Wing Design Project

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1. Abstract

The objection is this preliminary design report is designing a efficient wing's structure. An efficient wing has characteristics of low weight, low drag, high lift, high load, and reliable stable performance. This report is on the design of a wing meant from general aviation purposes. It will contain information on the design considerations, including lift and drag forces, maneuver loads, and structural analysis of preliminary design. In the preliminary design of this wing structure, there are aerodynamic analysis of the wing which is aerodynamic data collection, lift and drag generation, and lift-weight ratio in the critical conditions. Then, initial estimation of the wing structure which is choosing the materials, thickness, and positions of the spars, spar's caps, ribs, and springers inside of the wing. In order to accomplish this, a survey of the selected airfoil's lift profile will be studied in order to gain an understanding of lift loads produced on the structure. This will be followed by an analysis of 3D spanwise lift distribution under a distribution of angles of attack. Once a cursory analysis of all loads the wing must endure is completed an initial structural design will be modeled using matlab, as well as Solidworks for verification, to show the stress and strain which the wing will endure under loading.

2. Introduction

2.1 Background

The goal of this assignment is to produce a wing design for a small, utility class general aviation aircraft. The design will be similar to that of a cessna 172 or other aircraft of this category. All the preliminary design numbers (weight, load factor, speed, sizing) must satisfy the Federal Aviation Regulations (FAR) part 23. These requirements are meant to ensure that the structure can endure all aerodynamic forces induced during operation of the vehicle: both under steady and variable conditions.

2.2 Design Requirements

There are two sets of design requirements for this wing. One set by project guidelines and the other set by Federal Aviation Regulations. The project specifications are such that the wing be designed for a utility class aircraft of 2000 pounds. The aircraft will have a cruise velocity of 85.7 meters per second and a max allowed dive speed of 131.2 meters per second. The physical requirements are that it will have a total span of 37 feet, chord length of 4 feet, 5 inches, and be designed with a NACA 2412 series airfoil shape.

The Design requirements for the wing specified by the Federal Aviation Regulations, FAR 23 part C, for design of utility class aircraft. The regulations are meant to be used as a guide to ensure safely designed structures which can withstand operating flight conditions. The FAR 23 specifies that all structures be able to withstand loads from 4.4Gs to -1.76Gs. In addition to this an extra safety margin of 1.5 is needed to ensure the wings can withstand random gust loads.

3. Aerodynamic Load Analysis

3.1 Load Calculation

3.1.1 Lift Calculation

In order to begin lift calculation and analysis, the lift and drag for the NACA 2412 airfoil must be analyzed. This is accomplished using the program XFOIL. Flight conditions must also be considered for calculating reynold's numbers corresponding to those flight conditions. For operating conditions at 12000 feet, for example, the plane must be able to cruise at 85 MPH. Given the chord length of 53 inches, a reynold's number of roughly 3 million can be calculated. Using this data, a table of data can then be generated with XFOIL that gives angle of attack (AoA), Cl, Cd, and Cp for the airfoil. This is how our raw theoretical data was generated.

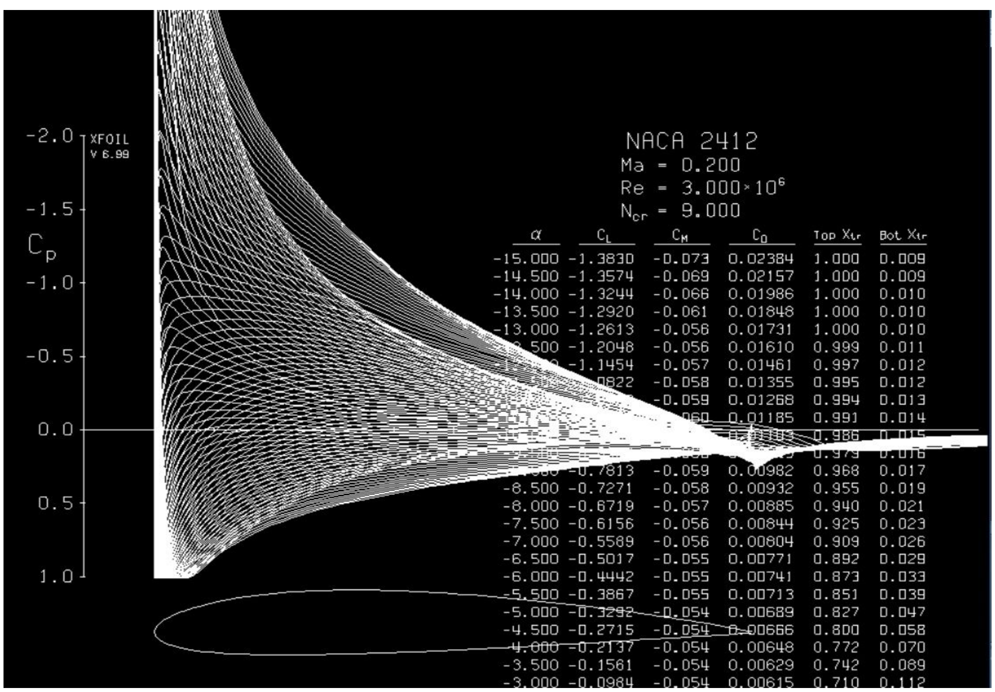

Fig 3.1.1.1 XFoil Data Collection

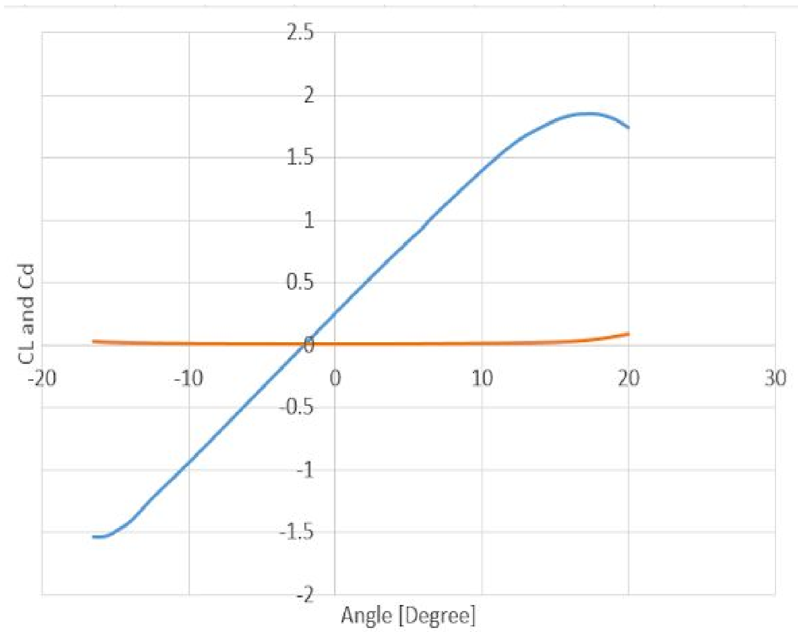

Fig 3.1.1.2: Plot of CL and Cd Data versus AoA in Degree at Sea Level

From this data set, a 2D lift curve slope for the airfoil was calculated and from that a 3D corrected lift curve slope as well. The 3D lift curve slope, using **Eqn. 3.1**, CL_alpha was found to be 4.87 per radian.

$$CL_{\alpha} = \frac{Cl_{\alpha}}{1 + \frac{Cl_{\alpha}}{\pi A e}}$$ **Eqn. 3.1**

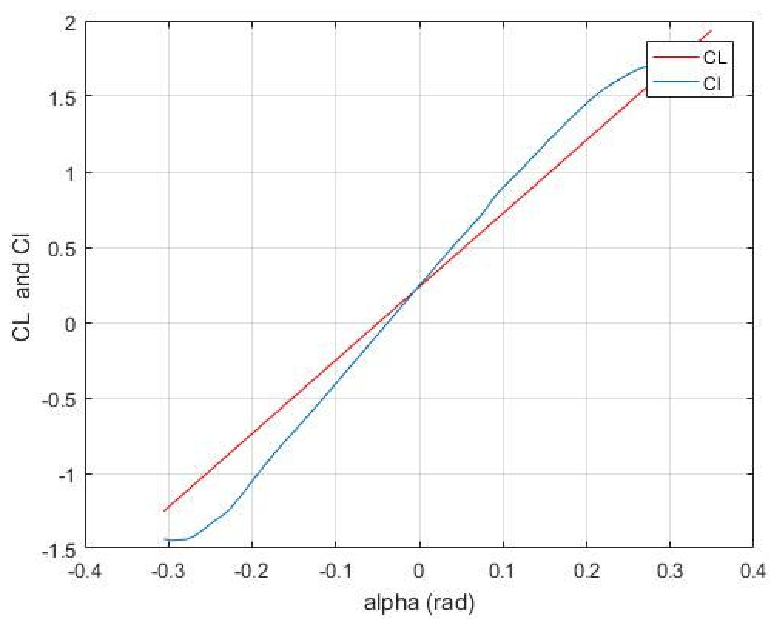

Fig 3.1.1.3: 2D and 3D Lift Data versus AoA in Radian at 12000 ft

From the data of alpha, Cl, and Cd, the lift and drag force on the wing is obtained at sea level and 12000 ft altitude in different angles of attack which are listed in **Appendix A**. After analyzing the results of lift and drag, the angle of attack (AoA) of 0.5 degree give the best performance for the wing which has the highest lift and lowest drag. Therefore, the plots of lift distribution in Figure 3.1.1.3 and Figure 3.1.1.4 uses the data of 0.5 degree AoA.

