Can one hear the shape of gender?

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Abstract

Prior work showed that the modal human body has seven holes, irrespective of biological gender, which showed that gender is not a topological invariant. We conjecture that genders can be classified according to torsion and curvature, up to rigid motion, but that only spectral information is not enough. We check this conjecture empirically by training a machine learning model to classify meshes of male/female bodies based on topological and geometric features, and compute pairs of non-isogender isospectral bodies.

1 Introduction

The human body is known to display a wide range of shapes while having broadly the same general topology. This intuition was formalized in [1] which first gave a rigorous proof that the modal human body is homeomorphic to a seven-holed torus. Their proof also showed that the number of holes in the human body does not depend on biological gender, meaning that gender is not a topological invariant.

In this work, we investigate whether biological gender can be characterized as a geometric invariant. We investigate this problem empirically using a dataset of meshes of human bodies generated from high quality scans [2]. We generate several feature sets from these meshes based on topological, geometric and spectral information and train machine learning (ML) models to classify genders using these feature sets.

2 Related work

2.1 Gender independence of topological genus

We restate here the proof that the modal human body is homeomorphic to a seven-holed torus. By hole, we mean a *through-hole*, like the handle of a cup, through which a string could be passed. Cavities like the inside of a cup are *blind holes*, which can be continuously filled up, and are therefore topologically irrelevant.

The most obvious hole in the human body is the digestive track, which connects the mouth to the rectum. The nostrils form two additional through-holes connected to the mouth cavity. In addition the *lacrimal canaliculi* (lacrimal ducts) connect the eyelids through the *lacrimal puncta* to the nose. There is one duct per eyelid, adding four through-holes

[1] EleutherAI Discord #off-topic

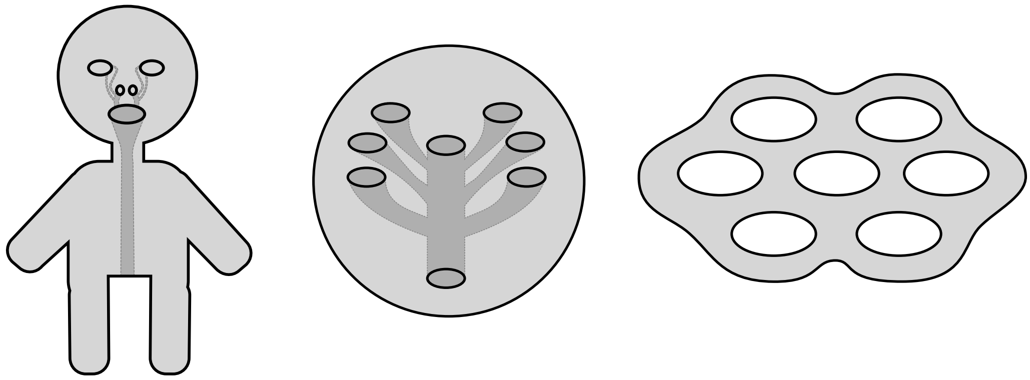

Figure 1: Schematic illustration of a continuous transformation of a human body into a seven-holed torus.

together, bringing the total to seven holes. Figure 1 shows an illustration of the continuous deformation of a human body into a seven holed torus.

Other holes like the ears and urinary tract are in fact blind holes not connected to the holes listed in the previous paragraph. In addition, we ignore cavities located inside the human body, such as the brain cavity or the circulatory system. Importantly, the reproductive organs do not contribute any through-holes, which implies that biological gender is not a topological invariant.

It is important to note that seven is only the *most common* number of through-holes in the human body. Any other holes resulting from injury, or cosmetic modifications (such as ear rings or piercings) will add to this count. In addition, people can be born with additional lacrimal puncta [3]. While it might be argued that holes made for cosmetic purposes could be used to determine one's gender, it would only do so imperfectly, as the practice of body piercing varies across cultures, and is not restricted to a single gender.

