62 Chapter 3 Growth of Functions

b. Give an example of a single nonnegative function f(n) such that for all functions $g_i(n)$ in part (a), f(n) is neither $O(g_i(n))$ nor $\Omega(g_i(n))$.

3-4 Asymptotic notation properties

Let f(n) and g(n) be asymptotically positive functions. Prove or disprove each of the following conjectures.

a.
$$f(n) = O(g(n))$$
 implies $g(n) = O(f(n))$.

b.
$$f(n) + g(n) = \Theta(\min(f(n), g(n))).$$

c.
$$f(n) = O(g(n))$$
 implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) \ge 1$ and $f(n) \ge 1$ for all sufficiently large n .

d.
$$f(n) = O(g(n))$$
 implies $2^{f(n)} = O(2^{g(n)})$.

e.
$$f(n) = O((f(n))^2).$$

f.
$$f(n) = O(g(n))$$
 implies $g(n) = \Omega(f(n))$.

$$\mathbf{r} = f(\mathbf{n}) = \Omega(f(\mathbf{n}/2))$$