Snakebonnnnt

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1 Definitions

$$
\begin{aligned}
S &= \text{Number of Segments} \\
\theta_n &= \text{Relative angle of segment } n \\
\Theta_n &= \text{Absolute angle of segment } n \\
C_n &= \text{Center (base) point of segment } n \\
L_n &= \text{Left point of segment } n \\
R_n &= \text{Right point of segment } n \\
W &= \text{Width of segment from center to edge} \\
H &= \text{Height of segment from center to top} \\
X &= \text{Cord displacement (per segment)}
\end{aligned}
$$

$$
Rot(\begin{bmatrix} a \\ b \end{bmatrix}, \theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{1}
$$

$$
= \begin{bmatrix} a \cdot cos\theta - b \cdot sin\theta \\ a \cdot sin\theta + b \cdot cos\theta \end{bmatrix} \tag{2}
$$

2 Models

$$
\Theta_n = \sum_{i=0}^{S} \theta_i \tag{3}
$$

$$
C_{n+1} = C_n + Rot(\begin{bmatrix} 0 \\ H \end{bmatrix}, \Theta_n) \tag{4}
$$

$$
L_n = C_n - Rot(\begin{bmatrix} W \\ 0 \end{bmatrix}, \Theta_n) \tag{5}
$$

$$
R_n = C_n + Rot(\begin{bmatrix} W \\ 0 \end{bmatrix}, \Theta_n) \tag{6}
$$

3 Initial Conditions

$$C_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{7}$$

$$\theta_0 = 0 \tag{8}$$

$$L_0 = C_0 - Rot(\begin{bmatrix} W \\ 0 \end{bmatrix}, \Theta_0) = \begin{bmatrix} -W \\ 0 \end{bmatrix} \tag{9}$$

$$R_0 = C_0 + Rot(\begin{bmatrix} W \\ 0 \end{bmatrix}, \Theta_0) = \begin{bmatrix} W \\ 0 \end{bmatrix} \tag{10}$$

$$C_1 = C_0 + Rot(\begin{bmatrix} 0 \\ H \end{bmatrix}, \Theta_0) = \begin{bmatrix} 0 \\ H \end{bmatrix} \tag{11}$$

$$L_1 = C_1 - Rot(\begin{bmatrix} W \\ 0 \end{bmatrix}, \Theta) = \begin{bmatrix} -W \cdot cos\Theta \\ H - W \cdot sin\Theta \end{bmatrix} \tag{12}$$

$$R_1 = C_1 + Rot(\begin{bmatrix} W \\ 0 \end{bmatrix}, \Theta) = \begin{bmatrix} W \cdot cos\Theta \\ H + W \cdot sin\Theta \end{bmatrix} \tag{13}$$

4 Inverse solution

Let $\Theta = \Theta_1$ for readability, then

$$\|L_1 - L_0\|_2 = H - X \tag{14}$$

$$\|R_1 - R_0\|_2 = H + X \tag{15}$$

$$\|R_1 - R_0\|_2 = \|L_1 - L_0\| + 2X \tag{16}$$

$$\left\| \begin{bmatrix} W \cdot cos\Theta - W \\ H + W \cdot sin\Theta \end{bmatrix} \right\|_2 = \left\| \begin{bmatrix} W - W \cdot cos\Theta \\ H - W \cdot sin\Theta \end{bmatrix} \right\|_2 + 2X$$

$$\sqrt{(W \cdot cos\Theta - W)^2 + (H + W \cdot sin\Theta)^2} = \tag{17}$$

$$\sqrt{(W - W \cdot cos\Theta)^2 + (H - W \cdot sin\Theta)^2} + 2X \tag{18}$$

Apply two approximations for small angles:

$$cos(\theta) = 1 - \frac{\theta^2}{2} \tag{19}$$

$$sin(\theta) = \theta \tag{20}$$

$$\sqrt{W^2 \frac{\Theta^4}{4} + H^2 + W^2\Theta^2 + 2HW\Theta} = \tag{21}$$

$$\sqrt{W^2 \frac{\Theta^4}{4} + H^2 + W^2\Theta^2 - 2HW\Theta} + 2X$$

Simplify further and remove terms that are negligible at small angles:

$$\sqrt{H^2 + 2HW\Theta} = \sqrt{H^2 - 2HW\Theta} + 2X \tag{22}$$

$$H^2 + 2HW\Theta = H^2 - 2HW\Theta + 4X(X + \sqrt{H^2 - 2HW\Theta}) \tag{23}$$

$$4HW\Theta = 4X(X + \sqrt{H^2 - 2HW\Theta}) \tag{24}$$

$$HW\Theta \approx X(X + H) \tag{25}$$

$$\Theta \approx \frac{X(X + H)}{WH} \tag{26}$$