UNIVERSIDADE DE SÃO PAULO
DEPARTAMENTO DE ENGENHARIA ELÉTRICA
ESCOLA DE ENGENHARIA DE SÃO CARLOS

SEL5755 - Sistemas Fuzzy

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EPC 3

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São Carlos,
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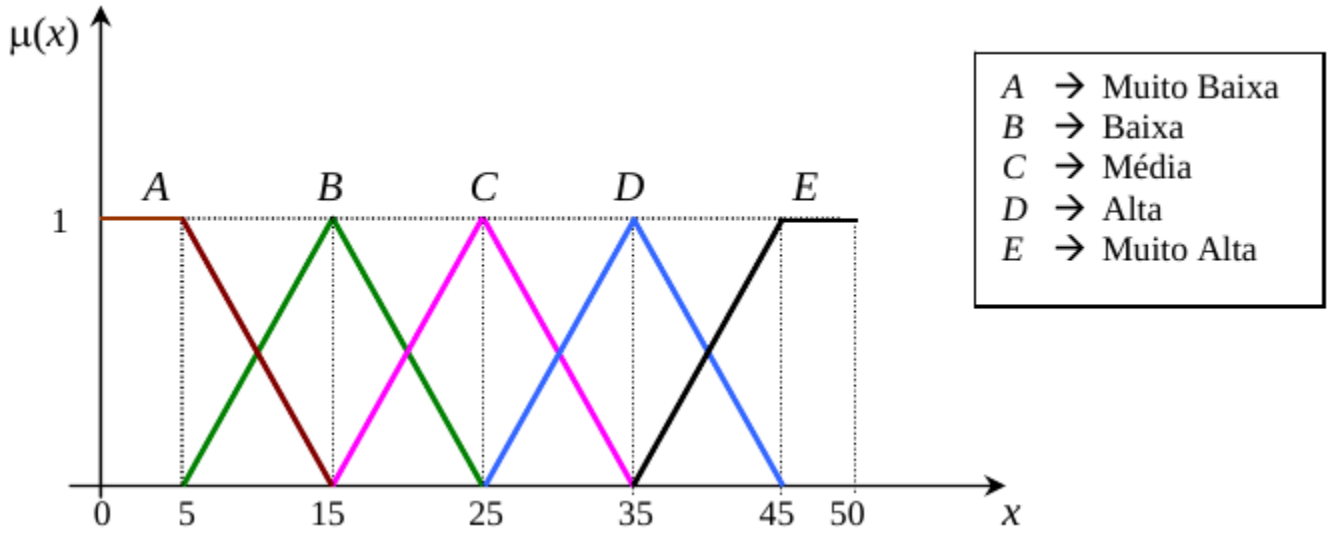
1 Considere a função de pertinência abaixo a qual está descrevendo 5 conjuntos *fuzzy* que representam a temperatura de um processo industrial.



a) Encontre as expressões analíticas referentes a cada um dos conjuntos *fuzzy*.

$$\mu_A(x) = \begin{cases} 1 & \text{se } 0 \leq x \leq 5 \\ -0,1x + 1,5, & \text{se } 5 < x \leq 15 \\ 0, & \text{caso contrário} \end{cases}$$

$$\mu_B(x) = \begin{cases} 0,1x - 0,5, & \text{se } 5 \leq x \leq 15 \\ -0,1x + 2,5, & \text{se } 15 < x \leq 25 \\ 0, & \text{caso contrário} \end{cases}$$

$$\mu_C(x) = \begin{cases} 0,1x - 1,5, & \text{se } 15 \leq x \leq 25 \\ -0,1x + 3,5, & \text{se } 25 < x \leq 35 \\ 0, & \text{caso contrário} \end{cases}$$

$$\mu_D(x) = \begin{cases} 0,1x - 2,5, & \text{se } 25 \leq x \leq 35 \\ -0,1x + 4,5, & \text{se } 35 < x \leq 45 \\ 0, & \text{caso contrário} \end{cases}$$

$$\mu_E(x) = \begin{cases} 0,1x - 3,5, & \text{se } 35 \leq x \leq 45 \\ 1 & \text{se } 45 < x \leq 50 \\ 0, & \text{caso contrário} \end{cases}$$

b) Elabore os procedimentos computacionais que permitam mapear os conjuntos *fuzzy* acima, utilizando para tanto 1000 pontos de discretização. Sugestão: Utilize Arrays.

