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Evolution of holographic dark energy models in $f(Q,T)$ gravity and cosmic constraint

Xuwei Zhang [1] and Xiaofeng Yang [1]

[1] *Xinjiang Astronomical Observatory*
1667 K Street NW, Suite 800
Urumqi, Xinjiang, 830046, China

ABSTRACT

We study a holographic dark energy model in $f(Q,T)$ gravity

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1. INTRODUCTION

Over the past decaeds, a series of discoveries in cosmology have profoundly changed our understanding of the universe. In 1998, the accelerated expansion of the universe was discovered through the study of Type Ia supernovae(Perlmutter et al. (1998); Riess et al. (1998)). This fact had been later confirmed by many other cosmological observations, such as the measurement of temperature anisotropy and polarization in the cosmic microwave background (CMB) radiation(Smoot et al. (1992); Aghanim et al. (2020)); Baryon acoustic oscillations (Baryon Acoustic Oscillations, BAO) peak length scale(Eisenstein et al. (2005); Blake et al. (2011)); the study of the large-scale structure (LSS) of the universe(Dodelson et al. (2002); Percival et al. (2007)) and use Cosmic Chronometers to direct measurement of Hubble parameter(Stern et al. (2010); Moresco (2015)). These observations suggest the existence of a mysterious energy in our universe, also named dark energy (DE) who has high negative pressure and increasing density. Dark energy behaves as anti-gravity, but its nature remains unknown.

Theoretical predictions and astronomical observations indicate that there may be a mysterious form of energy in the universe. This energy has the characteristics of negative pressure, and its density increases over time. This is considered to be the key factor driving the accelerated expansion of the universe, accounting for about three-quarters of the total energy of the universe (Ratra & Peebles (1988); Armendariz-Picon et al. (2001); Tomita (2001)). In order to achieve such accelerated expansion, this form of energy needs to produce an anti-gravitational effect throughout the observable universe. However, ordinary baryonic matter neither has this equation of state nor can it explain such a large proportion of the cosmic energy component. Therefore, scientists have proposed and studied a variety of alternative theories and models to explore the nature of this cosmic acceleration phenomenon.

The simplest and most widely accepted theory is ΛCDM model, where Λ means cosmological constant predicted by Einstein(Carroll (2001)). Based on ΛCDM model, the lastest observations suggest that our universe consists of 68.3% dark energy, 26.8% cold dark matter and 4.9% ordinary matter (Aghanim et al. (2020)). However, this model is not free from problems and the problems it is facing are cosmic coincidence, fine-tuning and the Hubble tension—a discrepancy between the value of the Hubble constant H_0 inferred from the CMB by the Planck satellite and that obtained from local measurements using Type Ia supernovae—has sparked significant debate.

Another interesting attempt is to deviate from general ralativity toward a modified form (detailed research progress can be reviewid in Clifton et al. (2012)). These theories assume that general relativity not work in large scale requiring a modification in action rather than standard Einstein-Hilbert action. The most well-known is $F(R)$ gravity which replaces the Ricci scalar R in the action by a general function $f(R)$ (Buchdahl (1970)). Another modified theory of gravity $f(T)$ extends the teleparallel equivalent of General Relativity (TEGR). It replaces the curvature scalar R in action with the torsion scalar T, derived from the Weitzenböck connection. Also shows some interpretations for the accelerating phases of our Universe(Cai et al. (2016); Bengochea & Ferraro (2009)). $f(Q)$ is generalized symmetric teleparallel gravity, with curvature and torsion both being zero, which is inspired by Weyl and Einstein's trial to unify electromagenetic and gravity. The geometric properties of gravity are described by "non-metricity". That is, the covariant derivative of the metric tensor is no longer zero (some detailed information can be found in review Heisenberg (2024)).

Holographic dark energy is an famous alternative theory for the interpretation of dark energy, originating from the holographic principle.