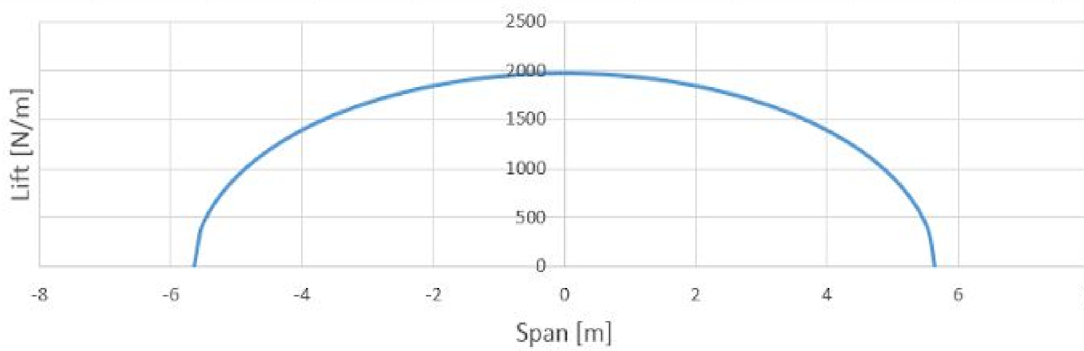

Fig 3.1.1.4: Lift Distribution at the Sea Level

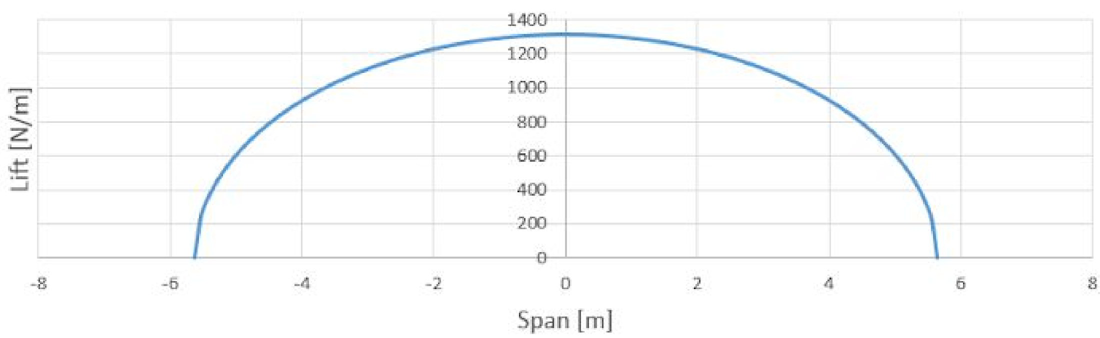

Fig 3.1.1.5: Lift Distribution at 12000 ft Altitude

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3.1.2 The Velocities and Load Factors in Critical Conditions

According to the regulations in FAR chapter 23, the limitation load factor for the utility aircrafts is between -1.76 to 4.4. Also, the positive and negative stall speeds are when the load factor equal to 1 or -1, which can be found using **Eqn. 3.2** and **Eqn. 3.3**.

$$V_{s_{pos}} = \sqrt{\frac{2W}{C_{Lmax}\rho S}}$$ **Eqn. 3.2**

$$V_{s_{neg}} = \sqrt{\frac{2W}{C_{Lmin}\rho S}}$$ **Eqn. 3.3**

The values are 39.84 m/s for positive stall speed and 49.61 m/s for negative stall speed. Therefore, the table of the critical conditions includes positive high angle of attack (PHAA), positive low angle of attack (PLAA), negative low angle of attack (NLAA), and negative high angle of attack (NHAA). Using the equations of load factor for both cases of positive and negative conditions to obtain their velocities. In the critical conditions, the positive load factor is 4.4 using **Eqn. 3.4**, and the negative load factor is -1.76 using **Eqn. 3.5**.

$$V_{pos} = \sqrt{\frac{2W n_{pos}}{C_{Lmax}\rho S}}$$ **Eqn. 3.4**

$$V_{neg} = \sqrt{\frac{2W n_{neg}}{C_{Lmin}\rho S}}$$ **Eqn. 3.5**

Table 3.1.2.0: The Values of Speed and Load Factor in Critical Conditions

Conditions	**Velocity [m/s]**	**Load Factor**
Positive Stall	39.84	1
Negative Stall	49.61	-1
PHAA	83.57	4.4
PLAA	87.50	4.4
NLAA	87.50	-1.76
NHAA	65.81	-1.76

3.2 V-n Diagram

3.2.1 Maneuver Limit Envelope

Based on the results of velocity and load factors in the table of critical conditions, the V-n diagram is constructed with the boundaries of these conditions. The positive and negative curves of load factors in the maneuver limit envelope are the functions of velocity, and they end at the intersections with the load factor limits at 4.4 and -1.76 found through **Eqn. 3.6** and **Eqn. 3.7** respectively. These intersections are the PHAA and NHAA condition points.

$$n_{pos} = \frac{0.5\rho C_{Lmax} V^2 S}{W} \quad \textbf{Eqn. 3.6}$$

$$n_{neg} = \frac{0.5\rho C_{Lmin} V^2 S}{W} \quad \textbf{Eqn. 3.7}$$

In addition, in negative load factor boundary, the curve is a straight line from the NLAA point to the dive speed of load factor of -1. This line is called the remaining lower limit. Therefore, the load factor at the dive speed is a vertical straight line which drops from 4.4 to -1.

3.2.2 Gust Load Envelope

In V-n diagram, the gust load can increase the boundary of the load factors. They are the straight lines which start from n=1 at zero speed to the higher value of load factor at the cruising speed and lower load factor at the dive speed. According to FAR 23, the gust speed at the cruising speed is 50 ft/s, and the gust speed at the dive speed is 25 ft/s. Then, the gust load factors at the cruising speed and the dive speed are obtained by **Eqn. 3.8**:

$$n = 1 + \frac{K_g a U_e V}{498(\frac{W}{S})}$$ **Eqn. 3.8**

Where:

K_g is given as: $K_g = \frac{0.88\mu}{5.3+\mu}$, and μ is given as: $\mu = \frac{2(\frac{W}{S})}{\rho c \alpha g}$.

a is the lift curve slope of from the aerodynamic data analysis, U_e is the gust speed by FAR 23 regulations, and V is the speed of the aircraft in unit of knot.

W is the weight of the aircraft which is estimated in the initial estimation, and **S** is the span of the wing.

ρ is the density of the atmosphere in $slug/ft^3$, c is the chord length of the wing, α is the angle of attack of the wing , and g is the gravitational acceleration.

All the value of gust load factors are calculated in imperial unit system; then, they are converted to SI unit for the V-n diagram. Therefore, the gust load factors are 5.032 for the cruising speed and 4.023 for the dive speed at the sea level, and the values are 4.844 for the cruising speed and 3.882 for the dive speed at 12000 ft altitude. Then, the total load factor limit envelope is the combination of maneuver limit envelope and gust load envelope. In the V-n diagram, the maneuver limit curves are represented by blue color, and the gust load factor lines are represented by green color, and the total envelopes curves are represented by purple color. The V-n diagram is constructed by using Microsoft Excel.

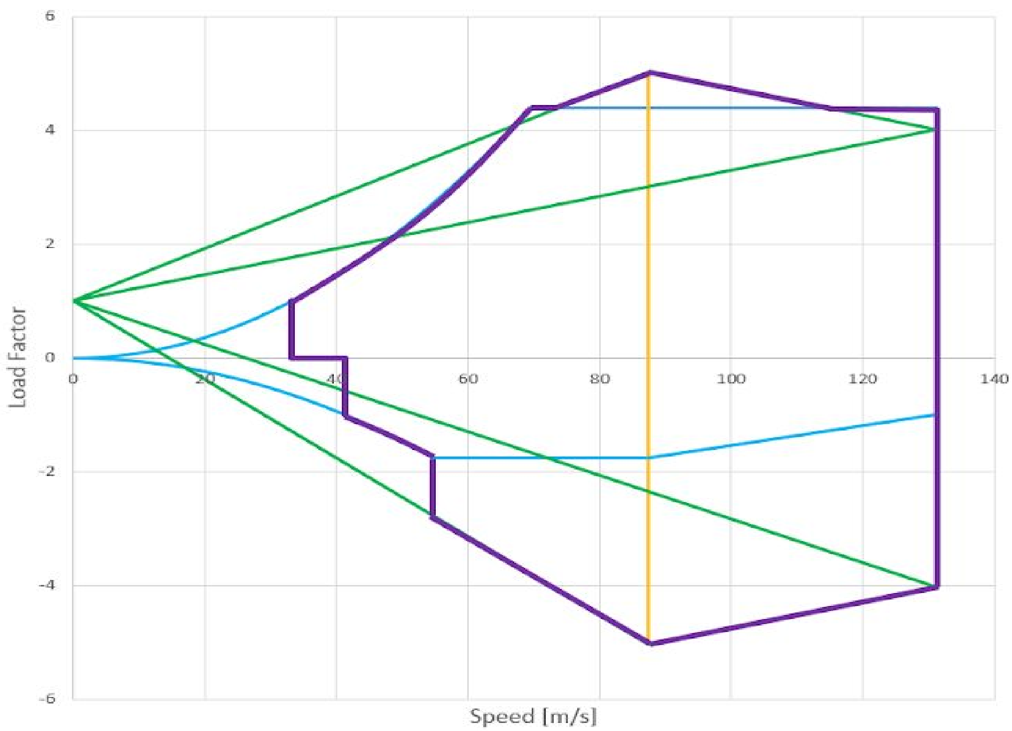

Fig 3.2.2.1: V-n diagram at the sea level.

