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# Holographic dark energy models in $f(Q,T)$ gravity and cosmic constraint

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ABSTRACT

We study a holographic dark energy model in $f(Q,T)$ gravity

*Keywords:* Cosmology

## 1. INTRODUCTION

Over the past decaeds, a series of discoveries in cosmology have profoundly changed our understanding of the universe. In 1998, the accelerated expansion of the universe was discovered through the study of Type Ia supernovae(Perlmutter et al. (1998); Riess et al. (1998)). This fact had been later confirmed by many other cosmological observations, such as the measurement of temperature anisotropy and polarization in the cosmic microwave background (CMB) radiation(Smoot et al. (1992); Aghanim et al. (2020)); Baryon acoustic oscillations (Baryon Acoustic Oscillations, BAO) peak length scale(Eisenstein et al. (2005); Blake et al. (2011)); the study of the large-scale structure (LSS) of the universe(Dodelson et al. (2002); Percival et al. (2007)) and use Cosmic Chronometers to direct measurement of Hubble parameter(Stern et al. (2010); Moresco (2015)). These observations suggest the existence of a mysterious energy in our universe, also named dark energy (DE) who has high negative pressure and increasing density. Dark energy behaves as anti-gravity, but its nature remains unknown.

Theoretical predictions and astronomical observations indicate that there may be a mysterious form of energy in the universe. This energy has the characteristics of negative pressure, and its density increases over time. This is considered to be the key factor driving the accelerated expansion of the universe, accounting for about three-quarters of the total energy of the universe (Ratra & Peebles (1988); Armendariz-Picon et al. (2001); Tomita (2001)). In order to achieve such accelerated expansion, this form of energy needs to produce an anti-gravitational effect throughout the observable universe. However, ordinary baryonic matter neither has this equation of state nor can it explain such a large proportion of the cosmic energy component. Therefore, scientists have proposed and studied a variety of alternative theories and models to explore the nature of this cosmic acceleration phenomenon.

The simplest and most widely accepted theory is ΛCDM model, where Λ means cosmological constant predicted by Einstein(Carroll (2001)). Based on ΛCDM model, the lastest observations suggest that our universe consists of 68.3% dark energy, 26.8% cold dark matter and 4.9% ordinary matter (Aghanim et al. (2020)). However, this model is not free from problems and the problems it is facing are cosmic coincidence, fine-tuning and the Hubble tension—a discrepancy between the value of the Hubble constant $H_0$ inferred from the CMB by the Planck satellite and that obtained from local measurements using Type Ia supernovae—has sparked significant debate.

Another interesting attempt is to deviate from general ralativity toward a modified form (detailed research progress can be reviewid in Clifton et al. (2012)). These theories assume that general relativity not work in large scale requiring a modification in action rather than standard Einstein-Hilbert action. The most well-known is $F(R)$ gravity which replaces the Ricci scalar $R$ in the action by a general function $f(R)$ (Buchdahl (1970)). The $f(G)$ gravity theory is also a modified theory of gravity that introduces a correction to the Gauss–Bonnet (GB) term $G$, allowing it to be arbitrary function $f(G)$ rather than remaining a constant(Nojiri & Odintsov (2005, 2007)). Another modified theory of gravity $f(T)$ extends the teleparallel equivalent of General Relativity (TEGR). It replaces the curvature scalar $R$ in action with the torsion scalar $T$, derived from the Weitzenböck connection. Also shows some interpretations for the accelerating phases of our Universe(Cai et al. (2016); Bengochea & Ferraro (2009)). $f(Q)$ is generalized symmetric teleparallel gravity, with curvature and torsion both being zero, which is inspired by Weyl and Einstein's trial to unify electromagenetic and gravity. The geometric properties of gravity are described by "non-metricity". That is, the covariant derivative of the metric tensor is no longer zero (some detailed information can be found in review Heisenberg (2024)). Harko et al. have proposed a new theory known as $f(R,T)$ gravity, where $R$ stands for the Ricci scalar and $T$ denotes the trace of energy-momentum tensor which presents a non-minimum coupling between geometry and matter(Harko et al. (2011)). Similar theories are introduced,$f(R,G)$ gravity proposed by Bamba et al.(Bamba et al. (2009)); $f(Q,T)$ proposed by Xu et al.(Xu et al. (2019)); $f(Q,C)$ gravity(De et al. (2024)); $f(\mathcal{T},T)$ proposed by(Harko et al. (2014)); $f(T,B)$ gravity(Bahamonde et al. (2015); Bahamonde & Capozziello (2017)); $f(R,T^2)$ proposed by Katırcı et al.(Katırcı & Kavuk (2014)), etc.

