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Enhanced analysis and design of eccentrically loaded weld connections

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ABSTRACT

The American Institute of Steel Construction (AISC) provides conservative elastic and more realistic instantaneous center of rotation (IC) methods to analyze eccentrically loaded weld groups in plane. The current AISC design manual provides tables for coefficient *C*, which are used to obtain the design strength of weld group patterns, at only six inclined angles (θ = 0°, 15°, 30°, 45°, 60°, and 75°). For other angles, either a direct analysis must be performed or using the values for the next lower angle increment in the tables is recommended in the manual except for straight-line interpolation. This work develops an iterative algorithm to implement a direct analysis, and proposes a rational interpolation method between *C* values for loads at various angles not tabulated in manual tables without the tediousness of the IC method. The proposed method is easy to implement but reasonably accurate instead of using the *C* values for the next lower angle increment in the tables or direct analysis. This work eliminates the current limitations on AISC design manuals. It provides a quick but reliable improvement for the analysis and design of eccentrically loaded weld groups in plane.

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KEYWORDS
Instantaneous center of rotation; eccentric load; weld connection; ultimate analysis

1. Introduction

Steel structures generally have eccentric joints. Common examples of such joints are bracket-type connections in columns and girders. In analysis and design for a welded joint subjected to eccentric load in plane as shown in Figure 1(a), both the eccentric load and the induced torsion contribute to welds. The AISC design manuals (1986, 1989, 1993, 1999, 2005, and 2010) provide two practical methods for calculating the design strength of an eccentrically loaded weld group in plane. The first method is an elastic method and is regarded as a conservative one. Before limit state design rules were adopted, the design of eccentrically loaded weld connection was based on a simple elastic analysis where it was assumed that the weld element furthest from the center of gravity of the weld group controlled the capacity of the welded connection. But it was not appropriate for a limit state design approach since it represented only the first yield of the weld group rather than its ultimate capacity. The second is based on the concept of the instantaneous center of rotation (IC), and is an ultimate strength-based method that provides more realistic results. The effect of the combined load is equivalent to a rotation of the welds about a specific point, which is called the instantaneous center of rotation (IC). The exact position of IC is important in the analysis and design of eccentrically loaded joints. The location of an IC depends on the weld patterns, eccentric conditions and inclined angle of the load. Theoretically, the force equilibrium equations are satisfied at the correct IC.

In the manuals of the AISC, the design strength of an eccentrically loaded weld connection is evaluated using a tabulated non-dimensional coefficient *C*, which is proportional to the required strength of the weld group. Since the load-deformation relationship for the weld segments has developed with the time, the coefficients *C* obtained by the IC method are enhanced with the successive editions of the manuals. The AISC (1989) allowable stress design (ASD) manual contains the *C* coefficients of only vertical loads based on the load-deformation relationship for the weld segments proposed by Butler and Kulak (1971). For other inclined loads, Iwankiw (1987) proposed an equation to represent capacity of connection as a function of load angle. Tables 8–4 through 8–11 in the AISC load and resistance factor design (LRFD) manuals (2010 and previous editions) provide the *C* coefficients for only six inclination angles of the load (0°, 15°, 30°, 45°, 60°, and 75°), which were calculated using the IC method. The IC method was based on the load-deformation relationship for the weld segments proposed by Lesik and Kennedy (1990). For other angles not tabulated in these tables, either a direct analysis should be performed or it is recommended that the values of *C* for the next lower angle increment in the tables should be used. And straight-line interpolation between values of *C* for loads at different angles is not recommended. The direct analysis method has been developed with the goal of more accurately determining the load effects in the structure and is a major step forward in the design of steel moment frames from past editions of the AISC specification. For weld group patterns not treated in AISC design tables, a direct analysis is required if the instantaneous center of rotation (IC) method is to be used. However, the direct implementation of the IC method is difficult because it involves a tedious trial-and-error process. Additionally, no design table

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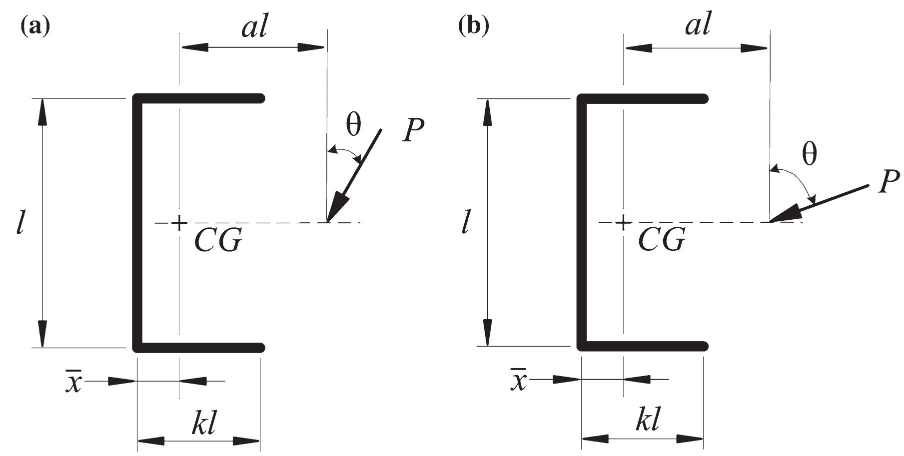

Figure 1. Loading cases for (a) (θ = 0°, 15°, 30°, 45°, 60°, and 75°) covered in AISC manuals, (b) (75° < θ ≤ 90°) not covered in AISC manuals.

is recommended in the manual for θ > 75° (Figure 1(b)), even though this angle exists in practice. A more effective analysis or design aid for an eccentrically loaded weld group without the above limitations in the manuals is therefore needed.

This work develops an iterative algorithm to implement a direct analysis, and proposes a rational interpolation method between *C* values for loads at various angles not tabulated in manual tables without a direct analysis. The optimum algorithm is also included to provide a means of automatically determining the step length for fast searching to obtain the IC point. The results thus obtained are compared with those using other available methods, including the elastic method, the IC method, the AISC manual-based straight-line interpolation method, and the method of Iwankiw (1987).

2. Proposed method for analyzing eccentric weld connections

Iwankiw (1987) presented a computationally simple but rather conservative method to evaluate the coefficients *C* for weld patterns under inclined loads. This work proposes an enhanced method that is based on the research of Iwankiw (1987) and yields sufficiently accurate results for loads at various angles between 0° and 90° without the complex iteration of the IC method.

According to the 2010 AISC specification, the nominal strength of the eccentrically loaded weld groups is given as $R_n = CC_1Dl$, where *C* is the tabulated non-dimensional coefficient and represents the effective resultant that resists the eccentric force, C_1 is the electrode strength coefficient, *D* denotes the number of sixteenths-of-an-inch in the fillet weld size, and *l* is characteristic length of weld group.

Coefficient *C* is proportional to the nominal strength of the eccentrically loaded weld groups. Therefore, coefficient C_γ represents the resistance at the six specified angles (0°, 15°, 30°, 45°, 60°, and 75°), as indicated in the AISC design manuals. Any inclined load is divided into vertical and horizontal components conventionally. Now, to be consistent with tabulated *C* coefficients in AISC design manuals, an applied inclined load (P_u) can be divided into two components (P'_γ and $P'_{\gamma+15}$), where P'_γ and $P'_{\gamma+15}$ are parts of the connection capacities of P_γ and $P_{\gamma+15}$. P_γ and $P_{\gamma+15}$ are proportional to C_γ and $C_{\gamma+15}$, which are listed in the manual in increments of 15°, as displayed in Figure 2. Since an algebraic addition of two components is greater than vector addition, these two components are added algebraically to provide a conservative estimate of connector strength. To put this proposed approach into mathematical form which is compatible with the AISC Manual *C* tables, part of the connection capacity C_γ resists P'_γ, and the remainder resists $P'_{\gamma+15}$, based on the algebraic addition. The magnitudes of P'_γ and $P'_{\gamma+15}$ are assumed to be as follows.

$$P'_\gamma = C'_\gamma C_1 Dl, \tag{1}$$

$$P'_{\gamma+15} = C'_{\gamma+15} C_1 Dl = \left(\frac{C_\gamma - C'_\gamma}{C_\gamma}\right) C_{\gamma+15} C_1 Dl, \tag{2}$$

where C_γ and $C_{\gamma+15}$ are the coefficients at the six specified angles (0°, 15°, 30°, 45°, 60°, and 75°), as tabulated in the 2010 AISC design manual, and C'_γ is a derived eccentricity coefficient to resist a part of P_γ.