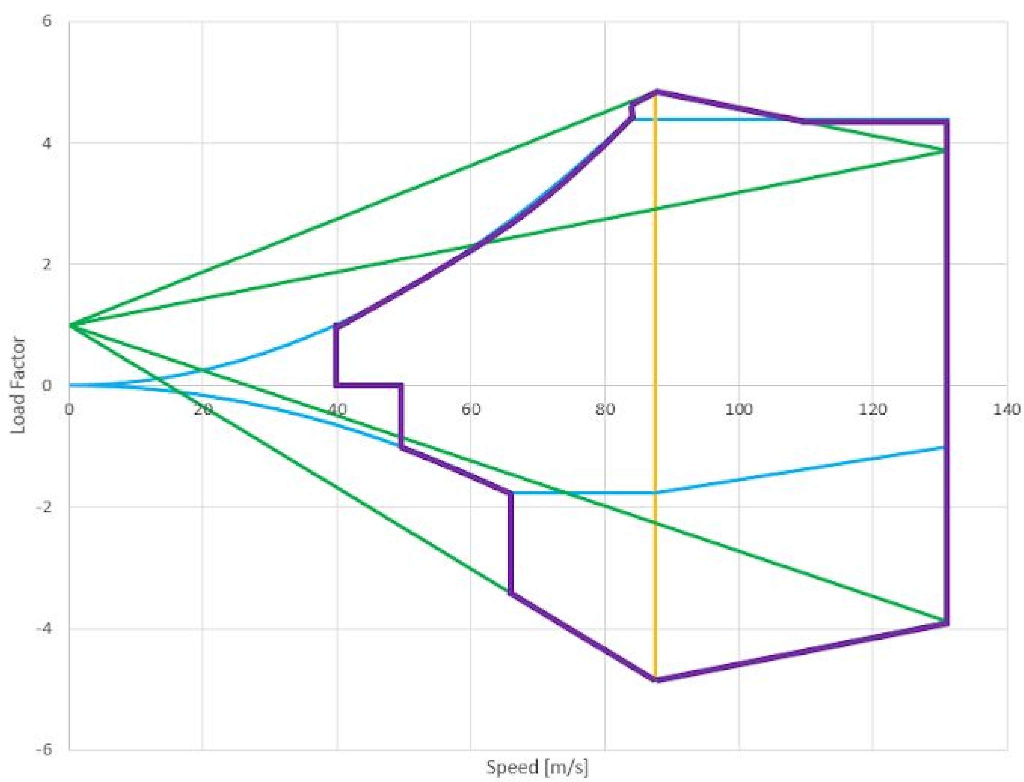

Fig 3.2.2.2: V-n diagram at the 12000 ft altitude.

4. Preliminary Structural Design

Most of the aerodynamic loads calculated in the previous sections will be carried by the wings of the airplane. A basic wing structure contains skin, wing cap, spars, ribs, stringers, wing flap and aileron as shown in **Figure 4.1**. The skin forms the airfoil of the wing, it is usually made out of thin sheet of material and generally not load bearing. Most of the loads experienced by the wing are carried by the spars, ribs and stringers. The spars are the pieces of metal that runs along the wingspan inside the wing. They carries the majority of the load that the wing experiences. The ribs runs across from the wingspan, they help hold the shape of the skin as well as help stiffen the wing. The stringers are small ribs that are just underneath the skin, they help hold the skin together while strengthen the wing in local areas.

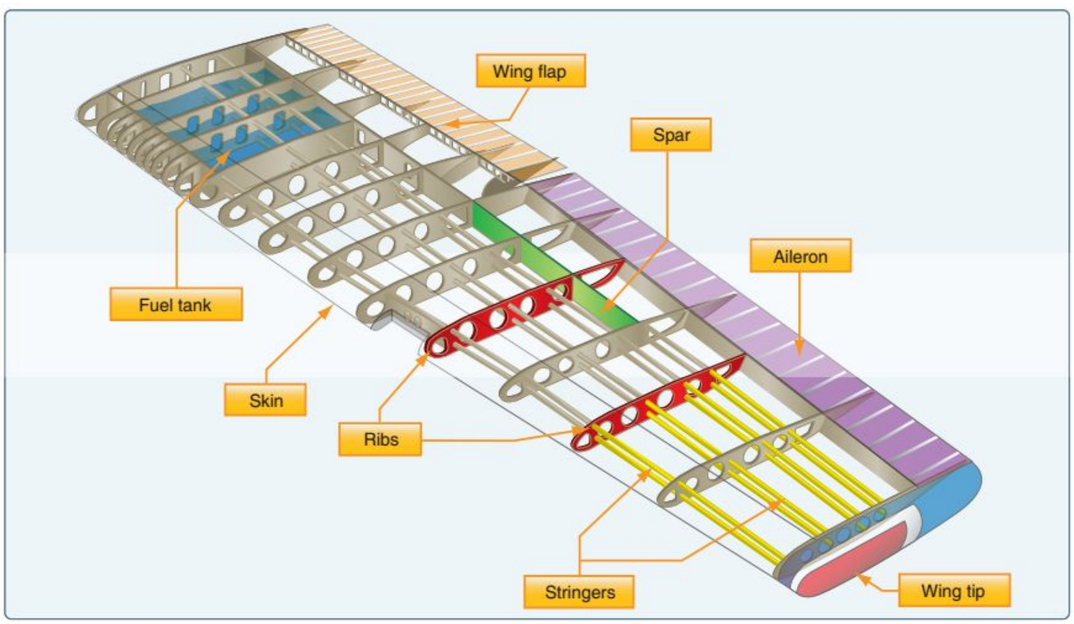

Figure 4.1: Typical Wing Structure

Based on the NACA 2412 airfoil, a preliminary layout of the wing structure was designed. Using SolidWorks, a computer aided design (CAD) model of the wing was created as seen in **Figure 4.2**. The geometry of the wing is based on the design requirement listed in Section 2.2. The skin thickness was set at 3 mm for the preliminary design, this thickness could change based on the analysis to save weight in the improved design.

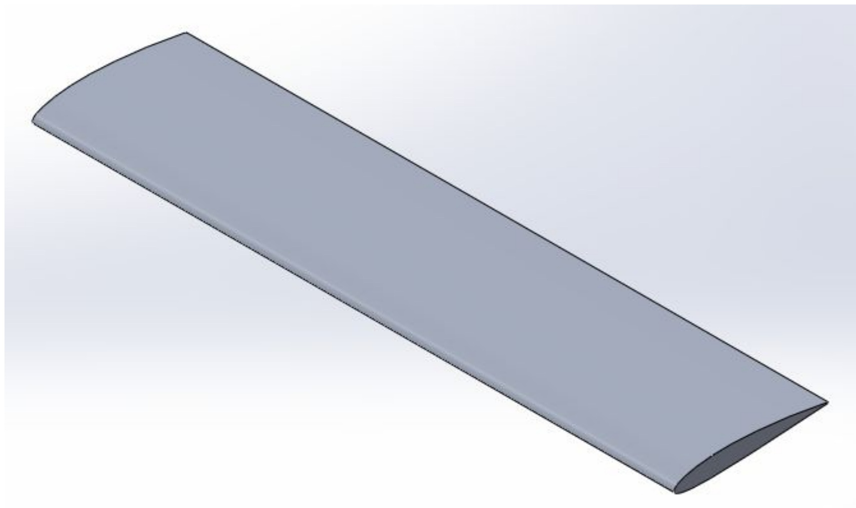

Figure 4.2: Preliminary Wing Design

As for the internal components, two spars and eight total ribs were added including the wingcap for the preliminary design as shown in the section view in **Figure 4.3**. All the thickness of these components were set to 3 mm as a starting point. Based on detailed analysis the number of ribs and spars and their length could be tuned to optimize the design.

