Modern Physics Letters A

Holographic dark energy models in $f(Q,T)$ gravity

Xuwei Zhang*
University Department, University Name, Address
City, State ZIP/Zone, Country†
author@emailaddress

Second Author
Group, Laboratory, Address
City, State ZIP/Zone, Country

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1. Introduction

Over the past decaeds, a series of discoveries in cosmology have profoundly changed our understanding of the universe. In 1998, the accelerated expansion of the universe was discovered through the study of Type Ia supernovae([?,?]). This fact had been later confirmed by many other cosmological observations, such as the measurement of temperature anisotropy and polarization in the cosmic microwave background (CMB) radiation([?,?]); Baryon acoustic oscillations (Baryon Acoustic Oscillations, BAO) peak length scale([?,?]); the study of the large-scale structure (LSS) of the universe([?,?]) and use Cosmic Chronometers to direct measurement of Hubble parameter([?,?]). These observations suggest the existence of a mysterious energy in our universe who has high negative pressure and increasing density.

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2. $f(Q,T)$ gravity theory and Holographic dark energy

In Weyl-Cartan geometry the connection can be decomposed into three parts: the Christoffel symbol $\hat{\Gamma}^{\alpha}{}_{\mu\nu}$, the contortion tensor $K^{\alpha}{}_{\mu\nu}$ and the disformation tensor $L^{\alpha}{}_{\mu\nu}$, so that the general affine connection can be expressed as

$$\Gamma^{\alpha}{}_{\mu\nu} = \hat{\Gamma}^{\alpha}{}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu} \tag{1}$$

The first term $\hat{\Gamma}^{\alpha}{}_{\mu\nu}$ is the Levi-civita connection of metric $g_{\mu\nu}$, given by

$$\hat{\Gamma}^{\alpha}{}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\partial_{\mu}g_{\beta\nu} + \partial_{\nu}g_{\beta\epsilon} - \partial_{\beta}g_{\epsilon\nu}) \tag{2}$$

The second term $K^{\alpha}{}_{\mu\nu}$ is the contortion tensor

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2}T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{;\alpha}{}_{\nu)} \tag{3}$$

The Last term is distortion tensor $L^{\alpha}{}_{\mu\nu}$, given by

$$L^{\alpha}{}_{\mu\nu} = \frac{1}{2}Q^{\alpha}{}_{\mu\nu} - Q_{(\mu}{}^{;\alpha}{}_{\nu)} \tag{4}$$

Enclose $Q^{\alpha}{}_{\mu\nu}$with nonmetricity tensor

$$Q_{\rho\mu\nu} \equiv \nabla_{\rho}g_{\mu\nu} = \partial_{\rho}g_{\mu\nu} - \Gamma^{\beta}{}_{\rho\ mu}g_{\beta\nu} - \Gamma^{\beta}{}_{\rho\nu}g_{\mu\beta}\ , \tag{5}$$

$$T^{\lambda}{}_{\mu\nu} \equiv \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}\ , \tag{6}$$

$$R^{\sigma}{}_{\rho\mu\nu} \equiv \partial_{\mu}\Gamma^{\sigma}{}_{\nu\rho} - \partial_{\nu}\Gamma^{\sigma}{}_{\mu\rho} + \Gamma^{\alpha}{}_{\nu\rho}\Gamma^{\sigma}{}_{\mu\alpha} - \Gamma^{\alpha}{}_{\mu\rho}\Gamma^{\sigma}{}_{\nu\alpha}\ , \tag{7}$$

In weyl geometry, affine connection is not compatible with the metric tensor as

$$Q_{\alpha\mu\nu} = \nabla_{\alpha}g_{\mu\nu} = -w_{\alpha}g_{\mu\nu} \tag{8}$$

In gravity, we consider the action is

$$S = \int (\frac{1}{2}f(Q,T) + \mathcal{L}_m)\sqrt{-g}d^4x \tag{9}$$

where f is an arbitrary function of the non-metricity and T is the trace of the matter-energy-momentum tensor, $\mathcal{L}_m$ is known as matter Lagrangian and $g = \det(g_{\mu\nu})$ denotes determinant of metric tensor.

Vary the action we can get

$$\delta S = \int \left(\frac{1}{2}\delta[f(Q,T)\sqrt{-g}] + \delta(\mathcal{L}_m\sqrt{-g})\right) d^4x \tag{10}$$

$$= \int \frac{1}{2}\left(-\frac{1}{2}fg_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu} + f_Q\sqrt{-g}\delta Q + f_T\sqrt{-g}\delta T - \frac{1}{2}T_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\right) d^4x \tag{11}$$

Field equation is

$$-\frac{2}{\sqrt{-g}}\nabla_{\alpha}(f_Q\sqrt{-g}P^{\alpha}{}_{\mu\nu}) - \frac{1}{2}fg_{\mu\nu} + f_T(T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q(P_{\mu\alpha\beta}Q^{\nu}{}_{\alpha\beta} - 2Q^{\alpha\beta}{}_{\mu}P_{\alpha\beta\nu}) = T_{\mu\nu} \tag{12}$$