- Para checar o nível de generalização do seu sistema, verifique se haverá necessidade de modificações acentuadas quando passarmos a trabalhar com 500 ou 2000 pontos de discretização.

c) Elabore os procedimentos computacionais que, dado um valor qualquer de x, permitam indicar quais dos conjuntos *fuzzy* acima estarão ativos, ou sejam, aqueles que possuem $\mu(x) \neq 0$.

d) Elabore os procedimentos computacionais que, dado um conjunto *fuzzy* que está ativo, retorne o respectivo valor do grau de pertinência em relação a um valor de temperatura x pertencente ao universo de discurso.

e) Elabore os procedimentos computacionais que dado um conjunto *fuzzy*, bem como um valor de grau de pertinência, retorne então o respectivo conjunto crisp representando o $\alpha - corte$ efetuado.

2 Baseado nos procedimentos computacionais realizados acima faça:

a) Imprima numa mesma folha os gráficos (conforme a figura anterior) dos cinco conjuntos *fuzzy* quando utilizamos 50 e 1000 pontos de discretização para representá-los, explicando ainda a importância de se especificar corretamente este parâmetro.

b) Imprima o conjunto *fuzzy* resultante da União dos cinco conjuntos *fuzzy* definidos acima, utilizando para tanto 1000 pontos de discretização e o operador Máximo.

c) Imprima o conjunto *fuzzy* resultante da Interseção dos cinco conjuntos *fuzzy* definidos acima, utilizando para tanto 500 pontos de discretização e o operador Mínimo.

d) Imprima o conjunto *fuzzy* resultante da operação de Complemento efetuado sobre o conjunto *fuzzy* C.

3 Baseado nos procedimentos computacionais realizados no primeiro e segundo exercício, considerando-se ainda apenas os conjuntos *fuzzy* ativos para uma determinada temperatura, faça os seguintes gráficos:

a) Imprima o conjunto *fuzzy* resultante da União dos conjuntos *fuzzy* ativos em x = 16,75.

b) Imprima o conjunto *fuzzy* resultante da União dos conjuntos *fuzzy* ativos em x = 37,29.

c) Imprima o conjunto *fuzzy* resultante da Interseção dos conjuntos *fuzzy* ativos em x = 20.

d) Imprima o conjunto *fuzzy* resultante da Interseção dos conjuntos *fuzzy* ativos em x = 40.

4 Refaça o exercício anterior adotando os operadores Soma Algébrica (União) e Produto Algébrico (Interseção).

5 Baseado nos procedimentos computacionais anteriores, imprima então os gráficos resultantes das seguintes operações: ««««< HEAD