Holographic dark energy is an famous alternative theory for the interpretation of dark energy, originating from the holographic principle proposed by 't Hooft('t Hooft (1993)). Cohen et al. introduced the "UV-IR" relationship, highlighting that in effective quantum field theory, a system of size $L$ has its entropy and energy constrained by the Bekenstein entropy bound and black hole mass, respectively. This implies that quantum field theory is limited to describing low-energy physics outside black holes(Cohen et al. (1999)). After that, Li et al. proposed that the infrared cut-off relevant to the dark energy is the size of the event horizon and obtained the dark energy density can be described as $\rho_{\mathrm{de}} = 3c^2 M_p^2 R_h^2$ where $R_h$ is future horizon of our universe(Li (2004)). Although choose Hubble cut-off is a natural thought, but Hsu found it might lead to wrong state equation and be strongly disfavored by observational data(Hsu (2004)).

Various attempts to reconstruct or discuss HDE in modified gravity have been completed by several authors. Wu and zhu reconstructed HDE in $f(R)$ gravity (Wu & Zhu (2008)). Shaikh et al. discussed HDE in $f(G)$ gravity with Bianchi type 1 model (Shaikh et al. (2020)). Zubair et al. reconstructed Tsallis holographic dark energy models in modified $f(T,B)$ gravity (Zubair et al. (2021)). Sharif et al. studied the cosmological evolution of HDE in $f(\mathcal{G},T)$ gravity (Sharif & Ikram (2019)) and Alam et al. investigated Renyi HDE in the same gravity (Alam et al. (2023)). Myrzakulov et al. reconstructed Barrow HDE in $f(Q,T)$ gravity (Myrzakulov et al. (2024)). Singh et al. and Devi et al. discussed HDE models respectively in $f(R,T)$ gravity and take cosmic constraint (Singh & Kumar (2016a); Devi et al. (2024)).

In this article, we assume that our universe is in a $f(Q,T)$ gravity and HDE exists as one component of fluid, we will breifly introduce $f(Q,T)$ gravity and HDE model in section 2.

## 2. $F(Q,T)$ GRAVITY THEORY

Weyl in 1918 introduced an extension of Riemannian geometry, using a non-metricity tensor $Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu} = -w_\alpha g_{\mu\nu}$, which describes how the length of a vector changes during parallel transport where $w_\alpha$ coincides with those of the electromagnetic potentials (Weyl (1918)). Weyl geometry can also be extended to so-called Weyl-Cartan geometry by considering the torsion of spacetime.

In Weyl-Cartan geometry, a connection can be decomposed into three independent parts: the Christoffel symbol $\hat{\Gamma}^\alpha{}_{\mu\nu}$, the contortion tensor $K^\alpha{}_{\mu\nu}$ and the disformation tensor $L^\alpha{}_{\mu\nu}$, so that the general affine connection can be expressed as (Järv et al. (2018))

$$\Gamma^\alpha{}_{\mu\nu} = \hat{\Gamma}^\alpha{}_{\mu\nu} + K^\alpha{}_{\mu\nu} + L^\alpha{}_{\mu\nu} \tag{1}$$

whereas

$$\hat{\Gamma}^\alpha{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}) \tag{2}$$

$$K^\alpha{}_{\mu\nu} = \frac{1}{2} T^\alpha{}_{\mu\nu} + T_{(\mu}{}^\alpha{}_{\nu)} \tag{3}$$

$$L^\alpha{}_{\mu\nu} = \frac{1}{2} Q^\alpha \mu\nu - Q_{(\mu}{}^\alpha{}_{\nu)} \tag{4}$$

are the standard Levi-civita connection of metric $g_{\mu\nu}$, contortion and disformation tensors respectively. In the above definitions, the torsion tensors and the non-metric tensor are introduced as follow

$$Q_{\rho\mu\nu} \equiv \nabla_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - \Gamma^\beta{}_{\rho\mu} g_{\beta\nu} - \Gamma^\beta{}_{\rho\nu} g_{\mu\beta} \tag{5}$$

$$T^\alpha{}_{\mu\nu} \equiv 2\Gamma^\alpha{}_{[\mu\nu]} = \Gamma^\alpha{}_{\mu\nu} - \Gamma^\alpha{}_{\nu\mu} \tag{6}$$

