Lambda Calculus

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1 Sel2013 Ex. 1,2,3

1. Evaluate the lambda-expression
   $(((\lambda f.\lambda x.f(f(f(x))))(\lambda g.\lambda y.g(g(y))))(\lambda z.z+1))(0)$
   Let $\lambda g.\lambda y.g(g(y)) =$ meow
   $= (\lambda x.meow(meow(meow(x)))(\lambda z.z+1))(0)$
   $= meow(meow(meow(\lambda z.z+1)))(0)$
   $= meow(meow(\lambda z.(\lambda z.z+1)+1))(0)$
   $= meow(\lambda z.(\lambda z.(\lambda z.z+1)+1)+1)(0)$
   $= meow(0+1+1+1)$
   $= meow(3)$
   Or something ...
2. What is $\omega(\omega)$
   ω
3. (a) Write the following terms with as few parenthesis as possible, without changing the meaning or structure of the terms:
   i. $(\lambda x.(\lambda y.(\lambda z.((xz)(yz)))))$
   $\lambda xyz.xyz^2$
   ii. $(((ab)(cd))((ef)(gh)))$
   $abcdefgh$
   iii. $(\lambda x.((\lambda y.(yx))(\lambda v.v)z)u)(\lambda w.w)$
   $\lambda x.(\lambda y.yx\lambda v.vz)u\lambda w.w$

   (b) Restore all the dropped parentheses in the following terms, without changing the meaning or structure of the terms:
   i. $xxxx$
   $(((x)x)x)x$
   ii. $\lambda x.x\lambda y.y$
   $\lambda x.x(\lambda y.y)$
   iii. $\lambda x.(x\lambda y.yxx)x$
   $(\lambda x.(\lambda y.y(((x)x)x)x)$

2 Ker Comp Practice 1a,1b

1a. Which of the following are either terms or terms with parentheses unambiguously removed?

i term

ii unambiguously removed

iii terms

iv terms

v unambiguously removed

1b. Write these terms with the minimum necessary parentheses:

i $(\lambda x.(\lambda y.(\lambda z.(z(xy)))))$
$\lambda x.\lambda y.\lambda z.zxy$

ii $(\lambda y.(((yy)y)y)$
$\lambda y.yyyy$

iii $(\lambda y.(yy)((\lambda x.x)(\lambda x.x)))$
$\lambda y.yy(\lambda x.x(\lambda x.x))$

3 Draw Construction trees for Ker Comp Practice 1b

i)

λx
|
λy
|
λz
/ | \
z x y

ii)

λy
/ | | \
y y y y

iii)

λy
/ | |
y y λx
/ |
x λx
|
x

4 Ker Exercise 1.1, 1.2

1.1 i Rewrite $((xy)(\lambda y.(\lambda z.(z(xy)))))$ using the minimum number of parentheses.
$xy(\lambda y.\lambda z.zxy)$

ii Write the term $(\lambda xyz.xy(xz))\lambda xy.x$ in full syntax
$\lambda x.(\lambda y.(\lambda z.xy))(xz)\lambda x.(\lambda y.x)$

1.2 Draw the construction tree of the combinator y, and list all its subterms.