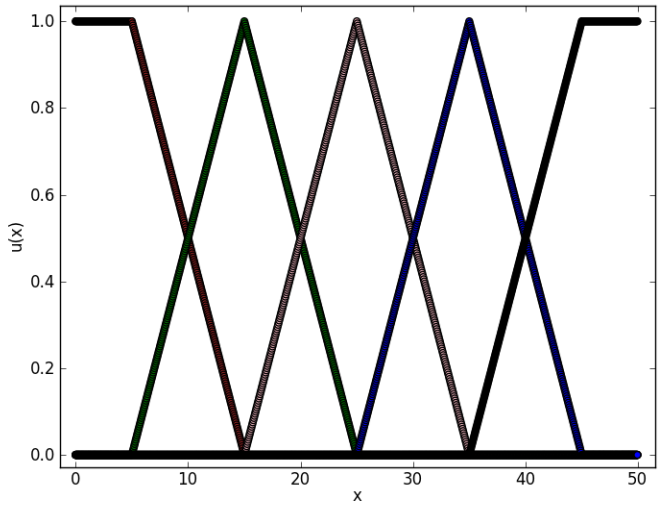
a) $A \cup B \cup C$

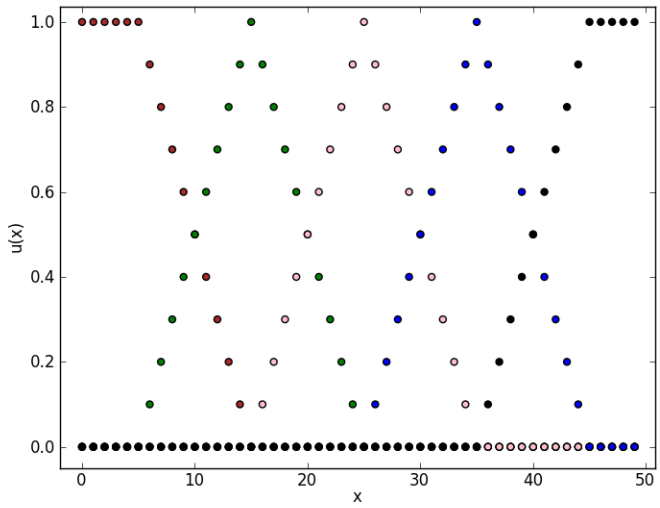
b) $B \cap (C \cup D)$

c) $(A \cap B) \cup (B \cap C)$

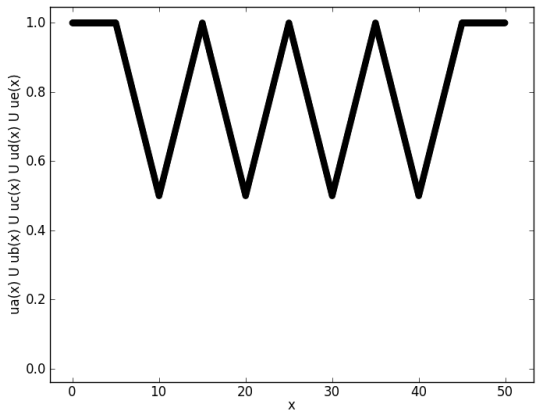
d) $\overline{A} \cup (B \cap C) \cup \overline{D}$

Ex 2a.