The non-metric tensor has two independent traces, namely $Q_\mu = Q_\mu{}^\alpha{}_\alpha$ and $\tilde{Q}^\mu = Q_\alpha{}^{\mu\alpha}$, so we can get quadratic non-metricity scalar as

$$Q = \frac{1}{4} Q_{\alpha\beta\mu} Q^{\alpha\beta\mu} - \frac{1}{2} Q_{\alpha\beta\mu} Q^{\beta\mu\alpha} - \frac{1}{4} Q_\alpha Q^\alpha + \frac{1}{2} Q_\alpha \tilde{Q}^\alpha \tag{7}$$

We consider the general form of the Einstein-Hilbert action for the $f(Q,T)$ gravity in the unit $8\pi G = 1$

$$S = \int (\frac{1}{2} f(Q,T) + \mathcal{L}_m) \sqrt{-g} d^4x \tag{8}$$

where $f$ is an arbitrary function of the non-metricity , $\mathcal{L}_m$ is known as matter Lagrangian, $g = \det(g_{\mu\nu})$ denotes determinant of metric tensor, and $T = g^{\mu\nu} T_{\mu\nu}$ is the trace of the matter-energy-momentum tensor, where $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}} \tag{9}$$

Vary the action (8) with respect to the metric tensor $g_{\mu\nu}$ we can get

$$\delta S = \int \left(\frac{1}{2}\delta[f(Q,T)\sqrt{-g}] + \delta(\mathcal{L}_m\sqrt{-g})\right) d^4x \tag{10}$$

$$= \int \frac{1}{2}\left(-\frac{1}{2}fg_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu} + f_Q\sqrt{-g}\delta Q + f_T\sqrt{-g}\delta T - \frac{1}{2}T_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\right) d^4x \tag{11}$$

In analogy to studies on torsionless $f(R)$ gravity and curvature-free $f(T)$ gravity, we can generalize the gravity to theories containing an arbitrary function of the non-metricity scalar i.e. $f(Q,T)$. Therefore, we con- sider the following action

$$-\frac{2}{\sqrt{-g}}\nabla_\alpha(f_Q\sqrt{-g}P^\alpha{}_{\mu\nu}) - \frac{1}{2}fg_{\mu\nu} + f_T(T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q(P_{\mu\alpha\beta}Q^\nu{}_{\alpha\beta} - 2Q^{\alpha\beta}{}_\mu P_{\alpha\beta\nu}) = T_{\mu\nu} \tag{12}$$

Where tensor $\Theta_{\mu\nu}$ are defined as $g^{\alpha\beta}\delta T_{\alpha\beta}/\delta g^{\mu\nu}$ and $P^\alpha_{\mu\nu}$ is the super-potential of the model (detailed discussion found in Xu et al. (2019)). In the case of a globally vanishing affine connections, the non-metricity tensor depends on the metric only and Einstein's GR action is recovered. This occurs under the choice of the coincidence gauge, in which the origin of spacetime and that of the tangent space coincide. In the coincident gauge with $\Gamma^\alpha_{\mu\nu} = 0$ we have $Q = 6H^2$ (detailed discussion can be found in Lu et al. (2019)).

Assuming that the Universe is described by the isotropic, homogeneous and spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime, given by as line element

$$ds^2 = -N^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j \tag{13}$$

where $a(t)$ is the cosmic scale factor used to define the Hubble expansion rate $H = \dot{a}/a$ and the lapse function $N(t)$ used to define dilation rates $\tilde{T} = \dot{N}/N$ (for standard case $N(t) = 1$). To derive Friedmann equations describing the cosmological evolution, we assume that the matter content of the Universe consists of a perfect fluid, whose energy-momentum tensor is given by $T^\mu_\nu = \mathrm{diag}(-\rho, p, p, p)$ and tensor $\Theta^\mu_\nu$ is expressed as $\mathrm{diag}(2\rho + p, -p, -p, -p)$. Then use the line element (13) and field equation (12), we can get Friedmann equations

$$\rho = \frac{f}{2} - 6f_Q H^2 - \frac{2f_T}{1+f_T}(\dot{f}_Q H + f_Q\dot{H}) \tag{14}$$