In this article, we assume that

2. $F(Q,T)$ GRAVITY THEORY AND HOLOGRAPHIC DARK ENERGY

In Weyl-Cartan geometry the connection can be decomposed into three parts: the Christoffel symbol $\hat{\Gamma}^{\alpha}{}_{\mu\nu}$, the contortion tensor $K^{\alpha}{}_{\mu\nu}$ and the disformation tensor $L^{\alpha}{}_{\mu\nu}$, so that the general affine connection can be expressed as

$$\Gamma^{\alpha}{}_{\mu\nu} = \hat{\Gamma}^{\alpha}{}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu} \tag{1}$$

The first term $\hat{\Gamma}^{\alpha}{}_{\mu\nu}$ is the Levi-civita connection of metric $g_{\mu\nu}$, given by

$$\hat{\Gamma}^{\alpha}{}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\epsilon} - \partial_\beta g_{\epsilon\nu}) \tag{2}$$

The second term $K^{\alpha}{}_{\mu\nu}$ is the contortion tensor

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2}T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{,\alpha}{}_{\nu)} \tag{3}$$

The Last term is distortion tensor $L^{\alpha}{}_{\mu\nu}$, given by

$$L^{\alpha}{}_{\mu\nu} = \frac{1}{2}Q^{\alpha}{}_{\mu\nu} - Q_{(\mu}{}^{;\alpha}{}_{\nu)} \tag{4}$$

Enclose $Q^{\alpha}{}_{\mu\nu}$with nonmetricity tensor

$$Q_{\rho\mu\nu} \equiv \nabla_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - \Gamma^{\beta}{}_{\rho\ mu} g_{\beta\nu} - \Gamma^{\beta}{}_{\rho\nu} g_{\mu\beta} \ , \tag{5}$$

$$T^{\lambda}{}_{\mu\nu} \equiv \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu} \ , \tag{6}$$

$$R^{\sigma}{}_{\rho\mu\nu} \equiv \partial_\mu \Gamma^{\sigma}{}_{\nu\rho} - \partial_\nu \Gamma^{\sigma}{}_{\mu\rho} + \Gamma^{\alpha}{}_{\nu\rho}\Gamma^{\sigma}{}_{\mu\alpha} - \Gamma^{\alpha}{}_{\mu\rho}\Gamma^{\sigma}{}_{\nu\alpha} \ , \tag{7}$$

In weyl geometry, affine connection is not compatible with the metric tensor as

$$Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu} = -w_\alpha g_{\mu\nu} \tag{8}$$

In gravity, we consider the action is

$$S = \int (\frac{1}{2}f(Q,T) + \mathcal{L}_m)\sqrt{-g}d^4x \tag{9}$$

where f is an arbitrary function of the non-metricity and T is the trace of the matter-energy-momentum tensor, $\mathcal{L}_m$ is known as matter Lagrangian and $g = \det(g_{\mu\nu})$ denotes determinant of metric tensor.

Vary the action we can get

$$\delta S = \int \left(\frac{1}{2}\delta[f(Q,T)\sqrt{-g}] + \delta(\mathcal{L}_m\sqrt{-g})\right) d^4x \tag{10}$$

$$= \int \frac{1}{2}\left(-\frac{1}{2}f g_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu} + f_Q\sqrt{-g}\delta Q + f_T\sqrt{-g}\delta T - \frac{1}{2}T_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\right) d^4x \tag{11}$$

Field equation is

$$-\frac{2}{\sqrt{-g}}\nabla_\alpha(f_Q\sqrt{-g}P^{\alpha}{}_{\mu\nu}) - \frac{1}{2}f g_{\mu\nu} + f_T(T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q(P_{\mu\alpha\beta}Q^{\nu}{}_{\alpha\beta} - 2Q^{\alpha\beta}{}_{\mu}P_{\alpha\beta\nu}) = T_{\mu\nu} \tag{12}$$

In FLRW metric, given by

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \tag{13}$$

then we can get Friedmann equations

$$\rho = \frac{f}{2} - 6f_Q H^2 - \frac{2f_T}{1+f_T}(\dot{f}_Q H + f_Q \dot{H}) \tag{14}$$

$$p = -\frac{f}{2} + 6f_Q H^2 + 2(\dot{f}_Q H + f_Q \dot{H}) \tag{15}$$

EoS parameter

$$w = \frac{p}{\rho} = -1 + \frac{4f_Q H + f_Q \dot{H}}{(1+f_T)(f - 12f_Q H^2) - 4f_T(\dot{f}_Q H + f_Q \dot{H})} \tag{16}$$

In our universe $w < -1/3$, where ρ and p denote total fluid energy density and pressure of the universe, which $\rho = \rho_m + \rho_{de}$, $p = p_m + p_{de}$.