From the simple trigonometric relationship between P'_γ and $P'_{\gamma+15}$, we have:

$$\frac{P'_\gamma}{\sin(\gamma + 15 - \theta)} = \frac{P'_{\gamma+15}}{\sin(\theta - \gamma)}. \tag{3}$$

Substitution of Equations (1) and (2) into Equation (3) gives

$$C'_\gamma C_1 Dl = \frac{\sin(\gamma + 15 - \theta)}{\sin(\theta - \gamma)} \times \left(\frac{C_\gamma - C'_\gamma}{C_\gamma}\right) C_{\gamma+15} C_1 Dl. \tag{4}$$

With further simplification of Equation (4), the equation then becomes:

$$C'_\gamma = \frac{C_{\gamma+15}\sin(\gamma + 15 - \theta)}{\sin(\theta - \gamma) + \frac{C_{\gamma+15}}{C_\gamma}\sin(\gamma + 15 - \theta)} = \frac{C_{\gamma+15}}{\frac{\sin(\theta-\gamma)}{\sin(\gamma+15-\theta)} + \frac{C_{\gamma+15}}{C_\gamma}}, \tag{5}$$

where C_γ is the AISC-tabulated *C* coefficient that corresponds to the angle γ (0°, 15°, 30°, 45°, 60°, or 75°), and γ < θ < γ + 15 in degrees.

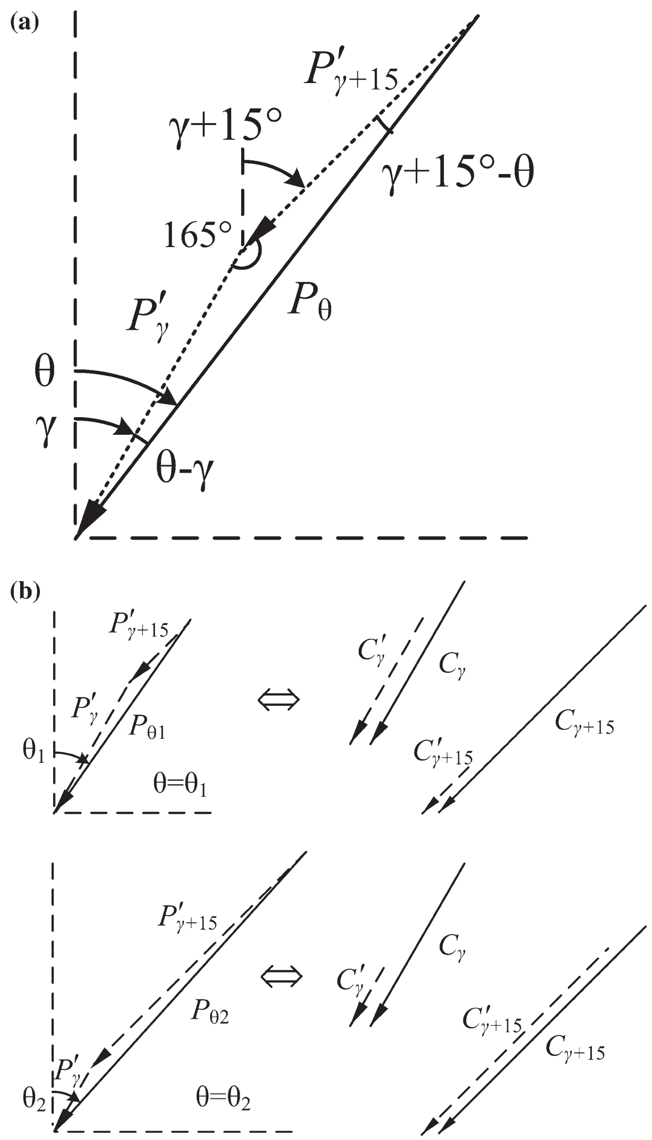

Figure 2. Illustrative relationships for, (a) P'_{γ}, $P'_{\gamma+15}$, P_{θ}, (b) θ_1 and θ_2.

Hence the derived eccentricity coefficient C'_{γ} can be given as:

$$C'_{\gamma} = \frac{C_{\gamma+15}}{\lambda + \frac{C_{\gamma+15}}{C_{\gamma}}} = \frac{C_{\gamma}C_{\gamma+15}}{C_{\gamma}\lambda + C_{\gamma+15}} \quad \text{if} \quad \lambda = \frac{\sin(\theta-\gamma)}{\sin(\gamma+15-\theta)}. \tag{6}$$

From the law of cosines, we have:

$$P_u^2 = P'^2_{\gamma} + P'^2_{\gamma+15} - 2P'_{\gamma}P'_{\gamma+15}\cos 165^{\circ} \tag{7}$$

Substituting $P'_{\gamma} = C'_{\gamma}C_1DI$ and $P'_{\gamma+15} = \frac{\sin(\theta-\gamma)}{\sin(\gamma+15-\theta)}P'_{\gamma} = \lambda C'_{\gamma}C_1DI$, the approximate coefficient at any inclination angle (θ) between 0° and 90° can be obtained as:

$$\begin{aligned} C_u &= \sqrt{\left(C'_{\gamma}\right)^2 + \left(C'_{\gamma}\lambda\right)^2 - 2C'_{\gamma}C'_{\gamma}\lambda\cos 165^{\circ}} \\ &= C'_{\gamma}\sqrt{1+\lambda^2-2\lambda\cos 165^{\circ}} \end{aligned} \tag{8}$$

The procedure for implementing the proposed method can be summarized as follows.

(1) Calculate C'_{γ}, which is based on Equation (5).
(2) Compute C_u, which is based on Equation (8) and is the approximate coefficient for any inclination angle (θ) between 0° and 90°.
(3) Obtain P_u, which is the maximum eccentric load for the weld group.

$$P_u = \phi(C_uC_1DI), \tag{9}$$

where ϕ is the resistance factor (= 0.75 based on the 2010 AISC specification).

To evaluate the accuracy of the proposed method, the exact results obtained by the IC method are developed as follows.

3. Procedure for implementing the IC method

The method of instantaneous center of rotation (IC method) was earlier developed by Crawford and Kulak (1971) for bolted connections and adapted by Butler, Pal, and Kulak (1972) for welded joints with in-plane eccentricity. The load vs. deformation relationship (Lesik and Kennedy 1990) for the weld segment can be represented by:

$$R_{ni} = 0.60\,F_{exx}(1.0 + 0.50\sin^{1.5}\theta_i)\left[p_i(1.9-0.9p_i)\right]^{0.3}, \tag{10}$$

where R_{ni} is the nominal shear stress in the ith weld element at a deformation Δ_i (ksi) (1 ksi = 6.89 MPa), F_{exx} denotes the electrode strength (ksi), θ_i is the load angle measured relative to the weld longitudinal axis (degrees), Δ_i is the deformation of the ith weld element at an intermediate stress level (in.) (1 inch = 2.54 cm), Δ_{mi} is the deformation of the ith weld element at maximum stress (in.), and $p_i = \Delta_i/\Delta_{mi}$.

3.1. Iterative algorithm

The AISC design manuals (2010 and previous editions) used the IC method to analyze eccentric weld connections by specifically evaluating the C coefficients. However, the location of IC is an essential issue for the implementation of the IC method. Brandt (1982a, 1982b) presented a technique for locating the IC. The present work develops a different algorithm for locating the IC, as follows.

Three force equilibrium equations ($\sum F_x = 0$, $\sum F_y = 0$, and $\sum M = 0$) are required to locate the instantaneous center of rotation in the IC method. The analytic solution to the problem is difficult to obtain exactly. To solve the problem, an iterative algorithm that is based on the concept of gradient descent method with a line search (Nocedal and Stephen 1999; Su and Siu 2007), is developed.

Referring to Figure 3(a), based on $\Sigma M = 0$, the trial nominal strength (P_n) of the eccentrically loaded weld group is given by $P_n = \Sigma M_n/r_o = \Sigma(R_{ni}d_i)/r_o$, where ΣM_n is the resultant moment of all weld segments, d_i is the radial distance from IC to the center of weld segment, and r_o is the distance between the IC and the line of P_n. Then P_n can be resolved into horizontal component, P_{nx}, and vertical component, P_{ny}.

