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# Holographic dark energy models in $f(Q,T)$ gravity and cosmic constraint

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ABSTRACT

We study a holographic dark energy model in $f(Q,T)$ gravity

*Keywords:* Cosmology

## 1. INTRODUCTION

Over the past decaeds, a series of discoveries in cosmology have profoundly changed our understanding of the universe. In 1998, the accelerated expansion of the universe was discovered through the study of Type Ia supernovae(Perlmutter et al. (1998); Riess et al. (1998)). This fact had been later confirmed by many other cosmological observations, such as the measurement of temperature anisotropy and polarization in the cosmic microwave background (CMB) radiation(Smoot et al. (1992); Aghanim et al. (2020)); Baryon acoustic oscillations (Baryon Acoustic Oscillations, BAO) peak length scale(Eisenstein et al. (2005); Blake et al. (2011)); the study of the large-scale structure (LSS) of the universe(Dodelson et al. (2002); Percival et al. (2007)) and use Cosmic Chronometers to direct measurement of Hubble parameter(Stern et al. (2010); Moresco (2015)). These observations suggest the existence of a mysterious energy in our universe, also named dark energy (DE) who has high negative pressure and increasing density. Dark energy behaves as anti-gravity, but its nature remains unknown.

Theoretical predictions and astronomical observations indicate that there may be a mysterious form of energy in the universe. This energy has the characteristics of negative pressure, and its density increases over time. This is considered to be the key factor driving the accelerated expansion of the universe, accounting for about three-quarters of the total energy of the universe (Ratra & Peebles (1988); Armendariz-Picon et al. (2001); Tomita (2001)). In order to achieve such accelerated expansion, this form of energy needs to produce an anti-gravitational effect throughout the observable universe. However, ordinary baryonic matter neither has this equation of state nor can it explain such a large proportion of the cosmic energy component. Therefore, scientists have proposed and studied a variety of alternative theories and models to explore the nature of this cosmic acceleration phenomenon.

The simplest and most widely accepted theory is $\Lambda$CDM model, where $\Lambda$ means cosmological constant predicted by Einstein(Carroll (2001)). Based on $\Lambda$CDM model, the lastest observations suggest that our universe consists of 68.3% dark energy, 26.8% cold dark matter and 4.9% ordinary matter (Aghanim et al. (2020)). However, this model is not free from problems and the problems it is facing are cosmic coincidence, fine-tuning and the Hubble tension—a discrepancy between the value of the Hubble constant $H_0$ inferred from the CMB by the Planck satellite and that obtained from local measurements using Type Ia supernovae—has sparked significant debate.

Another interesting attempt is to deviate from general ralativity toward a modified form (detailed research progress can be reviewid in Clifton et al. (2012)). These theories assume that general relativity not work in large scale requiring a modification in action rather than standard Einstein-Hilbert action. The most well-known is $F(R)$ gravity which replaces the Ricci scalar $R$ in the action by a general function $f(R)$ (Buchdahl (1970)). The $f(G)$ gravity theory is also a modified theory of gravity that introduces a correction to the Gauss–Bonnet (GB) term $G$, allowing it to be arbitrary function $f(G)$ rather than remaining a constant(Nojiri & Odintsov (2005, 2007)). Another modified theory of gravity $f(T)$ extends the teleparallel equivalent of General Relativity (TEGR). It replaces the curvature scalar $R$ in action with the torsion scalar $T$, derived from the Weitzenböck connection. Also shows some interpretations for the accelerating phases of our Universe(Cai et al. (2016); Bengochea & Ferraro (2009)). $f(Q)$ is generalized symmetric teleparallel gravity, with curvature and torsion both being zero, which is inspired by Weyl and Einstein's trial to unify electromagenetic and gravity. The geometric properties of gravity are described by "non-metricity". That is, the covariant derivative of the metric tensor is no longer zero (some detailed information can be found in review Heisenberg (2024)). Harko et al. have proposed a new theory known as $f(R,T)$ gravity, where $R$ stands for the Ricci scalar and $T$ denotes the trace of energy-momentum tensor which presents a non-minimum coupling between geometry and matter(Harko et al. (2011)). Similar theories are introduced, $f(Q,T)$ proposed by Xu et al.(Xu et al. (2019)); $f(Q,C)$ gravity(De et al. (2024)); $f(\mathcal{T},T)$ proposed by(Harko et al. (2014)); f(T,B) gravity(Bahamonde et al. (2015); Bahamonde & Capozziello (2017)) ;$f(R,T^2)$ proposed by Katırcı et al.(Katırcı & Kavuk (2014)), etc.

