Exam Seat no. 20102 6

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–I (Feb 2022)

Engineering Mathematics– I (CC101 – N)

08/02/2022

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are compulsory.
4) Indicate clearly, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for rough work.

Section- I

Q.1 **(a)** Evaluate (i) $\lim\limits_{x\to 0} \frac{\log(\sin x)}{\log(\tan x)}$ (ii) $\lim\limits_{x\to 0} \frac{x^2+2\cos x-2}{x\sin x}$ [5]

(b) Expand $\log_e x$ in power of $(x-1)$ upto fourth power. [5]

(c) Trace the curve $r=a(1+\cos\theta)$. [5]

OR

(c) If $u=\frac{x^2+y^2}{x+y}$ show that $\left[\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right]^2=4\left[1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right]$. [5]

Q.2 **(a)** Evaluate $\int_0^1\int_{x^2}^{x} xy(x+y)\,dy\,dx$. [5]

(b) Evaluate (i) $\int_0^{\frac{\pi}{2}} \cos^6 x\,\sin^5 x\,dx$ (ii) $\int_0^9 x^{3/2}(9-x)^{1/2}dx$ [5]

OR

(a) Use spherical coordinates to evaluate $\iiint_V \frac{dxdydz}{(x^2+y^2+z^2)^{3/2}}$ where V is the Region bounded by the spheres $x^2+y^2+z^2=16$ and $x^2+y^2+z^2=25$. [5]

(b) Check the convergence of the following series: [5]

(i) $\sum_{n=0}^{\infty}\frac{n}{(2n+1)(2n-1)}$ (ii) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$

Q.3 **(a)** If $u=f(x-y, y-z, z-x)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. [5]

(b) Find the maximum and minimum values of $x^2+y^2+xy+x-4y+5$. [5]

OR

(a) Prove that $\log(1+\tan x)=x-\frac{x^2}{2}+\frac{2x^3}{3}+\cdots$ [5]

(b) Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$. [5]

Section -II

Q. 4 (a) If $u=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that [5]

$$x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}+2xy\frac{\partial^2 u}{\partial x\partial y}=2\cos(3u)\sin(u).$$

(b) If $x= r\sin\theta\cos\phi$, $y= r\sin\theta\sin\phi$ and $z = r\cos\theta$, evaluate $J=\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. [5]

(c) Evaluate $\iiint_D e^{x+y+z}dV$ where D: $0\le x\le a$, $0\le y\le x$, $0\le z\le x+y$. [5]

OR

(c) Trace the curve $x^3+y^3=3axy$. [5]

Q. 5 (a) Find the volume of the solid generated by rotating the region bounded by $y=\sqrt{x}$ and the line $y=2$, $x=0$ about the line $y=2$. [5]

(b) Evaluate $\iint xy\,dA$, where R is the region bounded by x-axis, $x=2a$ and the curve $x^2=4ay$. [5]

OR

(a) Find the area included between the parabola $y^2=4ax$ and its latus rectum. [5]

(b) Test the convergence of the series $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\cdots\infty$. [5]

Q. 6 (a) Check the convergence of the following sequences $\{a_n\}$ where [5]
(i) $a_n=\frac{2n-7}{3n+2}\quad\forall n\in N$ (ii) $a_n=1+(-1)^n\quad\forall n\in N$

(b) Using the transformation $x+y=u$, $2x-3y=v$, evaluate $\iint_R \sqrt{x+y}\,dxdy$ where R is the region bounded by the lines $x+y=0$, $x+y=1$, $2x-3y=0$ and $2x-3y=4$. [5]

OR

(a) Define Absolute convergence and Conditional convergence and Test the convergence of the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots$.. [5]

(b) If $u=e^x\cos y$, $v=e^x\sin y$ then prove that [5]

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=(u^2+v^2)\left(\frac{\partial^2 f}{\partial u^2}+\frac{\partial^2 f}{\partial v^2}\right)$$

Exam Seat No : ....