$$p = -\frac{f}{2} + 6f_Q H^2 + 2(\dot{f}_Q H + f_Q\dot{H}) \tag{15}$$

where $f(Q,T)$ is simplified to $f$, and $f_Q = \partial f/\partial Q$, $f_T = \partial f/\partial T$, $\dot{f}_Q = \partial f_Q/\partial t$. By the usage of Eq. (7) and the line element (13), there exists following relationship (The detailed derivation can be found in the appendix of Xu et al. (2019))

$$Q = 6H(t)^2 \tag{16}$$

The equation of state (EoS) parameter is given by

$$w = \frac{p}{\rho} = -1 + \frac{4f_Q H + f_Q\dot{H}}{(1+f_T)(f - 12f_Q H^2) - 4f_T(\dot{f}_Q H + f_Q\dot{H})}, \tag{17}$$

where $\rho$ and $p$ denote the total energy density and pressure of the universe. Since we mainly focus in the late universe, the contribution of radiation can be ignored, so we only care about baryonic matter and holographic dark energy fluid.

$$\rho = \rho_m + \rho_{de}, \quad p = p_m + p_{de} \tag{18}$$

In our universe, the condition $w < -1/3$ ensures accelerated expansion.

Effective EoS parameter denote geometry qualities.

$$\rho_{\rm eff} = 3H^2 = \frac{f}{4f_Q} - \frac{1}{2f_Q}[(1+f_T)\rho + f_T p] \tag{19}$$

$$-p_{\rm eff} = 2\dot{H} + 3H^2 = \frac{f}{4f_Q} - \frac{2\dot{f}_Q H}{f_Q} + \frac{1}{2f_Q}[(1+f_T)\rho + (2+f_T)p] \tag{20}$$

The effective energy density $\rho_{\rm eff}$ and pressure $p_{\rm eff}$ described above highlight the coupling between geometry and matter within the $f(Q,T)$ gravity framework. The presence of $f_T$ explicitly links the geometric modifications to the energy density $\rho$ and pressure $p$ of the matter content. Additionally, the term involving $\dot{f}_Q$ introduces time dependence in the coupling, suggesting a dynamical interplay between the evolution of spacetime geometry and matter distribution.

Furthermore, the effective EoS using Eq.(19)(20) can be written as

$$w_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = \frac{-2\dot{H} - 3H^2}{3H^2} = -\frac{f - 8\dot{f}_Q H + 2[(1+f_T)\rho + (2+f_T)p]}{f - 2[(1+f_T)\rho + f_T p]} \tag{21}$$

## 3. COSMIC SOLUTIONS WITH HOLOGRAPHIC DARK ENERGY

The holographic principle sets an upper limit on the entropy of the universe. In the HDE model, the energy density of dark energy is typically expressed as (Li (2004))

$$\rho_{\rm de} = 3c^2 M_p^2 L^{-2} \tag{22}$$

where $L$ is the characteristic length scale of the universe, and $c$ is free parameter, $M_p$ denotes planck mass here we set it as 1. The Hubble horizon is considered the simplest option. In addition, the particle horizon $L_p$ or the future event horizon $L_F$ are also considered reasonable options. Consider a simple case, the HDE energy density in Hubble cut-off can be described as

$$\rho_{de} = 3c^2 H(t)^2 \tag{23}$$

Another HDE model called barrow holographic dark energy (BHDE) generalizes holographic entropy that arises from quantum-gravitational effects which deform the black-hole surface by giving it an intricate, fractal form . In this case, the HDE energy density can be define as

$$\rho_{de} = 3c^2 H(t)^{2-\Delta} \tag{24}$$

here a new exponent $\Delta$ quantifies the quantum-gravitational deformation, with $\Delta = 0$ coming back to the standard Bekenstein-Hawking entropy, and with $\Delta = 1$ corresponding to the most intricate and fractal structure (Saridakis (2020)).

In order to incorporate holographic dark energy in the modified gravitational universe, we consider a simple form of $f$

$$f(Q,T) = mQ^n + \alpha T \tag{25}$$

where $Q = 6H^2$, $T = -\rho + 3p$, $m$, $n$ and $\alpha$ are constants. So that we can derive $f_Q = \alpha n Q^{n-1} = \alpha n 6^{n-1} H^{2n-2}$, $f_T = \beta$, $\dot{f}_Q = 2\alpha n(n-1)6^{n-1}H^{2n-3}\dot{H}$.