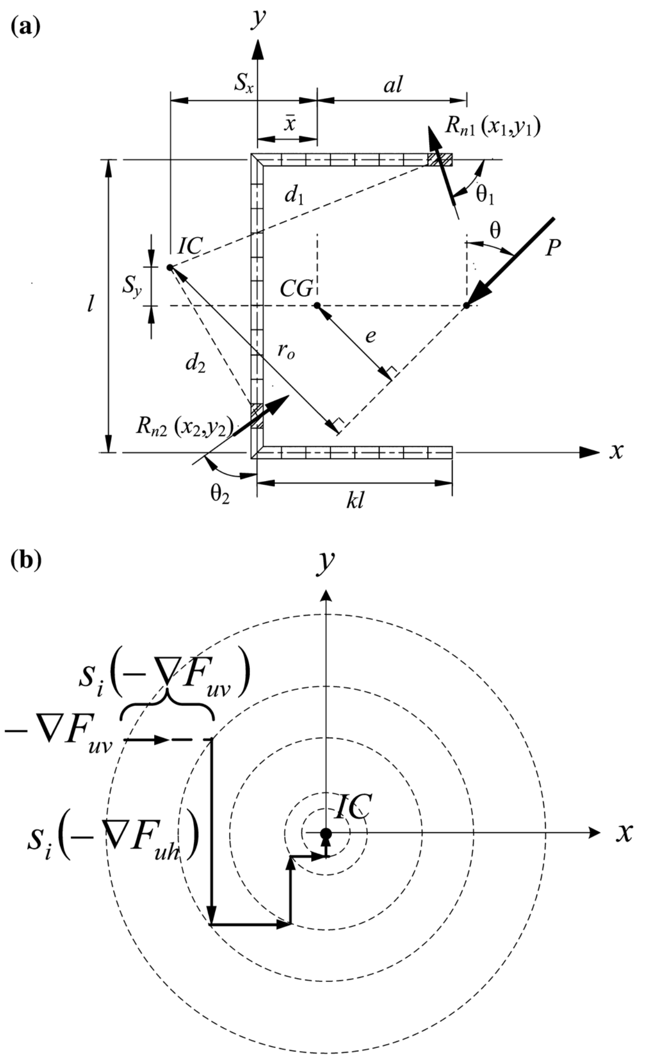

Figure 3. Schematic plots for, (a) IC method, (b) iterative algorithm to locate the IC.

At static equilibrium ($\sum F_x = 0,\ \sum F_y = 0$), the resultants are expressed as:

$$\sum F_x = F_{uh} = P_{nx} + \sum R_{nix}, \text{ and} \tag{11}$$

$$\sum F_y = F_{uv} = P_{ny} + \sum R_{niy}, \tag{12}$$

where F_{uh} and F_{uv} are called the unbalanced forces if $\sum F_x \neq 0$ or $\sum F_y \neq 0$.

Let F be the magnitude of the unbalanced force at a trial IC point (x_o,y_o).

$$\nabla F(x_o, y_o) = \frac{\partial F}{\partial x}\hat{\mathbf{i}} + \frac{\partial F}{\partial y}\hat{\mathbf{j}} = \Sigma F_x\,\hat{\mathbf{i}} + \Sigma F_y\,\hat{\mathbf{j}} = F_{uh}\hat{\mathbf{i}} + F_{uv}\hat{\mathbf{j}} \tag{13}$$

where $F = \sqrt{F_{uh}^2 + F_{uv}^2}$.

The magnitude of unbalanced force $F(x_o,y_o)$ increases rapidly in the direction of positive gradient (∇F), and falls rapidly in the direction of negative gradient ($-\nabla F$). The negative gradient ($-\nabla F$) specifies the direction of descent of the resultant $F(x_o,y_o)$. In the iterative algorithm, this negative gradient is applied to reduce the *x*- and *y*-components of unbalanced forces, F_{uh} and F_{uv}, in each direction of descent. Gradient is perpendicular to the force vector. Accordingly, the direction of descent opposes the normal to the force vector, so F_{uv} declines in the positive *x* direction and F_{uh} falls in the negative *y* direction. Then, step length, s_iF_{uv} or s_iF_{uh}, is adjusted as a shift along each direction of descent, with reference to Figure 3(b). The positive s_i is the step-length parameter, which can be set for each iterative process. Therefore, the iterated coordinates are given by:

$$x_{i+1} = x_i + s_i(-\nabla F_{uv}) = x_i + s_iF_{uv}, \text{ and} \tag{14}$$

$$y_{i+1} = y_i + s_i(-\nabla F_{uh}) = y_i - s_iF_{uh}. \tag{15}$$

The algorithm requires that the initially guessed IC position is the centroid of the section; the iterative process generates the next point by moving one step length in the direction of negative gradient from the preceding IC point. The computational procedure is described in detail herein using an illustrative example.

The iterative process stops when the unbalanced forces F_{uv} and F_{uh} have been obtained within the desired accuracy, which is approaching zero. The required design force (P_u) and available weld strength (ϕCC_1Dl) are then set, yielding the coefficient $C = P_u/(\phi C_1Dl)$. The detailed computational procedure is in the following:

(1) Determine the sectional properties and geometry of the weld group and choose the center of gravity (CG) of the weld group as the first trial location of IC.
(2) Refer to Figure 4(a), the perpendicular distance (e) from the CG to l can be given in normal form (Sisam and Atchison 1955) as:

$$e = (x_p - x_{cg})\cos\beta + (y_p - y_{cg})\sin\beta, \tag{16}$$

and the perpendicular distance (r_o) from the IC to l can be calculated as:

$$r_o = (x_{ic} - x_{cg})\cos\beta + (y_{ic} - y_{cg})\sin\beta - e, \tag{17}$$

where β is the angle between the horizontal line and the normal to the load direction (= 180°–θ)

(3) Calculate the resistance (R_{ni}), which is based on Equation (10), for each weld element based on the load-deformation relationship given in the 2010 AISC manual.
(4) Calculate the resultant moment (ΣM_n) and force components (ΣR_{nix} and ΣR_{niy}) for the weld group:

$$\sum R_{nix} = \sum (R_{ni}y_i)/d_i,\quad \sum R_{niy} = \sum (R_{ni}x_i)/d_i, \text{ and} \tag{18}$$

$$\sum M_n = \sum (R_{ni}d_i), \tag{19}$$

where (x_i, y_i) are the coordinates of each weld element.

(5) Calculate the corresponding applied load (P_n) and the load components (P_{nx} and P_{ny}) by considering the static equilibrium.

$$P_n = \sum M_n/|r_o| = \sum (R_{ni}d_i)/|r_o|, \tag{20}$$

$$P_{nx} = P_n\cos\alpha, \text{ and } P_{ny} = P_n\sin\alpha, \tag{21}$$

where α is the inclined angle of P_n with respect to the horizontal line and r_o is the load eccentricity.

(6) Check the force equilibrium

$$\sum F_x = F_{uh} = P_{nx} + \sum R_{nix} \text{ and} \tag{22}$$

$$\sum F_y = F_{uv} = P_{ny} + \sum R_{niy}. \tag{23}$$

(7) If an equilibrium condition is violated (i.e. $F_{uh} \neq 0$ or $F_{uv} \neq 0$), the next IC coordinates can be adjusted to:

$$x_{i+1} = x_i + s_i(-\nabla F_{uv}) = x_i + s_i F_{uv} \text{ and} \tag{24}$$

$$y_{i+1} = y_i + s_i(-\nabla F_{uh}) = y_i - s_i F_{uh}, \tag{25}$$

where s_i is the step-length parameter, which can be chosen for each iterative process.

(8) Repeat Steps 2–7 until the convergence criterion of 0.1 percent for unbalanced forces F_{uh} and F_{uv} is reached.

3.2. Optimum algorithm of step length selection

The iterative algorithm presented here is developed based on the concept of line search, which is a one-dimensional optimization. In determination of the step-length parameter, the ideal choice would be the global minimizer of the objective function, F_{uh} or F_{uv}. To find even a local minimizer of objective function to moderate precision generally requires too many evaluations of the objective function (F_{uh} or F_{uv}) and its first derivative. To implement the algorithm in practice simply, the step-length parameter (s_i) could be automatically determined by golden-section search (Kiefer 1953; Kahaner, Moler, and Nash 1989), which is a reliable algorithm for one-dimensional optimization and uses only function values to reduce many evaluations of the first derivative of objective function. The optimum algorithm presented in the manuscript takes no advantage whatever of the possible smoothness of F_{uh} or F_{uv}, but it is guaranteed to work in the worst possible cases. Referring to Figure 5(a), the golden-section search can be used to determine the minimum of a function $f(s)$, where $f(s)$ stands for function of unbalanced force, $F_{uv}(s)$ or $F_{uh}(s)$.