KADI SARVA VISHWAVIDYALAYA
B.E 1st SEMESTER EXAMINATION (DECEMBER 2023)
SUBJECT: Engineering Mathematics-I (New)(Code: CC101-N)

Date: 28/12/2023 | Time: 3 hour | Marks: 70

Instruction:
1. Answer each section in separate Answer Sheet
2. Use of scientific Calculator is permitted.
3. All questions are compulsory.
4. Indicate clearly, the option you attempted along with its respective question number.
5. Use the last page of main supplementary for rough work.

Section:1

Q.1 (a) Use L'Hospital's rule to evaluate $\lim_{x \to 0} \frac{2x - x\cos(x) - \sin(x)}{2x^3}$. **[05]**

(b) Evaluate $\int_0^1 \int_x^{x^2} (xy)\, dx\, dy$. **[05]**

(c) Find the Jacobian $J = \frac{\partial(u, v)}{\partial(x, y)}$ for $u = x\sin(y)$ and $v = y\sin(x)$. **[05]**

OR

(c) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$. **[05]**

Q.2 (a) If $u = x^2y^3$, where $x = \log(t)$, $y = e^t$ then find $\frac{du}{dt}$. **[05]**

(b) Find the equation of tangent plane and normal line at point $p(1, 2, 4)$ on the surface $x^2 + y^2 + z = 9$. **[05]**

OR

Q.2 (a) Find the area between the parabola $y^2 = 4ax$ and line $x^2 = 4ay$. **[05]**

(b) Trace the curve $y^2(a - x) = x^3$, $a > 0$. **[05]**

Q.3 (a) If $u = \tan^{-1}\left(\frac{x^4+y^4}{x^2+y^2}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin(2u)$. **[05]**

(b) If $u = f(x - y, y - z, z - x)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **[05]**

OR

Q.3 (a) Determine the convergence or divergence of $\sum_{n=1}^{\infty} \frac{2^n}{n^3+1}$ by D'Alembert's ratio test. **[05]**

(b) Using reduction formulae evaluate integral $\int_0^{\infty} \frac{1}{(1+x^2)^n}\, dx$. **[05]**

Section:2

Q.4 (a) Expand $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x-1)$ by using Taylor's series. [05]

(b) If $u = e^{xyz}$ then show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$. [05]

(c) Evaluate $\iiint\limits_E 2x \, dz dy dx$, where E is the region under the plane $2x + 3y + z = 6$ that lies in the first octant. [05]

OR

(c) Evaluate $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ [05]

Q.5 (a) If $u = \log(x^2 + y^2)$ then verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. [05]

(b) Evaluate $\iint\limits_R xy \, dx \, dy$, where R is the region bounded by $x - axis$, line $x = 2a$ and the curve $x^2 = 4ay$. [05]

OR

Q.5 (a) Examine the convergence of the series $\sum\limits_{n=1}^{\infty} \left[\frac{1}{(1+\frac{1}{n})^{n^2}}\right]$ by using Cauchy's root test. [05]

(b) Trace the curve $r = 2a\cos\theta$. [05]

Q.6 (a) Evaluate $\int\limits_0^2 \int\limits_{\frac{y}{2}}^1 e^{x^2} \, dx \, dy$ by changing order of integration. [05]

(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$ [05]

OR

Q.6 (a) Evaluate $\int\limits_0^2 \int\limits_2^4 \int\limits_0^3 x^2yz^2 \, dydzdx$. [05]

(b) Discuss the maxima and minima of $x^2 + y^2 + 6x + 12$. [05]

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KADI SARVA VISHWAVIDHYALAYA,

Exam Seat no. ....

BE Semester–I (June 2023)

Date: 16/6/23

Engineering mathematics-I (New) (CC101-N)

Time: 3 hour

Marks: 70

Instruction:
1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are compulsory.
4) Indicate clearly, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for rough work.
6) Use of table for area under the standard normal curve is permitted.

