Ultimate strength of fillet welded connections loaded in plane

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Fillet welded connections are frequently loaded eccentrically in shear with the externally applied load in the same plane as the weld group. While some current design tables are based on ultimate strengths, methods of analysis that incorrectly mix inelastic and elastic approaches are still used. These methods give conservative and variable margins of safety. Design standards generally use a lower-bound approach basing strengths on the longitudinal value neglecting, conservatively, the increase in strength for other directions of loading. The factored resistance of fillet welds, as a function of the direction of loading, is established based on ultimate strength expressions developed herein and using geometric, material variations, and test-to-predicted ratios reported in the literature. Factored resistances of eccentrically loaded fillet weld groups are established. These are basesd on the method of instantaneous centres, ultimate strengths, and the load–deformation expressions developed herein that are functions of the angle of loading. Also, statistical data on geometry, material variations, and the comparison of predicted strengths with the full-scale test results of others are used. Tables of design coefficients giving factored resistances for various eccentrically loaded fillet welded connections are developed. The coefficients, on the average, are essentially the same as those in current design tables.

Key words: connections, design tables, eccentric, fillet welds, limit states, ultimate strength.

Les assemblages avec soudure d'angle sont fréquemment soumis à des charges excentriques en cisaillement, la charge appliquée à l'externe se trouvant sur le même plan que le groupe de soudures. Bien que certaines tables de calcul actuelles soient basées sur les résistances ultimes, des méthodes d'analyse qui confondent injustement des approches élastiques et inélastiques sont encore utilisées. Ces méthodes proposent des marges de sécurité trop prudentes et variables. Les normes de calcul font généralement appel à la méthode de la limite inférieure qui base les résistances sur la valeur longitudinale et néglige l'accroissement de la résistance dans les autres directions de chargement. La résistance pondérée de soudures d'angle, comme fonction de la direction du chargement, est établie selon les équations de résistance ultime élaborées dans cet article et fait appel aux variations géométriques des matériaux ainsi qu'aux rapports essai/prévision mentionnées dans la documentation. Les résistances pondérées de groupes de soudures d'angle soumis à des charges excentriques sont établies. Elles sont basées sur la méthode des centres instantanés, sur les résistances ultimes et sur les équations charge–déformation élaborées dans cet article, qui sont des fonctions de l'angle de chargement. De plus, des données statistiques sur la géométrie, les variations des matériaux et la comparaison des résistances prévues et des résultats d'essais pleine grandeur sont utilisées. Des tables de coefficients de calcul établissant les résistances pondérées de divers assemblages avec soudure d'angle soumis à des charges excentriques sont élaborées. En moyenne, les coefficients sont essentiellement les mêmes que ceux qui apparaissent dans les tables de calcul existantes.

Mots clés : assemblage, table de calcul, excentrique, soudure d'angle, états limites, résistance ultime.

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1. Introduction

Fillet welded connections are frequently loaded eccentrically in shear with the externally applied load in the same plane as the weld group as shown in Fig. 1. Methods of analysis in which the shear stresses due to the concentric load are assumed to be distributed uniformly, and those due to the moment to be distributed as a function of the elastic section modulus of the weld configuration and then added, are basically incorrect. The first assumption is based on inelastic behaviour and the second on elastic behaviour. These methods give very conservative and variable margins of safety. However, some current design tables, such as those in the Canadian Institute of Steel Construction Handbook (CISC 1985), are based on ultimate strength considerations; in this case the work of Butler *et al.* (1972).

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Nonetheless current design standards such as CSA Standard CAN3-S16.1-M84, Steel Structures for Buildings (Limit States Design) (CSA 1984*a*) and CSA Standard W59-M1984, Welded Steel Construction (Metal Arc Welding) (CSA 1984*b*) generally use a lower-bound approach. The strength of fillet welds subject to shear is based on the strength of longitudinal fillet welds regardless of the loading angle, i.e., the angle the applied load makes with the axis of the weld. This approach is conservative because the ultimate strength increases as the direction of loading changes from parallel to the weld axis (longitudinal weld) to perpendicular to the weld axis (transverse weld) (Butler and Kulak 1971; Miazga and Kennedy 1986). The standards (e.g., Clause 13.13.3 of S16.1) do, however, allow an ultimate strength analysis to be used, but do not prescribe it.

Miazga and Kennedy (1986, 1989) developed a rational analytical model to predict the ultimate strength of fillet welds loaded in shear as a function of the angle of loading. The analysis was corroborated by tests when the connections were

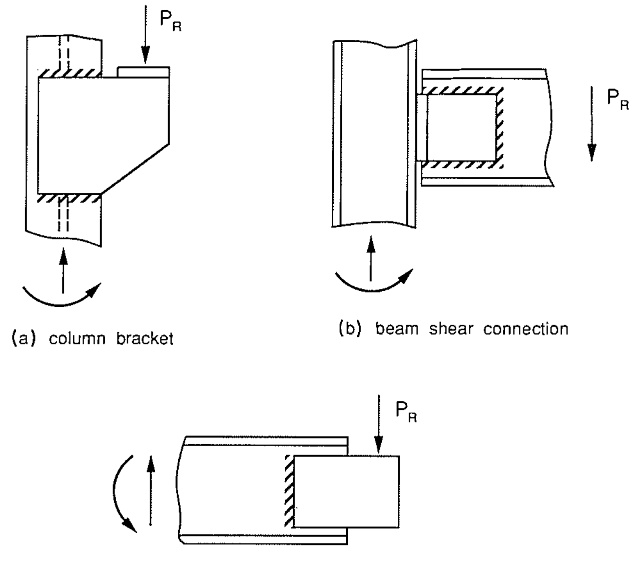

FIG. 1. Eccentrically loaded welded connections.

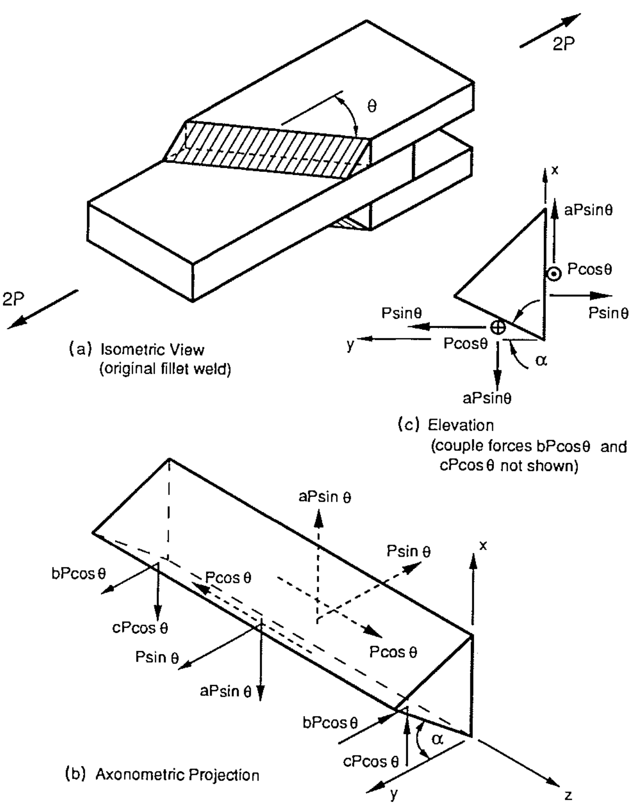

FIG. 2. Equilibrium of fractured weld.