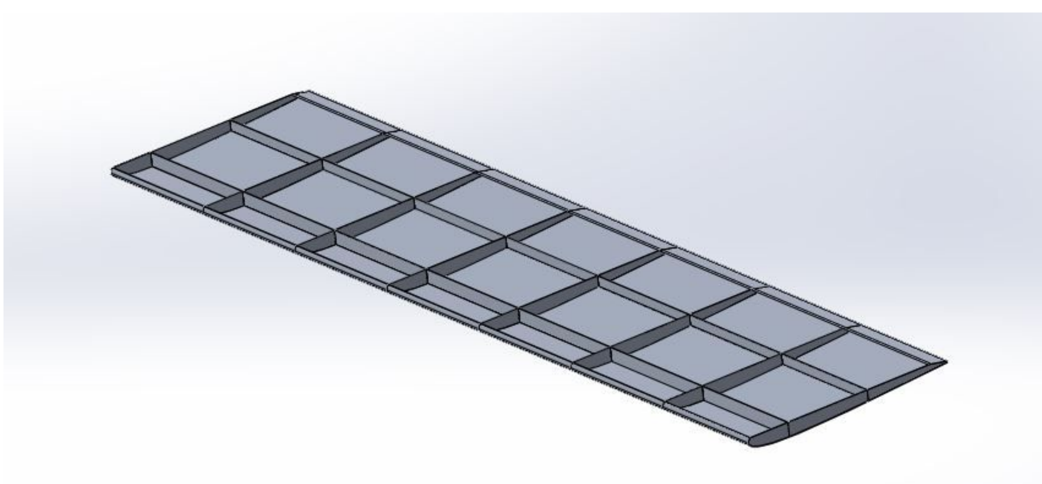

Figure 4.3: Top Section View of Preliminary Wing Design

5. Preliminary Design Analysis

In order to study the stresses and moments the wings experience under different loading condition, numerical method and CAD software were used to help analyze the structure of the wing. The analysis will include calculating the bending stress and deflection of the wing under different loading based on different configuration and state of the plane.Through those analysis, high stress area and components will be identified so design improvements can be made on the structure of the wing. The purpose of the preliminary design analysis is to verify and validates the analysis method as well as getting some rough ideas on what order of magnitude of loading and stress the wings will experience.

The coordinate system and sign convention for force and moment will be used for the analysis is shown in **Figure 5.0.1**. The positive X-axis is pointed from head to tail of the wing along the cord of the wing, the positive Y-axis is pointed perpendicular upward relative to the cord and the positive Z-axis is pointing from root to tip of the wing.

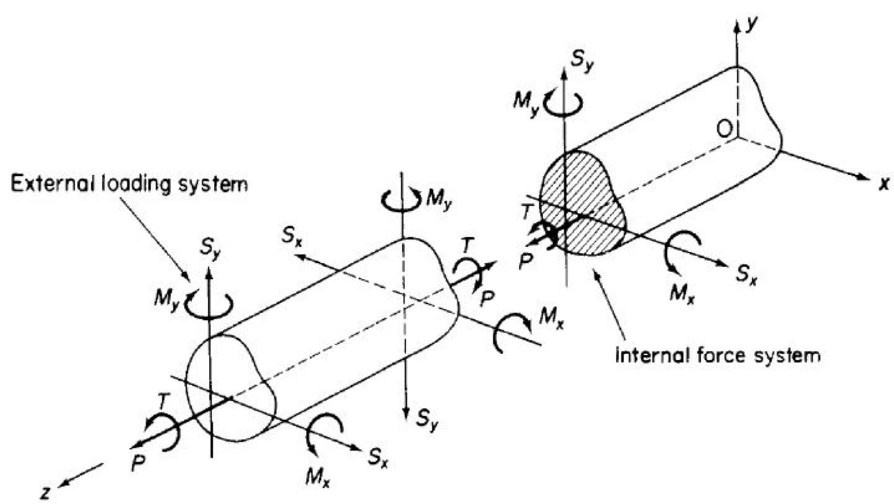

Figure 5.0.1: Coordinate System and Sign Convention Used in Analysis

5.1 Centroid and Area Moment of Inertia Calculation

To calculate the bending stress and deflection of the wing, the centroid and area moment of inertia of the cross section of the wing have to be calculated. In the numeric method, the cross section of the wing is break into quadrilateral elements. Using **Eqn. 5.1** and **Eqn. 5.2**, the centroid of each element is calculated and the result is shown in **Figure 5.1.1**. The code for numerical analysis could be found in **Appendix B**.

$$\bar{x} = \frac{1}{2}\left[\frac{x_1+x_2}{2} + \frac{x_3+x_4}{2}\right] \quad \textbf{Eqn. 5.1}$$

$$\bar{y} = \frac{1}{2}\left[\frac{y_1+y_2}{2} + \frac{y_3+y_4}{2}\right] \quad \textbf{Eqn. 5.2}$$

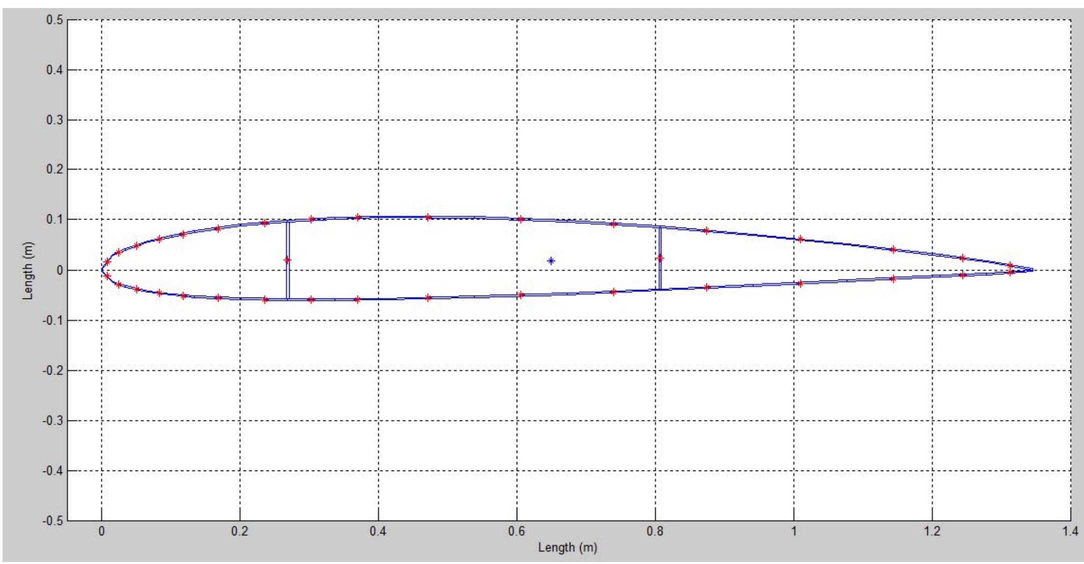

Figure 5.1.1: Centroid Calculated Using Numeric Method

With each elements' centroid already calculated, the centroid of the cross section area of the wing can be calculated by using **Eqn. 5.3** and **Eqn. 5.4**. The overall centroid is then added in **Figure 5.1.1** to show the relative location the geometry of the cross section

$$C_x = \frac{\sum_i \overline{x_i} A_i}{\sum_i A_i}$$ **Eqn. 5.3**

$$C_y = \frac{\sum_i \overline{y_i} A_i}{\sum_i A_i}$$ **Eqn. 5.4**

Meanwhile, using CAD software, the location of the centroid for the cross section can be easily found by using an evaluation tool. The location of the centroid measured from the CAD is shown in **Figure 5.1.2.**

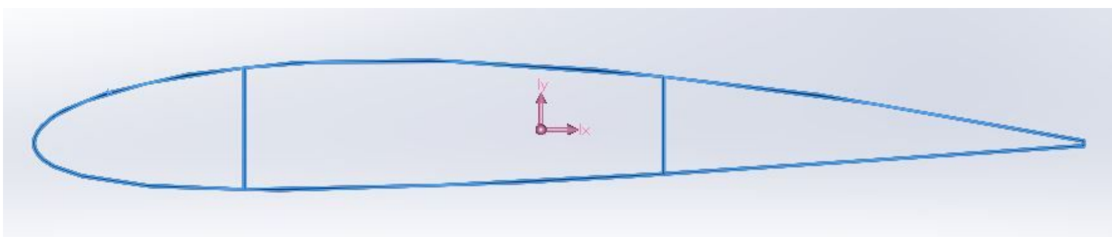

Figure 5.1.2: Centroid Calculated Using CAD Software

With centroid calculated, the area moment of inertia for the cross section at centroid can also be calculated. Using convention labeled in **Figure 5.1.3** along with **Eqn. 5.5**, **Eqn. 5.6**, **Eqn. 5.7** to calculate the area moment of inertia for each element on the skin of the wing in the numeric method. Combined with parallel axis theorem as well as adding the moment of inertia of other components, the Ixx, Iyy and Ixy of the cross sectional area can be calculated.