Initialization: Determine s_l and s_u which are assumed to be lower and upper limits of step-length parameters and known to contain the minimum of the function $f(s)$.

Step 1 Determine two intermediate points s_1 and s_2 such that:

$$s_1 = s_l + 0.382(s_u - s_l), \tag{26}$$

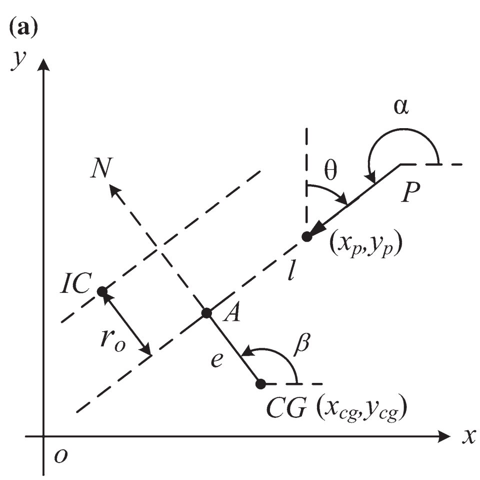

Figure 4. General analysis of weld groups, (a) perpendicular distance (r_o) from IC to the line of inclined load, (b) C-shaped weld for the illustrative example.

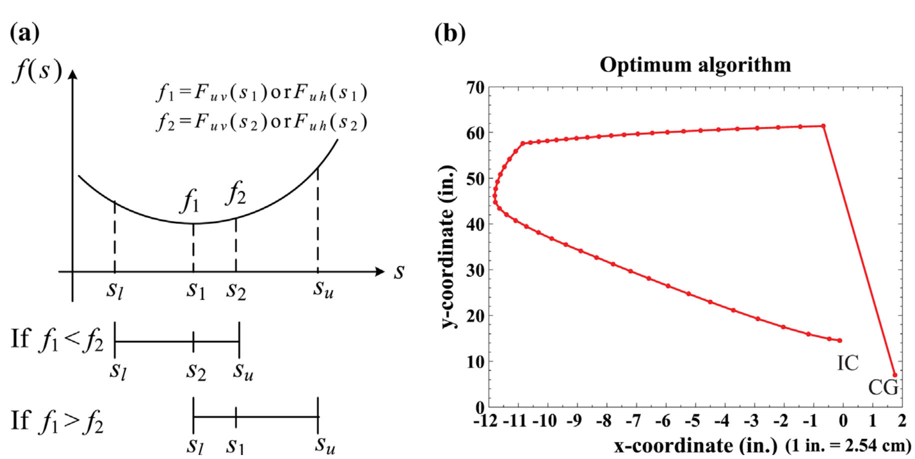

Figure 5. Optimum algorithm, (a) golden-section search, (b) traceable plots for IC searching by optimum algorithm.

$$s_2 = s_l + 0.618(s_u - s_l), \quad (27)$$

where the number 0.618 is called the golden ratio and the value of 0.382 = 1−0.618.

Step 2 Evaluate $f(s_1)$ and $f(s_2)$.

If $f(s_1) < f(s_2)$, then determine new s_u, s_1, and s_2 as shown in Equations (28)–(31). Note that the only new calculation is done to determine the new s_1.

$$s_u = s_2, \quad (28)$$

$$s_2 = s_1, \quad (29)$$

$$\text{and} \quad f(s_2) = f(s_1). \quad (30)$$

$$\text{The new} \quad s_1 = s_l + 0.382(s_u - s_1). \quad (31)$$

If $f(s_1) > f(s_2)$, then determine new s_l, s_1, and s_2 as shown in Equations (32)–(35). Note that the only new calculation is done to determine the new s_2.

$$s_l = s_1, \quad (32)$$

$$s_1 = s_2, \quad (33)$$

$$\text{and} \quad f(s_1) = f(s_2). \quad (34)$$

$$\text{The new} \quad s_2 = s_l + 0.618(s_u - s_l). \quad (35)$$

Step 3 If $s_u - s_l < \varepsilon$ (a sufficiently small number), then the minimum occurs at $(s_u + s_l)/2$ and stop iterating, else go to Step 2.

A design example is presented below to illustrate the available methods that are described above, whose results are compared with the proposed method. A computer program was developed to perform the above iterative process. The outputs (coefficients C) of this program were confirmed against those in the tables in the 2010 AISC design manual.

4. Illustrative example

Figure 4(b) displays a welded connection under an applied load (P_u) (l = 14 in., k = 0.5, a = 0.5). Use 1/4-in. fillet weld with E70 electrodes. The load acts at an angle of 70° with respect to vertical. The objective is to determine the maximum load (P_u) by using the various available methods and the proposed method. (Conversion of units: 1 in. = 2.54 cm, 1 kip = 4.45 kN, 1 k-in. = 0.113 kN-m, and 1 ksi = 6.89 MPa).

(a) Elastic method

With reference to Figure 4(b), the center of gravity (CG) of the given welds is found to be located at:

$x_{cg} = (\sum x_i A_w)/\sum A_w = 1.75$ in. and $y_{cg} = (\sum y_i A_w)/\sum A_w = 7.0$ in., where A_w is the cross-sectional area of welds. By calculation, $\Sigma I_p = 1057.583$ in.^{4} with respect to the CG of the weld group.

The shear per linear inch of weld due to the concentric force, r_{pu}, is determined as:

$$r_{pu} = P_u/\left[(1+2k)l\right] = P_u/28 = -0.0357P_u(\leftarrow)$$

The components of the direct shear are:

$$r_{pux} = r_{pu}\sin\theta = P_u \sin\theta/28 = P_u \sin 70^\circ/28 = -0.03356P_u(\leftarrow)$$

$$r_{puy} = r_{pu}\cos\theta = P_u\cos\theta/28 = P_u\cos 70^\circ/28 = -0.012215P_u(\downarrow)$$

The torque at the CG, $M_{CG} = P_u \cos\theta \times e_x = (-0.34202P_u \times 7) = -2.3941P_u$ (clockwise).

The shear per linear inch of weld due to the moment, $P_u e$, is r_{mu}:

$$r_{mux} = \frac{M_{CG} d_y}{\sum I_p} = \frac{-2.3941\,P_u \times 7}{1057.583} = -0.0158P_u\ (\leftarrow)$$

$$r_{muy} = \frac{M_{CG} d_x}{\sum I_p} = \frac{-2.3941P_u \times 5.25}{1057.583} = -0.01189P_u\ (\downarrow)$$

Thus, the required strength per linear inch of weld, r_u, is:

$$\begin{aligned} r_u &= \sqrt{\left(r_{pux}+r_{mux}\right)^2 + \left(r_{puy}+r_{muy}\right)^2} \\ &= \sqrt{\left(-0.03356\,P_u - 0.0158\,P_u\right)^2 + \left(-0.0122\,P_u - 0.01189\,P_u\right)^2} \\ &= 0.0549\,P_u \end{aligned}$$

The design strength of E70 electrode with 4/16-in. fillet weld per linear in. is:

$$\begin{aligned} \phi(0.6F_{exx})(0.707)(1/16)D &= 0.75(0.6)(70)(0.707)(1/16)4) \\ &= 5.5676 \text{ k/in.} \end{aligned}$$

According to the LRFD design principle, required strength ≤ design strength:

$$r_u \le 5.5676 \Rightarrow 0.0549\,P_u = 5.5676 \Rightarrow P_u = 101.41 \text{ kips}$$

As per the 2010 AISC LRFD method, C_1 = 1.0 for E70 electrode and D = 4 sixteenths,

$$P_u \le \phi R_n = \phi C C_1 D l = \phi C_e C_1 D l$$

$$\Rightarrow C_e = P_u/\phi C_1 D l = 101.41/(0.75 \times 1.0 \times 4 \times 14) = 2.41$$

With E70 electrodes, the obtained maximum P_u value is $(P_u)_{max} = 101.41$ kips, $C_e = 2.41$.