Section I

Q.1 (i) Evaluate $\lim_{x\to 0}\frac{x-\sin x}{x^3}$ [5]

(ii) Expand $f(x) = \log x$ in powers of (x-2) by Taylor series method. [5]

(iii) Find the $J = \frac{\partial(x,y)}{\partial(r,\theta)}$ for $x = r\cos\theta$ and $y = r\sin\theta$. [5]

OR

(iii) If $u = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$ [5]

Q.2 (i) Find the equations of tangent plane and the normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point (2,1,-3). [5]

(ii) Test the convergence of the series $\sum_{n=0}^{\infty}\frac{2^n-1}{3^n}$. [5]

OR

(i) Trace the curve $r = a(1+\cos\theta)$. [5]

(ii) Evaluate $\int_0^{\pi}(1+\cos\theta)^4\,d\theta$. [5]

Q.3 (i) If $u = f(x-y, y-z, z-x)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. [5]

(ii) Evaluate $\int_{-1}^{1}\int_0^2(1-6x^2y)\,dx\,dy$. [5]

OR

(i) Expand $e^x\cos y$ in powers of x and y up to second degree. [5]

(ii) Evaluate $\int_0^2\int_1^3\int_1^2 xy^2z\;dz\,dy\,dx$. [5]

Section II

Q.4 (i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial y}{\partial x}$ at $(1,-1,2)$ if $x^2 + y^2 + z^2 = a^2$.

(ii) Evaluate $\lim_{x \to \frac{\pi}{4}} (1 - \tan x) \sec 2x$

(iii) Trace the curve $y^2(2a - x) = x^3$.

OR

(iii) Prove that $\int_0^1 \int_1^2 (xy)\, dy dx = \frac{3}{4}$.

Q.5 (i) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in powers of $(x-3)$.

(ii) Show that $\sum_{n=1}^{\infty} \left(\frac{\log n}{1000}\right)^n$ diverges.

OR

(i) Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$

(ii) If $u = \log(x^2 + y^2)$ then verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

Q.6 (i) Find the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

(ii) Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\, dy\, dx$ by changing the order of the integration.

OR

(i) Trace the curve $r^2 = a^2 \cos 2\theta$.

(ii) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

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Exam Seat no. ....

KADI SARVA VISHWAVIDHYALAYA,

Gandhinagar

BE Semester–I (Feb 2022)

Engineering Mathematics- I (CC101 – N)

08/02/2022

Max Marks: 70
Duration: 3 hr.

Instruction: 1) Answer each section in separate Answer sheet.
2) Use of Scientific calculator is permitted.
3) All questions are **compulsory**.
4) Indicate **clearly**, the options you attempt along with its respective question number.
5) Use the last page of main supplementary for **rough work**.

Section- I

Q.1 (a) Evaluate (i) $\lim\limits_{x\to 0} \frac{\log(\sin x)}{\log(\tan x)}$ (ii) $\lim\limits_{x\to 0} \frac{x^2+2\cos x-2}{x\sin x}$ [5]

(b) Expand $\log_e x$ in power of $(x-1)$ upto fourth power. [5]

(c) Trace the curve $r=a(1+\cos\theta)$. [5]

OR

(c) If $u=\frac{x^2+y^2}{x+y}$ show that $\left[\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right]^2=4\left[1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right]$. [5]

Q.2 (a) Evaluate $\int_0^1\int_{x^2}^{x} xy(x+y)\,dy\,dx$. [5]

(b) Evaluate (i) $\int_0^{\frac{\pi}{2}} \cos^6 x\, \sin^5 x\, dx$ (ii) $\int_0^9 x^{3/2}(9-x)^{1/2}dx$ [5]

OR

(a) Use spherical coordinates to evaluate $\iiint_V \frac{dxdydz}{(x^2+y^2+z^2)^{3/2}}$ where V is the Region bounded by the spheres $x^2+y^2+z^2=16$ and $x^2+y^2+z^2=25$. [5]

(b) Check the convergence of the following series: [5]