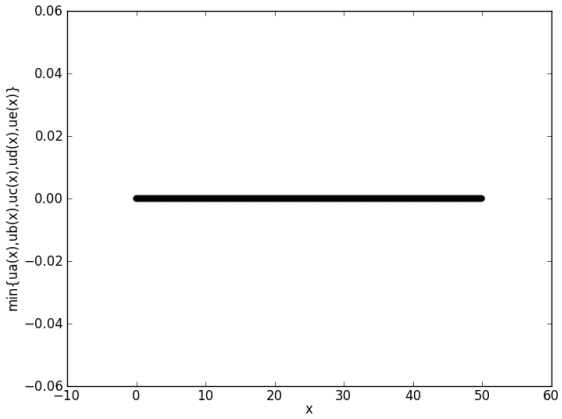

1.0
0.8
0.6
0.4
0.2
0.0
u(x)
0
10
20
30
40
50
x


1.0
0.8
0.6
0.4
0.2
0.0
u(x)
0
10
20
30
40
50
x

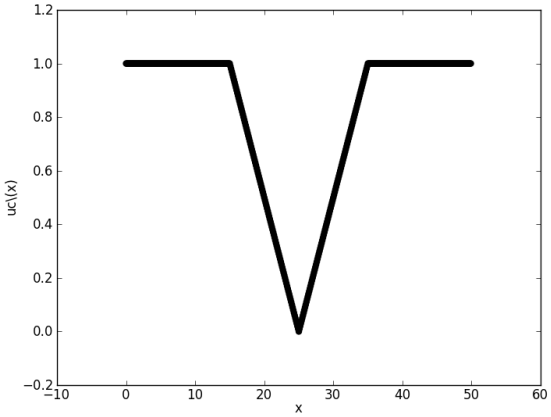
Ex 2 b,c,d



(a) 2b

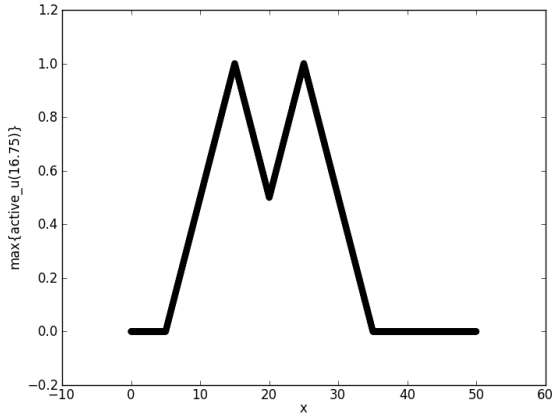


(b) 2c

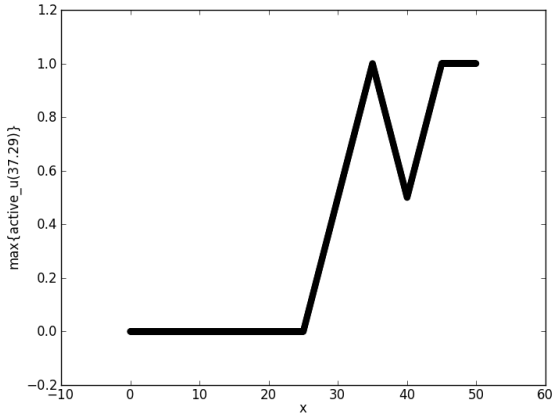


(c) 2d

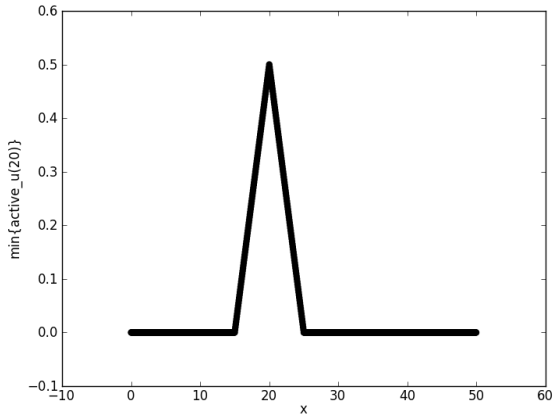
Ex 3



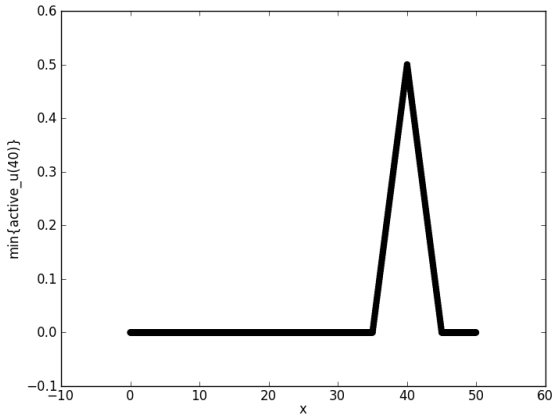
(d) 3a



(e) 3b

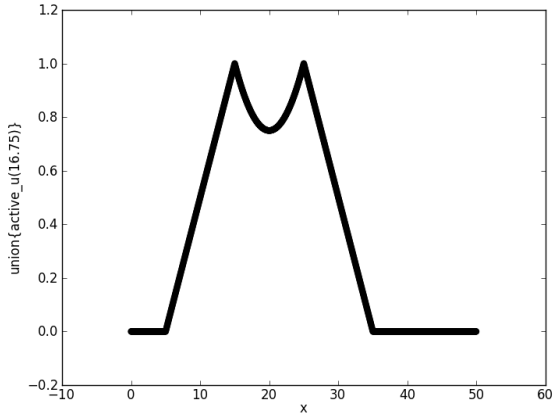


(f) 3c

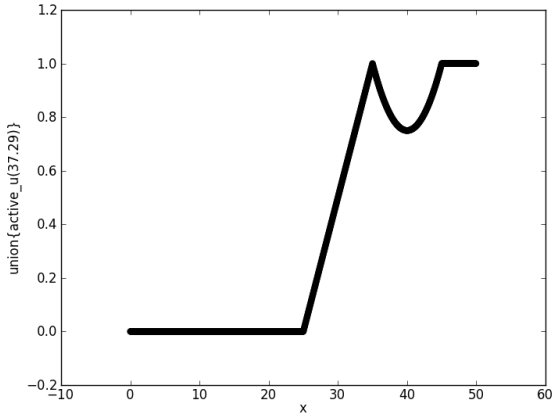


(g) 3d

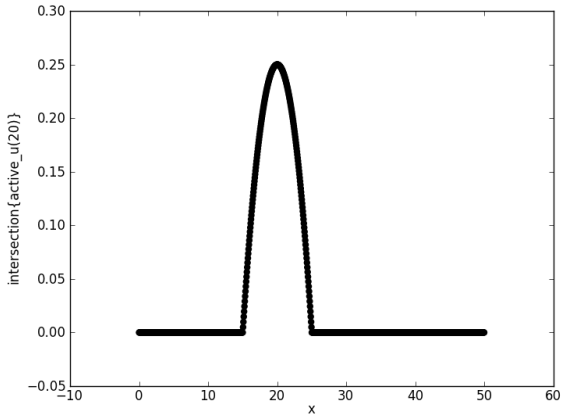
Ex 4



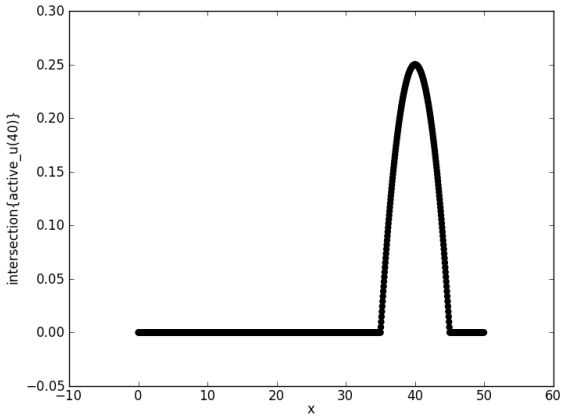
(h) 4a



(i) 4b

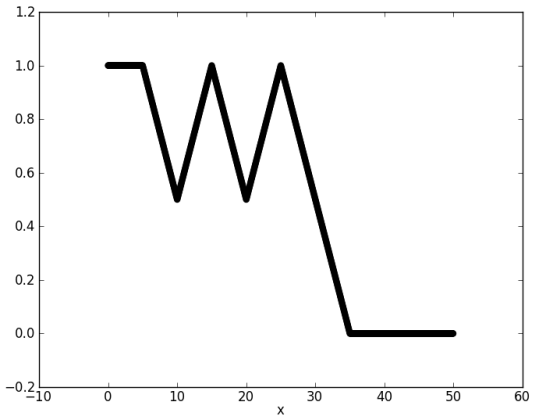


(j) 4c

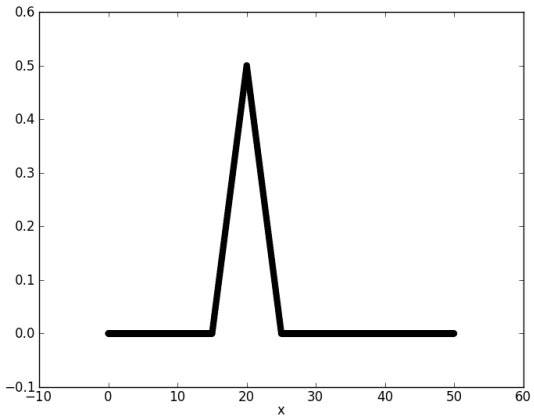


(k) 4d

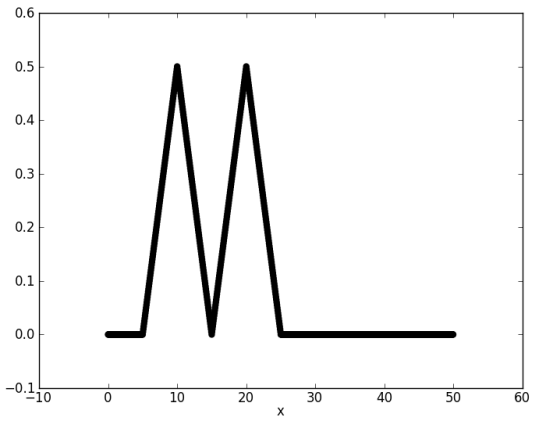
Ex 5



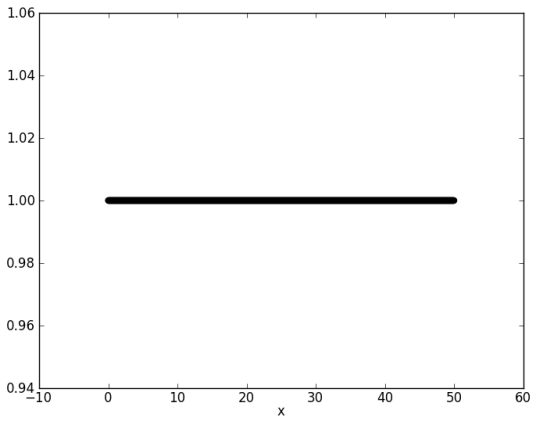
(l) 5a



(m) 5b



(n) 5c



(o) 5d

```
import matplotlib
import matplotlib.pylab
import numpy

N_POINTS = 1000

# Exercise 1b
def ua(x):
    if 0<=x and x <= 5:
        return 1
    elif 5<x and x<=15:
        return -0.1*x+1.5
    return 0

# Exercise 1b
def ub(x):
    if 5<=x and x<=15:
        return 0.1*x-0.5
    elif 15<x and x<=25:
        return -0.1*x+2.5
    return 0

# Exercise 1b
def uc(x):
    if 15<=x and x<=25:
        return 0.1*x-1.5
    elif 25<x and x<=35:
        return -0.1*x+3.5
    return 0

# Exercise 1b
def ud(x):
    if 25<=x and x<=35:
        return 0.1*x-2.5
    elif 35<x and x<=45:
        return -0.1*x+4.5
    return 0

# Exercise 1b
def ue(x):
    if 35<=x and x<=45:
        return 0.1*x-3.5
    elif 45<x and x<=50:
        return 1
    return 0

# Exercise 1b
def plot_u():
    xa = numpy.arange(0,50,50.0/N_POINTS)
    ya = map(ua, xa)
    xb = numpy.arange(0,50,50.0/N_POINTS)
```

```
    yb = map(ub, xb)
    xc = numpy.arange(0,50,50.0/N_POINTS)
    yc = map(uc, xc)
    xd = numpy.arange(0,50,50.0/N_POINTS)
    yd = map(ud, xd)
    xe = numpy.arange(0,50,50.0/N_POINTS)
    ye = map(ue, xe)
    matplotlib.pyplot.scatter(xa, ya, c = "brown", marker = 'o')
    matplotlib.pyplot.scatter(xb, yb, c = "green", marker = 'o')