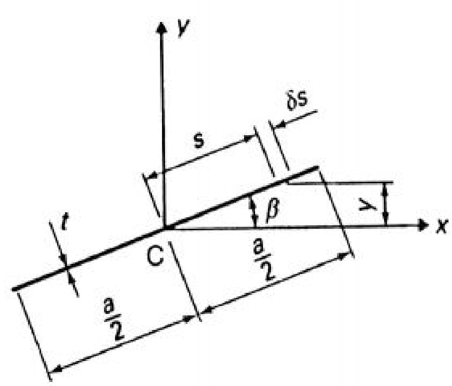

Figure 5.1.3: Convention to Calculate Moment of Inertia of Skin Element

$$I_{xx} = \frac{a^3 t sin^2\beta}{12}$$ **Eqn. 5.5**

$$I_{yy} = \frac{a^3 t cos^2\beta}{12}$$ **Eqn. 5.6**

$$I_{xy} = \frac{a^3 t sin2\beta}{24}$$ **Eqn. 5.7**

Using the same evaluation tool in the CAD software for finding the centroid, the area moment of inertia could also be found in the CAD model. Comparing the results from the two different method to verify the numerical method is working properly. As seen in **Table 5.1**, the numerical method calculation is really close to the measured result from the CAD model which means the numerical method is valid.

Table 5.1: Centroid and Moment of Inertia Result Comparison

	Cx (m)	Cy (m)	Ixx (m^4)	Iyy (m^4)	Ixy (m^4)
Numerical Method	$6.498 * 10^{-1}$	$1.742 * 10^{-2}$	$3.207 * 10^{-5}$	$1.357 * 10^{-3}$	$-3.963 * 10^{-6}$
Measured from CAD	$6.503 * 10^{-1}$	$1.758 * 10^{-2}$	$3.45 * 10^{-5}$	$1.461 * 10^{-3}$	$-3.42 * 10^{-4}$
Percent Error	0.077%	0.91%	7.0%	7.1%	15.9%

5.2 Stress Calculation

Using force and moment equilibrium about the root of the wing, the stress and moment could be calculated across the wingspan. The wing can be treated as a simple cantilever beam with the root is fixed. The forces applied on the wing can be seen as distributed loads in two directions; W_x is the sum of the forces that applied in direction along the cord, and W_y is the sum of forces applied perpendicular to the cord. Using **Eqn. 5.8** through **Eqn. 5.11**, the shear and moment distribution along the wingspan can be calculated in both directions.

$$S_y = -\int W_y \, dz \qquad \textbf{Eqn. 5.8}$$

$$S_x = -\int W_x \, dz \qquad \textbf{Eqn. 5.9}$$

$$M_x = \int S_y \, dz \qquad \textbf{Eqn. 5.10}$$

$$M_y = \int S_x \, dz \qquad \textbf{Eqn. 5.10}$$

Using the results from the aerodynamic load calculation, the stress and moment distribution along the wing can be calculated using numerical method. To verify the numerical analysis that's developed, a simple load case was implemented to test the method. As seen in **Figure 5.2.1**, the shear and moment distributions are plotted of a cantilever beam under uniformly distributed load. The result are as expected, the shear is linear and the moment is parabolic and it satisfy the end condition where the shear and moment equal to zero at the tip of the cantilever beam. Therefore, the numerical method developed is valid.

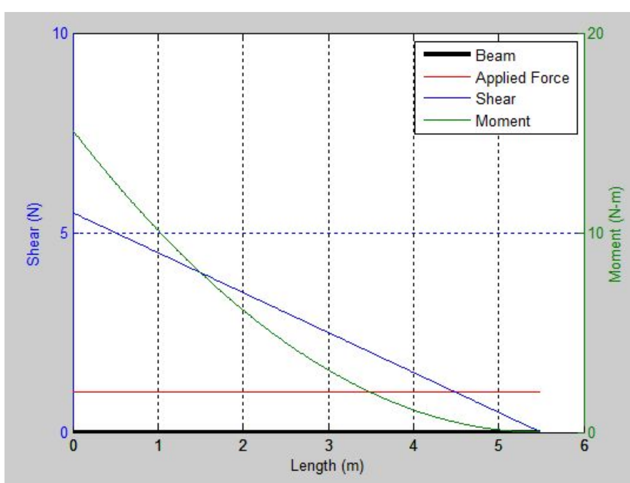

Figure 5.2.1: Shear and Moment Graph for a Cantilever Beam
Under Uniform Distributed Load

5.3 Preliminary FEA

Furthermore, using the CAD software, some preliminary finite element analysis (FEA) was done on the preliminary design of the wing. With a uniform distributed load applied under the wing, the stress distribution plot can be seen in **Figure 5.3.1** and a top section view in **Figure 5.3.2** to see the stress distribution inside the wing structure. For the simulation, the fix boundary condition was set at the root of the wing. A total of 10000 N was applied uniformly in the under side of the wing. All the component in the wing are using Al 2024-T5 as the material.

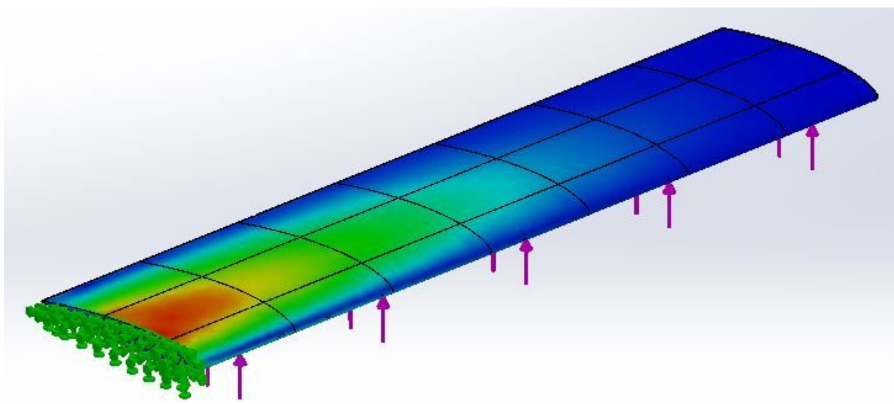

Figure 5.3.1: Stress Plot from Preliminary FEA of Wing Design

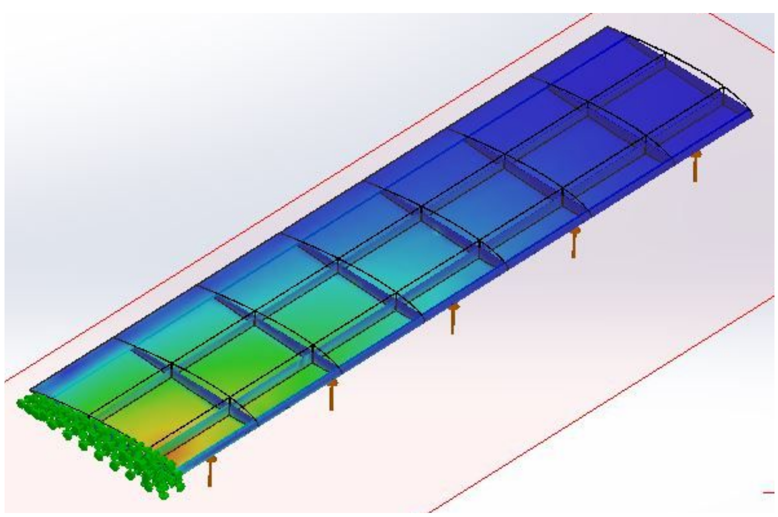

Figure 5.3.2: Top Section View of Stress Plot of Preliminary FEA

Although this is a rough design and simulation, there are a lot information could help to improve the design in the upcoming design optimization. As seen in the figures, the high stress area are near the root of the wing and there is not much stress near the tip of the wing. Currently, the lowest factor of safety is about 1.2 on the skin near the root of the wing. To improve the factor of safety and save weight, further design could bring the rib more closer together near the root instead of equally placed. Another design change could be made learning from the FEA result is that the spar might not need to run all the way to the tip of the wing since there's not much stress there.

7. Future Work, Discussion and Conclusion

Through the analysis of the preliminary structure design, the numerical method was validated for finding the centroid and moment of inertia of the cross section as well as the shear and moment distribution calculation. The preliminary FEA simulation also shows how the stress distribution looks like in the wing under load. For the next step, there are some refining need to be done to model closer to the real design.

Initial considerations for the aerodynamic forces on the wing are given to be a good first order approximation. Through use of Xfoil, to find the 2D lift data on a particular section and equation 3.1, we can get good approximations for the lift and drag coefficients on the wing. Given dynamic pressures for flight condition we were then able to find lift and drag forces, however true lift force varies as a function of the location of the wing. In later stages of the design process, it may be useful to create a 3D model, for use in a finite elements software like comsol, in order to find a more accurate model of the lift and drag forces induced on the plane's structure. By doing this would be able to verify structural integrity at critical operating conditions.

