Formal Languages week2

Michelle Bergin

October 16, 2018

1 Sudkamp CH1 14, 20, 30, 33, 40, 47

14 Let X_1,...,X_n be a partition of a set X. Define an equivalence relation $\equiv$ on X whose equivalence classes are precisely the sets X_1,...,X_n.

$X_1 \cup X_2 \cup ... \cup X_n = X$

So all X_i unioned together equal the total set X

And X would be to it's self Reflexive, Symmetric and Transitive.

20 Prove that there are an uncountable number of total functions from N to $\{0, 1\}$

So I think I need to prove that it IS countable first.

So I say that

	n	...	2	1	0
$f(n)$	$f(n)_n$	$f(n)_{...}$	$f(n)_2$	$f(n)_1$	$f(n)_0$
$f(...)$	$f(...)_n$	$f(...)_{...}$	$f(...)_2$	$f(...)_1$	$f(...)_0$
$f(2)$	$f(2)_n$	$f(2)_{...}$	$f(2)_2$	$f(2)_1$	$f(2)_0$
$f(1)$	$f(1)_n$	$f(1)_{...}$	$f(1)_2$	$f(1)_1$	$f(1)_0$
$f(0)$	$f(0)_n$	$f(0)_{...}$	$f(0)_2$	$f(0)_1$	$f(0)_0$

So I then assume $f(n)_1 + 1$ which doesn't appear in the list in a numerical order.

30 Give a recursive definition of the relation *greater than* on $N \times N$ using the successor operator s

We are saying that for any cross product that it will be greater than any other in the map of cross products?

B.C. My base case will be the set $\{1, 2, 3\}$

	1	2	3
1	(1,1)	(1,2)	(1,3)
2	(2,1)	(2,2)	(2,3)
3	(3,1)	(3,2)	(3,3)

So I am saying that for x from top to bottom or y from left to right is increasing.

So x_i is greater than or equal to x_{i+1}

$s(x_i) \geq x_i$

$s(s(x_i)) > x_i$

I am not really sure what I am doing...

each cross product is in a double for loop... how do I change that to recursion?

Would the base case for that be $(3,3)$?

In math words that would be (x_n, x_n)

So I don't know what i am doing but I won't stop. I am going to keep writing things that come to my head.

if x or $y = 3$ end

else

RC1 $(x++, y)$

RC2 $(x, y++)$

I am getting closer. I can print out most cross multiplications except (3,3) ... I really need to practice my recursion :(BUT The rules I have made are:

BC

if $x == 3$ or $y == 3$

print(x,y)

RC

RC1(s(x),y)

RC2(x,s(y))

Now I am stuck again so moving along...

33 Give a recursive definition of the operation of multiplication of natural numbers using the operations s and addition.

Thoughts:

A base case may be when a var gets to 0. A recursive case would be adding to it's self... while maybe we use operation s to count the number of times we have done the multiplication. So for example if I have $3 \cdot 4$ That would be 3 adding to 3 4 times.

So ...

Text Def

multiply(X,s(Y),Z)

BC

multiply(X,Y,Z)

Y == 0

RC

multiply(X,Y,add($Z + X$))

Something like this.

40 Prove $1 + 2^n < 3^n$ for all $n > 2$

Proving for a small number

$1 + 2^3 = 9$

and

$3^3 = 27$

$1 + 2^{n+1} < 3^n$

$1 + 2 \cdot 2^n < 3^n$

Not sure what to do but I do know that if I am saying that the first term is less than the second and I also show that the n+1 is still less than the second term then it would prove that for all next terms.

47 Prove that a strictly binary tree with n leaves contains $2n - 1$ nodes.