforcement.

· The approximations for the average tensile stress of reinforcement, $\bar{f_t}$, and the effective gross area enclosed by shear flow path, A_{oe}, were proposed for design application. The accuracy of the proposed approximations depends on the number of existing test beams. Since the number of the specimens with $\rho_t f_{ty}/f_c'$ greater than 0.2 is relatively small in this range, the accuracy of the analytical results for RC beams may be slightly degraded. However, it was found that the approximations give more reasonable analytical results in a broad range of f_{ly}+$\rho_t f_{ty}$)/$f_{c'}$, compared to the ACI 318-19 code. The approximations are simple enough to be applied to torsional design and can give practitioners valid analytical results without iterative calculations.

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