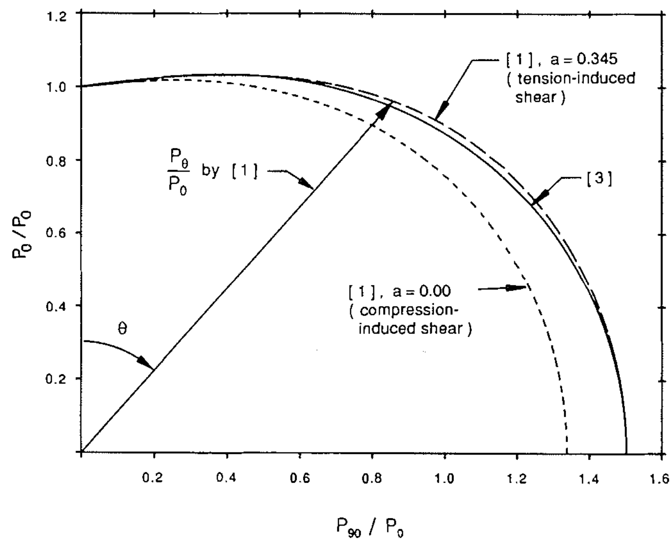

FIG. 3. Normalized ultimate strength.

loaded in tension and extended to connections loaded in compression.

In this work, the rationale and test results of Miazga and Kennedy are used to develop expressions for the ultimate strength of fillet welds loaded in shear and for the load–deformation characteristics as a function of the angle of loading. These are, in turn, used to predict analytically the ultimate strength of eccentrically loaded fillet welded connections using the method of instantaneous centres of rotation. This method, as discussed subsequently, takes equilibrium into consideration as well as deformation compatibility. Appropriate resistance factors are determined by taking into account geometric and material variations and the comparison of the predicted strengths with the full-scale test results of others. Tables giving the factored resistances for various eccentrically loaded fillet weld configurations are developed and compared with current design tables.

2. The method of instantaneous centres of rotation

This method, documented fully by others (Butler *et al.* 1972; CISC 1985; Lesik and Kennedy 1988), is based on the following assumptions:

1. At any load level, a fillet weld group subject to an eccentric load tends to rotate about an instantaneous centre of rotation such that equilibrium is satisfied.
2. The fillet weld group is considered to be divided into a number of finite weld elements. The resisting force of each element acts through its centroid and perpendicular to the radius to the instantaneous centre.
3. To satisfy deformation compatibility, the deformation of a weld element is considered to vary linearly with the distance to the instantaneous centre. (This implies that the deformation of the connecting plates is negligibly small.)
4. Knowing the angle of loading from assumption 2, and the deformation from 3, the resistance of a weld element is obtained from the average load–deformation response determined empirically for the appropriate angle of loading.
5. Summing, vectorially, the resistances of the weld elements gives the resistance of the weld group, that is, a shear and moment or an equivalent eccentric load.
6. The ultimate strength of the weld group is obtained when the maximum deformation of some weld element in the group is reached.

Because of the nonlinear behaviour, the ultimate strength of a weld group is determined iteratively. The location of the instantaneous centre is assumed. The most remote element generally is taken to have the maximum deformation, though

TABLE 1. Ultimate strengths of fillet welds (tests of Miazga and Kennedy 1989)

Test	Angle of loading (°)	Ultimate strength, P_θ ($N \cdot mm^{-1} \cdot mm^{-1}$)	Normalized ultimate strength, P_θ/P_0: Test	Normalized ultimate strength, P_θ/P_0: Predicted [3]	$\frac{\text{Test}}{\text{Predicted}}$
00.1	0	328.3	1.129	1.000	1.129
.2		301.8	1.038		1.038
.3		297.4	1.023		1.023
.11		263.8	0.908		0.908
.12		281.8	0.970		0.970
.13		270.8	0.932		0.932
15.1	15	304.7	1.048	1.066	0.984
.2		296.4	1.020		0.957
.3		303.8	1.045		0.981
.11		293.3	1.009		0.947
.12		266.9	0.918		0.862
.13		305.4	1.051		0.986
30.1	30	393.7	1.355	1.177	1.151
.2		376.6	1.296		1.101
.3		391.6	1.347		1.145
.11		351.8	1.210		1.029
.12		357.6	1.230		1.045
.13		345.2	1.188		1.009
45.1	45	407.6	1.402	1.297	1.081
.2		425.5	1.464		1.129
.3		418.5	1.440		1.110
.11		328.2	1.129		0.870
.12		325.6	1.120		0.863
.13		336.6	1.158		0.893
60.1	60	482.6	1.660	1.403	1.183
.2		484.3	1.666		1.188
.3		490.1	1.686		1.202
.11		419.9	1.445		1.030
.12		403.7	1.389		0.990
.13		396.5	1.364		0.972
75.1	75	422.1	1.452	1.475	0.985
.2		427.6	1.471		0.998
.3		437.3	1.505		1.020
.11		424.8	1.461		0.991
.12		431.6	1.485		1.007
.13		417.0	1.435		0.973
90.1	90	400.6	1.378	1.500	0.919
.2		404.0	1.390		0.927
.3		382.7	1.317		0.878
.11		441.0	1.517		1.011
.12		435.1	1.497		0.998
.13		432.0	1.487		0.991
				Mean, μ	1.0096
				Standard deviation, σ	0.0901
				Coefficient of variation, V	0.0892
				Sample size, n	42

care must be exercised as the deformation sustainable depends on the direction of loading. With the deformations of other elements in proportion to their radius vectors and providing no element exceeds its maximum deformation capability, the resisting forces in each element are determined. If the sum of the forces and moments is not in equilibrium with the external forces, a new instantaneous centre is selected and the procedure repeated.

It is common to assume that the load–deformation response of a fillet weld element loaded in compression-induced shear is the same as for tension-induced shear. (Most tests on fillet welds at various angles of loading have been tension tests in

TABLE 2. Ultimate strengths of fillet welds in compression (tests of Swannell and Skewes 1979*a*)

Test	Angle of loading (°)	Ultimate strength, P_θ ($N \cdot mm^{-1} \cdot mm^{-1}$)	Normalized ultimate strength, P_θ/P_0 test	Eq. [1], a = 0.0 P_θ/P_0 predicted	Eq. [1], a = 0.0 Test/Predicted	Eq. [3] P_θ/P_0 predicted	Eq. [3] Test/Predicted
1	0	322.8	0.928	1.000	0.928	1.000	0.928
2		326.0	0.937		0.937		0.937
3		355.9	1.023		1.023		1.023
4		365.4	1.050		1.050		1.050
5		370.1	1.064		1.064		1.064
6	30	357.5	1.027	1.134	0.906	1.177	0.873
7		373.2	1.072		0.946		0.911
8		363.8	1.045		0.922		0.888
9		374.8	1.077		0.950		0.915
10		384.3	1.104		0.974		0.938
11	60	390.6	1.122	1.283	0.875	1.403	0.800
12		392.1	1.127		0.878		0.803
13		390.6	1.122		0.875		0.800
14		393.7	1.131		0.882		0.806
15	90	406.3	1.168	1.337	0.873	1.500	0.778
16		407.9	1.172		0.877		0.781
17		409.5	1.177		0.880		0.784
18		415.7	1.195		0.894		0.796
19		422.0	1.213		0.907		0.809
				Mean, μ	0.928		0.878
				Standard deviation, σ	0.060		0.094
				Coefficient of variation, V	0.065		0.107
				Sample size, n	19		19

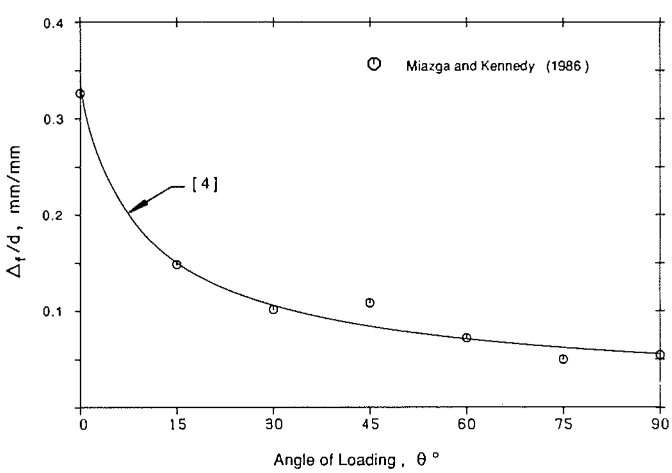

FIG. 4. Normalized deformation at fracture.