Holographic dark energy is an famous alternative theory for the interpretation of dark energy, originating from the holographic principle proposed by 't Hooft('t Hooft (1993)). Cohen et al. introduced the "UV-IR" relationship, highlighting that in effective quantum field theory, a system of size $L$ has its entropy and energy constrained by the Bekenstein entropy bound and black hole mass, respectively. This implies that quantum field theory is limited to describing low-energy physics outside black holes(Cohen et al. (1999)). After that, Li et al. proposed that the infrared cut-off relevant to the dark energy is the size of the event horizon and obtained the dark energy density can be described as $\rho_{\rm de} = 3c^2M_p^2R_h^2$ where $R_h$ is future horizon of our universe(Li (2004)). Although choose Hubble cut-off is a natural thought, but Hsu found it might lead to wrong state equation and be strongly disfavored by observational data(Hsu (2004)).

Some scholars combine or reconstruct HDE and modified theory

In this article, we assume that our universe is in a $f(Q,T)$ gravity and HDE exists objectively, we will...

## 2. $F(Q,T)$ GRAVITY THEORY AND HOLOGRAPHIC DARK ENERGY

In Weyl-Cartan geometry the connection can be decomposed into three parts: the Christoffel symbol $\hat{\Gamma}^{\alpha}{}_{\mu\nu}$, the contortion tensor $K^{\alpha}{}_{\mu\nu}$ and the disformation tensor $L^{\alpha}{}_{\mu\nu}$, so that the general affine connection can be expressed as

$$\Gamma^{\alpha}{}_{\mu\nu} = \hat{\Gamma}^{\alpha}{}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu} \tag{1}$$

The first term $\hat{\Gamma}^{\alpha}{}_{\mu\nu}$ is the Levi-civita connection of metric $g_{\mu\nu}$, given by

$$\hat{\Gamma}^{\alpha}{}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\epsilon} - \partial_\beta g_{\epsilon\nu}) \tag{2}$$

The second term $K^{\alpha}{}_{\mu\nu}$ is the contortion tensor

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2}T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{,\alpha}{}_{\nu)} \tag{3}$$

The Last term is distortion tensor $L^{\alpha}{}_{\mu\nu}$, given by

$$L^{\alpha}{}_{\mu\nu} = \frac{1}{2}Q^{\alpha}{}_{\mu\nu} - Q_{(\mu}{}^{,\alpha}{}_{\nu)} \tag{4}$$

Enclose $Q^{\alpha}{}_{\mu\nu}$with nonmetricity tensor

$$Q_{\rho\mu\nu} \equiv \nabla_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - \Gamma^{\beta}{}_{\rho\ mu} g_{\beta\nu} - \Gamma^{\beta}{}_{\rho\nu} g_{\mu\beta}\ , \tag{5}$$

$$T^{\lambda}{}_{\mu\nu} \equiv \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}\ , \tag{6}$$

$$R^{\sigma}{}_{\rho\mu\nu} \equiv \partial_\mu \Gamma^{\sigma}{}_{\nu\rho} - \partial_\nu \Gamma^{\sigma}{}_{\mu\rho} + \Gamma^{\alpha}{}_{\nu\rho}\Gamma^{\sigma}{}_{\mu\alpha} - \Gamma^{\alpha}{}_{\mu\rho}\Gamma^{\sigma}{}_{\nu\alpha}\ , \tag{7}$$

