Formal Languages week1

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1 Sudkamp CH1 1,4,6,10,22,29,34,38,42,46

1. Let $X = 1, 2, 3, 4$ and $Y = 0, 2, 4, 6$. Explicitly define the sets described in parts (a) to (e).

   (a) $X \cup Y$

   $\{0, 1, 2, 3, 4, 6\}$

   (b) $X \cap Y$

   $\{2, 4\}$

   (c) $X - Y$

   $\{1, 3\}$

   (d) $Y - X$

   $\{0, 6\}$

   (e) $\mathcal{P}(X)$

   $\{\{\},$

   $\{1\}, \{2\}, \{3\}, \{4\},$

   $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\},$

   $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$

   $\{1, 2, 3, 4\}\}$

4. Let $X = \{n^3 + 3n^2 + 3n \mid n \geq 0\}$ and $Y = \{n^3 - 1 \mid n > 0\}$ Prove that $X = Y$

   B.C.

   Proving for 0:

   When $n = 0$ for X

   $0^3 + 3 \cdot 0^2 + 3 \cdot 0 = 0$

   When $n = 1$ for Y

   $1^3 - 1 = 0$

   I.H.

   For Y let $n = n - 1$

So $n \geq 0$

$(n-1)^3 - 1$

$n^3 - 3n^2 + 3n - 1 - 1$

$n^3 - 3n^2 + 3n - 2$

I am missing something

6. Give functions $f : N \to N$ that satisfy the following

   (a) f is total and one-to-one but not onto.

   $f(x) = x^2$

   (b) f is total and onto but not one-to-one.

   $f(x) = \sqrt{x}$

   (c) f is total, one-to-one and onto but not the identity

   $f(x) = x + 1$

   (d) f is not total but is onto.

   $f(x) = \frac{1}{x}$

10. Let $\equiv$ be the binary relation on N defined by $n \equiv m$ if, and only if, $n \equiv m$. Prove that $\equiv$ is an equivalence relation. Describe the equivalence classes of $\equiv$.

    For $n \equiv m$ there are 3 rules that must be met, reflexivity, transitivity, Symmetry.

22. A total function f from N to N is *monotone increasing* if $f(n) < f(n+1)$ for all $n \in$ N. Prove that there are an uncountable number of monotone increasing functions.

    :(

29. Give a recursive definition of the relation *is equal to* on $N \times N$ using the operator s.

    Let $Eq \subseteq N \times N$

    i) Basis: $(0,0) \in Eq$

    ii) Recursive step: $(m,n) \in Eq$ iff $m = n$

    Honestly I do not know what I am doing. Which part is supposed to be equal? is equal to what?

34. Give a recursive definition of the predecessor operation

    i) Basis: 0 if $n = 0$

    ii) Recursive step: $s(n) = n - 1$

    iii) Closure: $n - 1 = 0$ Only if this equality can be obtained from $s(n)$

38. Prove that $2 + 5 + 8 + ... + (3n-1) = n(3n+1)/2$ for all $n > 0$

    $\sum_{i=1}^{n} 3i - 1 = \frac{n(3n+1)}{2}$

    B.C.

    $\sum_{i=1}^{1} 3i - 1 = 2$

    $\frac{1(3 \cdot 1+1)}{2} = 2$

I.H.
$\sum_{i=1}^{k} 3i-1 = \frac{k(3k+1)}{2}$
The same would work with $k+1$
$\sum_{i=1}^{k+1} 3i-1$
$= \sum_{i=1}^{k} 3i-1+3(k+1)-1$
$\frac{k(3k+1)}{2}+3(k+1)-1$
$\frac{k(3k+1)+2(3(k+1)-1)}{2}$
$\frac{k(3k+1)+2(3k+1)-2}{2}$
$\frac{(k+2)(3k+1)-2}{2}$

42. Let $P = \{A, B\}$ be a set of consisting of two proposition letters (Boolean variables). The set E of well-formed conjunctive and disjunctive Boolean expressions over P is defined recursively as follows:

    i Basis: $A, B \in E$

    ii Recursive step: If $u, v \in E$, then $(u \vee v) \in E$ and $(u \wedge v) \in E$.

    iii Closure: An expression is in E only if it is obtained from the basis by a finite number of iterations of the recursive step.

    (a) Explicitly give the Boolean expressions in the sets E_0, E_1, and E_2
    $E_0 = \{A, B\}$
    $E_1 = \{A, B, (A \vee B), (A \wedge B)\}$
    $E_2 = \{\}??$

    (b) Prove by mathematical induction that for every Boolean expression in E, the number of occurrences of proposition letters is one more than the number of operators. For an expression u, let $n_p(u)$ denote the number of proposition letters in u and $n_0(u)$ denote the number of operators in u.
    I don't think I did a right so.. I am not understanding this.

    (c) Prove by mathematical induction that, for every Boolean expression in E, the number of left parentheses is equal to the number of right parentheses.

46. Using the tree (in the book), give the values of each of the items in parts (a) to (e).

    (a) the depth of the tree
    If the tree starts at 0 then the depth is 4

    (b) the ancestors of x_{11}
    x_7, x_2, x_1

    (c) the minimal common ancestors of x_{14} and x_{11}, of x_{15} and x_{11}
    f