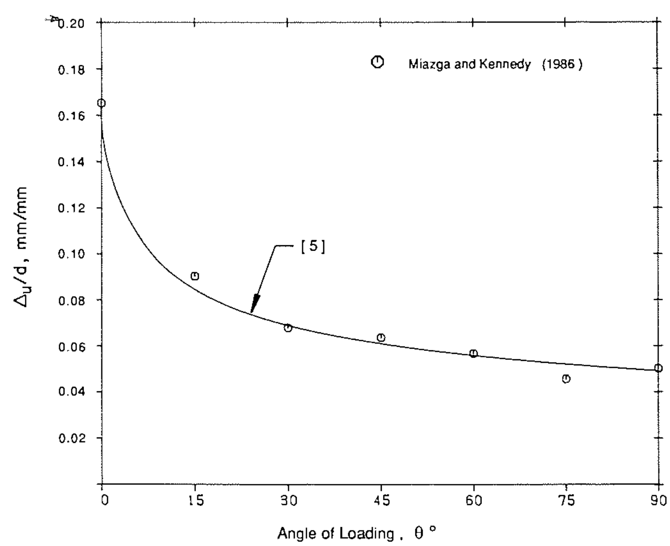

FIG. 5. Normalized deformation at ultimate load.

that the plates joined by the fillet welds have been pulled apart. This is tension-induced shear. Relatively few have been tested by pushing the plates together to produce compression-induced shear.) This assumption, particularly as related to ultimate strength, is discussed in the next section.

3. Fillet welds loaded in shear

3.1. Ultimate strength

Based on 42 tests on fillet weld specimens with seven angles of loading varying in 15° increments from 0° (longitudinal) to 90° (transverse), Miazga and Kennedy (1989) proposed that the ultimate load carried by a fillet weld loaded at any angle θ, normalized by dividing by the ultimate load for $\theta = 0°$, is

$$[1] \quad \frac{P_\theta}{P_0} = \frac{1 + 0.141 \sin \theta}{\sin(45 + \alpha) \sqrt{(\sin \theta \cos \alpha - a \sin \theta \sin \alpha)^2 + \cos^2 \theta}}$$

where the coefficient 0.141, determined experimentally,

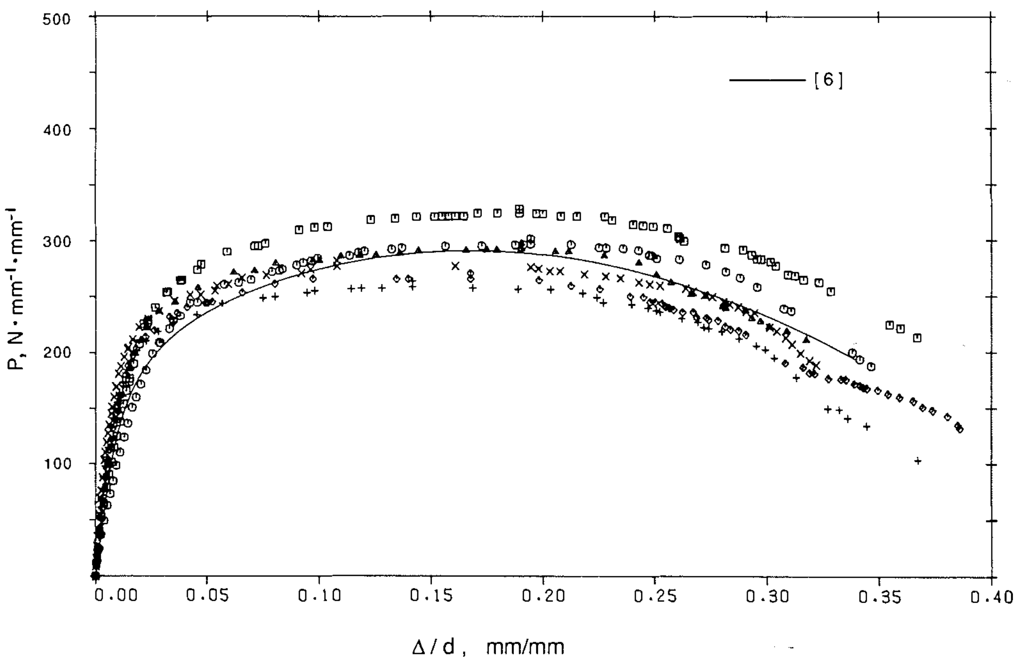

FIG. 6. Load–deformation curve, $\theta = 0°$.

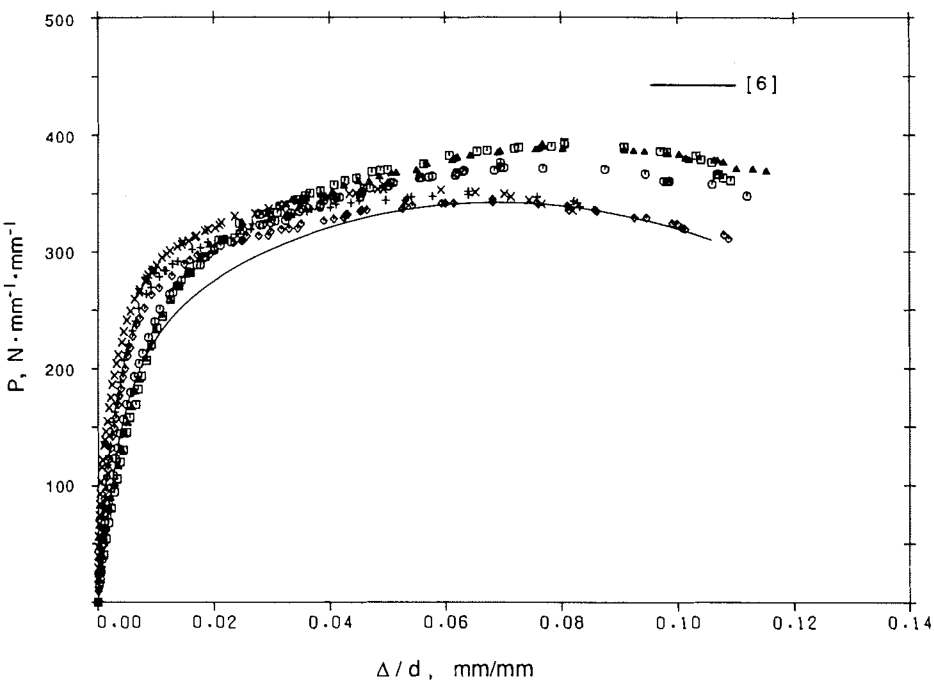

FIG. 7. Load–deformation curve, $\theta = 30°$.

reflects the increased strength due to lateral restraint at the root of the weld. The coefficient a in [1], determined experimentally to be 0.345 for welds loaded in tension-induced shear, when multiplied by the stress resultant $P \sin \theta$, as shown in Fig. 2c, maintains moment equilibrium with the equal and opposite forces $P \sin \theta$. Miazga and Kennedy also proposed that the angle of the fracture surface (see Fig. 2) is

$$[2] \quad \tan(45 + \alpha) = \frac{(\cos \alpha - a \sin \alpha)^2 + \cot^2 \theta}{(\cos \alpha - a \sin \alpha)(\sin \alpha + a \cos \alpha)}$$

By finding the value of α from [2] and substituting in [1], the ultimate strength curve marked [1] in Fig. 3 is obtained. In Fig. 3, as shown, the radial distance from the origin at an angle θ to an ultimate strength curve gives the value of P_θ/P_0, that is, the ultimate strength normalized by dividing by the value for $\theta = 0°$. Thus the vertical axis and horizontal axis give values of P_θ/P_0 when $\theta = 0°$ (P_0/P_0) and when $\theta = 90°$ (P_{90}/P_0) respectively.