In weyl geometry, affine connection is not compatible with the metric tensor as

$$Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu} = -w_\alpha g_{\mu\nu} \tag{8}$$

We consider the general form of the Einstein-Hilbert action for the $f(Q,T)$ gravity in the unit $8\pi G = 1$

$$S = \int (\frac{1}{2}f(Q,T) + \mathcal{L}_m)\sqrt{-g}d^4x \tag{9}$$

where $f$ is an arbitrary function of the non-metricity , $\mathcal{L}_m$ is known as matter Lagrangian, $g = \det(g_{\mu\nu})$ denotes determinant of metric tensor, and $T = g^{\mu\nu}T_{\mu\nu}$ is the trace of the matter-energy-momentum tensor, where $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}} \tag{10}$$

Vary the action (9) with respect to the metric tensor $g_{\mu\nu}$ we can get

$$\delta S = \int \left(\frac{1}{2}\delta[f(Q,T)\sqrt{-g}] + \delta(\mathcal{L}_m\sqrt{-g})\right) d^4x \tag{11}$$

$$= \int \frac{1}{2}\left(-\frac{1}{2}fg_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu} + f_Q\sqrt{-g}\delta Q + f_T\sqrt{-g}\delta T - \frac{1}{2}T_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\right) d^4x \tag{12}$$

Field equation is

$$-\frac{2}{\sqrt{-g}}\nabla_\alpha(f_Q\sqrt{-g}P^\alpha{}_{\mu\nu}) - \frac{1}{2}fg_{\mu\nu} + f_T(T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q(P_{\mu\alpha\beta}Q^\nu{}_{\alpha\beta} - 2Q^{\alpha\beta}{}_\mu P_{\alpha\beta\nu}) = T_{\mu\nu} \tag{13}$$

In FLRW metric, given by

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \tag{14}$$

then we can get Friedmann equations

$$\rho = \frac{f}{2} - 6f_Q H^2 - \frac{2f_T}{1+f_T}(\dot{f}_Q H + f_Q\dot{H}) \tag{15}$$

$$p = -\frac{f}{2} + 6f_Q H^2 + 2(\dot{f}_Q H + f_Q\dot{H}) \tag{16}$$

EoS parameter

$$w = \frac{p}{\rho} = -1 + \frac{4f_Q H + f_Q\dot{H}}{(1+f_T)(f - 12f_Q H^2) - 4f_T(\dot{f}_Q H + f_Q\dot{H})} \tag{17}$$

In our universe $w < -1/3$, where $\rho$ and $p$ denote total fluid energy density and pressure of the universe, which $\rho = \rho_m + \rho_{de}$, $p = p_m + p_{de}$.

Effective EoS parameter denote geometry qualities.

$$\rho_{\rm eff} = 3H^2 = \frac{f}{4f_Q} - \frac{1}{2f_Q}[(1+f_T)\rho + f_T p] \tag{18}$$

$$-p_{\rm eff} = 2\dot{H} + 3H^2 = \frac{f}{4f_Q} - \frac{2\dot{f}_Q H}{f_Q} + \frac{1}{2f_Q}[(1+f_T)\rho + (2+f_T)p] \tag{19}$$

EoS parameter for equivalent dark energy

$$w_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = -\frac{f - 8\dot{f}_Q H + 2[(1+f_T)\rho + (2+f_T)p]}{f - 2[(1+f_T)\rho + f_T p]} \tag{20}$$

Combine, we can get the evolution equation of Hubble parameter $H$ Deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(1+3w) = \frac{1}{2}\left(1 + 3\frac{p_{\rm eff}}{\rho_{\rm eff}}\right) = -1 + \frac{3(4\dot{f}_Q H - f + p)}{f - [(1+f_T)\rho + f_T p]} \tag{21}$$