In FLRW metric, given by

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \tag{13}$$

then we can get Friedmann equations

$$\rho = \frac{f}{2} - 6f_Q H^2 - \frac{2f_T}{1+f_T}(\dot{f}_Q H + f_Q \dot{H}) \tag{14}$$

$$p = -\frac{f}{2} + 6f_Q H^2 + 2(\dot{f}_Q H + f_Q \dot{H}) \tag{15}$$

EoS parameter

$$w = \frac{p}{\rho} = -1 + \frac{4f_Q H + f_Q \dot{H}}{(1+f_T)(f - 12f_Q H^2) - 4f_T(\dot{f}_Q H + f_Q \dot{H})} \tag{16}$$

In our universe $w < -1/3$, where ρ and p denote total fluid energy density and pressure of the universe, which $\rho = \rho_m + \rho_{de}$, $p = p_m + p_{de}$.

Effective EoS parameter denote geometry qualities.

$$\rho_{\text{eff}} = 3H^2 = \frac{f}{4f_Q} - \frac{1}{2f_Q}[(1+f_T)\rho + f_T p] \tag{17}$$

$$-p_{\text{eff}} = 2\dot{H} + 3H^2 = \frac{f}{4f_Q} - \frac{2\dot{f}_Q H}{f_Q} + \frac{1}{2f_Q}[(1+f_T)\rho + (2+f_T)p] \tag{18}$$

EoS parameter for equivalent dark energy

$$w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = -\frac{f - 8\dot{f}_Q H + 2[(1+f_T)\rho + (2+f_T)p]}{f - 2[(1+f_T)\rho + f_T p]} \tag{19}$$

Combine, we can get the evolution equation of Hubble parameter H Deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(1+3w) = \frac{1}{2}\left(1 + 3\frac{p_{\text{eff}}}{\rho_{\text{eff}}}\right) = -1 + \frac{3(4\dot{f}_Q H - f + p)}{f - [(1+f_T)\rho + f_T p]} \tag{20}$$

We assume that

$$f(Q,T) = \alpha Q^n + \beta T \tag{21}$$

where $Q = 6H^2$, $T = -\rho + 3p$, so that we can derive $f_Q = \alpha n Q^{n-1} = \alpha n 6^{n-1} H^{2n-2}$, $f_T = \beta$, $\dot{f}_Q = 2\alpha n(n-1)6^{n-1}H^{2n-3}\dot{H}$

3. Cosmic solutions

Hybrid Expansion Law (HEL)

$$H(z) = \frac{H_0}{\sqrt{2}}\sqrt{1 + (1+z)^{2m}} \tag{22}$$

$$\dot{H}(z) = \frac{H_0}{\sqrt{2}}\frac{2m(1+z)^{2m-1}}{\sqrt{1+(1+z)^{2m}}} \tag{23}$$

use deceleration parameter to simplify calculation

$$\dot{H} = -H^2(1+q) \tag{24}$$

get

$$\rho = \frac{f}{2} - 2f_Q(-H^2(1+q) + 3H^2 + \frac{f_T}{1+f_T}\frac{\dot{f}_Q}{f_Q}H) \tag{25}$$

$$p = -\frac{f}{2} + 2f_Q(-H^2(1+q) + 3H^2 + \frac{\dot{f}_Q}{f_Q}H) \tag{26}$$

Holographic dark energy with Hubble cutoff is

$$\rho_{de} = 3c^2H^2 \tag{27}$$

$$p_{de} = \frac{\alpha\left(6H(z)^2\right)^{n-1}\left(2n(-2\beta+(3\beta+2)n-1)\dot{H}(z) + 3(\beta+1)(2n-1)H(z)^2\right)}{(\beta+1)(2\beta+1)} \tag{28}$$

dark energy EoS parameter

$$w_{de} = \frac{p_{de}}{\rho_{de}} = \frac{\alpha\left(6H(z)^2\right)^{n-1}\left(2n(-2\beta+(3\beta+2)n-1)\dot{H}(z) + 3(\beta+1)(2n-1)H(z)^2\right)}{3c^2H^2(\beta+1)(2\beta+1)} \tag{29}$$

where $\dot{H}(z) = \frac{d}{dt}H(z) = -\frac{dH(z)}{dz}H(z)(1+z)$

stability parameter of in this situation can be described as

$$c_s^2 = \frac{\mathrm{d}p_{de}}{\mathrm{d}\rho_{de}} = \frac{\alpha(2n(-2\beta+(3\beta+2)n-1))6^{n-1}((2n-2)H^{2n-3}\dot{H}^2 + \ddot{H}H^{2n-2}) + \alpha 3(\beta+1)(2n-1)6^{n-1}2nH^{2n}}{6c^2H\dot{H}(\beta+1)(2\beta+1)} \tag{30}$$

4. Tsalis entropy dark energy in different IR cutoff

Tsallis proposed a modified black hole entropy

$$S_\delta = \gamma A^\delta \tag{31}$$

Tsallis holographic dark energy density

$$\rho_{de} = BH^{4-2\delta} \tag{32}$$

5. Age of the universe

6. Observational data and constraint

7. Conclusion

Appendix A. Appendix information