An empirical equation that closely approximates [1] as shown in Fig. 3 is

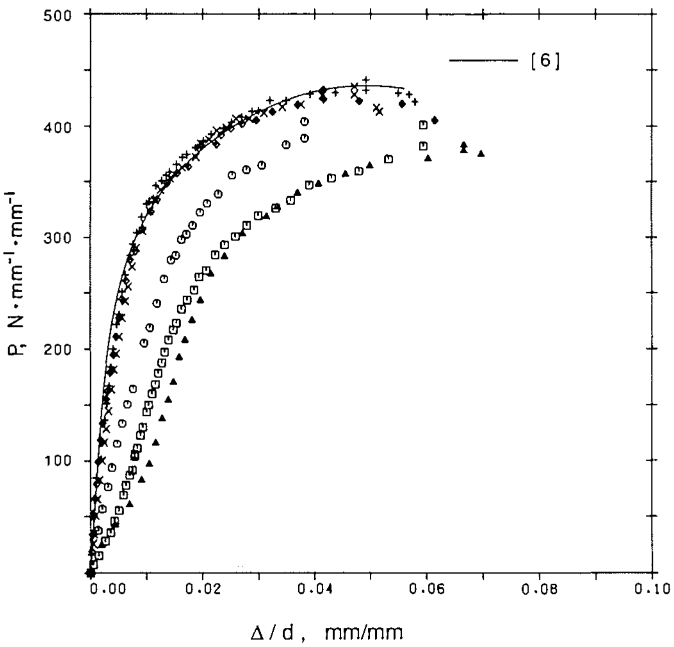

FIG. 8. Load–deformation curve, $\theta = 90°$.

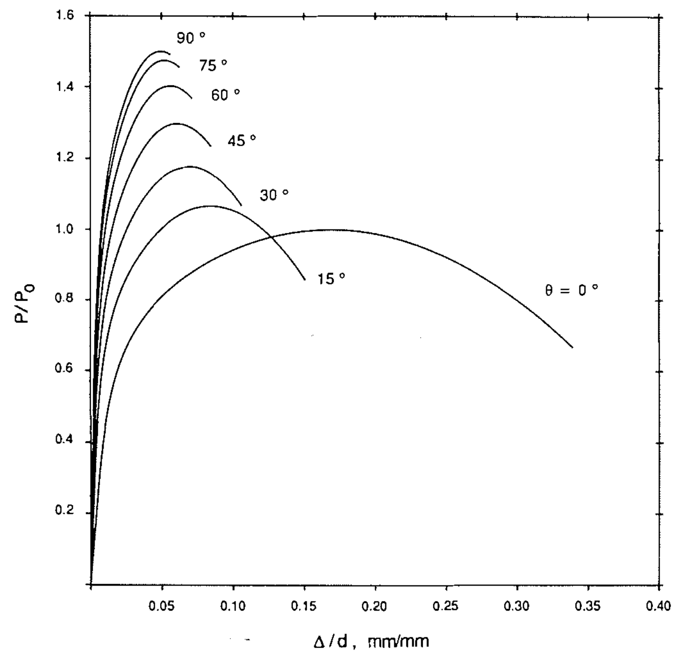

FIG. 9. Normalized load–deformation curves, eq. [6].

$$[3] \quad \frac{P_\theta}{P_0} = 1.00 + 0.50 \sin^{1.5}\theta$$

and underestimates [1] by a maximum amount of 1.5% for $\theta = 45°$.

Table 1 gives the test-to-predicted ratios for the ultimate strength of the 42 fillet weld tests reported by Miazga and Kennedy (1989) using [3]. The ultimate strengths are reported as N/mm of weld length/mm of weld size. They do not represent shear stresses on the throat or fracture surface. For longitudinal welds failing on a 45° throat, the ultimate shear stress would be 1/0.707 = 1.414 times the values given. The normalized ultimate strengths are determined by dividing by the average ultimate load for longitudinal welds of 290.7 $N \cdot mm^{-1} \cdot mm^{-1}$. The mean value of the test-to-predicted ratio for all 42 tests is 1.010 with a standard deviation of 0.090 and a coefficient of variation of 0.089. These values, and in particular the mean, are higher than those reported by Miazga and Kennedy based on [1], as would be expected, as the latter predicts higher strengths and therefore lower test-to-predicted ratios.

Also plotted, as a dotted line in Fig. 3, is the ultimate strength given by [1] when the value of the coefficient a is zero. This value was considered appropriate by Miazga and Kennedy (1989), for welds with compression-induced shear. The maximum difference between the strengths for compression-induced shear and tension-induced shear occurs for transverse welds when the compression-induced shear value is only 0.891 of the tension-induced value.

When assessing the strength of single fillet welds with compression-induced shear, it would be most appropriate to use [1] with a equal to zero as represented by the dotted line. Table 2 shows that 19 tests of Swannel and Skewes (1979*a*), with compression-induced shear, when compared to this, give a mean value of the test-to-predicted ratio of 0.928 with a coefficient of variation of 0.065. It is also possible to use [3] to determine test-to-predicted ratios for the compression-induced tests. In this case the mean value becomes 0.878, a decrease of about 6%, and the coefficient of variation is increased to 0.107. Both these values indicate that [3] does not predict the results as well as does [1] with a equal to zero. (It should be noted that Swannell and Skewes pointed out that the specimen configuration they used could be construed to influence the failure condition. Therefore, these data should be used with caution. There are, however, no other data available.)

When assessing the strength of weld groups, however, the circumstances are considerably different. One-half of the weld group will be in tension-induced shear and the other half in compression-induced shear, giving a mean test-to-predicted ratio of (1.010 + 0.878)/2 = 0.944 when using [3]. This is only 6.5% less than for tension-induced shear. The coefficient of variation can be shown to be 0.120. As well, the analyses of Miazga and Kennedy (1989) show that the angle of inclination of the fracture surface for compression-induced shear lies within the range for tension-induced shear, suggesting that the deformation capability would not be less. Furthermore, because in the final analysis the ratio of full-scale test results to predictions based on the tension-induced shear model are taken into consideration, then any prediction of too great a strength is automatically accounted for in the derived value of the mean test-to-predicted ratio and the coefficient of variation. (Had a lower strength been predicted, the test-to-predicted ratio would have been higher.) Thus the use of the tension-induced shear model is valid from the point of view of both strength and deformability.