    matplotlib.pyplot.scatter(xc, yc, c = "pink", marker =  'o')
    matplotlib.pyplot.scatter(xd, yd, c = "blue", marker =  'o')
    matplotlib.pyplot.scatter(xe, ye, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('u(x)')
    matplotlib.pyplot.show()

# Exercise 1c
def active_u(x):
    active = []
    if ua(x) > 0.0:
        print 'A',
        active.append('A')
    if ub(x) > 0.0:
        print 'B',
        active.append('B')
    if uc(x) > 0.0:
        print 'C',
        active.append('C')
    if ud(x) > 0.0:
        print 'D',
        active.append('D')
    if ue(x) > 0.0:
        print 'E'
        active.append('E')
    return active

# Exercise 1d
def u(active, x):
    if active == 'A':
        return ua(x)
    elif active == 'B':
        return ub(x)
    elif active == 'C':
        return uc(x)
    elif active == 'D':
        return ud(x)
    elif active == 'E':
        return ue(x)

# Exercise 1e
def crisp(active, value):
```

```
    x = numpy.arange(0,50,50.0/N_POINTS)

    if active == 'A':
        return filter (lambda a: ua(a) >= value, x)
    elif active == 'B':
        return filter (lambda a: ub(a) >= value, x)
    elif active == 'C':
        return filter (lambda a: uc(a) >= value, x)
    elif active == 'D':
        return filter (lambda a: ud(a) >= value, x)
    elif active == 'E':
        return filter (lambda a: ue(a) >= value, x)

def ex2b():
    x = numpy.arange(0,50,50.0/N_POINTS)
    ya = map(ua, x)
    yb = map(ub, x)
    yc = map(uc, x)
    yd = map(ud, x)
    ye = map(ue, x)

    matplotlib.pyplot.scatter(x, map(lambda a,b,c,d,e:
        max([a,b,c,d,e]), ya, yb, yc, yd, ye), c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('ua(x) U ub(x) U uc(x) U ud(x) U ue(x)')
    matplotlib.pyplot.show()

def ex2c():
    x = numpy.arange(0,50,50.0/N_POINTS)
    ya = map(ua, x)
    yb = map(ub, x)
    yc = map(uc, x)
    yd = map(ud, x)
    ye = map(ue, x)

    matplotlib.pyplot.scatter(x, map(lambda a,b,c,d,e:
        min([a,b,c,d,e]), ya, yb, yc, yd, ye), c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('min{ua(x),ub(x),uc(x),ud(x),ue(x)}')