(i) $\sum_{n=0}^{\infty}\frac{n}{(2n+1)(2n-1)}$ (ii) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$

Q.3 (a) If $u=f(x-y, y-z, z-x)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. [5]

(b) Find the maximum and minimum values of $x^2+y^2+xy+x-4y+5$. [5]

OR

(a) Prove that $\log(1+\tan x)=x-\frac{x^2}{2}+\frac{2x^3}{3}+\cdots$ [5]

(b) Evaluate $\lim_{(x,y)\to(0,0)} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$. [5]

Section -II

Q. 4 (a) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that [5]

$$x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}+2xy\frac{\partial^2 u}{\partial x\partial y}=2\cos(3u)\sin(u).$$

(b) If $x= r\sin\theta\cos\phi$, $y= r\sin\theta\sin\phi$ and $z = r\cos\theta$, evaluate $J=\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. [5]

(c) Evaluate $\iiint_D e^{x+y+z}dV$ where D: $0\le x\le a$, $0\le y\le x$, $0\le z\le x+y$. [5]

OR

(c) Trace the curve $x^3+y^3=3axy$. [5]

Q. 5 (a) Find the volume of the solid generated by rotating the region bounded by $y=\sqrt{x}$ and the line y=2, x=0 about the line y=2. [5]

(b) Evaluate $\iint xy\,dA$, where R is the region bounded by x-axis, x=2a and the curve $x^2=4ay$. [5]

OR

(a) Find the area included between the parabola $y^2=4ax$ and its latus rectum. [5]

(b) Test the convergence of the series $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\cdots\infty$. [5]

Q. 6 (a) Check the convergence of the following sequences $\{a_n\}$ where [5]
(i) $a_n=\frac{2n-7}{3n+2}\ \forall n\in N$ (ii) $a_n=1+(-1)^n\quad \forall n\in N$

(b) Using the transformation x+y=u, 2x-3y=v, evaluate $\iint_R \sqrt{x+y}\,dxdy$ where R is the region bounded by the lines x+y =0, x+y =1, 2x-3y=0 and 2x-3y=4. [5]

OR

(a) Define Absolute convergence and Conditional convergence and Test the convergence of the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots$. [5]

(b) If $u= e^x\cos y$, $v= e^x\sin y$ then prove that [5]

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=(u^2+v^2)\left(\frac{\partial^2 f}{\partial u^2}+\frac{\partial^2 f}{\partial v^2}\right)$$

Enrol. No.: [illegible]

KADI SARVA VISHWAVIDYALAYA

LDRP INSTITUTE OF TECHNOLOGY & RESEARCH, GANDHINAGAR

B.E. MID-SEMESTER EXAMINATION DECEMBER 2022

Date : 03/12/2022 **Branch : All**

Subject Name & Code : Engineering Mathematics-I(CC-101N) **Semester : I**

Time : 9:30 am to 11:00 am **Total Marks: 30**

Instruction:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Indicate **clearly** the options you attempt along with its respective question number.
4. Use the last page of main supplementary for rough work.

Q.1 (a) Evaluate $\lim_{x \to 0} \frac{\log(1+x^2)}{[illegible]}$. [05]

(b) Evaluate $\int_0^1 \int_1^3 (x^2y^2)\, dy\, dx$. [05]

Q.2 (a) Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^2y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ [05]

(b) If $u = \log(x^2+y^2)$ then verify $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$ [05]

OR

Q.2 (a) If $u = e^{xyz}$ then prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}(1 + 3xyz + x^2y^2z^2)$. [05]

(b) Expand $2x^3 + 7x^2 + x - 6$ in ascending powers of $(x-2)$ by using Taylor's series. [05]

Q.3 (a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dy\, dx$. [05]

(b) Trace the curve $x^3 + y^3 = 3axy$. [05]

OR

Q.3 (a) Evaluate $\int_0^\infty \frac{1}{(1+x^2)^2}\, dx$. [05]

(b) Trace the curve $r = a(1 - \cos\theta)$. [05]