We also introduce the deceleration factor, which describes the acceleration or deceleration of the late universe depending upon its value, is defined as

$$q = \frac{d}{dt}\frac{1}{H} - 1 = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\dot{H}}{H^2} - 1 = (1+z)\frac{1}{H(z)}\frac{dH(z)}{dz} - 1 \tag{26}$$

In order to understand the characteristic properties of the dark energy and more easier to get a analytical solution, we need to parameterize EoS of HDE as a constant parameter

$$w_{\rm de} = \frac{p_{\rm de}}{\rho_{\rm de}} \tag{27}$$

Use Eq.(19) and (20) we can solve the energy density of fluid component in field equation

$$\rho_{\rm de} = \frac{m(2n-1)\left(6H(t)^2\right)^{n-1}\left((3\alpha+2)nH'(t) + 3(\alpha+1)H(t)^2\right)}{\left(2\alpha^2+3\alpha+1\right)w_{\rm de}} \tag{28}$$

$$\rho_m = \frac{m(2n-1)\left(6H(t)^2\right)^{n-1}\left(n(\alpha(w_{\rm de}-3)-2)H'(t) - 3(\alpha+1)(w_{\rm de}+1)H(t)^2\right)}{\left(2\alpha^2+3\alpha+1\right)w_{\rm de}} \tag{29}$$

This is a second order differential equation and it depends on t. In order to get cosmological solution, there is also a simple relation between $H(t)$ and $H(z)$

$$\dot{H}(t) = \frac{d}{dt}H(t) = -\frac{dH(z)}{dz}H(z)(1+z) \tag{30}$$

Combine equation (28) (29) and (30) and relation (30), we can get the equation follows

$$3c^2H(z)^2 = \frac{m6^{n-1}(2n-1)\left(H(z)^2\right)^{n-1}\left(3(\alpha+1)H(z)^2 - (3\alpha+2)n(z+1)H(z)H'(z)\right)}{\left(2\alpha^2+3\alpha+1\right)w_{\rm de}} \tag{31}$$

In principle, we can get the form of $H(z)$ through the solution of above differential equation. However, solving higher-order differential equations analytically is difficult. So we consider $n = 1$ firstly to simplify calculation and set the initial condition $H(z=0) = H_0$ which denotes value of the Hubble parameter at present, get analytically solution as follow

$$H(z) = H_0(1+z)^{\frac{3(1+\alpha)\left(m - c^2 w_{\rm de}(1+2\alpha)\right)}{m(2+3\alpha)}} \tag{32}$$

We thus obtain the power-law evolution of the Universe which avoids the big-bang singularity similar to the $f(R,T)$ situation in (Singh & Kumar (2016b)). In this case, the deceleration factor is

$$q = -1 + \frac{3(\alpha+1)\left(m - (2\alpha+1)c^2 w_{\rm de}\right)}{(3\alpha+2)m} \tag{33}$$

Here we have a constant deceleration factor depending on the parameter. When we select a particular parameter value, it will show acceleration or deceleration characteristics. If $q < 0$, it will show the characteristics of accelerated expansion. However, since the deceleration factor is time independent, there is no phase transition in such a universe. So we can use Eq. (24) to obtain a tighter UV limit, if we set $\Delta = 1$ we can get

$$H(z) = H_0(1+z)^{\frac{3(\alpha+1)}{3\alpha+2}} - (1+2\alpha)c^2 w_{de}\left((1+z)^{\frac{3(\alpha+1)}{3\alpha+2}} - 1\right)\frac{1}{m} \tag{34}$$

In this case, the deceleration factor is

$$q = \frac{(2\alpha+1)c^2 w_{\rm de}\left(3\alpha + (z+1)^{\frac{3(\alpha+1)}{3\alpha+2}} + 2\right) - H_0 m (z+1)^{\frac{3(\alpha+1)}{3\alpha+2}}}{(3\alpha+2)\left((2\alpha+1)c^2 w_{\rm de}\left((z+1)^{\frac{3(\alpha+1)}{3\alpha+2}} - 1\right) - H_0 m (z+1)^{\frac{3(\alpha+1)}{3\alpha+2}}\right)} \tag{35}$$

in other situation, if $n \neq 1$ or $\Delta \neq 1$, higher-order differential equations are difficult to solve analytically, so we can only obtain numerical solutions through complex machine computing.