    matplotlib.pyplot.show()

def ex2d():
    x = numpy.arange(0,50,50.0/N_POINTS)
    yc = map(lambda a: 1-uc(a), x)

    matplotlib.pyplot.scatter(x, yc, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('uc\(x)')
    matplotlib.pyplot.show()

def ex3a():
```

```
    x = numpy.arange(0,50,50.0/N_POINTS)
    union = [0]*N_POINTS

    for i in active_u(16.75):
        union = map(lambda q,w: max([q,w]), union,
            map(lambda a: u(i, a), x))

    matplotlib.pyplot.scatter(x, union, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('max{active_u(16.75)}')
    matplotlib.pyplot.show()

def ex3b():
    x = numpy.arange(0,50,50.0/N_POINTS)
    union = [0]*N_POINTS

    for i in active_u(37.29):
        union = map(lambda q,w: max([q,w]), union,
            map(lambda a: u(i, a), x))

    matplotlib.pyplot.scatter(x, union, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('max{active_u(37.29)}')
    matplotlib.pyplot.show()

def ex3c():
    x = numpy.arange(0,50,50.0/N_POINTS)
    union = [1]*N_POINTS

    for i in active_u(20):
        union = map(lambda q,w: min([q,w]), union,
            map(lambda a: u(i, a), x))

    matplotlib.pyplot.scatter(x, union, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('min{active_u(20)}')
    matplotlib.pyplot.show()

def ex3d():
    x = numpy.arange(0,50,50.0/N_POINTS)
    union = [1]*N_POINTS

    for i in active_u(40):
        union = map(lambda q,w: min([q,w]), union,
            map(lambda a: u(i, a), x))

    matplotlib.pyplot.scatter(x, union, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('min{active_u(40)}')
    matplotlib.pyplot.show()
```

```
def ex4a():
    x = numpy.arange(0,50,50.0/N_POINTS)
    union = [0]*N_POINTS

    for i in active_u(16.75):
        union = map(lambda q,w: q+w-q*w, union,
            map(lambda a: u(i, a), x))

    matplotlib.pyplot.scatter(x, union, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('union{active_u(16.75)}')
    matplotlib.pyplot.show()

def ex4b():