(b) IC method

As stated above, the CG of the weld group is located at x_{cg} = 1.75 in. and y_{cg} = 7.0 in. With reference to Figures 3(a) and 4(a), the angle between the horizontal line and the normal to the load direction is:

$$\beta = 180^\circ - \theta = 180^\circ - 70^\circ = 110^\circ.$$

The perpendicular distance from the CG to the line along which the load is applied is:

$$\begin{aligned} e &= (x_p - x_{cg})\cos\beta + (y_p - y_{cg})\sin\beta \\ &= 7 \times \cos 110^\circ + 0 \times \sin 110^\circ \\ &= -2.3941 \text{ in.} \end{aligned}$$

where (x_p, y_p) = (8.75 in., 7.0 in.) and (x_{cg}, y_{cg}) = (1.75 in., 7.0 in.). The initial guess of IC at CG is (x_o, y_o) = (1.75 in., 7.0 in.). The perpendicular distance from the initially guessed IC to the line along which the load is applied is:

$$r_o = (x_o - x_{cg})\cos\beta + (y_o - y_{cg})\sin\beta - e$$
$$= (1.75 - 1.75)\times\cos 110^\circ + (7.0 - 7.0)\times\sin 110^\circ + 2.3941$$
$$= 2.3941$$

Utilizing the AISC load-deformation relationship for a weld element, namely:

$$R_{ni} = 0.60F_{exx}(1.0 + 0.50\sin^{1.5}\theta_i)[p_i(1.9 - 0.9p_i)]^{0.3}, p_i = \Delta_i/\Delta_{mi}$$

From the output of the developed program,

$\sum(R_{ni}\times d_i) = 1391.012$ k-in., $\sum R_{nix} = 0$, and $\sum R_{niy} = -36.221$ kips

Therefore,

$$P_n = \sum M_n/|r_o| = \sum(R_{ni}\times d_i)/2.3941 = 1391.012/2.3941 = 581.0167 \text{ kips},$$

The equilibrium of the unbalanced forces in the horizontal and vertical directions is checked as follows. The angle between the line along which the load is applied and the horizontal axis is:

$$\alpha = 270^\circ - \theta = 270^\circ - 70^\circ = 200^\circ.$$

$$F_{uh} = \sum F_x = P_{nx} + \sum R_{nix} = 581.0167\times\cos 200^\circ + 0$$
$$= -545.977 \text{ kips } (\leftarrow) \neq 0, NG$$

$$F_{uv} = \sum F_y = P_{ny} + \sum R_{niy} = 581.0167\times\sin 200^\circ - 36.221$$
$$= -234.94 \text{ kips } (\downarrow) \neq 0, NG$$

The step-length parameter (s_i) can be obtained using the golden-section search in each iteration. Table 1 shows the iterative process of optimum step-length parameter using the golden-section search in the direction of x-axis and y-axis, respectively. In the first iteration, the optimum step-length parameters are:

$$(s_i)_{opt} = (0.0102 + 0.0104)/2 = 0.0103 = 1.03\% \text{ in } x\text{-direction.}$$

$$(s_i)_{opt} = (0.0995 + 0.0997)/2 = 0.0996 = 9.96\% \text{ in } y\text{-direction.}$$

The iteration algorithm generates the next trial location of IC as:

$$x_{i+1} = x_i + s_iF_{uv} \Rightarrow x_1 = 1.75 + 0.0103(-234.94) = -0.6699 \text{ in.}$$

$$y_{i+1} = y_i - s_iF_{uh} \Rightarrow y_1 = 7.0 - 0.0996(-545.977) = 61.3793 \text{ in.}$$

The step length and descent direction in this iteration are given by −2.4199 in. (= −0.6699–1.75) in the negative x direction and 54.3793 in. (= 61.3793–7.0) in the positive y direction, respectively. The iteration algorithm generates the next location (x_1, y_1) = (−0.6699 in., 61.3793 in.) from the current point (x_o, y_o) = (1.75 in., 7.0 in.). Repeating the above steps, as described above, yields a sequence of IC locations. When $s_u - s_l < \varepsilon$ (such as 0.001 in this example), then the optimum occurs at $(s_u + s_l)/2$ and stop iterating. In this example, the correct IC coordinates, (x_{ic}, y_{ic}) = (−0.1223 in., 14.5227 in.) are obtained (Refer to Figure 5(b) and Table 2) after 78 iterations. The distance from the IC to the CG, S = 7.7522 in. (with S_x = −1.8723 in. leftward and S_y = 7.5227 in. upward). Table 3 presents in detail the calculations that are associated with, and the results that are obtained using, the iterative algorithm that is based on the IC method. Δ_i meets Δ_{ui} at segment 21 with minimum ratio Δ_{ui} / d_i. The critical segment at rupture is the one where the ratio of its Δ_{ui} to radial distance d_i is the smallest.

$$P_n = 2152.924/\left[(-0.1223 - 1.75)\cos 110^\circ + (14.5227 - 7.0)\sin 110^\circ + 2.3941\right]$$
$$= 213.0867 \text{ kips}$$

The equilibrium of the horizontal forces is confirmed as follows:

$$P_{nx} = P_n\cos\alpha = 213.0867\times\cos 200^\circ = -200.236 \text{ kips } (\leftarrow).$$

$$\sum F_x = P_{nx} + \sum R_{nix} = -200.236 + 200.237 = 0 \text{ kips, OK.}$$

The equilibrium of the vertical forces is confirmed as follows:

$$P_{ny} = P_n\sin\alpha = 213.0867\times\sin 200^\circ = -72.88 \text{ kips } (\downarrow).$$

$$\sum F_y = P_{ny} + \sum R_{niy} = -72.88 + 72.88 = 0 \text{ kips, OK.}$$

Table 1. Optimum algorithm of step-length parameters (Iteration i = 1).

Loop	$(s_i)_1$	$(s_i)_2$	y_1	y_2	$(F_{uh})_1$	$(F_{uh})_2$
1	0.0656	0.0788	31.2301	49.9967	14.6906	12.6170
2	0.0788	0.0869	31.2301	54.4265	12.6170	11.5955
3	0.0869	0.0919	31.2301	57.1656	11.5955	11.0405
4	0.0919	0.0950	31.2301	58.8578	11.0405	10.7226
5	0.0950	0.0969	31.2301	59.9041	10.7226	10.5350
6	0.0969	0.0981	31.2301	60.5505	10.5350	10.4221
7	0.0981	0.0988	31.2301	60.9502	10.4221	10.3536
8	0.0988	0.0993	31.2301	61.1971	10.3536	10.3117
9	0.0993	0.0995	31.2301	61.3498	10.3117	10.2859
10	0.0995	0.0997	31.2301	61.4441	10.2859	10.2701
$(s_i)_{opt}$ = (0.0995+0.0997)/2 = 0.0996 = 9.96% in y-direction						
Loop	$(s_i)_1$	$(s_i)_2$	x_1	x_2	$(F_{uv})_1$	$(F_{uv})_2$
1	0.0171	0.0215	−2.2633	−4.9551	−12.5175	8.4027
2	0.0144	0.0171	−1.6279	−3.2917	−30.8169	−12.5175
3	0.0127	0.0144	−1.2350	−2.2633	−45.0281	−30.8169
4	0.0117	0.0127	−0.9922	−1.6279	−55.2182	−45.0281
5	0.0110	0.0117	−0.8422	−1.2350	−62.1632	−55.2182
6	0.0106	0.0110	−0.7494	−0.9922	−66.7365	−62.1632
7	0.0104	0.0106	−0.6921	−0.8422	−69.6847	−66.7365
8	0.0102	0.0104	−0.6567	−0.7494	−71.5560	−69.6847
$(s_i)_{opt}$ = (0.0102 + 0.0104)/2 = 0.0103 in x-direction Parameters refer to Figure 5(a)						

Table 2. Variation of optimum step-length parameters (s_i) in each iteration.

	Optimum s_i			Optimum s_i			Optimum s_i	
Iteration (i)	x-dir.	y-dir.	Iteration (i)	x-dir.	y-dir.	Iteration (i)	x-dir.	y-dir.
1	0.0103	0.0996	…	…	…	53	0.0397	0.0104
2	0.0103	0.0104	29	0.0103	0.0996	54	0.0103	0.0104
3	0.0103	0.0104	30	0.0397	0.0996	…	…	…
…	…	…	31	0.0397	0.0996	76	0.0105	0.0110
22	0.0103	0.0104	…	…	…	77	0.0114	0.0110
23	0.0103	0.0996	52	0.0103	0.0996	78	0.0108	0.0104

Table 3. Results for the illustrative example (θ = 70°) as obtained using IC method.