3.2. *Deformations*

In Figs. 4 and 5 are plotted the values of the normalized deformation as a function of the angle of loading when the welds fractured and at ultimate load, respectively. The deformations have been normalized by dividing by the weld size. Each point represents the mean of six tests. By using linear regression analyses, equations

TABLE 3. Measured-to-nominal ratios of ultimate tensile strength of weld metal

Source	Sample size, n	Nominal tensile strength, X_u	Mean tensile strength, σ_u	Measured/Nominal (σ_u/X_u)	Standard deviation	Coefficient of variation
Miazga and Kennedy (1986)	3	480 MPa	537.7 MPa	1.120	0.0158	0.0141
Gagnon and Kennedy (1987)	10	480 MPa	579.9 MPa	1.208	0.0428	0.0355
Swannell and Skewes (1979*b*)	2	410 MPa	538.8 MPa	1.314	0.0262	0.0199
Fisher *et al.* (1978)	127	60 ksi	66.0 ksi	1.100	0.0427	0.0388
	138	70 ksi	74.9 ksi	1.070	0.0381	0.0356
	136	80 ksi	87.9 ksi	1.099	0.0543	0.0494
	16	90 ksi	100.2 ksi	1.113	0.0480	0.0431
	72	110 ksi	116.9 ksi	1.063	0.0426	0.0400
	128	70 ksi	85.4 ksi	1.220	0.0681	0.0559
	40	70 ksi	86.8 ksi	1.240	0.1411	0.1138
			Mean, μ	1.123		
			Standard deviation, σ	0.087		
			Coefficient of variation, V	0.077		
			Sample size, n	672		

*1 ksi = 6.89 MPa.

TABLE 4. Measured-to-nominal ratios of shear strength/ultimate tensile strength

Source	Sample size, n	Mean τ_u/σ_u	Measured/Nominal $\left(\frac{\tau_u/\sigma_u}{0.67}\right)$	Standard deviation	Coefficient of variation
Miazga and Kennedy (1986)	6	0.765	1.141	0.0925	0.0810
Swannell and Skewes (1979*b*)	5	0.686	1.024	0.0296	0.0289
Kato and Morita (1969)	11	0.656	0.979	0.0753	0.0770
Ligtenburg (1968)					
Appendix I-2	18	0.739	1.103	0.168	0.1525
Appendix I-3	17	0.809	1.207	0.108	0.0895
Appendix I-4	18	0.728	1.087	0.135	0.1240
Appendix I-6	18	0.743	1.108	0.134	0.1210
Appendix I-7	17	0.810	1.210	0.093	0.0766
Appendix I-9	16	0.741	1.105	0.115	0.1041
Mean, μ		0.749	1.118		
Standard deviation, σ		0.091	0.135		
Coefficient of variation, V		0.121	0.121		
Sample size, n		126	126		

[4] $$\frac{\Delta_f}{d} = 1.087(\theta + 6)^{-0.65}$$

and

[5] $$\frac{\Delta_u}{d} = 0.209(\theta + 2)^{-0.32}$$

where d is the leg size of the fillet weld, were obtained for the normalized deformations at fracture and at ultimate load, respectively, as shown in Figs. 4 and 5. The mean test-to-predicted ratios are 1.000 for both equations, considering all 42 test values in each case. The coefficients of variation are 0.194 and 0.172, respectively, as given in Lesik and Kennedy (1988).

3.3. Load–deformation response

Figures 6–8 give typical load–deformation data as determined by Miazga and Kennedy (1986) for angles of loading of 0, 30, and 90°. Data for other angles of loading (15, 45, 60, and 75°) are given in Lesik and Kennedy (1988).[1] These data are similar to that of Figs. 6–8. The data exhibit considerable scatter. Also plotted in each figure is

[6] $$P = P_0(1.00 + 0.50 \sin^{1.5}\theta) f(\rho)$$

or

[6*a*] $$P = 290.7(1.00 + 0.50 \sin^{1.5}\theta) f(\rho)$$

where ρ is defined in [8] below. In [6*a*], the value of 290.7 $N \cdot mm^{-1} \cdot mm^{-1}$ is the average ultimate strength of longitudinal welds from the test data. The second term comes from [3]. The function $f(\rho)$ gives the variation of load with respect to deformation. A polynomial was selected to describe $f(\rho)$ so

[1]These data may be purchased from the Depository of Unpublished Data, CISTI, National Research Council of Canada, Ottawa, Canada, K1A 0S2.

TABLE 5. Test-to-predicted ratios of ultimate loads of full-scale test specimens (Butler *et al.* 1972)

Specimen No.	Ultimate load, P (kN) Test	Predicted	$\frac{\text{Test}}{\text{Predicted}}$
V1	560.5	534.8	1.048
V2	1014.2	1043.8	0.972
V3	418.1	454.4	0.920
V4	1383.4	1445.9	0.957
V5	1036.4	1155.6	0.897
V6	720.6	802.2	0.898
V7	1450.1	1621.6	0.894
V8	1058.7	1069.6	0.990
C1	1321.1	1391.2	0.950
C2	1000.8	1219.1	0.821
C3	1401.2	1044.0	1.342
C4	1338.9	1325.6	1.010
C5	1294.4	1299.2	0.996
		Mean, μ	0.977
		Standard deviation, σ	0.125
		Coefficient of variation, V	0.128
		Sample size, n	13

TABLE 6. Test-to-predicted ratios of ultimate loads of full-scale test specimens (Swannell and Skewes 1979*b*)

Specimen No.	Ultimate load, P (kN) Test	Predicted	$\frac{\text{Test}}{\text{Predicted}}$
1	306.0	291.6	1.049
3	179.3	160.7	1.116
4	98.0	91.3	1.073
5	220.0	240.9	0.913
6a	180.6	184.5	0.979
7	211.3	202.1	1.046
8	218.6	228.7	0.956
10a	196.6	226.9	0.866
11	235.3	218.5	1.077
		Mean, μ	1.008
		Standard deviation, σ	0.084
		Coefficient of variation, V	0.083
		Sample size, n	9

that the descending portion of the curve could be modeled. The polynomial function was determined, using a computer program adapted from Gerald (1978) to perform a nonlinear regression analysis of all the data, to be

$$[7a]\quad f(\rho) = 8.234\rho; \qquad 0 < \rho \leqslant 0.0325$$

and

$$[7b]\quad f(\rho) = -13.29\rho + 457.32\rho^{\frac{1}{2}} - 3385.9\rho^{\frac{1}{3}} + 9054.29\rho^{\frac{1}{4}} - 9952.13\rho^{\frac{1}{5}} + 3840.71\rho^{\frac{1}{6}}; \qquad \rho > 0.0325$$

in which

$$[8]\quad \rho = \frac{\Delta/d}{\Delta_u/d} = \frac{\Delta}{\Delta_u}$$

Combining [8] and [5] gives ρ in terms of Δ/d as

TABLE 7. Test-to-predicted ratios of ultimate loads of full-scale test specimens (Kulak and Timler 1984)

Specimen No.	Ultimate load, P (kN) Test	Predicted	$\frac{\text{Test}}{\text{Predicted}}$
1	612.3	518.1	1.182
2	464.9	364.8	1.274
3	499.6	359.6	1.389
		Mean, μ	1.282
		Standard deviation, σ	0.104
		Coefficient of variation, V	0.081
		Sample size, n	3

$$[9]\quad \rho = \frac{\Delta}{\Delta_u} = \frac{\Delta}{0.209(\theta + 2)^{-0.32}d}$$

The first of the equations, [7*a*], applies to a very limited range of deformations. Imposed upon the graph of [6] in Figs. 6–8 is the maximum deformation at fracture given by [4]. Equation [6] is a best-fit curve to all of the data for the seven angles of loading tested. Overall, [6] fits the data reasonably well. For an angle of loading of 0° (as well as for 45 and 75° not given here), [6] falls in the middle of the test points; while for an angle of loading of 90° (as well as 15° not given here), it tends to the high side, and for an angle of loading of 30° (as well as 60° not given here), it tends to the low side. It is important to note that, for all angles of loading, data exist with deformations greater than predicted by [4], i.e., the end of the curve represented by [6]. Thus the deformability of the welds is not overestimated. Figure 9 shows [6] plotted as P/P_0 for the 7 different angles of loading tested by Miazga and Kennedy. The increased strength and reduced deformability as the loading direction changes from longitudinal to transverse ($\theta = 0°$ to $\theta = 90°$) are evident.