We assume that

$$f(Q,T) = \alpha Q^n + \beta T \tag{22}$$

where $Q = 6H^2$, $T = -\rho + 3p$, so that we can derive $f_Q = \alpha n Q^{n-1} = \alpha n 6^{n-1}H^{2n-2}$, $f_T = \beta$, $\dot{f}_Q = 2\alpha n(n-1)6^{n-1}H^{2n-3}\dot{H}$

## 3. COSMIC SOLUTIONS

Use Eq.(18) and (19) we can solve the energy density of fluid component in field equation

$$\rho_{\rm de}=\frac{m6^{n-1}(2n-1)\left(H(t)^2\right)^{n-1}\left((3\alpha+2)nH'(t)+3(\alpha+1)H(t)^2\right)}{\left(2\alpha^2+3\alpha+1\right)w_{\rm de}} \tag{23}$$

$$\rho_m=\frac{m6^{n-1}(2n-1)\left(H(t)^2\right)^{n-1}\left(n(\alpha(w_{\rm de}-3)-2)H'(t)-3(\alpha+1)(w_{\rm de}+1)H(t)^2\right)}{\left(2\alpha^2+3\alpha+1\right)w_{\rm de}} \tag{24}$$

and the HDE energy density in Hubble cut-off can be described as

$$\rho_{de}=3c^2H(t)^2 \tag{25}$$

In principle, we can get the form of $H(z)$ through the solution of differential equation. However, solving higher-order differential equations analytically is difficult. So we assume $n=1$ first to simplify calculation and get analytically solution as follow

$$H(z)=H_0(1+z)^{\frac{3(a+1)\left(c^2(2aw+w)+1\right)}{3a+2}} \tag{26}$$

In a general barrow entropy, the HDE energy density is

$$\rho_{de}=3c^2H(t)^{2-\Delta} \tag{27}$$

If we set $\Delta=1$ the

$$H(z)=H_0(1+z)^{\frac{3(\alpha+1)}{3\alpha+2}}-(1+2\alpha)c^2w_{de}\left((1+z)^{\frac{3(\alpha+1)}{3\alpha+2}}-1\right) \tag{28}$$

in other situation, if $n\neq 1$

dark energy EoS parameter

$$w_{de}=\frac{p_{de}}{\rho_{de}}=\frac{\alpha\left(6H(z)^2\right)^{n-1}\left(2n(-2\beta+(3\beta+2)n-1)\dot{H}(z)+3(\beta+1)(2n-1)H(z)^2\right)}{3c^2H^2(\beta+1)(2\beta+1)} \tag{29}$$

where $\dot{H}(z)=\frac{d}{dt}H(z)=-\frac{dH(z)}{dz}H(z)(1+z)$

stability parameter of in this situation can be described as

$$c_s^2=\frac{\mathrm{d}p_{de}}{\mathrm{d}\rho_{de}}=\frac{\alpha(2n(-2\beta+(3\beta+2)n-1))6^{n-1}((2n-2)H^{2n-3}\dot{H}^2+\ddot{H}H^{2n-2})+\alpha 3(\beta+1)(2n-1)6^{n-1}2nH^{2n-1}\dot{H}}{6c^2H\dot{H}(\beta+1)(2\beta+1)} \tag{30}$$

## 4. TSALIS ENTROPY DARK ENERGY IN DIFFERENT IR CUTOFF

Tsallis proposed a modified black hole entropy

$$S_\delta=\gamma A^\delta \tag{31}$$

Tsallis holographic dark energy density

$$\rho_{de}=BH^{4-2\delta} \tag{32}$$

## 5. OBSERVATIONAL DATA AND PARAMETER CONSTRAINT

## 6. COSMIC EVOLUTION

The deceleration facter is defined as

$$q(z)=-(1+z)H(z)\frac{d}{dz}\left(\frac{1}{H}\right)-1=(1+z)\frac{1}{H(z)}\frac{dH(z)}{dz}-1 \tag{33}$$

## 7. CONCLUSION

## APPENDIX

## A. APPENDIX INFORMATION

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