In structural design, spar caps and stringers need to be added in appropriate locations in both numerical analysis as well as the CAD model. In a real airplane wing structure, the wing flaps and ailerons are usually not load bearing, they should be removed from the structural analysis. The centroid and moment of inertia will be updated to reflect those changes. Given more detailed lift and drag force distribution, the resultant shear and moment distribution will be calculated using the current numerical method. Those loading condition will also be updated in the FEA simulation to produce more accurate analysis results. With the calculated moment distribution, the wing's stress from bending as well as the displacements can also be calculated using **Eqn. 7.1** and **Eqn. 7.2** respectively. This will be added to the numerical analysis method and compare the result with FEA simulation.

$$\sigma_z = \frac{M_x\left(I_{yy}y - I_{xy}x\right)}{I_{xx}I_{yy} - I_{xy}^2} + \frac{M_y\left(I_{xx}x - I_{xy}y\right)}{I_{xx}I_{yy} - I_{xy}^2}$$

Eqn. 7.1

$$u(z) = \sum \frac{du}{dz}\left(z_{i+1} - z_i\right)$$

Eqn. 7.2

Based on those updated simulation, design changes will be made to make the wing lighter while maintaining its strength under calculated different critical loading conditions.

8. Reference

1) Megson, T. H. G.. (2010). Introduction to Aircraft Structural Analysis - 15. Bending of Open and Closed, Thin-Walled Beams.
2) Lynch, C. (2018). MAE154B: Design of Aerospace Structures, Lecture 1A-3B [PowerPoint Slides]. Retrieved from https://ccle.ucla.edu.

9. Appendix

Appendix A: Table of Aerodynamic Data of NACA-2412 at the Sea Level

alpha	CL	CD	alpha_rad	Lift [N]	Lift [lbf]	Theta (vortex)	Drag [N]	Drag [lbf]
-16.5	-1.533	0.02837	-0.2879793266	-109141.6781	-24536.0253	-1018.231399	2019.797395	454.0631435
-16	-1.5339	0.02419	-0.2792526803	-109205.7534	-24550.43001	-1018.829187	1722.202996	387.1620529
-15.5	-1.5216	0.02164	-0.2705260341	-108330.057	-24353.56562	-1010.659424	1540.656173	346.3491867
-15	-1.4884	0.0196	-0.2617993878	-105966.3886	-23822.19182	-988.6077067	1395.418715	313.6988936
-14.5	-1.4533	0.01783	-0.2530727415	-103467.4499	-23260.40807	-965.2939936	1269.403862	285.3699629
-14	-1.4114	0.01612	-0.2443460953	-100484.3865	-22589.78872	-937.4636638	1147.660698	258.001335
-13.5	-1.3515	0.01473	-0.235619449	-96219.816	-21631.07514	-897.6775837	1048.699881	235.7543216
-13	-1.2887	0.01358	-0.2268928028	-91748.78052	-20625.94638	-855.9652994	966.8258241	217.3485192
-12.5	-1.224	0.0126	-0.2181661565	-87142.47487	-19590.40767	-812.9910192	897.0548883	201.6635745
-12	-1.167	0.01182	-0.2094395102	-83084.36942	-18678.10928	-775.1311433	841.5229191	189.1796389
-11.5	-1.1116	0.01096	-0.200712864	-79140.17571	-17791.41926	-738.3340007	780.2953632	175.4152997
-11	-1.0555	0.01027	-0.1919862177	-75146.14561	-16893.52557	-701.0719124	731.1709288	164.3718183
-10.5	-0.998	0.0097	-0.1832595715	-71052.44274	-15973.22456	-662.8799323	690.5898744	155.2489423
-10	-0.9398	0.00922	-0.1745329252	-66908.9035	-15041.71988	-624.2230064	656.4163548	147.5665204
-9.5	-0.8818	0.00869	-0.1658062789	-62779.60322	-14113.41625	-585.6989221	618.6830936	139.0838462
-9	-0.8224	0.00828	-0.1570796327	-58550.63017	-13162.70529	-546.2449462	589.4932123	132.5217775
-8.5	-0.7629	0.0079	-0.1483529864	-54314.53764	-12210.3938	-506.7245495	562.439176	126.4398602
-8	-0.7033	0.00753	-0.1396263402	-50071.32563	-11256.4818	-467.1377319	536.097088	120.5179933
-7.5	-0.6433	0.00723	-0.1308996939	-45799.63569	-10296.16769	-427.2852309	514.7386383	115.7164796
-7	-0.5834	0.00695	-0.1221730476	-41535.06523	-9337.454115	-387.4991508	494.8040852	111.2350669
-6.5	-0.5236	0.0067	-0.1134464014	-37277.61425	-8380.341061	-347.7794915	477.0053771	107.2338055
-6	-0.4637	0.00649	-0.1047197551	-33013.04379	-7421.627482	-307.9934114	462.0544623	103.8727459
-5.5	-0.4037	0.00631	-0.09599310886	-28741.35384	-6461.313381	-268.1409105	449.2393925	100.9918377
-5	-0.3438	0.00615	-0.0872664626	-24476.78338	-5502.599803	-228.3548304	437.8482193	98.4310304
-4.5	-0.2839	0.006	-0.07853981634	-20212.21292	-4543.886224	-188.5687503	427.1689945	96.03027356
-4	-0.2242	0.00586	-0.06981317008	-15961.88143	-3588.373693	-148.9155118	417.2017179	93.78956718
-3.5	-0.1644	0.00576	-0.06108652382	-11704.43045	-2631.260639	-109.1958526	410.0822347	92.18906262
-3	-0.1047	0.00567	-0.05235987756	-7454.098953	-1675.748107	-69.54261414	403.6746998	90.74860852
-2.5	-0.0451	0.00558	-0.0436332313	-3210.886942	-721.8360998	-29.95579654	397.2671648	89.30815441
-2	0.0147	0.00549	-0.03490658504	1046.564036	235.2769549	9.76386273	390.8596299	87.86770031
-1.5	0.0744	0.00541	-0.02617993878	5296.895531	1190.789486	49.41710117	385.1640433	86.58729666
-1	0.1342	0.00535	-0.01745329252	9554.346509	2147.902541	89.13676044	380.8923534	85.62699393
-0.5	0.1938	0.0053	-0.00872664626	13797.55852	3101.814548	128.723578	377.3326118	84.82674165
0	0.2538	0.00525	0	18069.24847	4062.12865	168.576079	373.7728701	84.02648937
0.5	0.3135	0.0052	0.00872664626	22319.57996	5017.641181	208.2293174	370.2131285	83.22623709
1	0.3728	0.0052	0.01745329252	26541.43352	5966.751618	247.6168725	370.2131285	83.22623709
1.5	0.4317	0.00522	0.02617993878	30734.80915	6909.459962	286.7387443	371.6370252	83.546338

2	0.49	0.00523	0.03490658504	34885.46788	7842.565164	325.462091	372.3489735	83.70638846
2.5	0.5482	0.00531	0.0436332313	39029.00713	8774.069842	364.1190169	378.0445601	84.9867921
3	0.6057	0.00548	0.05235987756	43122.70999	9694.370856	402.310997	390.1476816	87.70764986
3.5	0.6631	0.00568	0.06108652382	47209.29337	10613.07135	440.4365562	404.3866481	90.90865897
4	0.7198	0.00592	0.06981317008	51246.04037	11520.56817	478.0971696	421.4734079	94.74986992
4.5	0.7758	0.0062	0.07853981634	55232.95098	12416.86133	515.2928372	441.4079609	99.23128268
5	0.8314	0.00646	0.0872664626	59191.38366	13306.7524	552.2228214	459.9186174	103.3925945
5.5	0.8851	0.00677	0.09599310886	63014.54616	14166.23352	587.8908097	481.9890154	108.3541587
6	0.9394	0.00716	0.1047197551	66880.42556	15035.31779	623.9573231	509.755	114.5961265
6.5	1.0067	0.00759	0.1134464014	71671.83779	16112.4701	668.6585449	540.368778	121.4782961
7	1.0644	0.00804	0.1221730476	75779.77962	17035.97216	706.9833667	572.4064526	128.6805666
7.5	1.1192	0.00846	0.1308996939	79681.25643	17913.05904	743.3819842	602.3082822	135.4026857
8	1.1745	0.0089	0.1396263402	83618.33066	18798.14854	780.1127059	633.6340084	142.4449058
8.5	1.2296	0.00938	0.1483529864	87541.16593	19680.03699	816.7105859	667.807528	150.1273277
9	1.2843	0.00989	0.1570796327	91435.52326	20555.52335	853.0427826	704.1168925	158.2899009
9.5	1.339	0.0104	0.1658062789	95329.88059	21431.0097	889.3749793	740.426257	166.4524742
10	1.3923	0.01098	0.1745329252	99124.56516	22284.08873	924.7772843	781.7192598	175.7354006
10.5	1.445	0.01158	0.1832595715	102876.5328	23127.56462	959.7810643	824.4361593	185.338428
11	1.496	0.01226	0.1919862177	106507.4693	23943.8316	993.6556901	872.8486453	196.221859
11.5	1.5461	0.01295	0.200712864	110074.3304	24745.69388	1026.932528	921.9730797	207.2653404
12	1.5925	0.0138	0.2094395102	113377.7706	25488.33678	1057.751796	982.4886872	220.8696292
12.5	1.6372	0.0146	0.2181661565	116560.1796	26203.77079	1087.441909	1039.444553	233.6736657
13	1.6735	0.01551	0.2268928028	119144.552	26784.76082	1111.552672	1104.231851	248.2382572
13.5	1.7065	0.01667	0.235619449	121493.9815	27312.93358	1133.471548	1186.817856	266.8041101
14	1.7362	0.01812	0.2443460953	123608.468	27788.28906	1153.198536	1290.050363	290.0114262
14.5	1.7667	0.01966	0.2530727415	125779.9104	28276.44872	1173.45689	1399.690405	314.6591964
15	1.7936	0.02159	0.2617993878	127695.0514	28706.98955	1191.324095	1537.096432	345.5489344
15.5	1.8161	0.024	0.2705260341	129296.9351	29067.10733	1206.268783	1708.675978	384.1210943
16	1.8332	0.02706	0.2792526803	130514.3668	29340.79685	1217.626745	1926.532165	433.0965338
16.5	1.8441	0.03096	0.2879793266	131290.3904	29515.25391	1224.866616	2204.192011	495.5162116
17	1.8465	0.03596	0.2967059728	131461.258	29553.66648	1226.460716	2560.166173	575.5414396
17.5	1.8485	0.04143	0.3054326191	131603.6477	29585.67695	1227.789133	2949.601907	663.089039
18	1.844	0.04801	0.3141592654	131283.271	29513.65339	1224.800196	3418.063904	768.402239
18.5	1.8306	0.05585	0.3228859116	130329.2602	29299.18324	1215.899804	3976.23139	893.8817964
19	1.8087	0.06503	0.3316125579	128770.0934	28948.66859	1201.353641	4629.799952	1040.808115