4. Design applications

4.1. Single fillet welds loaded concentrically

The ultimate resistance of a concentrically loaded fillet weld under tension-induced shear at any angle of loading has been given as

$$P_\theta = P_0(1.00 + 0.50 \sin^{1.5}\theta)$$

If P_0 is written, as given in CSA Standard S16.1, as

$$[10]\quad P_0 = 0.67\phi A_w X_u$$

then combining [3] and [10] gives

$$[11]\quad P_\theta = 0.67\phi A_w X_u(1.00 + 0.50 \sin^{1.5}\theta)$$

where it only remains to establish the resistance factor, ϕ. Galambos and Ravindra (1973) write

$$[12]\quad \phi = \rho_R \exp(-\beta\alpha_R V_R)$$

and propose a value of 0.55 for the coefficient of separation, α_R. Ravindra and Galambos (1978) and Fisher *et al.* (1978) suggest that the reliability index, β, be taken as 4.5 for connections to ensure that the probability of failure of the connection is less than that of the member as a whole, for which a value of 3.0 is commonly used for buildings.

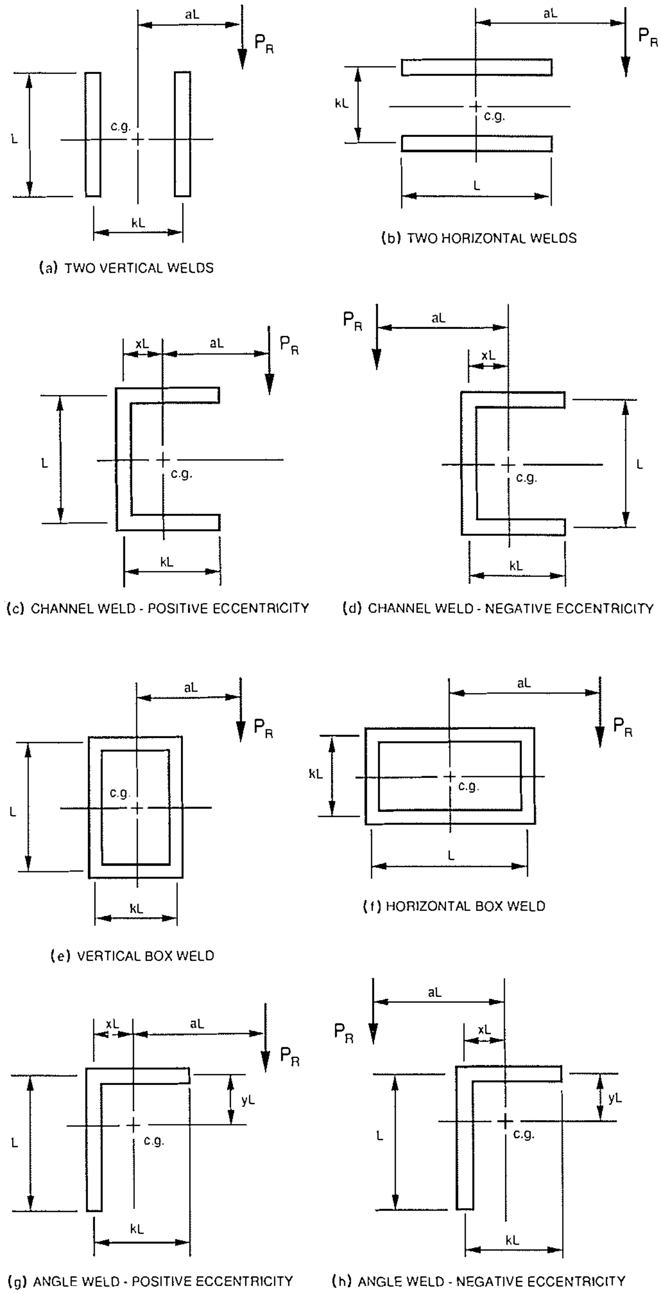

FIG. 10. Types of weld groups.

On the other hand, Gagnon and Kennedy (1989), based on the work with Buckland and Taylor Ltd. (1988), suggest that a value of 3.75 for β would be sufficiently large.

The mean value of the predicted-to-nominal resistance is

[13] $\rho_R = \rho_G \rho_{M1} \rho_{M2} \rho_P$

and the associated coefficient of variation is given by

[14] $V_R^2 = V_G^2 + V_{M1}^2 + V_{M2}^2 + V_P^2$

Using fillet weld leg dimensions for 42 test specimens as reported by Miazga and Kennedy (1986), the mean value of the measured-to-nominal ratio of effective throat area, ρ_G, is 1.034 and the coefficient of variation, V_G, is 0.026. The two material parameters accounting for the variation of material strength are, first, the variation of the ultimate tensile strength of the weld metal as compared to the electrode classification and, second, the ratio of the shear strength to the ultimate tensile strength of the weld metal as compared to that assumed in the expression. From Table 3 the mean value of the measured-to-nominal ratio of the ultimate tensile strength, ρ_{M1}, defined as σ_u/X_u, is 1.123 with a coefficient of variation, V_{M1}, of 0.077. Table 4 gives the ratios of shear strengths to ultimate strengths for a total of 126 tests reported in the literature. The mean value of the ratio of the measured shear strength to measured ultimate strength is 0.749 or, as shown in the table, 1.118 times the value of 0.67 assumed in [11] with a coefficient of variation of 0.121. The data of Miazga and Kennedy (1986) for tension-induced shear gave a mean test-to-predicted ratio, using [3], of 1.010 with a coefficient of variation of 0.089, and when combined equally with the respective quantities of Swannell and Skewes (1979*a*) for compression-induced shear, also using [3], of 0.878 and 0.107 give an overall value of mean test-to-predicted ratio of 0.944 with a coefficient of variation of 0.120. (Combining the two sets of data with equal weight suggests that, in practice, equal numbers of welds would be subject to compression-induced shear as to tension-induced shear.)

Using these values in [13] and [14] gives

$$\rho_R = 1.034 \times 1.123 \times 1.118 \times 0.944 = 1.226$$

$$V_R^2 = 0.026^2 + 0.077^2 + 0.121^2 + 0.120^2 = 0.189^2$$

The resistance factor is therefore, by [12],

$$\phi = 1.226 \exp(-4.5 \times 0.55 \times 0.189) = 0.768$$

for $\beta = 4.5$ and

$$\phi = 1.226 \exp(-3.75 \times 0.55 \times 0.189) = 0.830$$

for $\beta = 3.75$.

Equation [12] allows resistance factors to be computed separately from load factors which are found from the parallel expressison

[15] $\alpha = \rho_s \exp(\beta V_s \alpha_s)$

Obviously, as β increases, resistance factors decrease and load factors increase. To compute a consistent set of resistance factors, ϕ, and load factors, α, the same value of β must be used when evaluating [12] and [15]. Here, when resistance factors are calculated for $\beta = 4.5$ or 3.75, as considered appropriate, the load factors should also be computed for the same value of β, i.e., 4.5 or 3.75. As it is desirable for design purposes to maintain the load factors at the values determined for $\beta = 3.0$, the resistance factors determined for another value of β must be modified downwards to compensate for the lower load factors corresponding to $\beta = 3.0$. Fisher *et al.* (1978) show that for βs of 3.0 and 4.5 the adjustment factor can be taken as 0.88, while Gagnon and Kennedy (1989) give the adjustment factor as 0.93 for βs of 3.0 and 3.75. Therefore, the resistance factor for use in [11] is