## 4. OBSERVATIONAL DATA AND METHODOLOGY

In this work, we estimate the cosmological parameters of the model by employing a Markov Chain Monte Carlo (MCMC) method based on the minimization of the chi-square function, $\chi^2$ ((Padilla et al. (2021))). The chi-square function is given by:

$$\chi^2 = \sum_i \left(\frac{D_i - T_i(\theta)}{\sigma_i}\right)^2, \tag{36}$$

where $D_i$ represents the $i$-th data point, $T_i(\theta)$ is the theoretical prediction for the corresponding quantity, and $\sigma_i$ is the error associated with the $i$-th data point. Here, $\theta$ denotes the vector of model parameters.

For our analysis, we combine three independent observational datasets:

1. Baryon Acoustic Oscillations (BAO): The BAO measurements provide a standard ruler for distance measurements in the universe. We use the data from the SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS) and other relevant surveys to constrain the cosmological parameters. The comoving horizon distance, the transverse comoving distance and the volume-averaged distance ombineing line-of-sight and transverse distances defined as follow

$$D_H = \frac{c}{H(z)} \tag{37}$$

$$D_M = \frac{D_L}{1+z} \tag{38}$$

$$D_V = \left[\frac{cz}{H(z)}\right]^{1/3}\left[\frac{D_L}{1+z}\right]^{2/3} \tag{39}$$

Where $D_L$ is the luminosity distance. When scaled by the sound horizon at the drag epoch $r_d$ , ratios such as $D_H/r_d$, $D_M/r_d$, and $D_V/r_d$ serve as important observables for constraining cosmological models and testing the standard model of cosmology.

| Survey | $z_{\rm eff}$ | $D_{\rm M}/r_{\rm d}$ | $D_{\rm H}/r_{\rm d}$ | $D_{\rm V}/r_{\rm d}$ |
|---|---|---|---|---|
| 6dFGS | 0.106 | | | $2.98 \pm 0.13$ |
| SDSS MGS | 0.15 | | | $4.51 \pm 0.14$ |
| SDSS DR12 | 0.38 | $10.27 \pm 0.15$ | $24.89 \pm 0.58$ | |
| SDSS DR12 | 0.51 | $13.38 \pm 0.18$ | $22.43 \pm 0.48$ | |
| SDSS DR16 LRG | 0.70 | $17.65 \pm 0.30$ | $19.78 \pm 0.46$ | |
| SDSS DR16 ELG | 0.85 | $19.50 \pm 1.00$ | $19.60 \pm 2.10$ | |
| SDSS DR16 QSO | 1.48 | $30.21 \pm 0.79$ | $13.23 \pm 0.47$ | |
| SDSS DR16 Ly$\alpha$–Ly$\alpha$ | 2.33 | $37.60 \pm 1.90$ | $8.93 \pm 0.28$ | |
| SDSS DR16 Ly$\alpha$–QSO | 2.33 | $37.30 \pm 1.70$ | $9.08 \pm 0.34$ | |
| DESI BGS | 0.30 | | | $7.93 \pm 0.15$ |
| DESI LRG1 | 0.51 | $13.62 \pm 0.25$ | $20.98 \pm 0.61$ | |
| DESI LRG2 | 0.71 | $16.85 \pm 0.32$ | $20.08 \pm 0.60$ | |
| DESI LRG+ELG | 0.93 | $21.71 \pm 0.28$ | $17.88 \pm 0.35$ | |
| DESI ELG | 1.32 | $27.79 \pm 0.69$ | $13.82 \pm 0.42$ | |
| DESI QSO | 1.49 | | | $26.07 \pm 0.67$ |
| DESI Ly$\alpha$–QSO | 2.33 | $39.71 \pm 0.94$ | $8.52 \pm 0.17$ | |

**Table 1.** BAO dataset. This table is referenced fromt Luongo & Muccino (2024), including 6dFGS data (Beutler et al. (2011)), SDSS data (Alam et al. (2021)) and DESI2024 BAO data (Collaboration et al. (2024))

2. Chronometers Data (OHD): The Hubble parameter measurements, known as the chronometers data, provide independent estimates of the Hubble parameter $H(z)$ at various redshifts. These data serve as an important probe of the expansion rate of the universe. We choose the dataset from Favale et al. (2023)