Segment	x (in.)	y (in.)	θ_i (deg.)	Δ_{ui} (in.)	Δ_{ui}/d_i (in./in.)	Δ_i (in.)	Δ_{mi} (in.)	R_{ni} (kip)	R_{nix} (kip)	R_{niy} (kip)	$R_{ni}d_i$ (k-in.)
1	6.62	−0.52	85.4872	0.0144	0.0022	0.0067	0.0125	10.224	0.804	10.192	67.916
2	5.62	−0.52	84.6888	0.0145	0.0026	0.0057	0.0125	9.870	0.914	9.828	55.733
3	4.62	−0.52	83.5485	0.0146	0.0032	0.0047	0.0126	9.425	1.059	9.366	43.844
4	3.62	−0.52	81.7892	0.0148	0.0041	0.0037	0.0127	8.856	1.265	8.765	32.412
5	2.62	−0.52	78.7275	0.0152	0.0057	0.0027	0.0128	8.100	1.583	7.943	21.657
6	1.62	−0.52	72.1419	0.0160	0.0094	0.0017	0.0132	7.003	2.148	6.666	11.936
7	0.62	−0.52	49.9725	0.0199	0.0244	0.0008	0.0148	4.996	3.213	3.826	4.060
8	0.12	−1.02	83.1807	0.0147	0.0143	0.0010	0.0126	6.273	6.229	0.745	6.461
9	0.12	−2.02	86.5400	0.0143	0.0071	0.0020	0.0125	7.640	7.626	0.461	15.481
10	0.12	−3.02	87.6830	0.0142	0.0047	0.0030	0.0124	8.521	8.514	0.344	25.777
11	0.12	−4.02	88.2586	0.0142	0.0035	0.0040	0.0124	9.167	9.162	0.279	36.892
12	0.12	−5.02	88.6052	0.0141	0.0028	0.0050	0.0124	9.666	9.663	0.235	48.562
13	0.12	−6.02	88.8367	0.0141	0.0023	0.0060	0.0123	10.061	10.059	0.204	60.605
14	0.12	−7.02	89.0024	0.0141	0.0020	0.0070	0.0123	10.376	10.374	0.181	72.877
15	0.12	−8.02	89.1269	0.0141	0.0018	0.0080	0.0123	10.626	10.624	0.162	85.256
16	0.12	−9.02	89.2234	0.0141	0.0016	0.0090	0.0123	10.820	10.819	0.147	97.636
17	0.12	−10.02	89.3012	0.0141	0.0014	0.0100	0.0123	10.966	10.965	0.134	109.913
18	0.12	−11.02	89.3645	0.0141	0.0013	0.0110	0.0123	11.067	11.066	0.123	121.990
19	0.12	−12.02	89.4172	0.0140	0.0012	0.0120	0.0123	11.126	11.125	0.113	133.766
20	0.12	−13.02	89.4619	0.0140	0.0011	0.0130	0.0123	11.144	11.144	0.105	145.134
21	0.12	−14.02	89.5003	0.0140	0.0010	0.0140	0.0123	11.123	11.123	0.097	155.981
22	0.62	−14.52	2.4536	0.0425	0.0029	0.0146	0.0324	6.617	6.611	0.283	96.188
23	1.62	−14.52	6.3740	0.0425	0.0029	0.0146	0.0265	7.004	6.961	0.778	102.352
24	2.62	−14.52	10.2354	0.0425	0.0029	0.0148	0.0234	7.308	7.192	1.299	107.853
25	3.62	−14.52	14.0052	0.0388	0.0026	0.0150	0.0215	7.586	7.361	1.836	113.550
26	4.62	−14.52	17.6553	0.0348	0.0023	0.0153	0.0202	7.851	7.482	2.381	119.660
27	5.62	−14.52	21.1634	0.0318	0.0020	0.0156	0.0191	8.105	7.559	2.926	126.222
28	6.62	−14.52	24.5129	0.0295	0.0019	0.0156	0.0183	8.346	7.594	3.463	133.209
Sum									200.237	72.880	2152.924

Notes: The twenty-first segment is critical, $(\Delta_{ui}/d_i)_{min} = 0.001$, $d_{crit} = 14.02321$. Conversion of units: 1 in. = 2.54 cm, 1 kip = 4.45 kN, 1 k-in. = 0.113 kN-m, and 1 ksi = 6.89 MPa.

$$\Rightarrow P_u = \phi P_n = 0.75 \times 213.0867 = 159.815 \text{ kips}$$

$$\Rightarrow C = P_u/\phi C_1 Dl = 159.815/(0.75 \times 1.0 \times 4 \times 14) = 3.8051$$

(c) Straight-line interpolation

In this case, linear interpolation yields $C = 3.81$ for $\theta = 70°$ where $C = 3.5$ for $\theta = 60°$ and $C = 3.97$ for $\theta = 75°$ based on Table 8–8 in the AISC design Manual (AISC Manual 2010).

The available strength (P_u) is then calculated as:

$$P_u = C \times \phi C_1 Dl = 3.81 \times (0.75 \times 1.0 \times 4 \times 14) = 160.02 \text{ kips}$$

As expected, this result exceeds 0.22% relative to that obtained by the IC method.

(d) Method of Iwankiw

Iwankiw (1987) presented a simple method to evaluate the coefficients C as a function of load angle in his paper. Since the load versus deformation relationship for the weld segments has developed with the time, the coefficients C obtained by the IC method are enhanced with the successive editions of the manuals. Because the coefficients C in the original paper are outdated, the ones in the 2010 AISC manual are used in this example.

The available strength for concentrically loaded channel weld groups is

$$\phi R_n = 1.392Dl = 1.392 \times 4 \times 14 \times (0.825 \times 0.5 \times 2 + 1.5 \times 1) = 181.238 \text{ kips}$$

By $R_n = CC_1Dl \Rightarrow C = R_n/C_1Dl = (181.238/0.75)/(1.0 \times 4 \times 14) = 4.31$

$\Rightarrow$ Thus $C_{max} = 4.31$

Use $C_{max} = 4.31$ and $C_o = 2.85$ (2010 AISC tabulated C value for $\theta = 0°$), then:

$$A = \frac{C_{max}}{C_o} = \frac{4.31}{2.85} = 1.512 \geq 1.0$$

The approximate eccentricity coefficient for the inclined load (C_a) is determined by:

$$\frac{C_a}{C_o} = \frac{A}{(\sin\theta + A\cos\theta)} = \frac{1.512}{(\sin 70^{\circ} + 1.512\cos 70^{\circ})} = 1.0379 \geq 1.0$$

$$C_a = 1.0379 \times C_o = 1.0379 \times 2.85 = 2.958$$

The available strength (P_u) is then calculated as:

$$P_u = C_a \times \phi C_1 Dl = 2.958 \times (0.75 \times 1.0 \times 4 \times 14) = 124.24 \text{ kips}$$

(e) Proposed method

For $\gamma = 60°$, $C_\gamma = C_{60} = 3.5$, $C_{\gamma+15} = C_{75} = 3.97$ as obtained from the AISC Table (when $k = 0.5$, $a = 0.5$). So, we have:

$$\lambda = \frac{\sin(\theta - 60^{\circ})}{\sin(75^{\circ} - \theta)} = \frac{\sin(70^{\circ} - 60^{\circ})}{\sin(75^{\circ} - 70^{\circ})} = 1.9924$$

$$\Rightarrow C'_\gamma = \frac{C_{60}C_{75}}{C_{60}\lambda + C_{75}} = \frac{3.5 \times 3.97}{3.5 \times 1.9924 + 3.97} = 1.2697$$

$$C_u = C'_\gamma \sqrt{1 + \lambda^2 - 2\lambda\cos 165^{\circ}} = 3.771$$

Therefore, $P_u = C_u \times \phi C_1 Dl = 3.771 \times (0.75 \times 1.0 \times 4 \times 14) = 158.39$ kips

5. Discussions

The illustrative example performs the direct analysis and some findings can be discussed as follows:

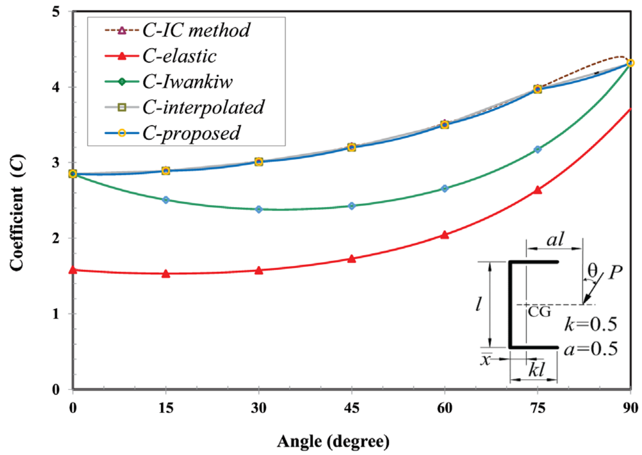

Figure 6. Computed coefficients C from the various methods.