$$\phi_w = 0.768 \times 0.88 = 0.68$$

for $\beta = 4.5$ and

$$\phi_w = 0.830 \times 0.93 = 0.77$$

for $\beta = 3.75$. Using the value of 0.67 currently given in CSA

TABLE 8. Coefficients C for channel welds with positive eccentricity

	k															
a	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.20	1.40	1.60	1.80	2.00
0.00	0.161	0.210	0.258	0.307	0.355	0.404	0.452	0.501	0.549	0.597	0.646	0.743	0.840	0.936	1.033	1.130
0.10	0.160	0.201	0.244	0.288	0.334	0.381	0.428	0.475	0.523	0.570	0.618	0.713	0.808	0.902	0.998	1.092
0.20	0.153	0.189	0.229	0.271	0.313	0.357	0.401	0.446	0.490	0.535	0.580	0.669	0.760	0.851	0.943	1.035
0.30	0.135	0.169	0.205	0.243	0.281	0.321	0.360	0.401	0.442	0.483	0.525	0.610	0.696	0.782	0.868	0.958
0.40	0.116	0.147	0.179	0.212	0.247	0.282	0.318	0.354	0.392	0.431	0.470	0.548	0.629	0.714	0.800	0.884
0.50	0.100	0.127	0.156	0.185	0.216	0.248	0.280	0.313	0.347	0.382	0.419	0.494	0.571	0.652	0.733	0.814
0.60	0.087	0.111	0.136	0.163	0.190	0.219	0.248	0.279	0.311	0.343	0.377	0.447	0.520	0.595	0.675	0.754
0.70	0.077	0.098	0.121	0.145	0.169	0.195	0.222	0.250	0.279	0.310	0.341	0.406	0.476	0.548	0.623	0.700
0.80	0.068	0.087	0.108	0.129	0.152	0.175	0.200	0.225	0.253	0.280	0.310	0.371	0.438	0.506	0.578	0.652
0.90	0.061	0.078	0.097	0.116	0.137	0.159	0.181	0.206	0.230	0.256	0.284	0.342	0.405	0.469	0.538	0.609
1.00	0.056	0.071	0.088	0.106	0.125	0.145	0.166	0.188	0.211	0.236	0.262	0.316	0.375	0.437	0.502	0.570
1.20	0.047	0.060	0.074	0.089	0.106	0.123	0.142	0.161	0.181	0.203	0.225	0.274	0.327	0.383	0.442	0.503
1.40	0.040	0.052	0.064	0.077	0.092	0.107	0.123	0.140	0.158	0.177	0.198	0.241	0.288	0.339	0.392	0.449
1.60	0.035	0.045	0.056	0.068	0.081	0.094	0.109	0.124	0.140	0.157	0.176	0.215	0.257	0.303	0.352	0.404
1.80	0.032	0.040	0.050	0.060	0.072	0.084	0.097	0.111	0.126	0.141	0.158	0.193	0.232	0.273	0.318	0.367
2.00	0.029	0.036	0.045	0.054	0.065	0.076	0.088	0.101	0.114	0.128	0.143	0.175	0.211	0.249	0.290	0.335
2.20	0.026	0.033	0.041	0.050	0.059	0.070	0.080	0.092	0.104	0.117	0.131	0.160	0.193	0.229	0.267	0.308
2.40	0.024	0.030	0.037	0.046	0.054	0.064	0.074	0.085	0.096	0.108	0.120	0.148	0.178	0.211	0.247	0.286
2.60	0.022	0.028	0.035	0.042	0.050	0.059	0.068	0.078	0.089	0.100	0.112	0.137	0.165	0.196	0.230	0.266
2.80	0.020	0.026	0.032	0.039	0.047	0.055	0.064	0.073	0.083	0.093	0.104	0.128	0.154	0.183	0.215	0.249
3.00	0.019	0.024	0.030	0.036	0.044	0.051	0.060	0.068	0.077	0.087	0.097	0.120	0.144	0.171	0.201	0.233
x	0.0	0.008	0.029	0.056	0.089	0.125	0.164	0.204	0.246	0.289	0.333	0.424	0.516	0.610	0.704	0.800

Standard S16.1 would be about 1% conservative, if it were desired to maintain β at a value of 4.5. With a β of 3.75 the current value is 13% conservative.

4.2. *Fillet weld groups loaded eccentrically*

The ultimate resistance of fillet weld groups can be predicted by using [6] in conjunction with the method of instantaneous centres of rotation. As discussed in Sect. 3.1, it is appropriate to use the tension-induced shear model on which [6] is based, provided that test-to-predicted ratios are determined on the same basis. Unlike the resistance of concentrically loaded single fillet welds, for which a simple equation can be written, the resistances of eccentrically loaded weld groups are determined iteratively. Design is facilitated by the use of tables of coefficients such as given in the CISC Handbook of Steel Construction (CISC 1985). The coefficients are presented as a function of the type of weld group, parameters describing its geometric configuration, and the eccentricity of the applied load.

The coefficients must incorporate an appropriate resistance factor determined, as for concentrically loaded fillet welds, from [12], [13], and [14]. In these equations, all the mean values of measured-to-nominal or predicted-to-nominal ratios and the coefficients of variation are the same as for single welds with the exception that the test-to-predicted ratio is now determined by comparisons with full-scale tests on different weld groups. Tables 5−7 give such data for tests on weld groups similar to those shown in Fig. 10 conducted by Butler *et al.* (1972), Swannell and Skewes (1979*b*), and Kulak and Timler (1984). The results for each series of tests were normalized by dividing by the actual mean value of the ultimate longitudinal shear resistance, P_0, for that series. Thus the variability due to welding procedures and materials can be considered separately. It was further considered that the ultimate resistance was obtained when the critical weld element had reached the fracture deformation. Tables 5−7 for the three sets of tests give mean test-to-predicted ratios of 0.977, 1.008, and 1.282 with coefficients of variation of 0.128, 0.083, and 0.081, respectively. For the 25 tests taken as a group, the mean test-to-predicted ratio is 1.025 with a coefficient of variation of 0.141. Equations [13] and [14] therefore give

$$\rho_R = 1.034 \times 1.123 \times 1.118 \times 1.025 = 1.331$$

$$V_R^2 = 0.026^2 + 0.077^2 + 0.121^2 + 0.141^2 = 0.203^2$$

These values result in

$$\phi = 1.331 \exp(-4.5 \times 0.55 \times 0.203) = 0.805$$

and

$$\phi = 1.331 \exp(-3.75 \times 0.55 \times 0.203) = 0.876$$

for $\beta = 4.5$ and 3.75 respectively. With the respective adjustment factors of 0.88 and 0.93, the value of resistance factors to be incorporated in the design tables are

$$\phi_w = 0.805 \times 0.88 = 0.71$$

and

$$\phi_w = 0.876 \times 0.93 = 0.81$$

The former value is 6% greater than ϕ_w given in CSA Standard S16.1 and the latter is 21% greater.