3. Type Ia Supernovae (SNIa) Data: Type Ia supernovae (SNIa) are considered standard candles because When the light curve reaches its maximum, the absolute luminosity is almost the same. The distance modulus $\mu$ can be obtained according to the following formula

$$\mu_{obs} = m - M \tag{40}$$

On the other hand, we can get the theoretical distance modulus from the cosmological model

$$\mu_{th}(z) = 5 \log_{10} d_L(z) + 25 + M_b \tag{41}$$

where $M_b$ denotes the absolute luminosity of SNIa and the luminosity distance is defined as

$$d_L(z) = \frac{c}{H_0}(1+z) \int_0^z \frac{dz'}{E(z')} \tag{42}$$

In this paper, we use Pantheon+ dataset who comprises 1701 SNIa samples, an increase from the 1048 samples in Pantheon dataset. Pantheon+ dataset consists of 1701 light curves of 1550 spectroscopically confirmed SNIa within the redshift range of $0.001 < z < 2.26$ (Scolnic et al. (2022); Brout et al. (2022)).

The combined likelihood function $\mathcal{L}$ is then constructed by multiplying the individual likelihoods of each dataset:

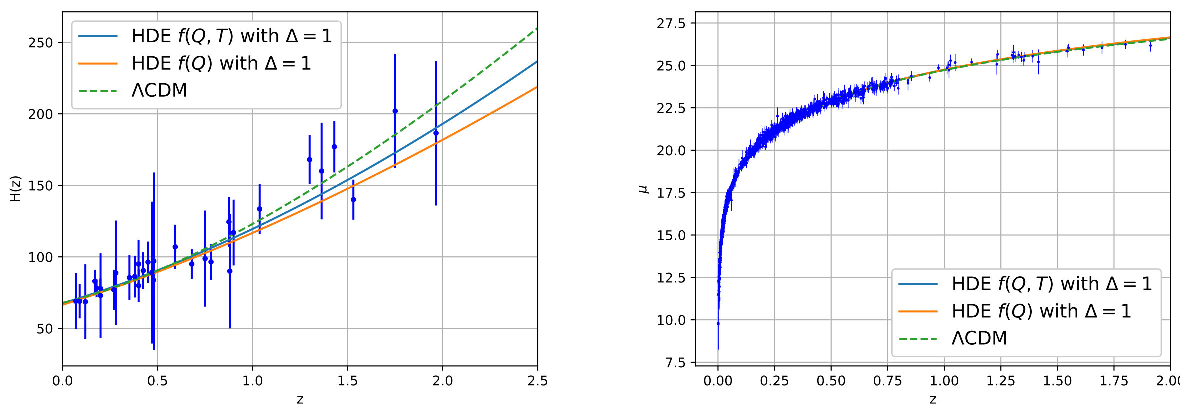
$$\mathcal{L} = \mathcal{L}_{\text{BAO}} \times \mathcal{L}_{\text{OHD}} \times \mathcal{L}_{\text{SNIa}} \tag{43}$$

it is implied that

$$\chi^2_{\text{tot}} = \chi^2_{\text{BAO}} + \chi^2_{\text{OHD}} + \chi^2_{\text{SNIa}} \tag{44}$$

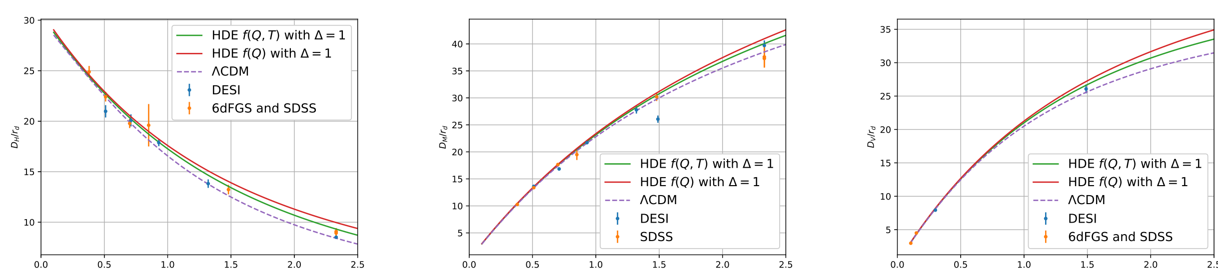
In order to test the statistical significance of our constraints we implement the AIC criterion. To complete our statistical study, we also use the selection method named the Bayesian Information Criterion (BIC). To calculate the value of BIC for each model we use