    x = numpy.arange(0,50,50.0/N_POINTS)
    union = [0]*N_POINTS

    for i in active_u(37.29):
        union = map(lambda q,w: q+w-q*w, union, map(lambda a: u(i, a), x))

    matplotlib.pyplot.scatter(x, union, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('union{active_u(37.29)}')
    matplotlib.pyplot.show()

def ex4c():
    x = numpy.arange(0,50,50.0/N_POINTS)
    union = [1]*N_POINTS

    for i in active_u(20):
        union = map(lambda q,w: q*w, union, map(lambda a: u(i, a), x))

    matplotlib.pyplot.scatter(x, union, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('intersection{active_u(20)}')
    matplotlib.pyplot.show()

def ex4d():
    x = numpy.arange(0,50,50.0/N_POINTS)
    union = [1]*N_POINTS

    for i in active_u(40):
        union = map(lambda q,w: q*w, union, map(lambda a: u(i, a), x))

    matplotlib.pyplot.scatter(x, union, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.ylabel('intersection{active_u(40)}')
    matplotlib.pyplot.show()

def ex5a():
    x = numpy.arange(0,50,50.0/N_POINTS)
```

```
    res = map(lambda q,w,e: max([q,w,e]), map(ua, x),map(ub, x),map(uc, x))

    matplotlib.pyplot.scatter(x, res, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.show()

def ex5b():
    x = numpy.arange(0,50,50.0/N_POINTS)

    res = map(lambda q,w,e: min([q, max([w,e])]),
        map(ub, x),map(uc, x),map(ud, x))

    matplotlib.pyplot.scatter(x, res, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.show()

def ex5c():
    x = numpy.arange(0,50,50.0/N_POINTS)

    res = map(lambda q,w,e: max([min([q,w]), min([w,e])]) ,
        map(ua, x),map(ub, x),map(uc, x))

    matplotlib.pyplot.scatter(x, res, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.show()

def ex5d():
    x = numpy.arange(0,50,50.0/N_POINTS)

    res = map(lambda q,w,e,r: max([1-q,min([w,e]),1-r]),
         map(ua, x),map(ub, x),map(uc, x), map(ud,x))

    matplotlib.pyplot.scatter(x, res, c = "black", marker = 'o')
    matplotlib.pyplot.xlabel('x')
    matplotlib.pyplot.show()


if __name__ == "__main__":
    ex5d()
```