Using $\phi_w = 0.71$, tables of coefficients C, similar to those given in the CISC Handbook of Steel Construction (CISC 1985), have been developed for the eight weld configurations shown in Fig. 10 (see footnote 1). Excerpts from typical tables, those for channel welds with positive eccentricity and

TABLE 9. Coefficients C for horizontal box welds

	k										
a	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.00	0.484	0.517	0.549	0.581	0.614	0.646	0.678	0.710	0.743	0.775	0.807
0.10	0.377	0.406	0.443	0.483	0.525	0.565	0.605	0.643	0.680	0.716	0.750
0.20	0.308	0.335	0.371	0.409	0.448	0.489	0.532	0.575	0.617	0.659	0.699
0.30	0.256	0.283	0.314	0.349	0.387	0.426	0.466	0.508	0.551	0.595	0.638
0.40	0.217	0.240	0.269	0.302	0.337	0.374	0.412	0.452	0.492	0.534	0.577
0.50	0.186	0.208	0.234	0.264	0.296	0.331	0.367	0.405	0.442	0.482	0.523
0.60	0.163	0.182	0.206	0.234	0.264	0.295	0.329	0.364	0.401	0.438	0.476
0.70	0.144	0.162	0.184	0.209	0.236	0.266	0.297	0.330	0.364	0.400	0.436
0.80	0.129	0.146	0.166	0.189	0.215	0.242	0.270	0.302	0.333	0.366	0.401
0.90	0.117	0.132	0.151	0.172	0.195	0.221	0.248	0.276	0.307	0.338	0.371
1.00	0.107	0.120	0.138	0.158	0.180	0.203	0.228	0.255	0.283	0.313	0.344
1.20	0.091	0.103	0.117	0.135	0.154	0.175	0.197	0.220	0.246	0.272	0.299
1.40	0.079	0.089	0.103	0.118	0.135	0.153	0.173	0.194	0.216	0.240	0.265
1.60	0.069	0.079	0.091	0.104	0.120	0.136	0.154	0.173	0.193	0.214	0.236
1.80	0.062	0.070	0.081	0.094	0.107	0.123	0.139	0.156	0.174	0.193	0.214
2.00	0.056	0.064	0.074	0.085	0.098	0.111	0.126	0.142	0.158	0.176	0.194
2.20	0.051	0.058	0.067	0.078	0.089	0.102	0.115	0.130	0.145	0.161	0.179
2.40	0.047	0.054	0.062	0.071	0.082	0.094	0.106	0.120	0.134	0.149	0.165
2.60	0.043	0.050	0.057	0.066	0.076	0.087	0.099	0.111	0.124	0.139	0.154
2.80	0.041	0.046	0.053	0.062	0.071	0.081	0.092	0.104	0.116	0.129	0.143
3.00	0.038	0.043	0.050	0.058	0.066	0.076	0.086	0.097	0.109	0.121	0.134

for horizontal box welds, are given in Tables 8 and 9 respectively. The coefficients are given for various values of the parameter, a, defining the eccentricity and the geometric parameter, k.

The factored resistance (with $\phi_w = 0.71$) is then given as

[16] $P_R = CDL$

The values listed in the eight tables (see footnote 1) are on the average essentially the same (about 1% greater) as those currently given in the CISC handbook computed in a similar manner, although individual differences in these tables of as much as 11% less exist. Furthermore, the maximum values of the coefficient C in the handbook tables (those above the horizontal lines) have been limited to that corresponding to a fillet weld loaded longitudinally. This limitation is not considered justified, as the increased strength of transversely loaded fillets is obviously considered in the remainder of the tables. For these cases, the new tables developed give values of C as much as 1.5 times the handbook values, as would be expected from the ratio of transverse weld capacity to longitudinal weld capacity.

If the value of ϕ is taken as 0.81, consistent with $\beta = 3.75$, then the tabulated values would simply be increased by 1.14 times.

5. Summary and conclusions

1. Based on the test results for fillet welds loaded at different angles of Miazga and Kennedy (1989) and other statistical data available in the literature, it is proposed that the factored shear resistance of concentrically loaded fillet welds at an angle to the load of θ degrees be taken as

$$V_r = P_\theta = 0.67\phi_w A_w X_u(1.00 + 0.50 \sin^{1.5}\theta)$$

$$\phi_w = 0.68 \quad \text{for } \beta = 4.5$$

and

$$\phi_w = 0.81 \quad \text{for } \beta = 3.75$$

The empirical relationship in this equation between the angle of the load and the weld strength is in good agreement with the theoretical relationship developed by Miazga and Kennedy.

2. The deformation capacity of fillet welds decreases as the angle of loading increases. Expressions have been developed to predict the deformation at ultimate load and the maximum deformation at fracture.

3. The load–deformation relationship for fillet welds due to tension-induced shear is represented by one nondimensionalized polynomial expression that is in reasonable agreement with the test data of Miazga and Kennedy and includes an unloading portion of the curve.

4. Using the load–deformation relationship and the method of instantaneous centres of rotation, the ultimate strength of weld groups is predicted. Comparison with 25 full-scale tests of others gives an overall test-to-predicted ratio of 1.025 with a coefficient of variation of 0.141.

5. Design tables, similar to those in the current CISC handbook, have been developed for eight weld groups based on the load–deformation relationships and other statistical data. The tables give values of the coefficient C as a function of the eccentricity parameter, a, and the geometric parameter, k.

6. The design tables, calculated for a reliability index of 4.5, on the average, give coefficients (ultimate strengths) essentially the same as those in the handbook with a maximum difference of minus 11%.

7. Reducing the value of β to 3.75 would result in coefficients 1.14 times those for $\beta = 4.5$.

8. Values in the design tables are not limited artificially as in the current handbook to the ultimate strength of longitudinal welds. Therefore, in a few cases where the strength of trans-

verse welds predominates, the coefficients proposed exceed those in the handbook considerably.

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List of symbols

a	coefficient in ultimate strength equation of fillet welds; parameter defining eccentricity of a weld group
A_w	effective throat area of a fillet weld, mm^2
C	coefficient to determine the ultimate strength of an eccentrically loaded fillet weld group
d	leg size of fillet weld, mm
D	leg size of fillet weld group, mm
k	parameter defining geometry of weld group
l	length of fillet weld, mm
L	basic length of fillet weld group, mm
n	sample size
P	load on fillet welds at any angle of loading, $N \cdot mm^{-1} \cdot mm^{-1}$
P_R	factored resistance of a fillet weld group
P_θ	ultimate strength of a fillet weld loaded in shear at angle of loading, θ, $N \cdot mm^{-1} \cdot mm^{-1}$
P_0	ultimate strength of a longitudinal fillet weld loaded in shear ($\theta = 0°$), $N \cdot mm^{-1} \cdot mm^{-1}$
P_{90}	ultimate strength of a transverse fillet weld loaded in shear ($\theta = 90°$), $N \cdot mm^{-1} \cdot mm^{-1}$
V	coefficient of variation
V_r	factored shear resistance of a longitudinal fillet weld
V_R	coefficient of variation of resistance
V_G	coefficient of variation of effective weld throat area
V_{M1}	coefficient of variation of ultimate tensile strength of weld metal
V_{M2}	coefficient of variation of shear strength to ultimate tensile strength
V_P	coefficient of variation of test-to-predicted ratio
V_S	coefficient of variation of effect of loads
x	parameter defining the location of the centre of gravity of a weld group in the x direction
X_u	ultimate tensile strength of weld metal as given by the electrode classification number, MPa
y	parameter defining the location of the centre of gravity of a weld group in the y direction
α	angle of inclination of fracture surface of a fillet weld loaded in shear, degrees; load factor
α_R; α_S	coefficient of separation of resistance; of load effect
β	reliability index
Δ	deformation of a fillet weld (element) at any angle of loading, mm
Δ_f	deformation at fracture of a fillet weld (element) at any angle of loading, mm
Δ_u	deformation at ultimate load of a fillet weld (element) at any angle of loading, mm
θ	angle of loading for a fillet weld (element), i.e., the angle between the direction of the load and the axis of the weld, degrees
μ	mean value
ρ	nondimensional ratio, Δ/Δ_u
ρ_R	mean value of measured-to-nominal ratio of resistance
ρ_G	mean value of measured-to-nominal ratio of effective fillet weld throat area

ρ_{M1} mean value of measured-to-nominal ratio of ultimate tensile strength of weld metal

ρ_{M2} Mean value of measured-to-nominal ratio of shear strength to ultimate tensile strength

ρ_P mean value of test-to-predicted ratio

ρ_S mean value of measured-to-nominal ratio of effect of loads

σ standard deviation

σ_u ultimate tensile strength of weld metal, MPa

τ_u ultimate shear strength of a longitudinal fillet weld, MPa

ϕ resistance factor

ϕ_w resistance factor for weld metal