Effective EoS parameter denote geometry qualities.

$$\rho_{\text{eff}} = 3H^2 = \frac{f}{4f_Q} - \frac{1}{2f_Q}[(1+f_T)\rho + f_T p] \tag{17}$$

$$-p_{\text{eff}} = 2\dot{H} + 3H^2 = \frac{f}{4f_Q} - \frac{2\dot{f}_Q H}{f_Q} + \frac{1}{2f_Q}[(1+f_T)\rho + (2+f_T)p] \tag{18}$$

EoS parameter for equivalent dark energy

$$w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = -\frac{f - 8\dot{f}_Q H + 2[(1+f_T)\rho + (2+f_T)p]}{f - 2[(1+f_T)\rho + f_T p]} \tag{19}$$

Combine, we can get the evolution equation of Hubble parameter H Deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(1+3w) = \frac{1}{2}\left(1 + 3\frac{p_{\text{eff}}}{\rho_{\text{eff}}}\right) = -1 + \frac{3(4\dot{f}_Q H - f + p)}{f - [(1+f_T)\rho + f_T p]} \tag{20}$$

We assume that

$$f(Q,T) = \alpha Q^n + \beta T \tag{21}$$

where $Q = 6H^2$, $T = -\rho + 3p$, so that we can derive $f_Q = \alpha n Q^{n-1} = \alpha n 6^{n-1} H^{2n-2}$, $f_T = \beta$, $\dot{f}_Q = 2\alpha n(n-1)6^{n-1}H^{2n-3}\dot{H}$

3. COSMIC SOLUTIONS

Hybrid Expansion Law (HEL)

$$H(z) = \frac{H_0}{\sqrt{2}}\sqrt{1+(1+z)^{2m}} \tag{22}$$

$$\dot{H}(z) = \frac{H_0}{\sqrt{2}}\frac{2m(1+z)^{2m-1}}{\sqrt{1+(1+z)^{2m}}} \tag{23}$$

use deceleration parameter to simplify calculation

$$\dot{H} = -H^2(1+q) \tag{24}$$

get

$$\rho = \frac{f}{2} - 2f_Q(-H^2(1+q) + 3H^2 + \frac{f_T}{1+f_T}\frac{\dot{f}_Q}{f_Q}H) \tag{25}$$

$$p = -\frac{f}{2} + 2f_Q(-H^2(1+q) + 3H^2 + \frac{\dot{f}_Q}{f_Q}H) \tag{26}$$

Holographic dark energy with Hubble cutoff is

$$\rho_{de} = 3c^2H^2 \tag{27}$$

$$p_{de} = \frac{\alpha\left(6H(z)^2\right)^{n-1}\left(2n(-2\beta+(3\beta+2)n-1)\dot{H}(z)+3(\beta+1)(2n-1)H(z)^2\right)}{(\beta+1)(2\beta+1)} \tag{28}$$

dark energy EoS parameter

$$w_{de} = \frac{p_{de}}{\rho_{de}} = \frac{\alpha\left(6H(z)^2\right)^{n-1}\left(2n(-2\beta+(3\beta+2)n-1)\dot{H}(z)+3(\beta+1)(2n-1)H(z)^2\right)}{3c^2H^2(\beta+1)(2\beta+1)} \tag{29}$$

where $\dot{H}(z) = \frac{d}{dt}H(z) = -\frac{dH(z)}{dz}H(z)(1+z)$

stability parameter of in this situation can be described as

$$c_s^2 = \frac{\mathrm{d}p_{de}}{\mathrm{d}\rho_{de}} = \frac{\alpha(2n(-2\beta+(3\beta+2)n-1))6^{n-1}((2n-2)H^{2n-3}\dot{H}^2+\ddot{H}H^{2n-2})+\alpha 3(\beta+1)(2n-1)6^{n-1}2nH^{2n-1}\dot{H}}{6c^2H\dot{H}(\beta+1)(2\beta+1)} \tag{30}$$

In principle, we can get the form of $H(z)$ through the solution of differential equation. However, solving higher-order differential equations analytically is difficult. So we assume $n = 1$ first to simplify calculation and get analytically solution as follow

$$H(z) = H_0(z+1)^{\frac{3(\alpha+1)}{3\alpha+2}} - (1+2\alpha)c^2w_{de}\left((z+1)^{\frac{3(\alpha+1)}{3\alpha+2}}-1\right) \tag{31}$$

in other situation, if $n \neq 1$

4. TSALIS ENTROPY DARK ENERGY IN DIFFERENT IR CUTOFF

Tsallis proposed a modified black hole entropy

$$S_\delta = \gamma A^\delta \tag{32}$$

Tsallis holographic dark energy density

$$\rho_{de} = BH^{4-2\delta} \tag{33}$$

5. OBSERVATIONAL DATA AND PARAMETER CONSTRAINT

6. CONCLUSION

APPENDIX

A. APPENDIX INFORMATION

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