## 5. RESULTS OF PARAMETER CONSTRAINT




(a) name1 (b) name2

**Figure 1.** Main name




(a) name1 (b) name2 (c) name3

**Figure 2.** Main name

The results show that our results may alleviate the Hubble tension

## 6. COSMIC EVOLUTION

The deceleration facter is defined as

$$q(z) = -(1+z)H(z)\frac{d}{dz}\left(\frac{1}{H}\right) - 1 \tag{45}$$

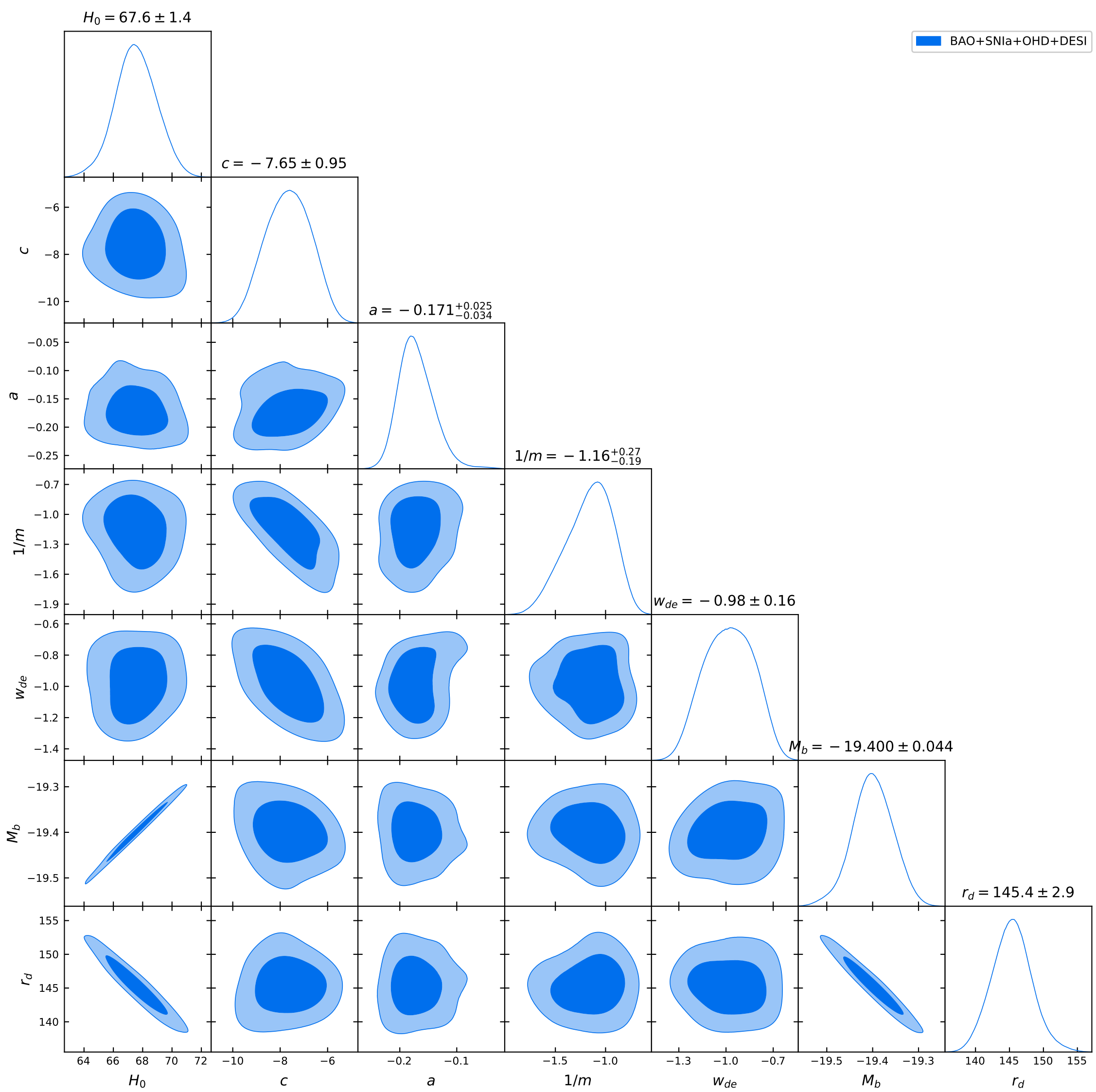



**Figure 3.** The $1\sigma$ and $2\sigma$ confidence contours and the 1D posterior distributions obtained from MCMC constraint of HDE in $f(Q,T)$ gravity using BAO+SNIa+OHD+DESI data

# 7. CONCLUSION

# APPENDIX

# A. APPENDIX INFORMATION

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