Appendix B: MatLab Code for Structural Analysis

```
%% MAE 154B Wing Design Structure Analysis
close all;
clear all;
clc;

%% Define Nodes
% Import Shape of NACA2412
TopNodes = importdata('NACA2412_Top.txt');
BottomNodes = importdata('NACA2412_Bottom.txt');
% Extract Node Points
Nx=TopNodes(:,1);
Nytop=TopNodes(:,2);
Nybot=BottomNodes(:,2);
% Scale points to desire size
xscalefactor=1.346;
topscalefactor=1.325;
botscalefactor=1.38;
nx=xscalefactor*Nx;
nytop=topscalefactor*Nytop;
nybot=botscalefactor*Nybot;

%% Create Elements
% All Nodes for airfoil are in the inner side of the elements
skint=0.003;  % Skin thickness

% Create Airfoil Elements
Topel=struct('posX',nx,'posY',nytop,...
        'Length',zeros(length(nx),1),'Area',zeros(length(nx),1),...
        'cx',zeros(length(nx),1),'cy',zeros(length(nx),1),...
        'Ixx',zeros(length(nx),1),'Iyy',zeros(length(nx),1),...
        'Ixy',zeros(length(nx),1),...
        'xcoord',zeros(length(nx),5),'ycoord',zeros(length(nx),5));
Botel=struct('posX',nx,'posY',nybot,...
        'Length',zeros(length(nx),1),'Area',zeros(length(nx),1),...
        'cx',zeros(length(nx),1),'cy',zeros(length(nx),1),...
        'Ixx',zeros(length(nx),1),'Iyy',zeros(length(nx),1),...
        'Ixy',zeros(length(nx),1),...
        'xcoord',zeros(length(nx),5),'ycoord',zeros(length(nx),5));

% Calculate Aera and Centroid of Each Element, Plot Results
figure()
hold on
XiAiairf=0;
YiAiairf=0;
for i=1:length(nx)-1
  % Top elements
  Topel.xcoord(i,:)=[Topel.posX(i) Topel.posX(i)...
```

```
        Topel.posX(i+1) Topel.posX(i+1) Topel.posX(i)];
    Topel.ycoord(i,:)=[Topel.posY(i) Topel.posY(i)+skint ...
        Topel.posY(i+1)+skint Topel.posY(i+1) Topel.posY(i)];
    % Calculate Area
    Topel.Length(i)=sqrt((Topel.posX(i+1)-Topel.posX(i))^2+...
            (Topel.posY(i+1)-Topel.posY(i))^2);
    Topel.Area(i)=skint*Topel.Length(i);
    % Calculate Element Centroid
    Topel.cx(i)=0.5*((Topel.xcoord(i,1)+Topel.xcoord(i,2))/2+...
             (Topel.xcoord(i,3)+Topel.xcoord(i,4))/2);
    Topel.cy(i)=0.5*((Topel.ycoord(i,1)+Topel.ycoord(i,2))/2+...
             (Topel.ycoord(i,3)+Topel.ycoord(i,4))/2);
    % Plot
    plot(Topel.xcoord(i,:),Topel.ycoord(i,:))
    plot(Topel.cx(i), Topel.cy(i),'r*')

    % Bottom elements
    Botel.xcoord(i,:)=[Botel.posX(i) Botel.posX(i)...
        Botel.posX(i+1) Botel.posX(i+1) Botel.posX(i)];
    Botel.ycoord(i,:)=[Botel.posY(i)-skint Botel.posY(i) ...
        Botel.posY(i+1) Botel.posY(i+1)-skint Botel.posY(i)-skint];
    % Calculate Area
    Botel.Length(i)=sqrt((Botel.posX(i+1)-Botel.posX(i))^2+...
            (Botel.posY(i+1)-Botel.posY(i))^2);
    Botel.Area(i)=skint*Botel.Length(i);
    % Calculate Element Centroid
    Botel.cx(i)=0.5*((Botel.xcoord(i,1)+Botel.xcoord(i,2))/2+...
             (Botel.xcoord(i,3)+Botel.xcoord(i,4))/2);
    Botel.cy(i)=0.5*((Botel.ycoord(i,1)+Botel.ycoord(i,2))/2+...
             (Botel.ycoord(i,3)+Botel.ycoord(i,4))/2);
    % Plot
    plot(Botel.xcoord(i,:),Botel.ycoord(i,:))
    plot(Botel.cx(i), Botel.cy(i),'r*')

    XiAiairf=XiAiairf+Topel.cx(i)*Topel.Area(i)+Botel.cx(i)*Botel.Area(i);
    YiAiairf=YiAiairf+Topel.cy(i)*Topel.Area(i)+Botel.cy(i)*Botel.Area(i);
end
% Centroid of Airfoil
AirfoilArea=sum(Topel.Area(:,1))+sum(Botel.Area(:,1));
Cxaf=XiAiairf/AirfoilArea;
Cyaf=YiAiairf/AirfoilArea;

% Create Spars Elements
spart=0.003; % Spar thickness
sparcapt=0.002; % Spar cap size
SparIndex=[8 13];
Spars=struct('posX',zeros(length(SparIndex),1),...
        'posY',zeros(length(SparIndex),1),...
        'Length',zeros(length(SparIndex),1),...
        'Area',zeros(length(SparIndex),1),...
```

```
        'cx',zeros(length(SparIndex),1), ...
        'cy',zeros(length(SparIndex),1),...
        'Ixx',zeros(length(SparIndex),1),...
        'Iyy',zeros(length(SparIndex),1),...
        'Ixy',zeros(length(SparIndex),1),...
        'xcoord',zeros(length(SparIndex),5),...
        'ycoord',zeros(length(SparIndex),5));

XiAiSpar=0;
YiAiSpar=0;
for i=1:length(SparIndex)
  ind=SparIndex(i);
  Spars.xcoord(i,:)=[nx(ind)-spart/2 nx(ind)-spart/2 ...
             nx(ind)+spart/2 nx(ind)+spart/2 ...
             nx(ind)-spart/2];
  Spars.ycoord(i,:)=[nybot(ind) nytop(ind) ...
             nytop(ind) nybot(ind) nybot(ind)];
  % Calculate Area
  Spars.Length(i)=(nytop(ind)-nybot(ind));
  Spars.Area(i)=spart*Spars.Length(i);
  % Calculate Element Centroid
  Spars.cx(i)=0.5*((Spars.xcoord(i,1)+Spars.xcoord(i,2))/2+...
            (Spars.xcoord(i,3)+Spars.xcoord(i,4))/2);
  Spars.cy(i)=0.5*((Spars.ycoord(i,1)+Spars.ycoord(i,2))/2+...
            (Spars.ycoord(i,3)+Spars.ycoord(i,4))/2);
  % Plot
  plot(Spars.xcoord(i,:),Spars.ycoord(i,:))
  plot(Spars.cx(i), Spars.cy(i),'r*')

  XiAiSpar=XiAiSpar+Spars.cx(i)*Spars.Area(i);
  YiAiSpar=YiAiSpar+Spars.cy(i)*Spars.Area(i);
end
% Centroid of Spars
SparArea=sum(Spars.Area(:,1));
Cxspar=XiAiSpar/SparArea;
Cyspar=YiAiSpar/SparArea;

% Calculate Total Centroid
Cx=(XiAiairf+XiAiSpar)/(AirfoilArea+SparArea);
Cy=(YiAiairf+YiAiSpar)/(AirfoilArea+SparArea);

plot(Cx,Cy,'*')
xlim([-0.05,1.4])
ylim([-0.5,0.5])
xlabel('Length (m)')
ylabel('Length (m)')
grid on
hold off

disp('The centroid is at')
```

```
disp([Cx,Cy])
%% Calculate Inertia

% For Airfoil Elements
for i=1:length(nx)-1
   %Top Element
   betatop=atan2(Topel.posY(i+1)-Topel.posY(i),...
              Topel.posX(i+1)-Topel.posX(i));
   Topel.Ixx(i)=Topel.Length(i)^3*skint*(sin(betatop))^2/12+...
             Topel.Area(i)*(Cy-Topel.cy(i))^2;
   Topel.Iyy(i)=Topel.Length(i)^3*skint*(cos(betatop))^2/12+...
             Topel.Area(i)*(Cx-Topel.cx(i))^2;
   Topel.Ixy(i)=Topel.Length(i)^3*skint*sin(2*betatop)/24+...
             Topel.Area(i)*(Cx-Topel.cx(i))*(Cy-Topel.cy(i));
   % Bottom Element
   betabot=atan2(Botel.posY(i+1)-Botel.posY(i),...
              Botel.posX(i+1)-Botel.posX(i));
   Botel.Ixx(i)=Botel.Length(i)^3*skint*(sin(betabot))^2/12+...
             Botel.Area(i)*(Cy-Botel.cy(i))^2;
   Botel.Iyy(i)=Botel.Length(i)^3*skint*(cos(betabot))^2/12+...
             Botel.Area(i)*(Cx-Botel.cx(i))^2;
   Botel.Ixy(i)=Botel.Length(i)^3*skint*sin(2*betabot)/24+...
             Botel.Area(i)*(Cx-Botel.cx(i))*(Cy-Botel.cy(i));
end

% Spars Elements
for i=1:length(SparIndex)
   Spars.Ixx(i)=spart*Spars.Length(i)^3/12+...
             Spars.Area(i)*(Cy-Spars.cy(i))^2;
   Spars.Iyy(i)=spart^3*Spars.Length(i)/12+...
             Spars.Area(i)*(Cx-Spars.cx(i))^2;
   Spars.Ixy(i)=Spars.Area(i)*(Cx-Spars.cx(i))*(Cy-Spars.cy(i));
end

Ixx=sum(Topel.Ixx(:))+sum(Botel.Ixx(:))+sum(Spars.Ixx(:));
Iyy=sum(Topel.Iyy(:))+sum(Botel.Iyy(:))+sum(Spars.Iyy(:));
Ixy=sum(Topel.Ixy(:))+sum(Botel.Ixy(:))+sum(Spars.Ixy(:));

format short e
disp('The Area Moment of Inertia are')
disp([Ixx Iyy Ixy])

%% Stress Analysis
% Constants
W=3200;        % m^2
E=20*10^6;     % Pa
g=9.8;         % kg*m/s^2
Wingspan=5.5;  % meters

% Set up
```

```
z=linspace(0,Wingspan,Wingspan*100+1);
Load=struct('Wy',zeros(6,length(z)), 'TotWy',zeros(6,length(z)),...
        'Wx',zeros(6,length(z)), 'TotWx',zeros(6,length(z)), ...
        'VWy',zeros(6,length(z)), 'MWy',zeros(6,length(z)),...
        'VWx',zeros(6,length(z)), 'MWx',zeros(6,length(z)),...
        'MM',zeros(6,length(z)), 'SigmaZ',zeros(6,length(z)));

% Critical Loads
% PHAA PLAA NHAA NLAA PosGust NegGust

% Speed
v=zeros(1,6);
% Load Factor
n=zeros(1,6);

% Calculate Lift Distribution
Load.Wy(1,:)=1;
WyZ=zeros(1,length(z));

% Calculate Drag Distribution
Load.Wx(1,:)=1;
WxZ=zeros(1,length(z));

% Total Lift and Drag
for ii=1:length(z)-1
   Load.TotWy(1,ii+1)=Load.TotWy(1,ii)+...
            0.5*(Load.Wy(1,ii)+Load.Wy(1,ii+1))*(z(ii+1)-z(ii));
   WyZ(1,ii)=0.5*(Load.Wy(1,ii)+Load.Wy(1,ii+1))*(z(ii+1)-z(ii))*...
        (z(ii)+(z(ii+1)-z(ii))/2);
   Load.TotWx(1,ii+1)=Load.TotWx(1,ii)+...
            0.5*(Load.Wx(1,ii)+Load.Wx(1,ii+1))*(z(ii+1)-z(ii));
   WxZ(1,ii)=0.5*(Load.Wx(1,ii)+Load.Wx(1,ii+1))*(z(ii+1)-z(ii))*...
        (z(ii)+(z(ii+1)-z(ii))/2);
end
Wyz=sum(WyZ(1,:));
WyZeq=Wyz/Load.TotWy(1,end);
Wxz=sum(WxZ(1,:));
WxZeq=Wxz/Load.TotWx(1,end);

% Calculate Shear and Moment
Load.VWy(1,1)= Load.TotWy(1,end);
Load.MWy(1,1)= Load.TotWy(1,end)*WyZeq;
Load.VWx(1,1)= Load.TotWx(1,end);
Load.MWx(1,1)= Load.TotWx(1,end)*WxZeq;
MWy=0;
MWx=0;

for k=2:length(z)
   % Shear/Moment from Lift
   Load.VWy(1,k)=Load.VWy(1,1)-Load.TotWy(1,k);
```

```
    MWy=MWy+0.5*(Load.VWy(1,k-1)+Load.VWy(1,k))*(z(k)-z(k-1));
    Load.MWy(1,k)=Load.MWy(1,1)-MWy;

    % Shear/Moment from Drag
    Load.VWx(1,k)=Load.VWx(1,1)-Load.TotWx(1,k);
    MWx=MWx+0.5*(Load.VWx(1,k-1)+Load.VWx(1,k))*(z(k)-z(k-1));
    Load.MWx(1,k)=Load.MWx(1,1)-MWx;
end

% Moment
CM=-0.007;

% Check
figure()
hold on
plot(z,zeros(1,length(z)),'k', 'Linewidth',3)
plot(z,Load.Wy(1,:),'r')
hAx=plotyy(z,Load.VWy(1,:),z,Load.MWy(1,:));
legend('Beam','Applied Force','Shear','Moment')
xlabel('Length (m)')
ylabel(hAx(1), 'Shear (N)')
ylabel(hAx(2), 'Moment (N-m)')
grid on
hold off
```