(1) In optimization algorithm, the positive s_i is the optimum step-length parameter, which can be determined automatically. Figure 5(b) shows the traceable plots for IC searching using the optimum algorithm. Variation of optimum step-length parameter in each iteration is given in Table 2.
(2) For C-shaped weld groups, Table 4 summarizes the results obtained using the various methods for θ = 70°. Coefficients C for other θ values can be computed similarly as shown in Table 5 and Figure 6. For the six specified values of θ (0°, 15°, 30°, 45°, 60°, and 75°), the C coefficients calculated using the proposed algorithm are very close to those tabulated in the 2010 AISC manual, verifying the high accuracy of the proposed iterative algorithm. As indicated in Table 5, for θ = 0–75°, the underestimations of the elastic method,

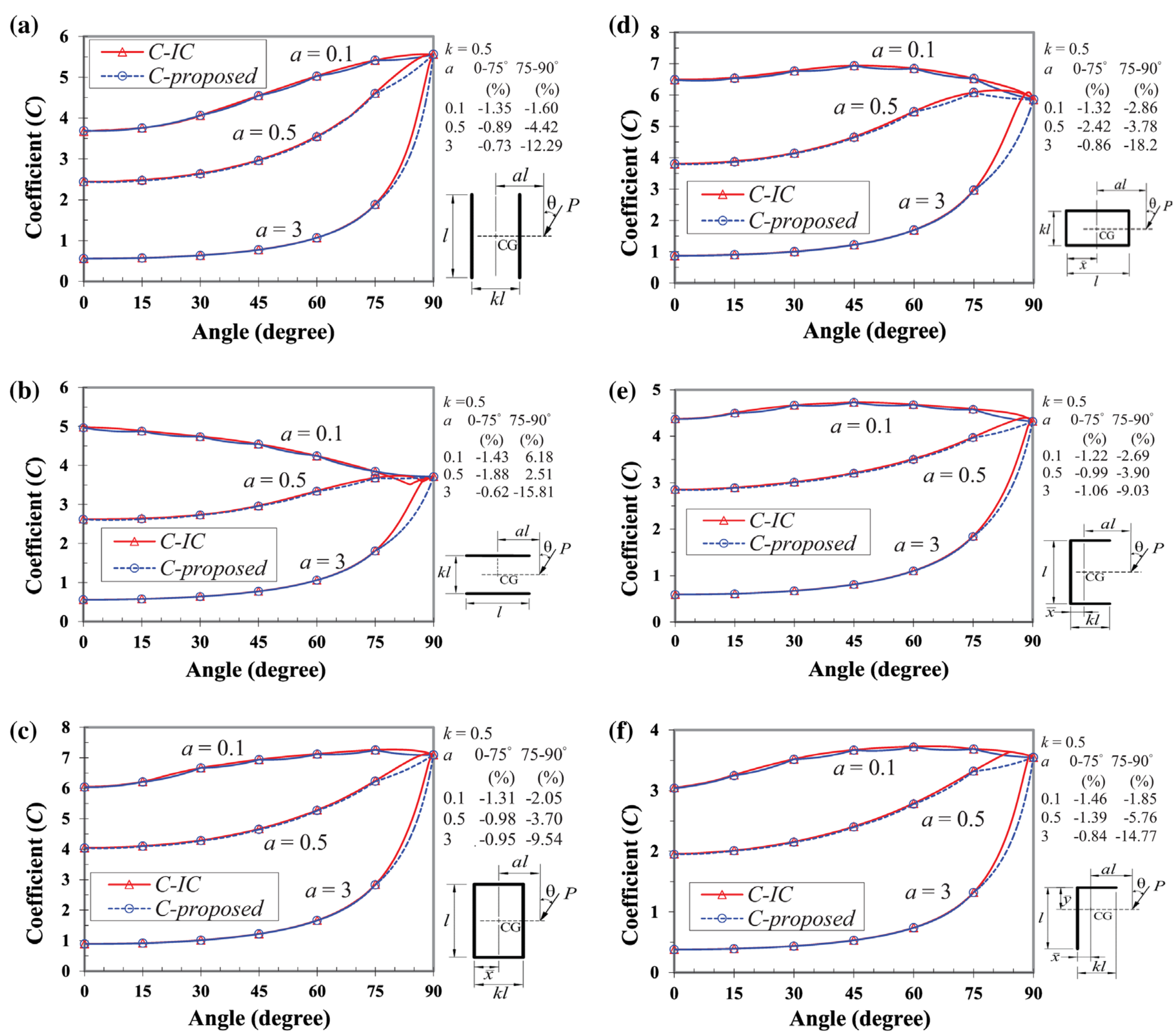

Figure 7. Comparison of C values and maximum error percentages for, (a) two vertical lines, (b) two parallel lines, (c) box-shaped, (d) horizontal box-shaped, (e) C-shaped, (f) inverted L-shaped weld patterns.

Table 4. Comparison of C coefficients as given in the illustrative example (θ = 70°).

	Elastic method (A)	IC method (B)	Linear interpolation (C)	Method of Iwankiw (D)	Proposed method (E)
Coefficient C	2.41	3.805	3.81	2.958	3.771
P_u (kips)	101.41	159.815	160.02	124.24	158.39
Difference (%)	−36.98 (A–B)/B	0.00 (B–B)/B	0.22 (C–B)/B	−22.25 (D–B)/B	−0.91 (E–B)/B

Table 5. Comparison of C coefficients by various methods.

Angle θ (degree)	$C_{elastic}$ [A]	$C_{IC\ method}$ [B]	$C_{interpolated}$ [C]	$C_{Iwankiw}$ [D]	$C_{proposed}$ [E]	(%) $\frac{[A]-[B]}{[B]}$	(%) $\frac{[C]-[B]}{[B]}$	(%) $\frac{[D]-[B]}{[B]}$	(%) $\frac{[E]-[B]}{[B]}$
0	1.582	2.854	2.850	2.850	2.850	−44.57	−0.15	−0.15	−0.15
5	1.554	2.857	2.863	2.704	2.841	−45.61	0.21	−5.35	−0.55
10	1.537	2.872	2.877	2.592	2.855	−46.49	0.15	−9.77	−0.62
15	1.531	2.896	2.890	2.506	2.890	−47.14	−0.21	−13.46	−0.21
20	1.535	2.929	2.930	2.445	2.907	−47.59	0.02	−16.55	−0.77
25	1.550	2.971	2.970	2.404	2.946	−47.83	−0.04	−19.10	−0.84
30	1.576	3.016	3.010	2.382	3.010	−47.76	−0.20	−21.03	−0.20
35	1.613	3.072	3.073	2.378	3.048	−47.48	0.05	−22.59	−0.79
40	1.664	3.136	3.137	2.393	3.110	−46.93	0.02	−23.70	−0.83
45	1.730	3.212	3.200	2.426	3.200	−46.14	−0.37	−24.46	−0.37
50	1.813	3.299	3.300	2.480	3.269	−45.06	0.02	−24.84	−0.91
55	1.916	3.400	3.400	2.555	3.368	−43.63	0.01	−24.83	−0.93
60	2.044	3.516	3.500	2.657	3.500	−41.86	−0.45	−24.43	−0.45
65	2.203	3.648	3.657	2.789	3.616	−39.63	0.23	−23.56	−0.87
70	2.410	3.805	3.810	2.958	3.771	−36.98	0.22	−22.25	−0.91
75	2.639	3.982	3.970	3.175	3.970	−33.72	−0.30	−20.26	−0.30
76	2.694	4.019	3.993	3.226	3.983	−32.96	−0.63	−19.73	−0.88
77	2.751	4.055	4.017	3.279	3.998	−32.16	−0.96	−19.15	−1.42
78	2.810	4.092	4.040	3.334	4.013	−31.32	−1.28	−18.51	−1.92
79	2.872	4.129	4.063	3.393	4.031	−30.44	−1.59	−17.81	−2.37
80	2.936	4.165	4.086	3.455	4.049	−29.50	−1.88	−17.04	−2.78
81	3.003	4.201	4.110	3.520	4.069	−28.51	−2.16	−16.19	−3.13
82	3.072	4.236	4.133	3.589	4.090	−27.47	−2.42	−15.25	−3.42
83	3.144	4.270	4.156	3.662	4.113	−26.36	−2.66	−14.22	−3.66
84	3.218	4.302	4.179	3.739	4.138	−25.19	−2.85	−13.08	−3.82
85	3.295	4.333	4.203	3.821	4.164	−23.95	−3.00	−11.81	−3.90
86	3.374	4.361	4.226	3.907	4.191	−22.62	−3.09	−10.39	−3.88
87	3.456	4.384	4.249	3.999	4.221	−21.16	−3.07	−8.77	−3.72
88	3.540	4.397	4.272	4.096	4.252	−19.51	−2.84	−6.85	−3.31
89	3.625	4.391	4.296	4.200	4.284	−17.44	−2.17	−4.35	−2.42
90	3.712	4.319	4.319	4.310	4.319	−14.05	0.00	−0.21	0.00