3 Methods

3.1 Differential Geometric approaches to anatomy

We model the surface of the human body as a *Riemannian manifold* $\mathcal{M}$. Riemannian surfaces can be completely described (up to isometry) by their Gaussian curvature K, which is a scalar quantity defined for each point $p \in \mathcal{M}$ as $K = \kappa_1\kappa_2$, where κ_i denote the *principal curvatures* at point p. Positive curvature indicates that $\mathcal{M}$ looks like the surface of a sphere around p, while negative curvature indicates that it looks like a saddle surface. If $K = 0$, $\mathcal{M}$ is said to be *flat* at p, meaning that around p it looks like a sheet of paper that could be flattened out.

We claim that Gaussian curvature can be used to identify biological gender, as secondary sexual characteristics in humans include different distributions of fat and muscle tissue, notably around the hips and breasts for females, and shoulders and larynx for males. These are not the only factors in the variation of body shapes, but we argue they are among the most important ones.

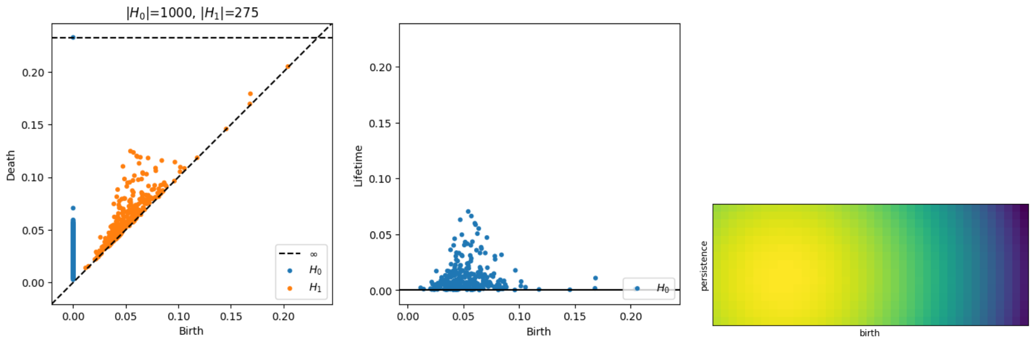

Figure 2: Typical topological features obtained from persistent homology on a mesh from the dataset.

3.2 Dataset

In the absence of a complete mathematical description of the human body that would allow for a formal proof of our claim, we instead adopt a data-driven approach. We use the dataset of [2], which consists of 3048 polygonal meshes obtained from scans of human subjects. Each mesh in the dataset is fit to the same topology with 12500 vertices and 25000 faces. The dataset is split equally between female and male subjects.

We process this dataset by smoothing each mesh to eliminate any artifacts which may erroneously change the curvature at specific points.

3.3 Topological Features

We use the Scikit-TDA package [4] to compute topological features from the vertex point cloud of each mesh. Specifically, we use Persistent Homology to record the emergence of one dimensional cycles over the Vietoris-Rips filtration of the mesh. Due to the large number of vertices and the high computational cost of persistent homology, we only use a small subset of 1000 vertices chosen randomly.

From the persistence homology process, we obtain a collection of birth and death times for each cycle that emerged during the filtration. Most such cycles are spurious and die shortly after their birth. Cycles that persist for a longer duration are typically indicative of actual topological features of the point cloud.

To turn the output of persistent homology into convenient features for statistical analysis, we take the birth times and lifetime of each cycle (second panel of 2) and take a two-dimensional histogram of the resulting point cloud over a coarse grid (third panel of 2). This yields 465 scalar values for each mesh, which we use as features for our analysis.

3.4 Geometric Features

Given a triangular mesh approximating a Riemannian manifold, we can estimate the Gaussian curvature at a vertex v by computing the *angle defect* at v, defined as $\pi - \sum_i \theta_i$, where the numbers θ_i are the angles at v for each triangle containing v. The interpretation of angle defect is the same as the gaussian curvature K. We compute the angle defect at each vertex to obtain our first geometric feature set.

In addition to curvature via the angle defect, we also use the euclidean coordinates of

	Train accuracy	Test accuracy
Topological features	0.5	0.5
Angular Defect	1.0	1.0
Vertex Coordinates	0.5	0.5
Spectral Features	0.5	0.5

Table 1: Train and test accuracies across feature sets

each vertex, concatenated into a single vector to form our second geometric feature set. In both cases, our geometric feature sets have many more variables than observations, which may pose a problem for statistical methods.