Appendix C: Matlab code for deriving 3d lift curve slope

```
% this script prints out data for 3d lift curve slope of the
% naca2412 airfoil at sealevel and 120000 ft.


% table of data on airfoil at 12000ft.
maxalt = naca_table_maxalt();

% table of data on airfoil at sealvl.
sealvl = naca_table_sealvl();

% vector of alpha values from Xfoil data

% note: only rows 18-54 are taken because this is the gaurenteed linear
% regime.

alpha_max = maxalt(18:54,1);
% vector of corresponding Cl values from Xfoil data
Cl_max = maxalt(18:54,2);

% vector of alpha values from Xfoil data
alpha_sea = sealvl(18:54,1);
% vector of corresponding Cl values from Xfoil data
Cl_sea = sealvl(18:54,2);
```

```
%getting 2d liftcurve slope from data
temp = polyfit(alpha_max,Cl_max,1);
Cl_alpha_max = (180/pi)*temp(1);
Cl_0_max = temp(2);

temp = polyfit(alpha_sea,Cl_sea,1);
Cl_alpha_sea = (180/pi)*temp(1);
Cl_0_sea = temp(2);


%calculate 3d liftcurve slope data

A = 8.3774;
e = 0.79;

CL_alpha_max = Cl_alpha_max/(1+(Cl_alpha_max/(pi*e*A)));
CL_alpha_sea = Cl_alpha_sea/(1+(Cl_alpha_sea/(pi*e*A)));

alpha_max = (pi/180)*maxalt(:,1);
alpha_sea = (pi/180)*sealvl(:,1);
CL_max = CL_alpha_max.*alpha_max + Cl_0_max;
CL_sea = CL_alpha_sea.*alpha_sea + Cl_0_sea;

CL_pmax = CL_alpha_max*(16)*(pi/180) + Cl_0_max;
CL_nmax = CL_alpha_max*(-14.5)*(pi/180) + Cl_0_max;

% plot CL vs alpha at sea level
plot(alpha_max,CL_max,'r')
hold on
plot(alpha_max,maxalt(:,2))


%display final results
disp('altitude    CL_alpha    CL_0    ');
fprintf('\n12000ft     %8.4f',CL_alpha_max);
fprintf('    %8.4f',Cl_0_max);
fprintf('\nsea lvl     %8.4f',CL_alpha_sea);
fprintf('    %8.4f\n',Cl_0_sea);
fprintf('CL_pmax %8.4f\n',CL_pmax);
fprintf('CL_nmax %8.4f\n',CL_nmax);
```