the approximation of Iwankiw, and the proposed approach are 34.4–47.8, 0–24.9, and 0–0.99%, respectively. For θ = 75–90°, not covered in any AISC manual, the degrees of conservatism are 14–33.7, 0.21–22.26, and 0.88–3.24%, respectively. Apparently, the proposed algorithm yields fairly accurate results.

(3) Besides C-shaped weld groups, the exact C curves obtained using IC method are various for other weld patterns listed in the 2010 AISC design manual. In general, the exact C curves may be convex at smaller eccentricity ratio (a) or concave at larger one for each weld pattern referred to in Figure 7. When the exact C curve is convex, the straight-line interpolation seems better than the proposed approach, but it may be unconservative for the concave curve of exact C values. However, the proposed approach is in excellent agreement with exact C values when θ ≤75° regardless of weld patterns. The proposed approach, whose results are compared with those obtained by IC method, are slightly conservative under 2% in each weld group. Rather than using the C values for the next lower angle increment in the tables or unconservative straight-line interpolation, it can be concluded that the proposed approach is precise and conservative for six weld patterns when θ ≤75° without tedious iteration.

(4) For filled weld groups concentrically loaded at the largest angle (θ = 90°), the nominal strengths which are obtained by multiplying the appropriate values of coefficients C from Table 8–1 in the 2010 AISC Manual are all the same regardless of eccentricity ratio of a, but not always the maximum value in each weld group. It may be smaller than the one with θ = 75° referred to Figure 7. The exact C curves when 75° < θ < 90°, which are not listed in any AISC manuals, are not predictable. The more detailed results and maximum error percentages for each weld patterns (k = 0.5 is used) are given as shown in Figure 7. It can be seen that the proposed approach yields conservative results when 75° < θ < 90°. However, there is an exception to the horizontal parallel lines with a small eccentricity ratio (a = 0.1) as shown in Figure 7(b).

(5) The recommendation by the AISC design manual, which uses the tabulated C for the next lower angle increment for design, is not appropriate when an inclined load occurs at 75° < θ < 90°. This is because it is possible that the C value at θ = 90° is smaller than the one at θ = 75° as shown in Figures 7(b), (c), (d), (e), and (f). However, a direct analysis method is the only option to be used when 75° < θ < 90°. The proposed approach yields reasonable but conservative results between C values

when $75° < \theta < 90°$ rather than using the tabulated C of $\theta = 75°$ as recommended by the manuals.

6. Conclusions

This work presents a procedure to implement a direct analysis, and a simple but reliable model to estimate the C values for loads at various angles is proposed. The results in this work support the following conclusions.

(1) The proposed iterative algorithm provides a general procedure for implementing the tedious IC method to find the exact IC location. The iterative procedure yields identical eccentricity coefficients C for the specified load angles ($\theta = 0°$, 15°, 30°, 45°, 60°, and 75°), which are tabulated in the AISC design manuals, further demonstrating the accuracy of the iterative algorithm. The iterative algorithm is guaranteed to work in the worst possible case. The proposed iterative algorithm also provides a general and rational analysis for any weld group patterns not treated in AISC design tables. Alternatively, the step-length parameter (s_i) and step size (s_iF_{uv} or s_iF_{uv}) in each iterative procedure can be automatically determined by optimization algorithm.

(2) This work provides a nonlinear interpolation between C values for loads at those angles which are not tabulated in the tables of AISC design manuals. The proposed interpolation yields fairly accurate results for inclined angles ($0° \leq \theta \leq 75°$) without a tedious iteration procedure. It is precise and conservative to employ the proposed model instead of the tedious IC method or unconservative straight-line interpolation for six weld patterns at $0° \leq \theta \leq 75°$. The accuracy of the proposed model has been proven to be excellent.

(3) The proposed approach is derived based on simple trigonometric relationship among applied load and two components of a force. Thus, the approach is rational to evaluate the strength of eccentrically loaded welds in plane without any restriction or tedious iteration process.

(4) The recommendation by the AISC design manual, which uses the C tabulated for the next lower angle increment for design, is not appropriate in the range of $75° < \theta < 90°$. The proposed approach yields reasonable but conservative results between C values when $75° < \theta < 90°$ rather than using the tabulated C of $\theta = 75°$ as recommended by the manuals. This research work overcomes the limitations (only for $0° \leq \theta \leq 75°$) that are evident in current AISC design manuals.

Nomenclature

- A constant which serves as the primary property by Iwankiw's method
- A_w area of weld segments, in^2
- C coefficient, as tabulated in the 2010 AISC manual
- C_a coefficient that is determined by Iwankiw's method
- C_e coefficient that is determined by the elastic method
- C_{max} coefficient which represent for total capacity of welds
- C_o tabulated C value that corresponds to vertical loading
- C_u coefficient that is determined by the proposed method
- C_γ AISC-tabulated C coefficient that corresponds to the angle γ
- C'_γ derived eccentricity coefficient that represents for a part of C_γ
- C_1 electrode strength coefficient (1.0 for E70XX electrodes)
- D weld size in sixteenths of an inch
- d_i radial distance from IC to the center of the ith weld segment, inches
- d_{cr} distance from IC to weld element with minimum Δ_{ui}/d_i ratio, inches
- e perpendicular distance from the CG to the line along which the load is applied, inches
- F magnitude of the unbalanced force, kips
- F_{exx} filler metal classification strength, ksi
- F_{uh} horizontal (x) component of the unbalanced force, kips
- F_{uv} vertical (y) component of the unbalanced force, kips
- I_p polar moment of inertia of the weld group, in^4
- l characteristic length of weld group, inches
- M_n torque due to the eccentric load, kips
- P_n nominal strength of the weld group, kips
- P_γ capacity of connection due to the angle γ, kips
- P'_γ parts of the connection capacities of P_γ that corresponds to the angle γ, kips
- R_n nominal strength of the weld group, ksi
- R_{ni} nominal shear stress in the ith weld element at a deformation Δ_i, ksi
- s_i step-length parameter in each i iteration
- s_l lower limit of step-length parameter in each i iteration
- s_u upper limit of step-length parameter in each i iteration
- s_1 intermediate point of step-length parameter in each i iteration
- s_2 intermediate point of step-length parameter in each i iteration
- s_{opt} optimum step-length parameter in each i iteration
- Δ_i $= d_i\Delta_{ucr} / d_{cr}$, deformation of the ith weld element at an intermediate stress level, inches
- Δ_{mi} $= 0.209\,(\theta_i + 2)^{-0.32}\,w$, deformation of the ith weld element at maximum stress, inches
- Δ_{ucr} deformation of the weld element with minimum Δ_{ui}/d_i ratio at ultimate stress, inches
- Δ_{ui} $= 1.087\,(\theta_i + 6)^{-0.65}\,w \leq 0.17w$, deformation of the ith weld element at ultimate stress, inches
- w weld leg size, inches
- θ_i angle of loading measured from the weld logitudinal axis, degrees
- α angle of the line along which the load is applied, degrees
- β angle of the normal of the line along which the load is applied, degrees
- γ angle, specified in the 2010 AISC manual (0°, 15°, 30°, 45°, 60°, or 75°), degrees

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