3.5 Spectral Features

A famous problem in differential geometry, posed by Kac in 1966 [5], asks whether the spectrum of the Laplace-Beltrami operator of a manifold can be used to identify it. This was answered in the negative by the discovery of non-isomorphic manifolds with the same Laplacian spectrum. Inspired by this classical results, we ask whether the spectrum of the Laplace-Beltrami operator can be used to differentiate between genders, or in more poetic words, "*Can one hear the shape of gender?*".

We compute the eigenvalues of the discrete approximation of the Laplace-Beltrami operator on the meshes in our dataset. Since computing the full spectrum of large (sparse) matrices is computationally expensive, we only compute the 20 largest eigenvalues in magnitude.

4 Results

We use each of the feature sets described in the previous section as features for performing logistic regression to predict the gender of each mesh. The results are summarized in table 1 We find that only the angular defect allows perfect classification, while all other feature sets perform as well as random chance. We interpret this as evidence to our claim that gender can be characterized using Gaussian curvature.

To further investigate this result, we perform principal component analysis on our angle defect dataset. As seen in figure 3, we find that nearly all of the variance across the dataset is explained by the first two components, and that both biological genders are clustered in the second component and perfectly linearly separated, which explains the excellent classification performance of the model trained on angular defect.

Regarding our other conjectures, we observe that the numerical ranks of feature matrices of both the topological and spectral datasets are equal to one. This indicates that the rows of these datasets are all essentially the same, which serves as evidence towards our claims that neither topological nor spectral information is enough to characterize gender.

5 Discussion

In this work, we have uncovered empirical evidence that biological gender can be characterized using the Gaussian curvature of the outer surface of the human body. While

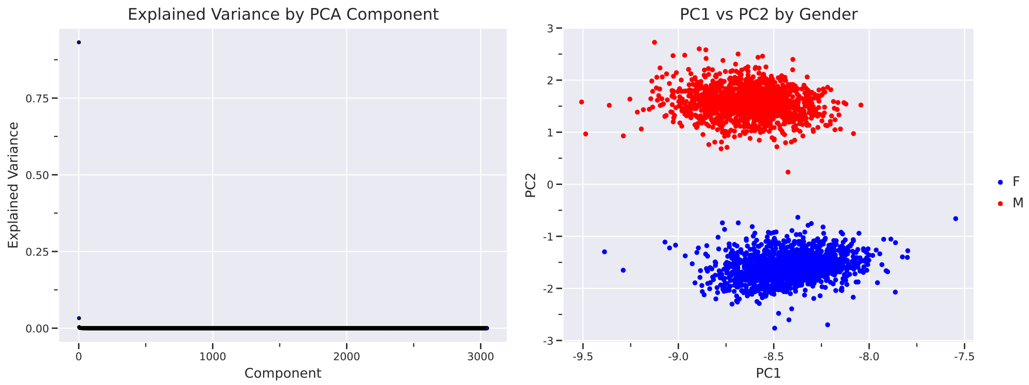

Figure 3: Principal Component Analysis of the Angle Defect feature set

this is encouraging and may serve as a stepping stone to a formal mathematical proof, it is important that we state the limitations of our results.

First, our discussion thus far has only been about *biological gender*, which is only a narrow subcategory of gender as a whole. To our knowledge, a proper mathematical description of the broader question of gender remains to be seen, and we do not believe that the tools used here are enough for this task. We leave such worthy task to more capable hands.

References

[1] M. Stevens. How many holes does a human have? `https://www.youtube.com/watch?v=egEraZP9yXQ`, 2020.

[2] Yipin Yang, Yao Yu, Yu Zhou, Sidan Du, James Davis, and Ruigang Yang. Semantic parametric reshaping of human body models. In *Proceedings of the 2014 Second International Conference on 3D Vision - Volume 02*, 3DV '14, page 41–48, USA, 2014. IEEE Computer Society.

[3] Al Saleh AS. Al Saleh A, Vargas JM. Supernumerary lacrimal puncta: Case series. *Saudi J Ophthalmol.*, 4(34), 2021.

[4] Nathaniel Saul and Chris Tralie. Scikit-tda: Topological data analysis for python, 2019.

[5] Mark Kac. Can one hear the shape of a drum? *The American Mathematical Monthly*, 73(4):1–23, 1966.